Fractionalization and Emergent Gauge Fields in Quantum Matter
ICTP, December 4-8, 2023

Deconfined Quantum Criticality in J-Q Models

Anders W Sandvik, Boston University

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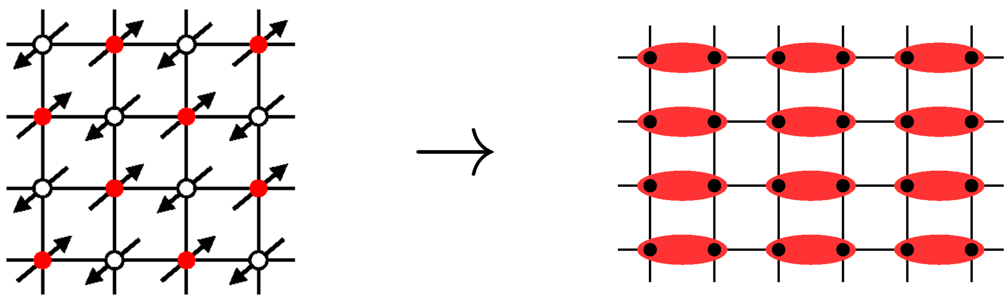

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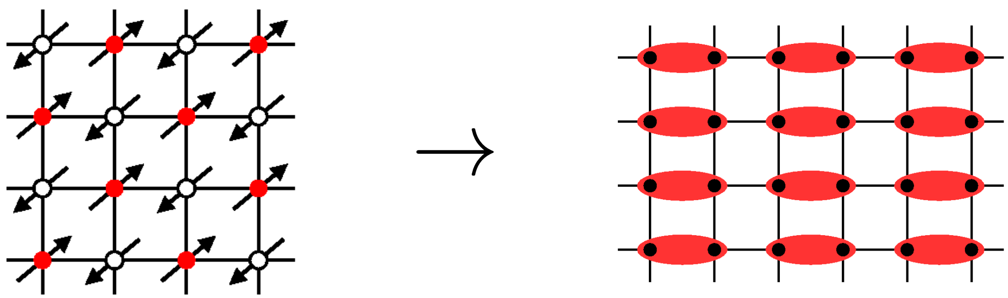

Recent main collaborators

Hui Shao, Wenan Guo, Beijing Normal University
Jon D'Emidio, Donostia International Physics Center, Spain
Jun Takahashi, University of New Mexico
Bowen Zhao, BU →Tencent Ltd

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J-Q Models; Designer Hamiltonians for DQC physics

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Heisenberg exchange = singlet-projector

$$P_{ij} = \frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_j \qquad H_{\text{Heisenberg}} = -J \sum_{\langle ij \rangle} P_{ij}$$

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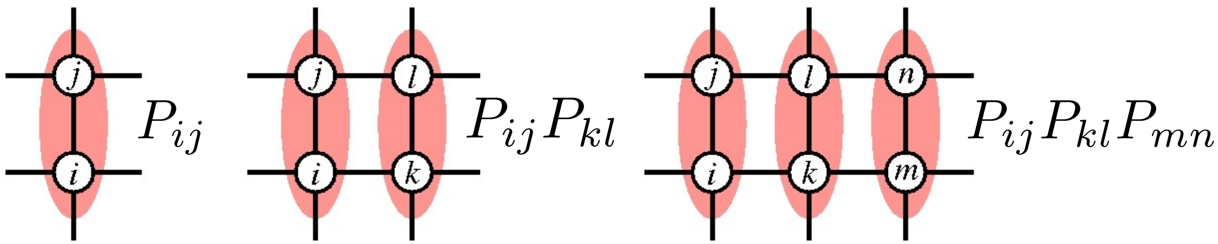

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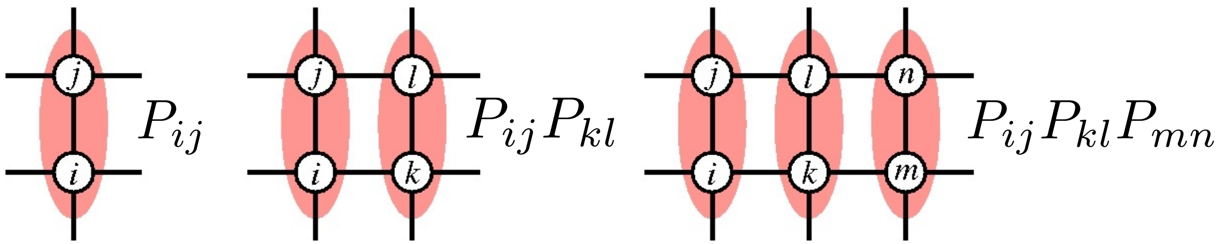

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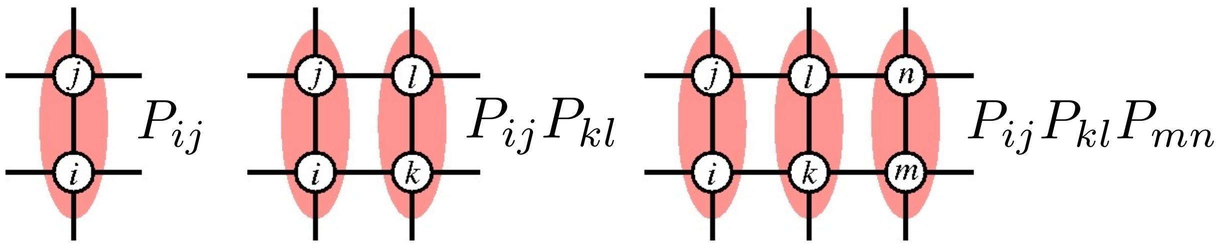

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- correlated singlet projection still competes with antiferromagnetism

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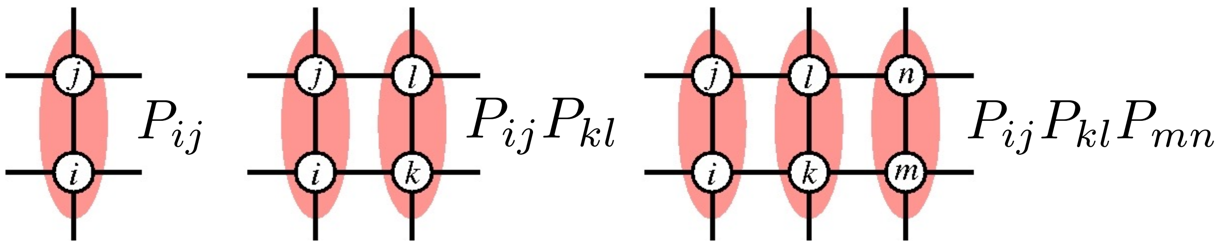

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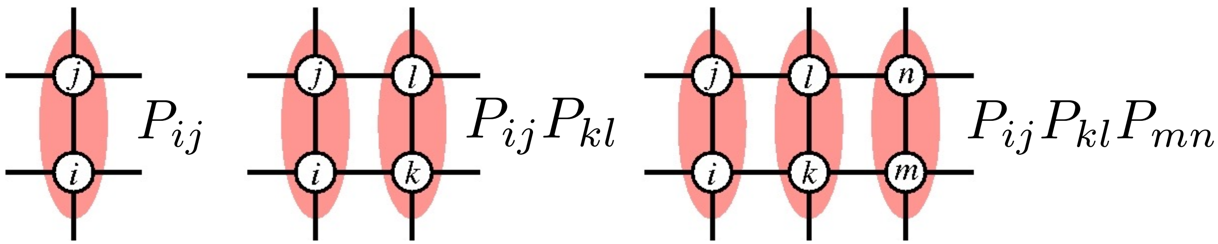

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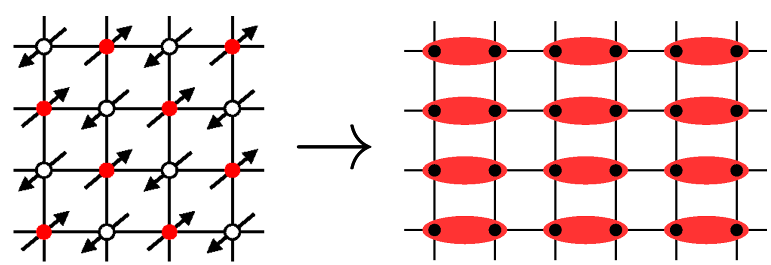

- Has **Néel-VBS transition of ground state**
- Sign-free in QMC simulations large-scale dqc tests possible

Phase transition in the J-Q_2 model

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QMC simulations

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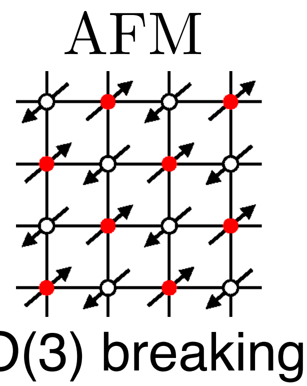

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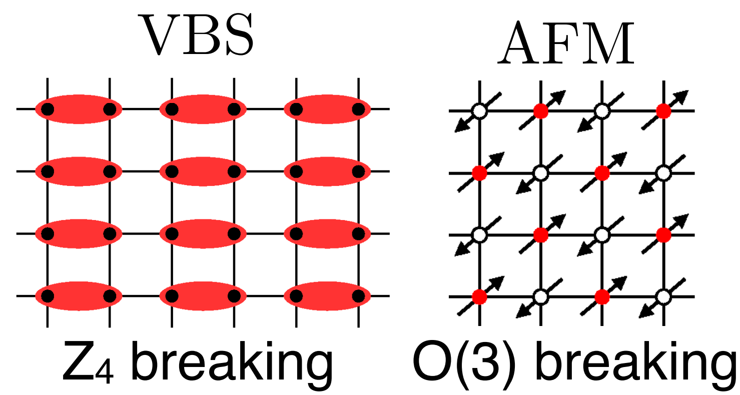

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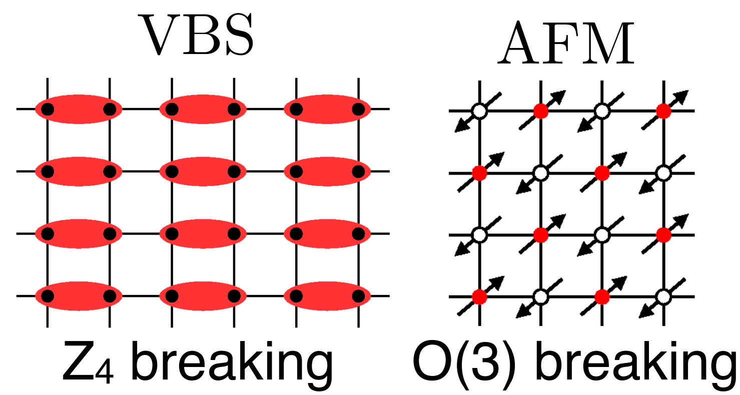

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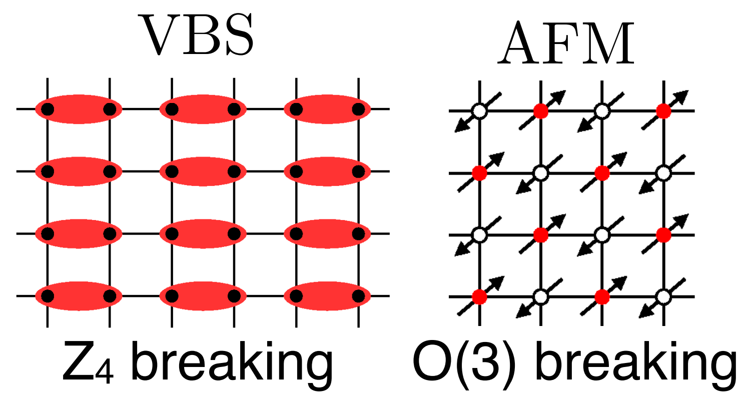

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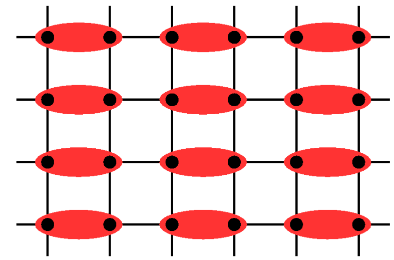
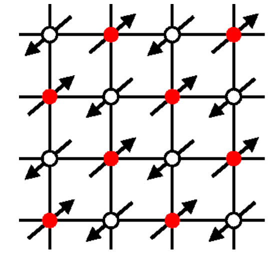


Z_4 breaking

AFM



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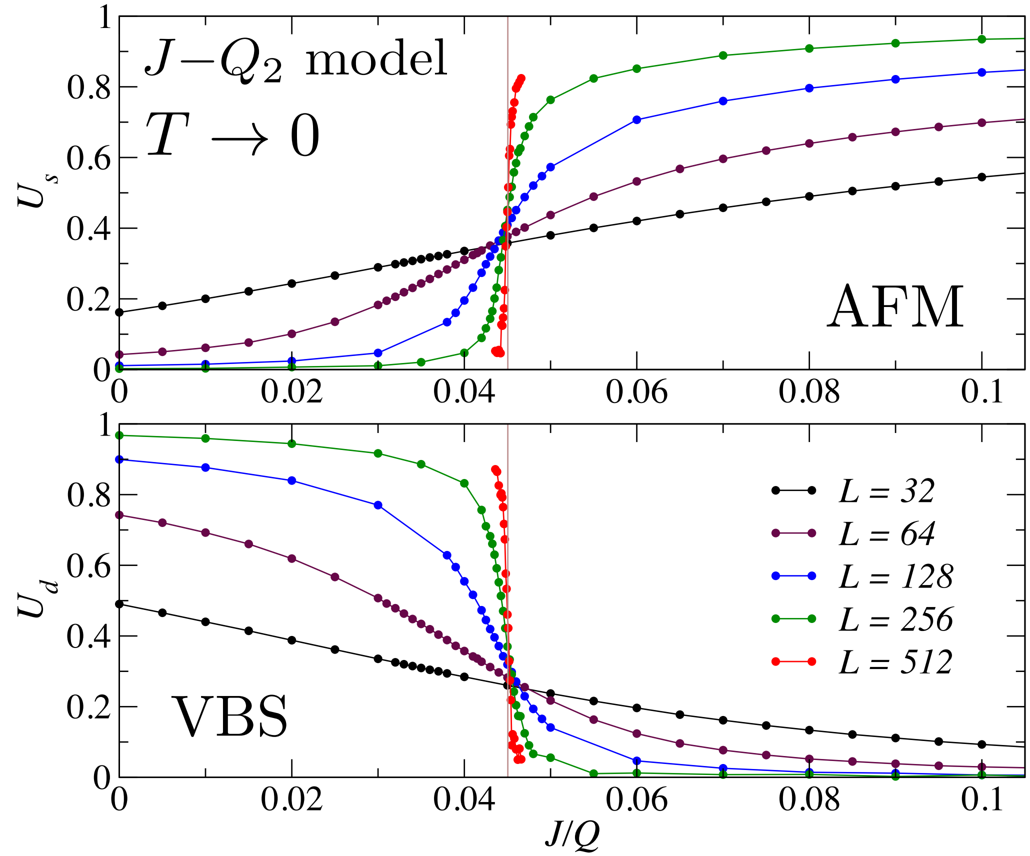

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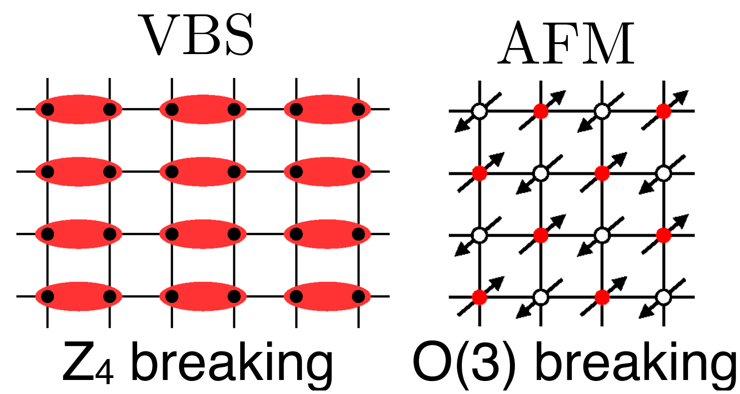

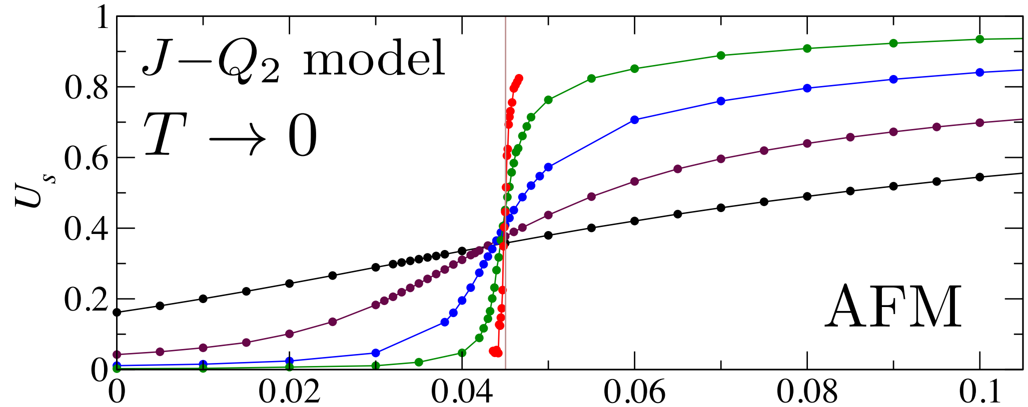

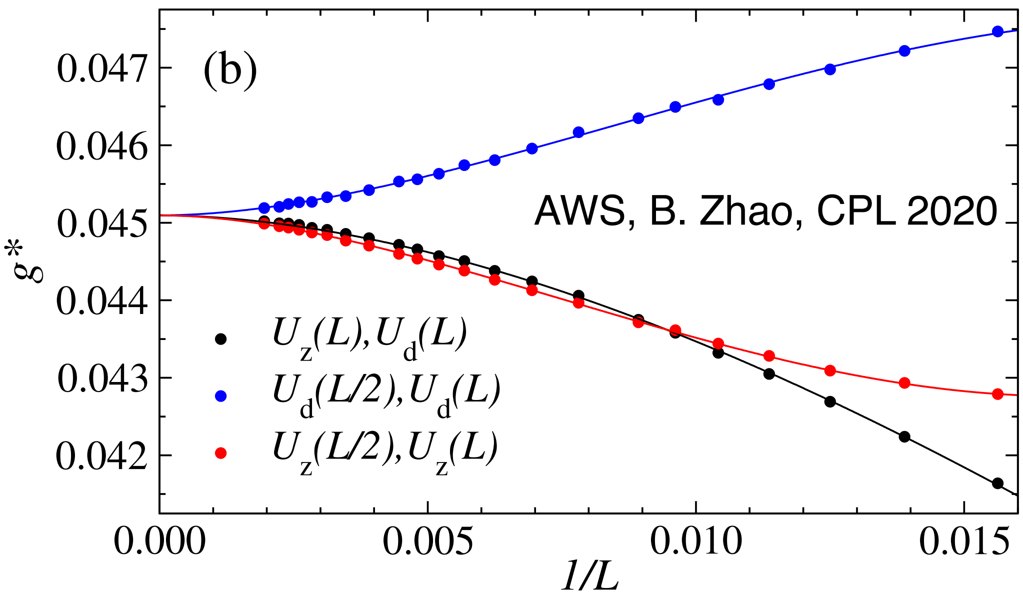

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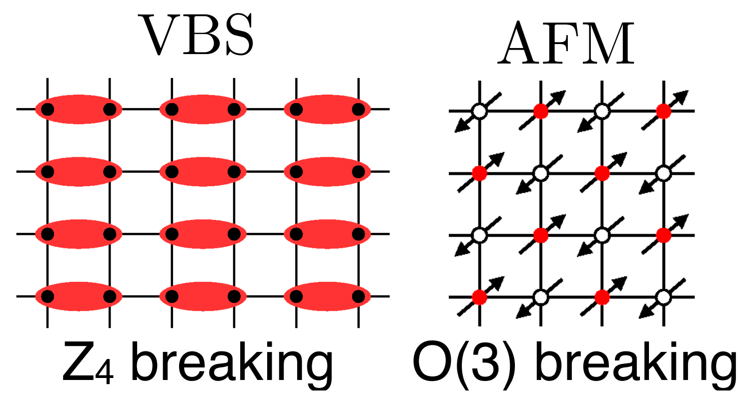

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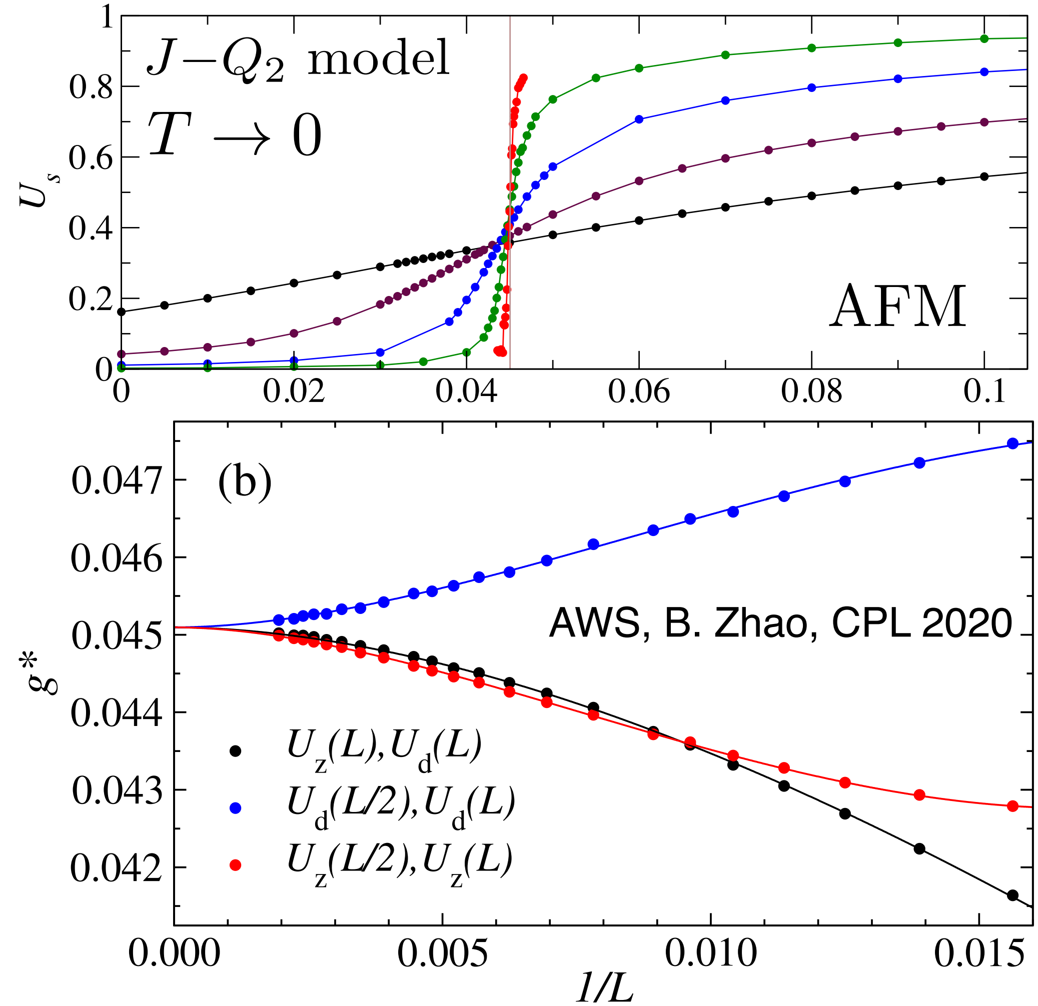

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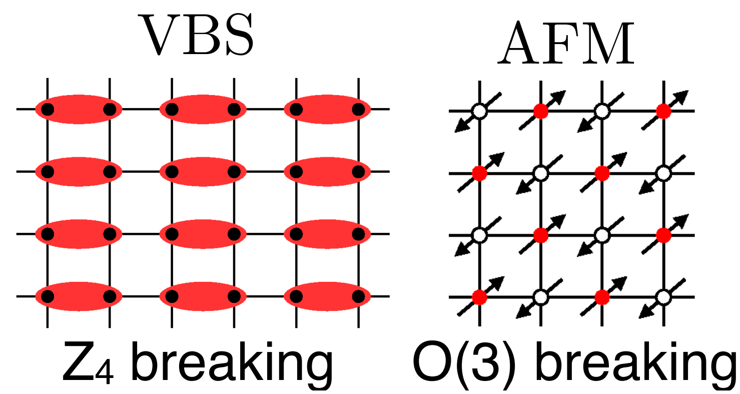

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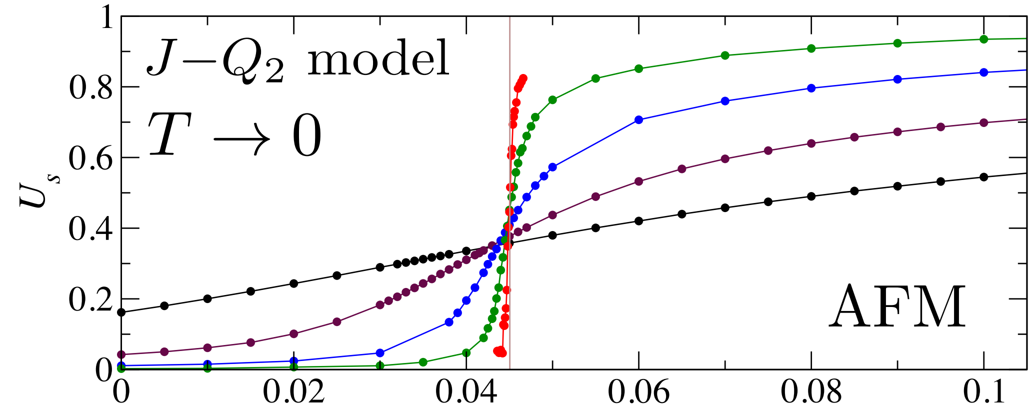

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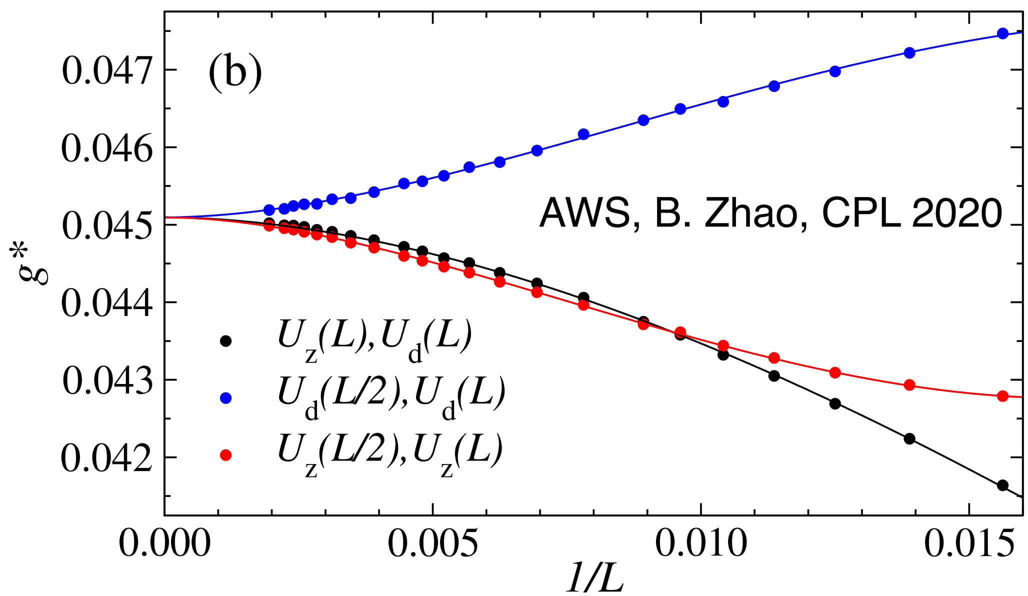

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- signals of first-order transitions have been ambiguous

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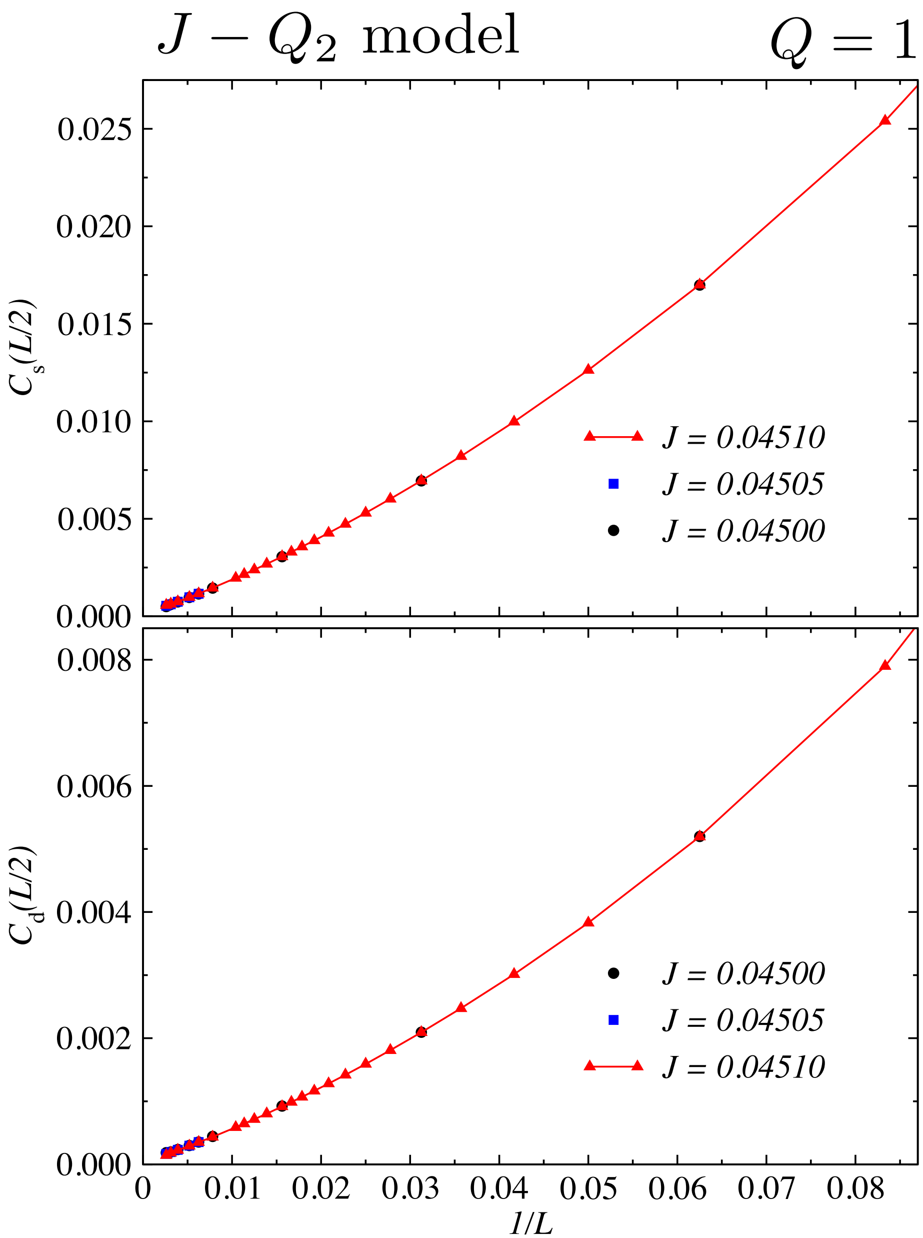

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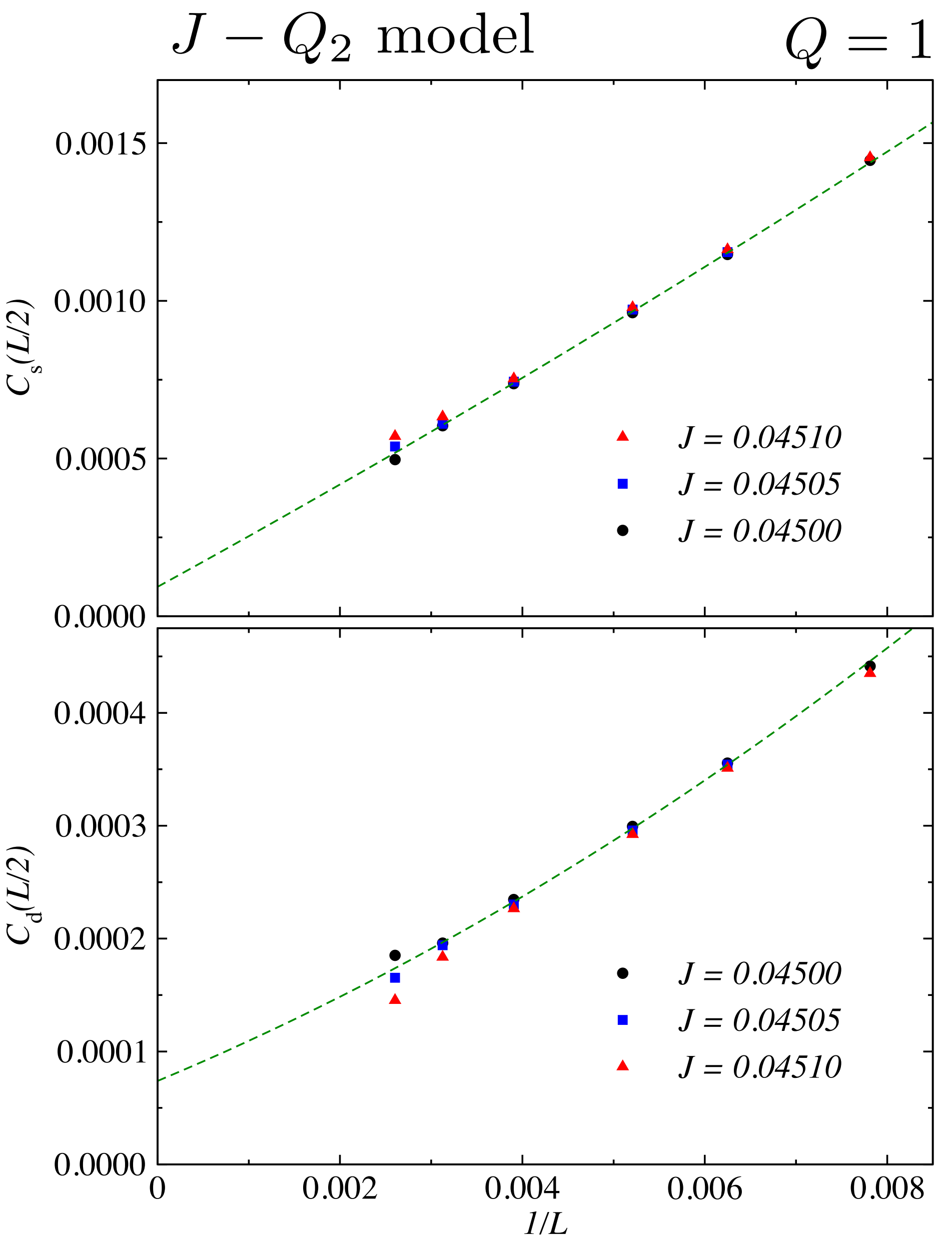

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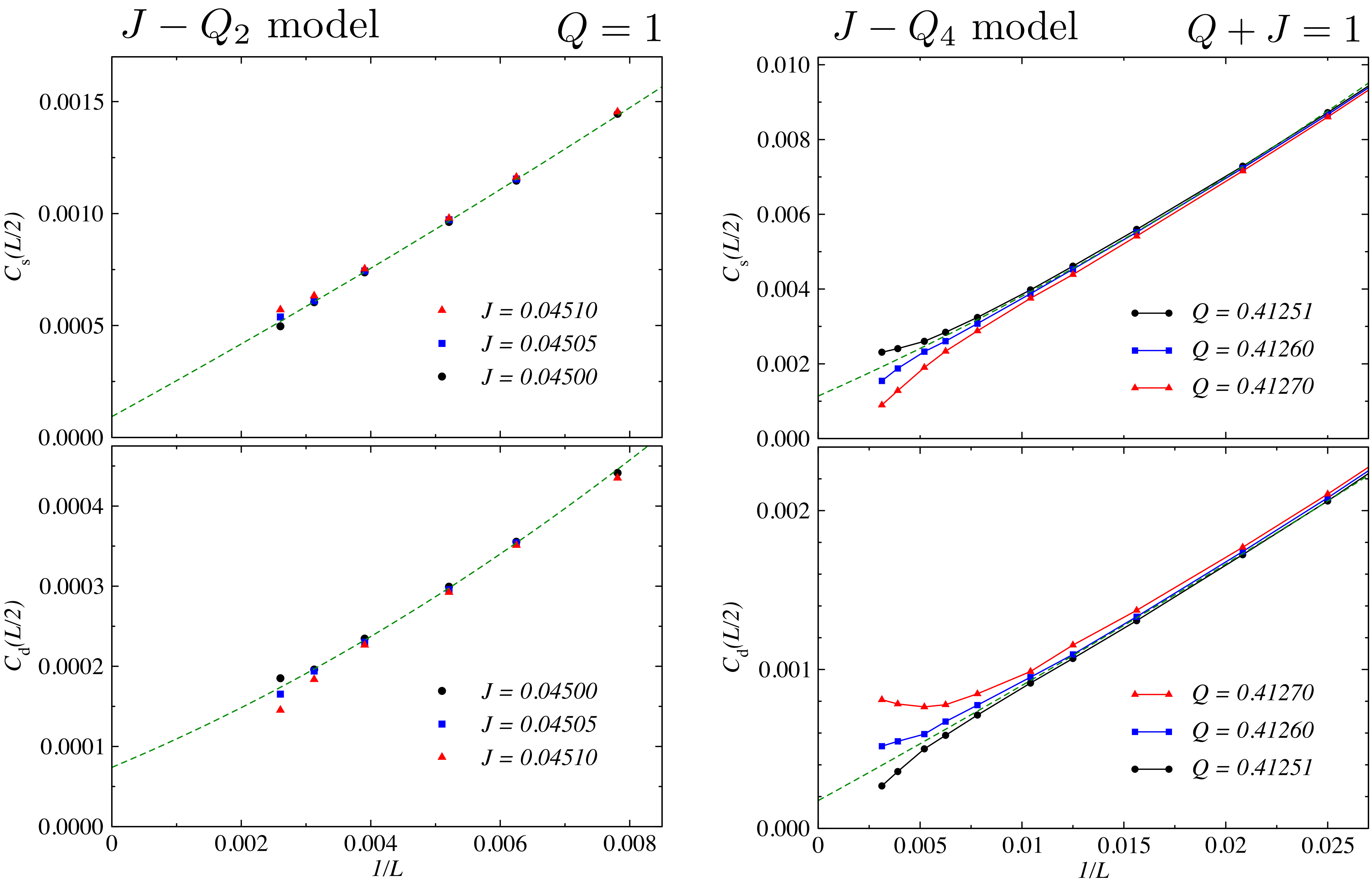

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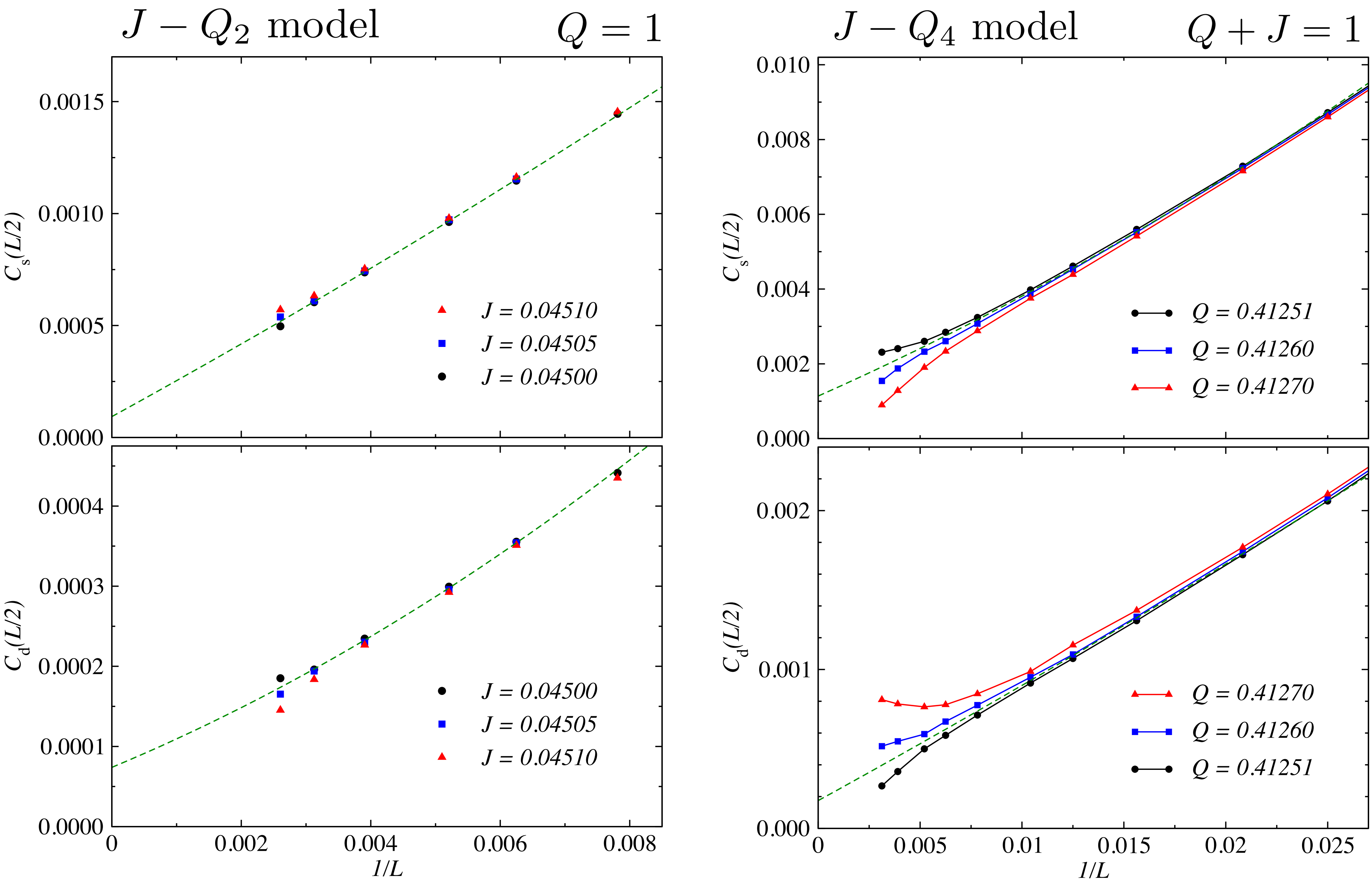

The J-Q_n models have first-order transitions - "pseudo critical" for n=2,3

- discontinuities increase with number of singlet projectors n

Critical scaling of order parameters? - spin and dimer correlations

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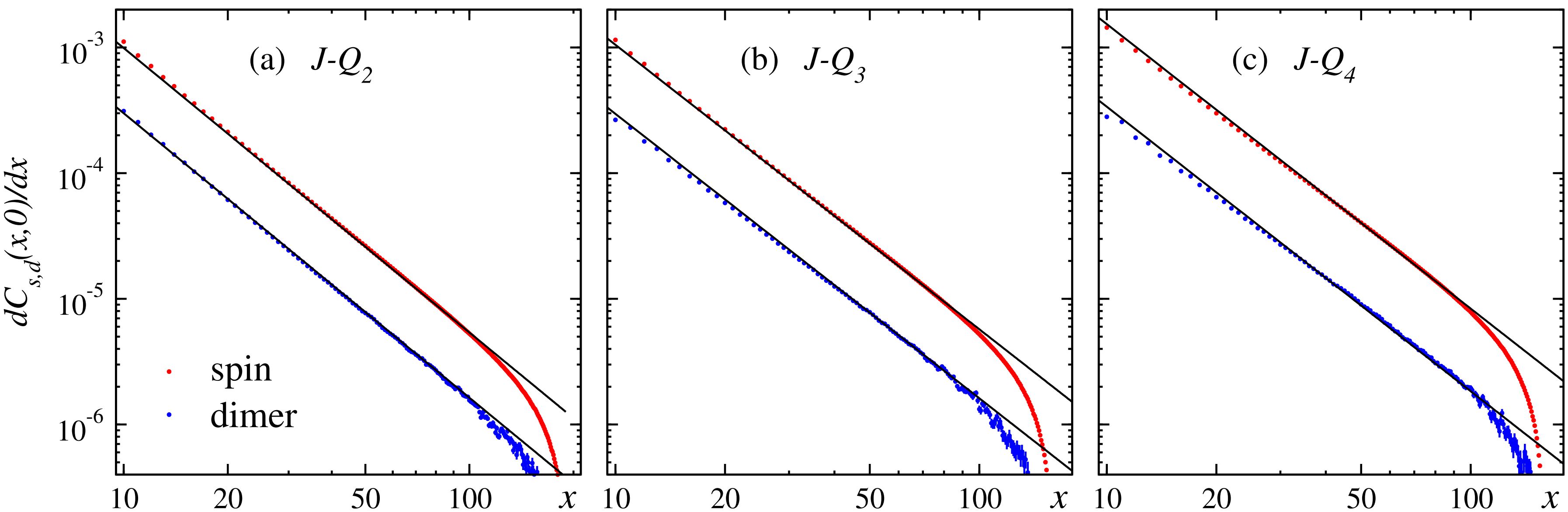

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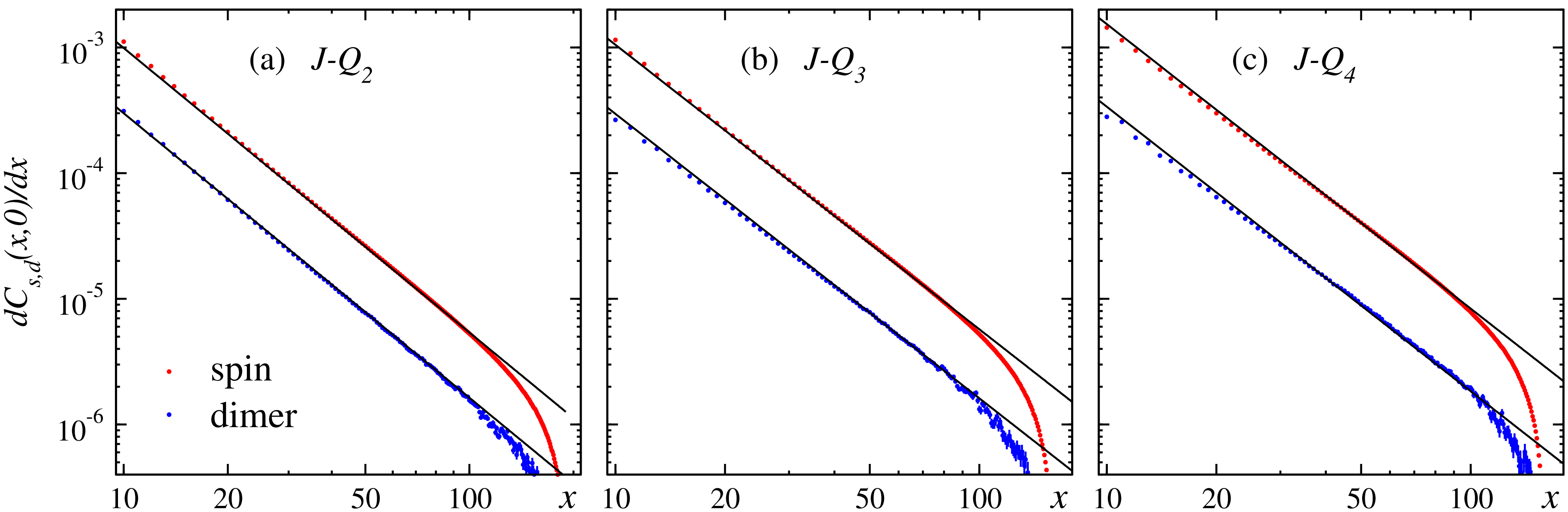

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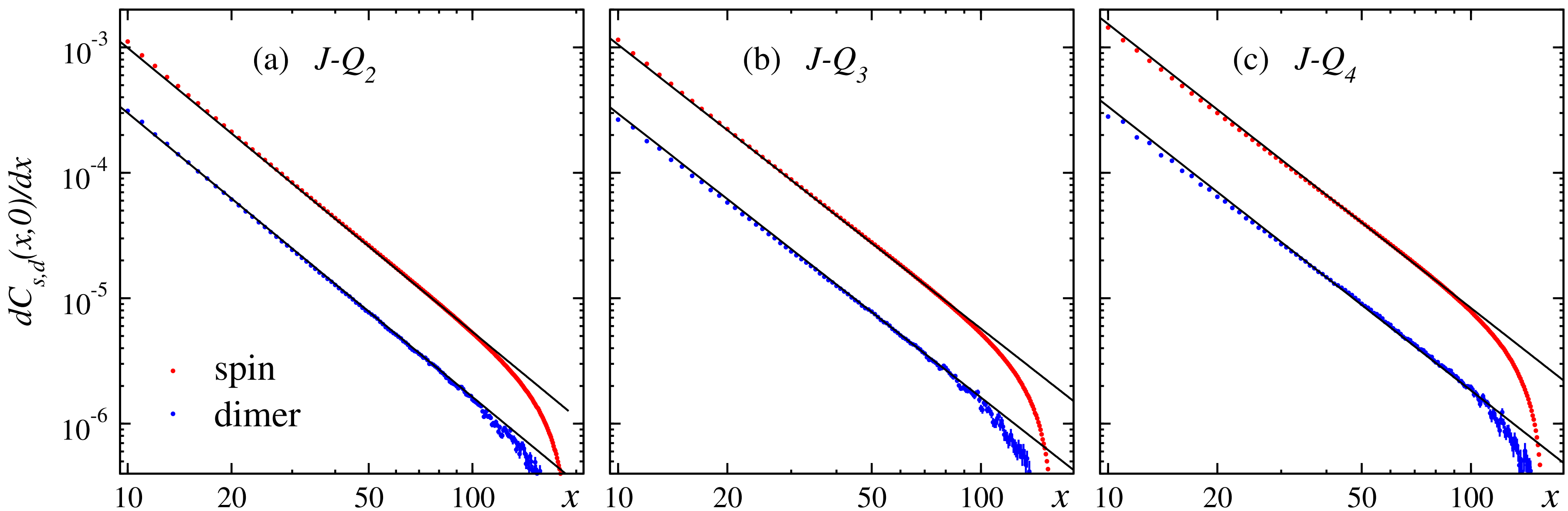

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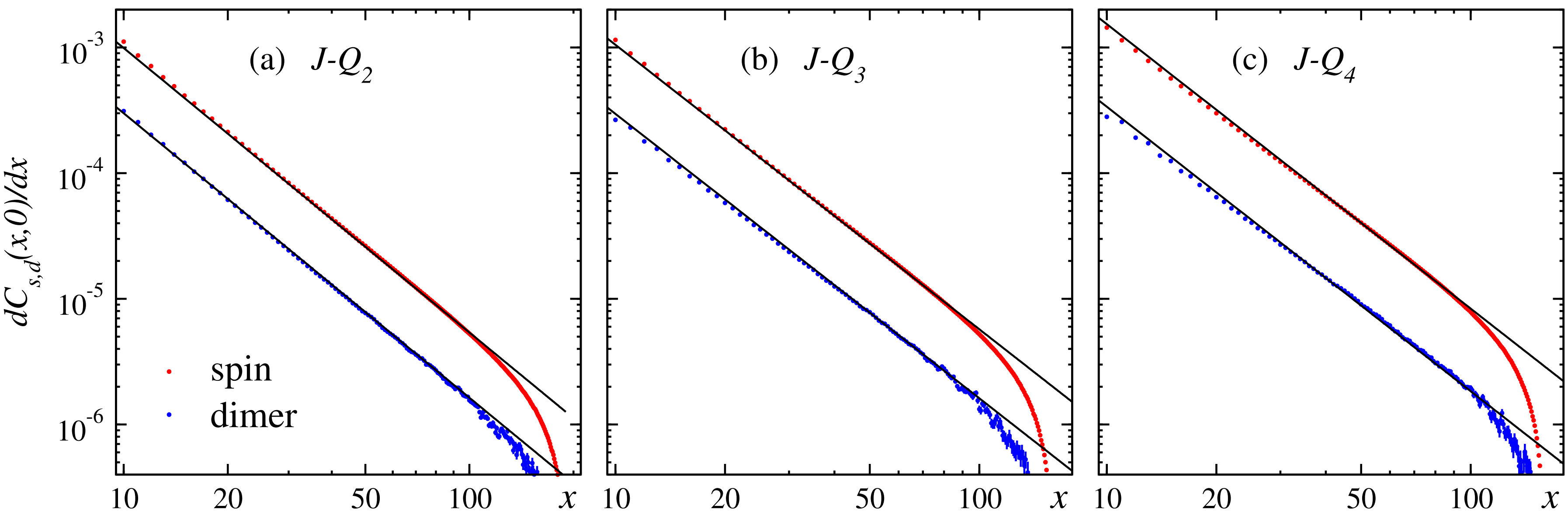

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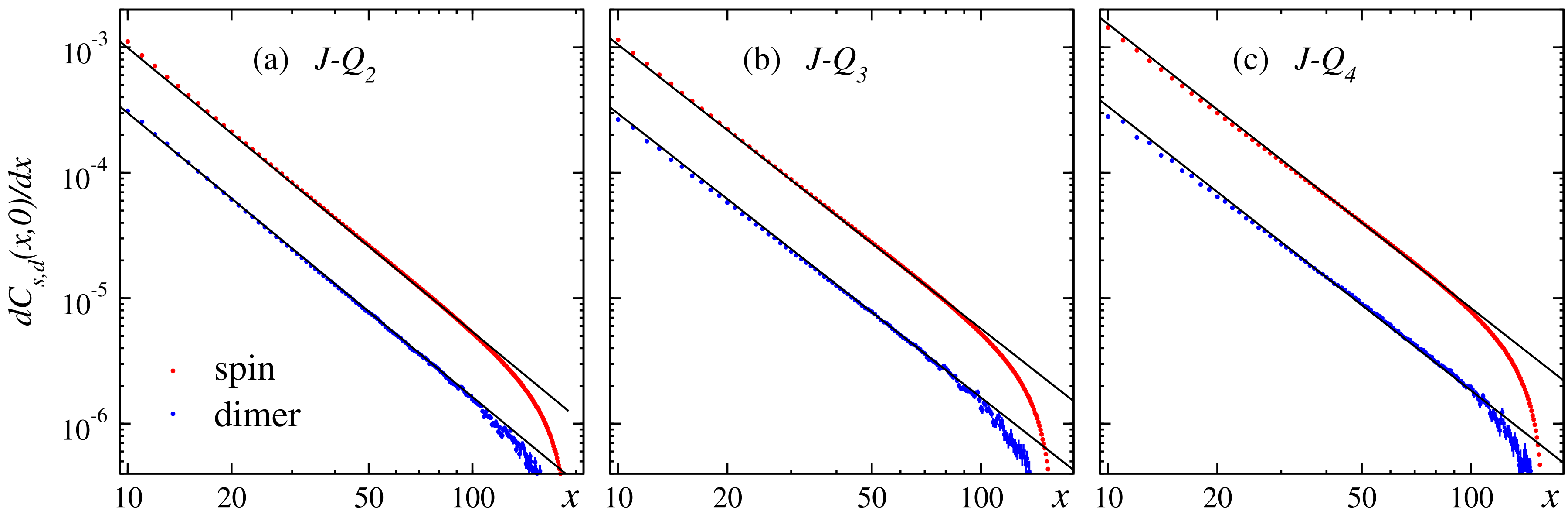

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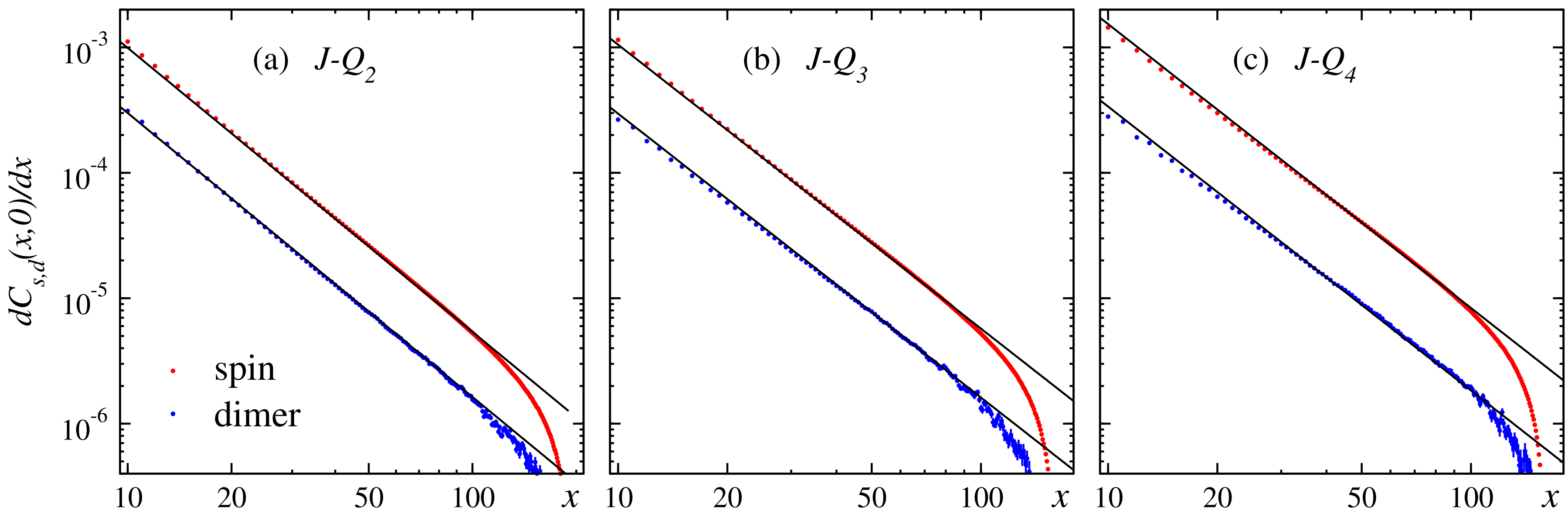

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← Relevant SO(5) singlet

- tuning this operator may correspond to going into 1st-order phase boundary

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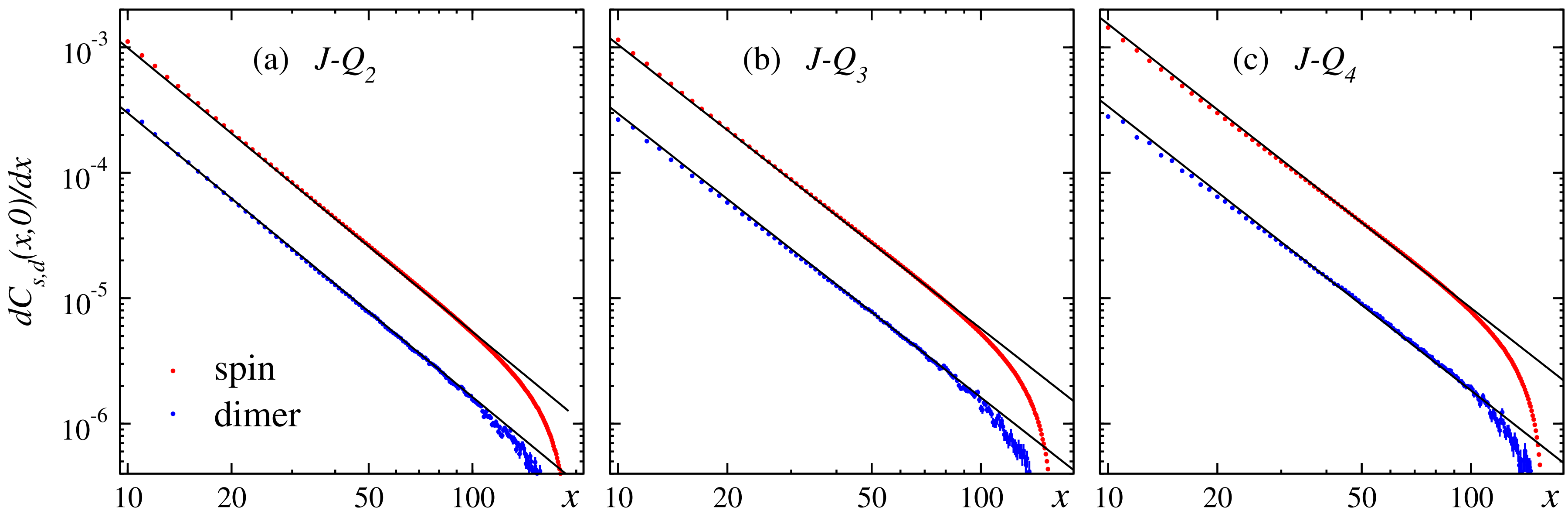

$\rightarrow \Delta_\phi \approx 0.63$ agrees with loop model (but somewhat smaller error bars)

Chester & Su, CFT numerical bootstrap, arXiv:2310.08343

- This scaling dim is consistent with tri-critical SO(5) point

	Δ_v	Δ_t	Δ_{t_3}	Δ_{t_4}	Δ_s
Bootstrap	0.630*	1.519	2.598	3.884	2.359
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Lattice	0.630(3)	1.5	–	–	–
Fuzzy Sphere	0.584	1.454	2.565	3.885	2.845

← Relevant SO(5) singlet (Δ_s)

- tuning this operator may correspond to going into 1st-order phase boundary

Fuzzy sphere: Zhou, Hu, Zhu, He, arXiv:2306.16435

Phase diagram of J-Q_2-Q_6 model

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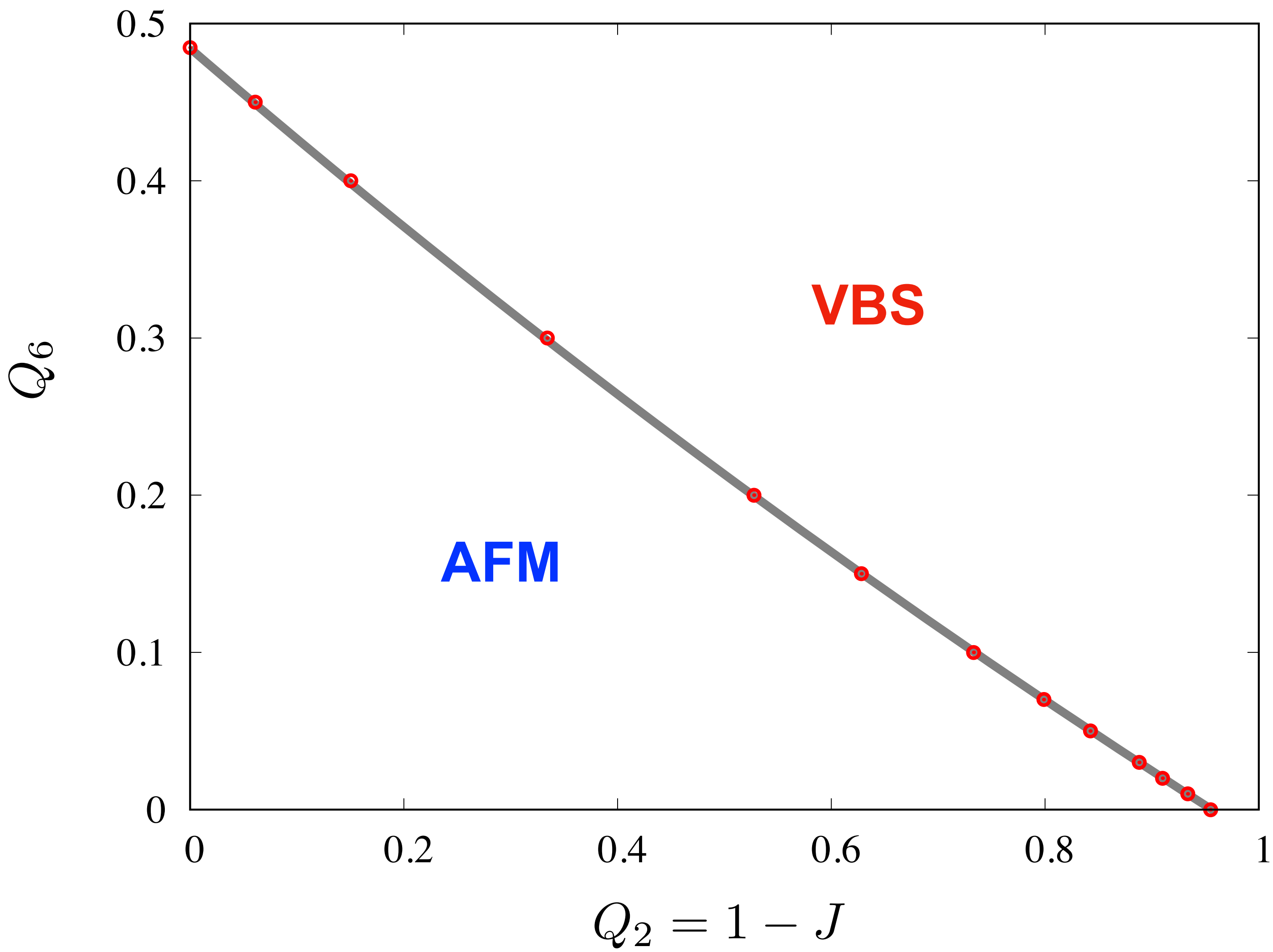

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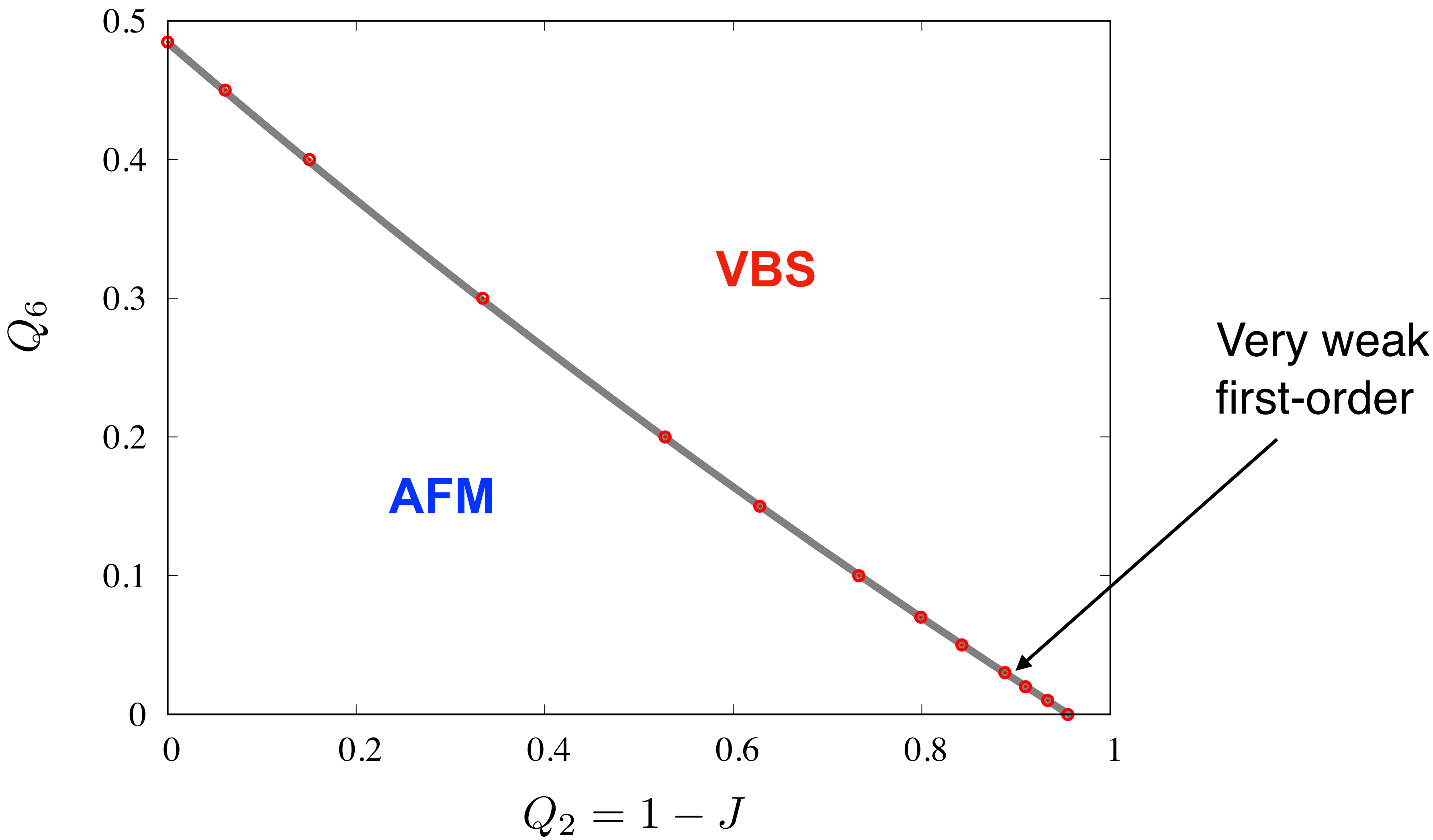

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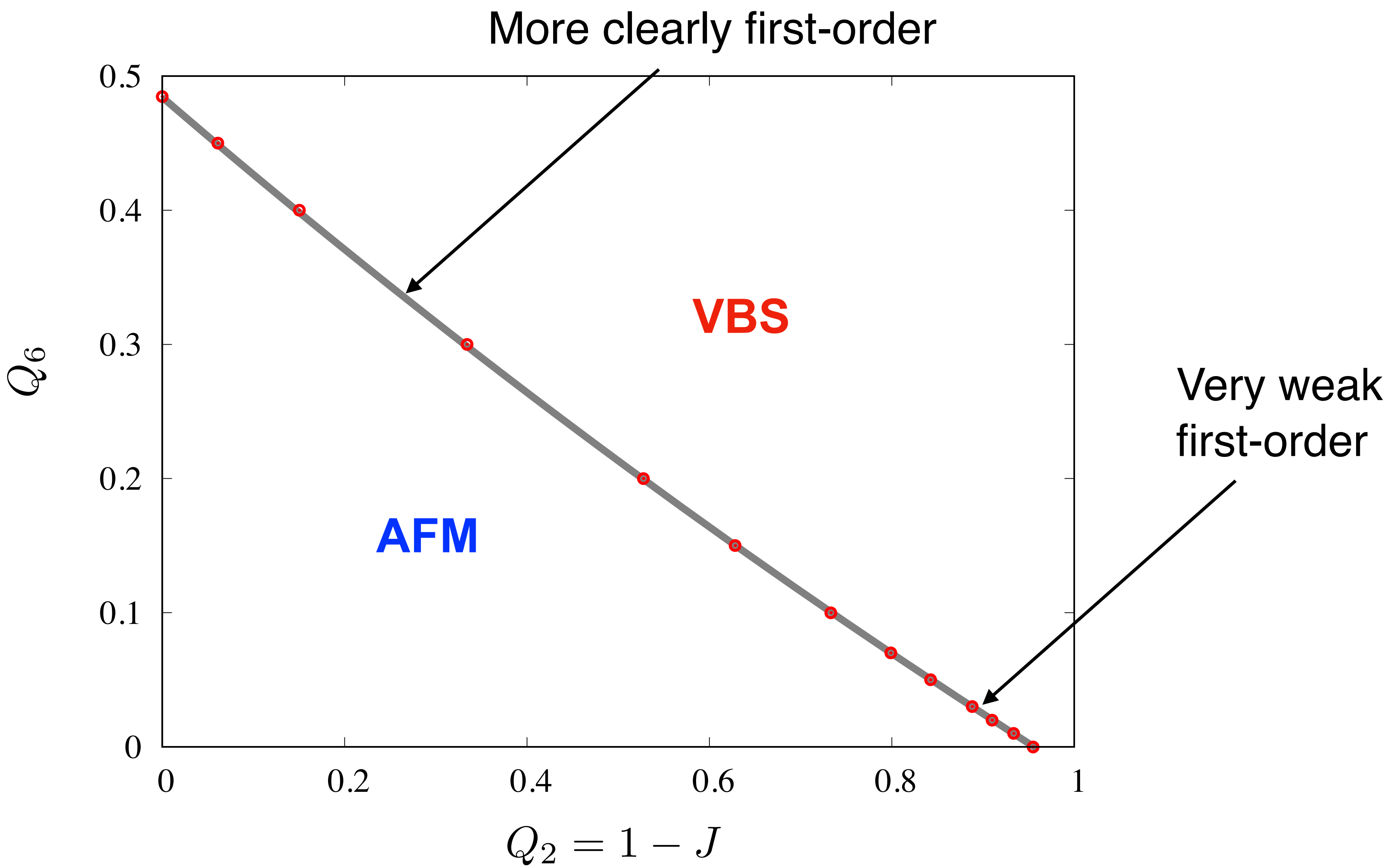

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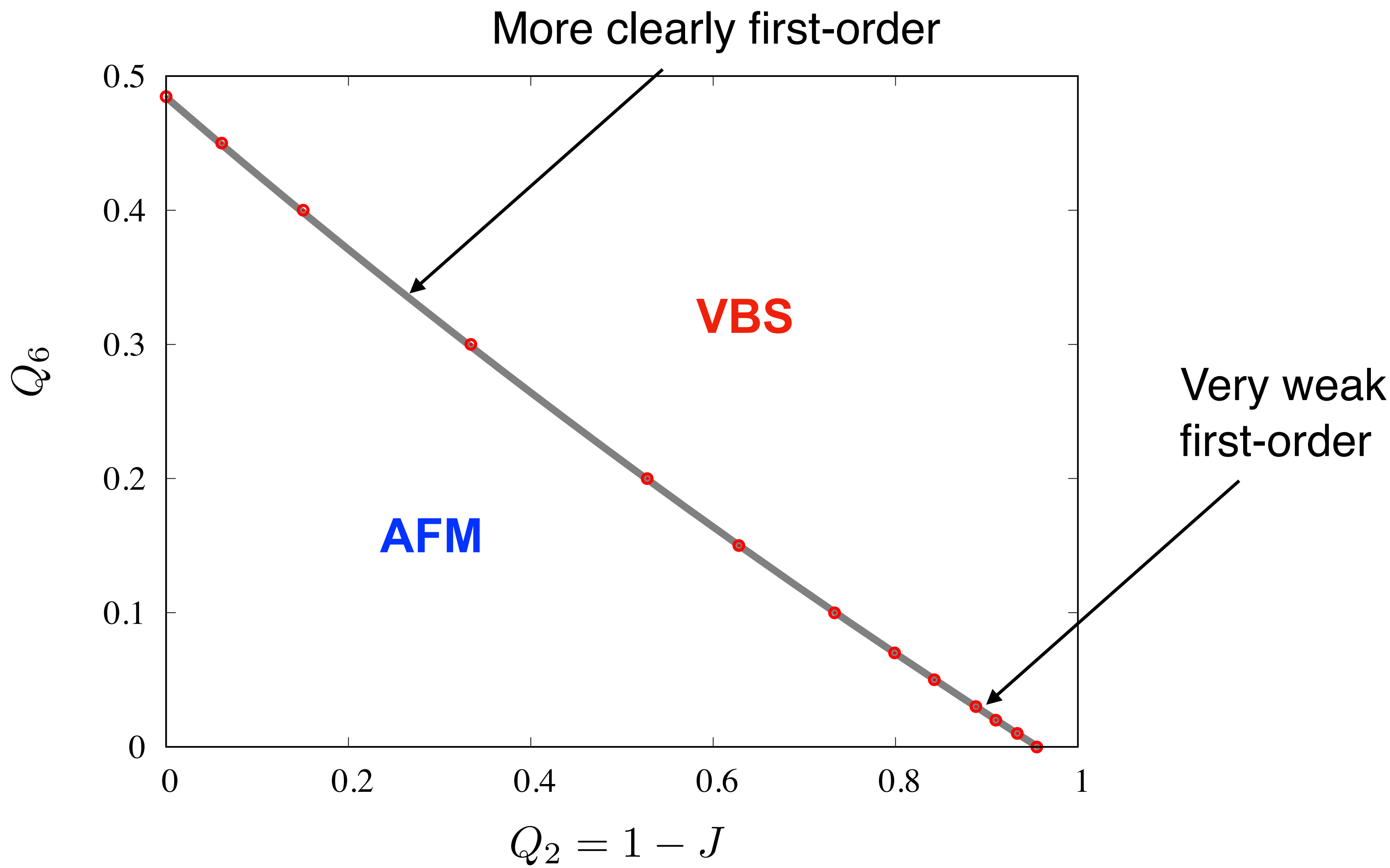

AFM and VBS orders coexist on phase boundary

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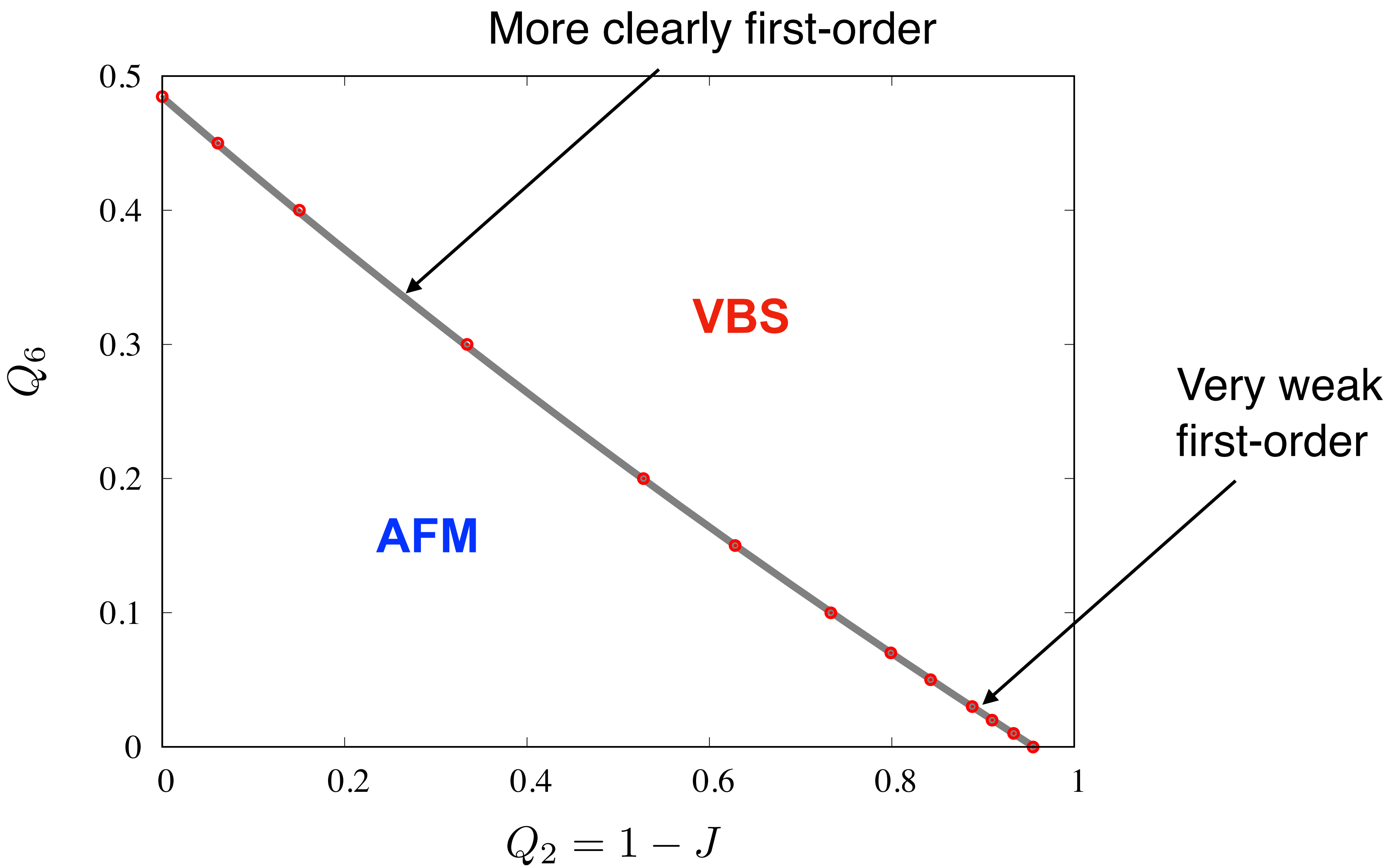

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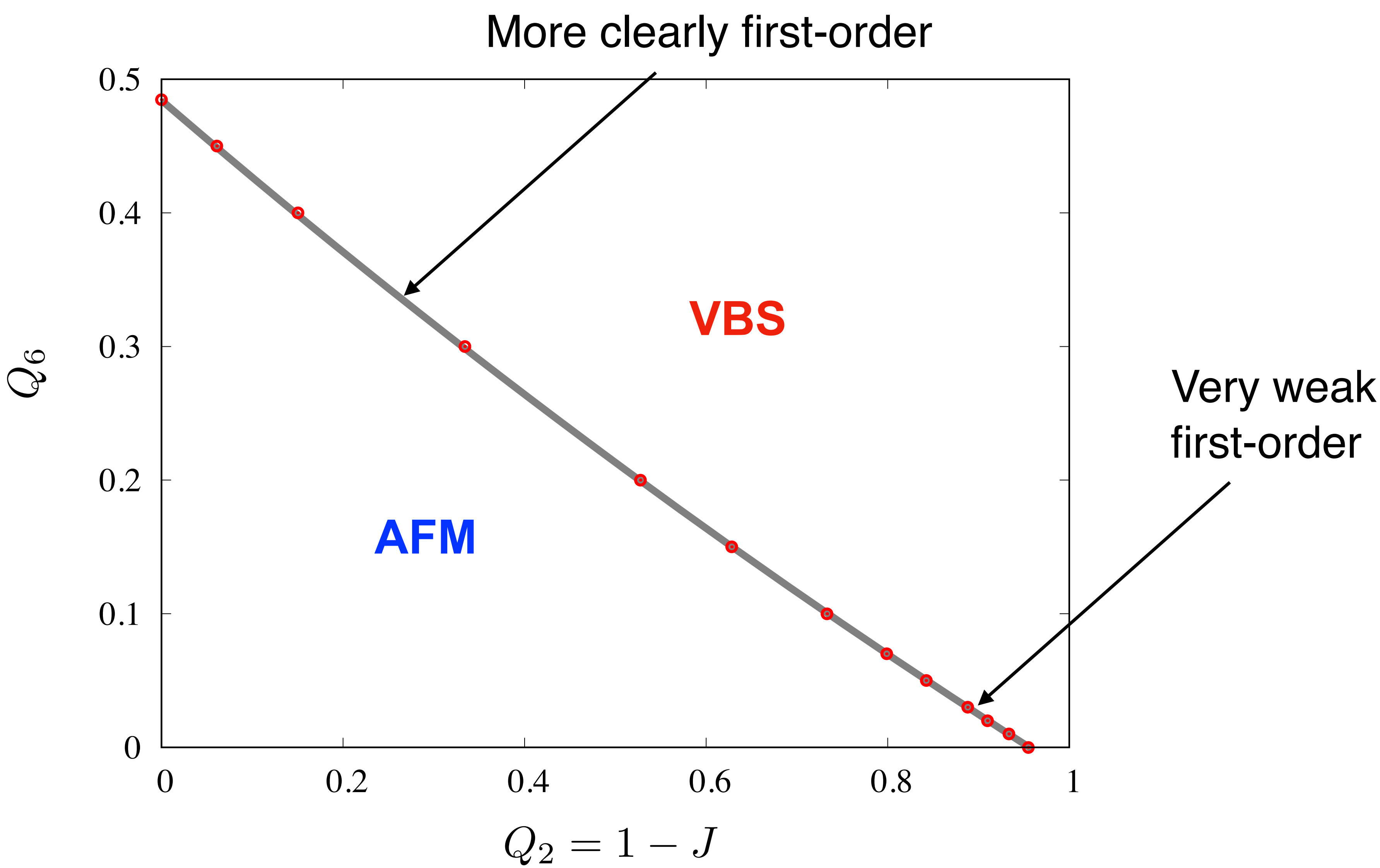

AFM and VBS orders coexist on phase boundary

- should show critical scaling m ~ δ^β in thermodynamic limit
- exponent β that of tri-critical SO(5) point?

L $\to\infty$ extrapolated order parameters on the coexistence curve

- using points where $m^2_{AFM}(L) = m^2_{VBS}(L)$

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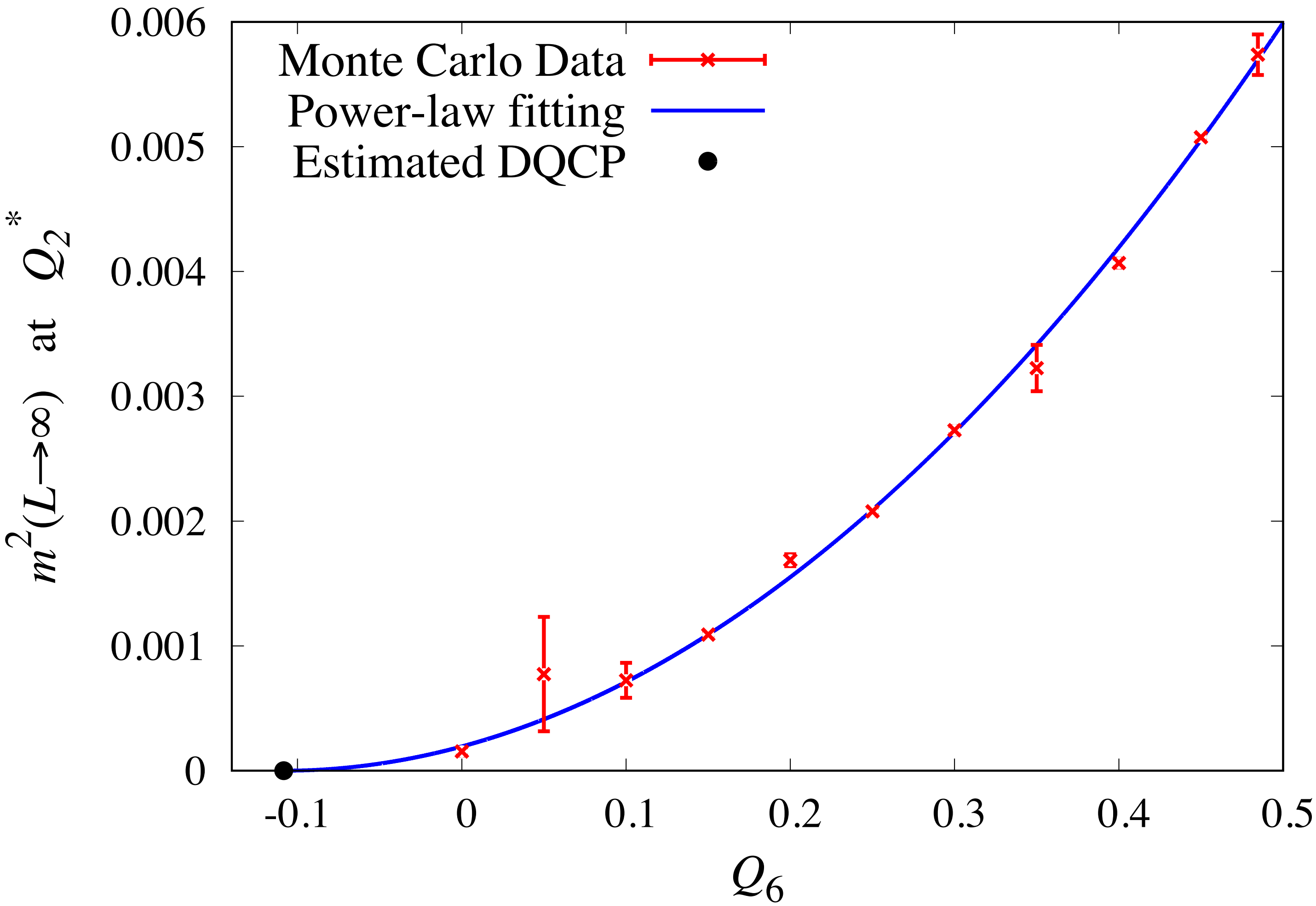

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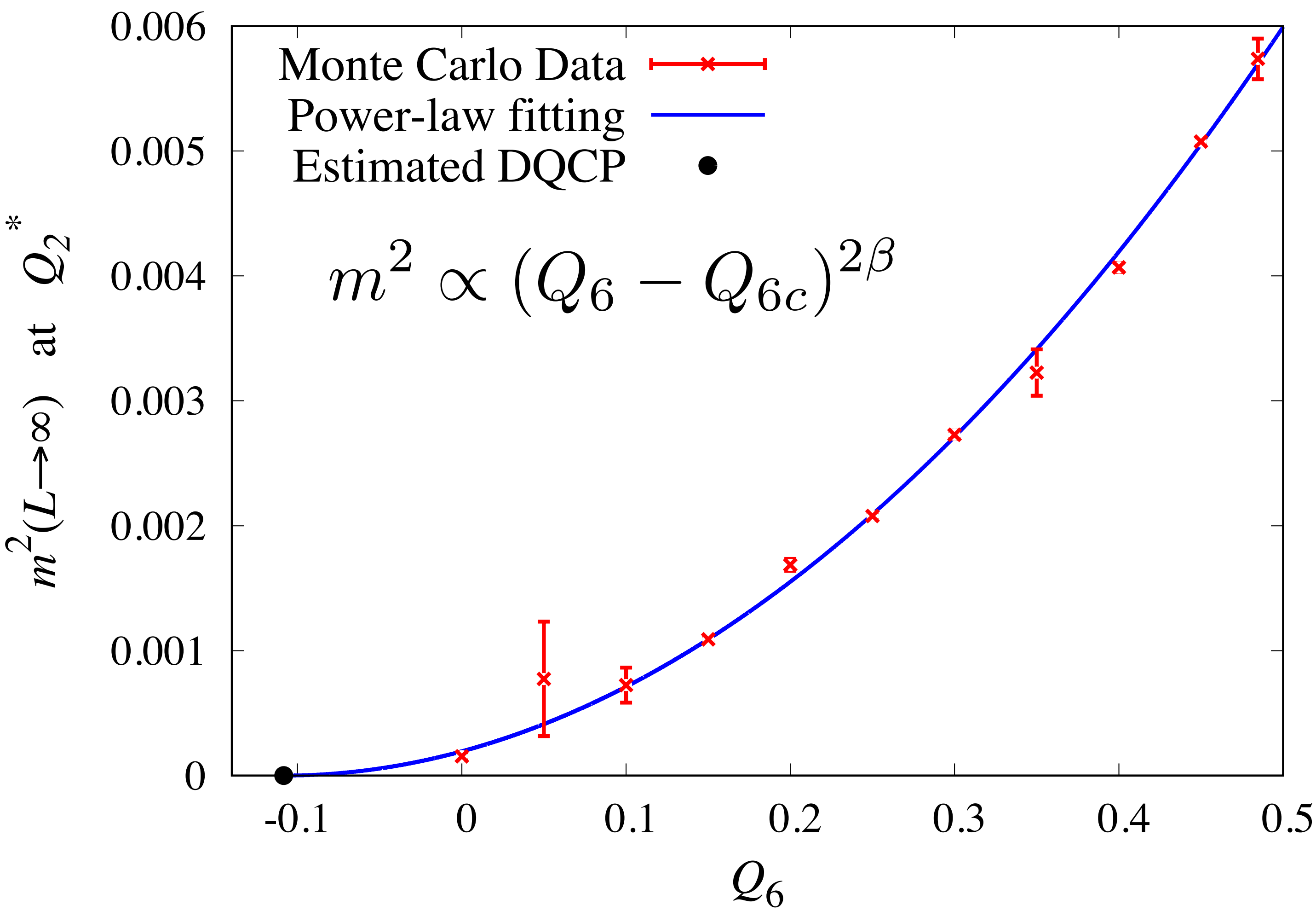

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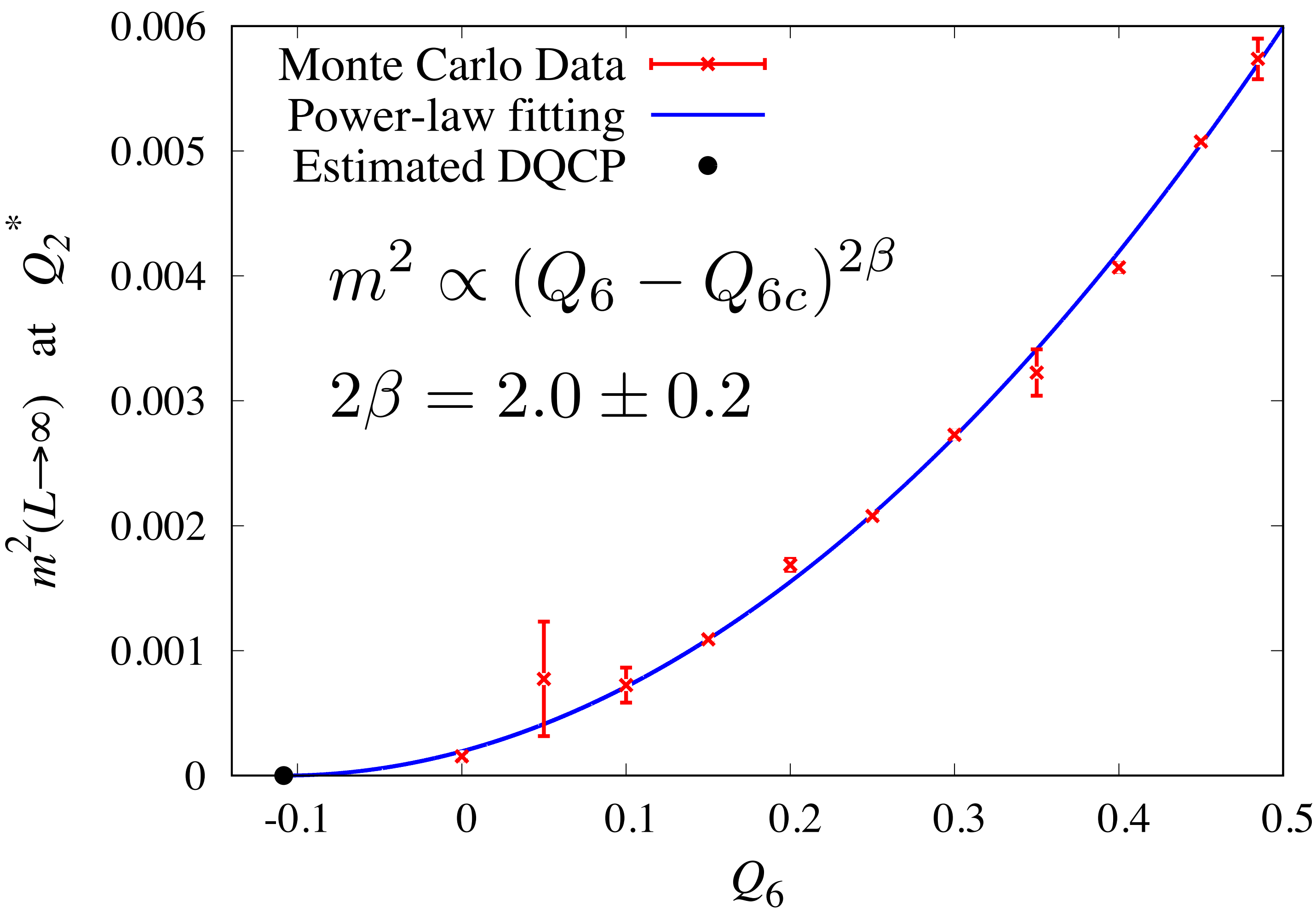

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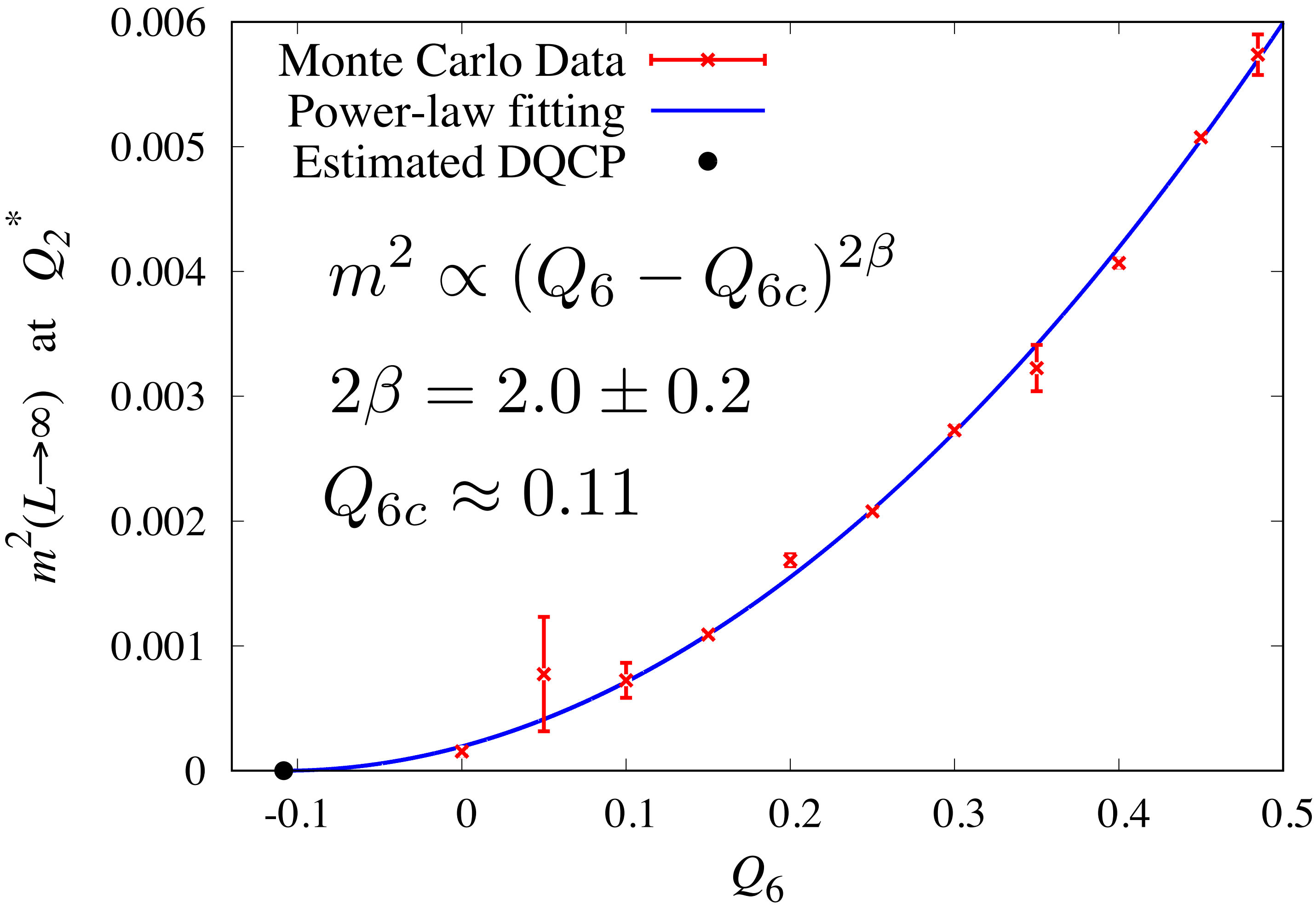

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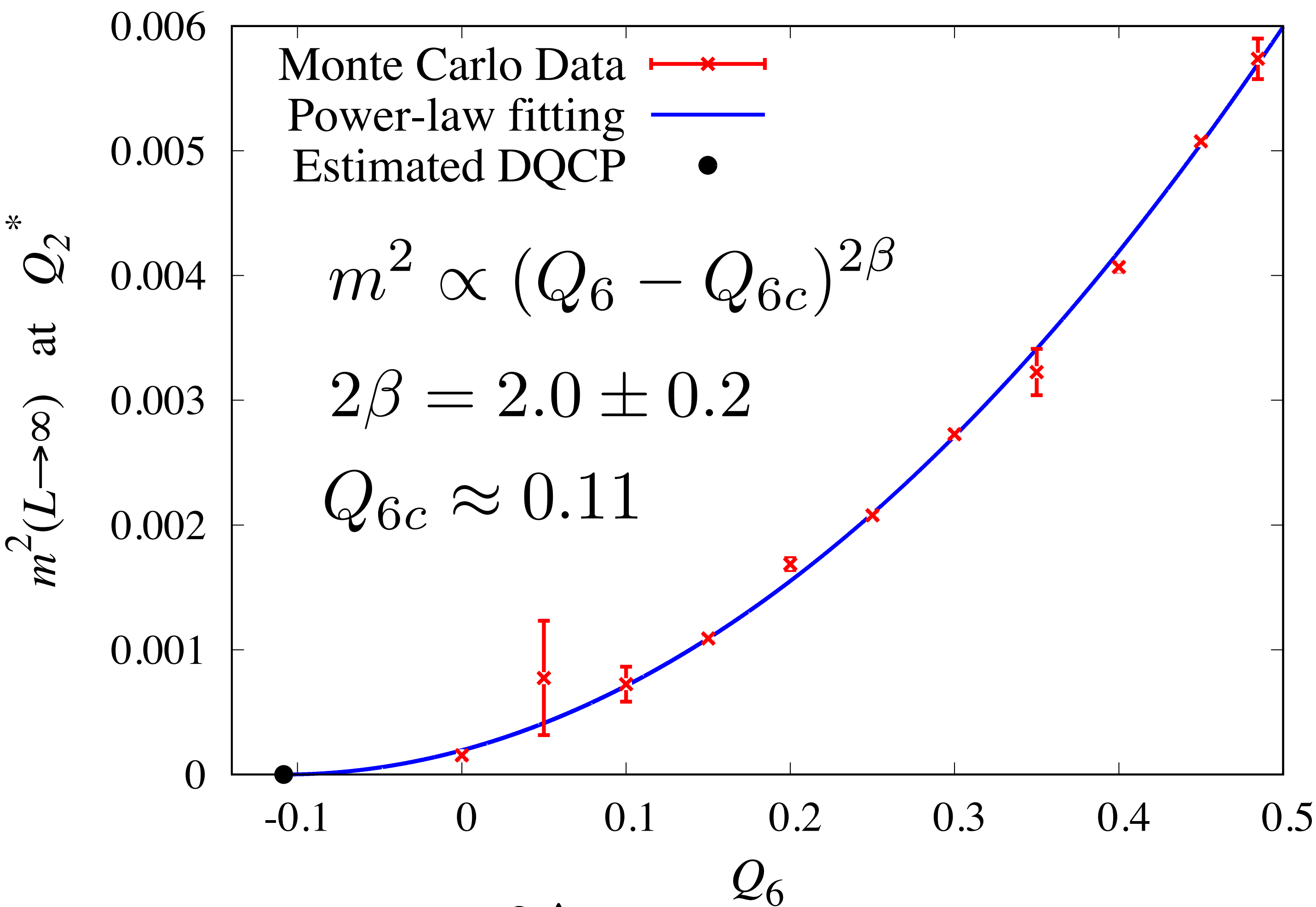

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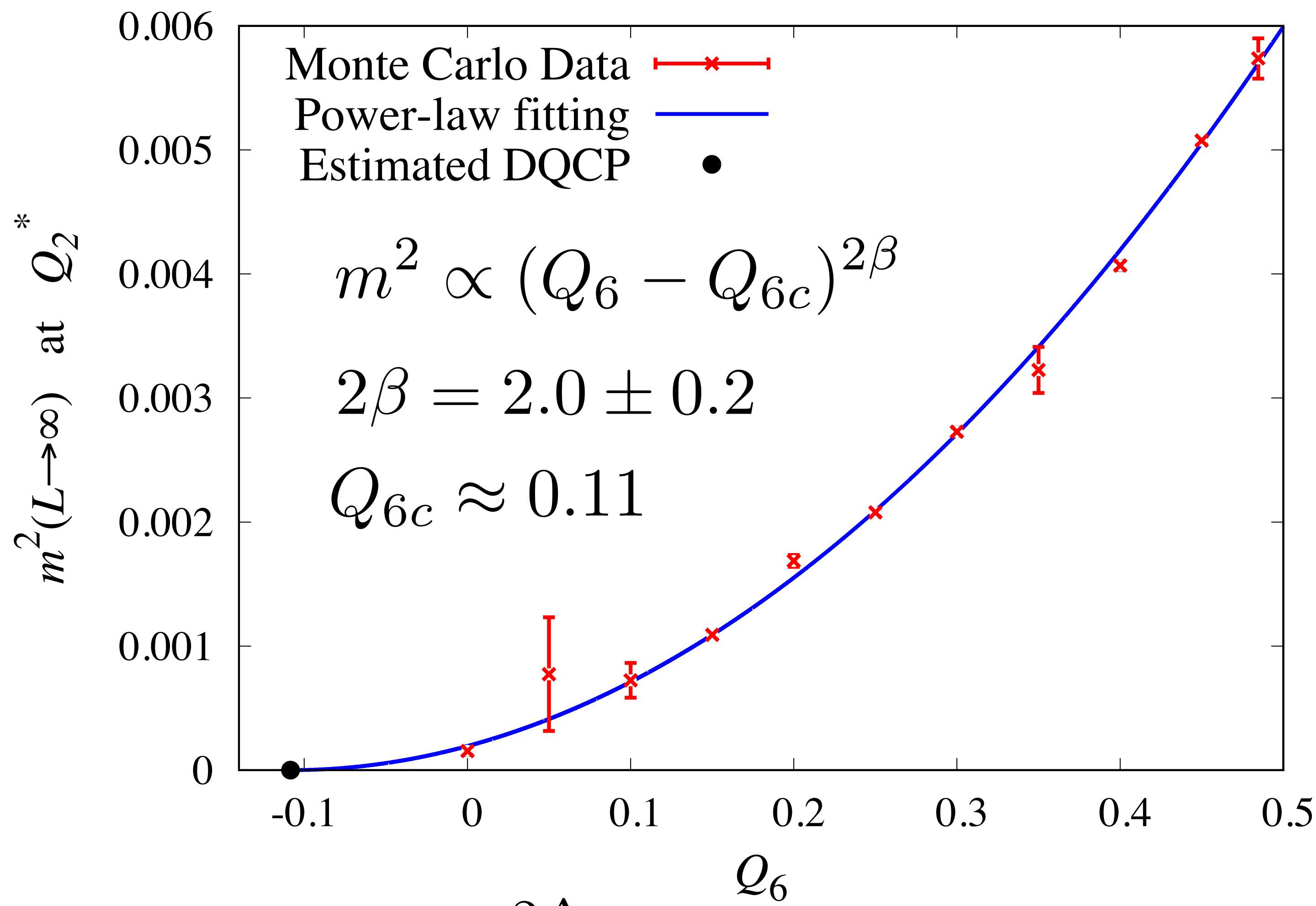

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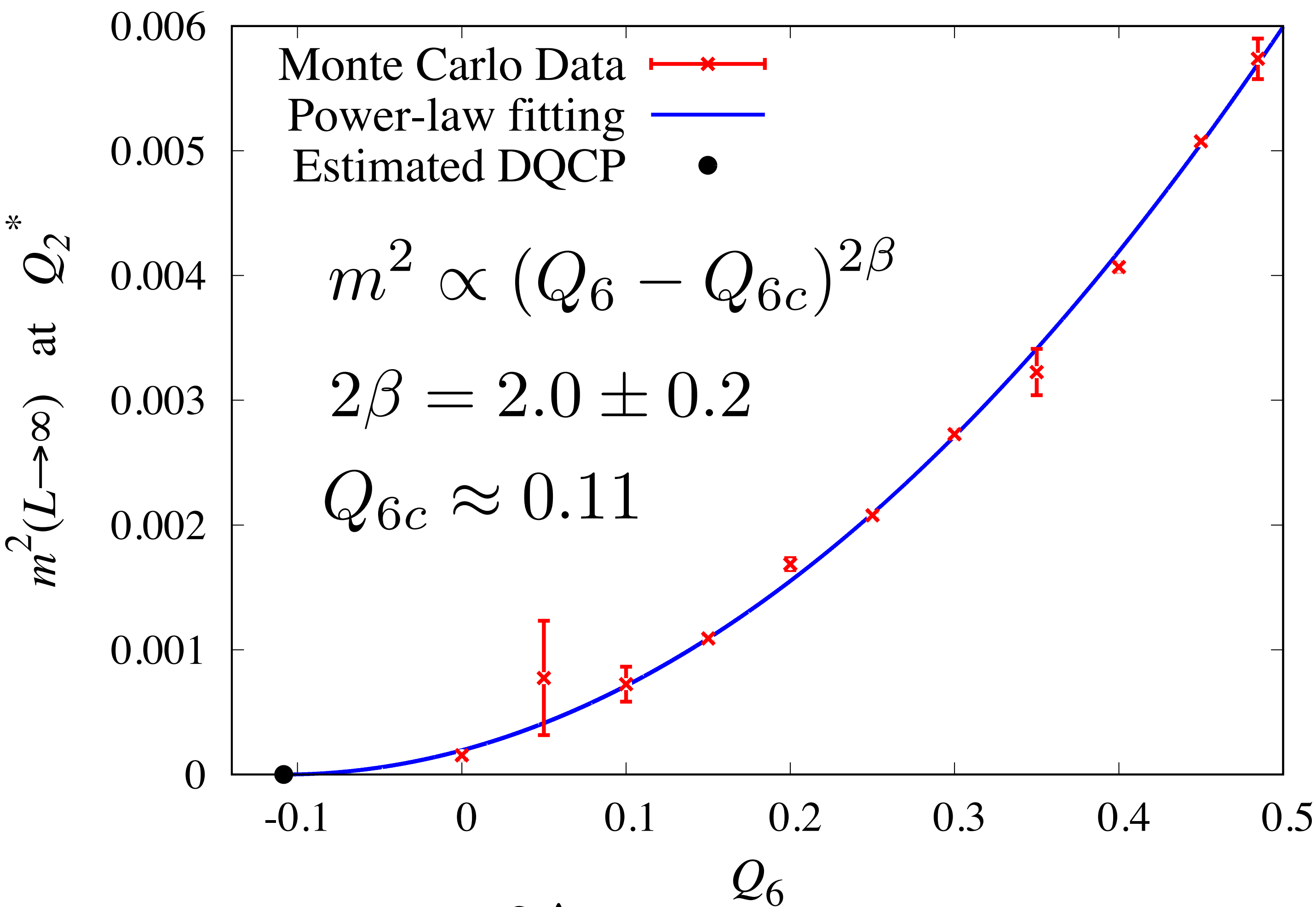

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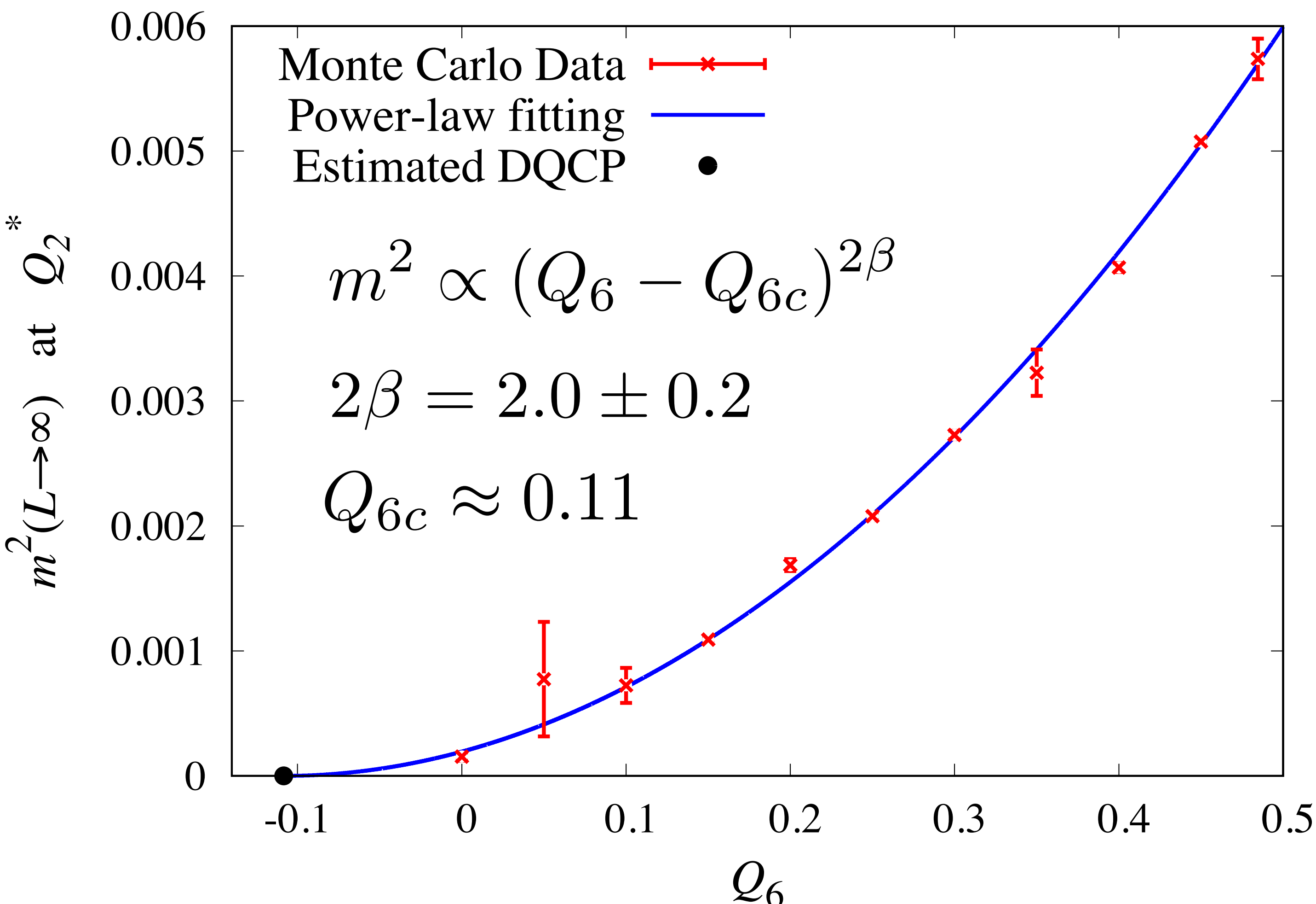

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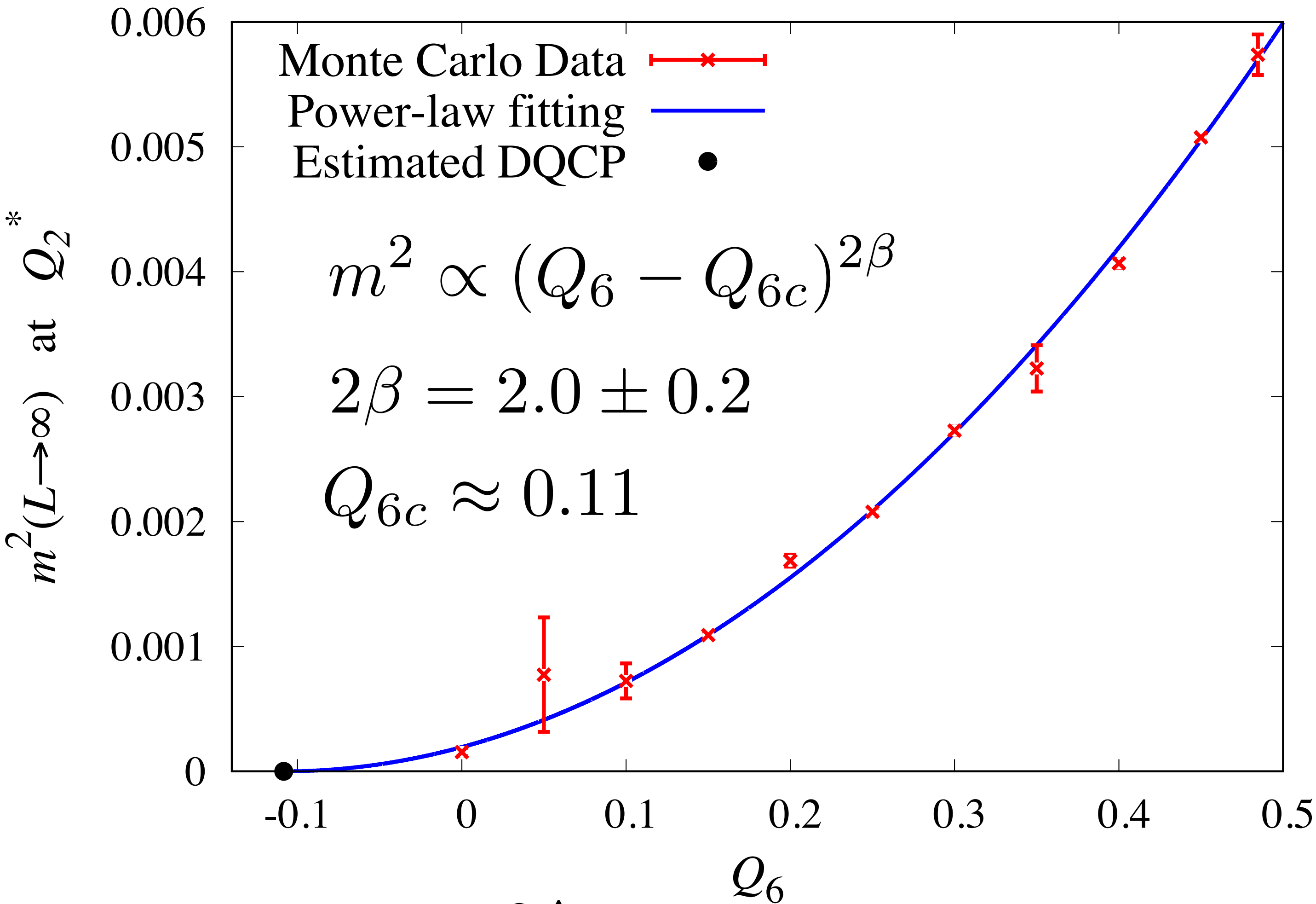

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- $Q_6 = 0 \sim 0.5$ is close enough to extract reliable exponent

Correlation-length exponent, J-Q_2

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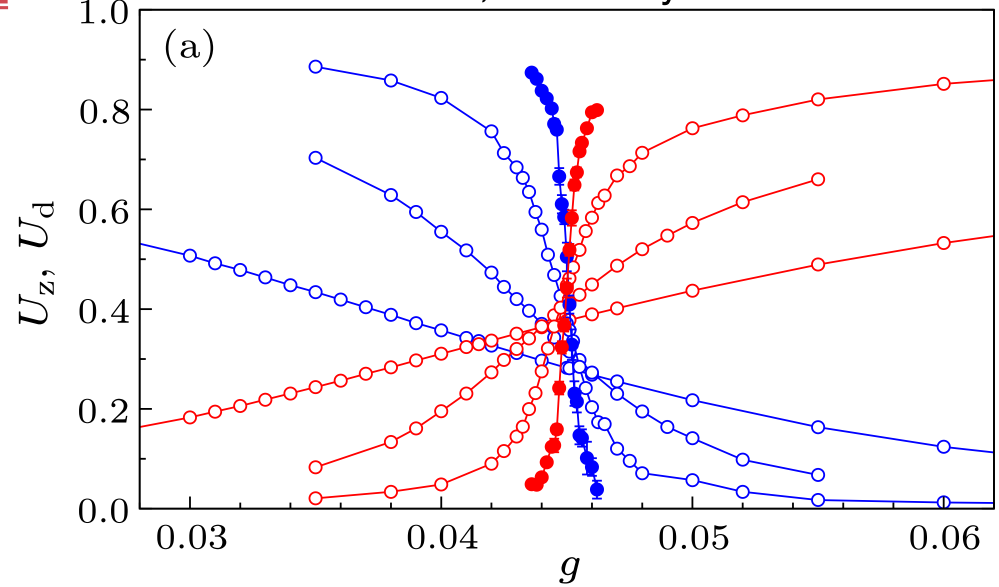
Binder cumulants slopes

Sandvik & Zhao, Chin. Phys. Lett. 2020



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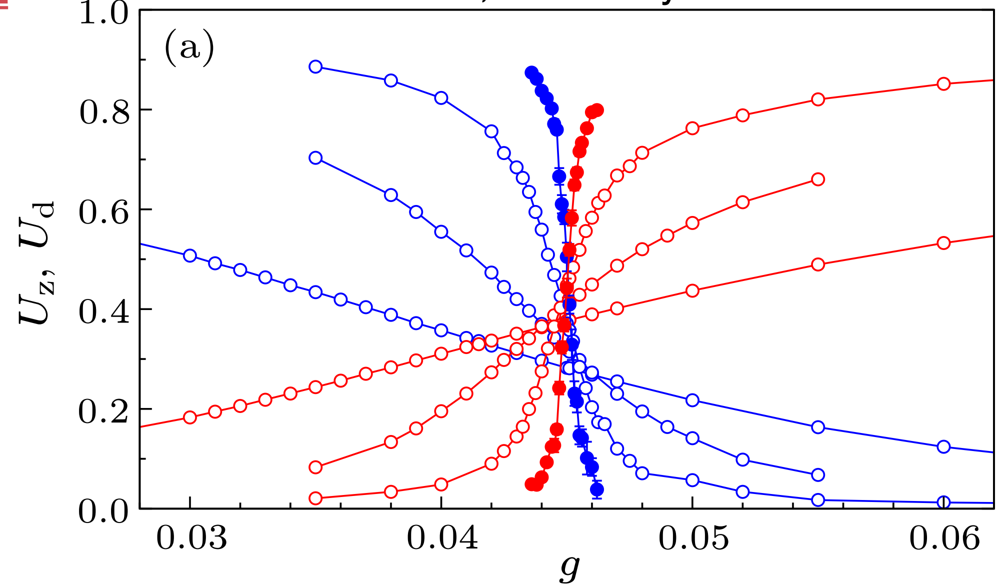

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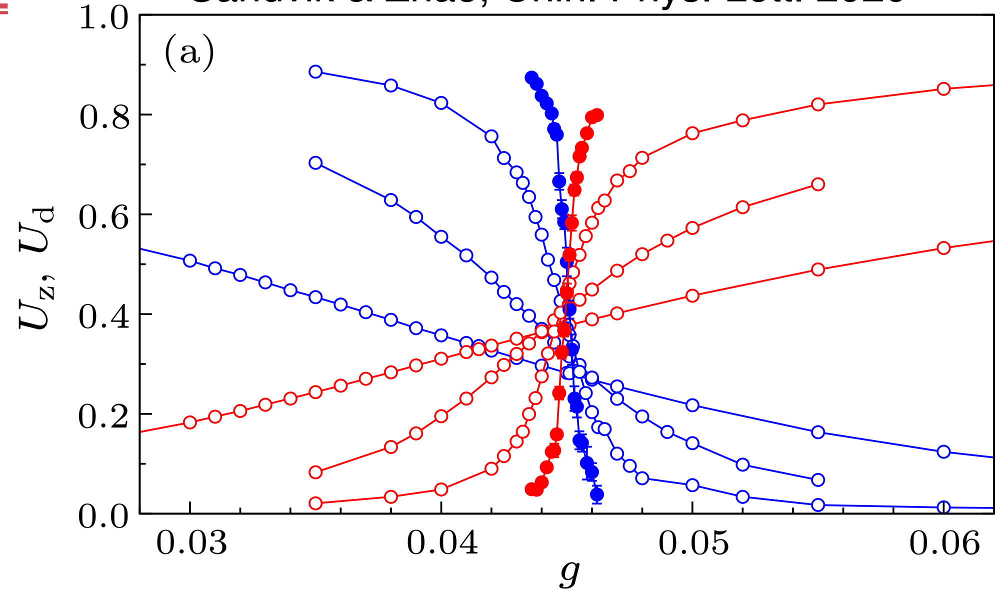

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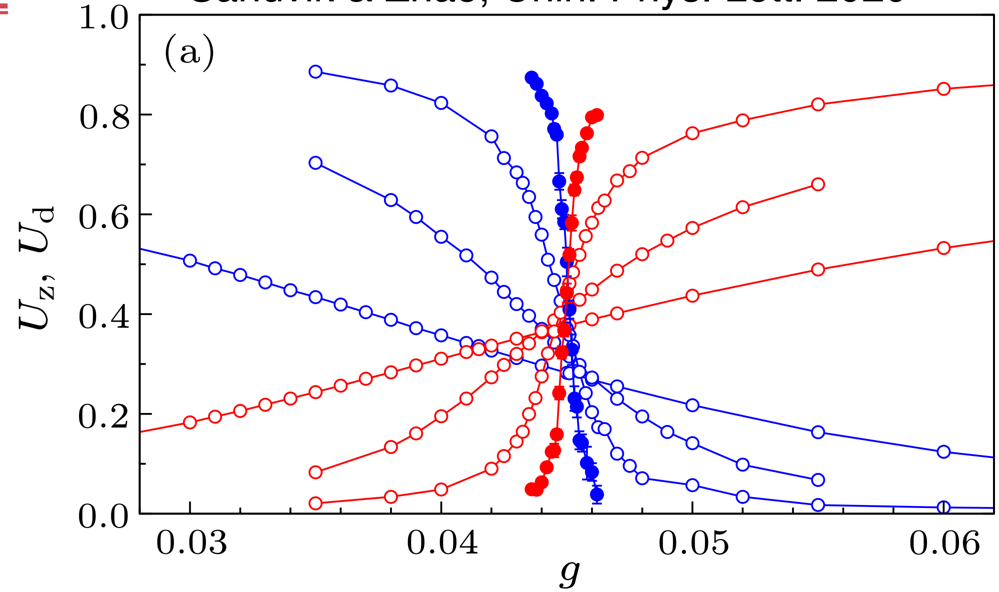

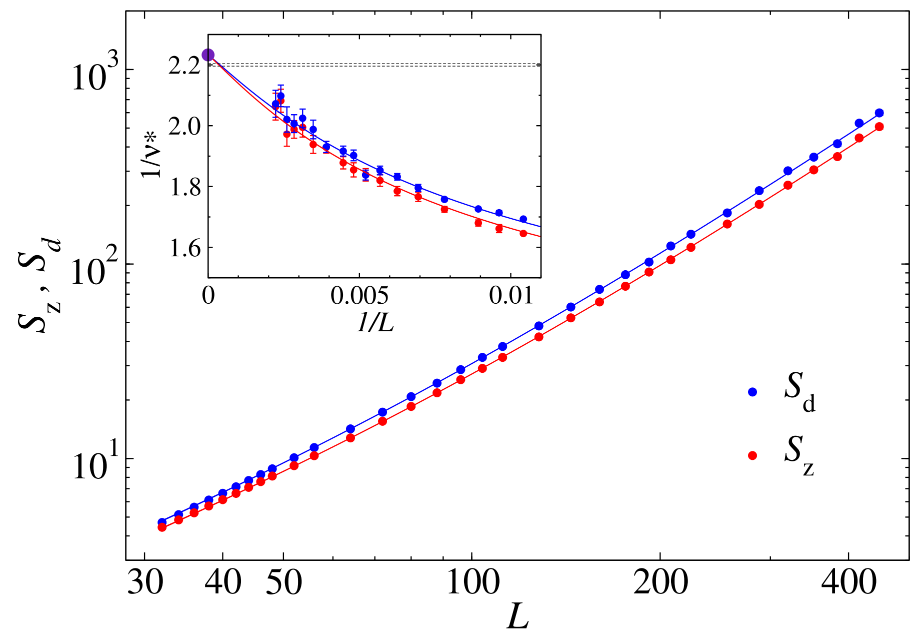

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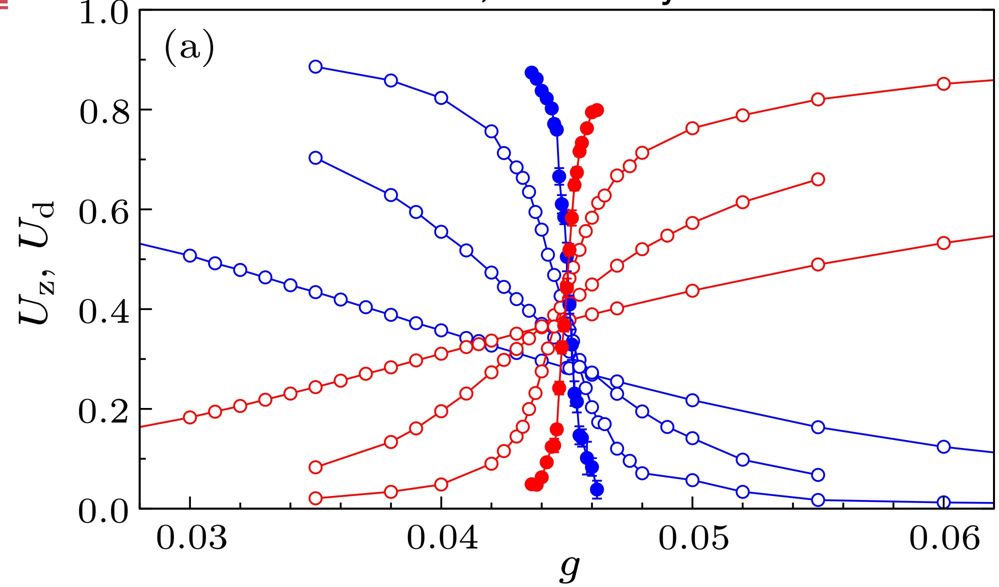

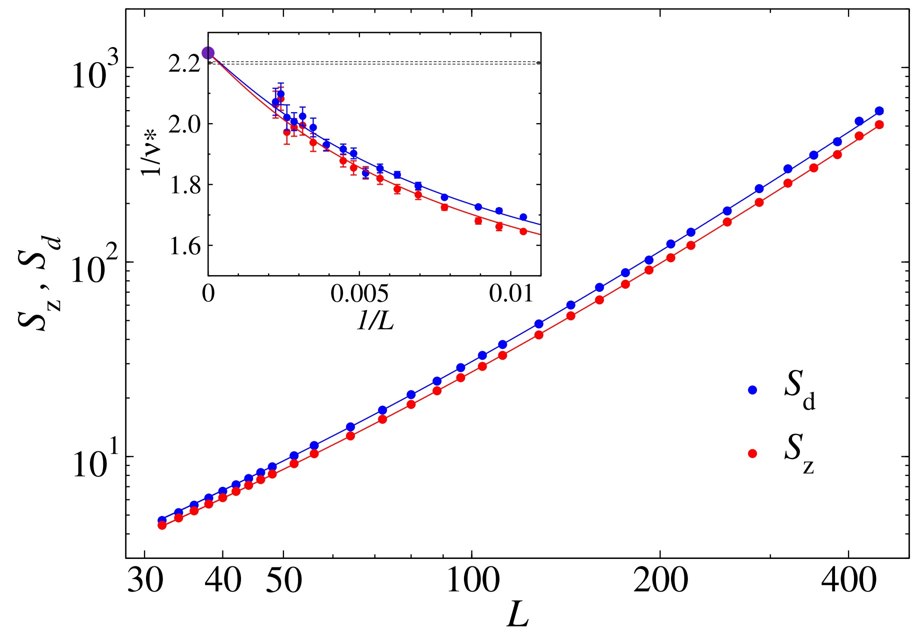

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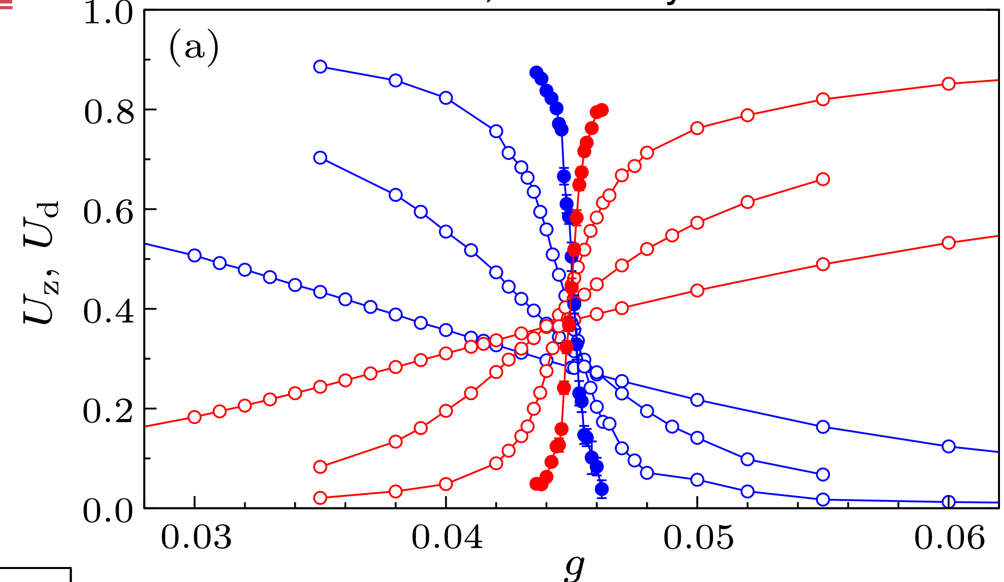
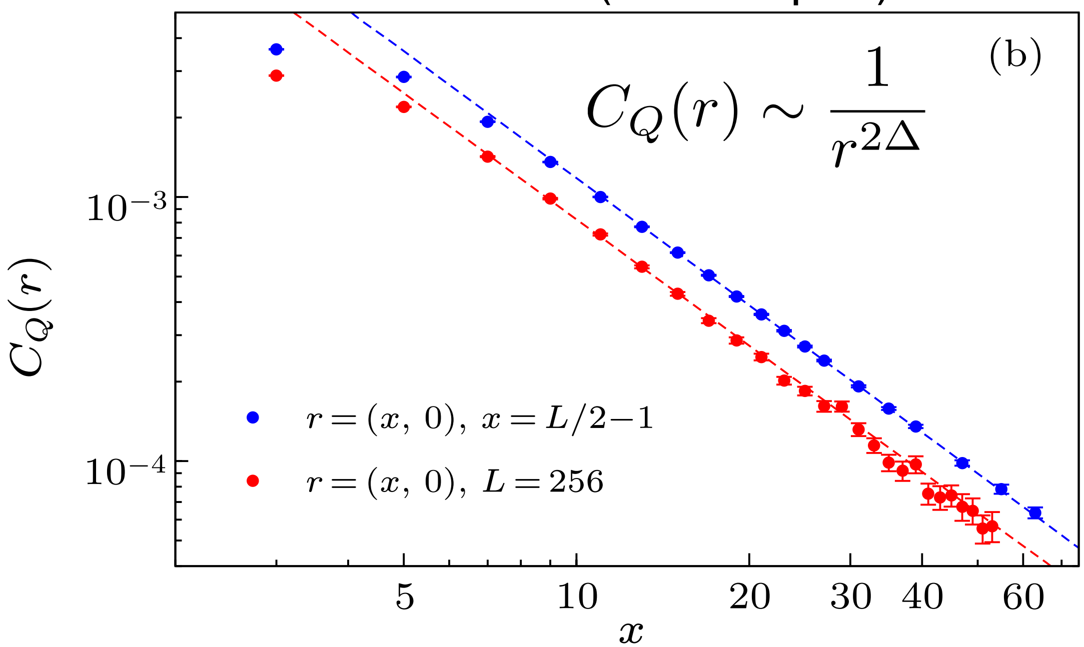
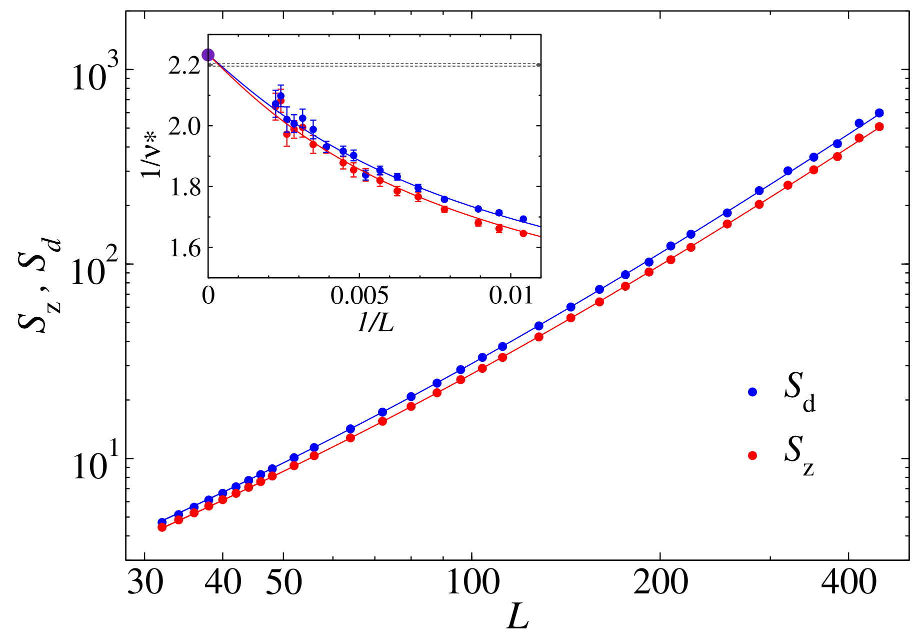


Q-Q correlations (uniform part)





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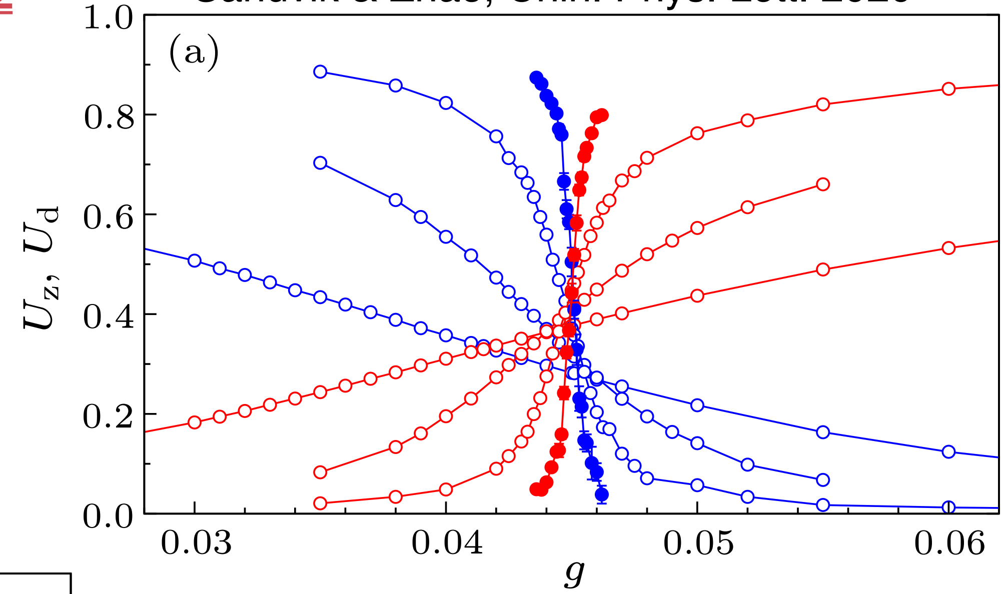

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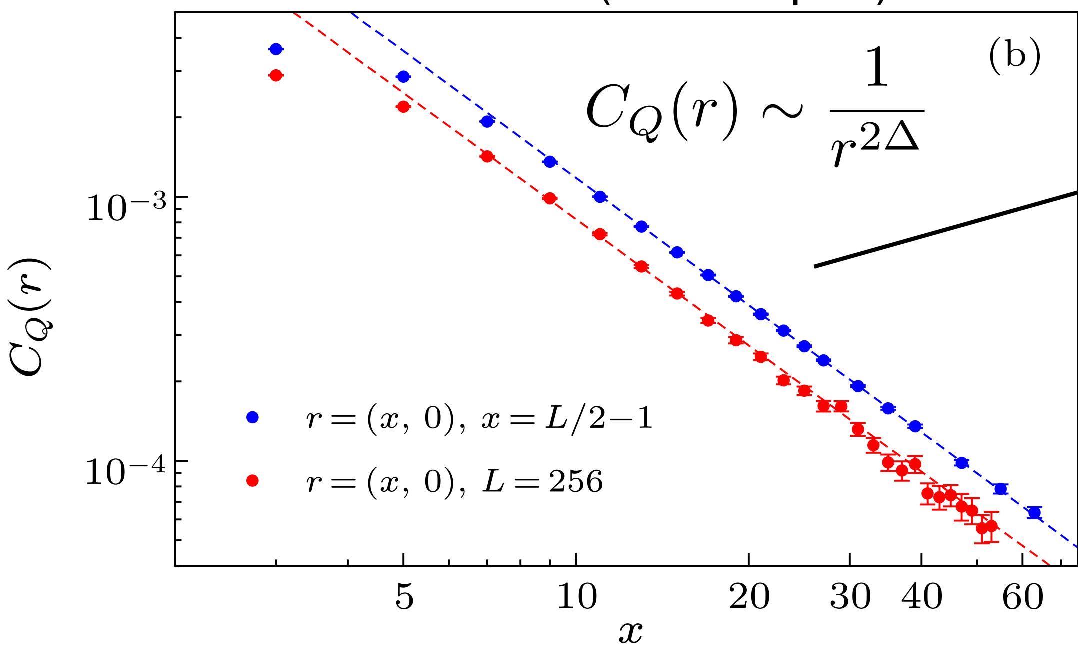

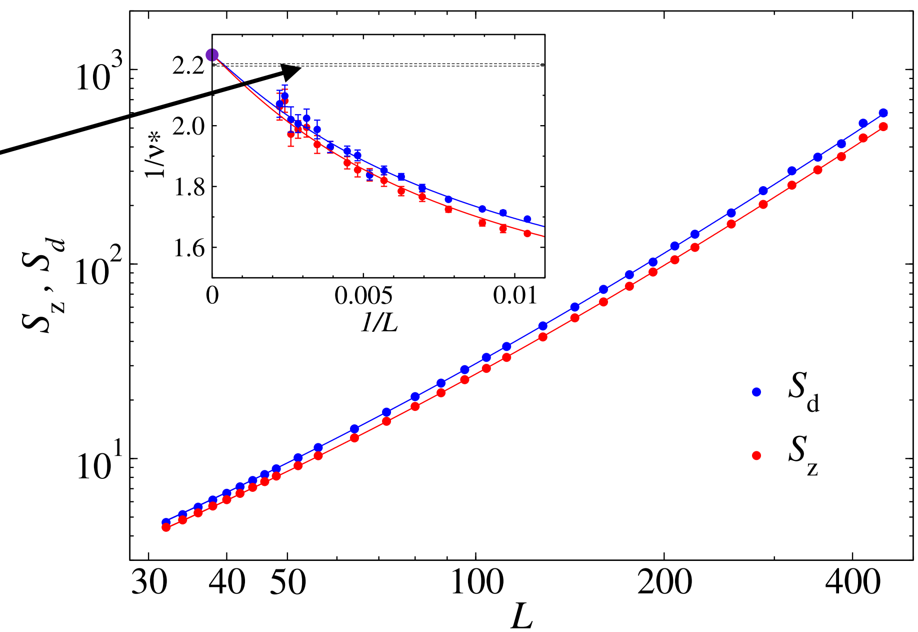

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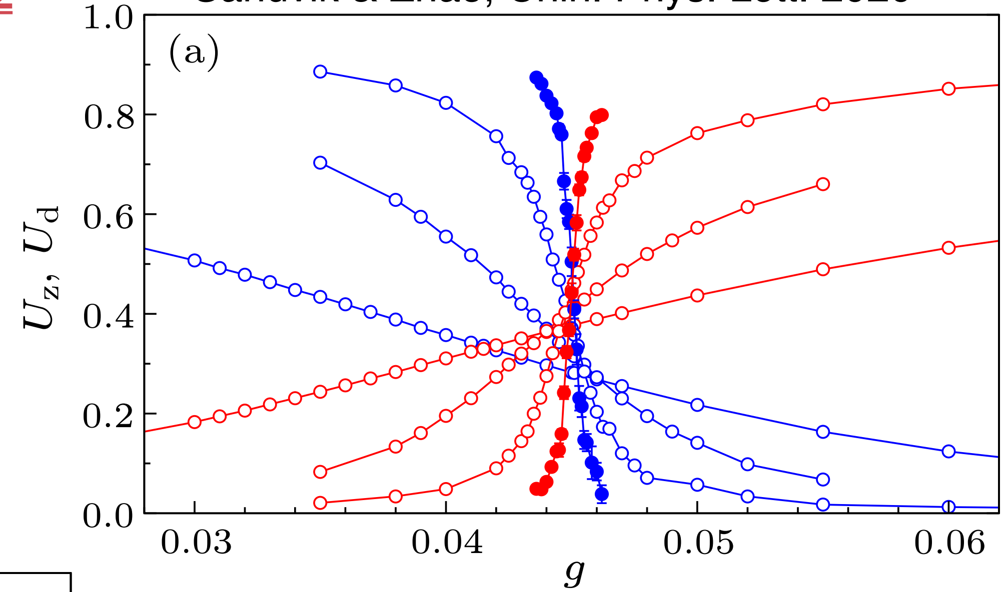

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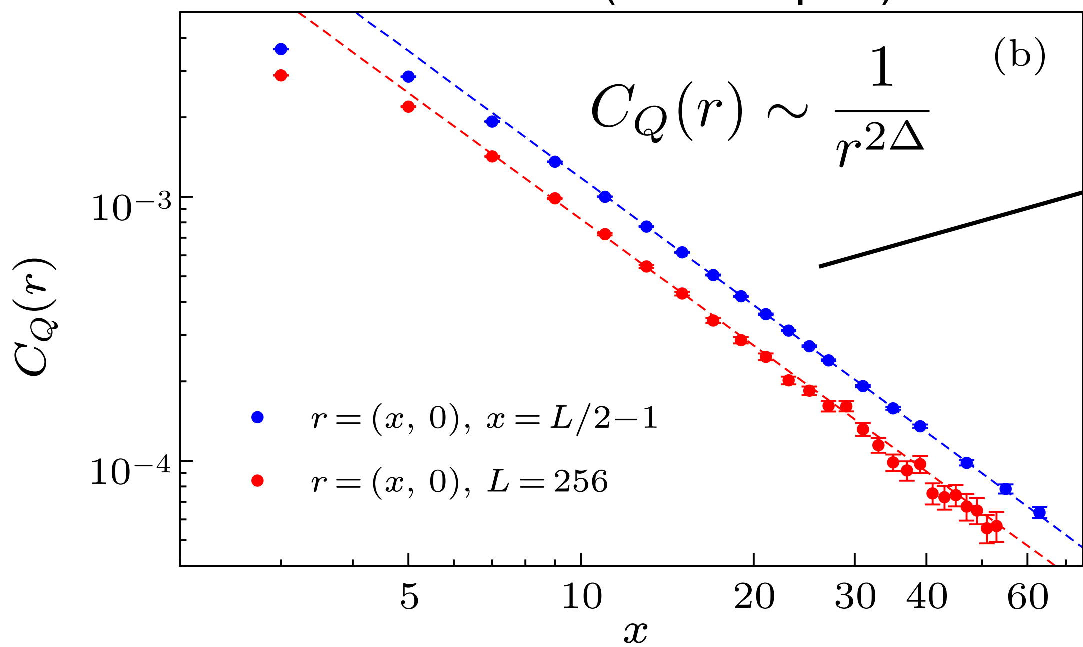

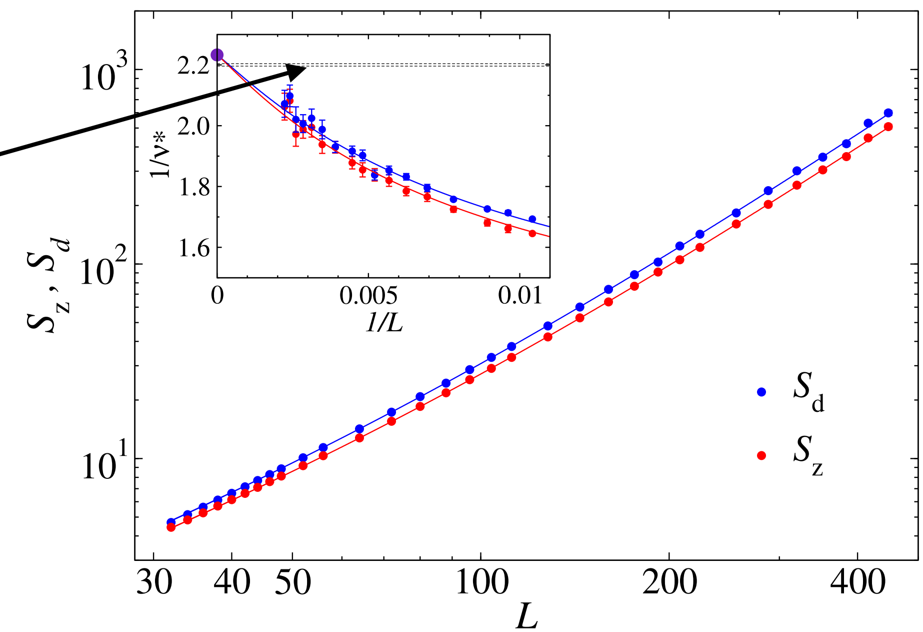

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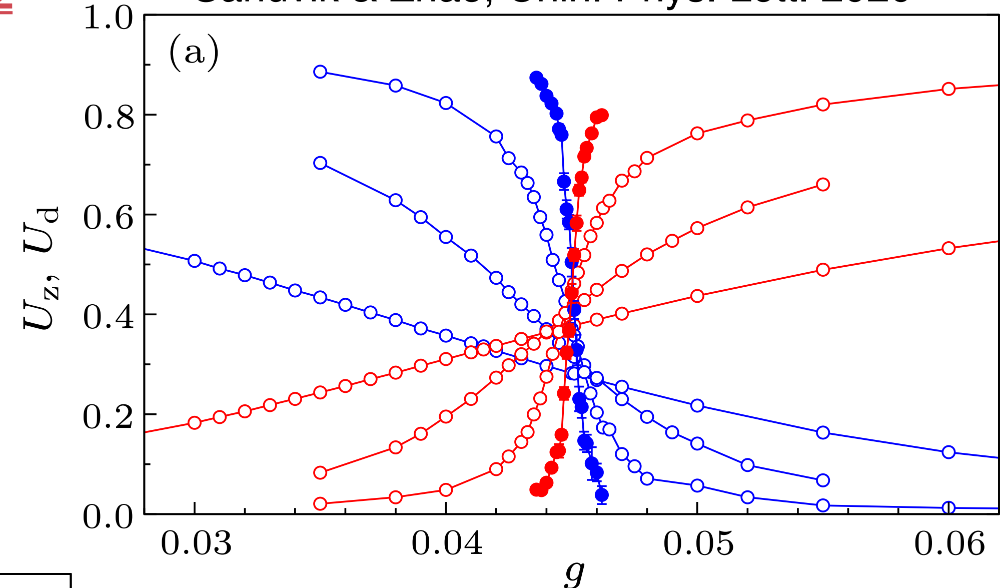

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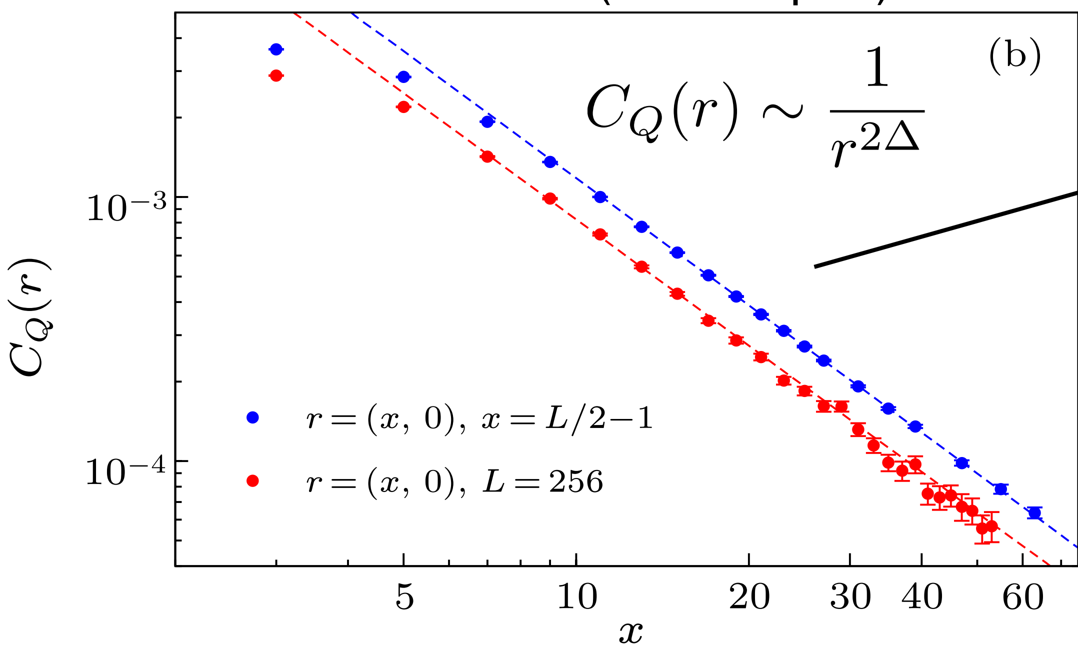

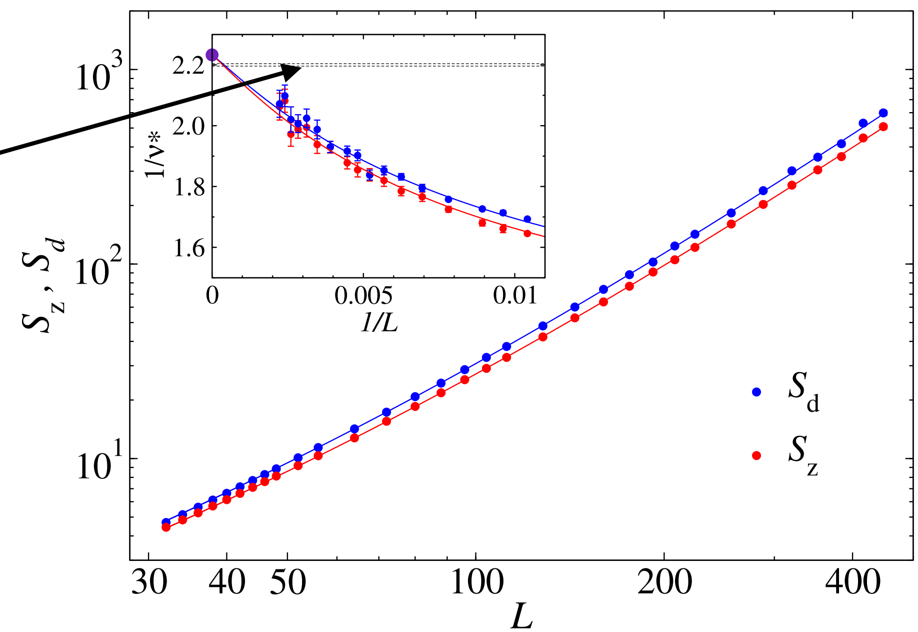

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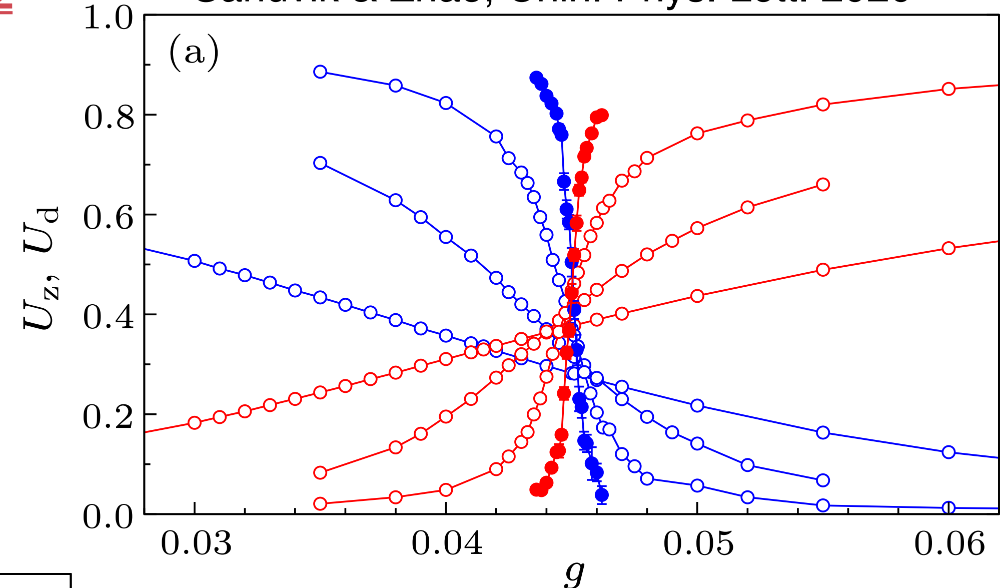

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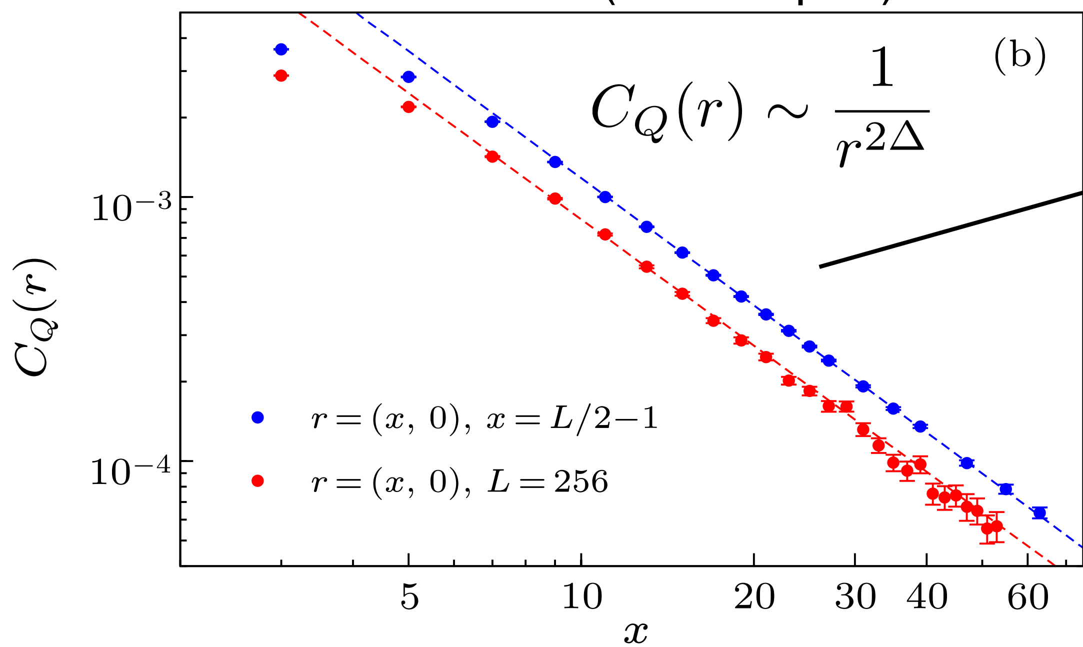

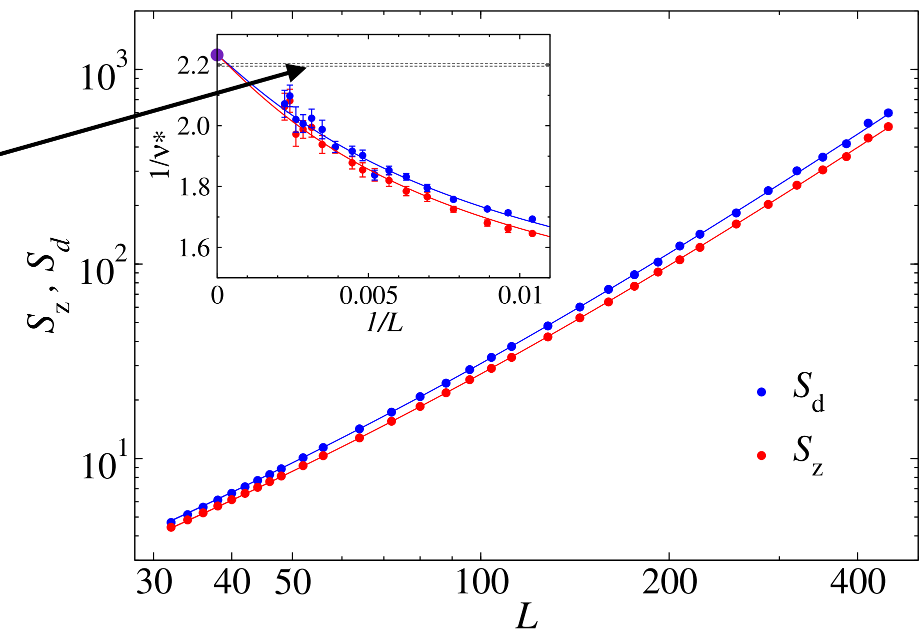

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- A length scale associated with some other criticality (before first-order flow)?

Correlations of staggered singlet projectors

[Zhao, Takahashi, Sandvik, PRL 2020] - new improved results

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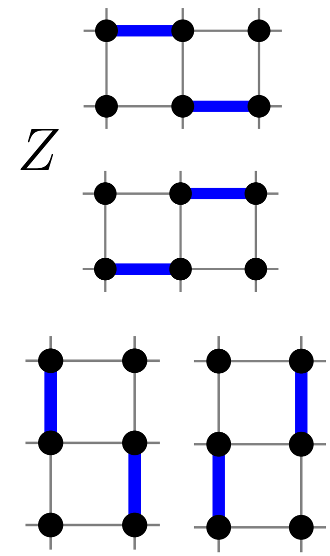

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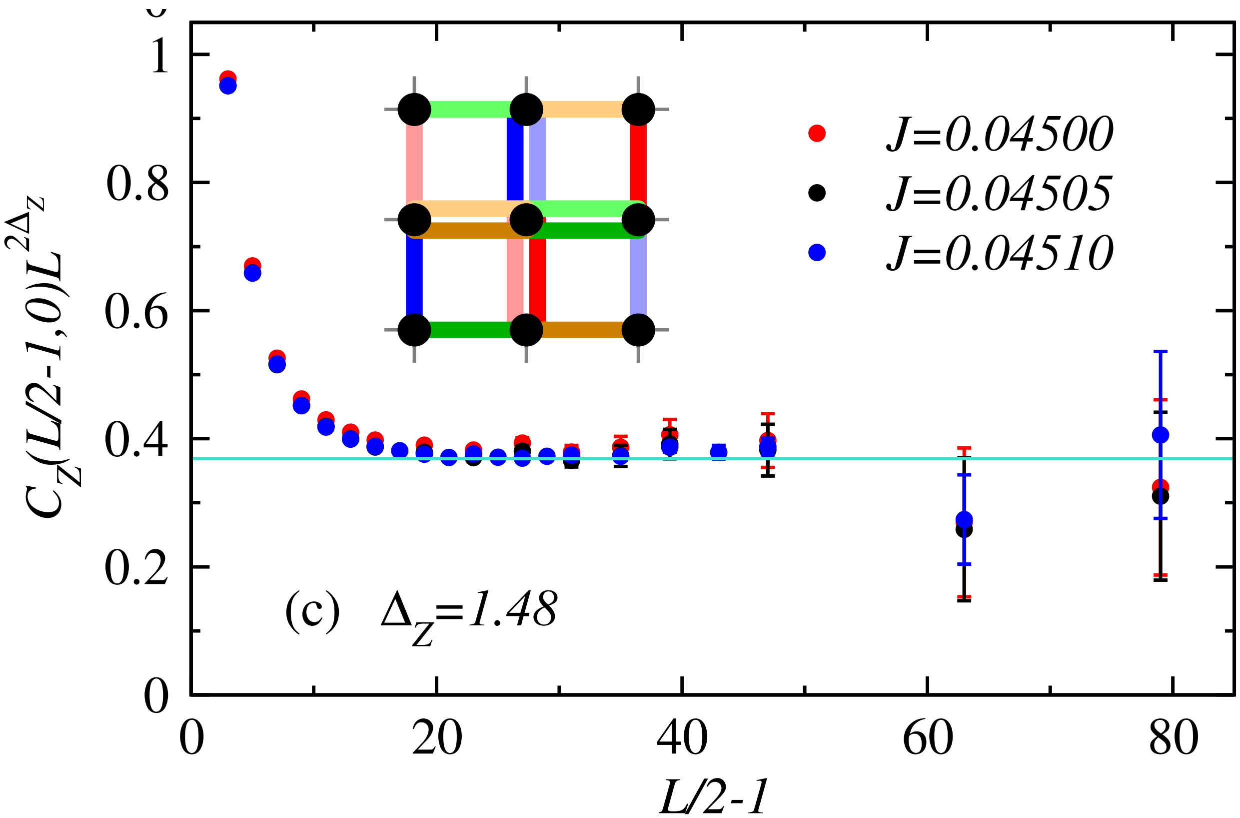

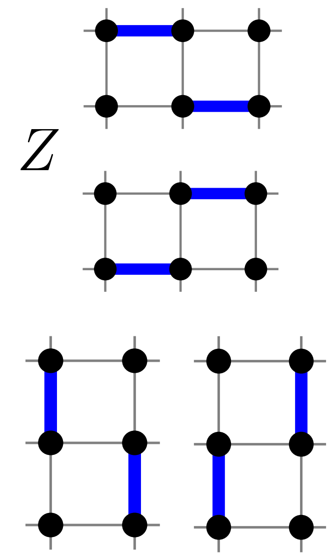

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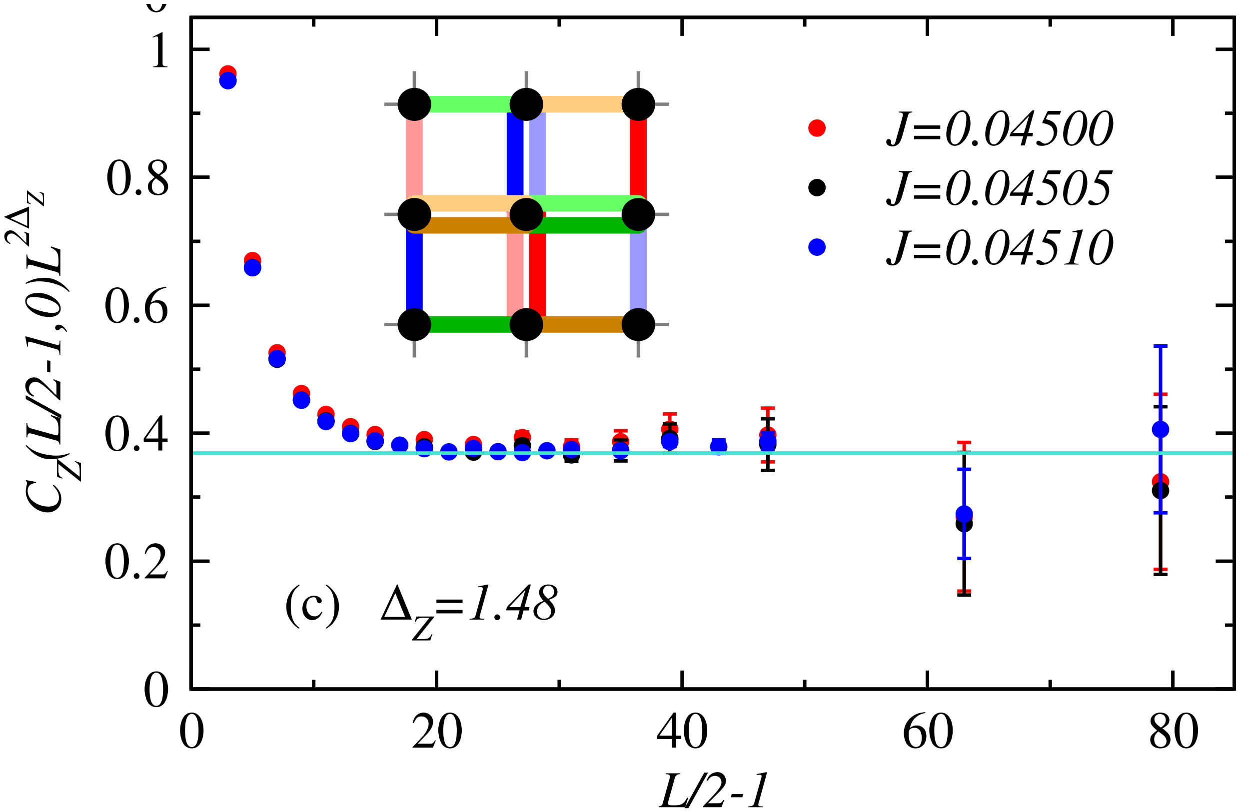

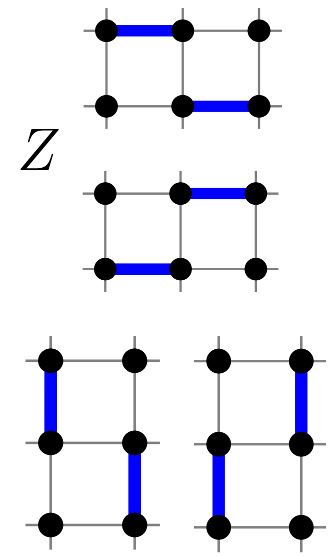

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- tuning the operator deforming the SO(5) sphere

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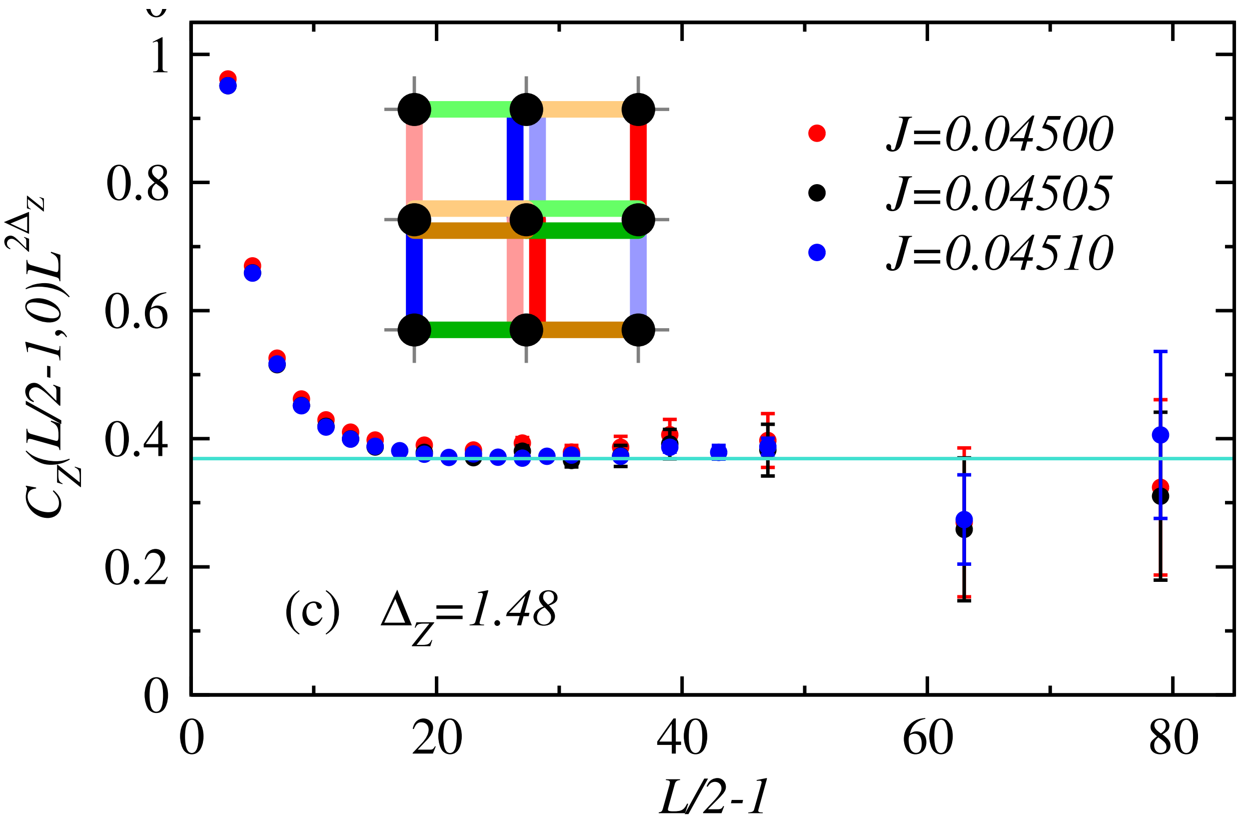

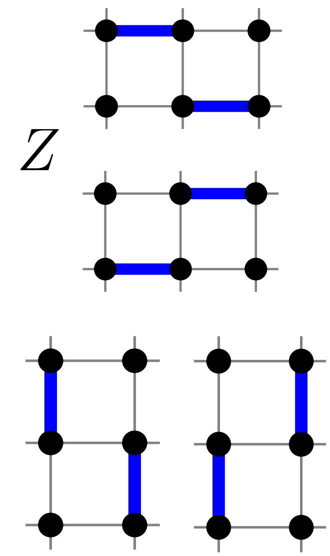

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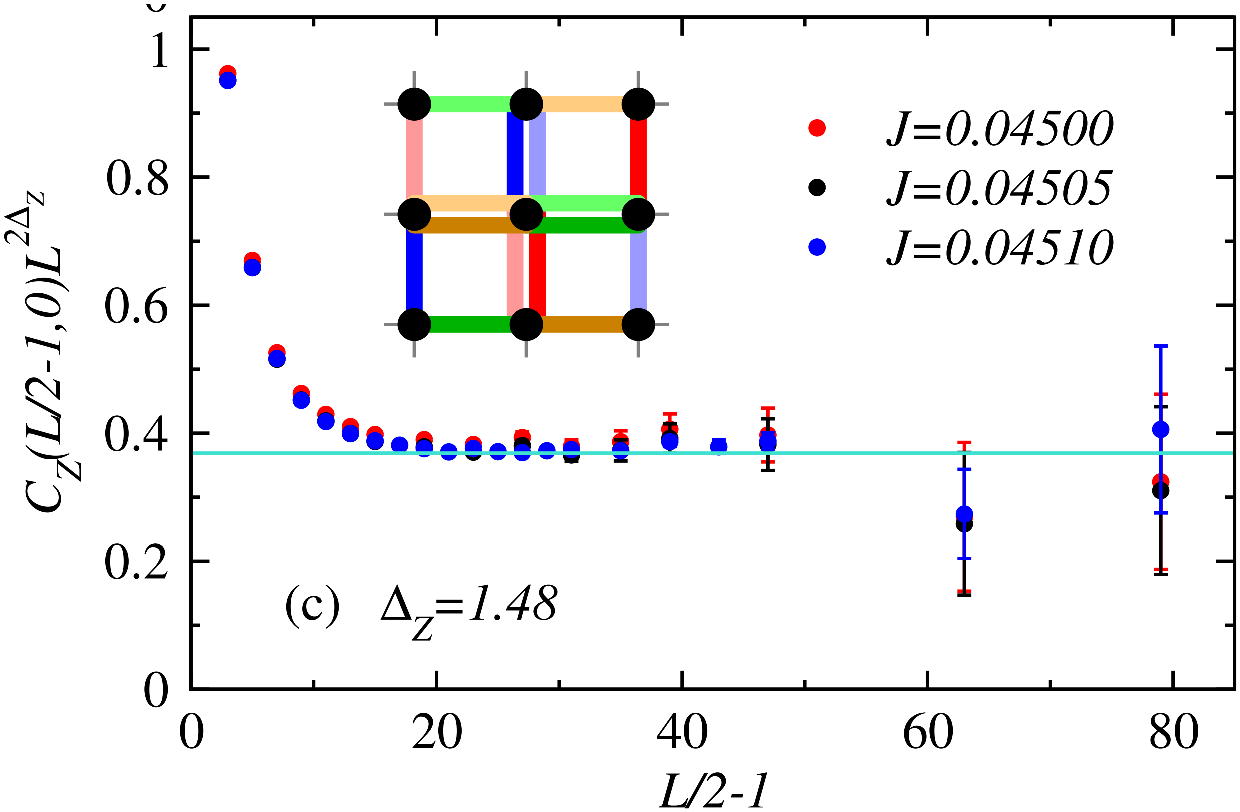

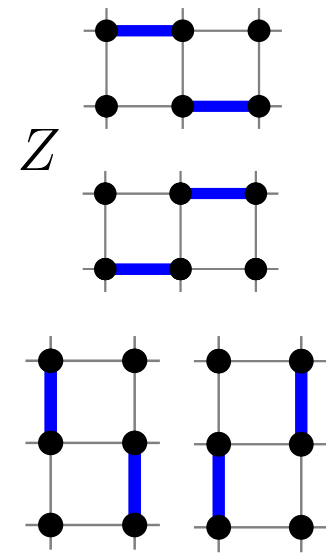

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But additional mystery operator affecting the J-Q (and loop) model

Phase diagrams; possible dqc scenarios
- in space of two relevant couplings (g,k)

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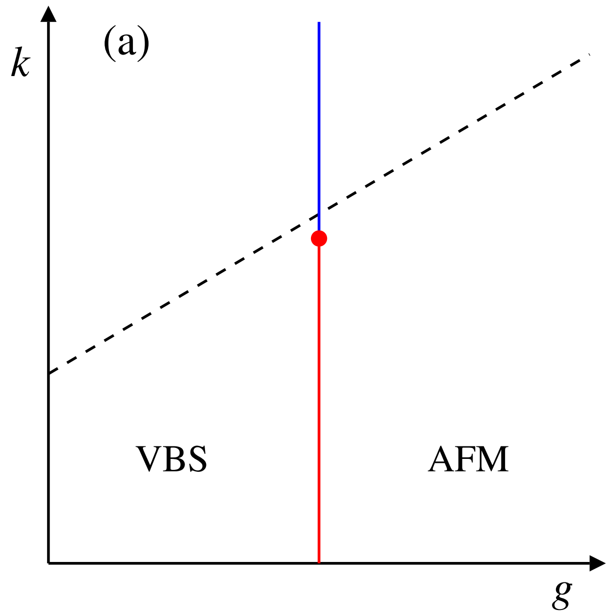

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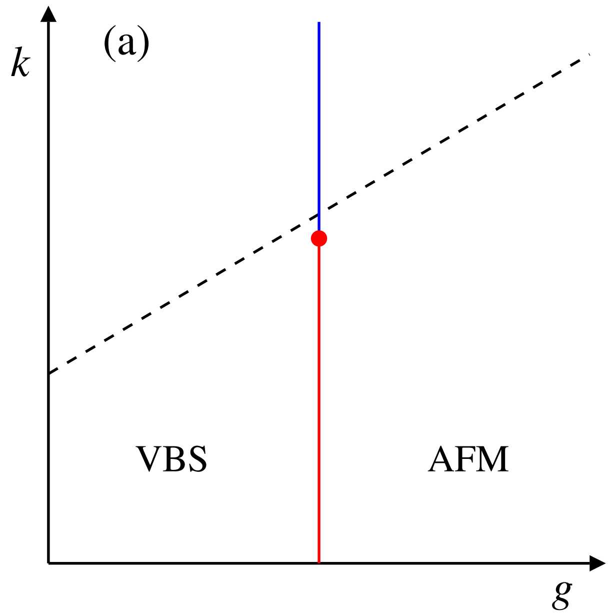
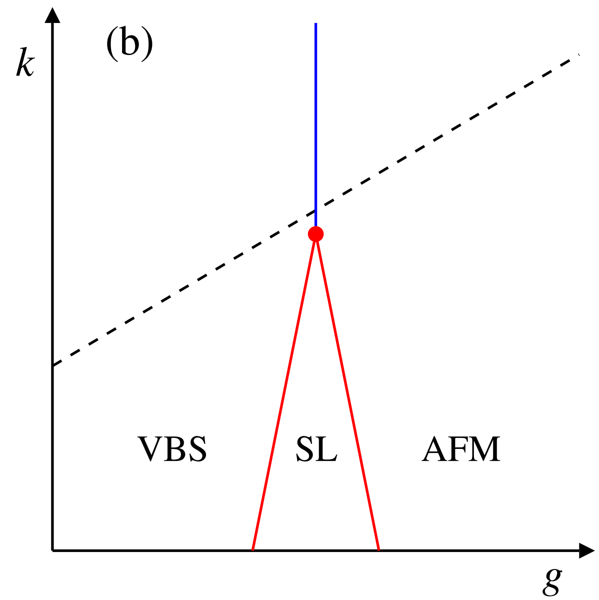
Phase diagrams; possible dqc scenarios - in space of two relevant couplings (g,k)

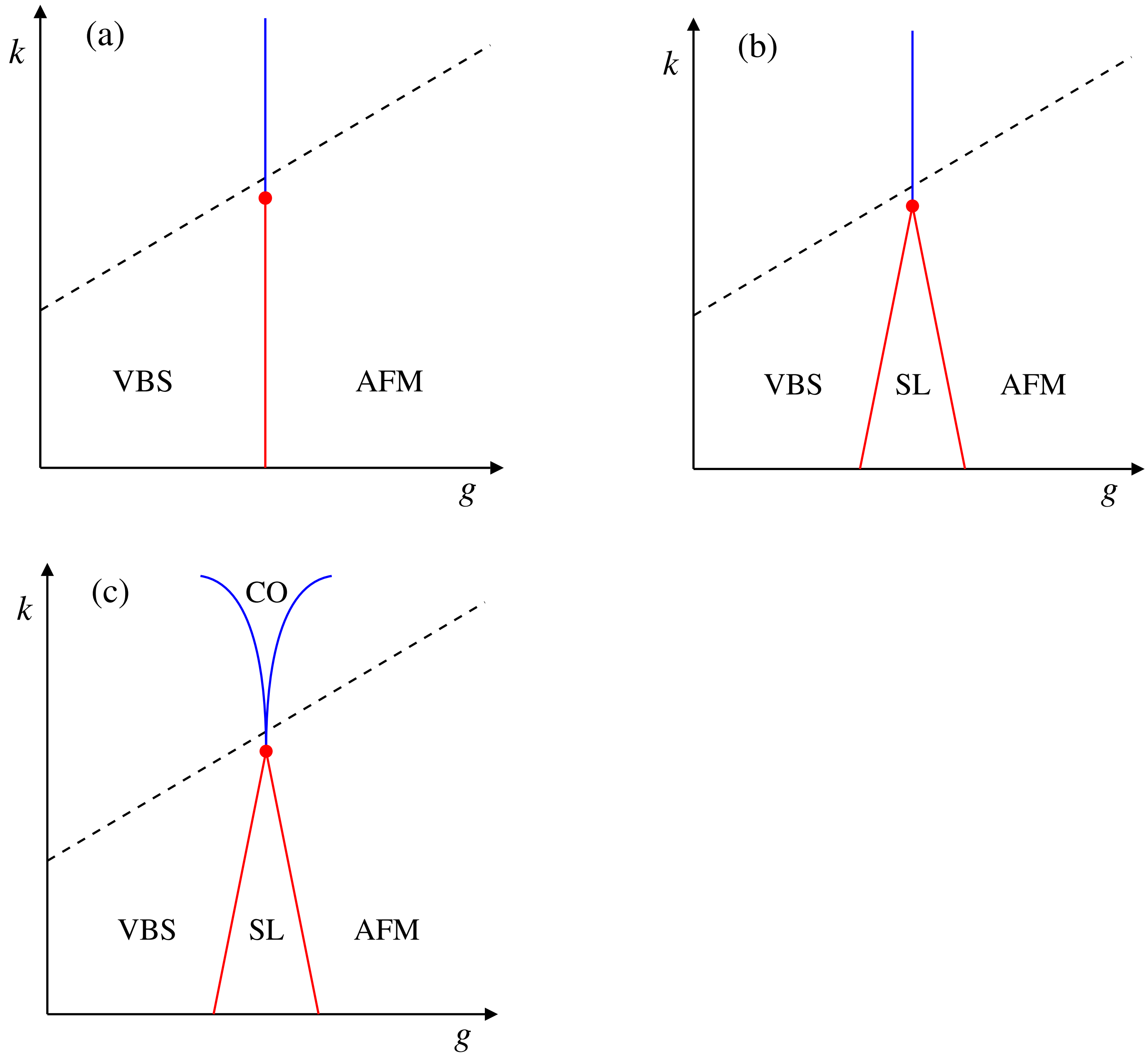




J-Q model with one tuning parameter crosses first-order line

Phase diagrams; possible dqc scenarios
- in space of two relevant couplings (g,k)

J-Q model with one tuning parameter crosses first-order line



Phase diagrams; possible dqc scenarios
- in space of two relevant couplings (g,k)

J-Q model with one tuning parameter crosses first-order line