Hilbert space and holography of information in de Sitter quantum gravity

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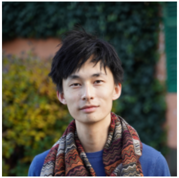
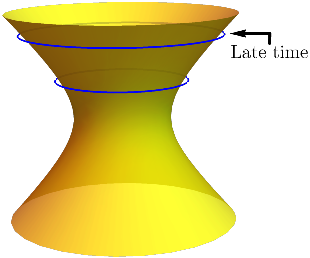


Victor Godet

- `2303.16316` and `2303.16315`, Joydeep Chakravarty, Tuneer Chakraborty, Victor Godet, Priyadarshi Paul, S.R.

Overview: Questions.

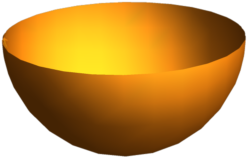


Cosmological correlators usually defined

$$\sim \int D\chi Dg |\Psi[g,\chi]|^2 \chi(x_1)\ldots\chi(x_n)$$

- **Hilbert space:** What are allowed $\Psi[g,\chi]$ in a theory of gravity?
- **Holography of information:** What are the properties of cosmological correlators in this Hilbert space?

Vacuum wavefunctional



$$|0\rangle \leftrightarrow \Psi_0[g,\chi]$$

Euclidean vacuum state can be computed by gravitational-path integral or via analytic continuation from AdS.

[Hartle,Hawking, 1983]

[Maldacena, 2001]

Cosmological correlators

$$\int |\Psi_0[g,\chi]|^2 \chi(x_1)\ldots\chi(x_n) DgD\chi$$

also computed via the in-in formalism.

[Weinberg, 2005]

Other states?

- But

$$|0\rangle$$

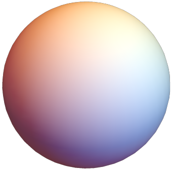
is one state.

- Try to build the Hilbert space using a Fock-space construction?

$$|\Psi\rangle = \int \chi(x_1)\ldots\chi(x_n) f(x_1,\ldots x_n)|0\rangle\,?$$

But this does not work in the presence of gravity.

dS invariance



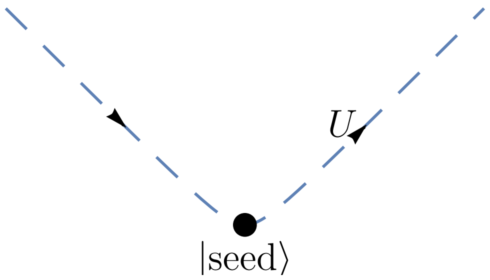
In gravity, charges can be measured at the boundary. But dS spatial slice has no boundaries.

Gauss law implies that even in the weak-gravity limit, all states must have zero charges,

$$U|\Psi\rangle = |\Psi\rangle, \qquad \forall U \in SO(d+1,1)$$

In original Hilbert space, the only such state is $|0\rangle$!

Higuchi's solution in the nongravitational limit



Starting with a "seed state", construct

$$|\Psi\rangle = \int [dU]\, U|\mathrm{seed}\rangle$$

[Higuchi, 1991]

Norm: $$\langle\Psi||\Psi\rangle = \frac{1}{\mathrm{vol}(SO(d+1,1))}\langle\Psi|\Psi\rangle$$

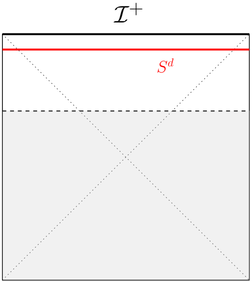
There is some evidence for this.

[Marolf, Morrison, 2008]

[Chandrasekaran,Longo,Penington,Witten, 2022]

But how does one derive this prescription? How is it corrected beyond $G_N \to 0$.

WDW Equation



$$\mathcal{H}\Psi[g,\phi] = 0; \qquad \mathcal{H}_i\Psi[g,\phi] = 0.$$

- **Technical simplification:** Focus on the regime

  $$\Lambda \gg R; \qquad \Lambda \gg V_{\text{matter}}$$

  (Large-volume/late-time regime)

- Sufficient to understand Hilbert space. (cf. asymptotic quantization).

- Insufficient for bulk dynamics/“earlier-time physics”.

Solution

At large volume all solutions of the WDW equation take the form

$$\Psi \longrightarrow e^{iS[g,\chi]} Z[g,\chi]$$

see AdS solutions by Freidel (2008),Regado, Khan, Wall (2022)

where

1. S is a divergent universal phase factor.
2. $Z[g,\chi]$ is diff invariant and almost Weyl invariant

$$\Omega \frac{\delta Z[g,\chi]}{\delta \Omega(x)} = \mathcal{A}_d[g] Z[g,\chi].$$

where Ω is the conformal factor and $\mathcal{A}_d$ is an imaginary local function of g in even d for dS_{d+1}.

$$|Z[g,\chi]|^2$$

is Weyl invariant.

Phase factor

The phase factor S contains terms familiar from holographic renormalization.

$$S = \frac{(d-1)}{\kappa^2} \int \sqrt{g} d^d x - \frac{1}{2\kappa^2(d-2)} \int \sqrt{g} R d^d x + \dots$$

[Papadimitriou, Skenderis, 2004]

It comprises integrals of local densities.

It doesn't depend on details of state.

Cancels out in $|\Psi[g,\chi]|^2$.

Expansion of $Z[g,\chi]$

After Weyl transformation to frame

$$g_{ij} = \delta_{ij} + \kappa h_{ij}$$

Expand

$$Z[g,\chi] = \exp[\sum_{n,m} \kappa^n \mathcal{G}_{n,m}]$$

with

$$\mathcal{G}_{n,m} = \int d\vec{y} d\vec{z}\, G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z}) h_{i_1 j_1}(z_1)\dots h_{i_n j_n}(z_n)\chi(y_1)\dots\chi(y_m),$$

Coefficient functions obey same Ward identities as CFT correlators.

$$G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z}) \sim \langle T^{i_1 j_1}(y_1)\dots T^{i_n j_n}(y_n)\phi(z_1)\dots\phi(z_m)\rangle^{\text{connected}}_{\text{CFT}} ,$$

“CFT” has imaginary central charge. Not necessarily local or unitary.

Hartle-Hawking state and other states

$$\Psi_0 = e^{iS} \exp[\sum_{n,m} \kappa^n \mathcal{G}_{n,m}]$$

[Pimentel, 2013]

Not just the Hartle-Hawking state but all states have this form.

Interactions do not constrain precise form of $\mathcal{G}_{n,m}$ beyond conformal invariance.

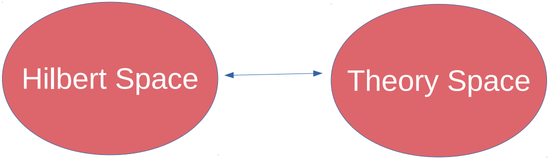
State space as theory space

Given a list of valid correlators

$$\{G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z})\}$$

we get a solution of the WDW equation.

But a list of such correlators can be thought of as defining a "theory".



(**Caution**: there might be additional constraints on allowed states beyond what we have found.)

Higuchi basis for states

Starting with $\mathcal{G}_{n,m}$ for H.H. state,

$$\mathcal{G}^{\lambda}_{n,m} = (1-\lambda)\mathcal{G}_{n,m} + \lambda\widetilde{\mathcal{G}}_{n,m}$$

Then

$$\frac{\partial \Psi_\lambda[g,\chi]}{\partial \lambda} = \sum_{n,m} \kappa^n \delta\mathcal{G}_{n,m}\Psi_0[g,\chi],$$

The Ward identities tell us

$$\delta\mathcal{G}_{n,m} \neq 0 \Rightarrow \delta\mathcal{G}_{n+1,m} \neq 0.$$

In general we require an infinite series to satisfy the constraints.

Higuchi states

- In the limit $\kappa \to 0$, Ward identities do not relate different values of n.

- This leads to a special class of states

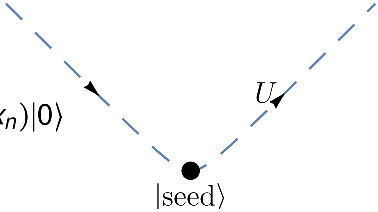
$$|\Psi_{\rm ng}\rangle = \int dx_i f(x_1, \ldots x_n)\chi(x_1)\ldots\chi(x_n)|0\rangle$$

  where f is not arbitrary but has the symmetries of a conformal correlator.

- These states are invariant under the dS isometries!

$$U|\Psi_{\rm ng}\rangle = |\Psi_{\rm ng}\rangle$$

Correction to Higuchi states



$$|\Psi_{\mathrm{ng}}\rangle = \int dx_i f(x_1, \ldots x_n)\chi(x_1)\ldots\chi(x_n)|0\rangle$$

The states $|\Psi_{\mathrm{ng}}\rangle$ are precisely Higuchi's states.

But away from $\kappa \to 0$ we need

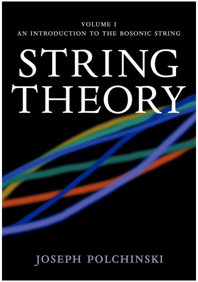
$$|\Psi\rangle = \sum \kappa^n \delta\mathcal{G}_{n,m}|0\rangle$$

Lowest order term is Higuchi's construction.

Our solution justifies Higuchi's construction and provides gravitational corrections to it.

Norm

$$(\Psi,\Psi) = \frac{1}{\text{vol(diff}\times\text{Weyl)}} \int DgD\chi \sum_{n,m,n',m'} \kappa^{n+n'} \delta\mathcal{G}^*_{n,m} \delta\mathcal{G}_{n',m'} |Z_0[g,\chi]|^2$$

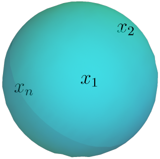


Actual computation requires us to fix gauge.

$$\partial_i g_{ij} = 0; \qquad \delta^{ij} g_{ij} = d$$

Normalizable states require at least two insertions. H.H. state is not normalizable, naively.

Cosmological correlators



Cosmological correlators usually computed as expectation value of

$$\chi(x_1)\dots\chi(x_n)$$

As written, these operators do not commute with the constraints.

Cosmological correlators can be interpreted as gauge-fixed operators

$$\langle\!\langle \Psi|\chi(x_1)\dots\chi(x_n)|\Psi\rangle\!\rangle_{\rm CC} = \int |\Psi|^2\chi(x_1)\dots\chi(x_n)\delta(\text{g.f})\Delta'_{FP} Dg D\chi$$

Cosmological correlators

$$\langle\langle \Psi_1 | \chi(x_1) \ldots \chi(x_n) | \Psi_2 \rangle\rangle_{CC} = \int \Psi_1^* \Psi_2 \chi(x_1) \ldots \chi(x_n) \delta(\text{g.f}) \Delta'_{FP} Dg D\chi$$

gives unambiguous prescription for the matrix elements.

There is some gauge invariant operator with the same matrix elements.

Gauge-fixing can be thought of as setting our reference frame as observers.

Residual gauge transformation

Some diff-and-Weyl transformations that preserve gauge conditions.

$$\begin{aligned} \text{translations}: &\quad \xi^i = \alpha^i; \\ \text{rotations}: &\quad \xi^i = M^{ij} x^j \\ \text{dilatations}: &\quad \xi^i = \lambda x^i \\ \text{SCTs}: &\quad \xi^i = 2(\beta \cdot x) x^i - x^2 \beta^i + v^i_j \beta^j \end{aligned}$$

SCTs are corrected by a metric-dependent term.

[Hinterbichler, Hui, Khoury, 2013]

[Ghosh, Kundu, S.R., Trivedi, 2014]

Symmetries of cosmological correlators

Residual gauge transformations turn into symmetries of cosmological correlators.

$$\langle\!\langle \Psi | \chi(\lambda x_1 + v) \dots \chi(\lambda x_n + v) | \Psi \rangle\!\rangle_{\mathrm{CC}} = \lambda^{-n\Delta} \langle\!\langle \Psi | \chi(x_1) \dots \chi(x_n) | \Psi \rangle\!\rangle_{\mathrm{CC}}$$

Under rotations

$$\langle\!\langle \Psi | \chi(M \cdot x_1) \dots \chi(M \cdot x_n) | \Psi \rangle\!\rangle_{\mathrm{CC}} = \langle\!\langle \Psi | \chi(x_1) \dots \chi(x_n) | \Psi \rangle\!\rangle_{\mathrm{CC}}$$

SCTs relate cosmological correlators of different orders.

Symmetries of cosmological correlators

All states display these symmetries.

Conformal invariance of cosmological correlators does not require choice of specific initial conditions. Generic prediction of inflation + Q.G.

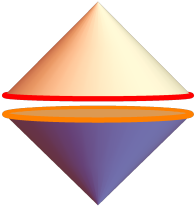
Conversely, conformal-invariance of early-Universe correlators does not provide evidence for Hartle-Hawking proposal.

Holography of information

Gravity localizes information unusually!

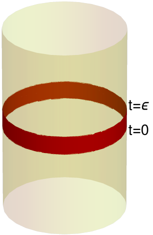
[Laddha, Prabhu, S.R., Shrivastava, 2020]

Asymptotically flat space



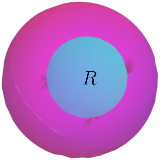
All information about massless particles is present near the past boundary of future null infinity.

Asymptotic AdS



Asymptotic correlators on an infinitesimal time band at the boundary completely fix the bulk state. (Does not assume AdS/CFT)

Holography of information in dS

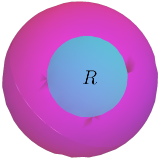


In dS, cosmological correlators in an arbitrarily small region on the asymptotic time slice are sufficient to determine them everywhere.

$$\langle\!\langle \langle\Psi|\chi(x_1)\ldots\chi(x_n)|\Psi\rangle\!\rangle_{\mathrm{CC}} = \lambda^{n\Delta}\langle\!\langle\Psi|\chi(\lambda x_1 + v)\ldots\chi(\lambda x_n + v)\rangle\!\rangle_{\mathrm{CC}}$$

Holography of information and cosmological correlators

$$\langle\langle \Psi_1 | \chi(x_1) \ldots \chi(x_n) | \Psi_1 \rangle\rangle_{\mathrm{CC}} = \langle\langle \Psi_2 | \chi(x_1) \ldots \chi(x_n) | \Psi_2 \rangle\rangle_{\mathrm{CC}} \, \forall \, n, x_i \in \mathcal{R},$$
$$\Rightarrow |\Psi_1\rangle = |\Psi_2\rangle$$

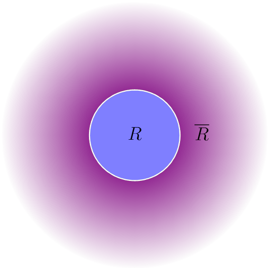


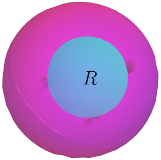
In sharp contrast to QFT.

Consequence of the gravitational constraints.

Persists in the nongravitational limit!

Holography of information





AdS and flat space: Whenever the complement of a region surrounds the region, it has information about the region.

In dS, the complement of every region surrounds the region and vice versa!

Cautionary remarks

Holography of information ⇒ sharp mathematical difference between QFT and QG.

Caution:

- "cosmological correlators" are secretly nonlocal since they are gauge fixed.
- Identifying the state requires all-point correlators.
- No claim that these gauge-fixed operators can all be "measured" by an "observer".

Conclusion

- **Hilbert space:** Solutions of WDW-eqn are of the form $e^{iS}Z[g,\chi]$, where $|Z[g,\chi]|^2$ is a diff and Weyl-invariant functional.
- All states are of this form, not just the Hartle-Hawking state. (HH state itself does not appear normalizable.)
- **Symmetries.** Cosmological correlators, after gauge-fixing covariant under scaling, rotations, translations in all states, not just the HH state.
- **Holography of information:** Cosmological correlators in an arbitrarily small region suffice to determine the state. Dramatic difference with QFT.