Backgrounds for celestial amplitudes

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ICTP WORKSHOP ON SCATTERING AMPLITUDES AND COSMOLOGY

20 APRIL 2023

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⚠ Disclaimer ⚠

Our universe has a (tiny) positive cosmological constant $\Lambda > 0$.

I will shamelessly ignore it! In the following: $\Lambda = 0$.

Consolidation: spacetime will only be *asymptotically* flat.

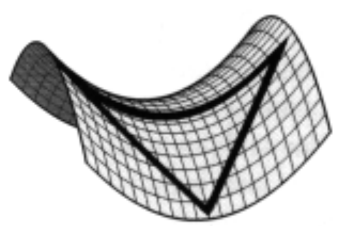
Some features may resonate with de Sitter space.

Modelling the Universe

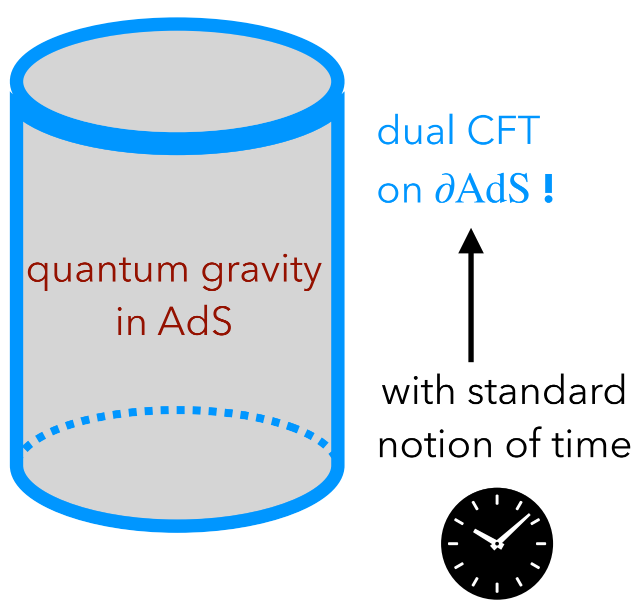
$\Lambda < 0$

Anti-de Sitter

negative curvature



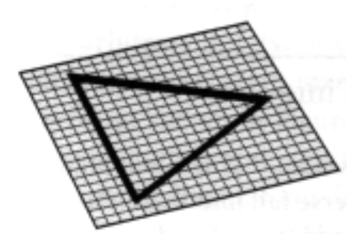
relevant scales: throats of highly rotating black holes



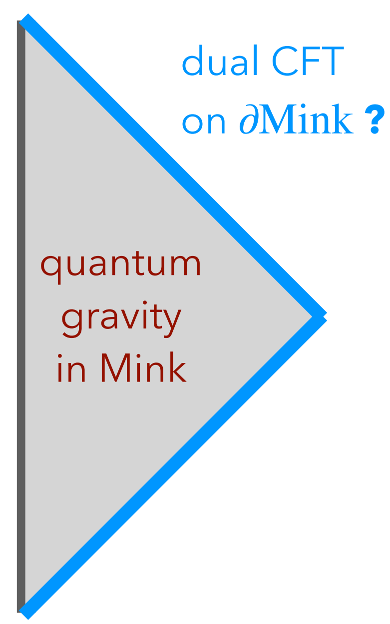
$\Lambda = 0$

Minkowski

flat curvature



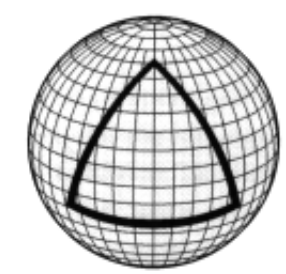
intermediate scales away from sources



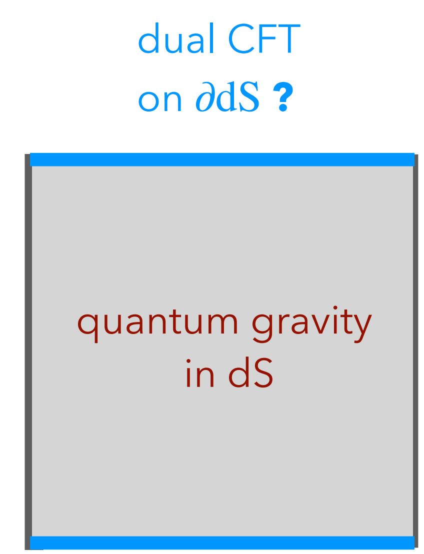
$\Lambda > 0$

de Sitter

positive curvature

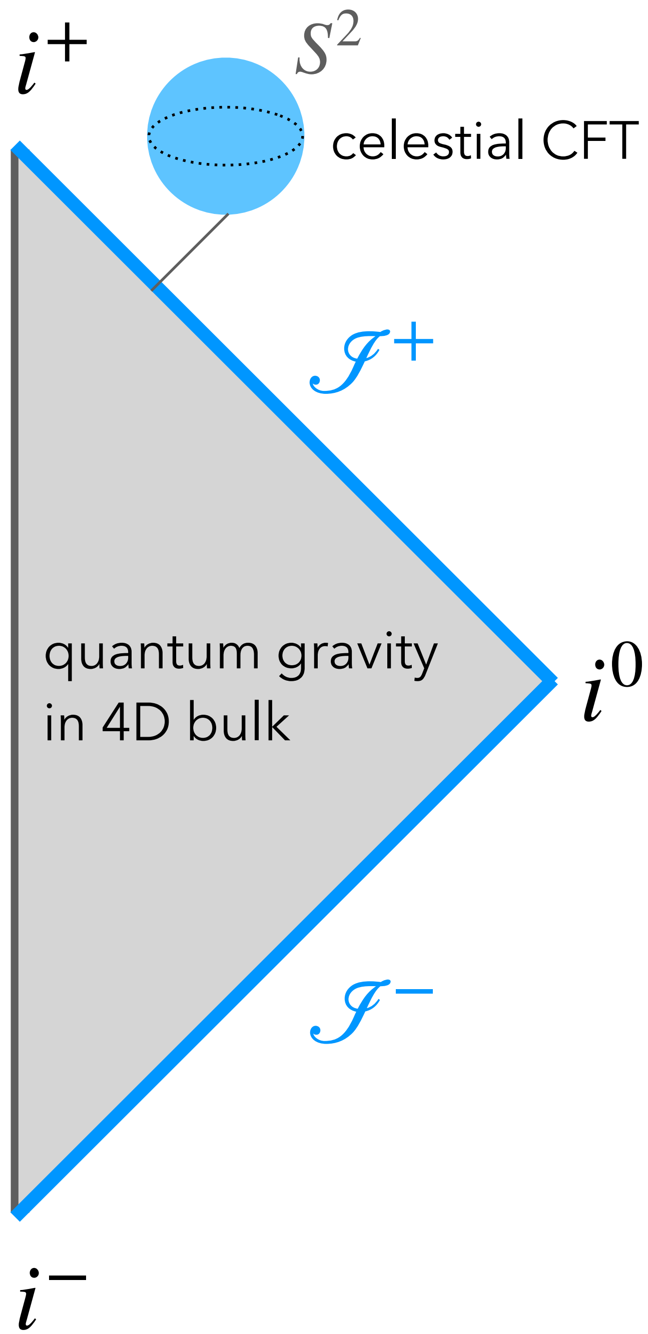


cosmological scales



No standard notion of time!
Bulk unitarity emergent.

Celestial Holography



Culmination of old and new insights on IR structure of gravity:

Proposal: *

Quantum gravity in 4D asymptotically flat bulk $\longleftrightarrow$ 2D ``celestial CFT'' on celestial sphere @ null infinity $\mathscr{I}^- \cup \mathscr{I}^+$

*extends to general spacetime dimensions

▶ New perspective on probing fundamental properties of the S-matrix.

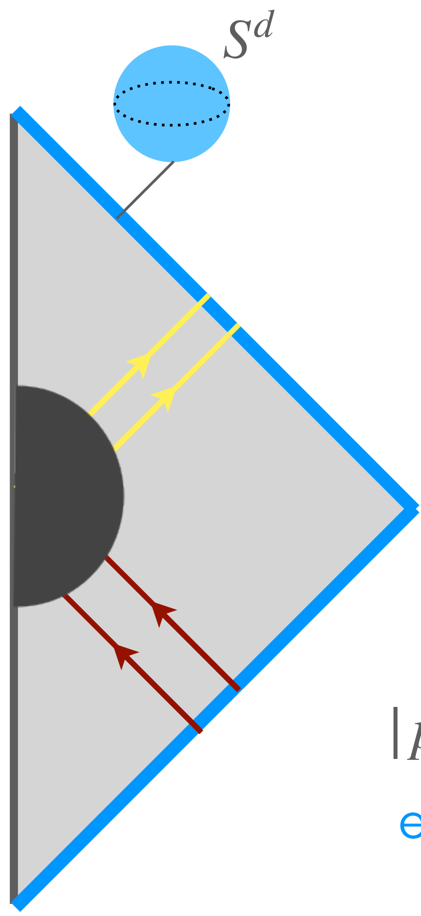
Outline

1. Aspects of Celestial CFT
    - Observables
    - Symmetries
2. Backgrounds in Celestial CFT
    - Boundary on shell action
    - Schwarzschild, Kerr, shockwaves
3. Dressings for backgrounds

Aspects of CCFT

Observables

in asymptotically flat spacetimes



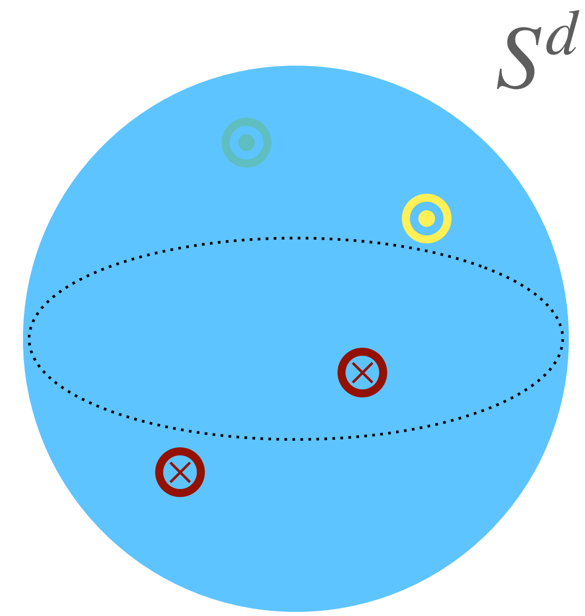
$\mathscr{M}$ellin

integrate over all energies

$$\int_0^\infty d\omega\, \omega^{\Delta-1}$$

$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis



$|\Delta_i, x_i\rangle$

boost-weight basis

Scattering amplitudes:

- basic observables in flat space

$$\mathscr{A}(p_1, \ldots p_n)$$

- constrained by analyticity & unitarity
- hidden structures e.g. double copy

gravity = gauge^2

Celestial amplitudes:

[Pasterski,Shao,Strominger'17] building on [de Boer, Solodukhin'03]

- natural observables in flat holography:

$$\langle \mathcal{O}_{\Delta_1}(x_1) \ldots \mathcal{O}_{\Delta_n}(x_n) \rangle$$

transform nicely under conformal transformations!

- constrained by symmetries e.g. BMS + ∞ tower

[Adamo,Mason,Sharma'19][AP'19] [Guevara'19]…

- operator-valued structures e.g. celestial double copy

[Casali,AP,'20]

Celestial amplitudes

$$ {}_{\text{boost}}\langle out \,|\, \mathcal{S} \,|\, in \rangle_{\text{boost}} = \langle \mathcal{O}_{\Delta_1,\ell_1}(x_1)\ldots\mathcal{O}_{\Delta_n,\ell_n}(x_n)\rangle_{\text{CCFT}} $$

plane wave $\Phi_\omega = e^{ip(\omega,x)\cdot X}$ $\xrightarrow{\int_0^\infty d\omega\omega^{\Delta-1}}$ $\dfrac{1}{(-q(x)\cdot X)^\Delta} = \Phi_\Delta$ conformal primary wavefunction

$p^\mu = \pm\, \omega q^\mu(x)$

[Pasterski,Shao'17]

$\pm i\varepsilon$ prescription

CCFT is not a garden variety CFT:

[Arkani-Hamed,Pate,Raclariu,Strominger'20]

- **no Wilsonian decoupling:** integrate over all energies

→ celestial amplitudes only finite if theory sufficiently well-behaved in UV

[Pasterski,Shao,Strominger'17]

- **distributional support** on the sphere from bulk $\delta^{(d+2)}(\sum_{i=1}^N p_i^\mu)$

Will see: celestial amplitudes on (≠ flat) backgrounds nicer behaved.

Universality

Important notion in QFT:

energetically soft limit

$$\omega \to 0$$

↓

universal behavior of scattering amplitudes

$\mathscr{M}$ellin

$$\int_0^\infty d\omega\, \omega^{\Delta-1}$$

Replaced in celestial CFT by:

[Donnay, AP, Strominger'18]

conformally soft limit

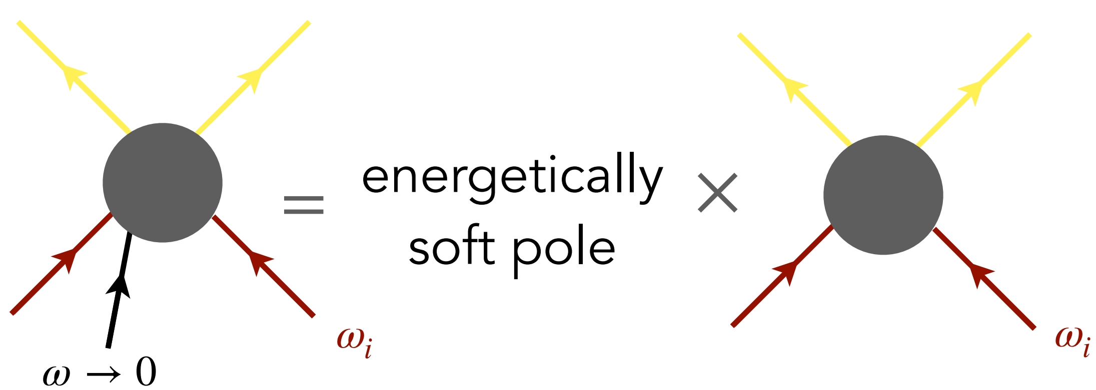
$$\Delta \in \mathbb{Z}$$

↓

universal behavior of celestial amplitudes

Weinberg's soft graviton theorem

[Weinberg'65]



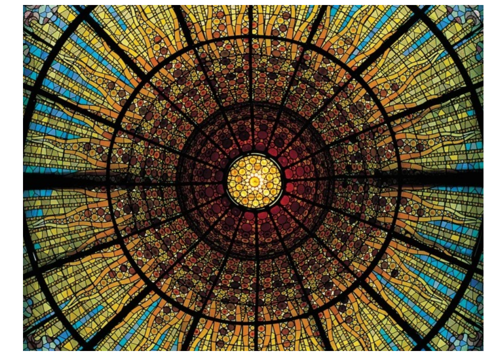
Conformally soft graviton theorem

[Adamo,Mason,Sharma'19][AP'19] [Guevara'19]

Does not stop here: subleading, subsubleading,… soft theorems!

Symmetries

What are all the symmetries (of nature)?



Key for any holographic dual construction.

Important in its own right.

Strategy: [Pasterski,AP,Trevisani'21a] [Pano,AP,Trevisani'23] uses work of [Penedones,Trevisani,Yamazaki'15]

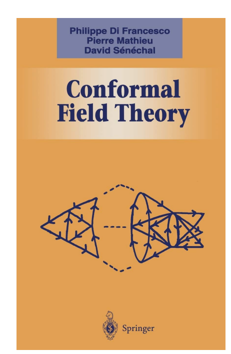
see also [Banerjee et al] [Kapec,Mitra'21-'22]

Soft theorems $\xrightarrow{\mathscr{M}\text{ellin}}$ CFT correlator for $\mathcal{O}_\Delta$ with conformally soft $\Delta \in \mathbb{Z}$

↓ CFT tool: conformal representation theory

Ward identity for $\mathcal{O}_\Delta$

↓



(+ a bit more advanced stuff)

Noether current → charge → symmetry group.

Soft symmetries

Noether current → charge → symmetry group:

[Pasterski,AP,Trevisani'21a]

CCFT_2

infinite-dimensional

symmetry group

↑

BMS supertranslations

2D local conformal symmetry ← superrotations

[Pano,AP,Trevisani'23]

$\text{CCFT}_{d>2}$

finite-dimensional

symmetry group

↑

translations

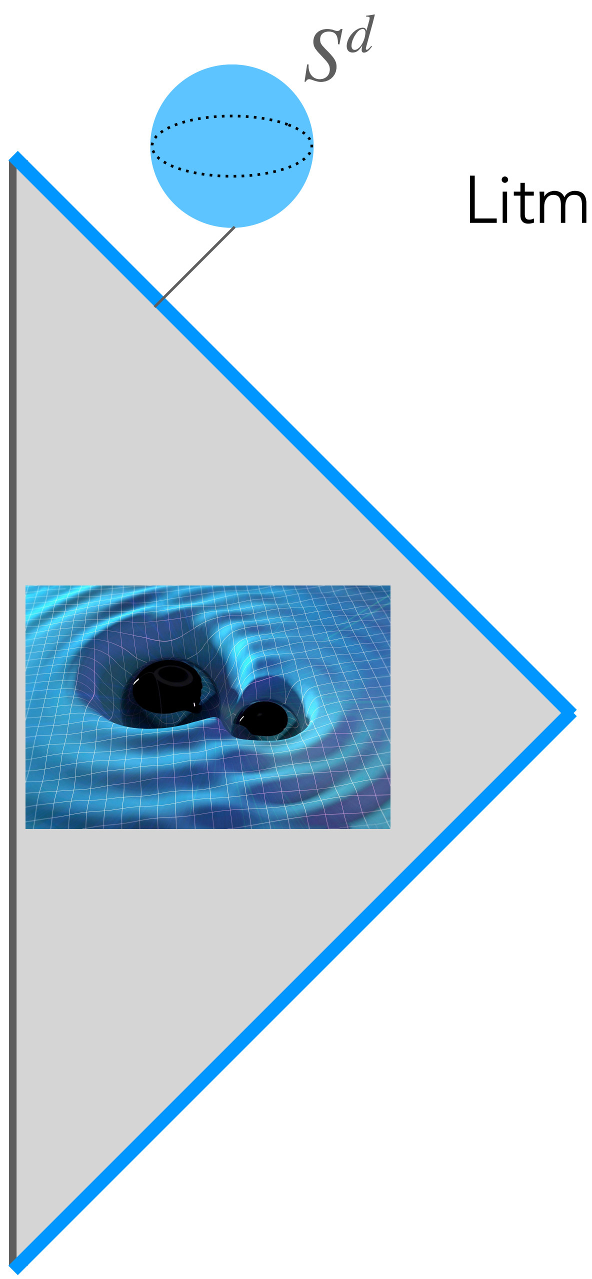
rotations

▶ Role of higher-dimensional BMS ?

BACKGROUNDS in CCFT

Backgrounds in celestial holography

e.g. black holes, Coulomb fields, shockwaves…



Litmus test: if boundary captures non-perturbative bulk physics.

[Gonzo,McLoughlin,AP'22]:

Imprint of bulk geometries on CCFT correlators?

- Bulk geometries from CCFT states?
- Backgrounds break isometries:
  structure of CCFT correlators?

Make use of old and new results relating classical backgrounds and amplitudes.

Amplitudes on backgrounds

Classical field Φ_{cl} produced by a source J is generating functional of tree-graph approximation to the corresponding QFT. [Boulware,Brown'68]

Classical limit of generating functional for connected correlators

$$W[J] \equiv -i \log Z[J] \qquad Z[J] = \int \mathscr{D}\Phi e^{i(S[\Phi]+J\Phi)}$$

is dominated by classical solutions of the eom, find relation:

$$\Phi_{cl}[J] = \frac{\delta W[J]}{\delta J}$$

Differentiate classical solution $(n-1)$ times wrt source → n-point correlator.

n-point amplitude:

Fourier transform ↓

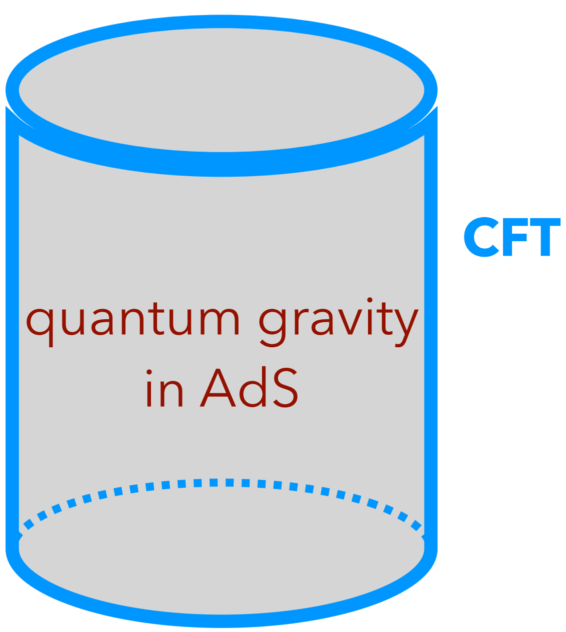
$$\mathscr{A}_n(p_1,\ldots,p_n) = i^n \prod_{k=1}^{n} \left(\lim_{p_k^2 \to 0} p_k^2 \right) \frac{\delta}{\delta J(p_1)} \cdots \frac{\delta}{\delta J(p_{n-1})} \bar{\Phi}_{cl}(-p_n) \Big|_{J=0}$$

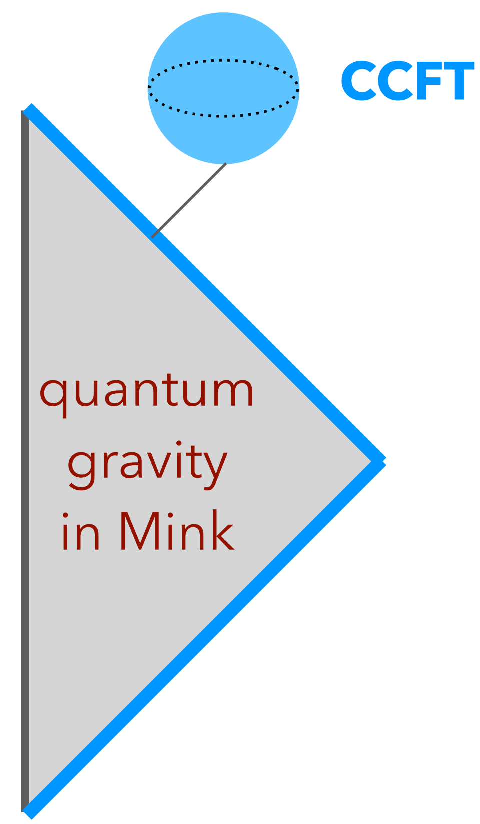
CFT correlators from on-shell action

$$S = \text{eom} + S_{\text{bdy}}$$

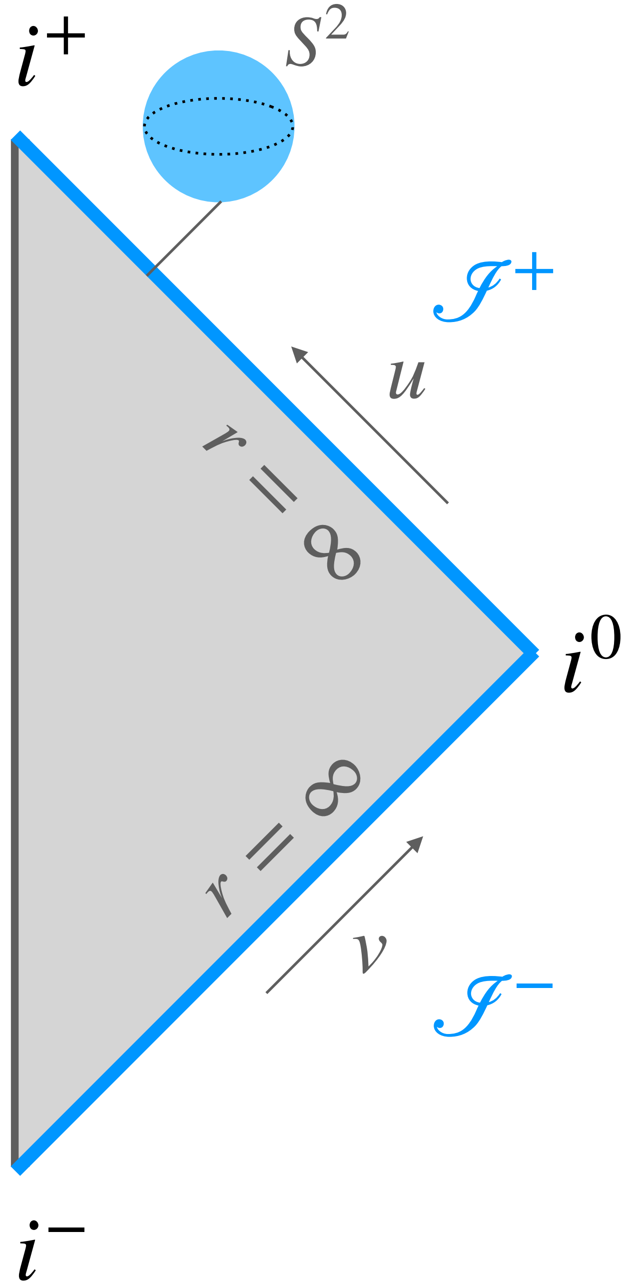
In AdS/CFT on-shell action generates CFT correlators.

Celestial holography: on-shell action generates CCFT correlators?





On-shell action



$$S = \text{eom} + S_{\mathscr{I}^- \cup \mathscr{I}^+}$$

E.g. complex massless scalar ϕ minimally coupled to gravity

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\phi(x) = J_{eff}(x) \qquad \phi = \phi_{in} + \phi_{out}$$

$\phi_{in} = e^{ip\cdot X}$

ϕ_{out} solve via Green's fct

At large r at fixed $v = t + r$ and $u = t - r$
saddle point approximation → **localization**:

$$S_{\mathscr{I}^- \cup \mathscr{I}^+}(p) = \#\bar{J}_{eff}(p)$$

[Gonzo, McLoughlin,AP'22]

On-shell action localizes on the boundary onto the Fourier transform of effective source evaluated along the incoming momentum.

* on-shell action also studied in [Fabbrichesi,Pettorini,Veneziano,Vilkovisky'93]

Generating CCFT correlators

Putting it all together:

Boulware-Brown

$$\mathscr{A}_2(p_1,p_2) = -\lim_{p_1^2\to 0}\lim_{p_2^2\to 0} p_1^2 p_2^2 \frac{\delta\bar{\phi}_{out}(-p_1)}{\delta\bar{J}(p_2)}$$

solving the eom to leading order

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\phi(x) = J_{eff}(x)$$

$$\bar{\phi}_{out}(p) = -\frac{\bar{J}_{eff}(p)}{p^2}$$

$$= \lim_{p_1^2\to 0}\lim_{p_2^2\to 0} p_2^2 \frac{\delta\bar{J}_{eff}(-p_1)}{\delta\bar{J}(p_2)}$$

large r limit + saddle point approximation

$$S_{\mathscr{I}^-\cup\mathscr{I}^+}(p) = \#\bar{J}_{eff}(p)$$

$$= \frac{1}{\#}\lim_{p_1^2\to 0}\lim_{p_2^2\to 0} p_2^2 \frac{\delta S_{\mathscr{I}^-\cup\mathscr{I}^+}(-p_1)}{\delta\bar{J}(p_2)}$$

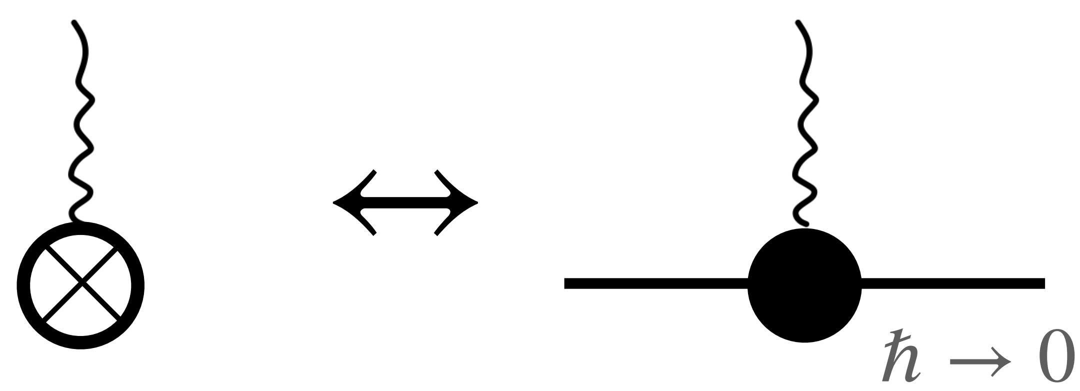
Boundary on-shell action generates CCFT correlators.

Particle-like backgrounds

Particle-like backgrounds generated by classical 3-point amplitudes with off-shell coherent emission of "messenger", e.g. photon or graviton.

[Monteiro,O'Connell,Penaidor Vega, Sergola'20]

[Kosower,Maybe,O'Connell'18]



Consider scattering particles in backgrounds generated by sources of mass and charge and interpret them in celestial CFT.

Coulomb field of static and spinning point charges & ultraboost limits.
Schwarzschild, Kerr & ultraboost limits: Aichelburg-Sexl and gyraton metrics.

[Duff'73] ['t Hooft'87] [Cristofoli'21]

All take the form of Kerr-Schild backgrounds.

Scattering on Kerr-Schild bgds

Kerr-Schild backgrounds:

$$h_{\mu\nu} = V k_\mu k_\nu \quad \rightarrow \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$ (exact sol!)

$$A_\mu = V k_\mu$$

Scalar fct V solves free wave equation

Kerr-Schild vector k null and geodesic wrt η and g

E.g. wave equation for complex scalar field minimally coupled to gravity in the presence of a source:

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\phi(X)) = J(X)$$

of order n in coupling G

Solve for $\bar{\phi}(p) = \sum_{n=0}^{\infty} \bar{\phi}^{(n)}(p)$ perturbatively in G and plug into

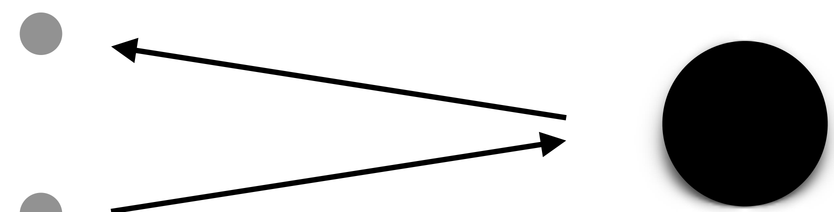
$$\mathscr{A}_2(p_1, p_2) = -\prod_{k=1}^{2}\left(\lim_{p_k^2\to 0} p_k^2\right)\frac{\delta}{\delta \bar{J}(p_1)}\bar{\phi}(-p_2)\Big|_{J=0}$$

$$= (2\pi)^4 \lim_{p_1^2\to 0}\lim_{p_2^2\to 0} p_1^2 \delta^{(4)}(p_1+p_2) - [(p_1)_\mu(p_2)_\nu - \tfrac{1}{2}\eta_{\mu\nu}p_1\cdot p_2]\bar{h}^{\mu\nu}(p_1+p_2) + \ldots$$

$n=0$ $\qquad$ $n=1$

Black hole avatars in CCFT

Compute celestial scattering on **Schwarzschild** & **Kerr**: * [Gonzo, McLoughlin,AP'22]



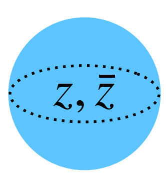
scalar (massless) — black holes (massive)

$\mathscr{M}$ellin ↓

in boost basis

*also Coulomb field of point charge

- Celestial amplitudes on **backgrounds** <u>nicer features</u> than in **flat** space.



Supported everywhere on the S^2. vs δ-function support on S^2 !

power-law in $z_{ij} = z_i - z_j$!

Classical spin acts as UV regulator. $\mathscr{M}$ellin integrals UV divergent.

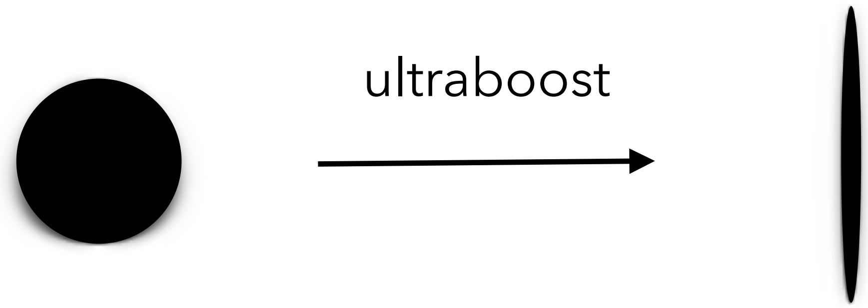
non-spinning spinning

$$\int_0^\infty d\omega\omega^{\Delta-1} \to \int_0^\infty d\omega\omega^{\Delta} H^{(2)}_{-1}(a\omega)$$ Hankel fct

$\downarrow$ $2\pi\delta(i\Delta)$ $\downarrow$ finite support

Shocks in CCFT

Ultraboost limit of black holes is special:



black holes

Schwarzschild metric

shockwaves

Aichelburg-Sexl metric $g_{\mu\nu} = \eta_{\mu\nu} + r_0 \log(X^2)\delta(q\cdot X)q_\mu q_\nu$

- **Shockwaves** are generated by **conformal primaries** in CCFT! [Pasterski,AP'20]

Scalar shockwaves: $\phi_{sw}(X) = -\log(X^2)\delta(q\cdot X)$ $\quad(\Delta,\ell) = (1{,}0)$ scalar primary

$q^\mu = (1+|z_{sw}|^2, z_{sw}+\bar{z}_{sw}, i(\bar{z}_{sw}-z_{sw}), 1-|z_{sw}|^2)$

null vector pointing at $(z_{sw},\bar{z}_{sw})$ on celestial sphere

Kerr-Schild double copy

Spinning shockwaves:

$A_\mu(X) = r_0\phi_{sw}(X)q_\mu$ $\quad(\Delta,\ell) = (0{,}0)$ vector primary

$h_{\mu\nu}(X) = r_0\phi_{sw}(X)q_\mu q_\nu$ $\quad(\Delta,\ell) = (-1{,}0)$ metric primary

Celestial shockwave correlators

[Gonzo,McLoughlin,AP'22]

electromagnetic: $A_\mu(X) = r_0 \phi_{sw}(X) q_\mu$ $(\Delta, \ell) = (0,0)$ vector primary

$$\mathcal{M}_3(\Delta_1, \Delta_2, \Delta_{sw}) = \frac{e(2\pi)^3 \delta(i(\Delta_1 + \Delta_2 - 2))}{|z_{12}|^{\Delta_1+\Delta_2} |z_{1sw}|^{\Delta_1-\Delta_2} |z_{2sw}|^{\Delta_2-\Delta_1}}$$

$\Delta_{sw} = 0$

Looks like 3-point correlator in standard CFT_2 !

gravitational: $h_{\mu\nu}(X) = r_0 \phi_{sw}(X) q_\mu q_\nu$ $(\Delta, \ell) = (-1,0)$ metric primary

$$\mathcal{M}_3(\Delta_1, \Delta_2, \Delta_{sw}) = \frac{r_0(2\pi)^3 \delta(i(\Delta_1 + \Delta_2 - 1))}{|z_{12}|^{\Delta_1+\Delta_2+1} |z_{1sw}|^{\Delta_1-\Delta_2-1} |z_{2sw}|^{\Delta_2-\Delta_1-1}}$$

$\Delta_{sw} = -1$

Looks like 3-point correlator in standard CFT_2 after continuing off the principal series $\mathrm{Re}(\Delta_1 + \Delta_2) = 1$!

DRESSINGS FOR BACKGROUNDS

IR divergences

So far only leading contribution to 2-point, at higher orders in perturbation theory: IR divergences due to non-trivial asymptotic dynamics of long-range massless interactions - can be seen by iteratively solving the wave equation.

$$\bar{\Phi}_{cl}(p) = \sum_{n=0}^{\infty} \bar{\Phi}_{cl}^{(n)}(p)$$

of order n in coupling

Infinite sums of products of terms exponentiate:

QED:

for Coulomb & shockwave

$$\mathscr{A}_2^{conn,IR}(p_1,p_2) = \exp\left[e\int \frac{d^4k}{(2\pi)^4}\frac{\bar{A}(-k)\cdot p_2}{k\cdot p_2}\right]\mathscr{A}_2^{(1)}(p_1,p_2) = \exp\left[\frac{ieQ}{8\pi\epsilon}\right]\mathscr{A}_2^{(1)}(p_1,p_2)$$

gravity:

for Schwarzschild & shockwave

$r_0 = M$ $\quad$ $r_0 = P^+$

$$\mathscr{A}_2^{conn,IR}(p_1,p_2) = \exp\left[-\int \frac{d^4k}{(2\pi)^4}\frac{\bar{h}^{\mu\nu}(-k)p_{2\mu}p_{2\nu}}{2k\cdot p_2}\right]\mathscr{A}_2^{(1)}(p_1,p_2) = \exp\left[-\frac{i(p_2\cdot u)Gr_0}{\epsilon}\right]\mathscr{A}_2^{(1)}(p_1,p_2)$$

Conformal Faddeev-Kulish dressings

IR divergent factors exponentiate & hard/soft factorization persists in CCFT

$$\mathscr{M}_2^{\mathrm{IR}} = \underline{\mathscr{M}_2^{soft}}\mathscr{M}_2^{(1)}$$

QED: $\exp\left\{\frac{iQe}{8\pi\epsilon}\right\}$ gravity: $\exp\left\{\frac{iP^+G}{\epsilon}\left[|z_{1\mathrm{sw}}|^2 e^{\partial_{\Delta_1}} - |z_{2\mathrm{sw}}|^2 e^{\partial_{\Delta_2}}\right]\right\}$

Conformal Faddeev-Kulish dressings for **particles**: choice of FK dressing that respects conformal invariance but not energy finiteness. CCFT interpretation as correlators of vertex operator of Goldstones for asymptotic symmetries.

[Arkani-Hamed,Pate, Raclariu,Strominger'21]

single particle dressing: $e^{-iR_k}|\omega_k, z_k, \bar{z}_k\rangle$

QED: $R_k = e_k\Phi_k(z, \bar{z})$

gravity: $R_k = \frac{\kappa}{2}C_k(z, \bar{z})$

$\xrightarrow{\mathscr{M}\text{ellin:}}$

$$\langle\Phi(z_i, \bar{z}_i)\Phi(z_j\bar{z}_j)\rangle = \frac{1}{8\pi^2\epsilon}\ln|z_{ij}|^2$$

Goldstones

$$\langle C(z_i, \bar{z}_i)C(z_j\bar{z}_j)\rangle = -\frac{1}{4\pi^2\epsilon}|z_{ij}|^2\ln|z_{ij}|^2$$

Conformal FK for backgrounds

[Gonzo,McLoughlin,AP'22]

Conformal Faddeev-Kulish dressings for **particles-like backgrounds**:

$$R_{\rm sw}^{\rm QED} = \frac{Q}{2}\left(\Phi^{+}(z_{\rm sw}, \bar{z}_{\rm sw}) + \Phi^{-}(z_{\rm sw}, \bar{z}_{\rm sw})\right) \qquad \langle \Phi^{\eta_i}(z_i, \bar{z}_i)\Phi^{\eta_j}(z_j\bar{z}_j)\rangle = \frac{\eta_i\eta_j}{8\pi^2\epsilon}(\ln|z_{ij}|^2 - i\pi\delta_{\eta_i\eta_j})$$

$$R_{\rm sw}^{\rm GR} = \frac{\kappa}{4}\left(C^{+}(z_{\rm sw}, \bar{z}_{\rm sw}) + C^{-}(z_{\rm sw}, \bar{z}_{\rm sw})\right) \qquad \langle C^{\eta_i}(z_i, \bar{z}_i)C^{\eta_j}(z_j\bar{z}_j)\rangle = -\frac{\eta_i\eta_j}{4\pi^2\epsilon}|z_{ij}|^2\ln(|z_{ij}|^2 - i\pi\delta_{\eta_i\eta_j})$$

Dressings for particle-like backgrounds which remove divergent terms at all orders in perturbation theory in shockwave 2-point functions.

For dressed shockwave operators $\hat{\mathcal{O}}_{\rm sw} = e^{-iR_{\rm sw}}\mathcal{O}_{\rm sw}$ get 3-point functions

$$\langle \hat{O}_{sw}(z_{sw}, \bar{z}_{\rm sw})\hat{O}^{-}_{\Delta_1}(z_1, z_2)\hat{O}^{+}_{\Delta_2}(z_2, \bar{z}_2)\rangle$$

which are IR finite to all orders in perturbation theory.

Conclusion

Celestial amplitudes offer a new perspective on probing fundamental aspects of the S-matrix:

- use powerful CFT toolkit (classify symmetries, bootstrap?)
- need to deal with UV behavior (anti-Wilsonian)

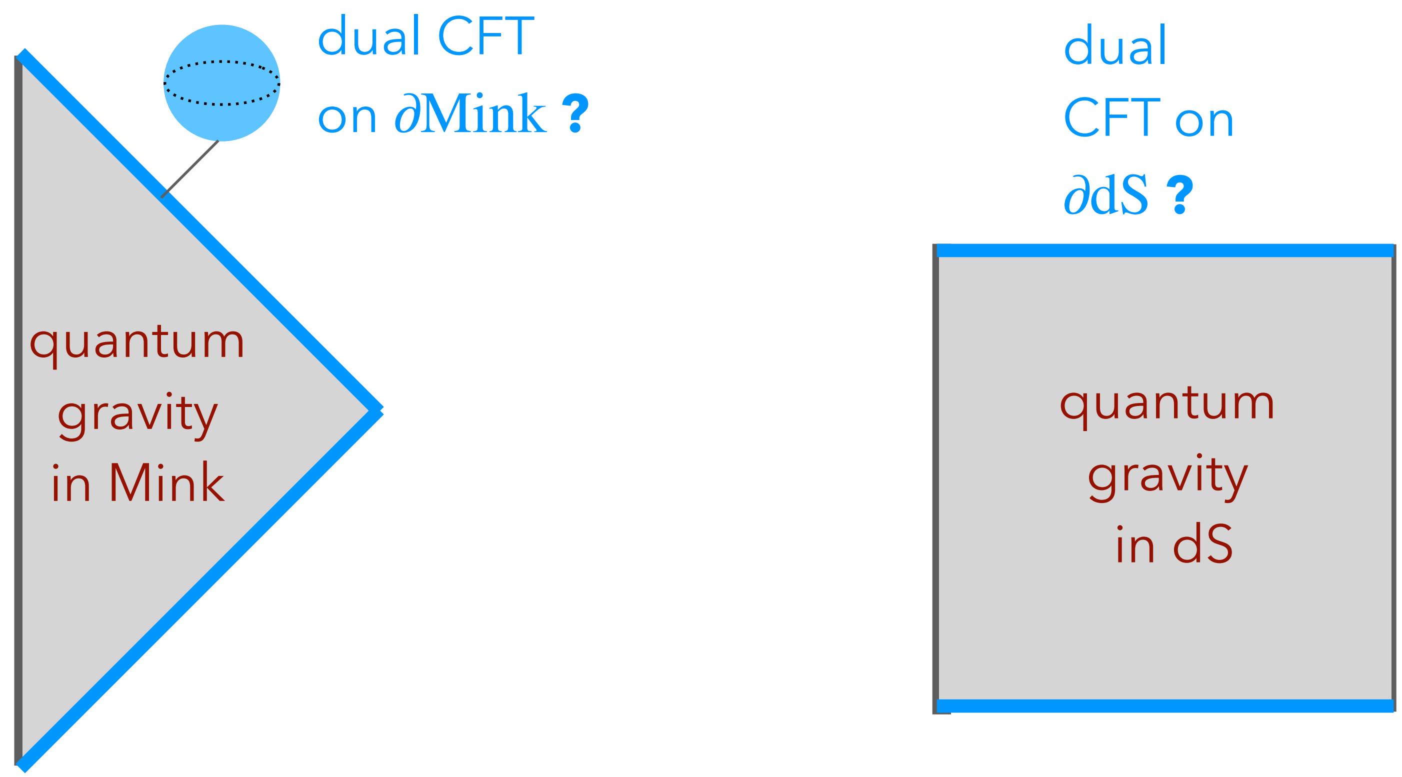
On backgrounds celestial amplitudes better behaved (power-law in angles, classical spin $\rightarrow$ UV regulator).

Conformal Fadeev-Kulish dressing for particle-like backgrounds.

Flat $\overset{?}{\leftrightarrow}$ de Sitter

- Can we use new tools from **celestial holography** to advance **de Sitter holography** and vice versa?

e.g. exploiting the AdS / dS slicing of Minkowski? [de Boer, Solodukhin'03]



Bulk unitarity emergent in both settings?

celestial bootstrap vs **cosmological bootstrap**

Thank you!