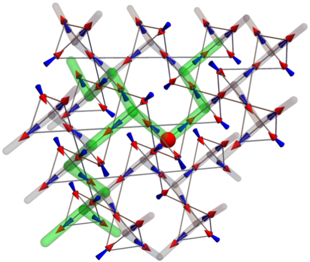
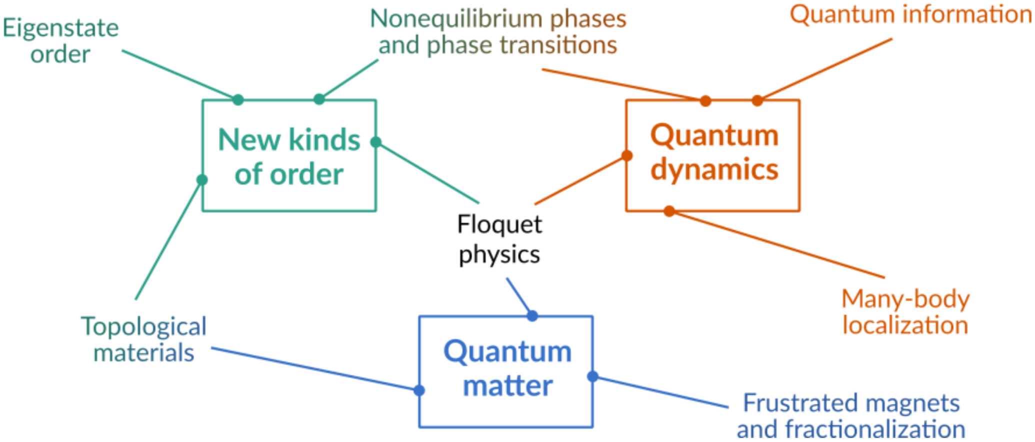
Dynamical fractal in a clean magnet

Roderich Moessner

J. Hallen, S. Grigera, A. Tennant, C. Castelnovo, R.M.

Science **378**, 1218 (2022)+unpublished





Eigenstate order
Nonequilibrium phases and phase transitions
Quantum information
New kinds of order
Quantum dynamics
Floquet physics
Many-body localization
Topological materials
Quantum matter
Frustrated magnets and fractionalization

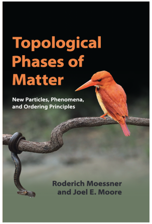
Outline

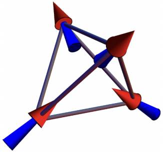
Emergent gauge fields in condensed matter

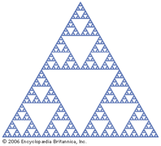
- ubiquitous
  - often arise from constraints
- physical consequences?
  - very rich, currently being explored topological physics

Spin ice: emergent QED

- history and material
- effective theory and consequences
  - emergence of dynamical fractal
  - persistent dynamical dichotomy
  - strong-coupling QED







VOLUME 79, NUMBER 13 PHYSICAL REVIEW LETTERS 29 SEPTEMBER 1997

Geometrical Frustration in the Ferromagnetic Pyrochlore $Ho_2Ti_2O_7$

M. J. Harris,[1] S. T. Bramwell,[2] D. F. McMorrow,[3] T. Zeiske,[4] and K. W. Godfrey[5]
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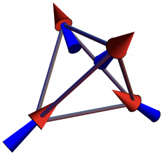
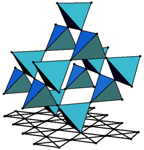
[2]*Department of Chemistry, University College London, 20 Gordon Street, London, WC1H 0AJ, United Kingdom*

Spin ice compounds Dy/$Ho_2Ti_2O_7$

- local [111] crystal field $\sim$ 200 K
- Ising spins $\sigma = \pm 1$
- classical spins (15/2 and 8)
  - magnetic moment $|\vec{\mu}| \approx 10\,\mu_B$





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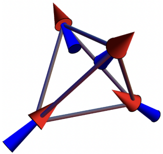
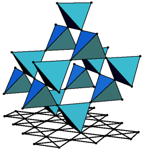




Ising model on pyrochlore lattice Anderson 1956

- ice rule: two-in two-out
- extensive ground state degeneracy
  - topological magnet

Classically: emergent magnetostatics

- fractionalised excitations–magnetic monopoles

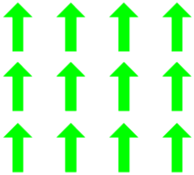
Quantumly: emergent QED

- strong coupling: $\alpha_e \sim 0.1 \gg 1/137$

Is spin ice ordered or not? Henley; Huse et al.; Hermele et al.

No order as in ferromagnet

- extensive degeneracy



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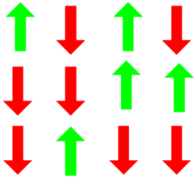
Not disordered like a paramagnet



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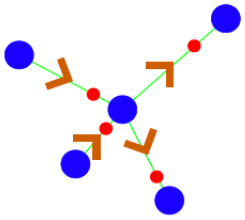
Not disordered like a paramagnet

- ice rules $\Rightarrow$ conservation law



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Not disordered like a paramagnet

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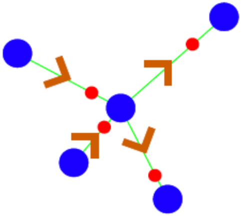
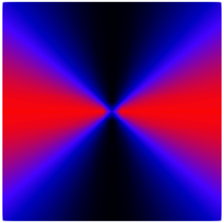
Magnetic moments $\vec{\mu}_i \Leftrightarrow$ (lattice) 'flux'

- Ice rules $\Leftrightarrow \nabla \cdot \vec{\mu} = 0 \;\Rightarrow\; \vec{\mu} = \nabla \times \vec{A}$
- Local constraint
  $\Rightarrow$ emergent gauge structure
    $\rightarrow$ algebraic spin correlations
    $\rightarrow$ 'bow-tie' structure factor

Effective action: $\mathcal{S} = (K/2) \int d^3 r |\nabla \times \vec{A}|^2$

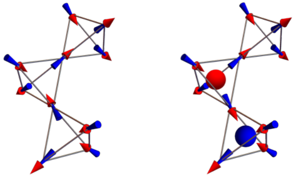


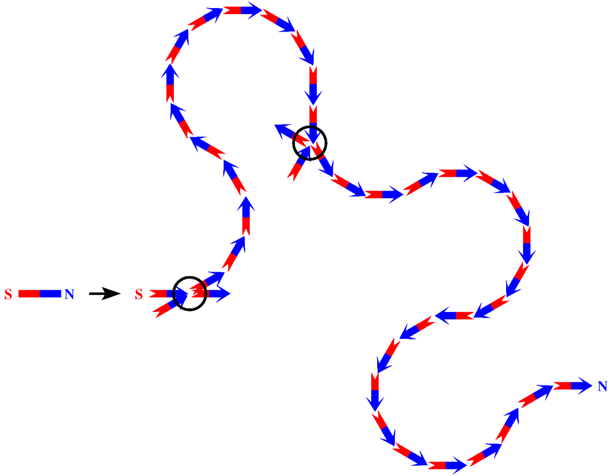


Fractionalisation: emergent magnetic monopoles

Flipping spin $\Rightarrow$ pair of defects

- separated by further spin flips





Magnetic Coulomb interaction

$$E(r) = -\frac{\mu_0}{4\pi} q_m^2 / r$$

$$q_m = 2|\vec{\mu}|/a_d \approx q_D/8000$$

- deconfined monopoles

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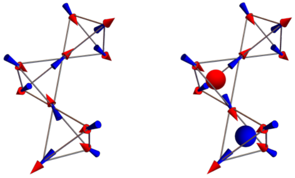

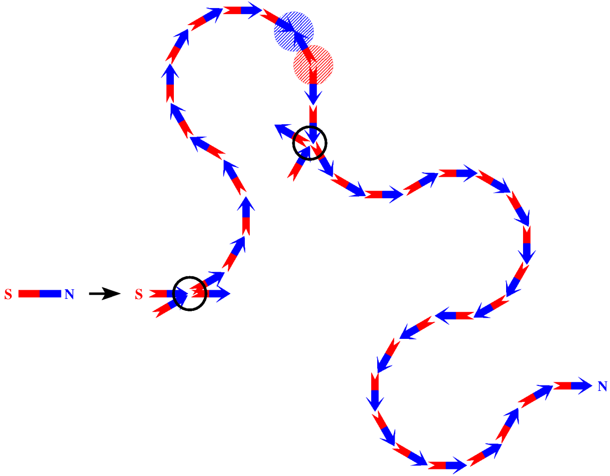

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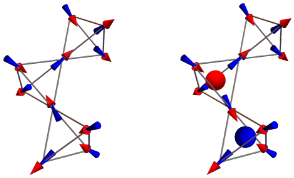

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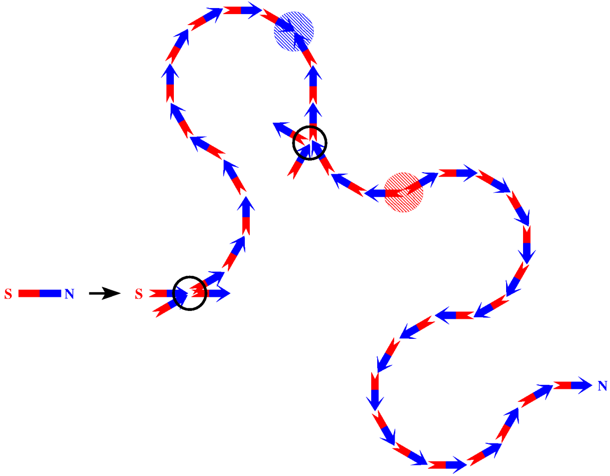


Flipped spins =(observable) ‘Dirac string’

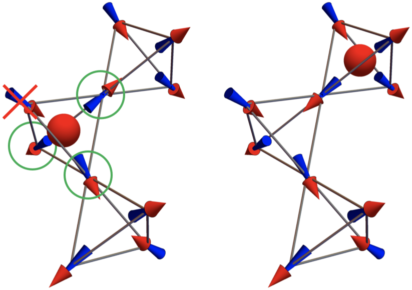
Standard model of classical spin ice dynamics Ryzhkin; Jaubert+Holdsworth

Spin flip = monpole motion

- monopoles sparse at low T

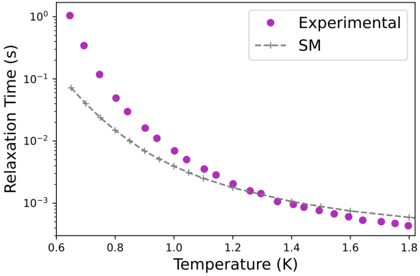
Incoherent 'Monte Carlo' dynamics

- Monte Carlo time $\propto$ real time
  - timescale set by attempt rate $1/\tau$
- hopping only possible in three directions
  - gauge field 'blocks' fourth direction



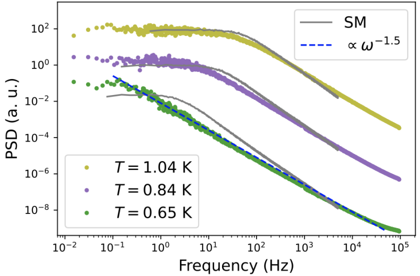
Experimental puzzles

Rapidly Diverging Relaxation Time



Previous explanations invoked extrinsic contributions (e.g. disorder, boundary effects).

Anomalous Magnetic Noise



SM → Lorentzian

Experiments → anomalous

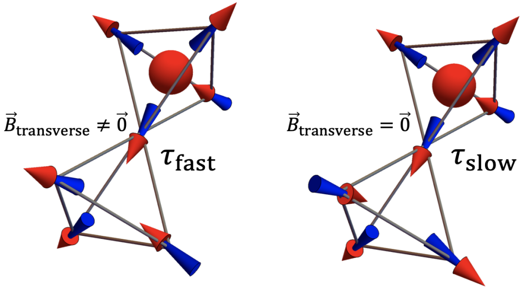
Experimental results from: A. M. Samarakoon, *et al.*, *Proceedings of the National Academy of Sciences* 119, e2117453119 (2022).

Beyond the standard model of classical spin ice dynamics

1/3 of all spins experience no net field

- lower spin flip attempt rate $1/\tau_{\text{slow}}$
- in $Dy_2Ti_2O_7$, $\tau_{\text{slow}}/\tau_{\text{fast}} \approx 1000$
  - we use $\tau_{\text{slow}}/\tau_{\text{fast}} = \infty$

revisit experimental puzzles with this model!



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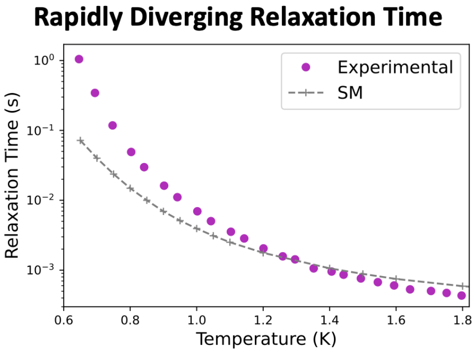
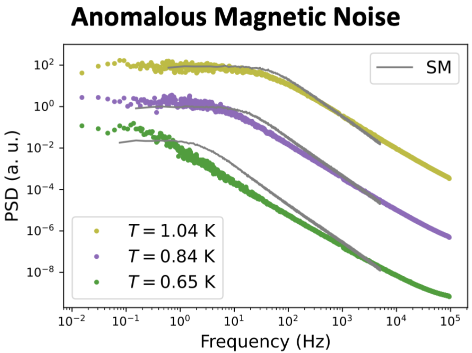
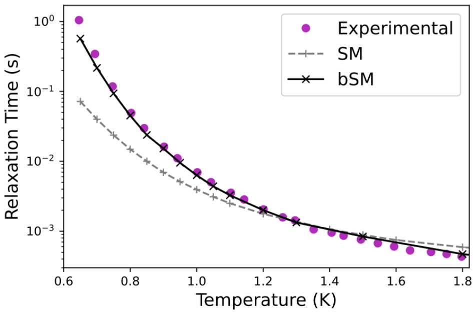




Experimental puzzles resolved

Rapidly Diverging Relaxation Time



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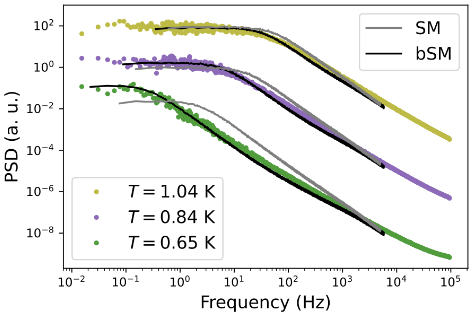

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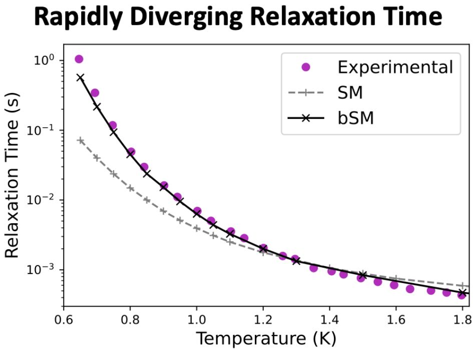
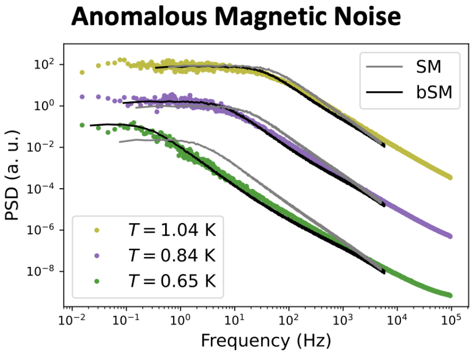




Fits: $\tau_{\mathrm{SM}} = 200\mu s$; $\tau_{\mathrm{bSM,fast}} = 85\mu s$

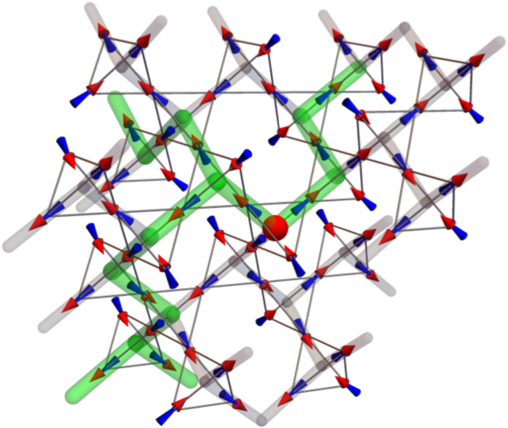
Explanation: critical percolation

Monopoles hop on bond-diluted diamond lattice

- average coordination: 2
  - close to percolation transition

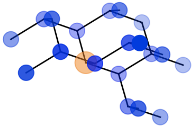
Random walk on percolation cluster

- looks subdiffusive when embedded in 3D
  - subdiffusion exponent yields anomalous noise exponent
- observable on short-medium timescales
  - invisible statically/thermodynamically

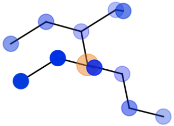


Emergent dynamical fractal in real space

SM

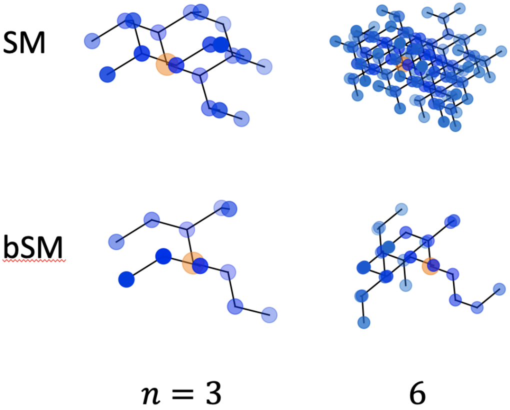


bSM



$n = 3$

Emergent dynamical fractal in real space



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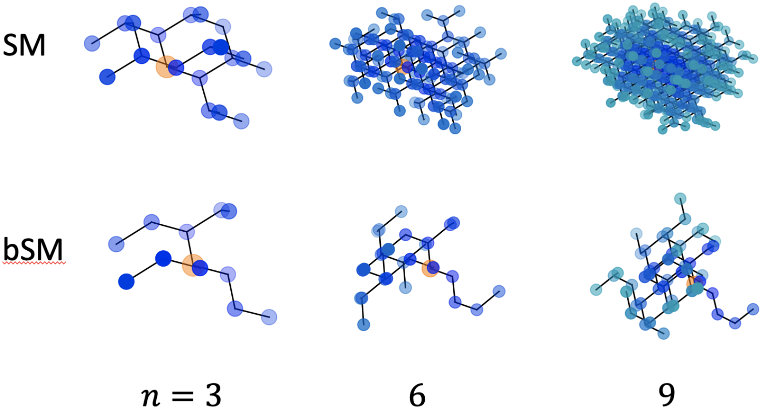

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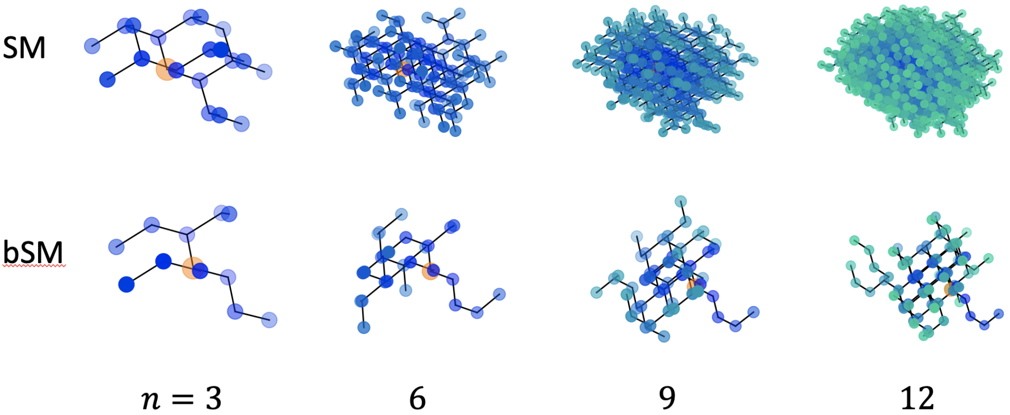
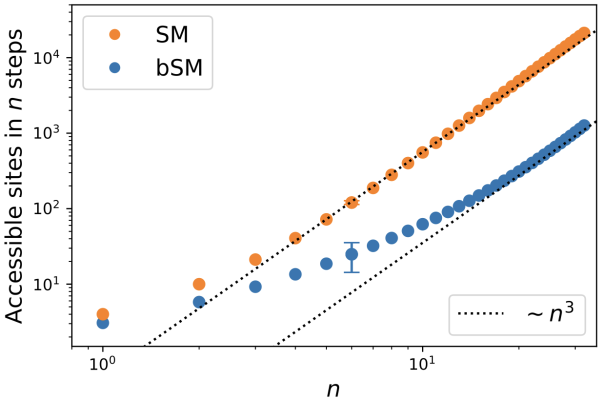


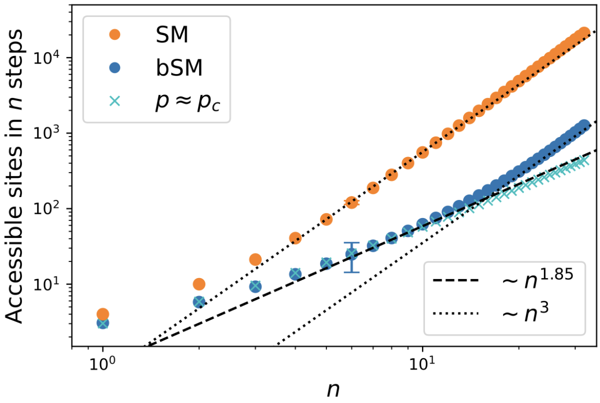
Cluster growth



Percolation theory:

$$S \sim \begin{cases} n^{1.85}, & n < n_\xi \\ n^3, & n > n_\xi \end{cases}$$

Cluster growth



Percolation theory:

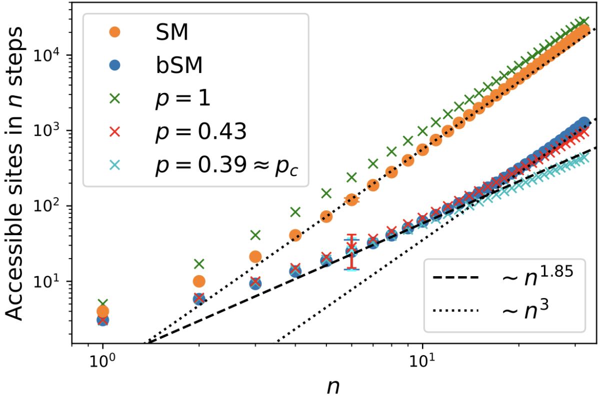
fractal exponent

$$S \sim \begin{cases} n^{1.85}, & n < n_\xi \\ n^3, & n > n_\xi \end{cases}$$

Fractal up to $n_\xi \approx 14$!

bSM monopoles can reach ~130/2000 sites in 14 steps

Cluster growth



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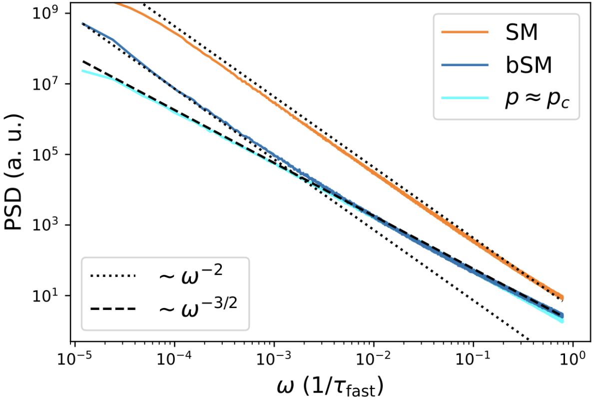
Monopole noise



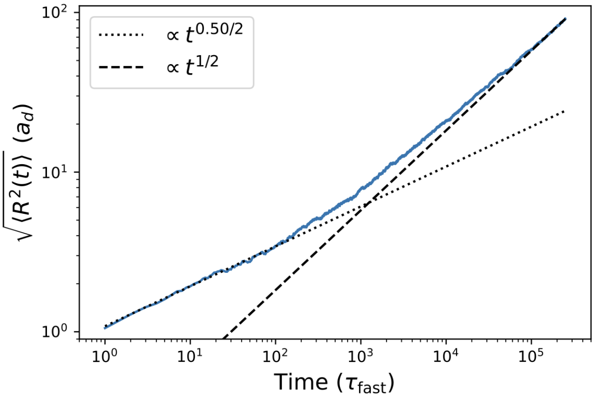
Percolation theory:

$$\text{PSD} \sim \begin{cases} \omega^{-2}, & \omega < \omega_\xi \\ \omega^{-1.50}, & \omega > \omega_\xi \end{cases}$$

fractal exponent

Explains anomalous exponent seen in experiments!

Monopole subdiffusion



Percolation theory:

fractal exponent

$$\langle R^2(t)\rangle \sim \begin{cases} t^{0.50}, & t < t_\xi \\ t, & t > t_\xi \end{cases}$$

Subdiffusive monopole motion on timescales up to $t_\xi \approx 10^3\ \tau_{\text{fast}}$.

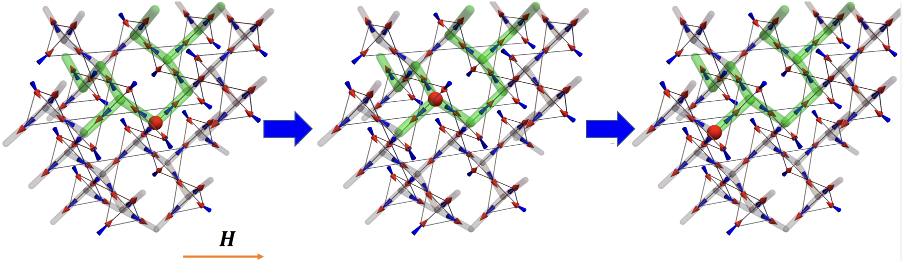
Nonequilibrium: polarisation and rearrangement current

Polarisation current: monopole motion builds up magnetisation

- comparatively slow, τ_1

Rearrangement current: change in field liberates from, and drives into new, obstacles

- comparatively fast, τ_2
- only present bSM: same origin as fractal, but distinct phenomenon



persistent dichotomy

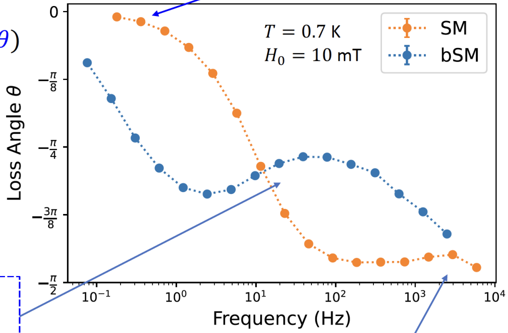
Characteristic frequency dependence of loss angle

Drive: $\boldsymbol{H}(t) = \hat{\boldsymbol{x}}\, H_0 \sin(2\pi\nu t)$

Response: $\boldsymbol{M}(t) = \hat{\boldsymbol{x}}\, M_0 \sin(2\pi\nu t + \theta)$

Loss angle: $\tan(\theta) = \dfrac{\chi''}{\chi'}$

Low Frequency – equilibrium maintained



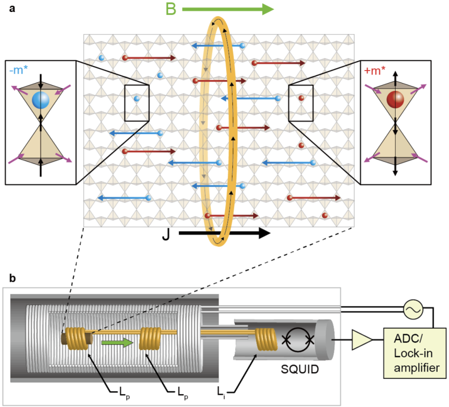
Intermediate frequency – Monopoles leaving dead-ends as field magnitude drops cause rearrangement currents opposing the field

High frequency – completely out of phase

Out-of-equilibrium experiments S. Davis group, Oxford/Cork

New set of experiments

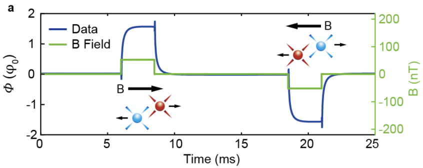
- time-dependent fields
- very sensitive
  - small signals
  - good time resolution
- intermediate $T \geq 1.7K$ regime

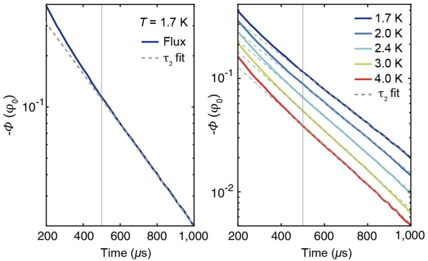


Out-of-equilibrium experiments: field quench work in progress

Magnetisation responds on two timescales

- fully consistent with modeling
  - ratio $\tau_1/\tau_2 \approx 4$
- persists to high $1.7K \leq T \leq 4K$
  - above spin ice proper





The frontier: quantum spin ice

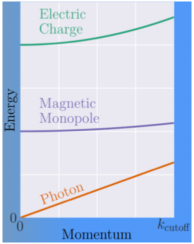
Very simple: nearest-neighbour Ising magnet + loop flip: $W_{\hexagon} = |\circlearrowleft\rangle\langle\circlearrowright|$:

$$H_{\mathrm{QSI}} = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z - g \sum_{\hexagon} \left(W_{\hexagon} + W_{\hexagon}^{\dagger} \right)$$

Topological 3+1D quantum spin liquid – effective theory: QED

- emergent electric/magnetic charges; photons (tunable c_{QSI})
- strong and tunable coupling: $\alpha_e \approx 0.1 \gg 1/137$
  - very different from our universe–but largely unknown
- Cerenkov radiation; constrained (quantum) diffusion; ...

Quasiparticle coherence?



Fine-structure constant

VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS 11 AUGUST 1980

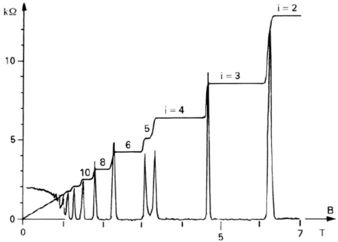
New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing

Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France

Familiar in topological condensed matter physics

- IQHE: same as in high-energy physics
- dimensionless
  - strength of electron-photon interaction

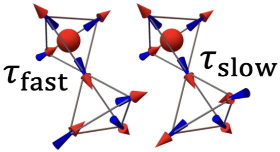


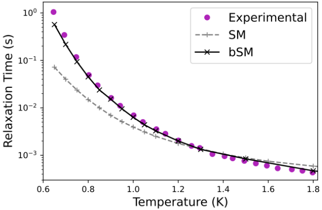
Value in quantum spin ice

- different and tunable

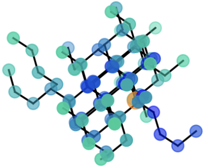
Dynamical fractal in spin ice: how things fit together

Bimodal distribution of internal transverse fields proves crucial to spin ice dynamics. *Thermodynamics unaffected.*





Explains dynamical properties of spin ice as a consequence of *intrinsic* effects.



Subdiffusion on emergent fractal structure in a disorder-free bulk crystal probed in uniform magnetisation dynamics.