Superconductivity of non-Fermi liquids described by Sachdev-Ye-Kitaev models

Darshan G. Joshi

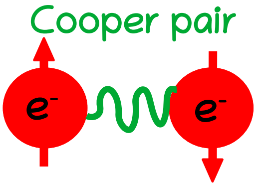
December 5, 2023

TIFR HYDERABAD

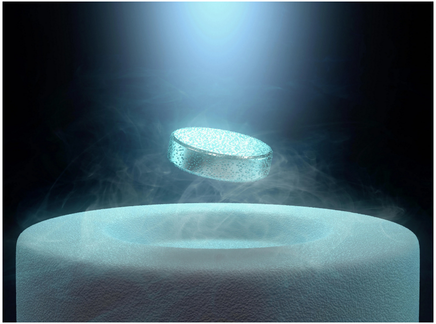
Collaborators: Chenyuan Li and Subir Sachdev

Phys. Rev. Research 5, 013045 (2023)

Superconductivity



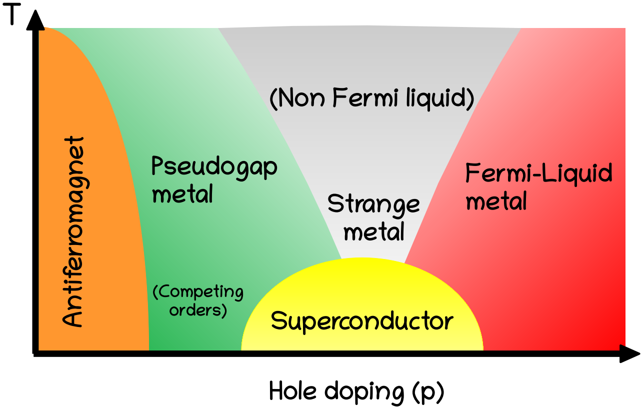
Electrons in a superconductor



Superconductor (Image: Scientific American)

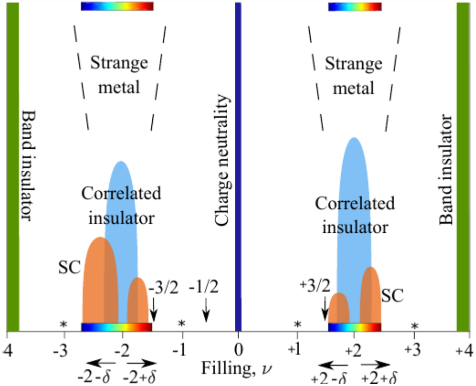
Phonon mediated Cooper-pair formation in conventional SC

Superconductivity



Modern quantum materials: SC emerging in vicinity of non-Fermi liquid

Superconductivity



Modern quantum materials: SC emerging in vicinity of non-Fermi liquid

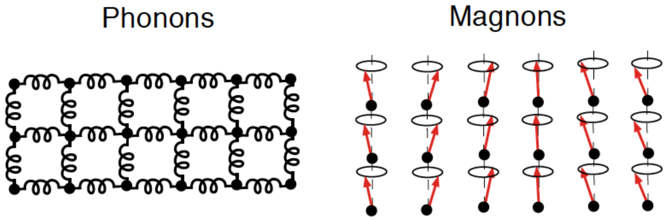
Y. Cao et. al Phys. Rev. Lett. 124, 076801 (2020)

Quasiparticles

- Non-interacting system: Energy spectrum known exactly

Quasiparticles

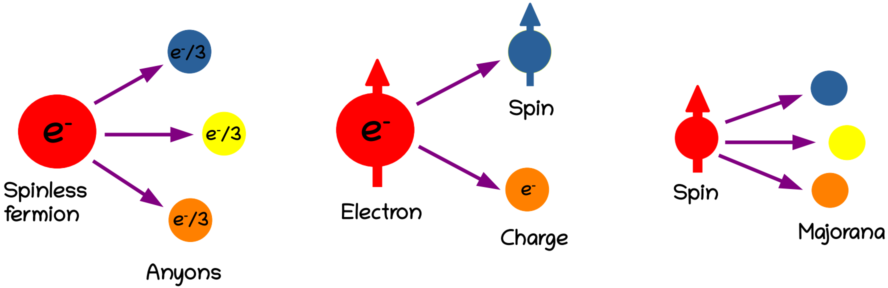
- Non-interacting system: Energy spectrum known exactly
- Interacting system: Effective low-energy description using quasiparticles



Common conventional quasiparticles

Quasiparticles

Fractionalized quasiparticles ...



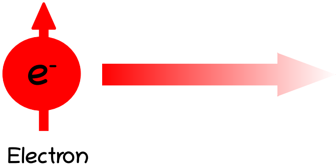
Fractional QHE

Spin-charge separation

Quantum spin liquid

Quasiparticles

... Or complete breakdown of quasiparticles



No quasiparticles!

Non-Fermi liquid

Unconventional transport properties in contrast to Fermi liquid
(eg. T-linear resistivity in strange metals)

Superconductivity from NFL

- Emergence of SC out of non-Fermi liquid (NFL) not well understood

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- One of the popular theories: Fermi surface coupled to a critical boson (ϕ)

$$\mathcal{L} = \psi^\dagger(\frac{\partial}{\partial \tau} + \epsilon)\psi + \alpha|\phi|^2 + g\psi^\dagger\psi\phi$$

M. Metlitski et al., Phys. Rev. B 91, 115111 (2015)
S. S. Zhang et al., Phys. Rev. B 104, 144509 (2021)
D. Pimenov and A. V. Chubukov, Ann. Phys. 447, 169049 (2022).

Superconductivity from NFL

- Emergence of SC out of non-Fermi liquid (NFL) not well understood

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ϕ could be order parameter field (magnetic fluctuation) or emergent excitation (spin liquid)

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S. S. Zhang et al., Phys. Rev. B 104, 144509 (2021)
D. Pimenov and A. V. Chubukov, Ann. Phys. 447, 169049 (2022).

Sachdev-Ye-Kitaev (SYK) models

- SYK models involve all-to-all and random interactions - no spatial structure

S. Sachdev and J. Ye, Phys. Rev. Lett. 70, 3339 (1993)
A. Kitaev, Talks at KITP
D. Chowdhury, A. Georges, O. Parcollet, and S. Sachdev, Rev. Modern Phys. 94, 035004 (2022)

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- Capture universal properties of certain strongly correlated systems

D. G. Joshi et al., Phys. Rev. X 10, 021033 (2020)
M. Christos et al., PNAS 119, e2206921119 (2022)

Sachdev-Ye-Kitaev (SYK) models

- SYK models involve all-to-all and random interactions - no spatial structure
- Simpler models for NFL with solvability and tractable
- Capture universal properties of certain strongly correlated systems
- Previous works on SC instability in SYK models: non-SU(2) or lattice of SYK islands

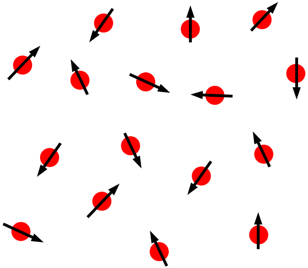
I. R. Klebanov et al., J. High Energy Phys. 11 (2020) 162
É. Lantagne-Hurtubise et al., Phys. Rev. B 104, L020509 (2021)
Y. Wang, Phys. Rev. Lett. 124, 017002 (2020)
H. Wang et al., Phys. Rev. Res. 2, 033025 (2020)
A. A. Patel et al., Phys. Rev. Lett. 121, 187001 (2018)
D. Chowdhury and E. Berg, Phys. Rev. Res. 2, 013301 (2020).
I. Esterlis et al., Phys. Rev. B 103, 235129 (2021)

Random matrix Bogoliubov-de Gennes theory

Random-matrix with attractive Hubbard interaction ($U < 0$)

$$H_{tU} = -\frac{1}{\sqrt{N}} \sum_{i<j} t_{ij} \left(c^\dagger_{i\alpha} c_{j\alpha} + c^\dagger_{j\alpha} c_{i\alpha} \right) + U \sum_i c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow} - \mu \sum_i c^\dagger_{i\alpha} c_{i\alpha}$$

where t_{ij} is a real random number with $\overline{t_{ij}} = 0$ and $\overline{t_{ij}^2} = t^2$



SU(2) $\rightarrow$ USp(M)

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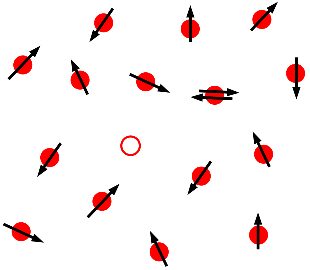


SU(2) $\rightarrow$ USp(M)

Saddle-point equations

Disorder average leads to single-site action giving saddle-point equations:

$$
\begin{aligned}
G_\Sigma(i\omega) &\equiv \frac{1}{i\omega + \mu - \Sigma(i\omega)} \\
\Sigma(i\omega) &= t^2 G(i\omega) = t^2 \frac{[G_\Sigma(-i\omega)]^{-1}}{|\Phi(i\omega)|^2 + [G_\Sigma(i\omega) G_\Sigma(-i\omega)]^{-1}} \\
\Delta &= -UT \sum_\omega \frac{\Phi(i\omega)}{|\Phi(i\omega)|^2 + [G_\Sigma(i\omega) G_\Sigma(-i\omega)]^{-1}} \\
F(i\omega) &= \frac{\Phi(i\omega)}{|\Phi(i\omega)|^2 + [G_\Sigma(i\omega) G_\Sigma(-i\omega)]^{-1}} \\
\Phi(i\omega) &= \Delta + t^2 F(i\omega)
\end{aligned}
$$

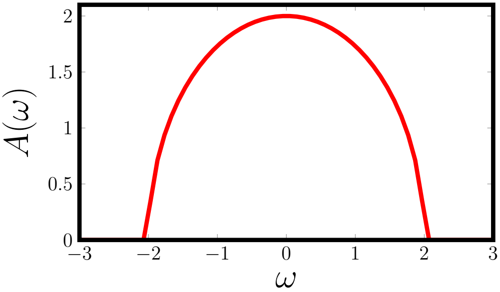
These have to be solved self-consistently

Normal state solution

Set $\Delta = F(i\omega) = 0$, which yields for $\mu < 2t$

$$G(i\omega) \equiv G_0(i\omega) = \frac{i\omega + \mu}{2t^2} - i\frac{\text{sgn}(\omega)}{2t^2}\sqrt{4t^2 + (\omega - i\mu)^2}\,,$$

Fermi-liquid solution



Superconducting solution

Linearized gap equation gives equation for critical temperature T_c

$$1 = -UT \sum_{\omega_n} \frac{G_0(i\omega_n)G_0(-i\omega_n)}{1 - t^2 G_0(i\omega_n)G_0(-i\omega_n)}$$

At small $|\omega_n|$,

$$t^2 G_0(i\omega_n)G_0(-i\omega_n) = 1 - \frac{2|\omega_n|}{\sqrt{4t^2 - \mu^2}} + \mathcal{O}(\omega_n^2)$$

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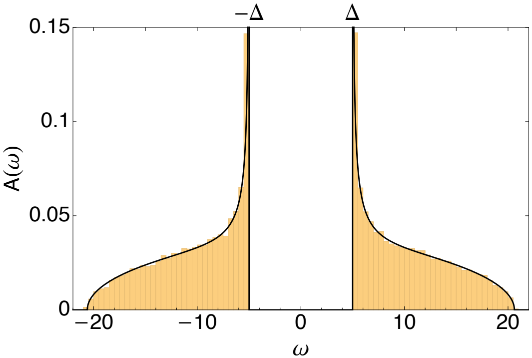
Singularity at $\omega_n = 0$ yields the BCS log divergence

$$\Downarrow$$

Superconductivity at $T = 0$ for infinitesimal negative U

SC phase



Spectral function

$$A(\omega) = \frac{|\omega|}{2\pi t^2} \frac{\sqrt{4t^2 + \Delta^2 - \omega^2}}{\sqrt{\omega^2 - \Delta^2}}, \quad \Delta < |\omega| < \sqrt{\Delta^2 + 4t^2}$$

Random t-J-L-U model

$$H = H_{tU} + H_J + H_L$$

$$H_{tU} = -\frac{1}{\sqrt{N}} \sum_{i<j} t_{ij} \left(c^\dagger_{i\alpha} c_{j\alpha} + c^\dagger_{j\alpha} c_{i\alpha} \right) + U \sum_i c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow} - \mu \sum_i c^\dagger_{i\alpha} c_{i\alpha}$$

$$H_J = \frac{1}{\sqrt{N}} \sum_{i<j} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

$$H_L = \frac{1}{\sqrt{N}} \sum_{i<j} L_{ij} c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c_{j\downarrow} c_{j\uparrow}$$

$$\overline{t_{ij}} = \overline{J_{ij}} = \overline{L_{ij}} = 0, \; \overline{t^2_{ij}} = t^2, \; \overline{J^2_{ij}} = J^2, \; \overline{L^2_{ij}} = L^2$$

SU(2) $\rightarrow$ USp(M)

C. Li, D. G. Joshi, and S. Sachdev, Phys. Rev. B 103, 115147 (2021)

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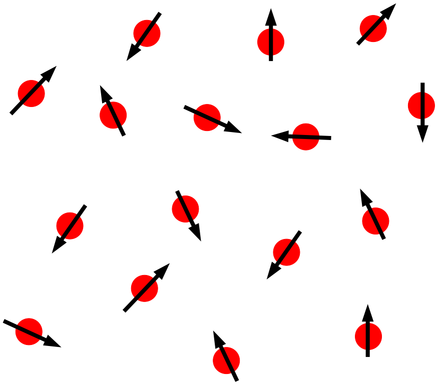

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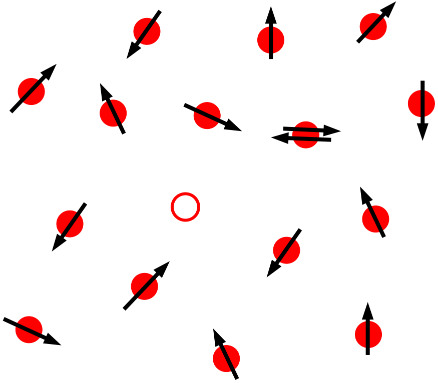

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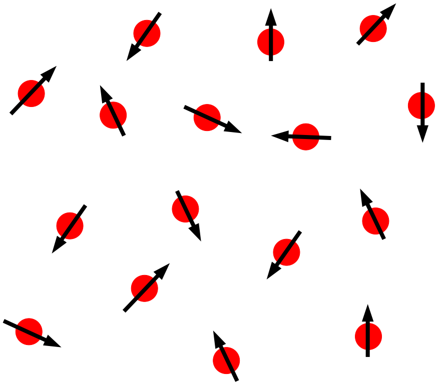

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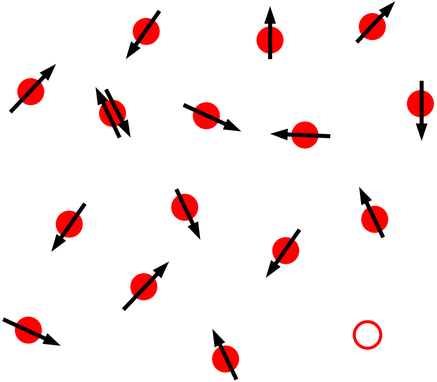

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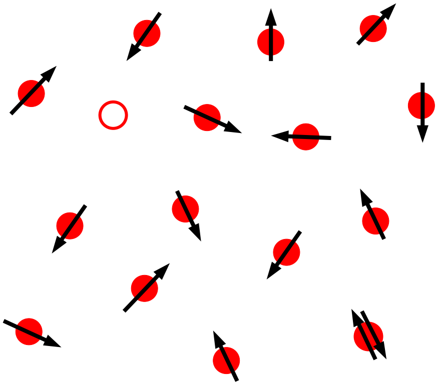


Saddle-point equations

$$G_\Sigma(i\omega) \equiv \frac{1}{i\omega + \mu - \Sigma(i\omega)}$$

$$\Sigma(\tau,\tau') = t^2 G(\tau,\tau') - (J^2 + L^2) G^2(\tau,\tau') G(\tau',\tau)$$

$$G(i\omega) = \frac{[G_\Sigma(-i\omega)]^{-1}}{|\Phi(i\omega)|^2 + [G_\Sigma(i\omega)G_\Sigma(-i\omega)]^{-1}}$$

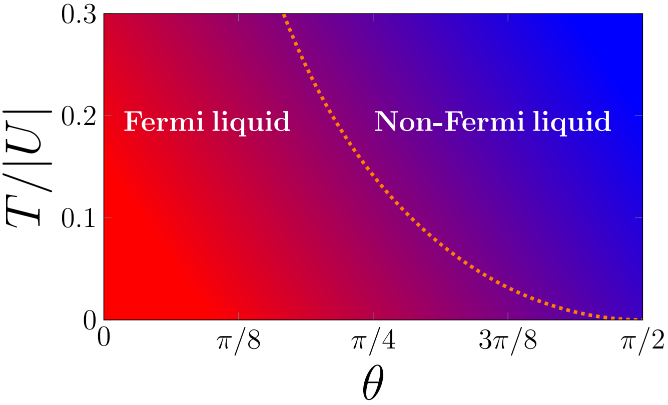
$$\Delta = -UT \sum_\omega \frac{\Phi(i\omega)}{|\Phi(i\omega)|^2 + [G_\Sigma(i\omega)G_\Sigma(-i\omega)]^{-1}}$$

$$F(i\omega) = \frac{\Phi(i\omega)}{|\Phi(i\omega)|^2 + [G_\Sigma(i\omega)G_\Sigma(-i\omega)]^{-1}}$$

$$\Phi(\tau,\tau') = -UF(\tau,\tau)\delta(\tau-\tau') + t^2 F(\tau,\tau') + (J^2 + L^2) F^2(\tau,\tau') F^*(\tau',\tau)$$

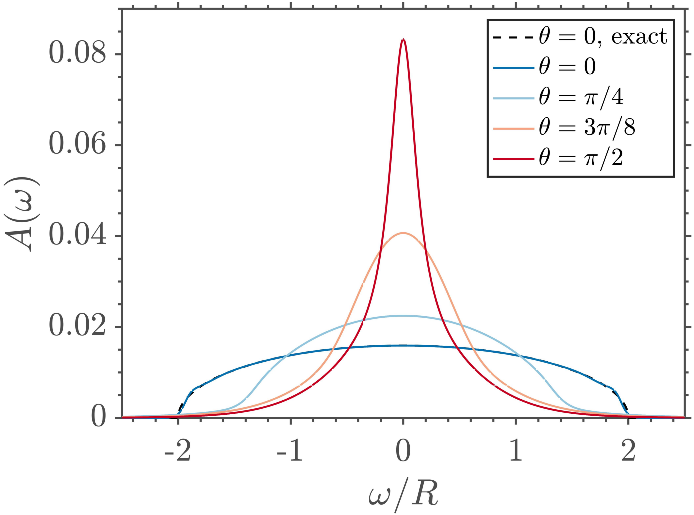
Normal state

$$t = R\cos\theta\,,\quad \widetilde{J} \equiv \sqrt{J^2 + L^2} = R\sin\theta\,,\quad R = \sqrt{t^2 + \widetilde{J}^2}$$



$$T_{coh} = t^2/\widetilde{J} = R\cos\theta_{coh}\cot\theta_{coh}$$

Normal state: Electron spectral function



Normal state: Spin susceptibility

Local spin-spin correlation,

$$\chi(\tau) = \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle$$

For the SYK-type conformal solutions,

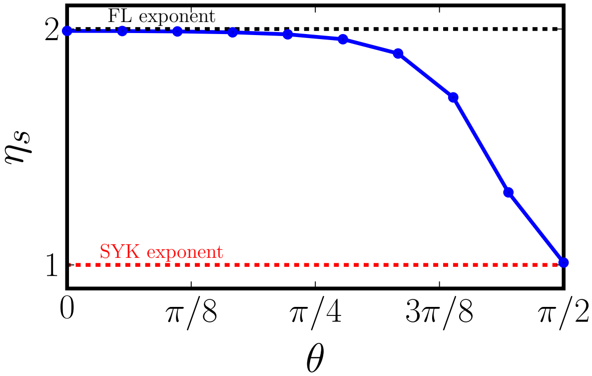
$$\chi''(\omega) \sim T^{\eta_s - 1} \Phi_{\eta_s} \left(\frac{\hbar\omega}{k_B T} \right)$$

For $\hbar\omega \ll k_B T$ we have,

$$\chi''(\omega) \sim \omega \, T^{\eta_s - 2}$$

Normal state: Spin susceptibility

Effective spin exponent



$$\frac{\chi''(\omega)}{\omega} \sim T^{\eta_s - 2}\,, \quad \hbar\omega \ll k_B T$$

Superconducting solution

The linearized gap equation remains same as before,

$$1 = -UT \sum_{\omega_n} \frac{G_0(i\omega_n)G_0(-i\omega_n)}{1 - t^2 G_0(i\omega_n)G_0(-i\omega_n)}$$

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We can write a quadratic equation for $G_0(0)$:

$$t^2[G_0(0)]^2 - (\mu - \Sigma_{in}(0))G_0 + 1 = 0$$

Thus,

$$\lim_{\omega \to 0} G_0(i\omega)G_0(-i\omega) = \frac{1}{t^2}$$

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Thus,

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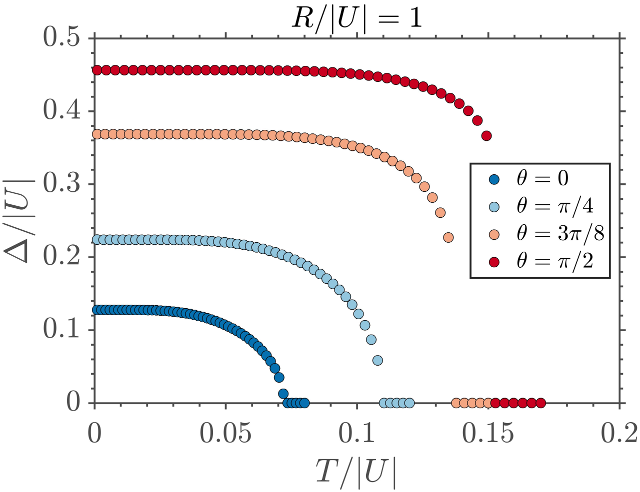
So even when J or L are non-zero SC at $T = 0$!

SC order parameter



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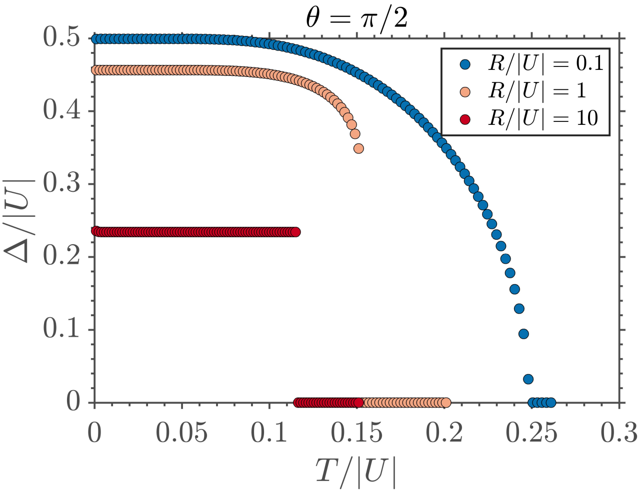
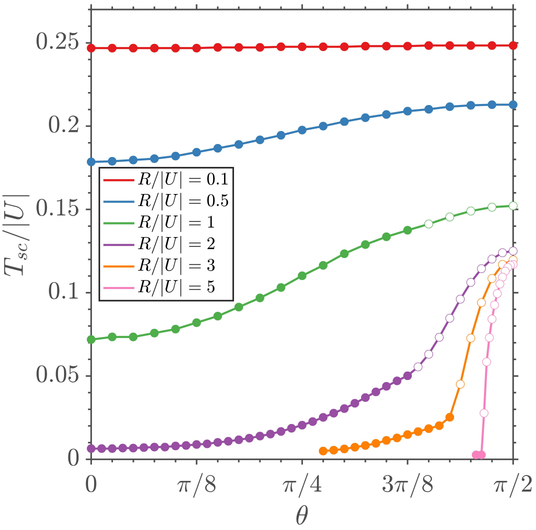
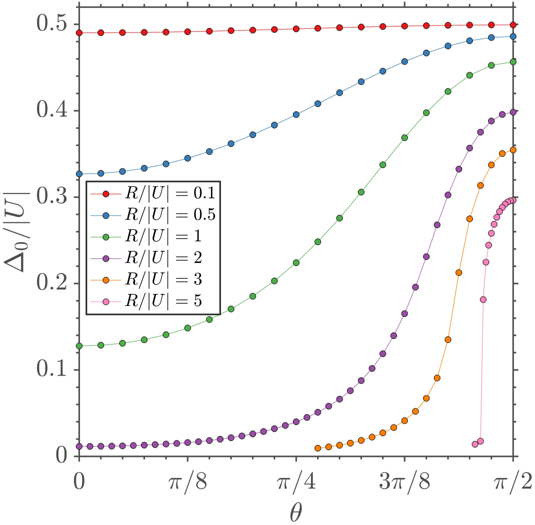


SC critical temperature (T_{sc})

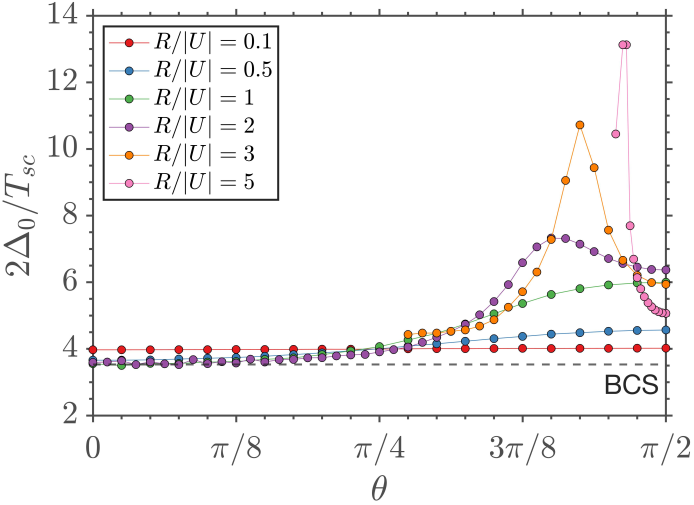


SC gap

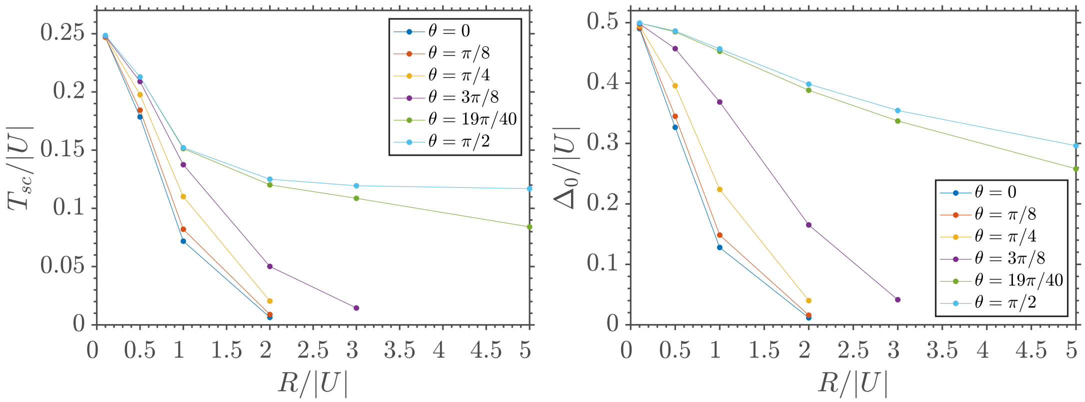


0.5
0.4
0.3
0.2
0.1
0
$\Delta_0/|U|$
$R/|U| = 0.1$
$R/|U| = 0.5$
$R/|U| = 1$
$R/|U| = 2$
$R/|U| = 3$
$R/|U| = 5$
0
$\pi/8$
$\pi/4$
$3\pi/8$
$\pi/2$
θ

Ratio of SC gap and critical temperature



Dependence on $R/|U|$ and deviation from BCS



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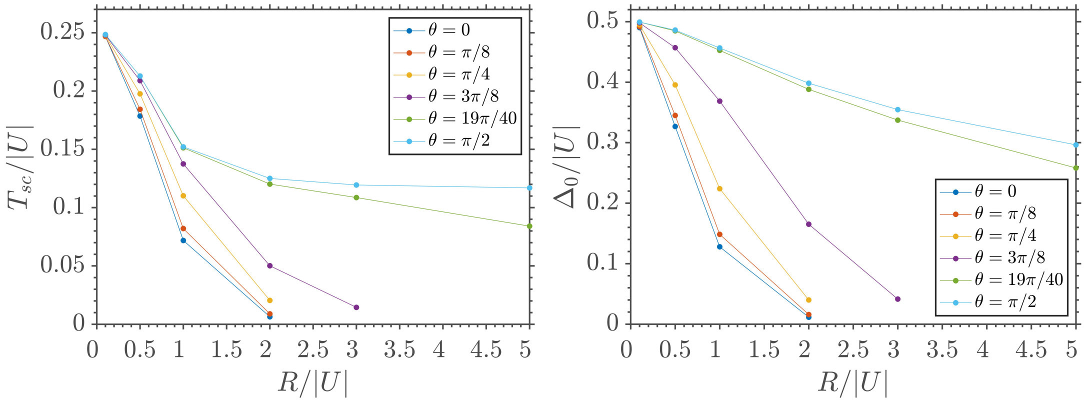

$$\theta \lesssim \theta_{coh} : \ \Delta_0 \sim A\, e^{-\nu R/|U|} , \ T_{sc} \sim B\, e^{-\nu R/|U|} \ \Rightarrow \ \Delta_0/T_{sc} \sim \text{constant}$$

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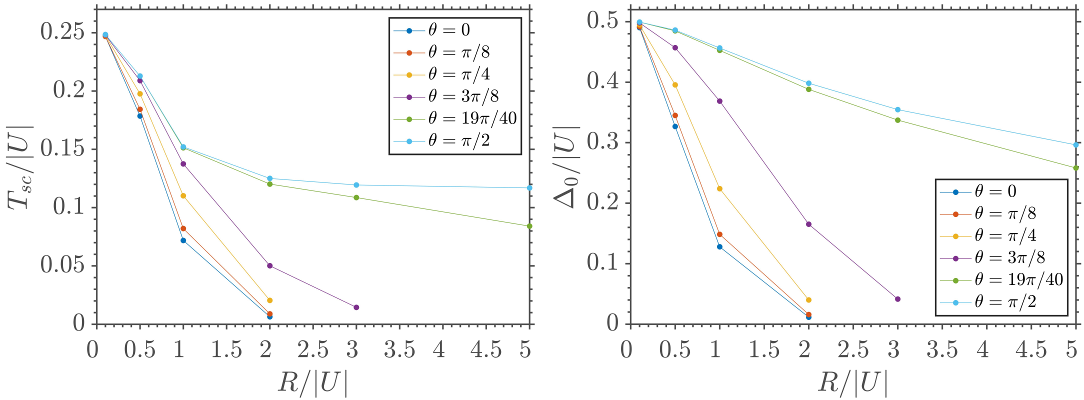

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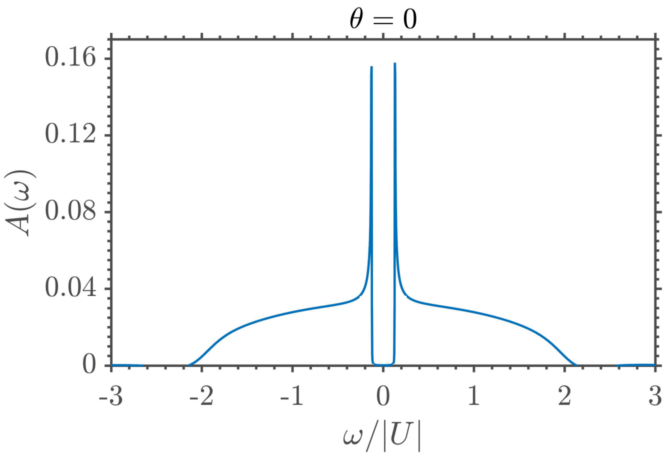
$$\theta \gtrsim \theta_{coh} : \; \Delta_0 \sim [C + R/|U|]^{-\alpha(\theta)} \,,\; T_{sc} \sim [D + R/|U|]^{-\beta(\theta)} \;;\;\; \alpha\,,\beta \leq 1$$

SC spectral function



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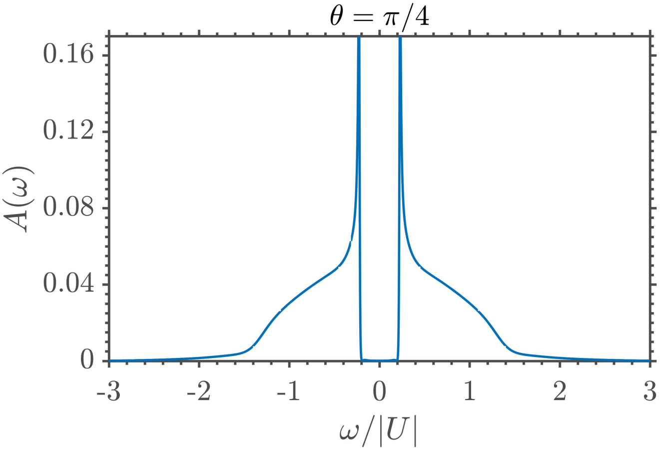

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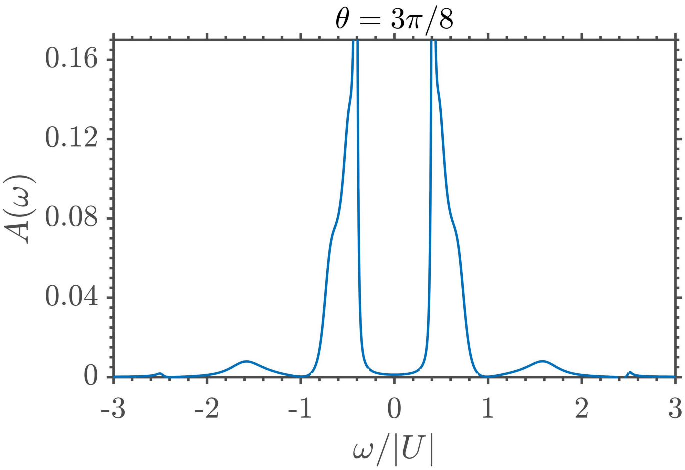

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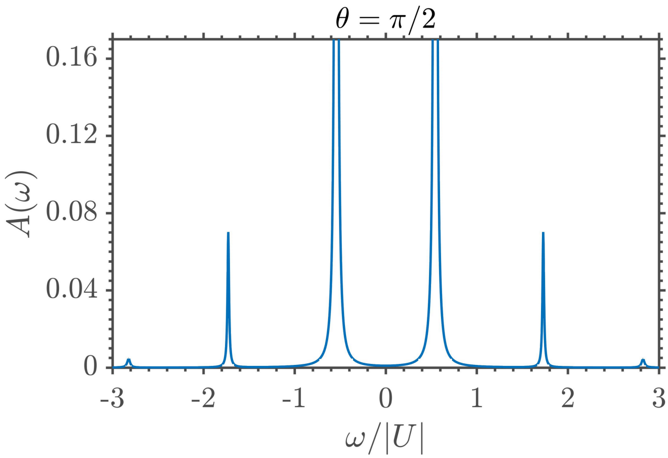
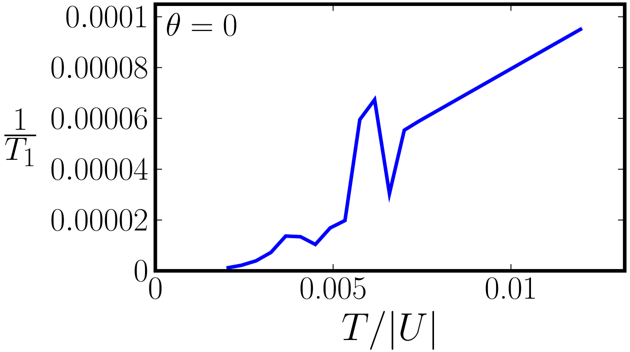


NMR relaxation rate

$$\frac{1}{T_1} = T \left. \frac{\chi''(\omega)}{\omega} \right|_{\omega=0}$$



Hebel-Slichter peak near T_{sc}

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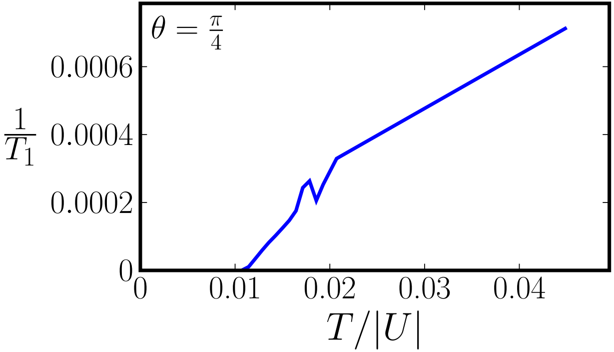

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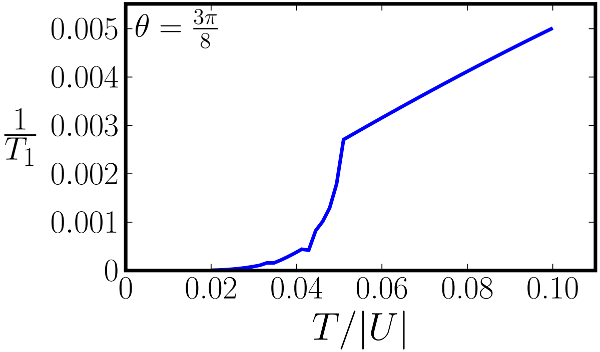
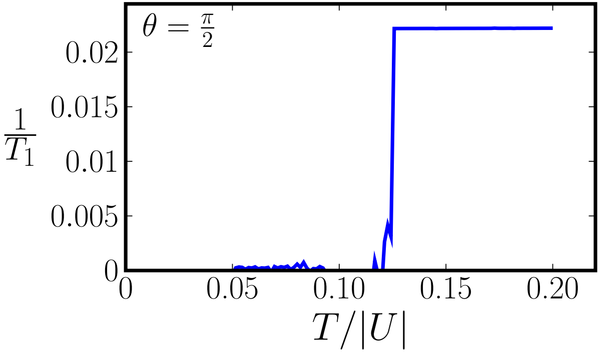


Hebel-Slichter peak replaced by a kink near T_{sc}

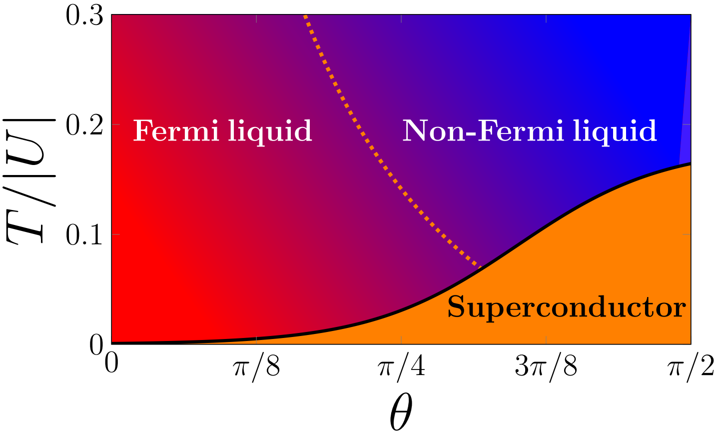
NMR relaxation rate

$$\frac{1}{T_1} = T \left. \frac{\chi''(\omega)}{\omega} \right|_{\omega=0}$$



Hebel-Slichter peak replaced by a kink near T_{sc}

Summary



C. Li, S. Sachdev, and D. G. Joshi, Phys. Rev. Research 5, 013045 (2023)