

# Dynamics of the $S=1/2$ Heisenberg Antiferromagnet on the Triangular Lattice

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## (i) Dynamical structure factor

[Drescher, Vanderstraeten, Moessner, FP, arXiv:2209.03344 (in print)]



Drescher



Vanderstraeten



Moessner

## (ii) Spin-Peierls instability

[Seifert, Willsher, Drescher, FP, Knolle, arXiv:2307.12295]



Seifert



Willsher



Drescher



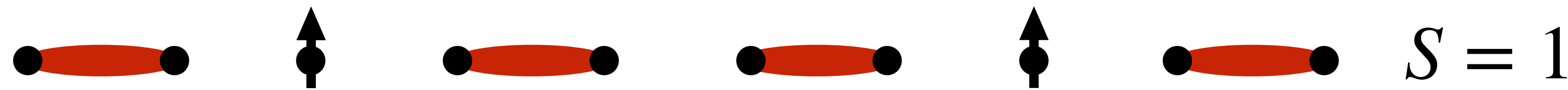
Knolle



Fractionalization and Emergent Gauge Fields in Quantum Matter  
ICTP Dec. 5 2023

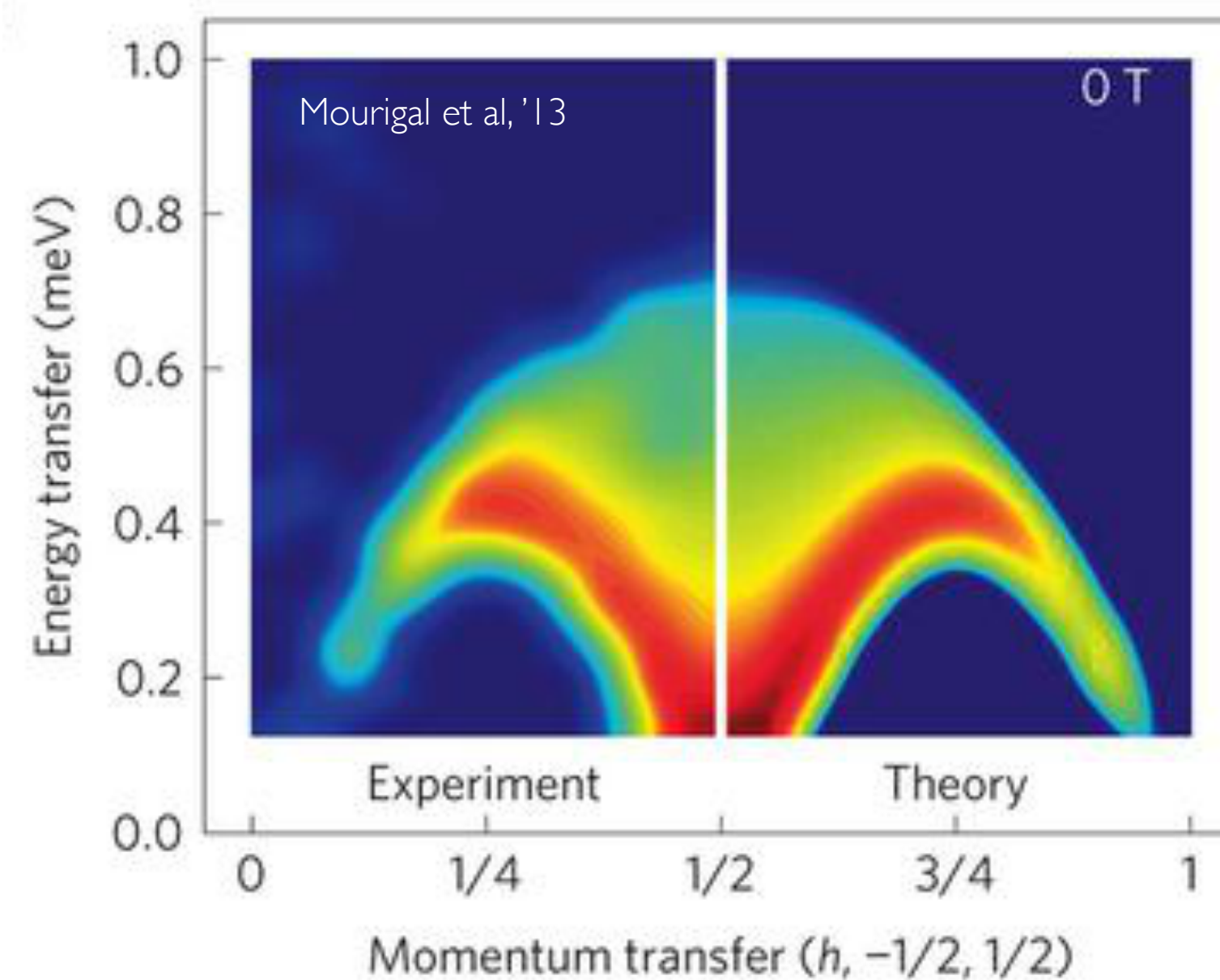
# Dynamical structure factor of Quantum Spin Liquids (QSL)

**1D quantum spin liquid:** Fractional **spinon excitations** in the Heisenberg antiferromagnetic chain



Copper Sulphate

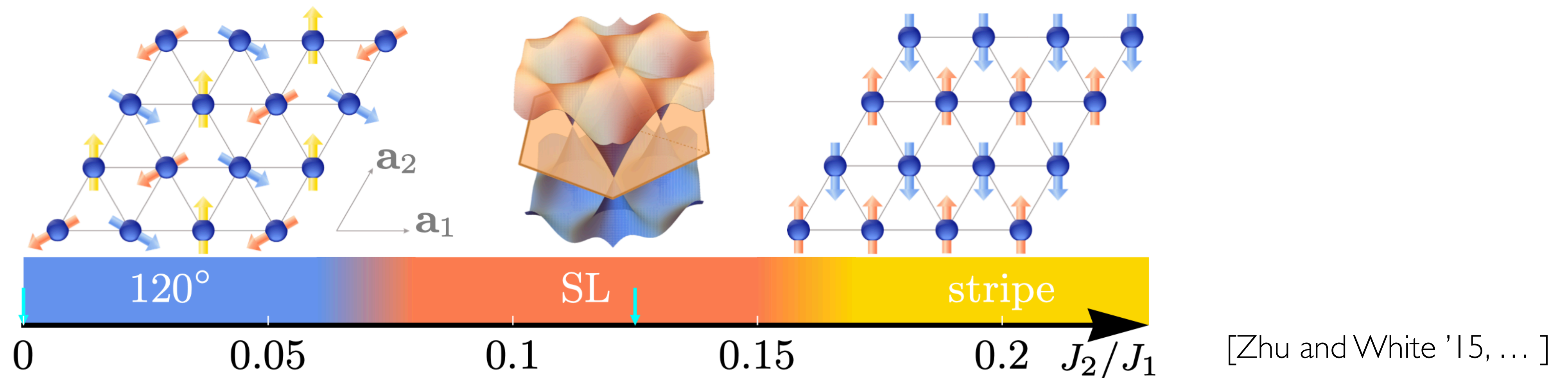
**Dynamical structure factor:** 
$$S(\vec{q}, \omega) = \sum_n \left| \langle \psi_n | S_{\vec{q}}^+ | \psi_0 \rangle \right|^2 \delta(\omega + \omega_0 - \omega_n)$$



# QSLs candidate: Triangular lattice Heisenberg materials

$J_1 - J_2$  model on the triangular lattice

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



► Many Candidate Materials:  $\text{Ba}_3\text{CoSb}_2\text{O}_9$ ,  $\text{YbMgGaO}_4$ ,  $\text{YbZn}_2\text{GaO}_5$ , ...

# Towards 2D quantum spin systems

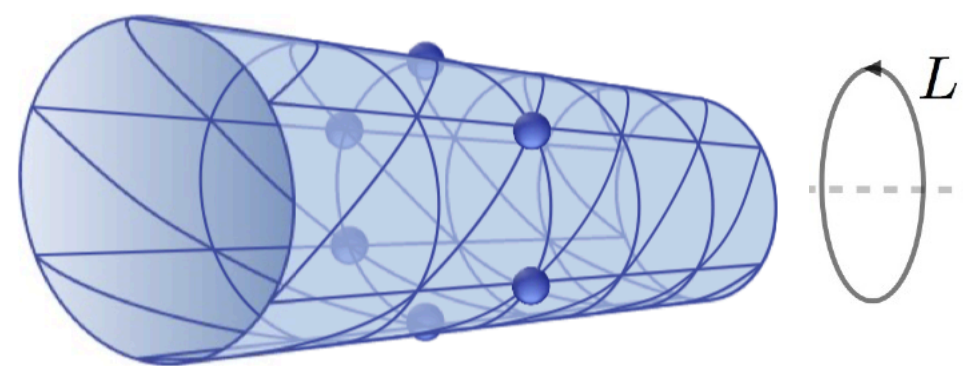
## Density Matrix Renormalization Group (DMRG)

to efficiently simulate 1D quantum systems [White '92, Schollwoeck '11]

$$|\psi\rangle = \sum_{\{j_n\}} \psi_{\sigma_1, \sigma_2, \dots, \sigma_N} |\sigma_1, \sigma_2, \dots, \sigma_N\rangle, \quad \sigma_n = \pm 1$$

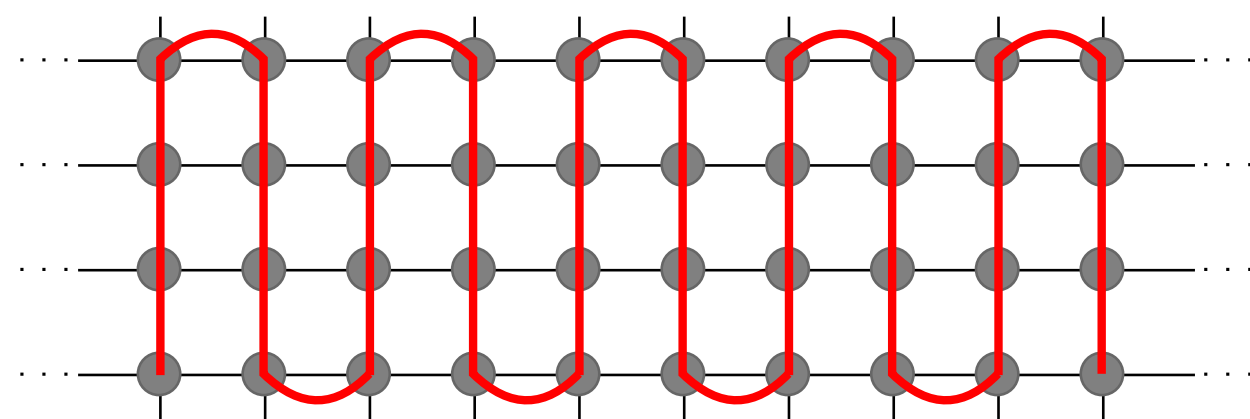
$$\psi_{\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_N} \approx \sum_{\alpha_1, \alpha_2, \dots, \alpha_{N-1}}^{\chi} A_{\alpha_1}^{\sigma_1} A_{\alpha_1, \alpha_2}^{\sigma_2} \dots A_{\alpha_{N-1}}^{\sigma_N} = \boxed{A - A - \dots - A - A} \quad \boxed{2^N \rightarrow N 2\chi^2}$$

## DMRG of 2D systems on cylinders



- ▶ 2D physics at the cost of **long range interactions** in 1D representation!

[Stoudenmire and White '12]



- ▶ Long ( $L_x \rightarrow \infty$ ) cylinders with moderate circumferences ( $L_y \approx 10$ )

# Dynamics of quantum spin systems

Numerical calculation of the **dynamical structure factor**

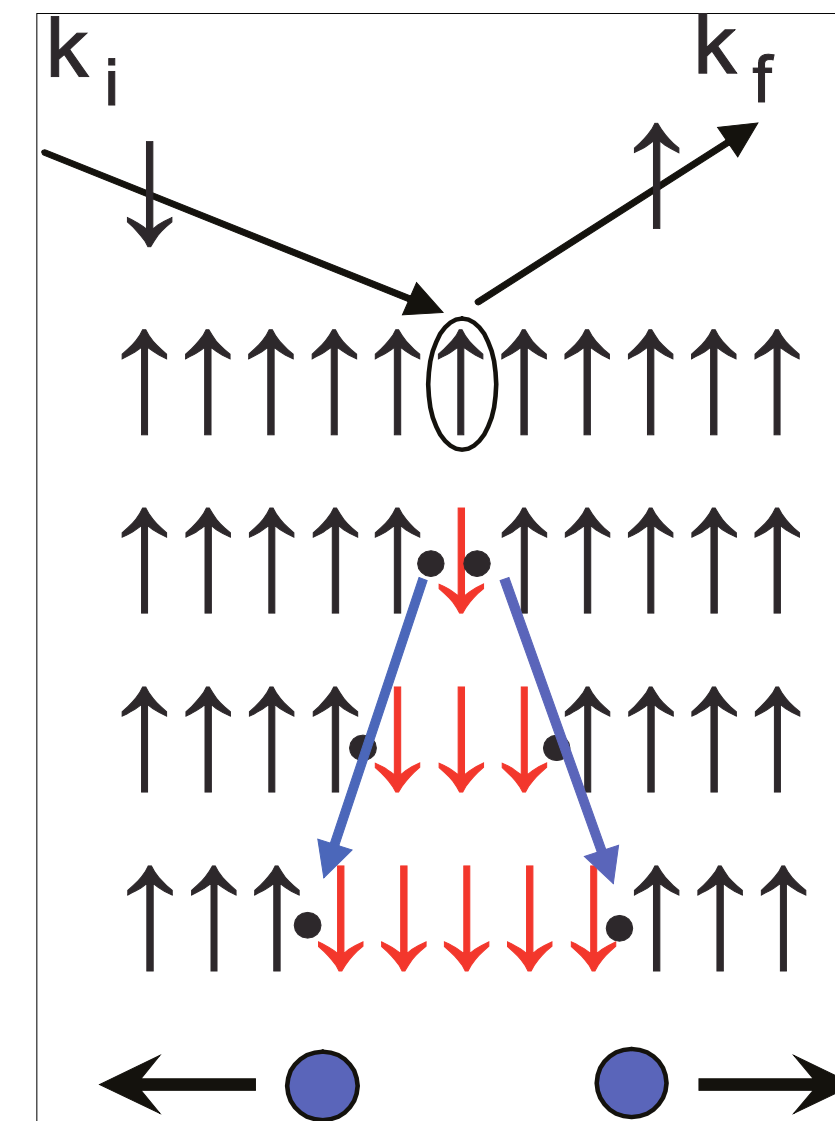
$$S(k, \omega) = \sum_x \int_{-\infty}^{\infty} dt e^{-i(kx + \omega t)} C(x, t)$$

with  $C(x, t) = \langle \psi_0 | S_x^+(t) S_0^-(0) | \psi_0 \rangle$

(1) Find the ground state  $|\psi_0\rangle$ : DMRG

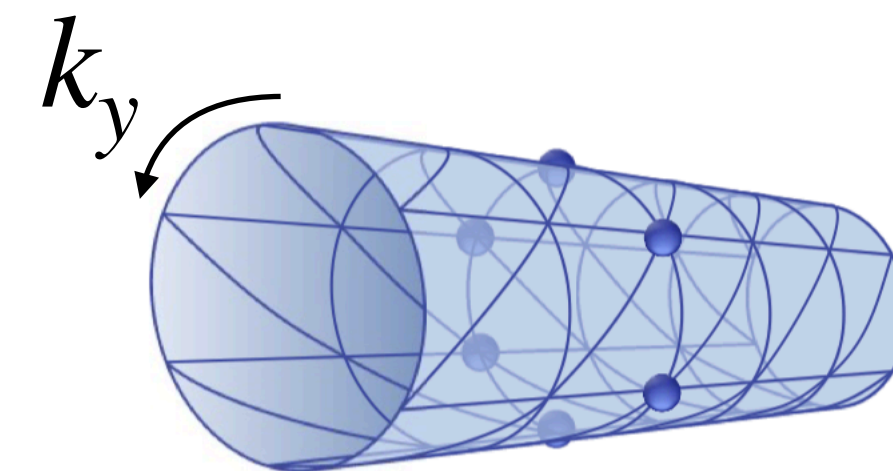
(2) Time evolve  $S_0^\alpha |\psi_0\rangle$  to obtain  $C(x, t)$

[Zaletel, Mong, Karrasch, Moore, FP '15]



**$k_y$  resolved**  $\hat{S}_{n_1}^-(k_y) = \frac{1}{\sqrt{L_y}} \sum_{j=0}^{L_y-1} e^{ij \cdot k_y} \hat{S}_{n_1 \cdot \mathbf{a}_1 + j \cdot \mathbf{a}_2}^-$

► Slow growth of entanglement: Long times!

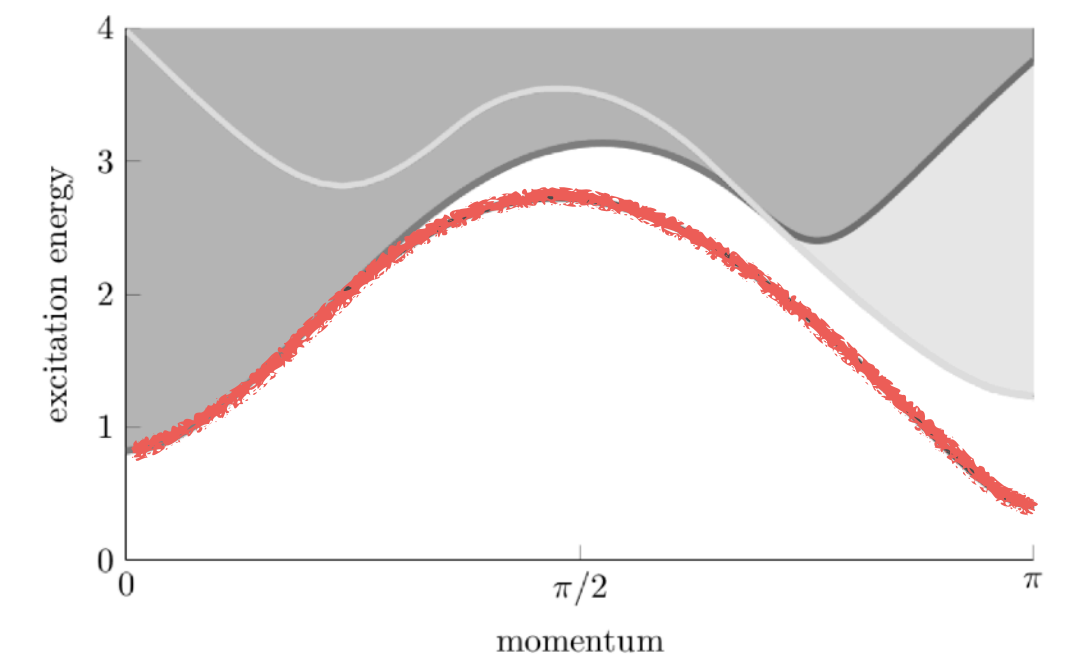


# Dynamics of quantum spin systems

## Quasi particle ansatz [Haegeman et al, '13]

$$|\psi_k(B)\rangle = \sum_n e^{ikn} \dots \begin{array}{c} \boxed{A_L} \\ | \\ \dots \end{array} \begin{array}{c} \boxed{A_L} \\ | \\ s_{n-1} \end{array} \begin{array}{c} \boxed{B} \\ | \\ s_n \end{array} \begin{array}{c} \boxed{A_R} \\ | \\ s_{n+1} \end{array} \begin{array}{c} \boxed{A_R} \\ | \\ \dots \end{array}$$

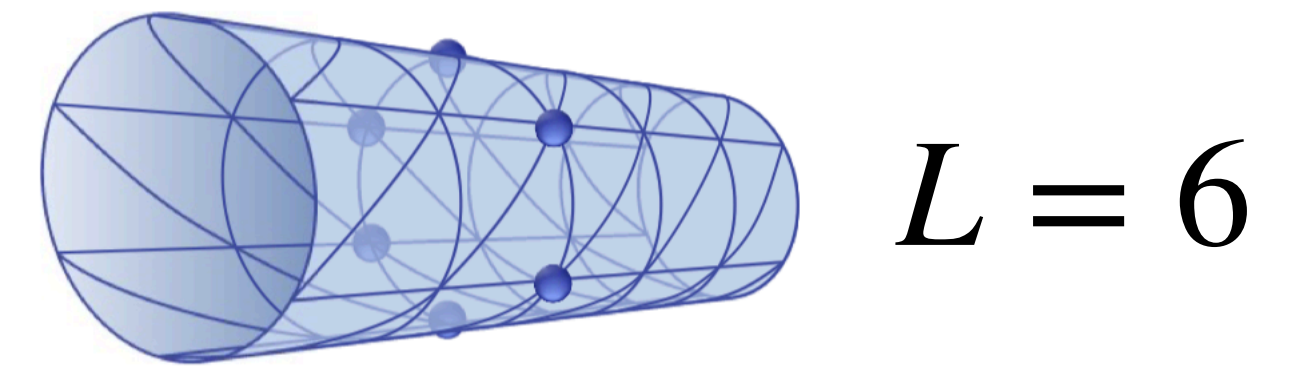
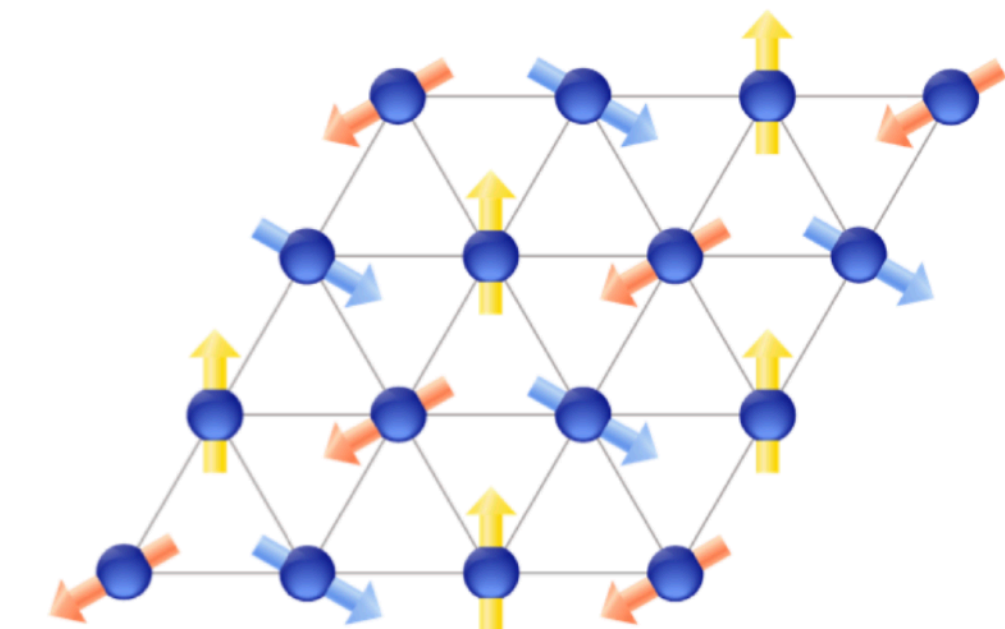
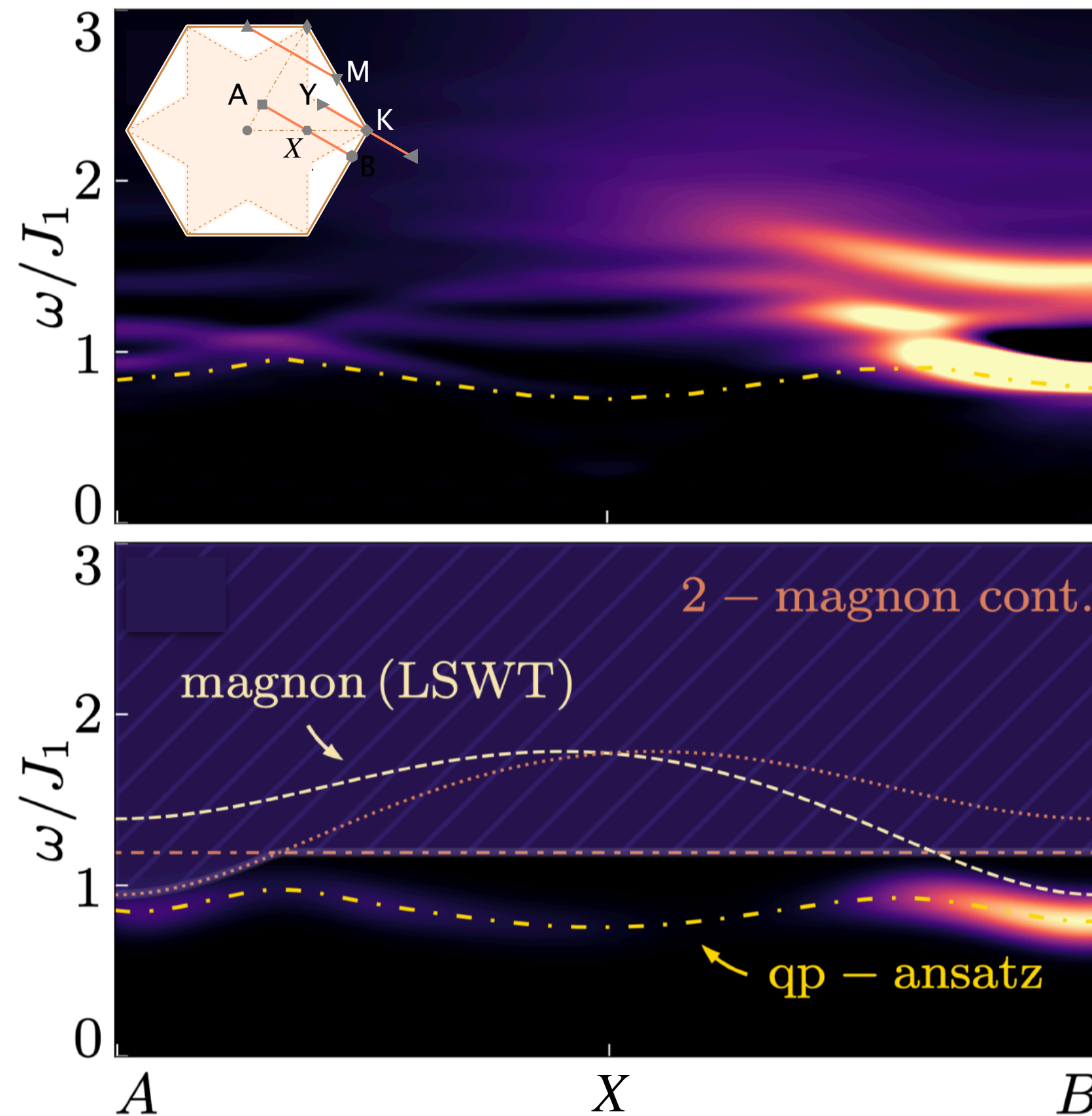
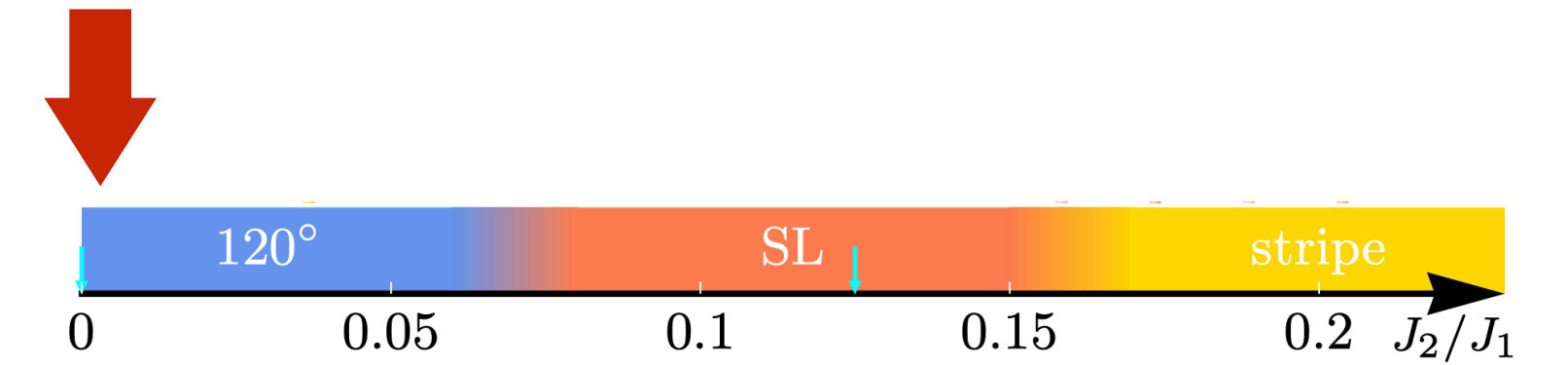
- ▶ Gauge fixing ensures a state **orthogonal to GS**
- ▶  $B$  contains all the variational parameters and perturbs the ground state



$$\begin{array}{c} \boxed{B} \\ | \\ \circ \text{ } l \\ | \\ \boxed{\bar{A}} \end{array} = \begin{array}{c} \boxed{A} \\ | \\ \circ \text{ } l \\ | \\ \boxed{\bar{B}} \end{array} = 0.$$

# Dynamics of the triangular lattice Heisenberg model

Heisenberg model with  $J_2 = 0$ :  $120^\circ$  order



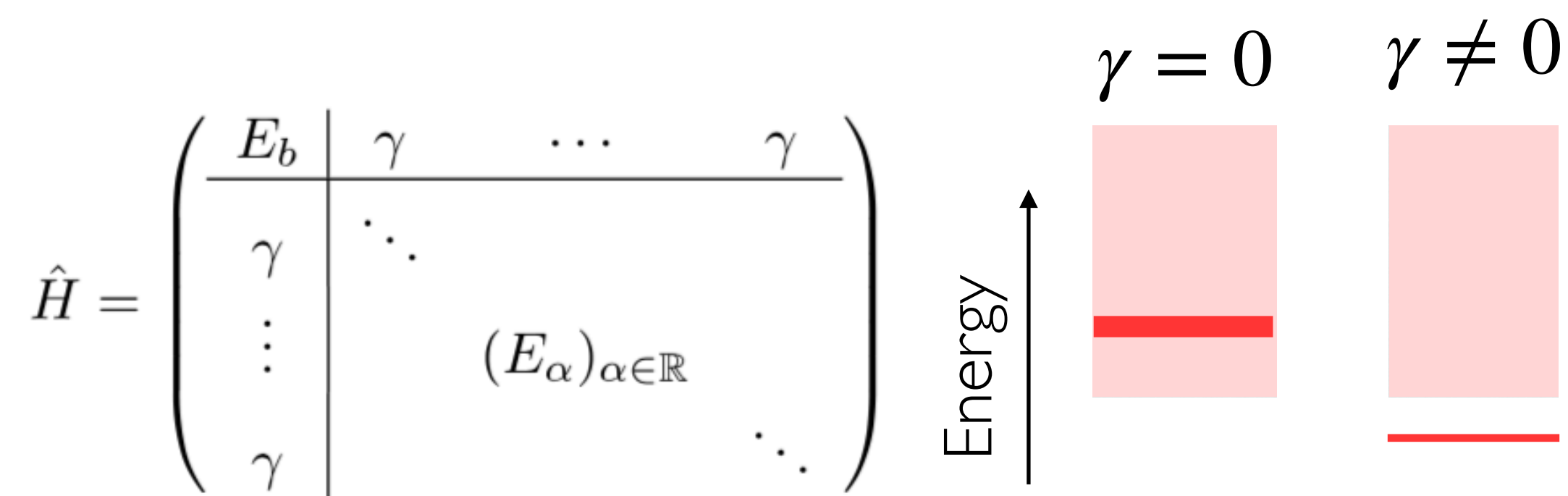
► Magnon repelled from continuum: **Decay prevented**

[Zheng et al. '06, Chernyshev et al, '06, '09, '13]

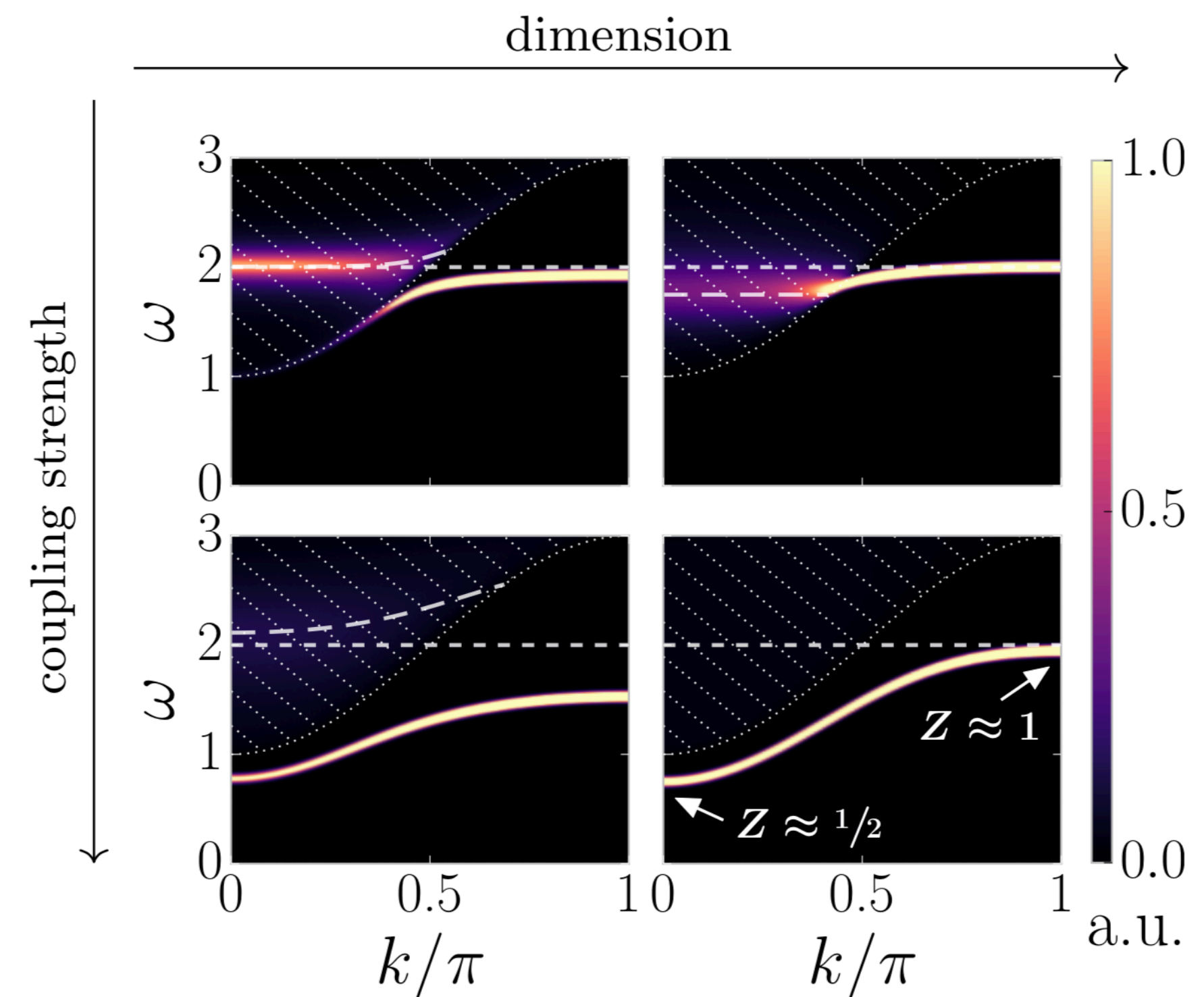
[Drescher, Vanderstraeten, Moessner, FP, arXiv:2209.03344]  
[Verresen, Moessner, FP, Nat. Phys. **15**, 750 (2019)]

# Strong interactions prevent quasiparticle decay

$$\hat{H} = E_b |\psi\rangle\langle\psi| + \int d\alpha \left( E_\alpha |\varphi_\alpha\rangle\langle\varphi_\alpha| + \gamma |\psi\rangle\langle\varphi_\alpha| + \gamma |\varphi_\alpha\rangle\langle\psi| \right),$$



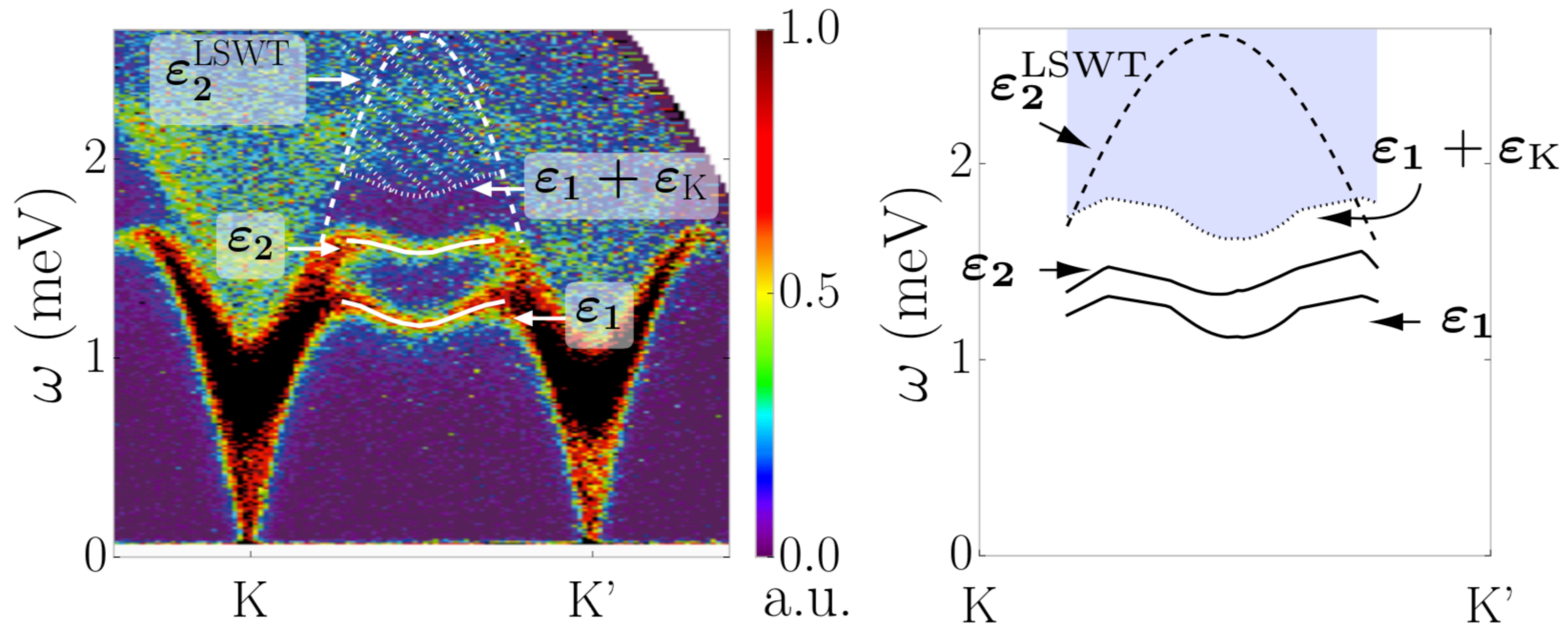
- ▶ **Sharp onset:** QP below the continuum for any  $\gamma \neq 0$
- ▶ **Soft onset:** QP below the continuum for  $\gamma > \gamma_0$





# Dynamics of the triangular lattice Heisenberg model

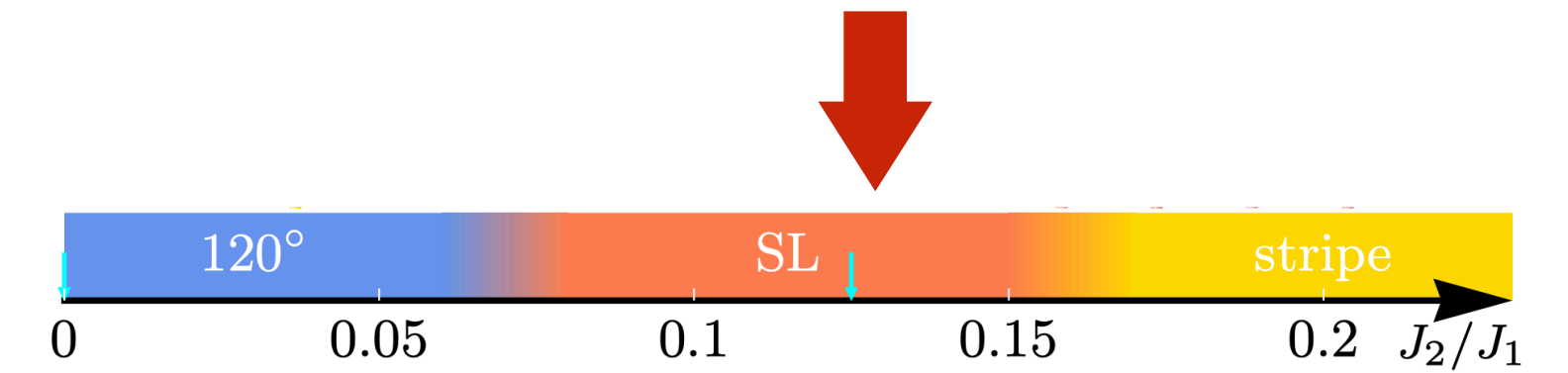
Inelastic neutron scattering for  $\text{Ba}_3\text{CoSb}_2\text{O}_9$  [Ito et al. '17]



- ▶ Magnon repelled from continuum: **Decay prevented**

# Dirac QSL

## $J_1 - J_2$ Heisenberg model in QSL regime



- ▶ Parton construction  $\vec{S}_i = \frac{1}{2} f_{i,\alpha}^\dagger \sigma_{\alpha,\beta} f_{i,\beta}$  with  $f_{i\uparrow}^\dagger f_{i\uparrow} + f_{i\downarrow}^\dagger f_{i\downarrow} = 1$

$$\text{Mean-field } \mathcal{H} = - \sum_{i,j,\alpha} \left( \chi_{ij} f_{i\alpha}^\dagger f_{j\alpha} + \text{h.c.} \right)$$

$$\text{Local gauge redundancy } f_{j,\alpha} \rightarrow e^{i\phi_j} f_{j,\alpha}$$

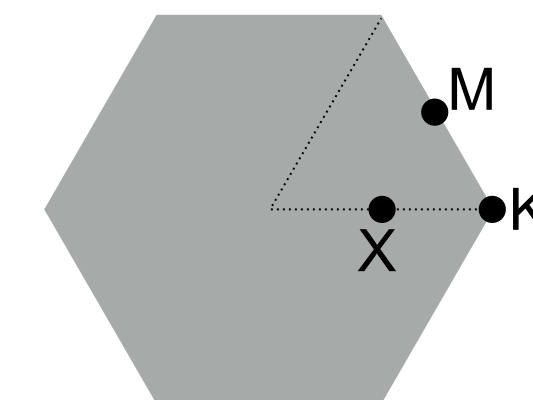
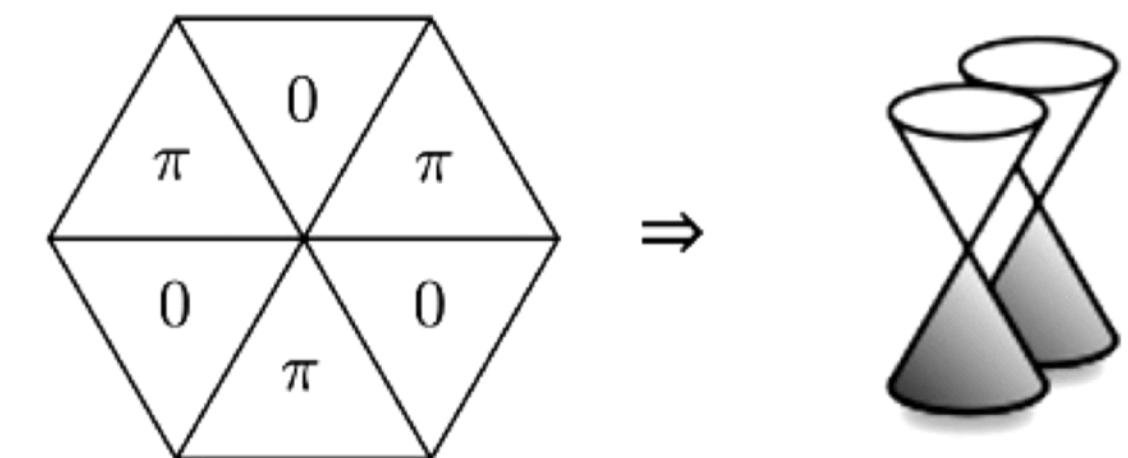
- ▶ Staggered flux: Dirac fermions  $\text{QED}_3$ ,  $N = 4$

[Song et al '19, Wietek et al. '23]

**Fermion bilinear** at M-points

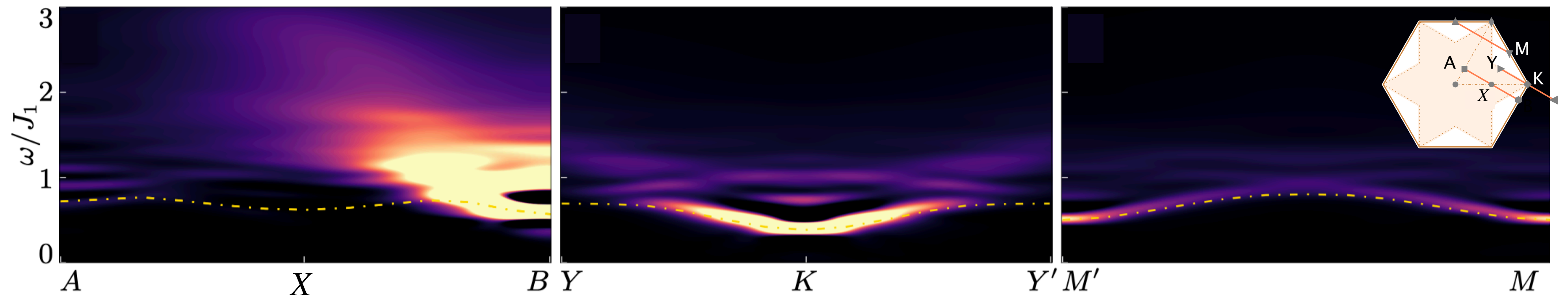
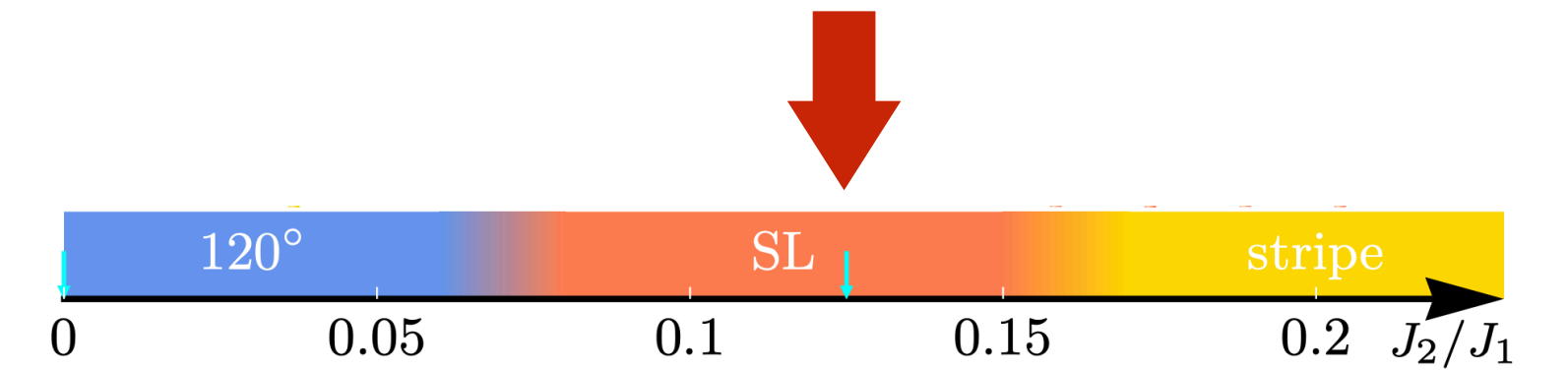
**Singlet monopoles** at X-Points

**Triplet monopoles** at K-points



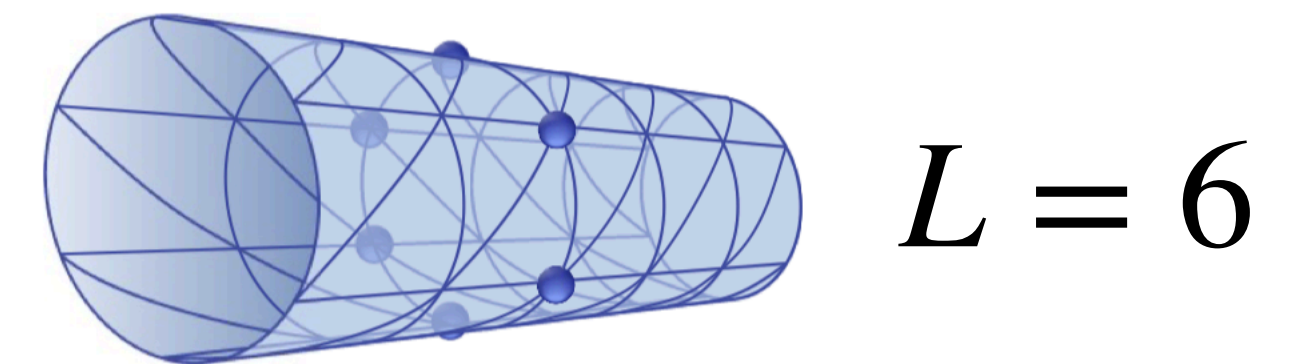
# Dynamics of the triangular lattice Heisenberg model

Heisenberg model with  $J_2/J_1 = 0.125$ : QSL



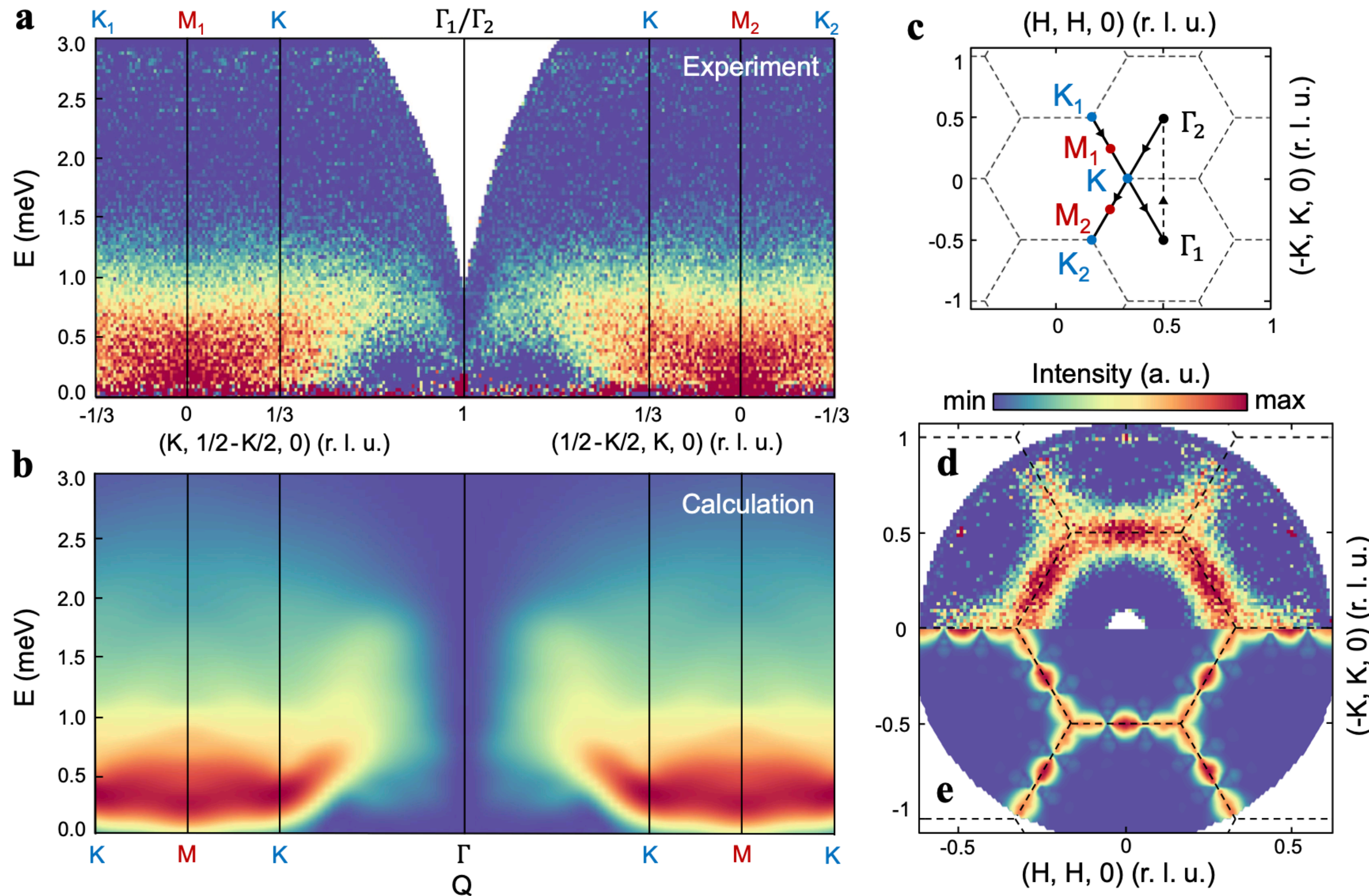
[see also Sherman et al '23]

- ▶ Triplet monopole (K) and Fermion (M) excitations [Song et al '19]
- ▶ Agreement with VMC using Dirac QSL ansatz [Ferrari, Becca '19]



# Sinatures of U(1) Dirac QSL in $\text{YbZn}_2\text{GaO}_5$

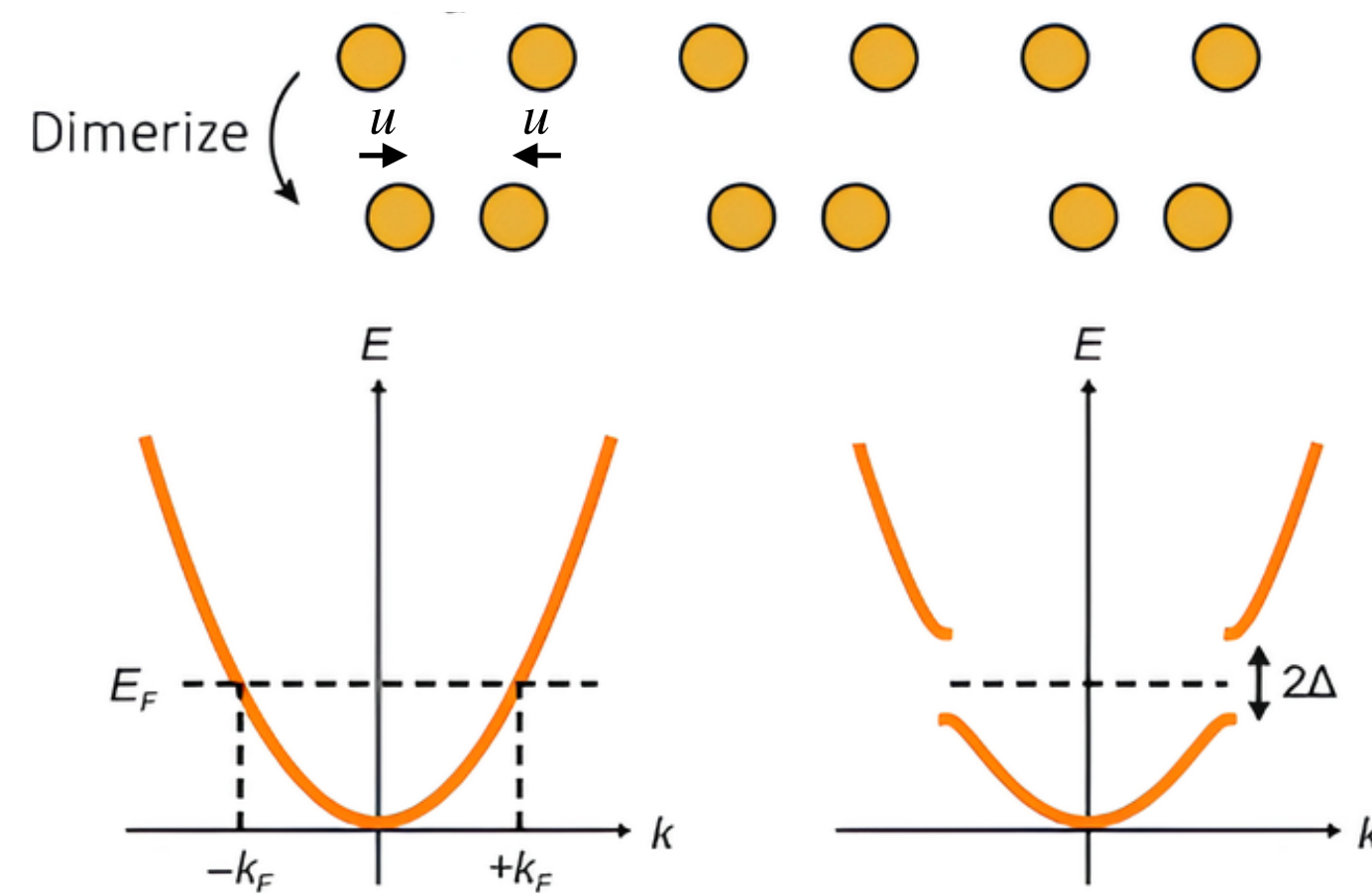
Dirac QSL with gapless excitations at M and K points [Xu et al, '23]



# Spin-Peierls instability of the U(1) Dirac QSL

Warmup: **Peierls instability in spin-1/2 chain:**

- ▶ AF Heisenberg chain is gapless and disordered
- ▶ Infinitesimally weak coupling to lattice deformations leads to dimerization and gap



Free Fermions  $E(u) = \frac{K}{2}u^2 - \eta u^2 \ln u$

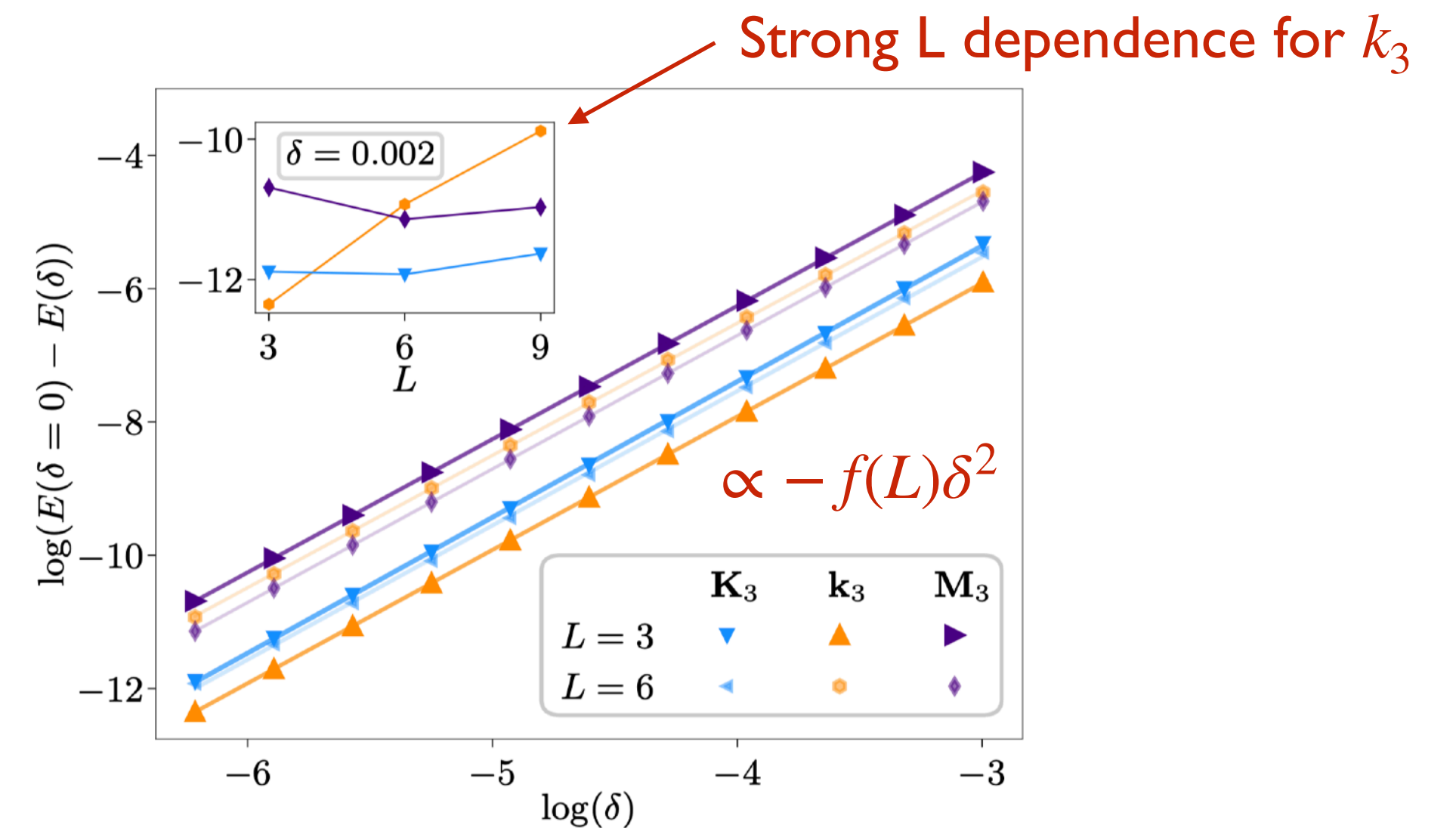
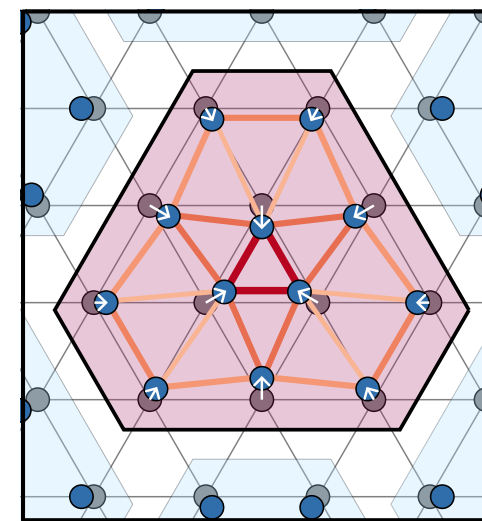
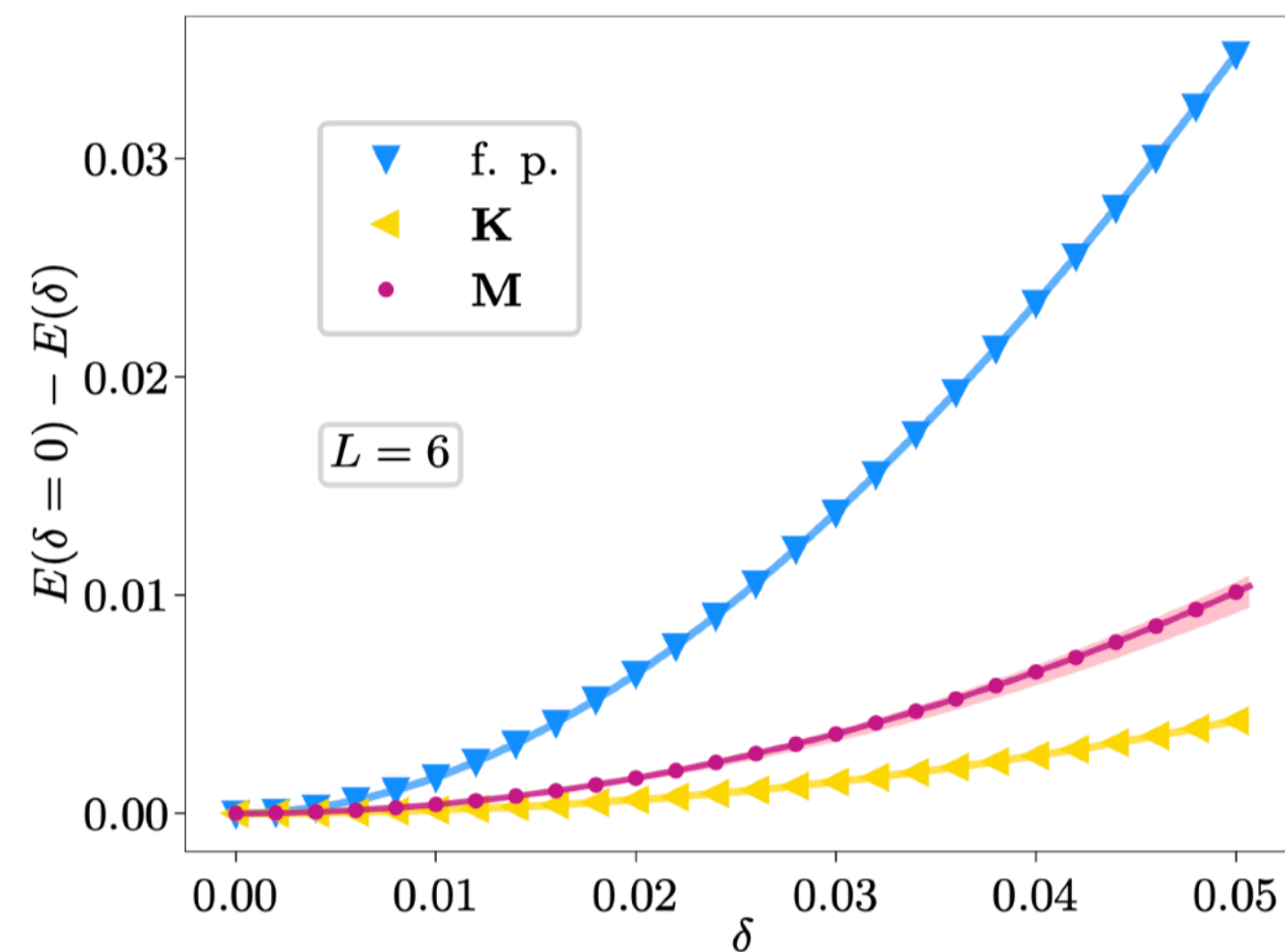
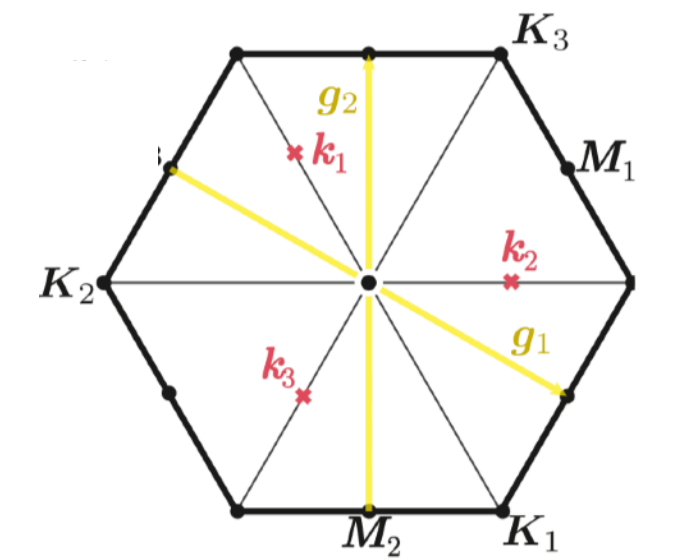
Interacting  $E(u) = \frac{K}{2}u^2 - \eta u^\chi,$   
 (e.g.,  $\chi = 4/3$  for Heisenberg)

👉 **P89 on Thursday by Willsher**

# Spin-Peierls instability of the U(1) Dirac QSL

## Coupling U(1) Dirac QSL to lattice distortions

- ▶ Relevant and symmetry allowed interaction between monopoles and lattice distortion at  $k_a = -K_a/2$
- ▶ Minimization of effective action: |2-site VBS-ordered state



👉 P89 on Thursday by Willsher

# Summary

## Dynamical properties of adjacent phases on the triangular lattice

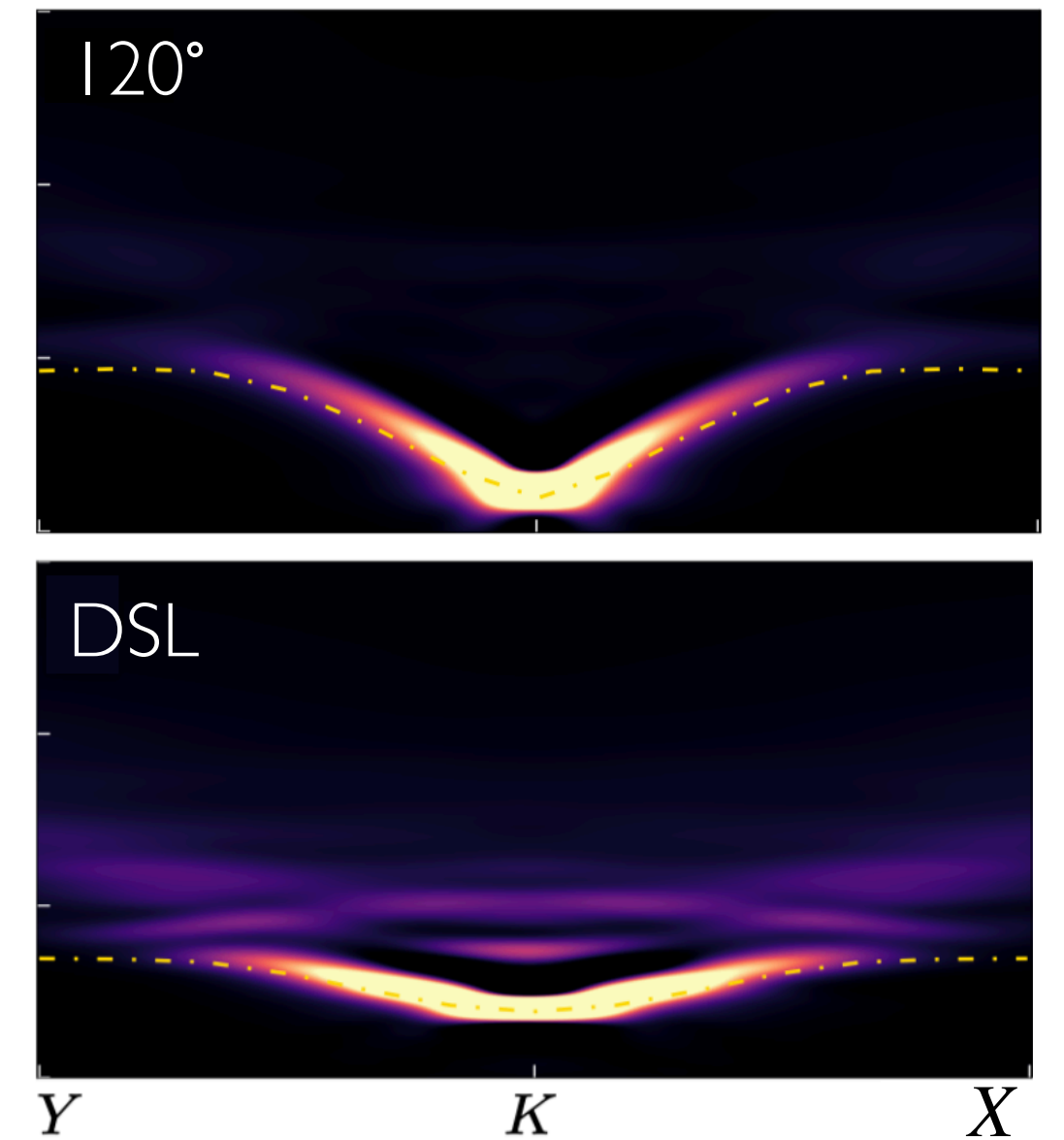
- ▶ DMRG dynamics
  - ▶ Avoided magnon-decay for  $120^\circ$  order
  - ▶ Candidate QSL: Supportive of U(1) DSL

[Drescher, Vanderstraeten, Moessner, FP, arXiv:2209.03344 (in print)]

- ▶ Spin-Peierls instability of the U(1) DSL

[Seifert, Willsher, Drescher, FP, Knolle, arXiv:2307.12295]

👉 **P89 on Thursday by Willsher**



Drescher



Vanderstraten



Moessner



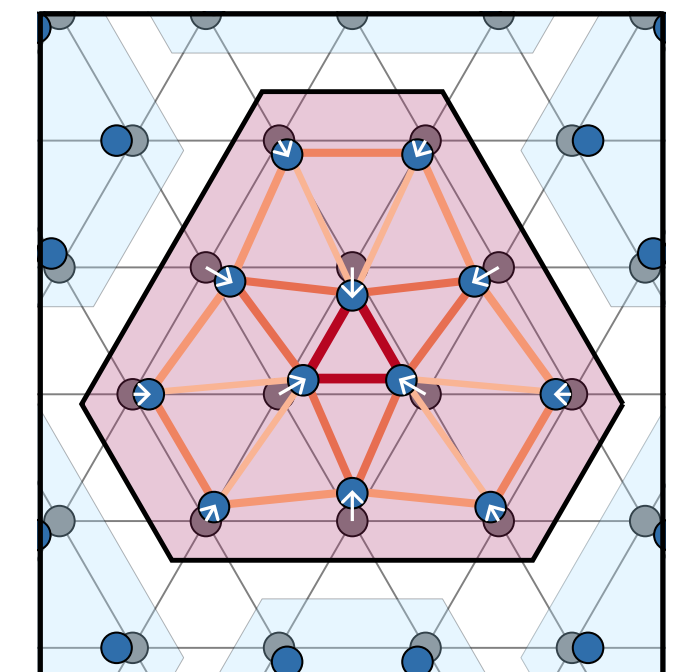
Seifert



Willsher

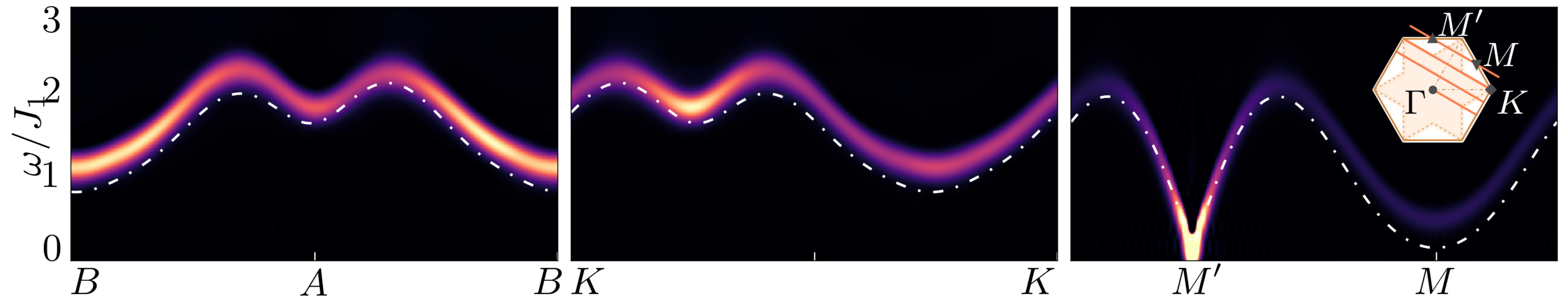
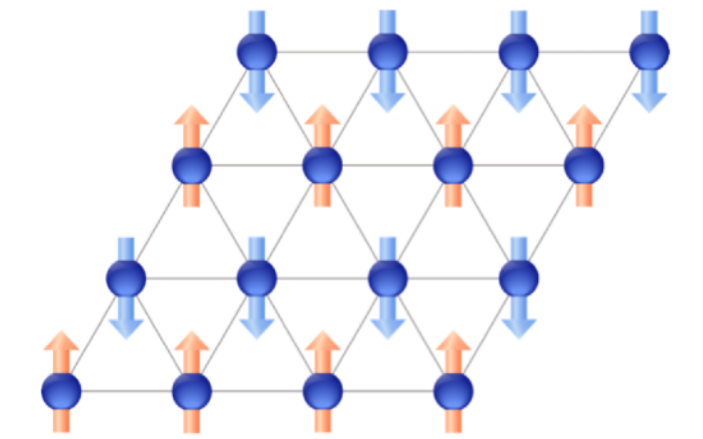


Knolle



# Dynamics of the Heisenberg model: Triangular lattice

Heisenberg model with  $J_2/J_1 = 0.55$ : Stripe order



► Good agreement with spin wave calculations

