On Classification of Effective Field Theories (soft bootstrap)

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Outline

- EFTs and Adler zero
- Amplitudology
- classification: (single) scalar theories
- generalizations

Effective field theories

- very broad subject
- focus on low energy dynamics of theories with SSB
- strictly massless theories
- ground state spontaneously breaks a global symmetry of the underlying theory

$$G \to H$$

- we have Nambu-Goldstone bosons ϕ in the spectrum with

$$\langle 0|J^\mu|\phi\rangle \neq 0$$

- $\Rightarrow$ the shift symmetry

$$\phi \to \phi + a$$

- $\Rightarrow$ Adler zero, i.e. vanishing of amplitudes in soft limit

Effective field theories

Our aim: classification of interesting EFTs

Usual steps:

Symmetry $\rightarrow$ Lagrangian $\rightarrow$ Amplitudes $\rightarrow$ physical quantities
(cross-section, masses, decay constants, ...)

Our method: Amplitudology

works done in collaborations with Christoph Bartsch, Johan Bijnens, Clifford Cheung, Jiri Novotny, Filip Preucil, Chia-Hsien Shen, Mikhail Shifman, Mattias Sjö, Jaroslav Trnka, Petr Vasko, Congkao Wen...

Amplitudology

Not to be confused with astrology...

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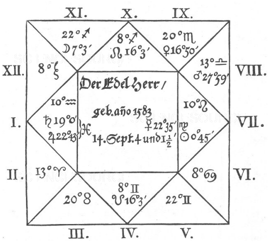
Not to be confused with astrology... well, maybe some similarities:
need for precise data [Tycho Brahe] led to

$\rightarrow$ horoscopes [e.g. Kepler for Wellenstein]



Wellenstein's death by K. Piloty

but more importantly to

$\rightarrow$ serious astrophysics [Kepler's laws]

Tycho Brahe's motto

By looking up I see downward

By looking down I see upward

EFT: simplest case

- focus on two derivatives: $\partial_\mu \phi \partial^\mu \phi \phi^n$
- Single field is a trivial case $\rightarrow$ have to consider multi-flavours $\phi_1, \phi_2 \ldots$
- case by case studies: of two, three, ... flavours

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^i \partial^\mu \phi^i + \lambda_{ijkl}\partial_\mu \phi^i \partial^\mu \phi^j \phi^k \phi^l + \lambda_{i_1 \ldots i_6}\partial_\mu \phi^{i_1} \partial^\mu \phi^{i_2} \phi^{i_3} \ldots \phi^{i_6} + \ldots$$

- Very complicated generally
- Assume some simplification using the group structure

$$\phi = \phi^a T^a$$

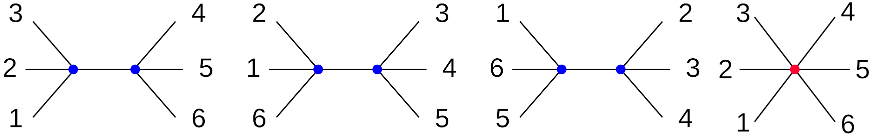
- similar to the 'gluon case': flavour ordering

$$A^{a_1 \ldots a_n} = \sum_{perm} \mathrm{Tr}(T^{a_1} \ldots T^{a_n}) A(p_1, \ldots p_n)$$

First example: NLSM

[KK, Novotny, Trnka '13]

bottom-up analysis, first non-trivial case, the 6pt amplitude:



power-counting:

$$\lambda_4^2 \; p^2 \frac{1}{p^2} p^2 + \lambda_6 \; p^2$$

in order to combine the pole and contact terms we need to consider some limit. The most natural candidate: we will demand soft limit, i.e.

$$A \to 0, \qquad \text{for} \quad p \to 0$$

$$\Rightarrow \quad \lambda_4^2 \sim \lambda_6 \qquad \text{corresponds to NLSM}$$

How to extend it to all orders (n-pt)? $\rightarrow$ new recursion relations

New recursion relations: modification of BCFW

[Cheung, KK, Novotny, Shen, Trnka '15]

The high-energy behaviour forbids a naive Cauchy formula

$$A(z) \neq 0 \qquad \text{for} \quad z \to \infty$$

Can we instead use the soft limit directly?

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Can we instead use the soft limit directly? $\to$ yes!
The standard BCFW not applicable, we propose new shifts:

$$p_i \to p_i(1 - za_i) \qquad \text{on all external legs}$$

This leads to a modified Cauchy formula

$$\oint \frac{dz}{z} \frac{A(z)}{\Pi_i (1 - a_i z)^\sigma} = 0$$

note there are no poles at $z = 1/a_i$ (by construction).

Natural classification: σ and ρ

Generalization of the soft limit:

$$A(tp_1, p_2, \ldots, p_n) = \mathcal{O}(t^{\sigma}), \qquad \text{as} \quad tp_1 \to 0$$

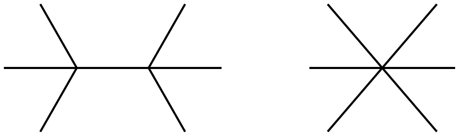
Interaction term

$$\mathcal{L} = \partial^m \phi^n$$

Then another natural parameter is:

$$\rho = \frac{m-2}{n-2} \qquad \text{"averaging number of derivatives"}$$

e.g. $\mathcal{L} = \partial^m \phi^4 + \partial^{\widetilde{m}} \phi^6$



so these two diagrams can mix if the same ρ

Non-trivial cases

$$\text{for: } \mathcal{L} = \partial^m \phi^n : \qquad m < \sigma n$$

or

$$\sigma > \frac{(n-2)\rho + 2}{n}$$

i.e.

ρ	σ at least
0	1
1	2
2	2
3	3

i.e. non-trivial regime for $\rho \leq \sigma$

First case: $\rho = 0$ (i.e. two derivatives)

Schematically for a single scalar case

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \sum_i \lambda_4^i(\partial^2\phi^4) + \sum_i \lambda_6^i(\partial^2\phi^6) + \ldots$$

similarly for multi-flavour (ϕ_i: $\phi_1, \phi_2, \ldots$).
non-trivial case

$$\sigma = 1$$

Outcome:

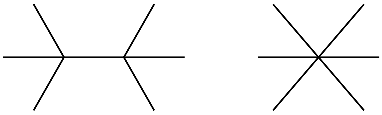
- single scalar: free theory
- multiple scalars (with flavour-ordering): non-linear sigma model

n.b. it represents a generalization of [Susskind, Frye '70], [Ellis, Renner '70]

Second case: $\rho = 1$, $\sigma = 2$ (double soft limit)

1. focus on the lowest combination and fix the form:

$$\mathcal{L}_{int} = c_2(\partial\phi\cdot\partial\phi)^2 + c_3(\partial\phi\cdot\partial\phi)^3 \qquad \text{condition: } c_3 = 4c_2^4$$



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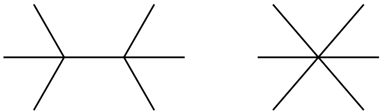

2. find the symmetry

$$\phi \to \phi - b_\rho x^\rho + b_\rho \partial^\rho \phi\, \phi \quad \text{(again up to 6pt so far)}$$

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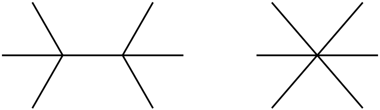

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3. ansatz of the form

$$c_n(\partial\phi \cdot \partial\phi)^n + c_{n+1}(\partial\phi \cdot \partial\phi)^n \partial\phi \cdot \partial\phi$$

4. in order to cancel: $2(n+1)c_{n+1} = (2n-1)c_n$
   i.e. $c_1 = \frac{1}{2} \Rightarrow c_2 = \frac{1}{8}, c_3 = \frac{1}{16}, c_4 = \frac{5}{128}, \ldots$

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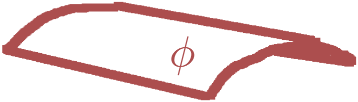
solution:

$$\mathcal{L} = -\sqrt{1 - (\partial\phi \cdot \partial\phi)}$$

This theory known as a scalar part of the Dirac-Born-Infeld [1934] – DBI action

Scalar field can be seen as a fluctuation of a 4-dim brane in five-dim Minkowski space



Third case: $\rho = 2$, $\sigma = 2$ (double soft limit)

Similarly to the previous case, we get a unique solution: the Galileon Lagrangian

$$\mathcal{L} = \sum_{n=1}^{d+1} d_n \phi \mathcal{L}^{\mathrm{der}}_{n-1}$$

$$\mathcal{L}^{\mathrm{der}}_n = \varepsilon^{\mu_1 \ldots \mu_d} \varepsilon^{\nu_1 \ldots \nu_d} \prod_{i=1}^{n} \partial_{\mu_i} \partial_{\nu_i} \phi \prod_{j=n+1}^{d} \eta_{\mu_j \nu_j} = -(d-n)! \det \left\{ \partial^{\nu_i} \partial_{\nu_j} \phi \right\}.$$

It possesses the Galilean shift symmetry

$$\phi \to \phi + a + b_\mu x^\mu$$

and leads to EoM of second-order in field derivatives.

Galileon itself is a remarkable theory: can be connected with a local modification of gravity [Nicolis, Rattazzi, Trincherini '09].

Surprise: $\rho = 2$, $\sigma = 3$ (enhanced soft limit)

- general galileon: three parameters (in 4D)
- only two relevant (due to dualities [de Rham, Keltner, Tolley '14] [KK, Novotny '14])

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- symmetry explanation: hidden symmetry [K. Hinterbichler and A. Joyce 1501.07600]

$$\phi \to \phi + s_{\mu\nu} x^\mu x^\nu - 12\lambda_4 s^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

- theory appears also in the context of CHY-type formulation [Cachazo, He, Yuan 1412.3479]

Summary of Classification of EFTs: “soft-bootstrap”

Non-trivial cases

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i.e.

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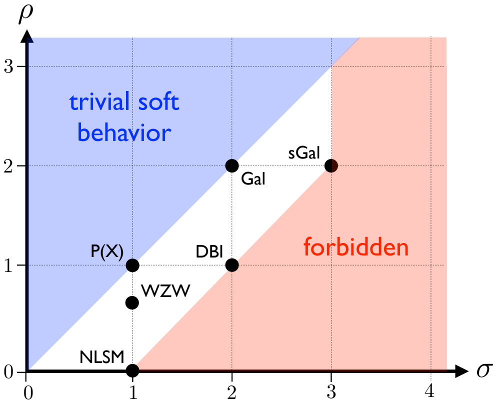
non-trivial regime for $\rho \leq \sigma$

[C. Cheung, K. Kampf, J. Novotny, C. H. Shen and J. Trnka '17]



Further directions of the soft amplitudology:

- vector effective field theories from soft limits [1801.01496]
- higher orders and more flavours [1909.13684 & 2109.11574] [2009.07940]
- generalization for Adler zero [1910.04766]
- scalar-vector galileon [2104.10693]

  $\rightarrow$ exceptional special scalar-vector galileon

$$\mathcal{L}_{\text{spec}} = (\text{spec.galileon}) + (\tilde{F}\tilde{F})(\partial\partial\phi + (\partial\partial\phi)^2)$$

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- beyond tree-level: [2206.04694] see poster of Christoph Bartsch
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Thank you!