Conference on Fractionalization and Emergent Gauge Fields in Quantum Matter

ICTP

Flux-fractionalization transition in anisotropic S=1 kagome antiferromagnets and dimer-loop models

Kedar Damle

Tata Institute, India

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Souvik Kundu and KD, arXiv:2305.07012

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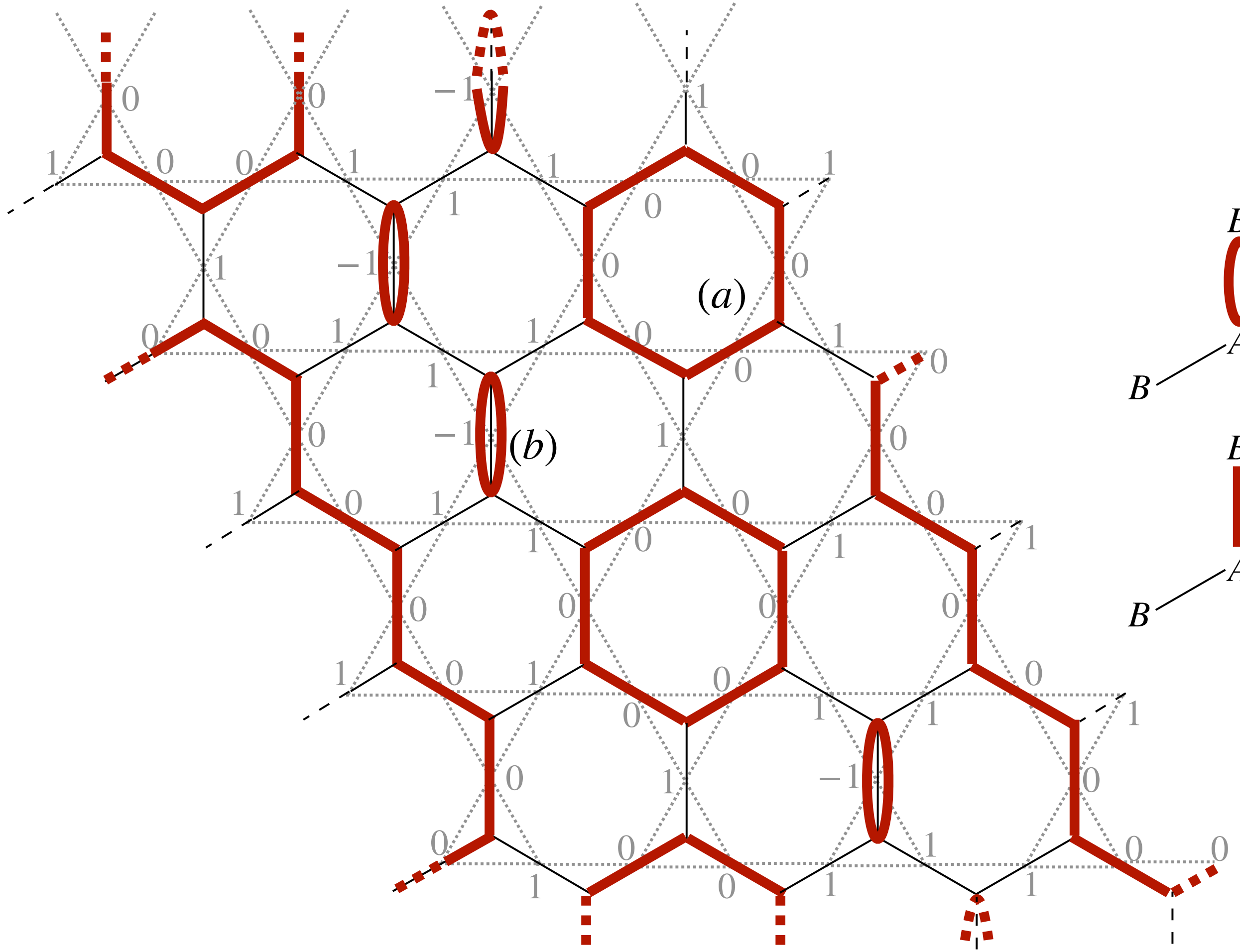
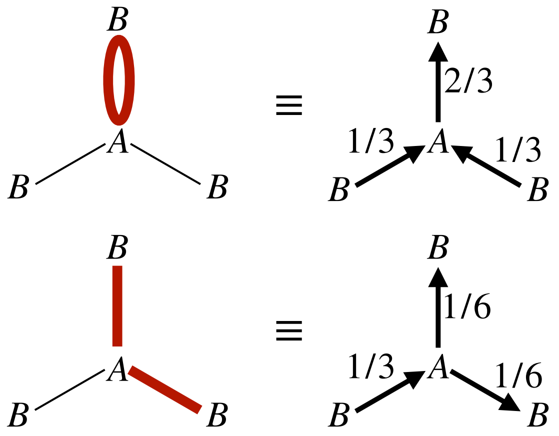
Fully-packed configurations of loops + trivial loops (dimers)

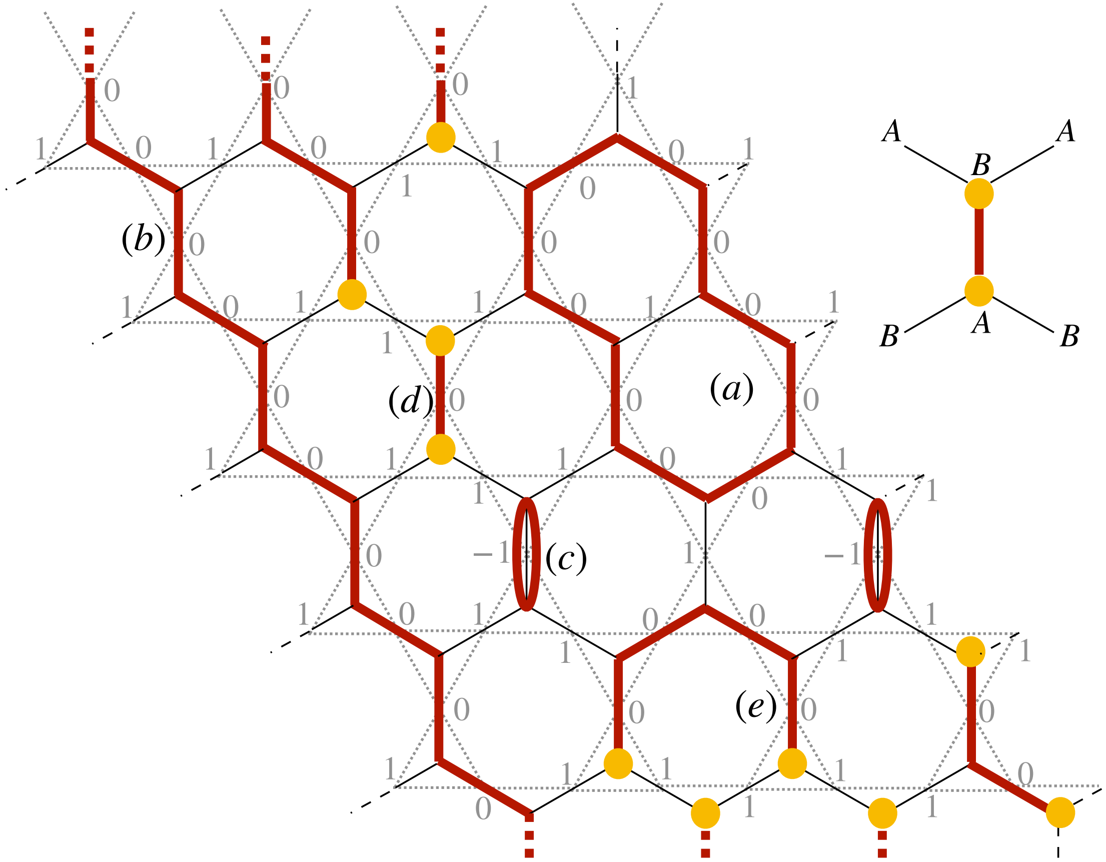


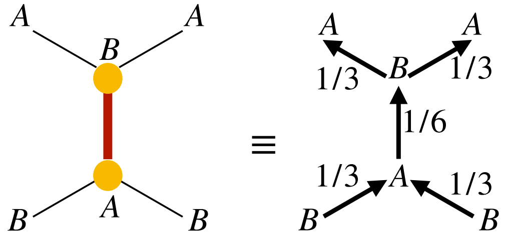
Divergence-free polarization



$$\Delta \cdot P = 0$$

All open strings disallowed





Half-charges (half-vortices) forbidden

Integer charges (unit-vortices) also forbidden

Two distinct objects of length 1 if half vortices allowed (quite different from variable dimer density situations)

Anisotropic S=1 kagome 1/3-magnetization plateau

Each kagome triangle has: $S^z_{tot} = 1$

(Large O(J) energy gap to other values)

Two ways to add up to $S^z_{tot} = 1$ (1,0,0) or (1,1, -1)

(With slightly different energies)

$$H_{eff} = J^z \sum_{rr'} S^z_r S^z_{r'} + \Delta \sum_r (S^z_r)^2 - B \sum_r S^z_r$$

$$J^z = J \;, \quad \Delta = J + \mu \qquad T,\ \mu \ll J$$

Implicit: Quantum fluctuations negligible $J_\perp \ll T$

Dimer-loop partition function

$$Z = \sum_{\mathcal{C}} w^{n_d(\mathcal{C})}$$

$$w = \exp(-2\mu/T)$$

Our focus: Classical phase diagram as function of w on honeycomb and square lattices

On square lattice, Z(w) describes half-magnetization plateau of anisotropic S=1 planar pyrochlore antiferromagnet.

Tool: Classical Monte Carlo using a worm algorithm

Some theoretical perspective

- w=0 is fully-packed O(1) honeycomb loops (loop fugacity is unity). Configurations in one-to-one correspondence with fully-packed dimers (empty links form loops)
- On square lattice, Z(w=0) is exactly solved fully packed O(1) loop model (Baxter)
- Power-law distribution of loop sizes and dipolar correlations between loop segments. (Baxter, Moessner-Tchernyshyov-Sondhi 2004, Jaubert-Haque-Moessner 2011, Jacobsen-Kondev 1998, Saleur-Duplantier 1987)
- Limit of infinite w is usual fully-packed dimer model.
- Warning: On honeycomb: w=0 and infinity have identical configurations and relative weights, but no obvious duality between w and 1/w for general w.

Coarse-grained height field-theory

$$S = \pi g \int (\nabla h)^2$$

- Valid both for loop model and for dimer model (Youngblood-Axe 1980, Henley, Fradkin et al 2004, Vishwanath-Balents-Senthil 2004, Alet et al 2005, Moessner-Tchernyshyov-Sondhi 2004 ...)
- Coarse-grained height is an angle: In pure dimer limit, integer shifts of h are a redundancy of description (“compactification radius”). In pure loop limit, half integer shifts are a redundancy of description. [in our normalization]
- Might expect: Dimer-loop system would have half-integer shifts as redundancy except in pure dimer limit, because of loops being present at any finite w (??)
- Might suggest: Pure loop limit controls behavior at any finite w (??)

Two-fold motivation for detailed study

Very natural interpolation between fully-packed dimers and fully-packed loops

Also describes interesting low-temperature plateau in class of kagome magnets

Caveat: “Realizability” needs further thought

Aside: Extension that allows half-vortices but forbids unit-vortices gives description of transition to next i.e. 2/3 magnetization plateau of kagome magnet (work in progress)

Numerics:

- Classical Monte Carlo using two worm updates
- One update creates a unit-vortex antivortex pair at random location, keeps one of them fixed, while other does random walk before annihilating antipartner
- Other update does the same with a pair of half-vortices
- Allows measurement of test vortex two-point functions
- Periodic boundary conditions
- Configurations characterized by two independent fluxes of polarization field (winding numbers). These are allowed to be half-integer in general (in our normalization) except in pure dimer limit.

Measurements

$$P_l(s, L)$$

$$S_m = \langle \sum_j s_j^m \rangle$$

Loop size distribution and moments

$$Q_2 = \langle \sum_{i \neq j} s_i^2 s_j^2 \rangle / S_2^2$$

Convenient Binder ratio characterization of loop sizes

$$P(\phi_x, \phi_y)$$

Flux (winding number) distribution

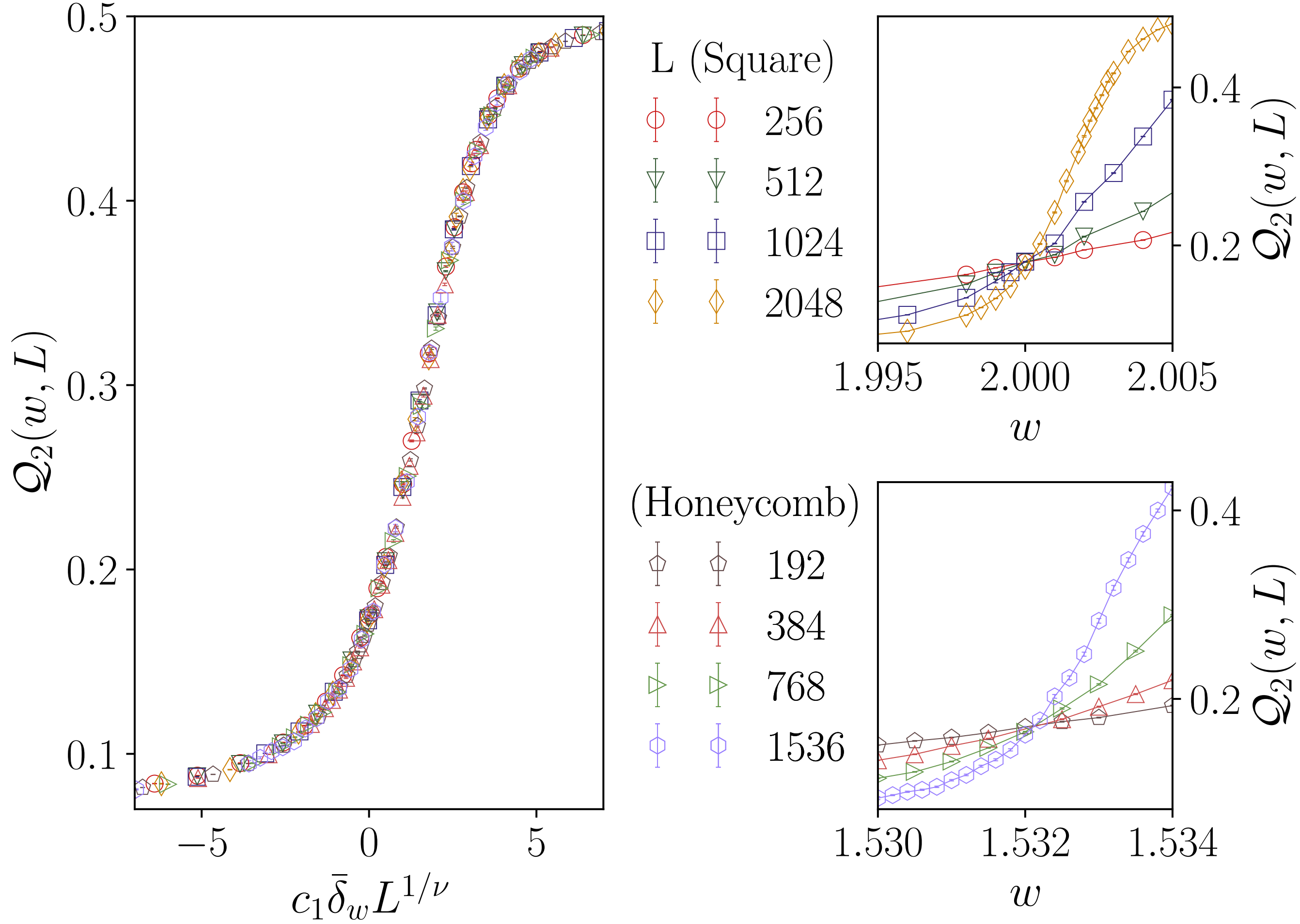
$$P_{\text{frac}} = 1 - \sum_{\phi_x \in Z, \phi_y \in Z} P(\phi_x, \phi_y)$$

Probability of having fractional fluxes

$C_\psi(r)$ and $C_v^q(r)$ for $q = 1/2, 1$

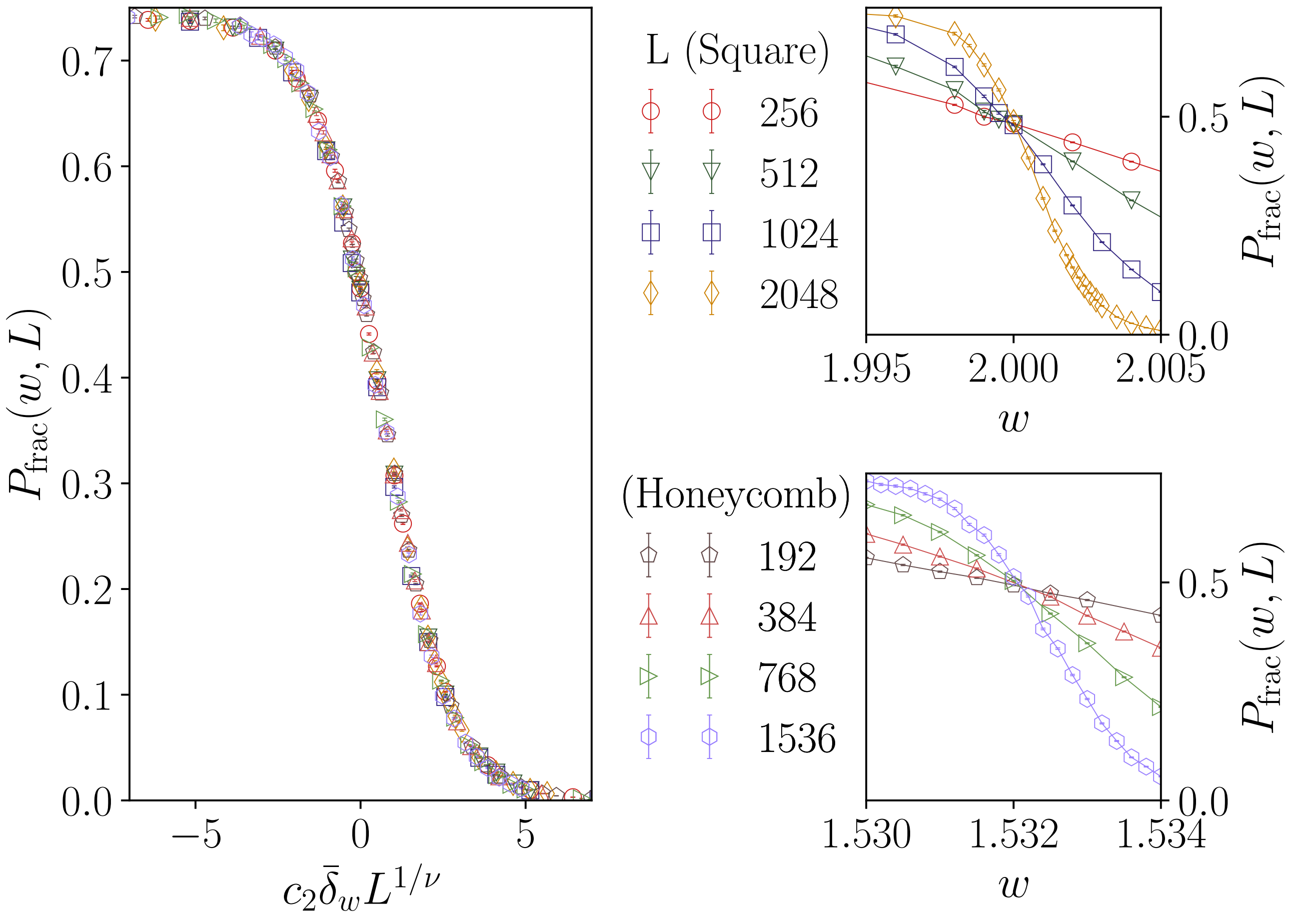
Three-sublattice spin order parameter and half/unit-vortex correlators

short-to-long loop phase transition



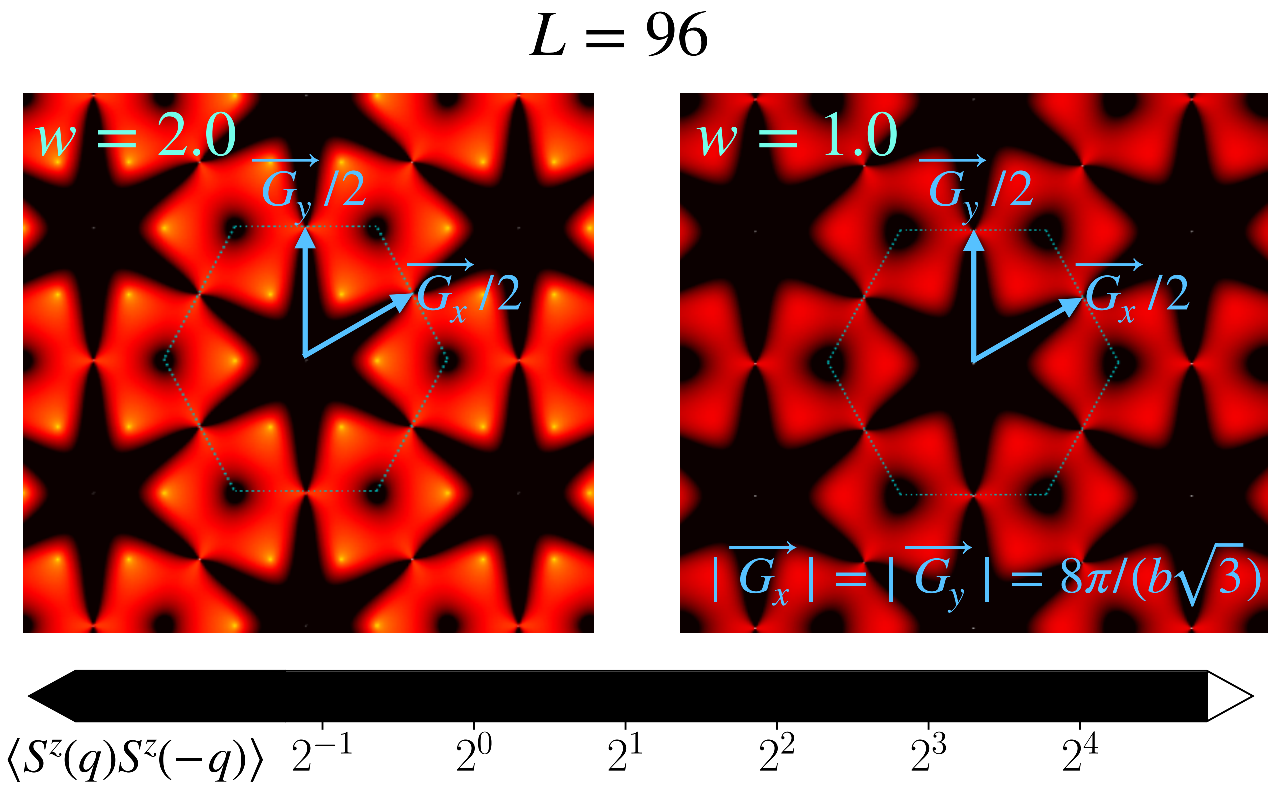
$\nu \approx 1.00(2)$

Flux-fractionalization character of transition



$\nu \approx 1.00(2)$

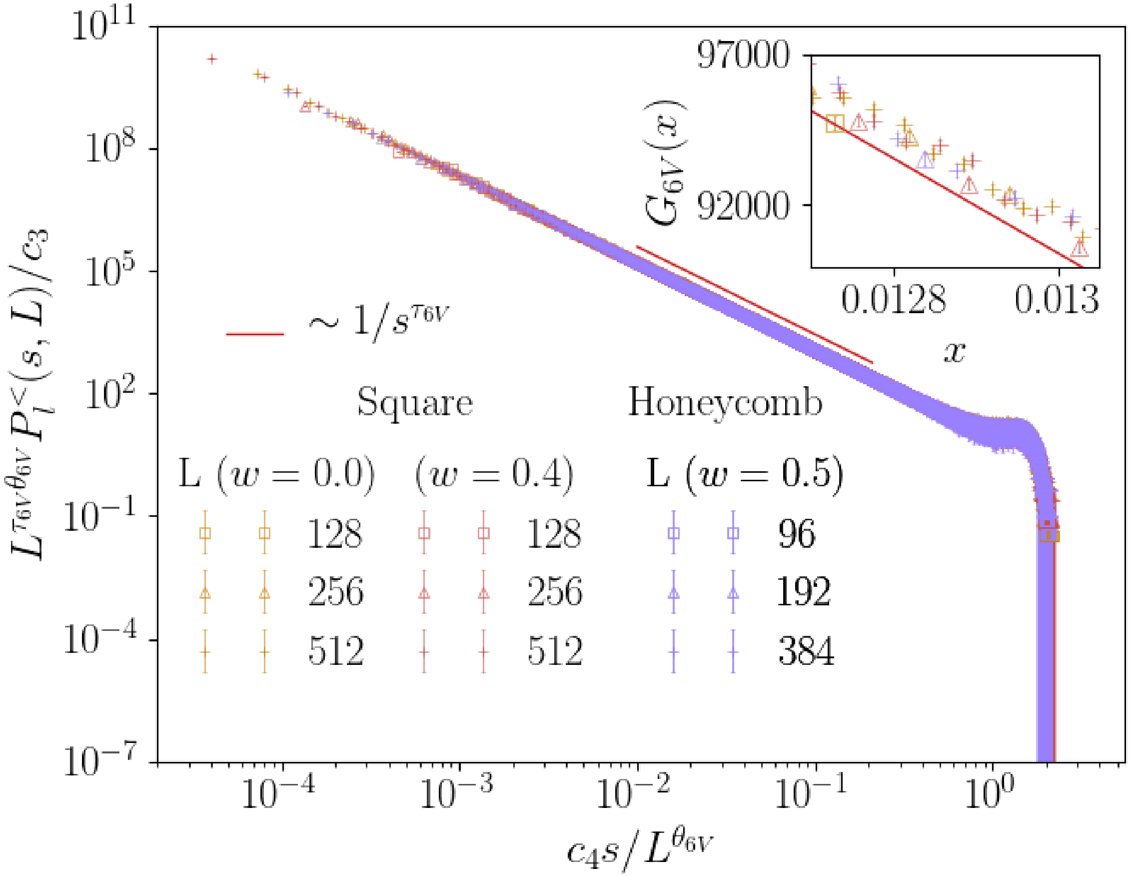
Transition observable in kagome spin structure factor



(a) (b)

Power-law feature at three sublattice wavevector absent in long-loop phase

Universal loop size distribution in long-loop phase



$$\tau_{6\mathrm{V}} = \frac{15}{7} \quad \theta_{6\mathrm{V}} = \frac{7}{4}$$

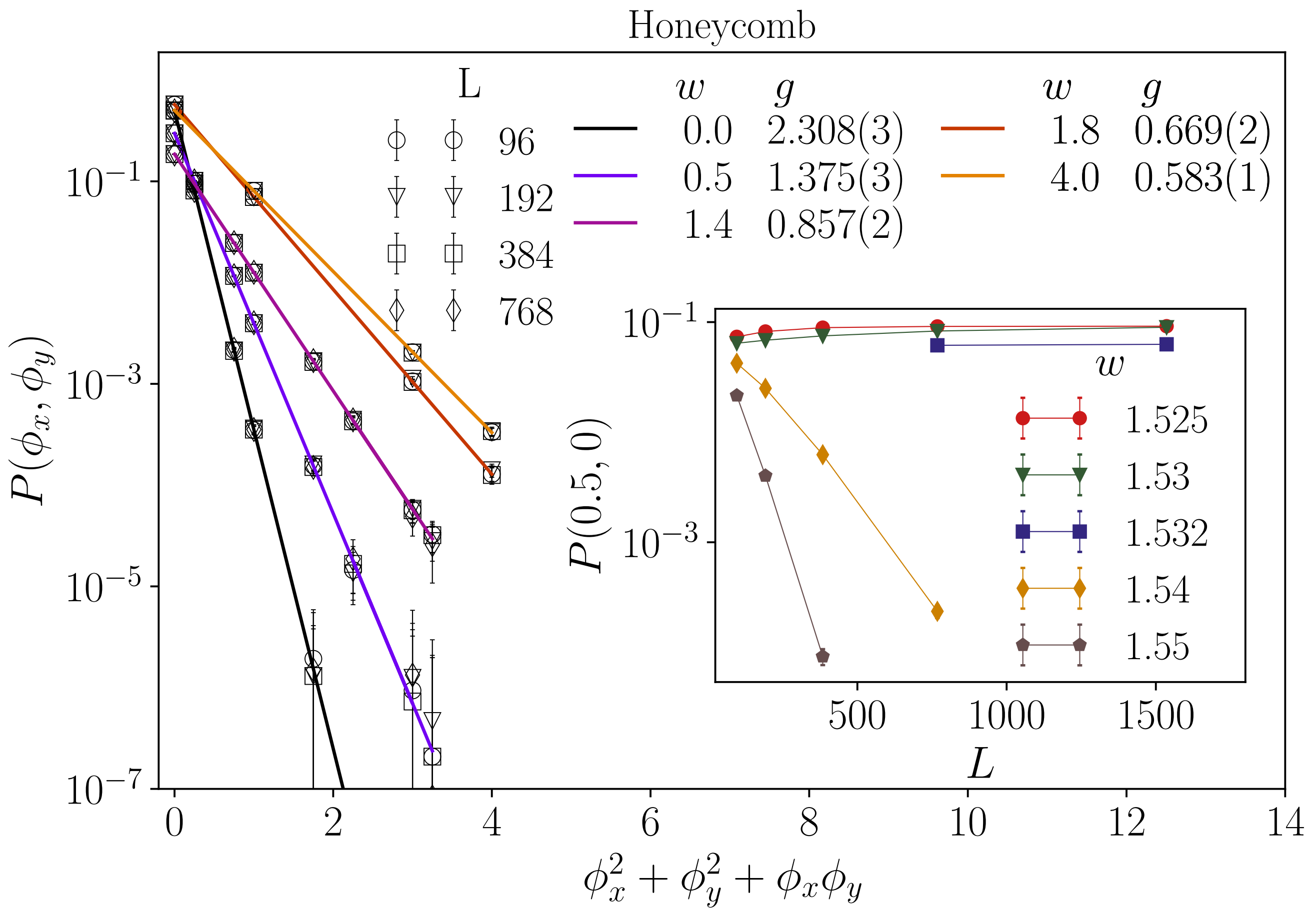
Note scaling relation:

$$\tau_{6\mathrm{V}}\theta_{6\mathrm{V}} = \theta_{6\mathrm{V}} + 2$$

O(1) loops with

$$s \sim L^{\theta_{6\mathrm{V}}}$$

Gaussian flux statistics in both phases

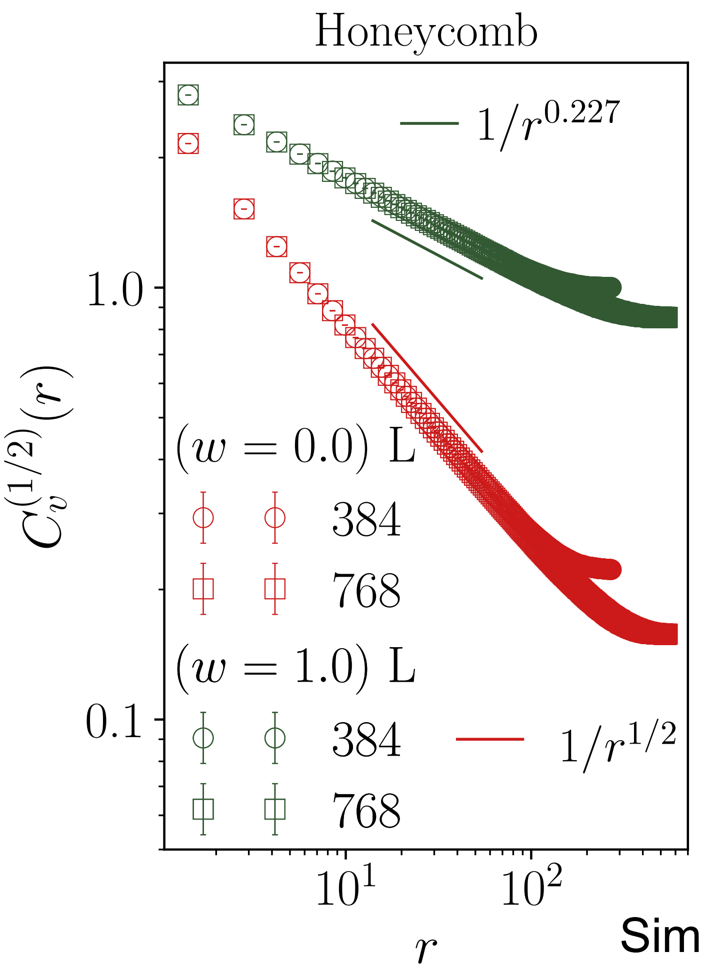


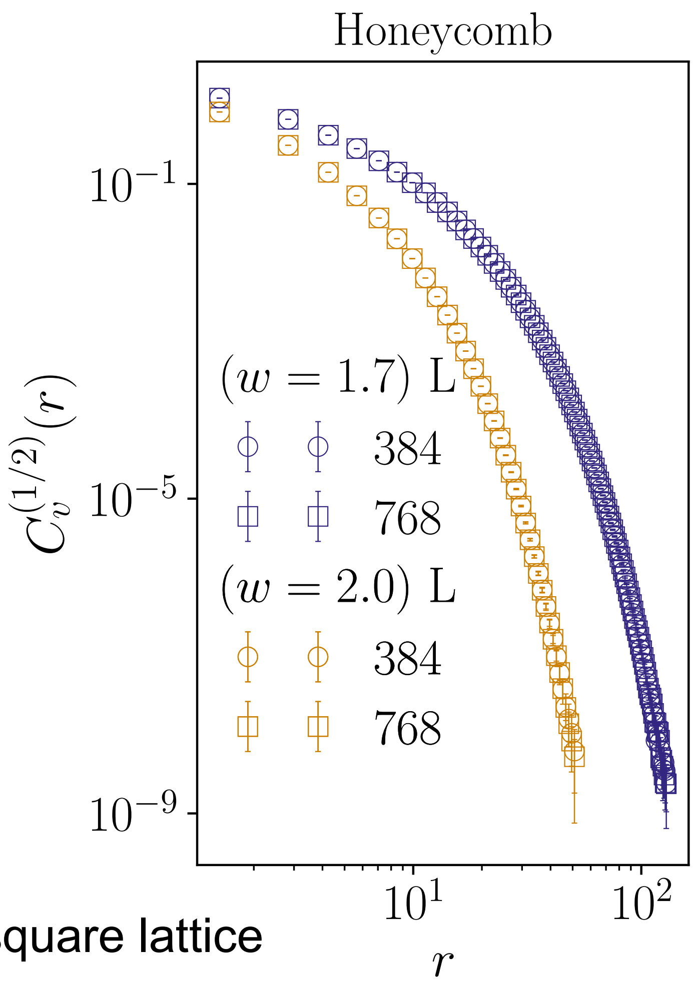
Can view as a "measurement" of the Gaussian action in both phases

But: In thermodynamic limit, half-integers "fall off" the Gausian in short-loop phase.

Similar picture on square lattice

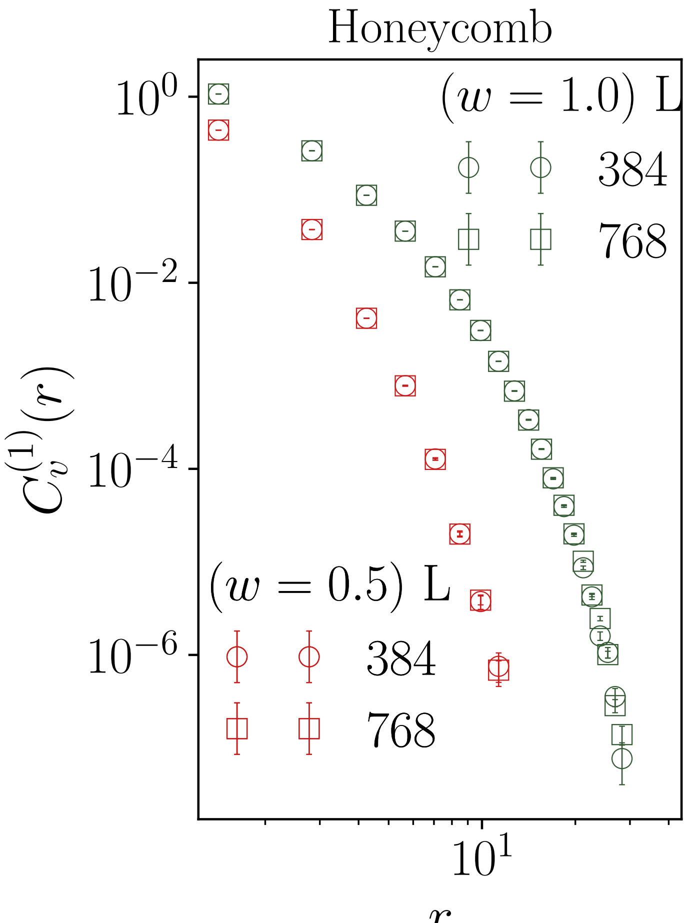
Half-vortex correlators in two phases

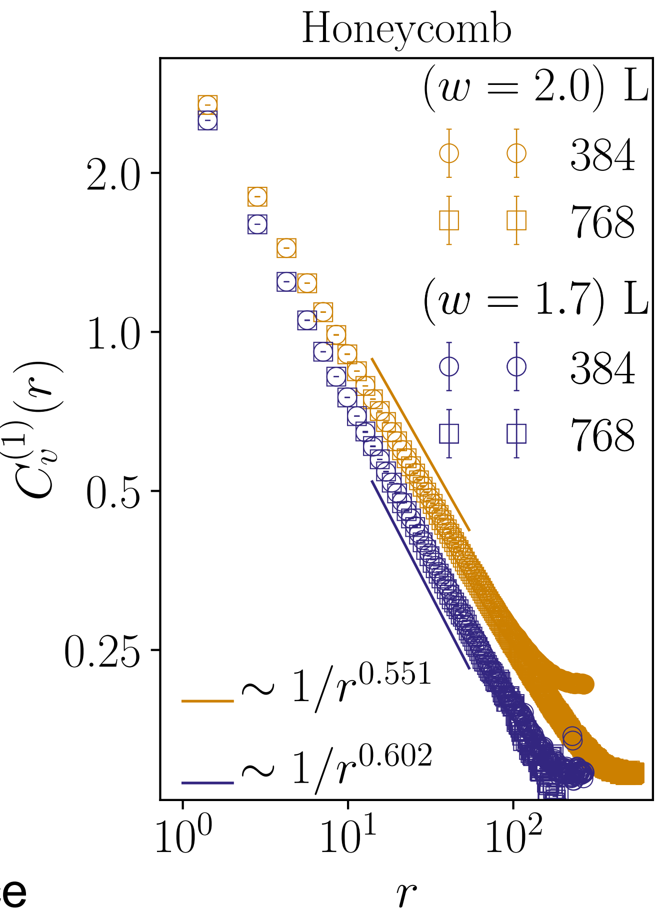




Similar picture on square lattice

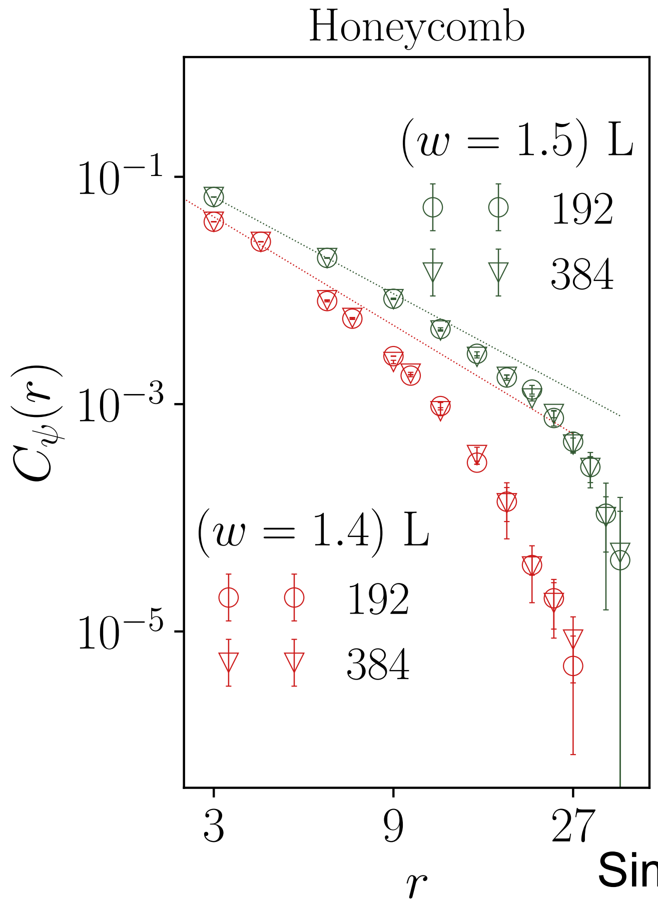
Unit-vortex correlators in both phases

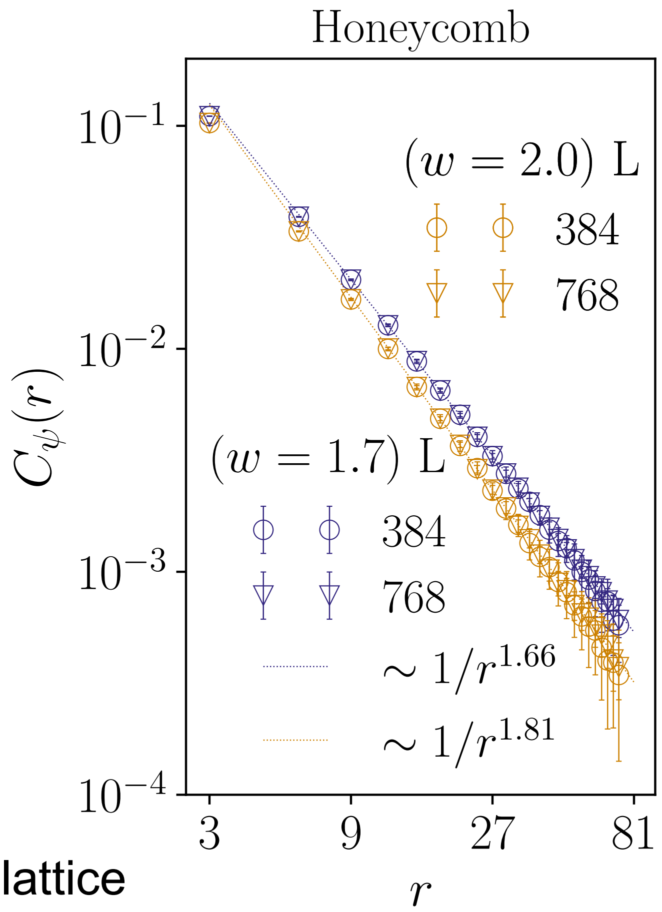




Similar picture on square lattice

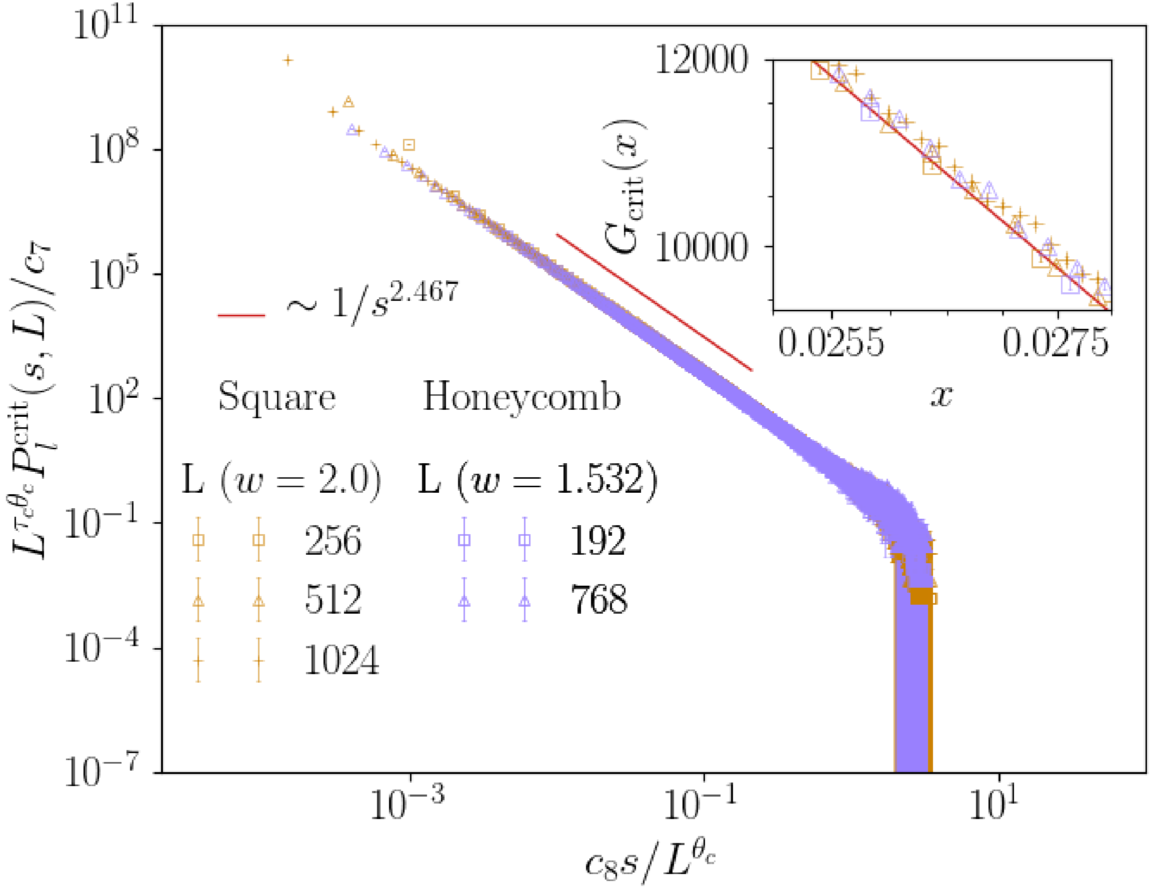
Columnar (three-sublattice) correlator in both phases





Similar picture on square lattice

Different loop size distribution at criticality

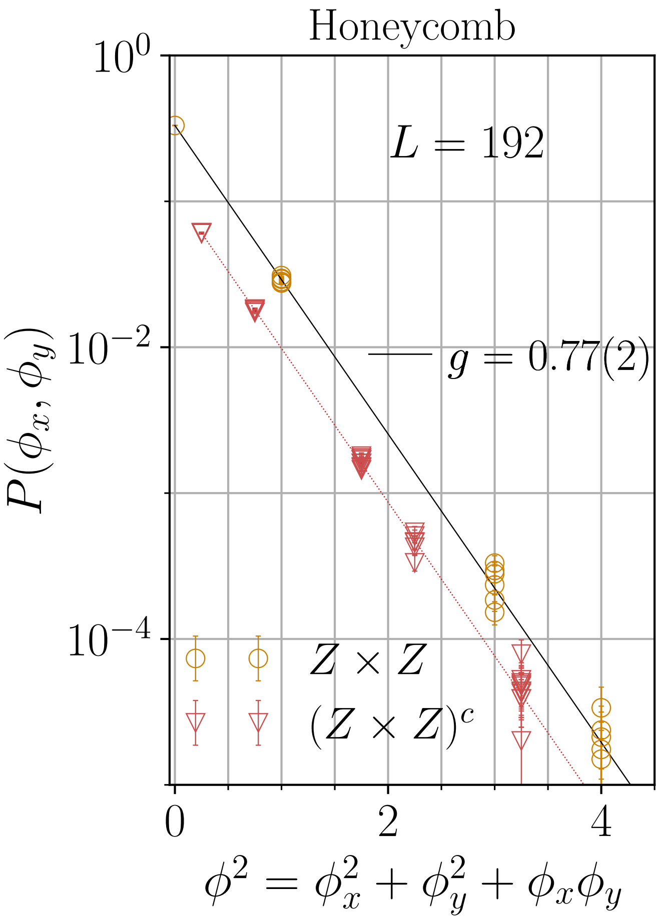


$$\tau_c \approx 2.47(1) \quad \theta_c \approx 1.38(1)$$

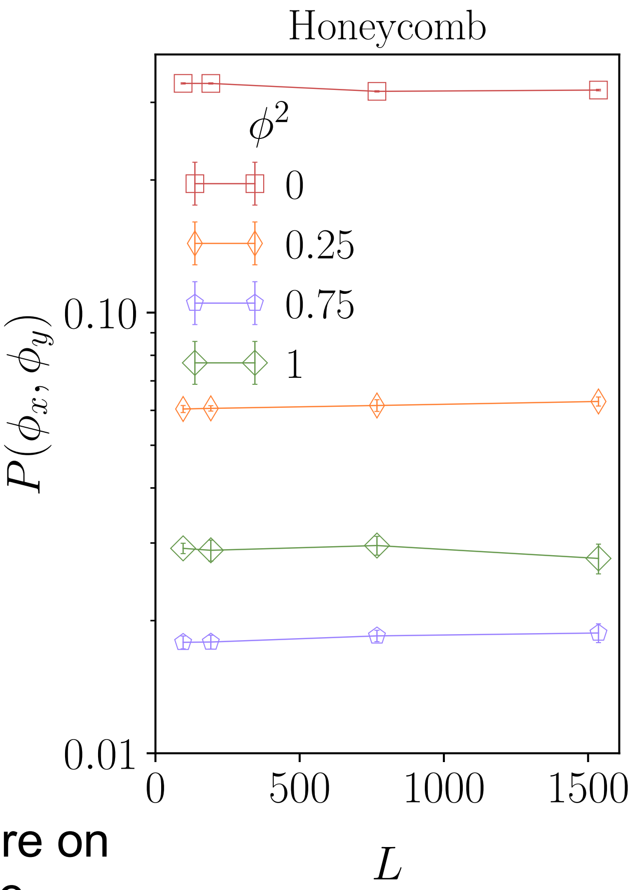
Scaling relation obeyed within errors

$$\chi_l^{\text{crit}} \equiv \frac{S_2}{L^2} \sim L^{2(\theta_c - 1)}$$

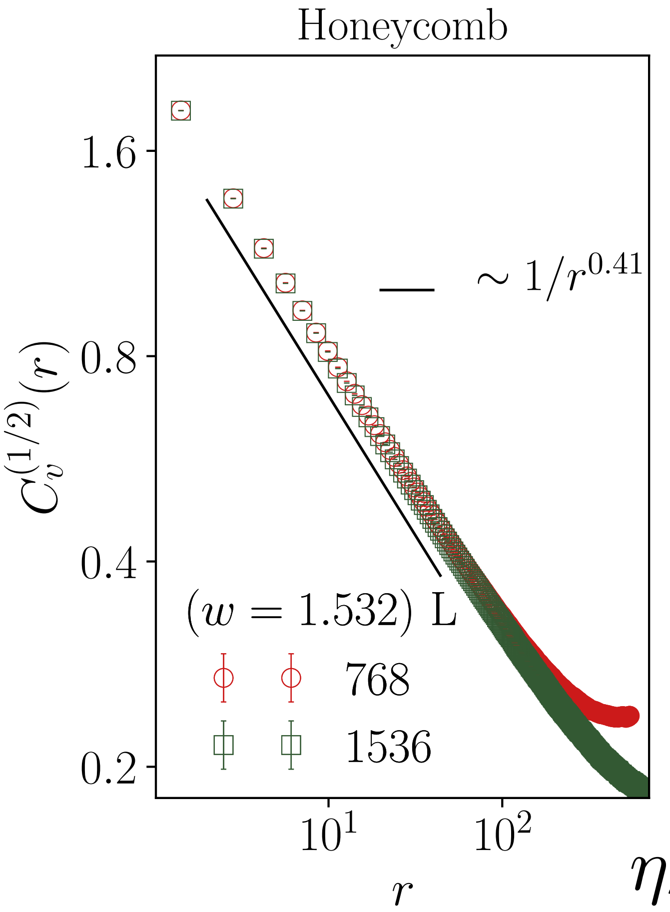
Critical flux distribution not single Gaussian



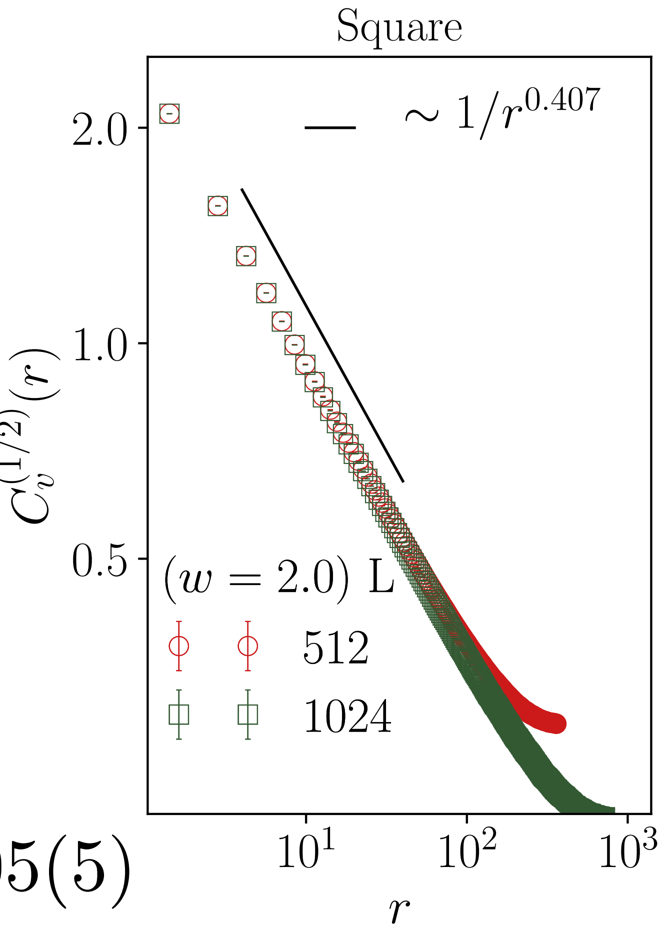
Similar picture on square lattice



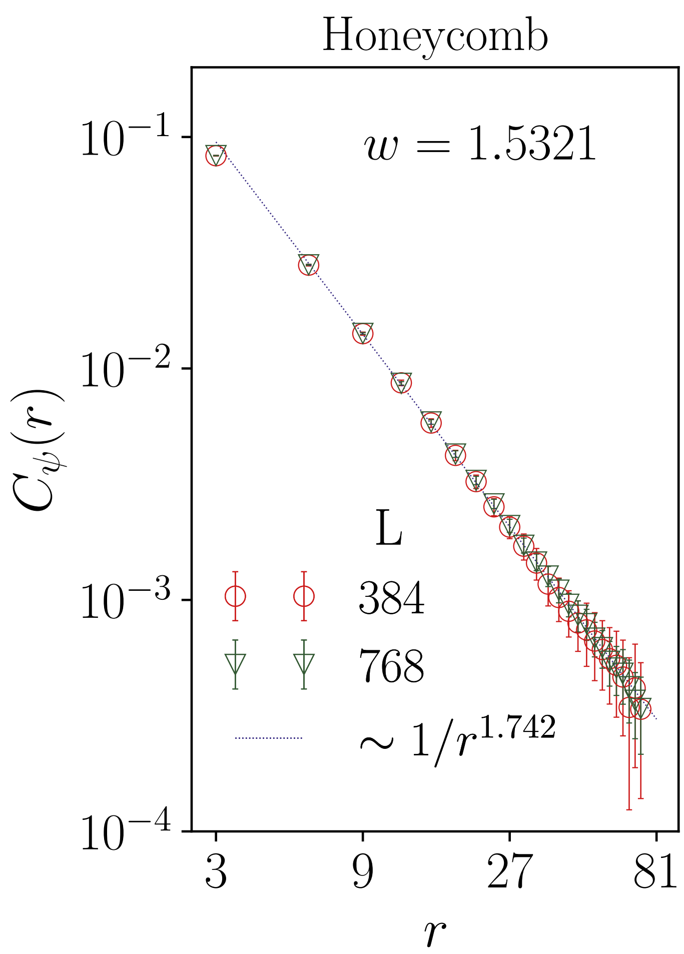
Critical half-vortex correlators

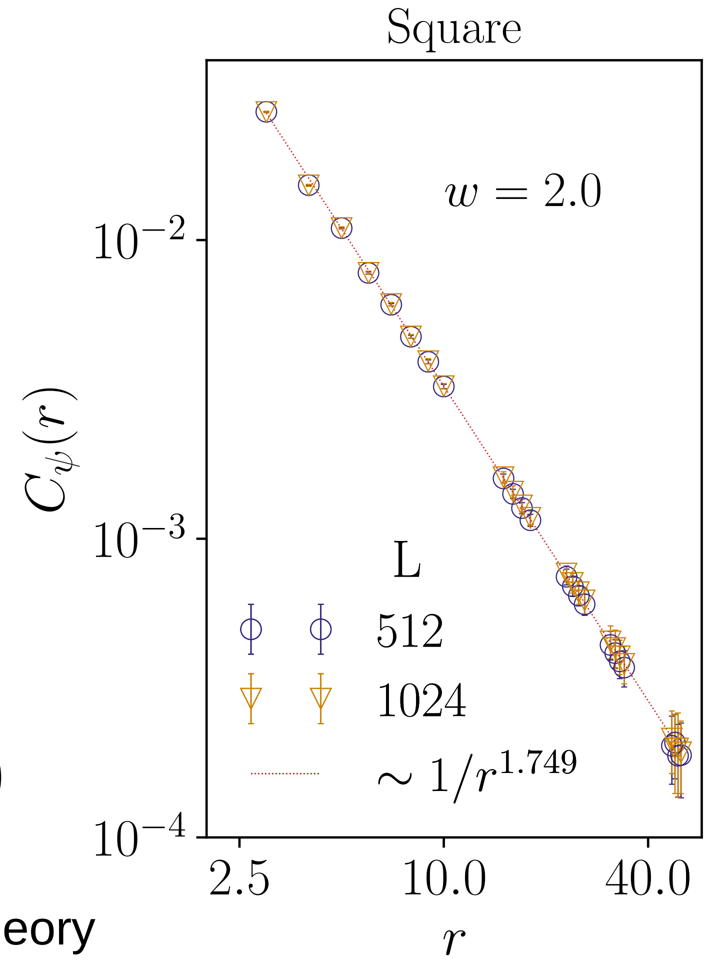


$$\eta_v^{1/2} \approx 0.405(5)$$



Columnar correlator at criticality

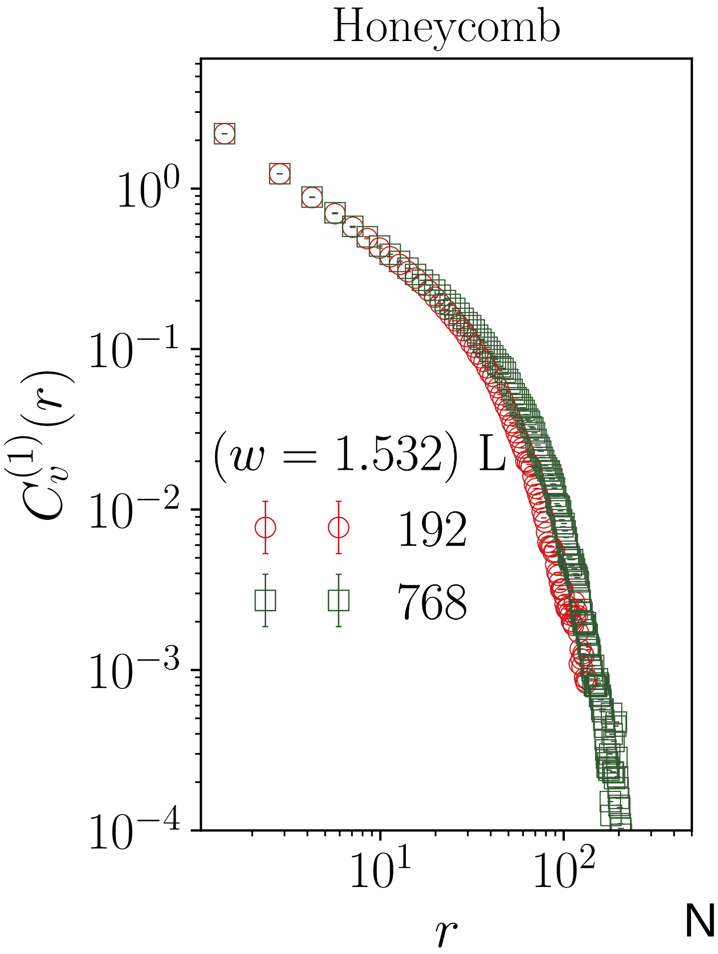


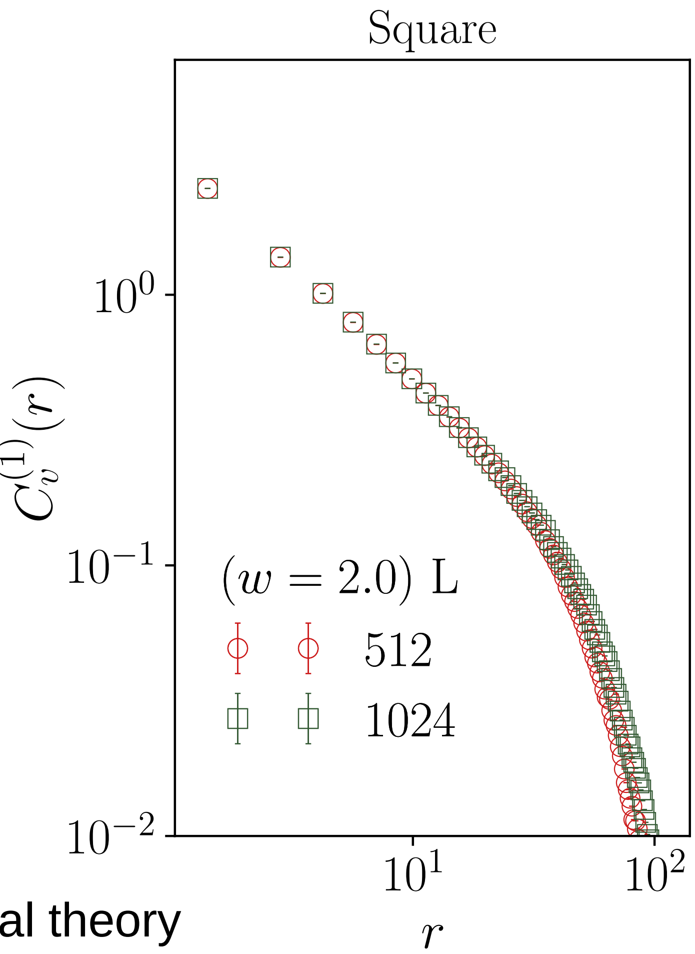


$$\eta_\psi \approx 1.745(10)$$

Non-Gaussian critical theory

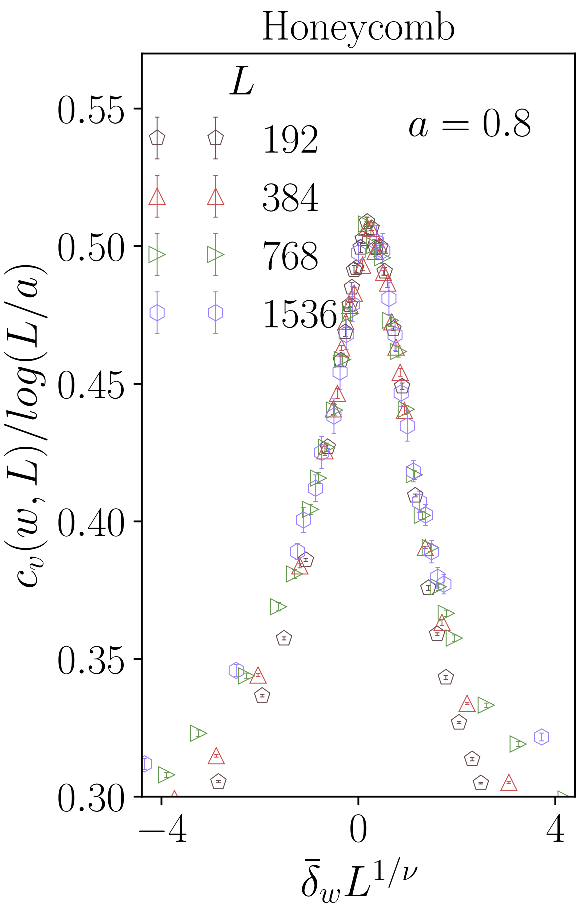
Unit-vortex correlator at criticality



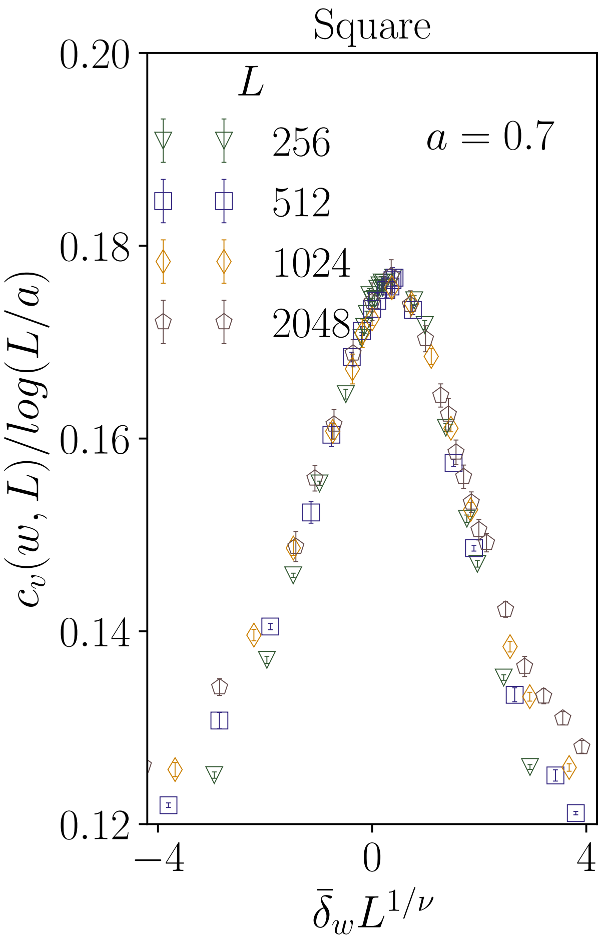


Non-Gaussian critical theory

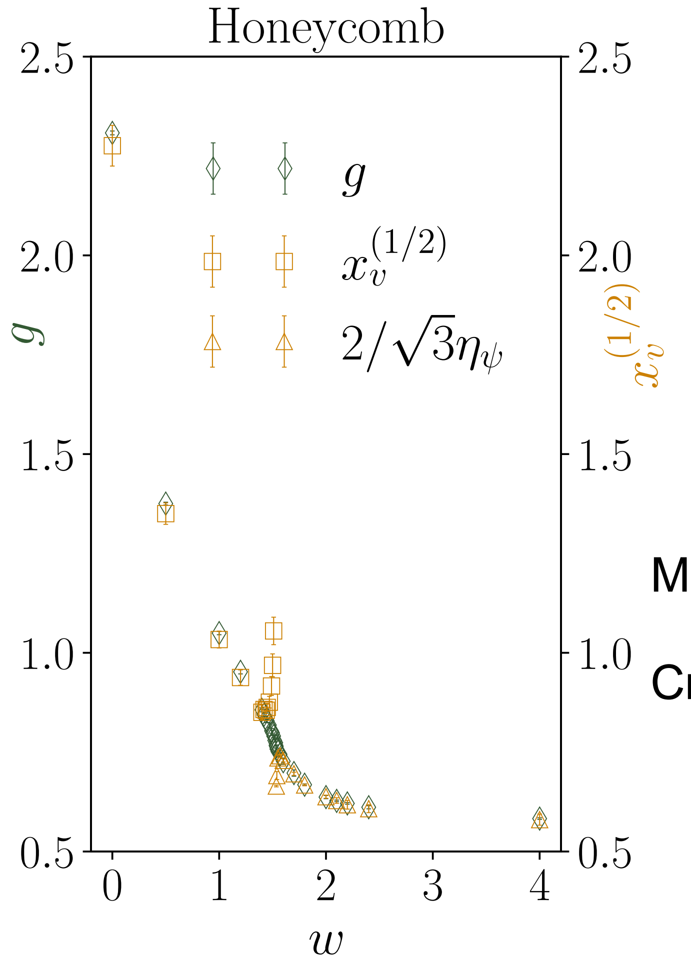
Specific heat near criticality



Ising-like (?)

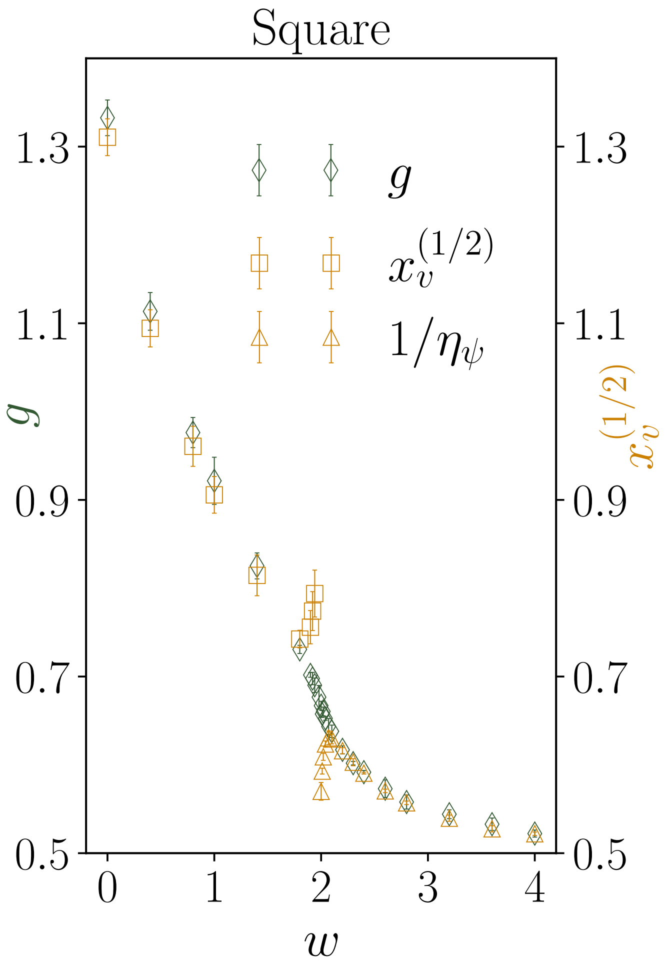


Stiffness extracted in multiple ways



Matches except near criticality

Critical theory not Gaussian



Overall picture from numerics

- Critical long-loop phase at small w separated by second order transition from a short loop (dimer rich) phase at large w.
- Entire long-loop phase controlled by w=0 fixed point.
- Fractional fluxes proliferate in the long-loop phase but combine into integers in short-loop phase.
- Short-loop phase has power-law three-sublattice spin order (columnar correlators of link segments/dimers), Destroyed in the long-loop phase when fractional fluxes proliferate.
- Mechanism not KT(!) . Correlation length exponent matches Ising (within errors)
- Test vortex correlators suggest: Half-vortex fugacity irrelevant perturbation in short-loop phase and relevant in long-loop phase.
- Prediction: Nonzero half-vortex fugacity destroys dipolar pinch-points in long-loop phase but not in short-loop phase.
- Interesting half-vortex driven transition out of short-loop phase as model for inter-plateau transition (ongoing work)

Acknowledgements

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