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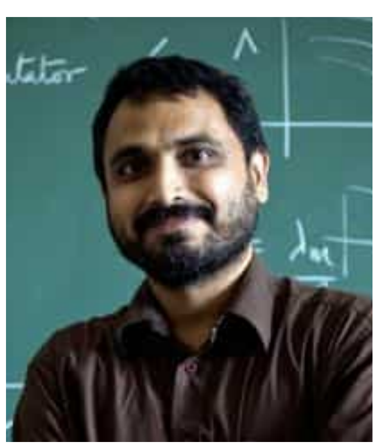
Spin-orbit coupled Dirac Fermions

Subhro Bhattacharjee

ICTS | International Centre for Theoretical Sciences

TATA INSTITUTE OF FUNDAMENTAL RESEARCH



Basudeb Mondal
ICTS

Vijay Shenoy
IISc

arXiv : 2304.07223

(To appear in PRB)

Department of Science and Technology (DST)
DST

MAX PLANCK GESELLSCHAFT

Embedding of UV symmetries on IR theory

$$SG_{IR} \geq SG_{UV}$$

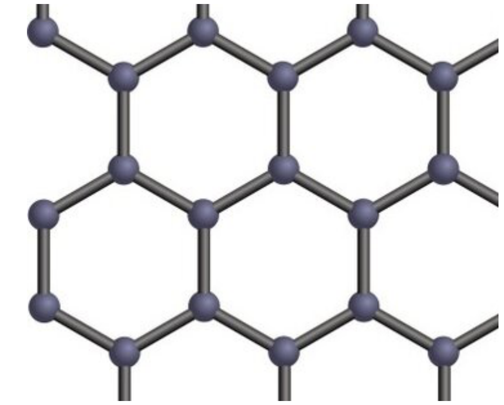
How are the UV/microscopic symmetries implemented on the IR degrees of freedom ?

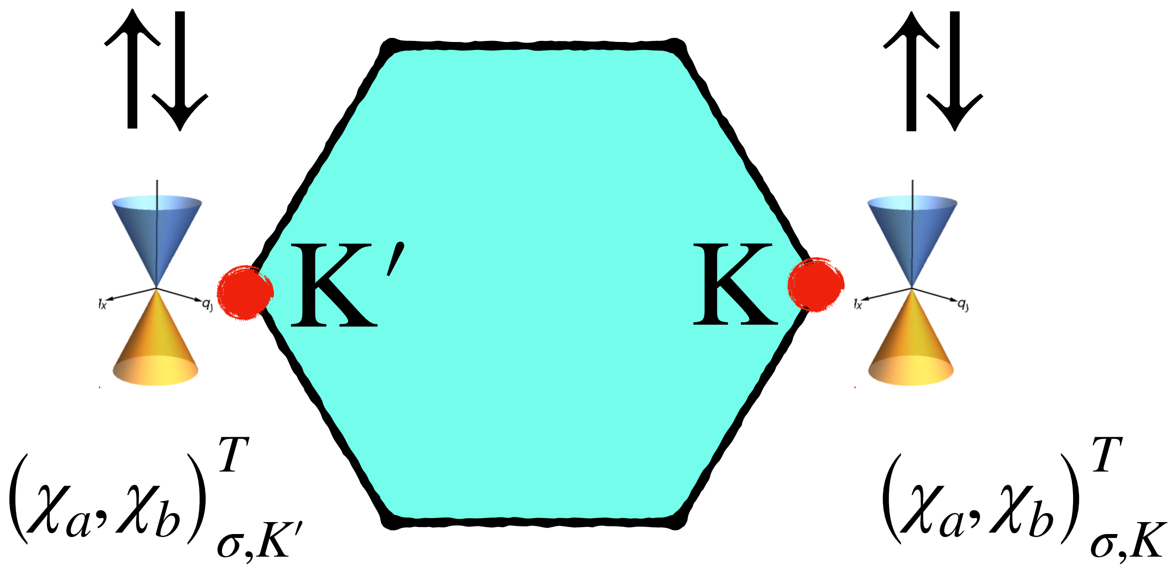
Text book example :

- Consider the nearest neighbour FM and AFM Ising model on square lattice.
- Both lead to ϕ^4 - theory in the coarse grained continuum limit.
- Lattice translations are differently implemented on the fields :
  - FM : $\phi \rightarrow \phi$: Ferromagnetic oder
  - AFM : $\phi \rightarrow -\phi$: Neel order

UV symmetry implementation is very important to understand the nature of phases and their experimental signatures

Another well known example : Monolayer graphene





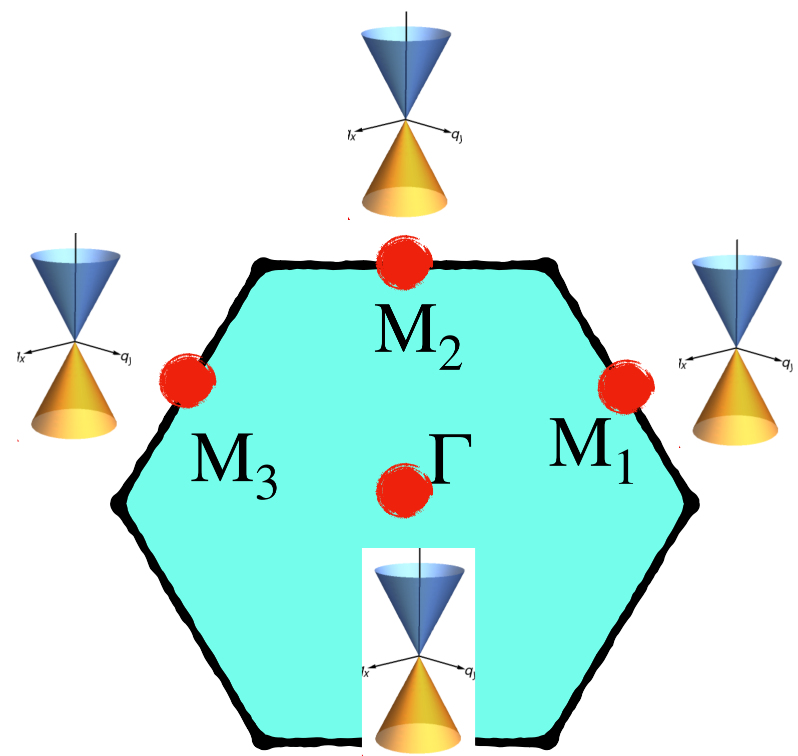
Low energy : SU(4) Dirac theory

The embedding of the lattice symmetries on the low energy Dirac fermions χ crucially depend on the position of the Dirac points in BZ

The above embedding then fixes the nature of all the phases proximate to the semimetal that can be obtained by condensing

$$\bar{\chi}_\alpha \chi_\beta$$

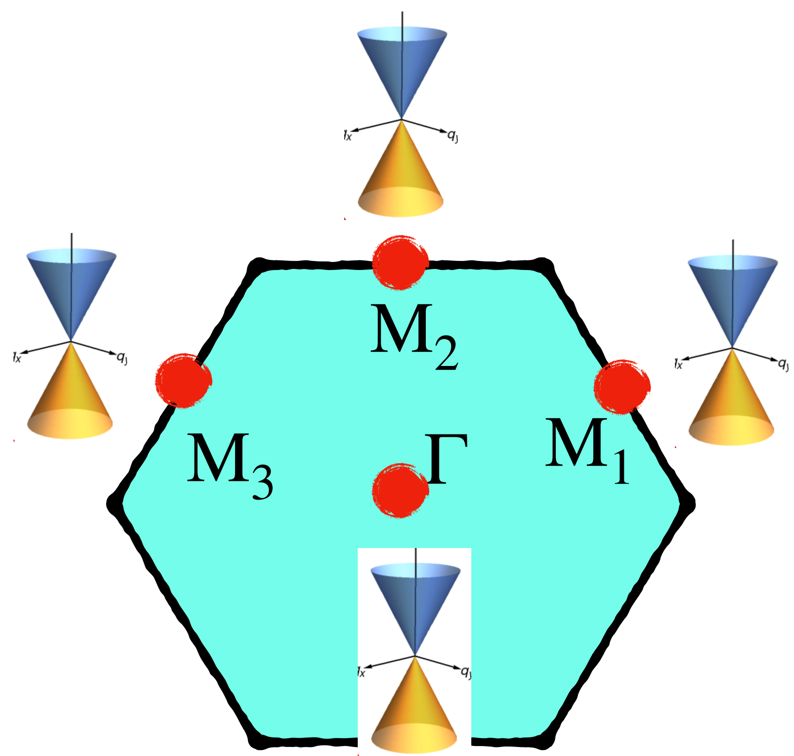
This Talk : J=3/2 electrons on honeycomb lattice



Due to Atomic spin orbit coupling

- Changes the position and number of the Dirac points
- Hence changes the embedding of the Honeycomb lattice symmetries on the low energy Dirac fermion

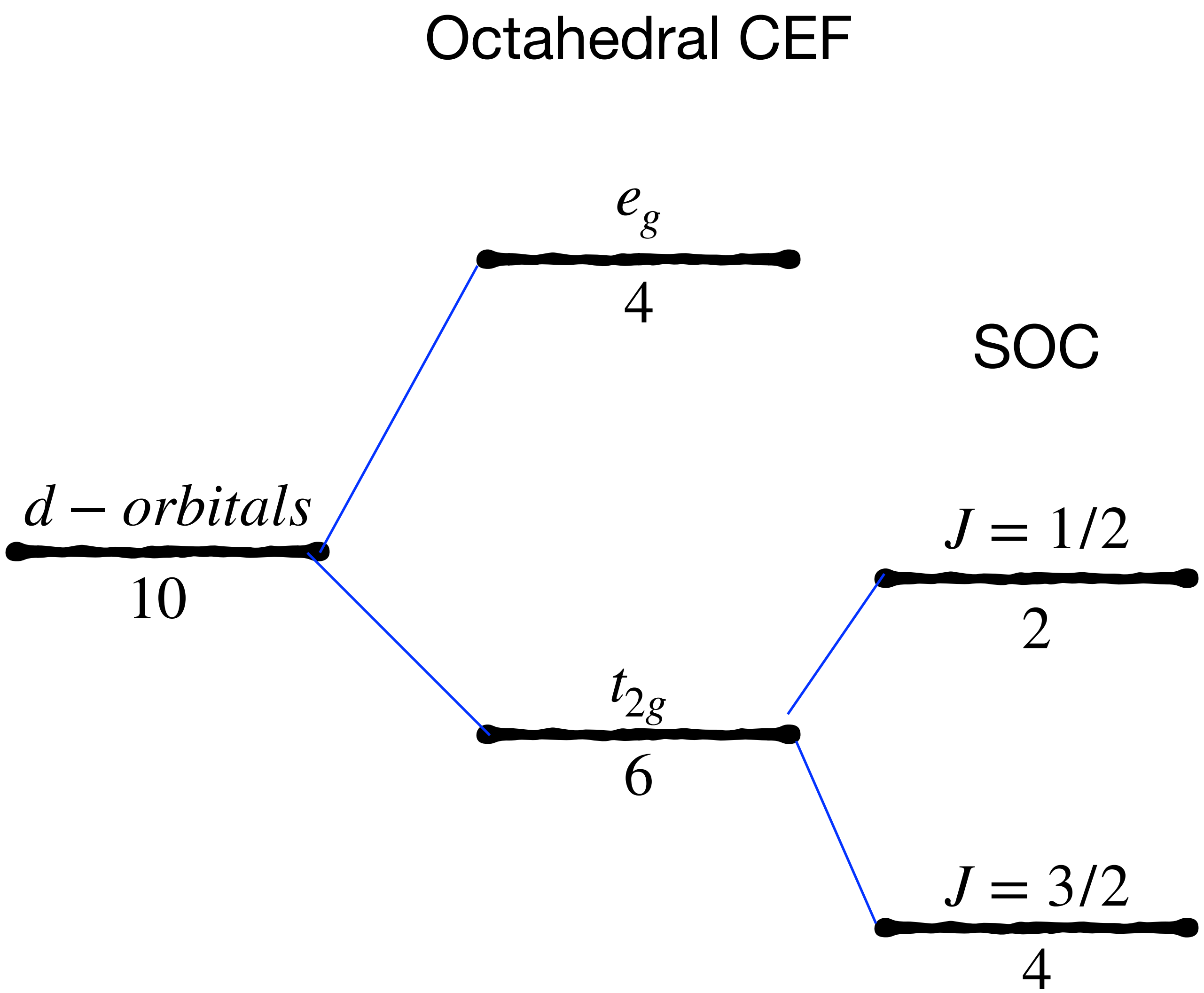
This Talk : J=3/2 electrons on honeycomb lattice



As a result :

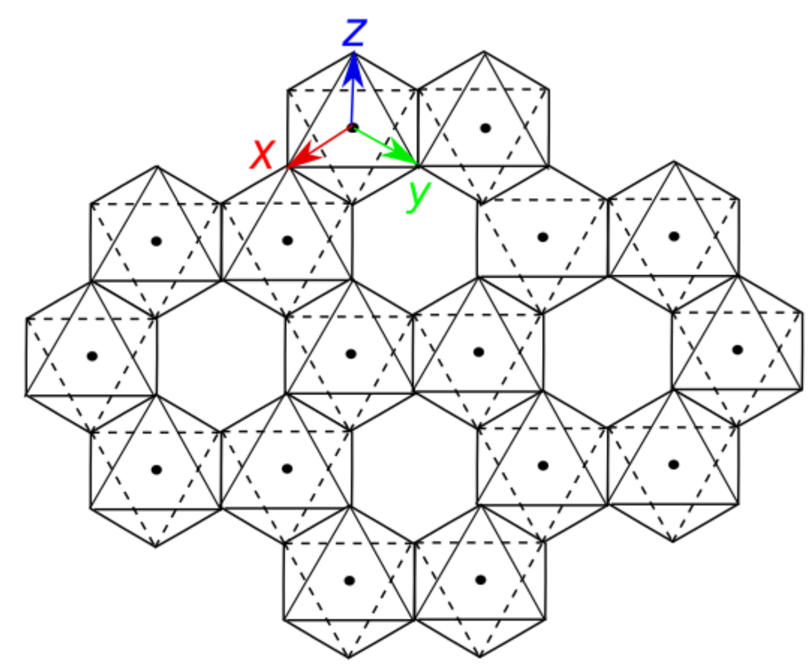
- (Non-interacting) low energy : SU(8) Dirac semimetal
- The nature of proximate phases are completely dictated by the newer embedding of UV symmetries
- Provides much richer set of phases and possible phase transitions.

Spin-orbit coupled electronic d-orbitals



[Romhanyi et. al. (2017)]

d^1 : quarter filled J=3/2 orbitals



Put them on a corner sharing octahedral geometry such that we get a honeycomb lattice

e.g. : Similar to $\alpha - \mathrm{ZrCl}_3$ but we shall consider the general weak to intermediate coupling physics

[Yamada, Oshikawa, Jackeli (2018)]

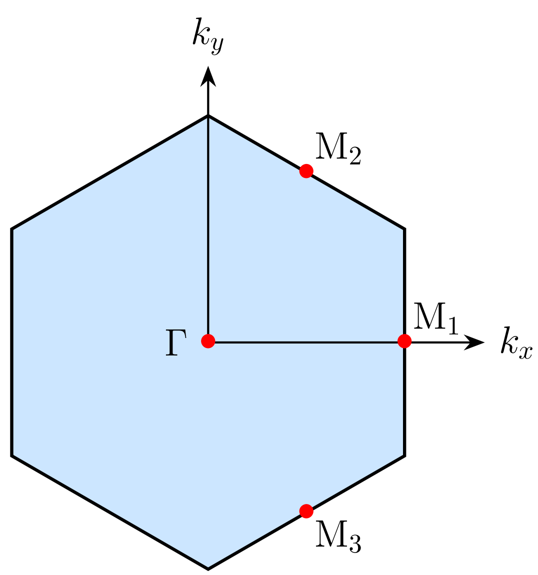
Hopping Hamiltonian and SU(4) Symmetry

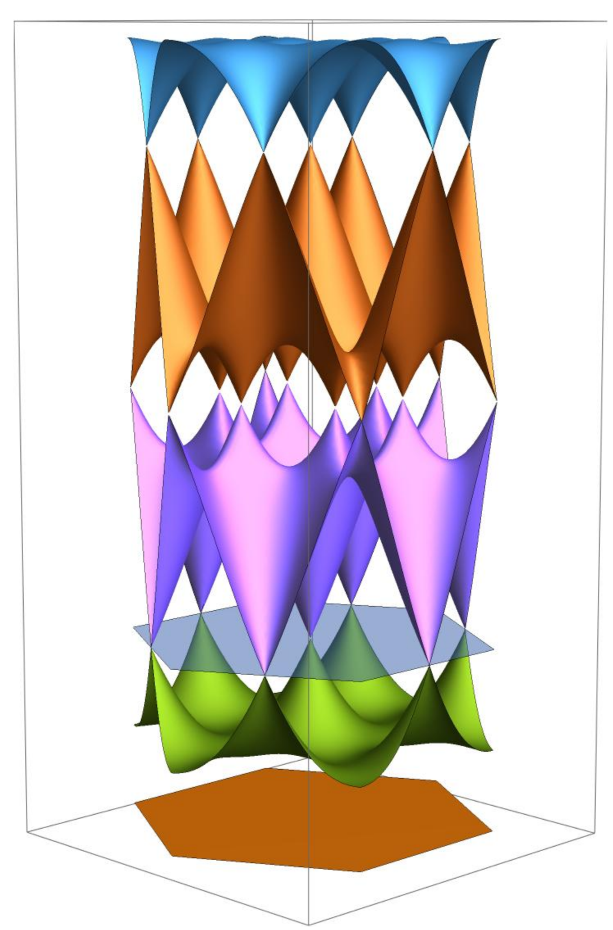
[Yamada, Oshikawa, Jackeli (2018)]

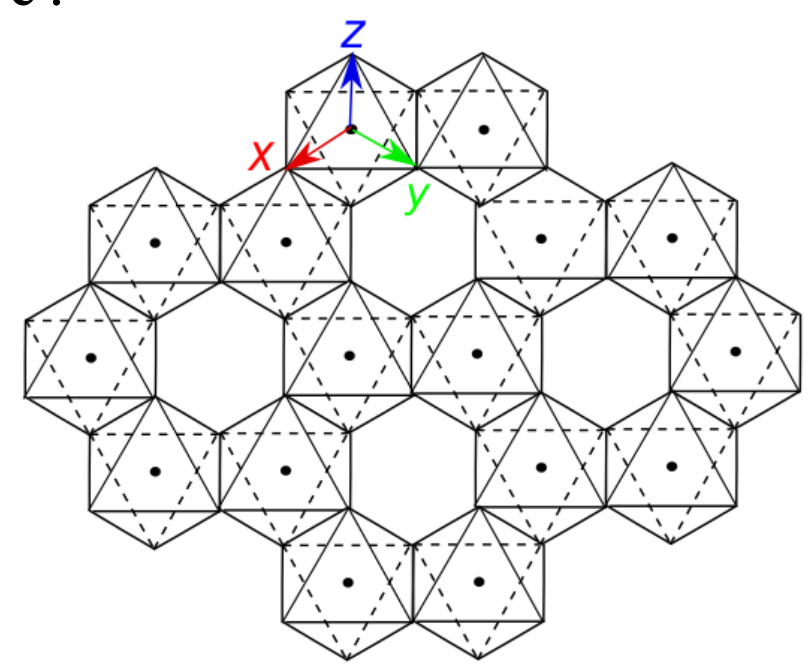
$$\left(\psi_{1/2}, \psi_{-1/2}, \psi_{3/2}, \psi_{-3/2}\right)^T$$

Indirect Hopping Model

$$H = -\frac{t}{\sqrt{3}} \sum_{\langle \mathbf{r},\mathbf{r}' \rangle} \psi^\dagger(\mathbf{r}) \mathbf{U}_{\mathbf{r}\mathbf{r}'} \psi(\mathbf{r}') + \mathrm{h.c.}$$





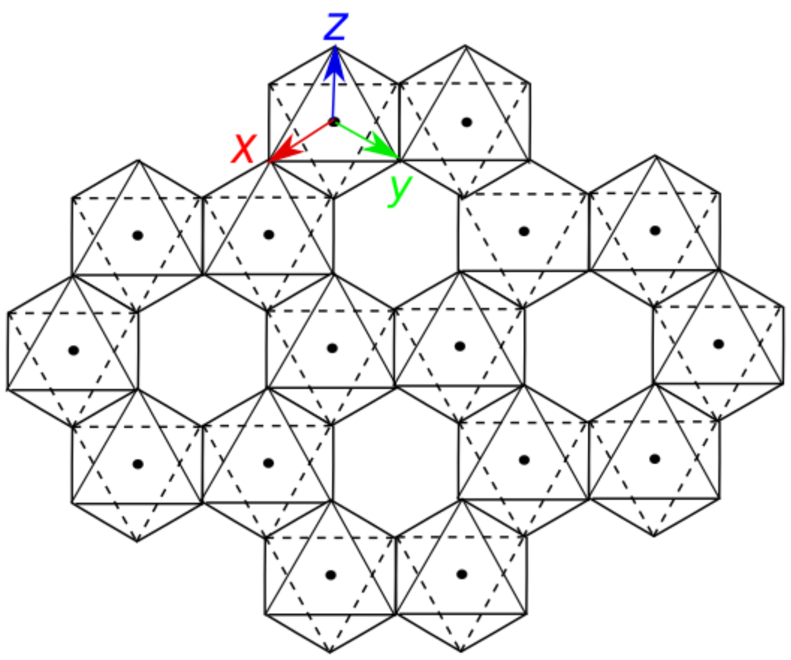


Hopping Hamiltonian and SU(4) Symmetry

[Yamada, Oshikawa, Jackeli (2018)]

$$\left(\psi_{1/2}, \psi_{-1/2}, \psi_{3/2}, \psi_{-3/2}\right)^T$$

$$H = -\frac{t}{\sqrt{3}} \sum_{\langle \mathbf{r},\mathbf{r}' \rangle} \psi^\dagger(\mathbf{r}) \mathbf{U}_{\mathbf{r}\mathbf{r}'} \psi(\mathbf{r}') + \mathrm{h}$$



Indirect Hopping Model

$$\prod_{\mathbf{r},\mathbf{r}' \in \, hexagon} U_{\mathbf{r}\mathbf{r}'} = -\mathbb{I}_4$$

The system has a SU(4) symmetry which becomes manifest in a local basis

$$\phi(\mathbf{r}) = g(\mathbf{r})^\dagger \psi(\mathbf{r})$$

$$H = -\frac{t}{\sqrt{3}} \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \eta_{\mathbf{r}\mathbf{r}'} \phi^\dagger(\mathbf{r}) \phi(\mathbf{r}') + \textbf{ h.c.}$$

$$\prod_{\mathbf{r},\mathbf{r}' \in \, hexagon} \eta_{\mathbf{r}\mathbf{r}'} = -1$$

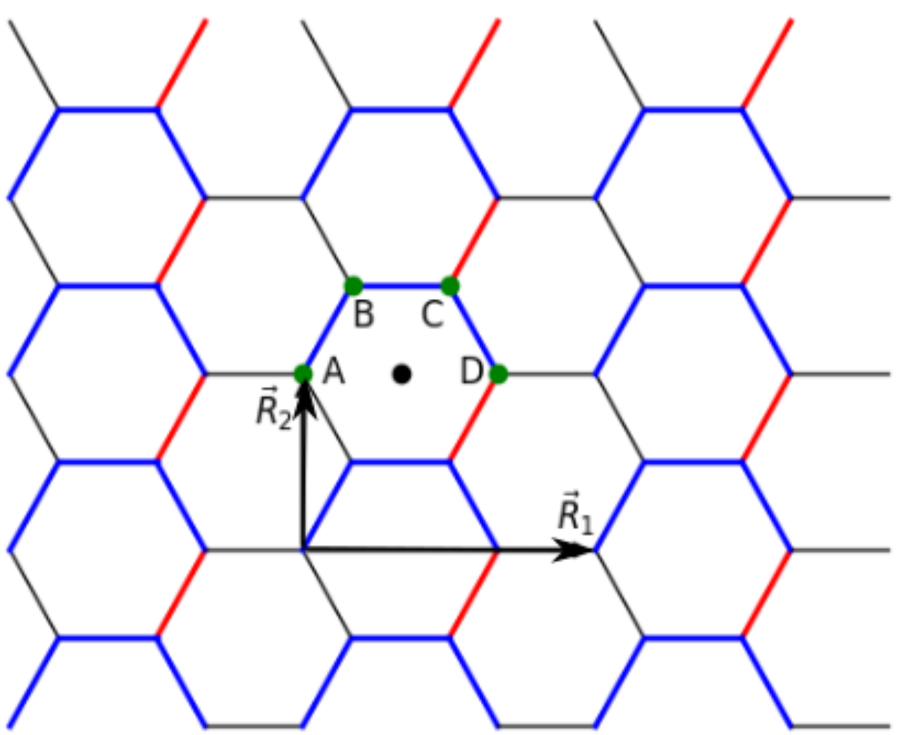
Hopping Hamiltonian and SU(4) Symmetry : Band structure

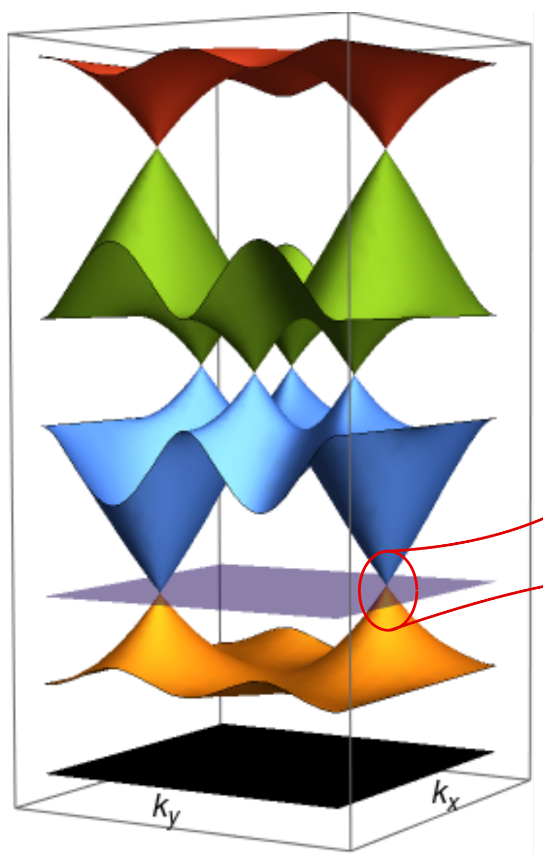
[2304.07223, Mondal, Shenoy, SB]

$$H = -\frac{t}{\sqrt{3}} \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \eta_{\mathbf{r}\mathbf{r}'} \phi^{\dagger}(\mathbf{r}) \phi(\mathbf{r}') + \text{ h.c.} \qquad \prod_{\mathbf{r},\mathbf{r}' \in \, hexagon} \eta_{\mathbf{r}\mathbf{r}'} = -1$$

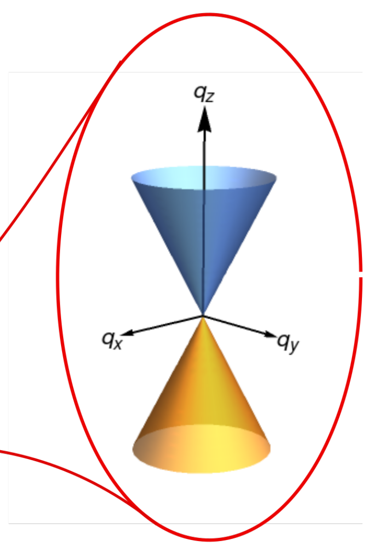
Honeycomb lattice in $\pi - flux$

Choose a 4-site unit cell

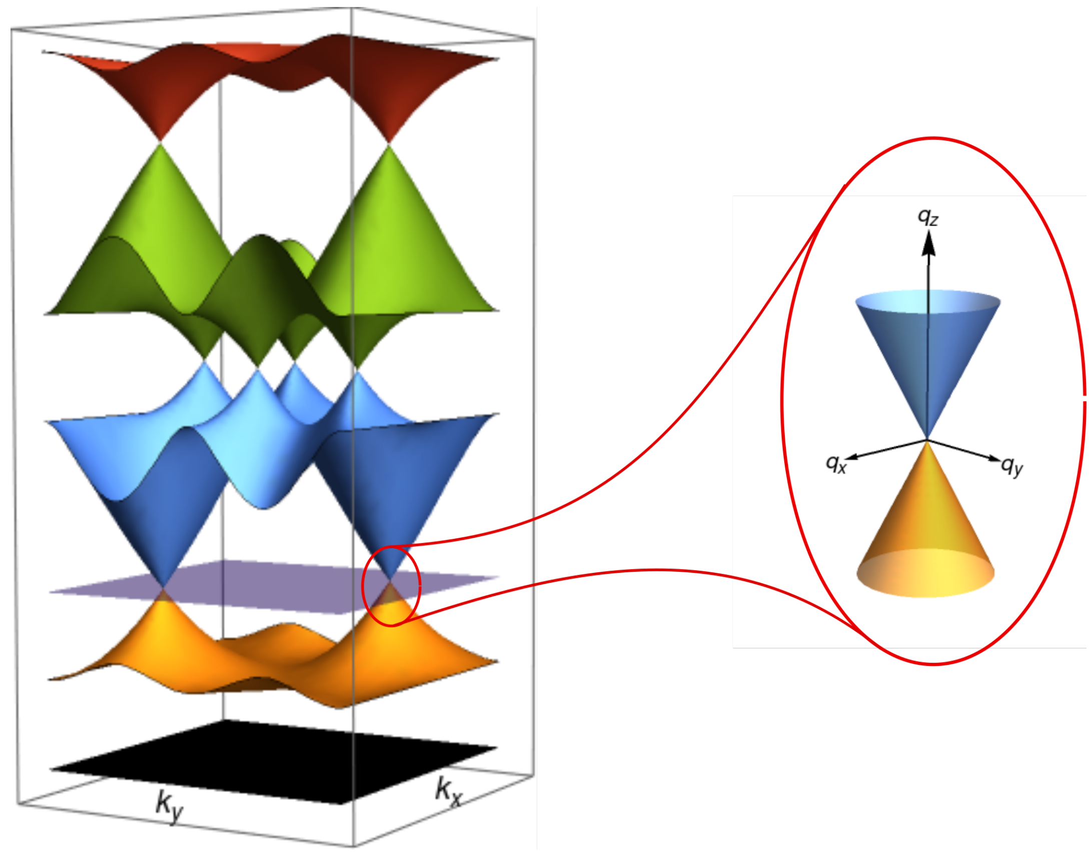




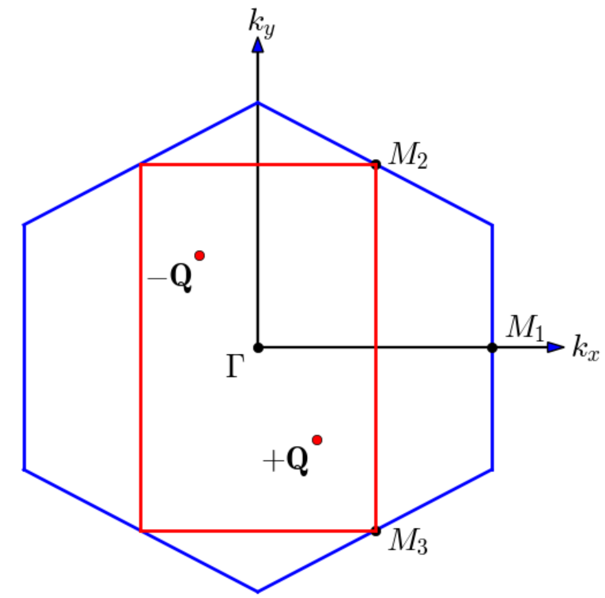
(a)



Hopping Hamiltonian and SU(4) Symmetry : Band structure



(a)



- Each band is four fold degenerate due to the manifest SU(4)
- for 1/4th filling there are two Dirac points at $\mathbf{Q} = \pm \left[\frac{\pi}{6}, -\frac{\pi}{2\sqrt{3}}\right]$
- Also similar case for 3/4th filling

Low energy Dirac Theory

The low energy physics is captured by the Dirac points and like graphene the low energy Hamiltonian is obtained by expanding about the two Dirac points

$$\chi_{SU(4)\,:\,1234;\; Valley\,:\,\pm}^{particle-hole\,:\,1/2}$$

χ : 16 component spinor

$$H_D = -iv_F \int d^2\mathbf{r}\ \chi^\dagger(\mathbf{r})(\alpha_x \partial_x + \alpha_y \partial_y)\chi(\mathbf{r})$$

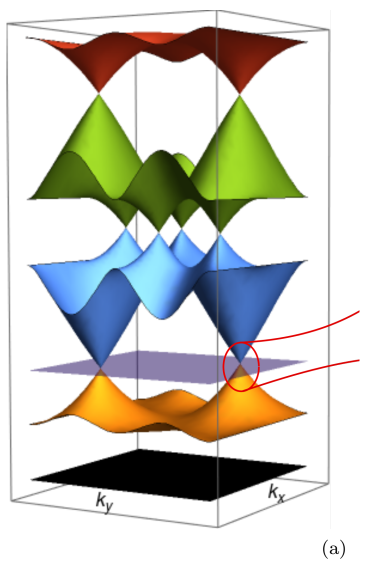
Introduce three matrices for the three spaces

$\Sigma_p \ \ (p = 0,1 \cdots 15) : 4 \times 4$ matrices that act in the SU(4) space

$\tau^a \ (a = 0,1,2,3)$: Pauli matrices acts in the valley space

$\sigma^a \ (a = 0,1,2,3)$: Pauli matrices acts in the particle-hole space

Emergent Global Symmetry : SU(8) Dirac fermions



[2304.07223, Mondal, Shenoy, SB]

$$\mathcal{S}_0 = \int d^2\mathbf{r}d\tau\ \bar{\chi}(\mathbf{r})(-i\partial_\mu\gamma^\mu)\chi(\mathbf{r})$$

$$SU(4) \otimes SU(2)$$

The SU(4) “flavour” symmetry arising from J=3/2

$$\Sigma_i \quad i = 1,\cdots,15$$

The “Chiral” symmetry (same as graphene) generated by

$$(\zeta_1, \zeta_2, \zeta_3)$$

(Combined rotations in valley and particle/hole space)

- **SU(8) Internal Symmetry : $\mathcal{P}_a = \Sigma_i\zeta_j$ (63 of them)**
- Emergent SO(2,1) Lorentz Invariance (Same as Graphene)
- CPT symmetries

SOC enhances the symmetry

Implementation of Microscopic Symmetries

$$\mathcal{S}_0 = \int d^2\mathbf{r}d\tau\ \bar{\chi}(\mathbf{r})(-i\partial_\mu\gamma^\mu)\chi(\mathbf{r})$$

IR Symmetry $SU(4) \otimes SU(2) \Rightarrow SU(8)$

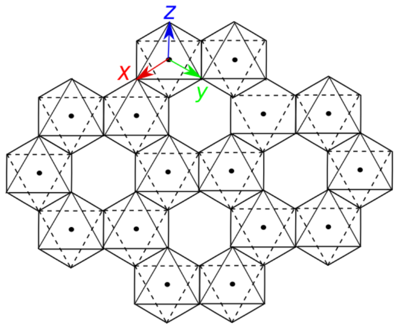
However the microscopic symmetry group is much smaller

Need to understand how the low energy fermions transform under microscopic symmetries

This is involved and at the same time interesting for the following reason

- There is a π-flux : Transformations are projective
- Due to spin-orbit coupling the spatial and spin transformations get intermixed

This leads to unconventional symmetry implementation on the low energy modes



Proximate Phases : microscopic Symmetries

[2304.07223, Mondal, Shenoy, SB]

- Time reversal symmetry, $\mathscr{T} \;:\; \chi(\mathbf{r}) \to \chi'(\mathbf{r}) = (i\Sigma_{13}) \otimes (\tau_1 \otimes \sigma_0)\, \chi(\mathbf{r}) \qquad [\mathscr{T}^2 = -1]$

- Two dimensional lattice translations of the honeycomb lattice : $\mathbf{T_1}, \mathbf{T_2}$:
  $\mathbf{T_1} \;:\; \chi(\mathbf{r}) \to \chi'(\mathbf{r}) = (-\mathbf{i}\boldsymbol{\Sigma}_{\mathbf{23}}) \otimes (-\mathbf{i}\tau_{\mathbf{2}} \otimes \sigma_{\mathbf{2}})\, \chi(\mathbf{r})$

- Rotations by angle by angle $2\pi/3$ about the centre of a honeycomb plaquette : C_3

- Rotations about the z axis by angle $\pi/3$ followed by a reflection about the plane of xy plane : $C_6\sigma_h$

- Rotations by angle π about the axes lying on the plane of the honeycomb lattice and passing through two opposite vertices of a honeycomb plaquette, C_2'

- Reflections about planes that are parallel to the z axis and that bisect angle between two consecutive C_2' axes, σ_d

- Inversion about the centre of a honeycomb plaquette, $\mathscr{I}$

All the symmetries non-trivially mixes the flavour and chiral sectors

Proximate Phases

$$\mathcal{S}_0 = \int d^2\mathbf{r}d\tau\ \bar{\chi}(\mathbf{r})(-i\partial_\mu\gamma^\mu)\chi(\mathbf{r})$$

$$H_{\text{int}} = \int d^2\mathbf{r}d^2\mathbf{r}'\ V_{\mu\nu\alpha\beta}(\mathbf{r}-\mathbf{r}')\chi_\mu(\mathbf{r})^\dagger\chi_\nu(\mathbf{r}')^\dagger\chi_\alpha(\mathbf{r})\chi_\beta(\mathbf{r}')$$

The semi-metal is perturbatively stable to short range 4-fermion interactions

But at large values of interactions fermion bilinears can condense

$-i\langle\bar{\chi}\mathscr{P}_a\chi\rangle$ **Charge Invariant for this talk**

[2304.07223, Mondal, Shenoy, SB]

possibly gapping out the fermions

Question : What are these masses ?

Proximate Phases : The symmetry of the fermion bilinear

$$-i\langle\bar{\chi}\mathscr{P}_a\chi\rangle$$

These break $SU(8)$ + Time Reversal in different ways $\Rightarrow$ 64 **ways (mass terms)**

- SU(8) Scalar : $-i\bar{\chi}\chi$
- SU(8) Adjoint multiplet :$-i\bar{\chi}\mathscr{P}_a\chi \qquad a = 1,\cdots,63$

Have same low energy correlations at the leading order

$\Rightarrow$ Many apparently unrelated orders naturally can compete

Including competition between the symmetry broken and SPTs

What are these phases ?

- (1) Scalar : $-i\bar{\chi}\chi$

[2304.07223, Mondal, Shenoy, SB]

- (63) Adjoint multiplet : $-i\bar{\chi}\mathscr{P}_a\chi$

$$\mathscr{P}_a = \Sigma_i \zeta_j$$

$\Sigma_i \quad i = 0,1,\cdots,15 \quad \text{SU(4) Flavour}$

$\zeta_j \quad j = 0,1,2,3 \quad \text{SU(2) Chiral}$

Group-1	$-i\bar{\chi}\chi$	1 : Scalar	(Integer Chern Insulator)
Group-2	$-i\bar{\chi}\Sigma_0\zeta_i\chi$	3 : Chiral Masses	(Charge Density wave)
Group-3	$-i\bar{\chi}\Sigma_i\zeta_0\chi$	15 : Flavour Masses	(Generalised QSH)
Group-4	$-i\bar{\chi}\Sigma_i\zeta_j\chi$	45 : Mixed Masses	Spin Density wave insulators and Semimetals

Break up (Irreducible representations) under the UV symmetries

Group-1

$$-i\bar{\chi}\chi$$

$$A^e_{1g} \otimes A^o_{2g} = A^o_{2g}$$

Group-2

$$-i\bar{\chi}\Sigma_0\zeta_i\chi$$

$$A^e_{1g} \otimes T^e_{1g} = T^e_{1g}$$

Group-3

$$-i\bar{\chi}\Sigma_i\zeta_0\chi$$

$$A^o_{2g} \otimes A^o_{2g} = A^e_{1g}$$

$$T^o_{2g} \otimes A^o_{2g} = T^e_{1g}$$

$$T^o_{1u} \otimes A^o_{2g} = T^e_{2u}$$

$$T^o_{2u} \otimes A^o_{2g} = T^e_{1u}$$

$$E^e_{u} \otimes A^o_{2g} = E^o_{u}$$

$$T^e_{1g} \otimes A^o_{2g} = T^o_{2g}$$

Group-4

$$-i\bar{\chi}\Sigma_i\zeta_j\chi$$

$$A^o_{2g} \otimes T^e_{1g} = T^o_{2g}$$

$$E^e_{u} \otimes T^e_{1g} = T^e_{1u} \oplus T^e_{2u}$$

$$T^e_{1g} \otimes T^e_{1g} = T^e_{1g} \oplus T^e_{2g} \oplus E^e_{g} \oplus A^e_{1g}$$

$$T^o_{2g} \otimes T^e_{1g} = T^o_{1g} \oplus T^o_{2g} \oplus E^o_{g} \oplus A^o_{2g}$$

$$T^o_{1u} \otimes T^e_{1g} = T^o_{1u} \oplus T^o_{2u} \oplus E^o_{u} \oplus A^o_{1u}$$

$$T^o_{2u} \otimes T^e_{1g} = T^o_{1u} \oplus T^o_{2u} \oplus E^o_{u} \oplus A^o_{2u}$$

A : *Singlet*

E : *Doublet*

T : *Triplet*

Break up (Irreducible representations) under the UV symmetries

Group-1	Group-2	Group-3	Group-4
$-i\bar{\chi}\chi$	$-i\bar{\chi}\Sigma_0\zeta_i\chi$	$-i\bar{\chi}\Sigma_i\zeta_0\chi$	$-i\bar{\chi}\Sigma_i\zeta_j\chi$
$A^e_{1g}\otimes A^o_{2g}=A^o_{2g}$	$A^e_{1g}\otimes T^e_{1g}=T^e_{1g}$	$A^o_{2g}\otimes A^o_{2g}=A^e_{1g}$	$A^o_{2g}\otimes T^e_{1g}=T^o_{2g}$
		$T^o_{2g}\otimes A^o_{2g}=T^e_{1g}$	$E^e_u\otimes T^e_{1g}=T^e_{1u}\oplus T^e_{2u}$
		$T^o_{1u}\otimes A^o_{2g}=T^e_{2u}$	$T^e_{1g}\otimes T^e_{1g}=T^e_{1g}\oplus T^e_{2g}\oplus E^e_g\oplus A^e_{1g}$
		$T^o_{2u}\otimes A^o_{2g}=T^e_{1u}$	$T^o_{2g}\otimes T^e_{1g}=T^o_{1g}\oplus T^o_{2g}\oplus E^o_g\oplus A^o_{2g}$
		$E^e_u\otimes A^o_{2g}=E^o_u$	$T^o_{1u}\otimes T^e_{1g}=T^o_{1u}\oplus T^o_{2u}\oplus E^o_u\oplus A^o_{1u}$
		$T^e_{1g}\otimes A^o_{2g}=T^o_{2g}$	$T^o_{2u}\otimes T^e_{1g}=T^o_{1u}\oplus T^o_{2u}\oplus E^o_u\oplus A^o_{2u}$

$A : Singlet$

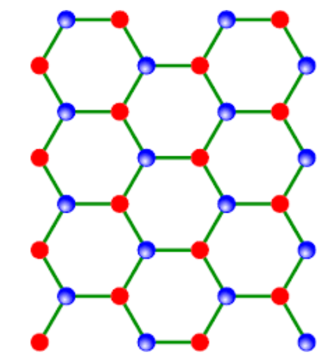
$E : Doublet$

$T : Triplet$

Singlet and Chiral Irreps

Three CHIRAL Masses $-i\langle \bar{\chi} \Sigma_0 \zeta_i \chi \rangle$

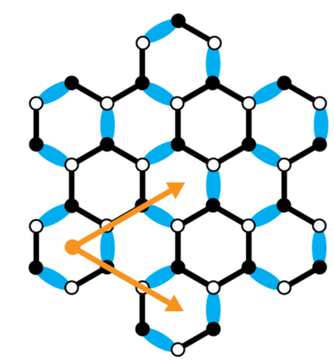
Such Chiral masses also exists for spinless graphene



In Graphene $3 = 1 \oplus 2$

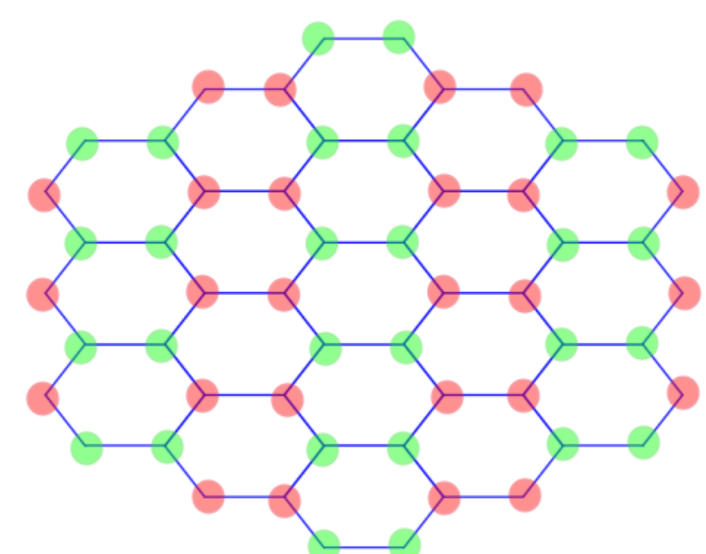
(1) Sub-lattice mass

(2) VBS (Kekule) mass



In the present case the Chiral triplet survives

$A^e_{1g} \otimes T^e_{1g} = T^e_{1g}$ $\Rightarrow$ **Three Stripy CDW masses**



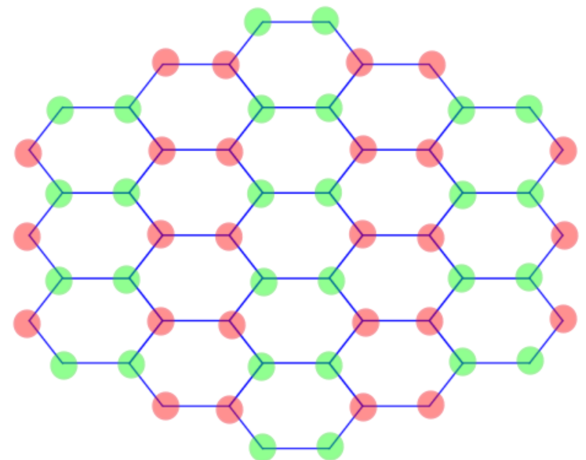
The same Lattice symmetries act differently at low energy

Break up (Irreducible representations) under the UV symmetries

Group-1	Group-2	Group-3	Group-4
$-i\bar{\chi}\chi$	$-i\bar{\chi}\Sigma_0\zeta_i\chi$	$-i\bar{\chi}\Sigma_i\zeta_0\chi$	$-i\bar{\chi}\Sigma_i\zeta_j\chi$
$A^e_{1g} \otimes A^o_{2g} = A^o_{2g}$	$A^e_{1g} \otimes T^e_{1g} = T^e_{1g}$	$A^o_{2g} \otimes A^o_{2g} = A^e_{1g}$	$A^o_{2g} \otimes T^e_{1g} = T^o_{2g}$
		$T^o_{2g} \otimes A^o_{2g} = T^e_{1g}$	$E^e_u \otimes T^e_{1g} = T^e_{1u} \oplus T^e_{2u}$
		$T^o_{1u} \otimes A^o_{2g} = T^e_{2u}$	$T^e_{1g} \otimes T^e_{1g} = T^e_{1g} \oplus T^e_{2g} \oplus E^e_g \oplus A^e_{1g}$
		$T^o_{2u} \otimes A^o_{2g} = T^e_{1u}$	$T^o_{2g} \otimes T^e_{1g} = T^o_{1g} \oplus T^o_{2g} \oplus E^o_g \oplus A^o_{2g}$
		$E^e_u \otimes A^o_{2g} = E^o_u$	$T^o_{1u} \otimes T^e_{1g} = T^o_{1u} \oplus T^o_{2u} \oplus E^o_u \oplus A^o_{1u}$
		$T^e_{1g} \otimes A^o_{2g} = T^o_{2g}$	$T^o_{2u} \otimes T^e_{1g} = T^o_{1u} \oplus T^o_{2u} \oplus E^o_u \oplus A^o_{2u}$

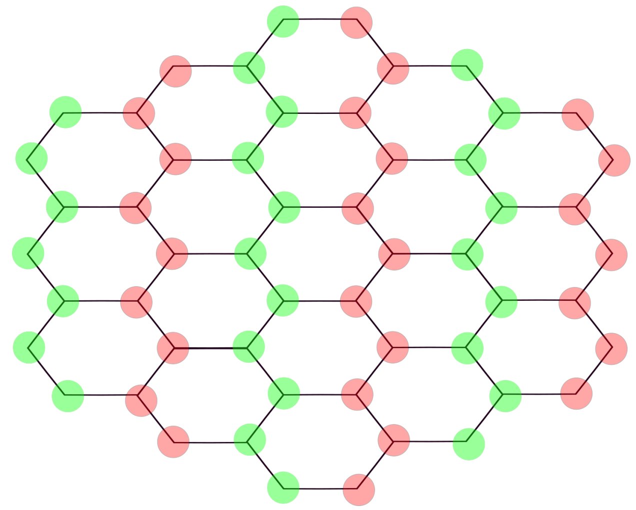
Density wave masses

$$A_{2g}^{o} \otimes T_{1g}^{e} = T_{2g}^{o}$$



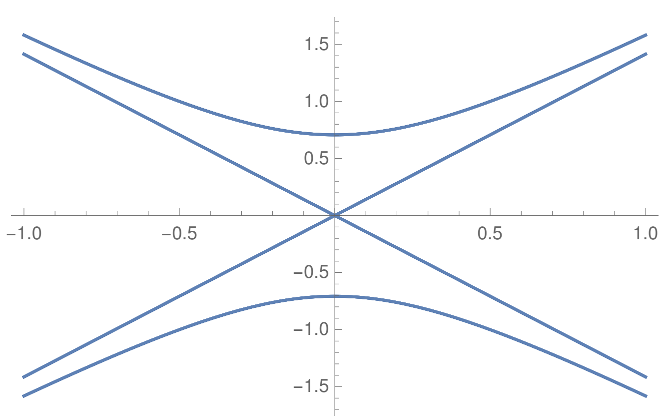
Stripy Octupolar DW

$$E_{u}^{e} \otimes T_{1g}^{e} = \boxed{T_{1u}^{e}} \oplus \quad T_{2u}^{e}$$



Zig-Zag Quadrupolar DW

Some of them : Density wave semi metals



A gapless Dirac mode is left at each valley

Doubly degenerate

protected by Inversion
and subgroup of SU(4)

Break up (Irreducible representations) under the UV symmetries

Group-1

$$-i\bar{\chi}\chi$$

$$A^{e}_{1g} \otimes A^{o}_{2g} = A^{o}_{2g}$$

Group-2

$$-i\bar{\chi}\Sigma_0\zeta_i\chi$$

$$A^{e}_{1g} \otimes T^{e}_{1g} = T^{e}_{1g}$$

Group-3

$$-i\bar{\chi}\Sigma_i\zeta_0\chi$$

$$A^{o}_{2g} \otimes A^{o}_{2g} = A^{e}_{1g}$$

$$T^{o}_{2g} \otimes A^{o}_{2g} = T^{e}_{1g}$$

$$T^{o}_{1u} \otimes A^{o}_{2g} = T^{e}_{2u}$$

$$T^{o}_{2u} \otimes A^{o}_{2g} = T^{e}_{1u}$$

$$E^{e}_{u} \otimes A^{o}_{2g} = E^{o}_{u}$$

$$T^{e}_{1g} \otimes A^{o}_{2g} = T^{o}_{2g}$$

Group-4

$$-i\bar{\chi}\Sigma_i\zeta_j\chi$$

$$A^{o}_{2g} \otimes T^{e}_{1g} = T^{o}_{2g}$$

$$E^{e}_{u} \otimes T^{e}_{1g} = T^{e}_{1u} \oplus T^{e}_{2u}$$

$$T^{e}_{1g} \otimes T^{e}_{1g} = T^{e}_{1g} \oplus T^{e}_{2g} \oplus E^{e}_{g} \oplus A^{e}_{1g}$$

$$T^{o}_{2g} \otimes T^{e}_{1g} = T^{o}_{1g} \oplus T^{o}_{2g} \oplus E^{o}_{g} \oplus A^{o}_{2g}$$

$$T^{o}_{1u} \otimes T^{e}_{1g} = T^{o}_{1u} \oplus T^{o}_{2u} \oplus E^{o}_{u} \oplus A^{o}_{1u}$$

$$T^{o}_{2u} \otimes T^{e}_{1g} = T^{o}_{1u} \oplus T^{o}_{2u} \oplus E^{o}_{u} \oplus A^{o}_{2u}$$

Time reversal and Inversion even

The generalised Spin-Hall masses

$$\mathcal{S} = -i\int d^2x d\tau \left[\bar{\chi}\gamma^\mu\partial_\mu\chi + \overrightarrow{\phi}\cdot\bar{\chi}\overrightarrow{M}\chi\right]$$

$\overrightarrow{\phi}$ **:** The triplet spin Hall order parameter

Gaps out all the fermions.

$$S_{CS} = i\frac{N_f}{2\pi}\ sgn(m_0)\ \int d^3x\ \epsilon^{\mu\nu\lambda}A_{c,\mu}\partial_\nu A_{o,\lambda}$$ $\Rightarrow$ **spin-Hall current**

Consider long wavelength fluctuations in $\overrightarrow{\phi}$

Integrate out the fermions to obtain the boson field theory :
O(3) Non linear sigma model

$$\mathcal{S}_{eff} = \frac{1}{2g}\int d^2x d\tau\,|\,\partial\overrightarrow{\phi}\,|^2$$

O(3) Non linear sigma model

$$\mathcal{S}_{eff} = \frac{1}{2g}\int d^2x d\tau |\partial \vec{\phi}|^2$$

However, the O(3) order parameter allows topological defects : Skyrmions

$$\mathcal{Q}_{topo}^{Sky} = \frac{1}{4\pi}\int d^2\mathbf{r}\ \vec{\phi} \cdot \partial_x \vec{\phi} \times \partial_y \vec{\phi}$$

Electric charge of skyrmions

$$\mathcal{Q}_e^{Sky} = eN_F \mathcal{Q}_{topo}^{Sky}$$

(elementary skyrmions carry 4e charge)

Consider sitting in the quantum spin-octuple Hall phase

[Goldstone, Wilczek (1960s); Abanov, Weigmann (2000s); Grover, Senthil (2006)]

[2304.07223, Mondal, Shenoy, SB]

Destroy it by proliferating and condensing Skyrmions.

The single fermion gap survives because the $\overrightarrow{\phi}$ is still locally non-zero

The resultant phase is therefore a charge 4e superconductor

$$\langle \chi\chi\chi\chi \rangle \neq 0$$

Cannot be obtained directly from a fermion bilinear condensation

Non-BCS

Break up (Irreducible representations) under the UV symmetries

Group-1

$$-i\bar{\chi}\chi$$

$$A^e_{1g} \otimes A^o_{2g} = A^o_{2g}$$

Group-2

$$-i\bar{\chi}\Sigma_0\zeta_i\chi$$

$$A^e_{1g} \otimes T^e_{1g} = T^e_{1g}$$

Group-3

$$-i\bar{\chi}\Sigma_i\zeta_0\chi$$

$$A^o_{2g} \otimes A^o_{2g} = A^e_{1g}$$

$$T^o_{2g} \otimes A^o_{2g} = T^e_{1g}$$

$$T^o_{1u} \otimes A^o_{2g} = T^e_{2u}$$

$$T^o_{2u} \otimes A^o_{2g} = T^e_{1u}$$

$$E^e_u \otimes A^o_{2g} = E^o_u$$

$$T^e_{1g} \otimes A^o_{2g} = T^o_{2g}$$

Group-4

$$-i\bar{\chi}\Sigma_i\zeta_j\chi$$

$$A^o_{2g} \otimes T^e_{1g} = T^o_{2g}$$

$$E^e_u \otimes T^e_{1g} = T^e_{1u} \oplus T^e_{2u}$$

$$T^e_{1g} \otimes T^e_{1g} = T^e_{1g} \oplus T^e_{2g} \oplus E^e_g \oplus A^e_{1g}$$

$$T^o_{2g} \otimes T^e_{1g} = T^o_{1g} \oplus T^o_{2g} \oplus E^o_g \oplus A^o_{2g}$$

$$T^o_{1u} \otimes T^e_{1g} = T^o_{1u} \oplus T^o_{2u} \oplus E^o_u \oplus A^o_{1u}$$

$$T^o_{2u} \otimes T^e_{1g} = T^o_{1u} \oplus T^o_{2u} \oplus E^o_u \oplus A^o_{2u}$$

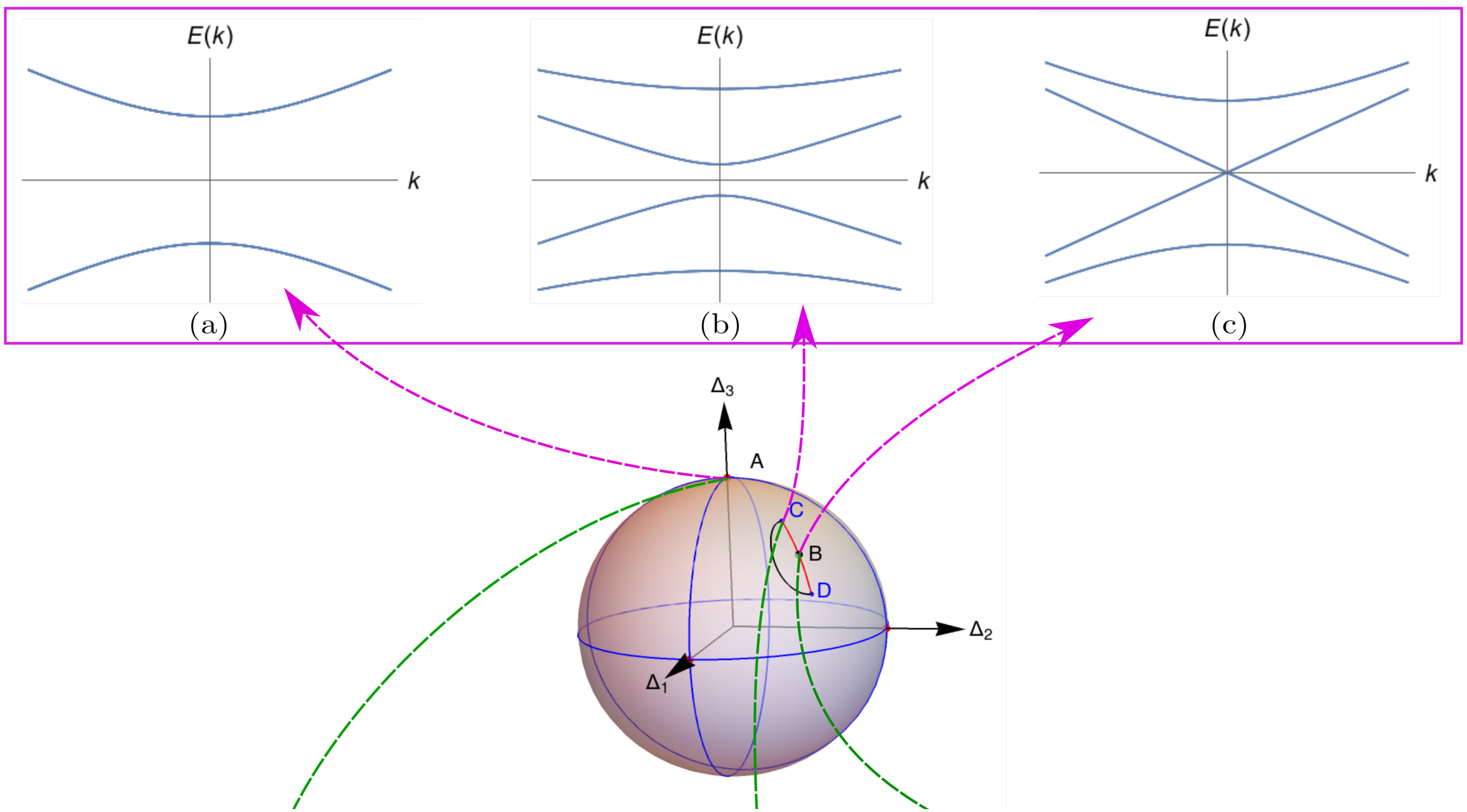
Two different time reversal and Inversion even triplets

Individual members are “non-compatible”

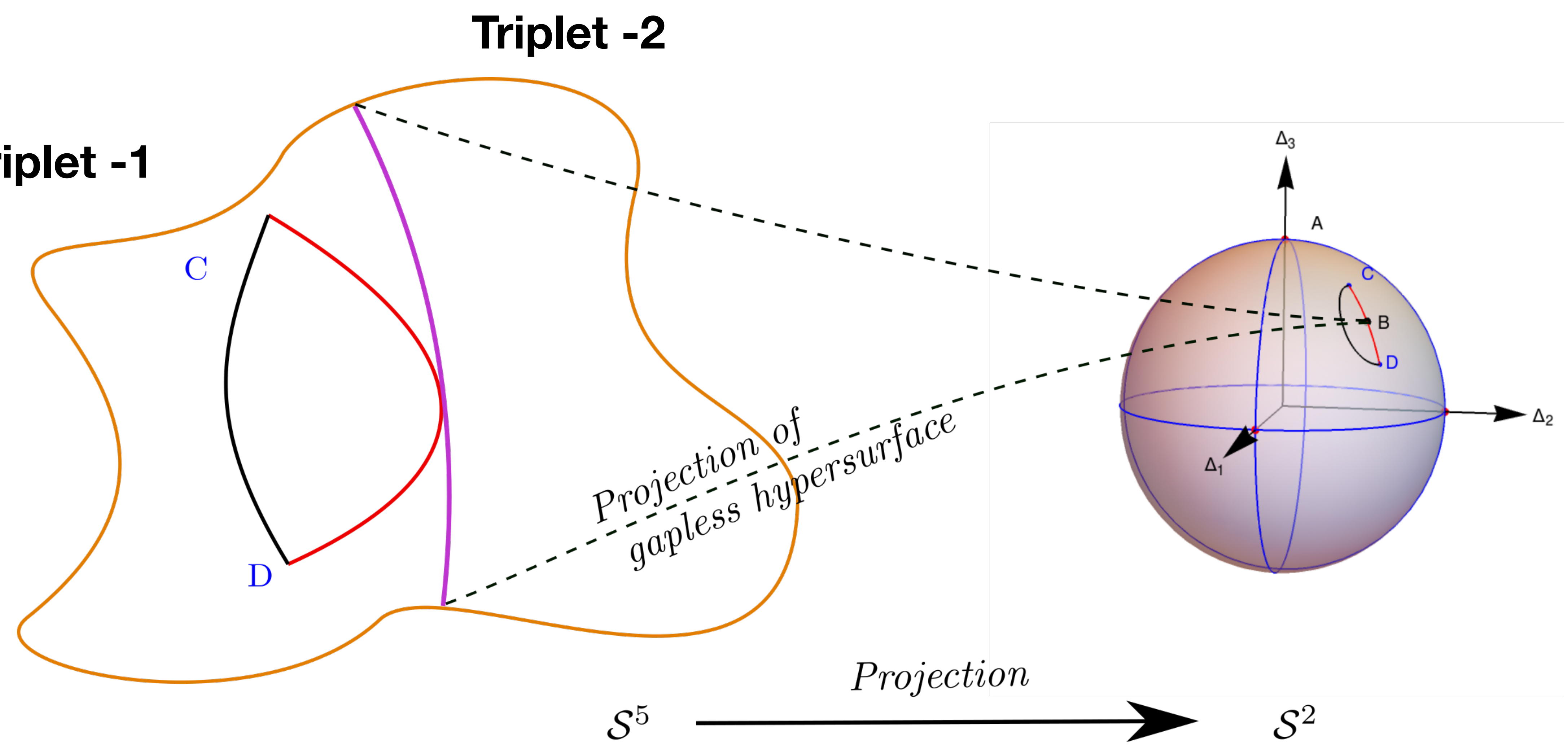
$$\{\Sigma_i, \Sigma_j\} \neq 0$$

$$\mathcal{S} = -\,i \int d^2x d\tau \left[\bar{\chi}\gamma^{\mu}\partial_{\mu} + \overrightarrow{\phi} \cdot \bar{\chi}\overrightarrow{M}\chi\right]$$

For each individual triplet we can parametric the masses on the sphere, S^2



The special isolated points are gapless ⇒ Protected by UV symmetries.



The special isolated points are gapless ⇒ Higher symmetry

⇒ Unnecessary "Multi-critical point" [Bi et. al, 2020]

[2304.07223, Mondal, Shenoy, SB]

Summary

- Microscopic symmetries can be embedded non-trivially in much larger IR symmetry group.
- This allows for many apparently unrelated orders to naturally compete and occur in close proximity
- Example : d^1 or d^3 systems on honeycomb lattice with SOC
    - SOC coupled Dirac fermions
    - Enlarged symmetry (good for materials where it is weakly broken)
    - Natural starting point to understand host of competing orders both symmetry broken and symmetry protected topological.
    - Interesting fallouts : 4e superconductivity, unconventional critical points
- Materials ? $ZrCl_3$ …….. ?

Thank You