Double Copy in (A)dS

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Based on:

2304.07141 with S. Nagy

2304.07206 with C. Armstrong, H. Goodhew, J. Mei

Motivation

- Double copy reduces complicated calculations in gravity to simpler ones in gauge theory.
- Can this progress be extended to cosmology?
- What does this teach us about flat space amplitudes?
- New connections between AdS/CFT and flat space holography?

Overview

- review of flat space double copy
- self-dual AdS_4 gravity
- color/kinematics duality and $w_{1+\infty}$ in AdS_4
- review of cosmological correlators
- cosmo boostrap
- 4-graviton wavefunction
- conclusion

AdS_4 metric

- Poincaré patch of Euclidean AdS_4 with unit radius:

$$ds^2_{\mathrm{AdS}} = \frac{dt^2 + dx^2 + dy^2 + dz^2}{z^2}$$

where $0 < z < \infty$

- Light cone coordinates:

$$u = it + z, \quad v = it - z$$
$$w = x + iy, \quad \bar{w} = x - iy,$$

$$\rightarrow \quad ds^2_{\mathrm{AdS}} = \frac{4\,(dw\,d\bar{w} - du\,dv)}{(u-v)^2}$$

- Wick rotation $z \to i\eta$ gives dS_4 metric. In practice we will Wick rotate conformal time integrals in dS_4 to AdS_4

Double Copy in Flat Space

- 3-point gluon amplitudes square into 3-point graviton amplitudes
- Color/kinematics duality can be used to extend double copy beyond three points. Take a 4-point gluon amplitude:

$$\mathcal{A}_4 = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}, \qquad c_s + c_u + c_t = 0$$

numerators obey kinematic Jacobi: $n_s + n_t + n_u = 0$

- Squaring these numerators gives 4-point graviton amplitude:

$$\mathcal{M}_4 = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

(Bern,Carrasco,Johansson)

Self-dual sector

- SDYM: $F_{\mu\nu} = \frac{\sqrt{g}}{2}\epsilon_{\mu\nu\rho\lambda}F^{\rho\lambda}$, SDG: $R_{\mu\nu\rho\sigma} = \frac{1}{2}\sqrt{g}\epsilon_{\mu\nu}{}^{\eta\lambda}R_{\eta\lambda\rho\sigma}$

- SDYM in lightcone gauge (Bardeen,Chalmers,Siegel):

$$\Box_{\mathbb{R}^4}\Phi - \frac{i}{2}[\{\Phi,\Phi\}] = 0, \quad \{f,g\} := \partial_w f\partial_u g - \partial_u f\partial_w g = \varepsilon^{\alpha\beta}\Pi_\alpha f\Pi_\beta g$$

- SDG in lightcone gauge (Plebanski):

$$\Box_{\mathbb{R}^4}\phi - \{\{\phi,\phi\}\} = 0, \quad \{\{f,g\}\} = \frac{1}{2}\varepsilon^{\alpha\beta}\{\Pi_\alpha f,\Pi_\beta g\}, \quad \Pi_\alpha = (\Pi_v,\Pi_{\bar{w}}) = (\partial_w,\partial_u)$$

- Color/kinematics duality manifest (Montiero,O’Connell)

SDG in AdS_4

- SDYM eom same as flat space (only knows about b.c.)

- SDG: $$T_{\mu\nu\rho\sigma} = \frac{1}{2}\sqrt{g}\epsilon_{\mu\nu}{}^{\eta\lambda}T_{\eta\lambda\rho\sigma}$$

  where $T_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \frac{1}{3}\Lambda(g_{\mu\rho}g_{\nu\sigma} - g_{\nu\rho}g_{\mu\sigma})$ (we will set $\Lambda = -3$)

- Contracting with inverse metric gives vacuum Einstein equation:

$$R_{\mu\rho} - \Lambda g_{\mu\rho} = \frac{1}{2}\sqrt{g}\epsilon_{\mu}{}^{\sigma\eta\lambda}R_{\eta\lambda\rho\sigma} = 0$$

Solution:

$$\frac{1}{u-v}\Box_{\mathbb{R}^4}\left(\frac{\phi}{u-v}\right)-\left\{\left\{\frac{\phi}{u-v},\frac{\phi}{u-v}\right\}\right\}_*=0$$

where $\{\{f,g\}\}_*=\frac{1}{2}\varepsilon^{\alpha\beta}\{\Pi_\alpha f,\Pi_\beta g\}_*$

- Deformed Poisson bracket:

$$\{f,g\}_*=\frac{1}{2}\varepsilon^{\alpha\beta}(\Pi_\alpha f\tilde{\Pi}_\beta g-\Pi_\alpha g\tilde{\Pi}_\beta f) \qquad \tilde{\Pi}=(\tilde{\Pi}_v,\tilde{\Pi}_{\bar{w}})=\left(\partial_w,\partial_u-\tfrac{4}{u-v}\right)$$

- Solutions: $\phi=(u-v)e^{ik\cdot x}$

CK Duality for SDG in AdS_4

- Feynman rules: $$V_{\mathrm{SDYM}} = \frac{1}{2} X(k_1, k_2) f^{a_1 a_2 a_3},$$ $$V_{\mathrm{SDG}} = \frac{1}{2} X(k_1, k_2) \tilde{X}(k_1, k_2)$$

  where $X(k_1, k_2) = k_{1u} k_{2w} - k_{1w} k_{2u}, \quad \tilde{X}(k_1, k_2) = X(k_1, k_2) - \frac{2i}{u-v}(k_1 - k_2)_w$

- Double copy: $f^{a_1 a_2 a_3} \to \tilde{X}(k_1, k_2)$

- Kinematic Jacobi: $$\begin{aligned} 0 &= X(k_1, k_2) X(k_3, k_1 + k_2) + \mathrm{cyclic} \\ &= \tilde{X}(k_1, k_2) \tilde{X}(k_3, k_1 + k_2) + \mathrm{cyclic} \end{aligned}$$

w-infinity algebras in AdS_4

- Expand plane waves: $e^{ik\cdot x} = \sum_{a,b=0}^{\infty} \frac{(ik_u)^a (ik_w)^b}{a!b!} \mathfrak{e}_{ab}$

  where $\mathfrak{e}_{ab} = (u + \rho\bar{w})^a (w + \rho v)^b$ (Monteiro)
- Let $w^p_m = \frac{1}{2}\mathfrak{e}_{p-1+m,p-1-m}$. Then

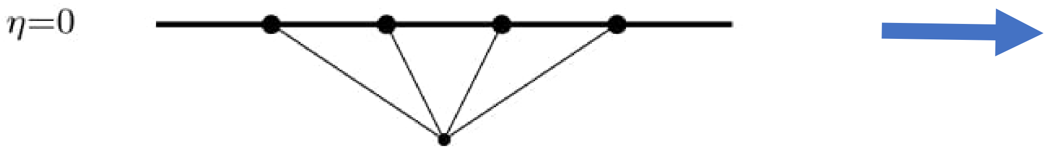
$$\{w^p_m, w^q_n\} = (n(p-1) - m(q-1))\, w^{p+q-2}_{m+n},$$
$$\{w^p_m, w^q_n\}_* = \{w^p_m, w^q_n\} + \frac{(m+q-p-n)}{u-v} w^{p+q-3/2}_{m+n+1/2}$$

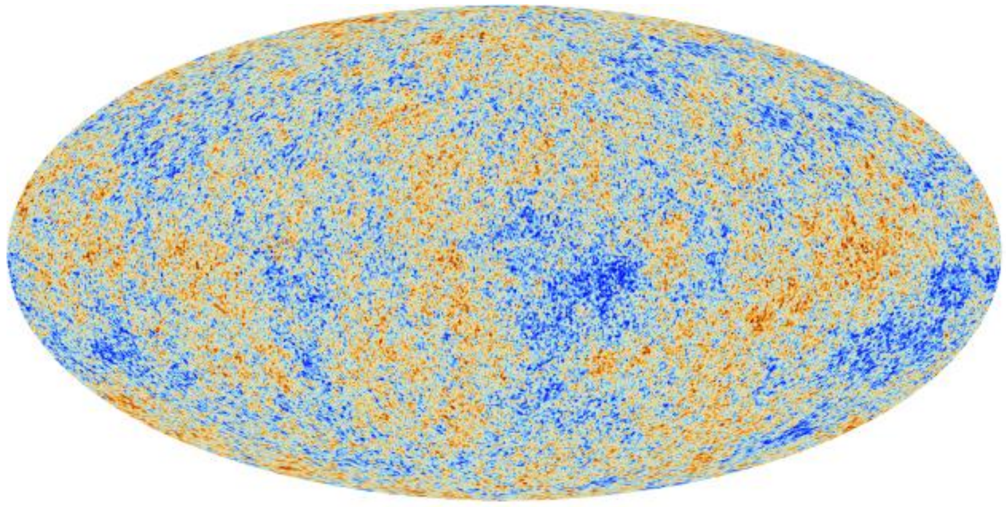
- First line is $w_{1+\infty}$ algebra, which plays an important role in flat space holography (Strominger). Second line contains a deformation

Cosmological Observables

- Inflation: early Universe approximately described by dS_4. CMB comes from correlations on future boundary

$$ds^2 = \frac{-d\eta^2 + (dx^i)^2}{\eta^2}, \quad -\infty < \eta < 0$$





Cosmological Wavefunction

- In-in correlators (Maldacena,Weinberg):

$$\left\langle \phi(\vec{k}_1)...\phi(\vec{k}_n) \right\rangle = \frac{\int \mathcal{D}\phi \, \phi(\vec{k}_1)...\phi(\vec{k}_n) \left|\Psi\left[\phi\right]\right|^2}{\int \mathcal{D}\phi \left|\Psi\left[\phi\right]\right|^2}$$

- Wavefunction:

$$\ln \Psi\left[\phi\right] = -\sum_{n=2}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} \frac{\mathrm{d}^d k_i}{(2\pi)^d} \psi_n \left(\vec{k}_1, ...\vec{k}_n\right) \phi(\vec{k}_1)...\phi(\vec{k}_n)$$

- ψ_n can be treated like CFT correlator in the future boundary and computed from Witten diagrams (Maldacena,Pimentel,McFadden,Skenderis)

Graviton Wavefuntion

- Tree-level wavefunction for 4 gravitons recently computed by (Bonifacio,Goodhew,Joyce,Pajer,Stefanyszyn)

- Bootstrapped using flat space limit (Maldacena,Pimentel,Raju), Cosmological Optical Theorem (Goodhew,Jazayeri,Melville,Pajer) and Manifestly Local Test (Jazayeri,Pajer,Stefanyszyn)

- De Sitter Feynman diagrams give hundreds of thousands of terms but result of bootstrap is only about a page long.

- Combining bootstrap with double copy reduces it down to only a few lines!

Cosmo Boostrap

- Flat space limit: $\lim_{E\to 0} \psi_4^{\gamma} \propto \frac{k_1k_2k_3k_4}{E^3}\mathcal{M}_4$, $E = k_1 + k_2 + k_3 + k_4$, $k_a = |\vec{k}_a|$

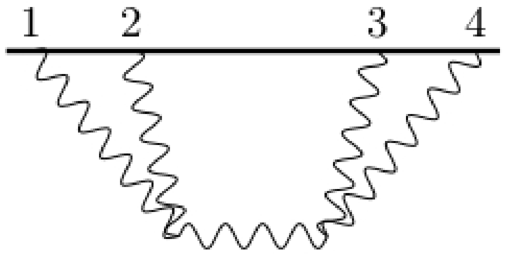
- COT: $\tilde{\psi}_4(k_1, k_2, k_3, k_4, k_s) + \tilde{\psi}_4^*(-k_1, -k_2, -k_3, -k_4, k_s) =$
$$\frac{1}{k_s^3}\left[\tilde{\psi}_3(k_1, k_2, k_s) - \tilde{\psi}_3(k_1, k_2, -k_s)\right]\left[\tilde{\psi}_3(k_3, k_4, k_s) - \tilde{\psi}_3(k_3, k_4, -k_s)\right]$$

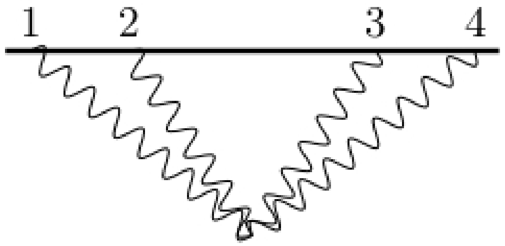
where $k_s = |\vec{k}_1 + \vec{k}_2|$ and $\tilde{\psi}_n$ is "trimmed" wavefunction

- MLT: $\lim_{k_1\to 0} \partial_{k_1}\tilde{\psi}_4(k_1, k_2, k_3, k_4, k_s) = 0$

Gluon Wavefuntion

- Diagrams:

$\psi_A =$ 

$+\ 2 \leftrightarrow 4\ +$ 

(axial gauge Feynman rules obtained by Liu,Tseytlin,Raju)

- s-channel wavefunction:

$$\psi_A^{(s)} = \int \frac{d\omega\, \omega}{k_s^2 + \omega^2} dz\, dz'\, (KKJ)_{12}^{1/2}(z)(KKJ)_{34}^{1/2}(z')N_s$$

where $N_s = V_{12}^i H_{ij} V_{34}^j + V_c^s(\omega^2 + k_s^2)$

$$(KKJ)_{ab}^{\nu} = \frac{2}{\pi}(k_a k_b z)^{\nu} z K_{\nu}(k_a z) K_{\nu}(k_b z) J_{\nu}(\omega z)$$

Double copy ansatz

- Ansatz: $\psi^{(s)}_{\gamma,\mathrm{DC}} = \int \frac{d\omega\,\omega}{k_s^2+\omega^2} dz\, dz' \,(KKJ)^{3/2}_{12}(z)(KKJ)^{3/2}_{34}(z')$

$$\times \left(N_s^2 - \frac{1}{2}\tilde{V}^{ij}_{12} H_{ij} \tilde{V}^{kl}_{34} H_{kl} + \frac{1}{2}(\epsilon_1 \cdot \epsilon_2)^2 (\epsilon_3 \cdot \epsilon_4)^2 (\omega^2 + k_s^2)^2 \right)$$

where $\tilde{V}^{ij}_{ab} = V^i_{ab} V^j_{ab}$

- Satisfies flat space limit, COT, and MLT
- Can be written in terms of deformed numerators: $N_s^\gamma = \frac{1}{2}\left(N^-_{12} N^+_{34} + N^+_{12} N^-_{34} \right)$

$$N^\pm_{12} = N_s + \frac{i}{\sqrt{2}} \epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4 \left(\omega^2 + k_s^2 \right) \pm \frac{1}{\sqrt{2}} \tilde{V}^{ij}_{12} H_{ij}$$

$$N^\pm_{34} = N_s - \frac{i}{\sqrt{2}} \epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4 \left(\omega^2 + k_s^2 \right) \pm \frac{1}{\sqrt{2}} \tilde{V}^{ij}_{34} H_{ij}$$

Corrections

- The double copy ansatz captures most terms in the 4-graviton wavefunction
- But it has spurious poles, so we add a term to cancel the poles and another to restore the MLT:

$$\psi_\gamma^{(s)} = \psi_{\gamma,\mathrm{DC}}^{(s)} + (\epsilon_1 \cdot \epsilon_2)^2 (\epsilon_3 \cdot \epsilon_4)^2 \left(\psi_{\mathrm{sp}}^{(s)} + \psi_{\mathrm{MLT}}^{(s)}\right)$$

$$\psi_{\mathrm{sp}}^{(s)} = -\frac{1}{2}\left(\frac{2k_1k_2k_3k_4}{(k_{12}+k_{34})^2}\left(\frac{\alpha^2}{k_{34}} + \frac{\beta^2}{k_{12}}\right) + \frac{\alpha^2 k_3 k_4}{k_{34}} + \frac{\beta^2 k_1 k_2}{k_{12}}\right)$$

$$\psi_{\mathrm{MLT}}^{(s)} = \frac{5k_1k_2k_3k_4}{E} + \frac{E}{2}(k_{12}k_{34} - 4k_1k_2 - 4k_3k_4) - \frac{1}{E}(k_1k_2 - k_3k_4)(\alpha^2 - \beta^2) - 3(\alpha^2 k_{12} + \beta^2 k_{34})$$

where $k_{ab} = k_a + k_b$, $\alpha = k_1 - k_2, \quad \beta = k_3 - k_4$

Conclusions I

- SDG in AdS_4 can be described by a simple scalar theory with a deformed Poisson bracket
- Can be derived from asymmetric double copy combining the flat space kinematic algebra with a new deformed kinematic algebra
- Encodes two $w_{1+\infty}$ algebras, one of which is deformed.
- Implies new connection between AdS/CFT and flat space holography!

Conclusions II

- Combining double copy with bootstrap gives a compact new formula for tree-level wavefunction of four gravitons in dS_4

- The double copy ansatz can be written in terms of asymmetric product of deformed numerators

- We do not yet have a systematic understanding of double copy in $(A)dS_4$ but it appears to be useful

Future

- Compute boundary correlators of SDYM and SDG in AdS_4
- Investigate how they encode the double copy and $w_{1+\infty}$
- Derive the double copy in AdS_4 by expanding around self-dual sector
- Translate double copy to Mellin space or differential notation
- 5-point graviton wavefunction, loops
- Susy in AdS_4: COT and MLT in supermomentum space?
- Double copy for gravity coupled to massive scalars, boost breaking?