A tale of two transitions

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References

Scipost Phys. 14, 004 (2023)
Scipost Phys. 14, 146 (2023)
arXiv:2311.12107

Outline of the talk

(I)

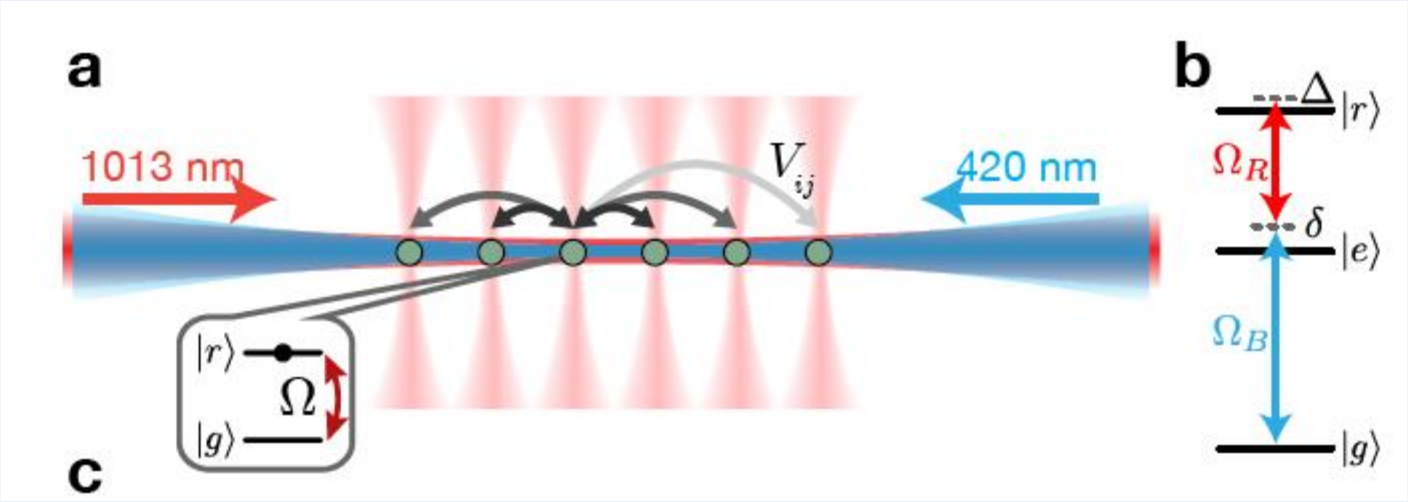
1. *Rydberg chains: Experiments and models*
2. *Rydberg ladder with staggered detuning*
3. *Order-by-disorder: an Ising transition*

(II)

4. *A constrained bosonic model with subsystem symmetry*
5. *Gapped and gapless phases of the model*
6. *Critical point: emergent Ashin-Teller universality*

Rydberg systems: Chains and arrays

Rydberg atom arrays



$$\delta \sim 2\pi \times 560\,\mathrm{MHz}$$

$$\Omega_B, \Omega_R \sim 2\pi \times 60, 36\,\mathrm{MHz}$$

$$\Omega = \Omega_B \Omega_R/(2\delta) \sim 2\pi \times 2\,\mathrm{MHz}.$$

$$|g\rangle = |5S_{1/2}, F = 2, m_F = -2\rangle$$

$$|r\rangle = |71S_{1/2}, J = 1/2, m_J = -1/2\rangle$$

System of ^{87}Rb atoms controllably coupled to their Rydberg excited state.

The van dar Walls interaction between two atoms in their excited (Rydberg) states is denoted by V and is a tunable parameter.

One can vary the detuning parameter Δ which allows one to preferentially put the atom in a Rydberg or ground state

Effective low-energy description

$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j,$$

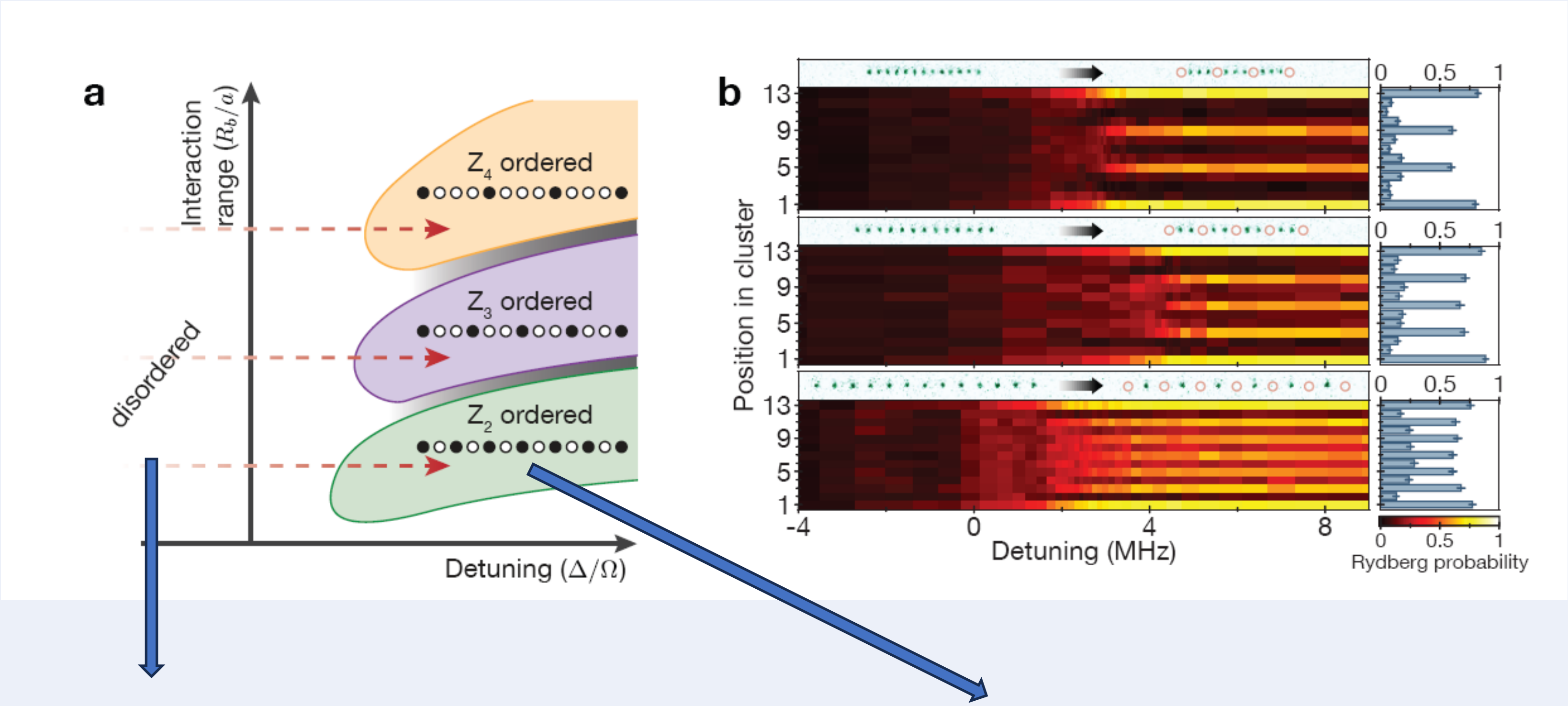
$n = (1+\sigma^z)/2$

$V_{ij} = V_0/|r_{ij}|^6$

V_0 can be tuned so that Rydberg excitations in neighbouring sites are forbidden.

H. Bernien at al. Nature 2017

H. Bernien at al. Nature 2017



Rydberg vacuum (|0>) for $\Delta >> \Omega$ and $\Delta < 0$

|Z_2> state for $\Delta >> \Omega$ and $\Delta > 0$.

These states are separated by an Ising transition

Similar to the transition found in tilted optical lattice

S. Sachdev et al, PRB 66, 075128 (2002).

Mapping to a constrained model

$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j,$$

Two states per site: Natural spin ½ representation

$$\sigma_\ell^z \;=\; 2n_\ell - 1, \quad \sigma_\ell^{x(y)} = (i)(d_\ell + (-)d_\ell^\dagger).$$

Rydberg blockade on neighboring sites: $V_{i,i+1}$ >> Δ, Ω >> $V_{i,i+2}$

$$P_\ell \;=\; (1 - \sigma_\ell^z)/2$$

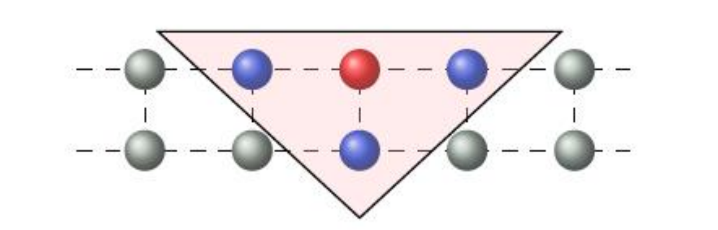
A up-spin (Rydberg excitation) can be created on a site if and only if there are no up-spins (excitations) on the neighboring sites

$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j,$$

→

$$\begin{aligned} H_{\mathrm{spin}} \;&=\; -w\sum_\ell P_{\ell-1}\sigma_\ell^x P_{\ell+1} + \lambda/2\sum_\ell \sigma_\ell^z \\ &=\; \sum_\ell (-w\tilde{\sigma}_\ell^x + \lambda/2\sigma_\ell^z) \end{aligned}$$

Rydberg Ladders



Two-leg Rydberg ladder

$$H = \sum_{j=1}^{2L}\sum_{\ell=1}^{2}\left(w\sigma^x_{j,\ell} - \frac{1}{2}[\Delta(-1)^j + \lambda]\sigma^z_{j,\ell}\right) + \sum_{\mathbf{r}\neq\mathbf{r}'} V_{\mathbf{r}\mathbf{r}'}\hat{n}_{\mathbf{r}}\hat{n}_{\mathbf{r}'},$$

Add a site-dependent detuning having a staggered component

$$H_a = \sum_{\mathbf{r},\mathbf{r}'} V_0 \hat{n}_{\mathbf{r}}\hat{n}_{\mathbf{r}'}\delta_{|\mathbf{r}-\mathbf{r}'|-1},$$

$$H_b = \sum_{j=1}^{2L}\sum_{\ell=1}^{2}\left(w\sigma^x_{j,\ell} - \frac{1}{2}[\Delta(-1)^j - \lambda]\sigma^z_{j,\ell}\right) + \frac{V_0}{2}\sum_{\mathbf{r}\mathbf{r}'}\frac{n_{\mathbf{r}}\hat{n}_{\mathbf{r}'}}{|\mathbf{r}-\mathbf{r}'|^6}(1-\delta_{|\mathbf{r}-\mathbf{r}'|-1}).$$

Work with an effective low-energy Hamiltonian (where H_a=0) where there are no nearest-neighbor Rydberg excitations

$$H^V_{\text{eff}} = \sum_{j=1}^{2L}\sum_{\ell=1}^{2} w\tilde{\sigma}^x_{j,\ell} - \sum_{j=1}^{2L}\sum_{\ell=1}^{2}\frac{1}{2}[\Delta(-1)^j + \lambda]\sigma^z_{j,\ell} + \frac{V_0}{2}\sum_{\mathbf{r}\mathbf{r}'}\frac{n_{\mathbf{r}}\hat{n}_{\mathbf{r}'}}{|\mathbf{r}-\mathbf{r}'|^6}(1-\delta_{|\mathbf{r}-\mathbf{r}'|-1}),$$

$$\tilde{\sigma}^x_{j,\ell} = P_{j-1,\ell}\left(\prod_{\ell'\neq\ell} P_{j\ell'}\sigma^x_{j,\ell}\right)P_{j+1,\ell},$$

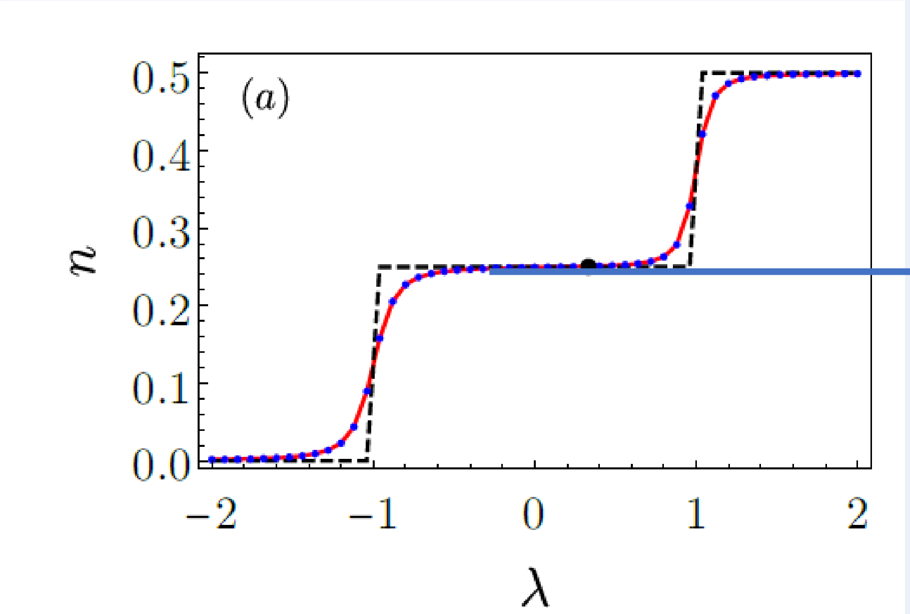
Effective Hamiltonian for the ladder where the second nearest-neighbor interaction are small

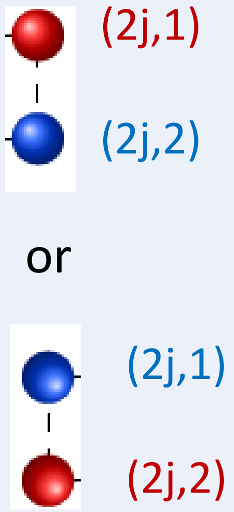
$$V_0 \gg \lambda, \Delta \gg w \gg V_0/(\sqrt{2})^6,$$

$$H_{\text{eff}} = \sum_{j=1}^{2L}\sum_{\ell=1}^{2} w\tilde{\sigma}^x_{j,\ell} - \frac{1}{2}\sum_{j=1}^{2L}\sum_{\ell=1}^{2}[\Delta(-1)^j + \lambda]\sigma^z_{j,\ell}.$$

D. Bluvestein et al., Science (2021)

Ground state phase diagram of the ladder





In the classical limit the energy of creating a Rydberg excitation on an even (odd) site is

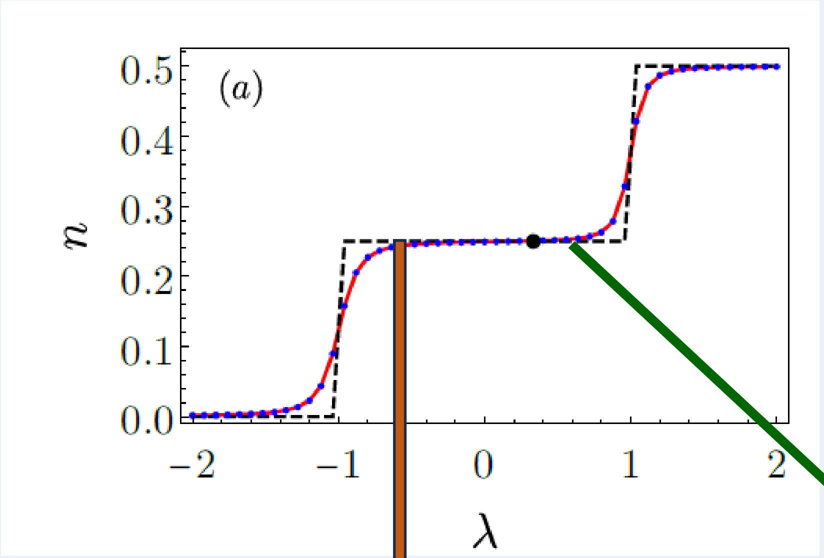
$$E_{even}/\Delta = -(\lambda/\Delta + 1)/2$$
$$E_{odd}/\Delta = -(\lambda/\Delta - 1)/2$$

For -1< λ /Δ <1, the ground state has a Rydberg excited atom on one out the two even sites of the ladder.

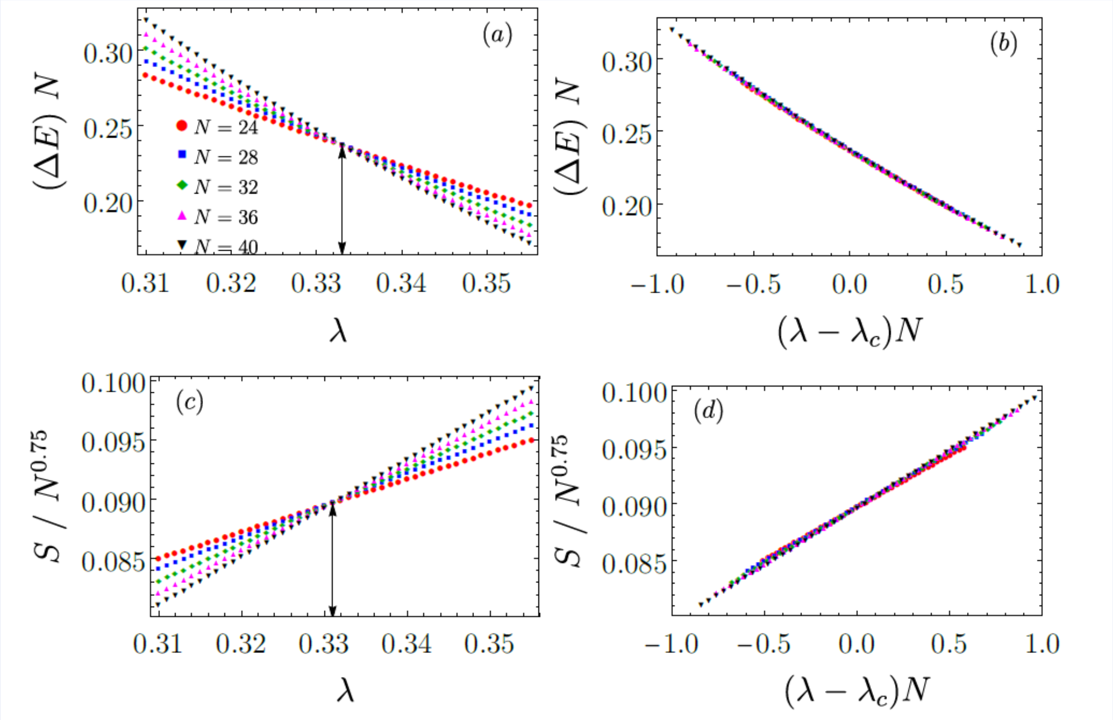
Macroscopic ground state degeneracy D= $2^{(L)}$ for chain length 2L

Order-by-disorder mechanism to determine the ground state

An Ising transition

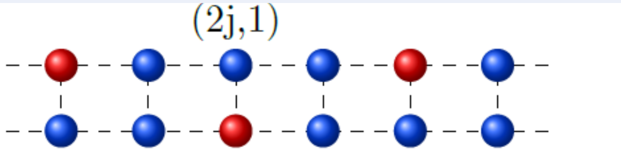


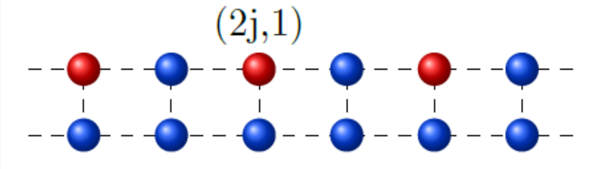
Finite-size scaling



$$\Delta E = N^{-z} f\left[N^{1/\nu}(\lambda-\lambda_c)\right],$$

$$S = \frac{1}{N}\langle\hat{O}^2\rangle = N^{2-z-\eta} g\left[N^{1/\nu}(\lambda-\lambda_c)\right]$$





Unique ground state superposition of many Fock states with Rydberg excitations randomly distributed over both ladders

Two fold degenerate ground state (which breaks Z_2 symmetry) where all Rydberg excitations occur on any one of the ladders

Both the phases and the transition is stabilized via an order-by-disorder mechanism

Perturbation theory: steps

Perturbative parameter w << λ, Δ, |λ–Δ| →

$$H_{\rm eff} = H_0 + H_1$$

$$H_1 = w\sum_{j,=1}^{2L}\sum_{\ell=1}^{2}\tilde{\sigma}^x_{j,\ell}.$$

Design a canonical transformation to obtain effective low energy Hamiltonian

$$H' = \exp[iS']H_{\rm eff}\exp[-iS'] = H_0 + H_1 + [iS', H_0 + H_1] + \frac{1}{2}[iS',[iS',H_0]] + \ldots,$$

Eliminate all processes that takes one out of the low-energy manifold to first order in w/λ

$$[iS', H_0] = -H_1$$

$$S' = -w\sum_{j=1}^{2L}\sum_{\ell=1}^{2}(\Delta(-1)^j + \lambda)^{-1}\tilde{\sigma}^y_{j,\ell}.$$

With this choice, the effective Hamiltonian is

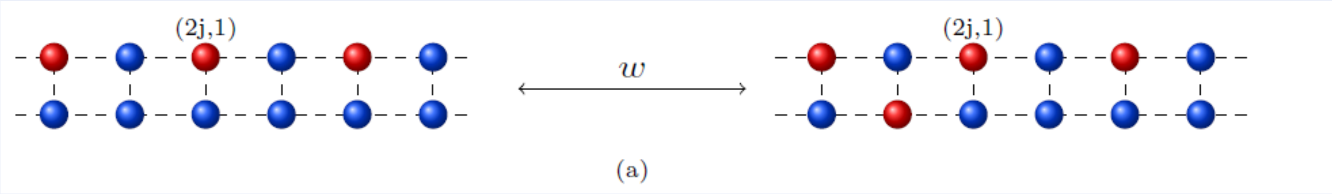
$$H' = \frac{-w^2}{\Delta-\lambda}\sum_{j=1}^{L}\Big[\sum_{\ell=1}^{2}P_{2j,\ell}P_{2j+2,\ell} + \frac{\alpha_0}{2}\sum_{\ell,\ell'=1,2;\ell\neq\ell'}P_{2j,\ell'}(\sigma^x_{2j,\ell}\sigma^x_{2j,\ell'} + \sigma^y_{2j,\ell}\sigma^y_{2j,\ell'})\Big]$$

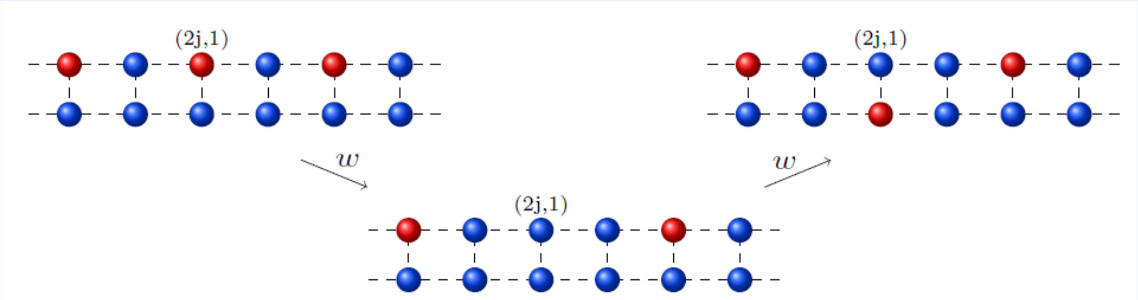
$\alpha_0 = (\Delta-\lambda)/(\Delta+\lambda)$

How do we understand this transition?

The ground state degeneracy is lifted by quantum fluctuations.

There are two important virtual processes





$$H' = \frac{-w^2}{\Delta-\lambda}\sum_{j=1}^{L}\Big[\sum_{\ell=1}^{2} P_{2j,\ell}P_{2j+2,\ell} + \frac{\alpha_0}{2}\sum_{\ell,\ell'=1,2;\ell\neq\ell'} P_{2j,\ell'}(\sigma^x_{2j,\ell}\sigma^x_{2j,\ell'}+\sigma^y_{2j,\ell}\sigma^y_{2j,\ell'})\Big],$$

$\alpha_0 = (\Delta-\lambda)/(\Delta+\lambda)$

The first term prefers Rydberg excitations on one of the ladders and hence leads to Z_2 symmetry broken ground state.

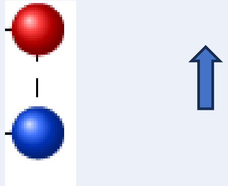
The second term prefers states having superposition of Rydberg excitations on both ladders

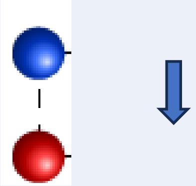
Example of an Ising transition stabilized purely by an order-by-disorder mechanism.

Effective Ising model

Define pseudospin states on even sites of the ladder

$$|1_{2j}\rangle \equiv |\uparrow_{2j,1};\downarrow_{2j,2}\rangle \qquad |-1_{2j}\rangle \equiv |\downarrow_{2j,1};\uparrow_{2j,2}\rangle$$





In this space of states, the projection operators can be written in terms of Ising variables s_j^z

$$P_{2j,1(2)} = (1-(+)s^z_{2j})/2.$$

The terms in the Hamiltonian maps to

(1)

$$\sum_{j=1}^{L}\sum_{\ell=1}^{2} P_{2j,\ell}P_{2j+2,\ell} \to \frac{1}{2}\sum_{j=1}^{L} s^z_{2j}s^z_{2j+2}.$$

(2)

$$\frac{1}{2}\sum_{j=1}^{L}\sum_{\ell,\ell'=1,2;\ell\neq\ell'} P_{2j,\ell'}(\sigma^x_{2j,\ell}\sigma^x_{2j,\ell'}+\sigma^y_{2j,\ell}\sigma^y_{2j,\ell'}) \to \sum_{j=1}^{L} s^x_{2j}.$$

This leads to an effective Ising model

$$H^I_{\mathrm{eff}} = \frac{-w^2}{\Delta-\lambda}\sum_{j=1}^{L}\left(\frac{1}{2}s^z_{2j}s^z_{2j+2}+\alpha_0 s^x_{2j}\right)$$

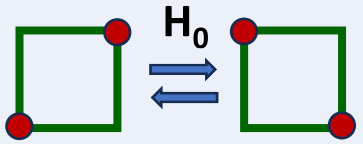
One can simply read off the critical point to be at

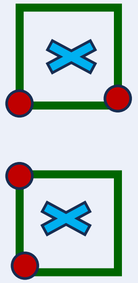
$$\alpha_0 = (\Delta-\lambda)/(\Delta+\lambda) = 1/2 \longrightarrow \lambda_c = \Delta/3.$$

Emergent Ashkin-Teller universality

A constrained spin or boson hopping model

Ring-exchange Hamiltonian with additional constraint of not having nearest-neighbor up-spins/bosons.





Can be expressed in terms of a simple boson or spin model

$$2b^{\dagger}_{j_x,j_y} b_{j_x,j_y} - 1 = \sigma^z_{j_x,j_y}$$
$$b^{\dagger}_{j_x,j_y} = \sigma^+_{j_x,j_y}$$

Mapping

$$\left(1+\sigma^z_{j_x,j_y}\right)\left(1+\sigma^z_{j_x\pm1,j_y}\right) = \left(1+\sigma^z_{j_x,j_y}\right)\left(1+\sigma^z_{j_x,j_y\pm1}\right) = 0$$

Constraint

$$H = J \sum_{j_x,j_y} \left(\sigma^+_{j_x,j_y}\sigma^+_{j_x+1,j_y+1}\sigma^-_{j_x+1,j_y}\sigma^-_{j_x,j_y+1} + \text{h.c.}\right)$$

Hamiltonian

$$S^z_{\text{tot}} = \sum_{j_x,j_y} \sigma^z_{j_x,j_y}.$$

$$D_x = \sum_{j_x,j_y} j_x \sigma^z_{j_x,j_y}, \qquad D_y = \sum_{j_x,j_y} j_y \sigma^z_{j_x,j_y}.$$

The model has conserved boson number and dipole moments and is an example of a 2D model showing strong Hilbert space fragmentation.

The model has subsystem symmetry: any state in a given fragment conserves total boson number on all vertical and horizonal lines of a square lattice.

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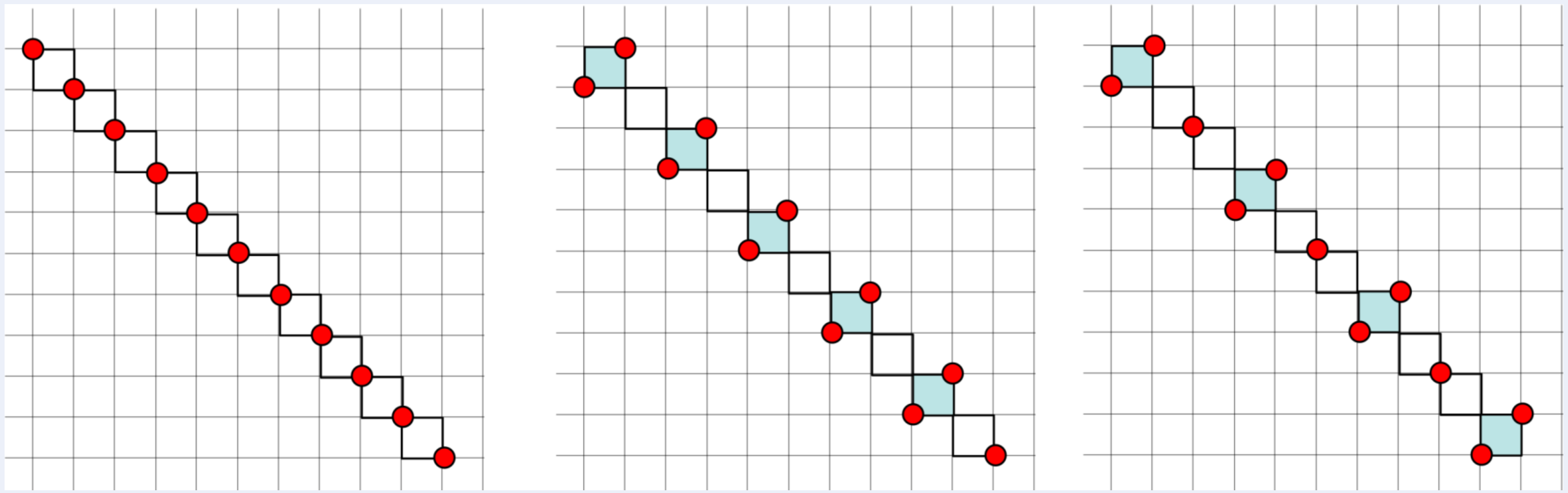
1D arrays of bosons

States of the model on 1D array of bosons

Boson numbers on all vertical and horizontal connected links are unity and are conserved



Modify the Hamiltonian to introduce an on-site competing term

Define states: $|\ell_j\rangle = b_j^\dagger b_{j+d_l}^\dagger |0_j; 0_{j+d_l}\rangle$ and $|t_j\rangle = b_j^\dagger b_{j+d_t}^\dagger |0_j; 0_{j+d_t}\rangle$

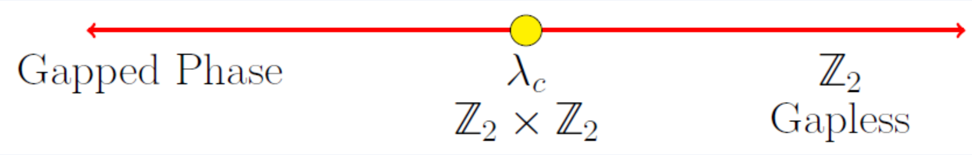
Constraint: $\hat{n}_j \hat{n}_{j+n} = 0$

$$H_0 = -w \sum_j (|t_j\rangle\langle \ell_j| + \text{h.c.})$$ → *The ring-exchange term*

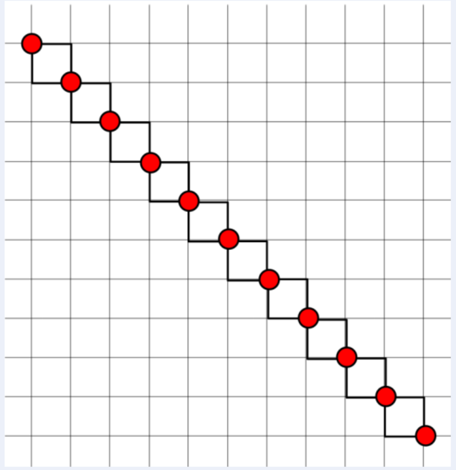
$$H_1 = \lambda \sum_j (|\ell_j\rangle\langle \ell_j| - |t_j\rangle\langle t_j|),$$ → *On-site terms*

$n_0 = \sum_{j\in q} n_j,$ *is conserved and equals unity on all connected links* } *Subsystem symmetry*

Main result



Large negative λ
|λ| /w >> 1



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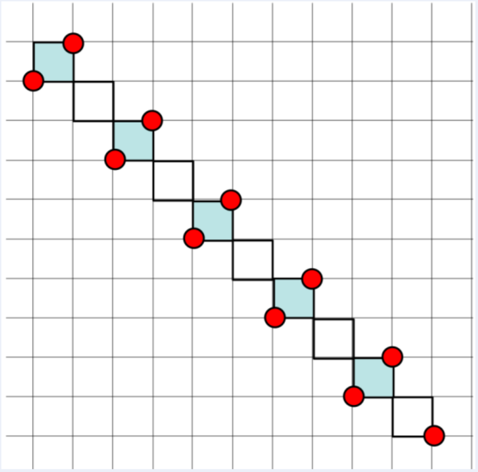


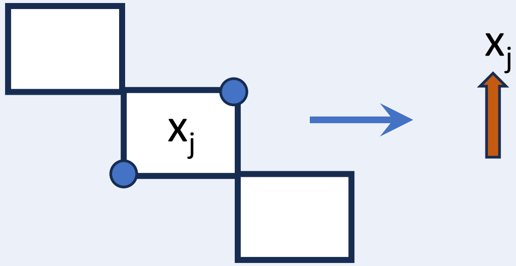
z=1
ν=1.56… +(-) 0.001…
c=1 +(-) 0.05..

QCP: Ashkin-Teller universality

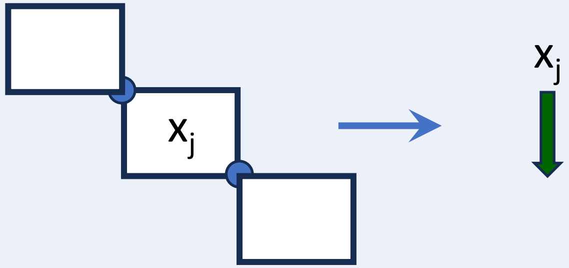
Schematic gapped ground state

Typical state in the ground state manifold
The ground state is a superposition of such states and breaks Z_2 symmetry

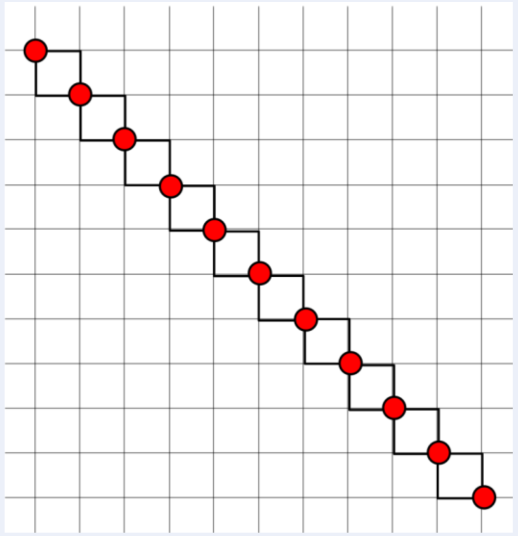
Effective PXP model with multi-spin interaction



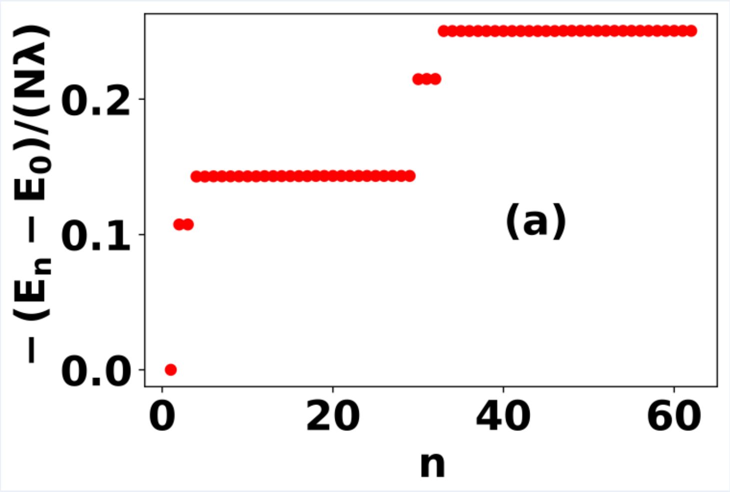
$$H_s = \sum_j [-w P_{x_j-1} \tau^x_{x_j} P_{x_j+1} + \frac{\lambda}{2}(\tau^z_{x_j} - P_{x_j-1} P_{x_j} P_{x_j+1})]$$

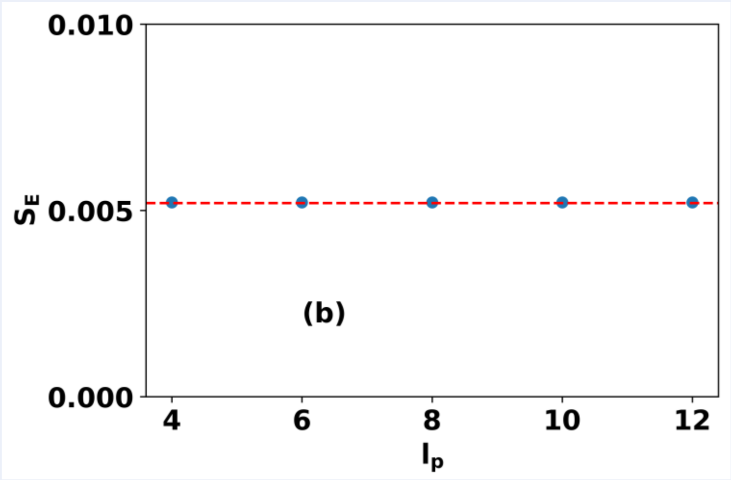


Large negative λ



Schematic ground state





The ground state is unique and gapped.

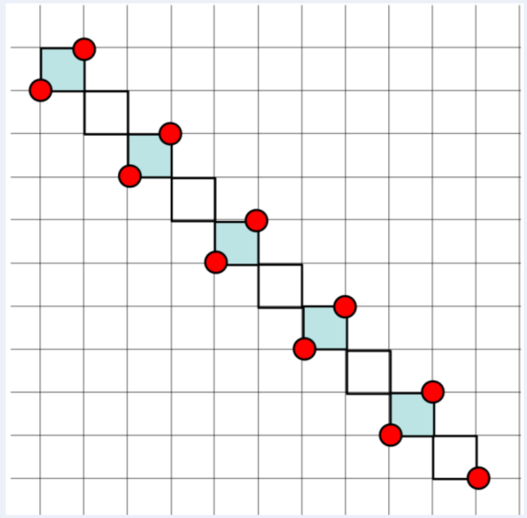
The first excited state corresponds to flipping bosons on a single plaquette.

Flipping the bosons on the edge plaquette has an energy cost of $2|\lambda|$ while that at the bulk has $3|\lambda|$. This leads to two edge states.

Standard area law entanglement for the ground state

Large positive λ

Flip all bosons from longitudinal to transverse diagonals preserving subsystem symmetry and total boson number.



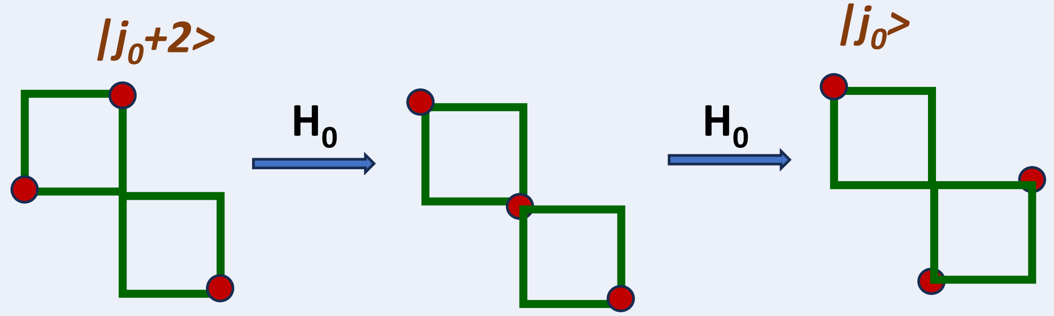
This leaves out a single boson on one of the plaquettes.

These states can be labelled by position of the lone boson: $|j_0\rangle$

The ground state is manifold is formed by $O(N_p)$ degenerate states.

This degeneracy of the ground state manifold is lifted by quantum fluctuations originating from H_0.

$$H_{\text{eff}} = \frac{-w^2}{3\lambda} \sum_{j_0} (|j_0\rangle\langle j_0 + 2| + \text{h.c.}) + \text{O}(w^4/\lambda^3)$$



Large positive λ

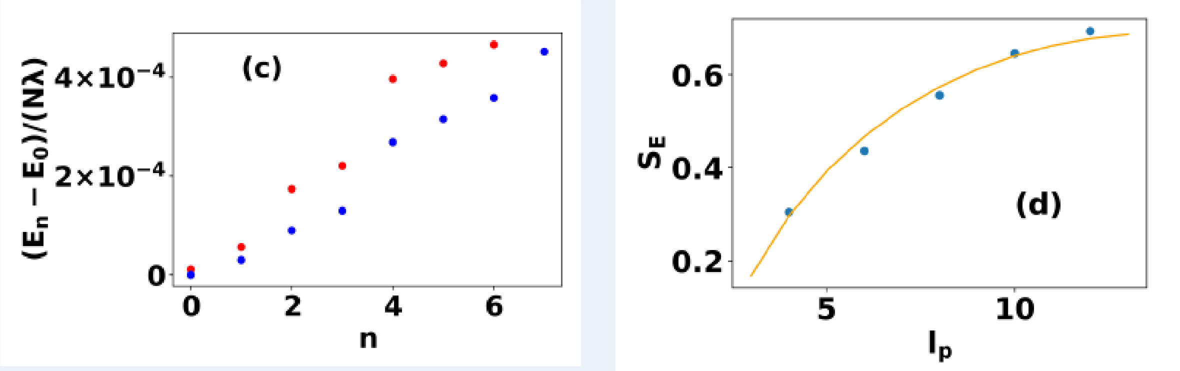
For arrays with large positive λ and PBC the energy dispersion in the thermodynamic limit

$$E(k) = -\frac{2w^2}{3\lambda}\cos 2k + \mathrm{O}(w^4/\lambda^3)$$

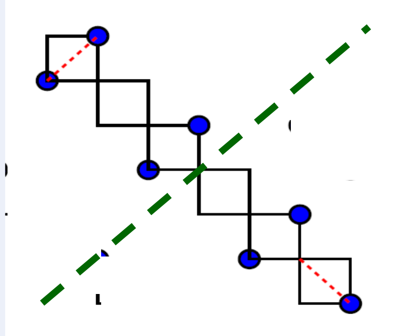
Phase with two-fold degenerate ground state manifold. States at k and π-k have same energy.

Higher order perturbative terms connecting states within the ground state manifold do not break this degeneracy.

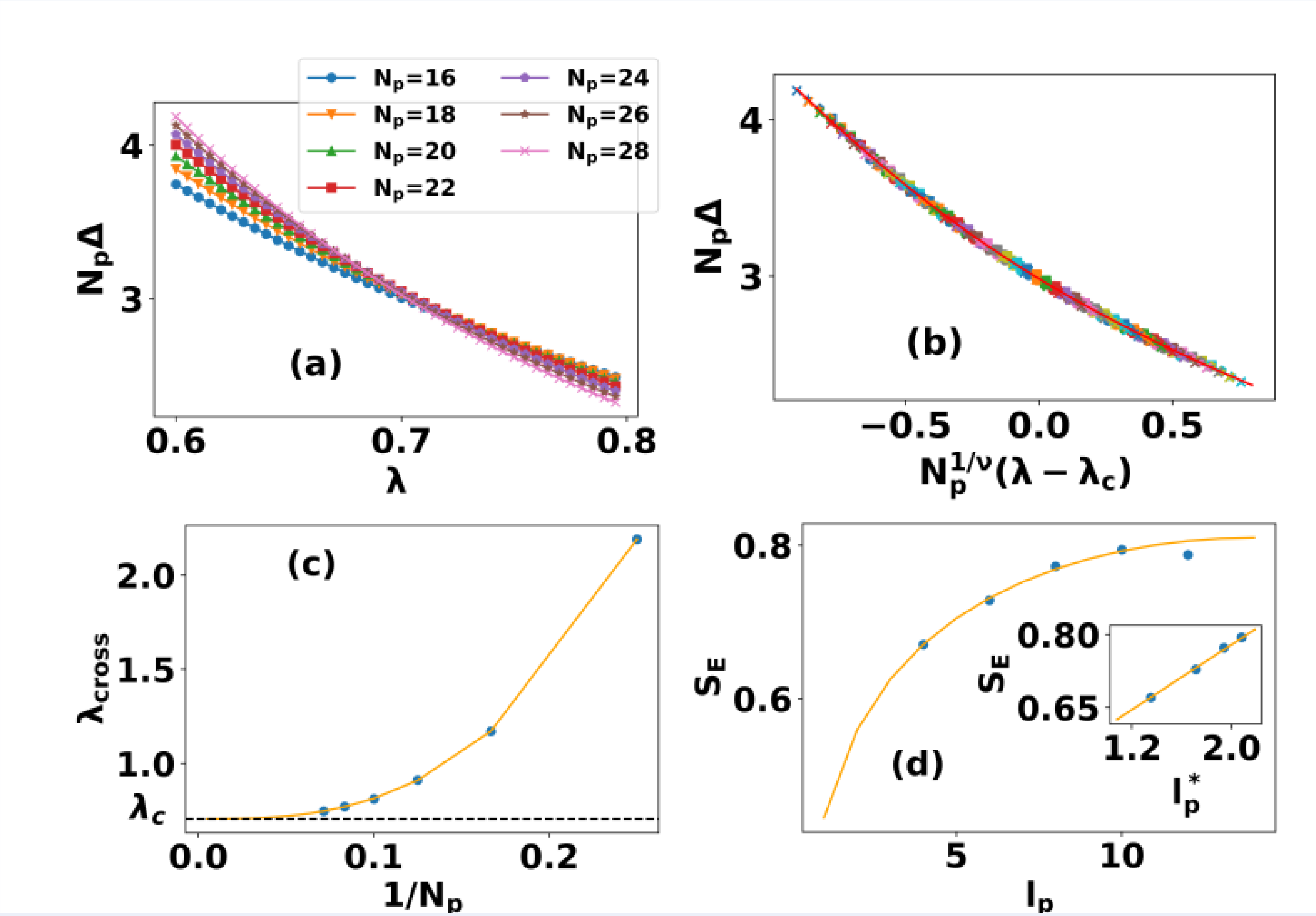
This leads to a Z_2 symmetry broken gapless ground state with logarithmic entanglement



In an open chain, a similar two-fold degeneracy occurs due to reflection symmetry R about an axis as shown: R is the only global symmetry that can be spontaneously broken.



Quantum Phase transition (Finite size scaling on open chain using ED)



Gap scaling

$$\Delta = N_p^{-z} f(N_p^{1/\nu}(\lambda - \lambda_c)),$$

Central charge of the CFT

$$S_{l_p} = (c/6)\ln[(N_p/\pi)\sin(\pi l_p/N_p)]$$

Finite sized scaling yields the following exponents consistent with Ashkin-Teller universality

z=1
ν=1.56… +(-) 0.001…
c=1 +(-) 0.05..

The critical theory is expected to have Z_2 X Z_2 symmetry and to be described by a c=1 orbifold CFT.

Additional emergent Z_2 symmetry

The large positive λ phase has a Z_2 symmetry since the momenta k and π-k are degenerate → *The critical theory needs to be described in terms of fluctuations around k_0 and π-k_0*

$$\Phi(x,t) \;=\; e^{ik_0 x}\varphi_1(x,t) + e^{i(\pi - k_0)x}\varphi_2(x,t)$$

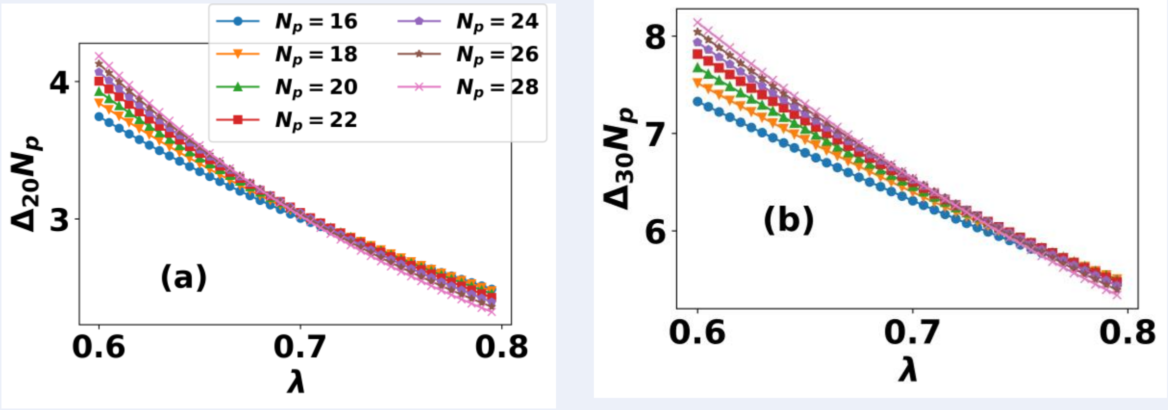
The invariance of Φ under k → π-k implies that the low energy effective theory must be invariant under $\varphi_{1(2)} \;\rightarrow\; \varphi_{2(1)}$ → *The simplest two-field L-G action consistent with this symmetry.*

$$\mathcal{L} \;=\; \sum_{\alpha=1,2} (|\partial_\mu \varphi_\alpha|^2 + r|\varphi_\alpha)|^2) + c_1(|\varphi_1|^4 + |\varphi_2|^4)$$
$$+c_2|\varphi_1|^2|\phi_2|^2 + c_3(\varphi_1^* \varphi_2 + h.c.)^2,$$

Both the fields condense if $c_2+4c_3 < 0$ and $c_3<0$ or $c_2<0$, $c_3>0$.

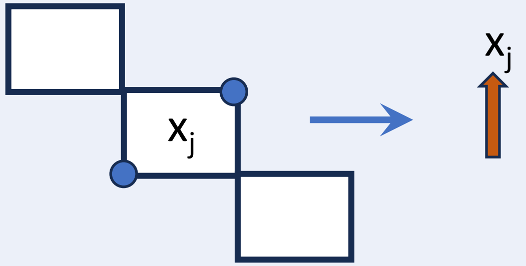
The relative phase between the fields is pinned to two values providing an extra Z_2 symmetry (0 or π if $c_3 < 0$ and $\pi/2$ and $3\pi/2$ if $c_3>0$).

This predicts three (two amplitude and one relative phase) gapless modes at the critical point.

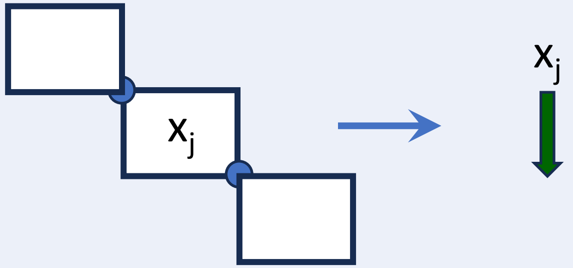


This leads to an additional emergent Z_2 symmetry of the low-energy effective action describing the transition leading to a Z_2 X Z_2 symmetry at the critical point

Relation to the PXP model



Map boson configurations to spins living on the center of the plaquette



The subsystem symmetry ensures that two up-spins can not live on neighboring sites

The spin model is PXP type as seen in Rydberg atoms

$$H_s = \sum_j [-w P_{x_j-1} \tau^x_{x_j} P_{x_j+1} + \frac{\lambda}{2}(\tau^z_{x_j} - P_{x_j-1} P_{x_j} P_{x_j+1})]$$

$$P_{x_j} = (1 - \tau^z_{x_j})/2.$$

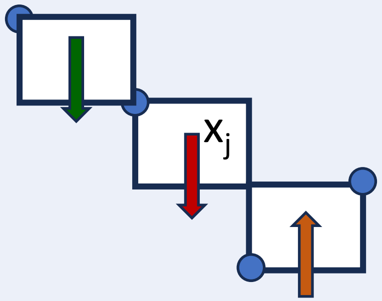
$$\tau^x_{x_j} = (|t_j\rangle\langle \ell_j| + \mathrm{h.c.})$$

$$\tau^z_{x_j} = |t_j\rangle\langle t_j| - |\ell_j\rangle\langle \ell_j|$$

$$P_{x_j} = (1 - \tau^z_{x_j})/2.$$

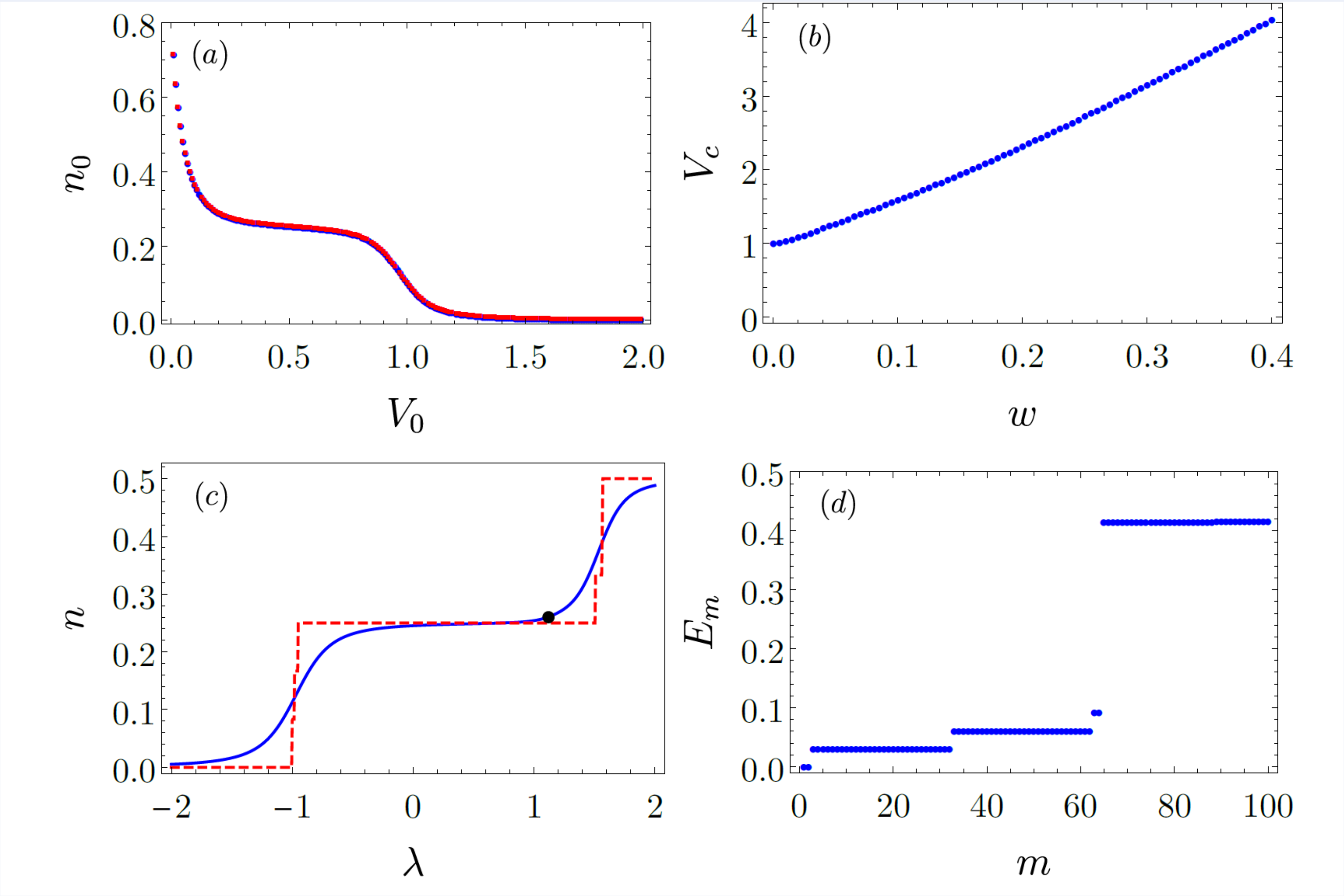
Effective PXP model with multi-spin interaction

Extra ingredient: One may have unflippable down spins



A down spin may be flipped if it is flanked by two neighboring down spins. The on-site term of the boson Hamiltonian only counts the flippable down spins

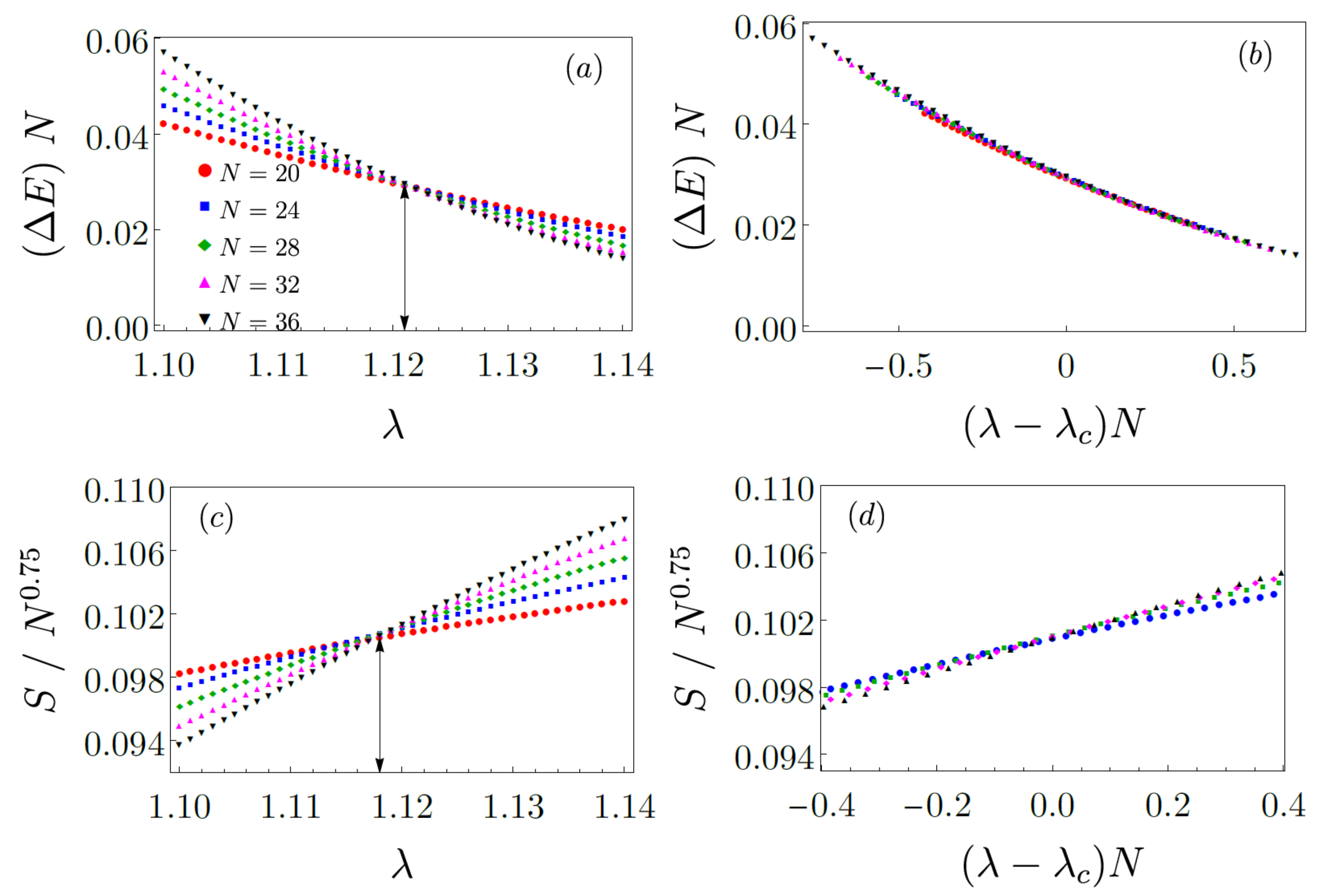
Role of next-nearest neighbor interaction



Choose V_0=2 for w=0.1 and keep the further-neighbor interactions

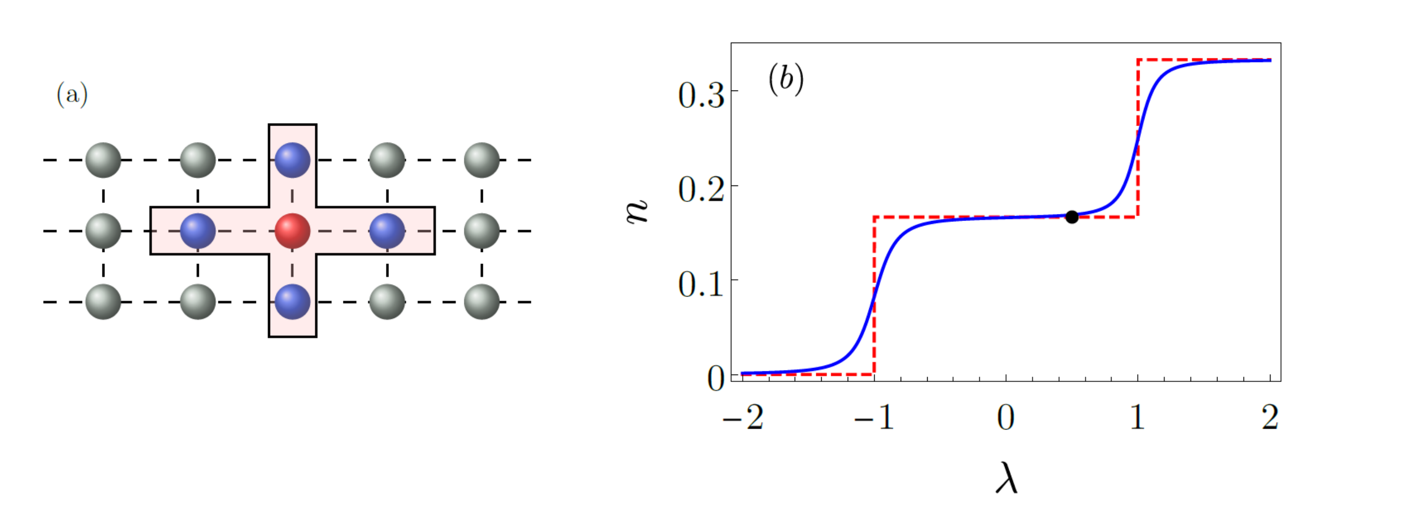
Low-energy spectrum for w=0

Numerical studies suggest that the effect of further-neighbor interaction is to shift the transition position.



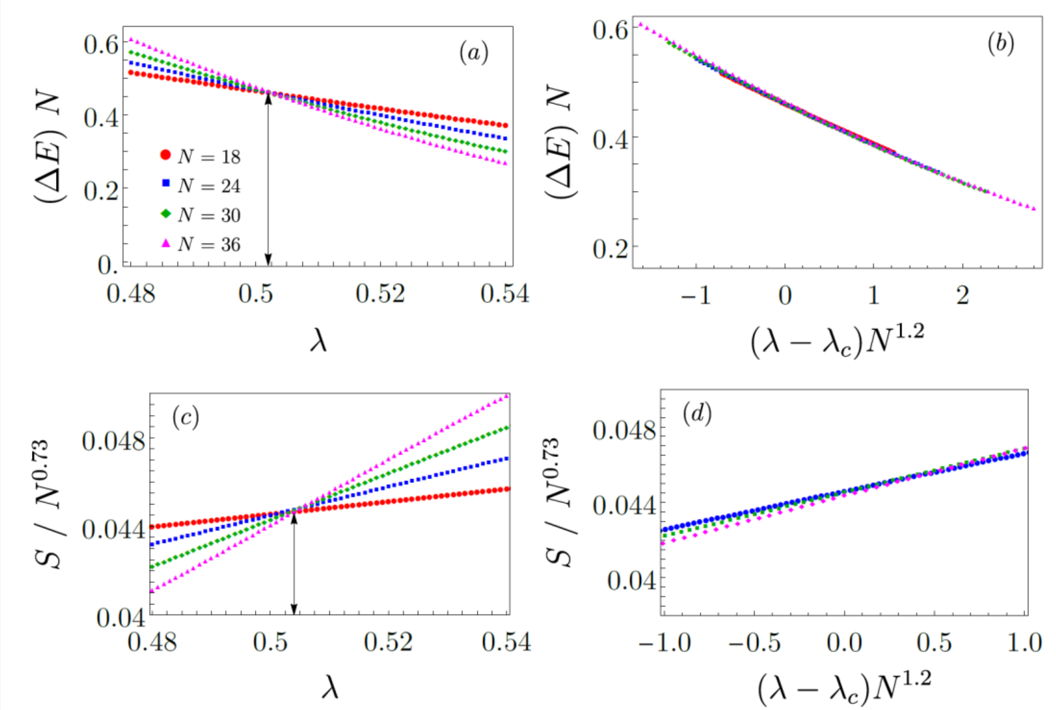
Finite-size diagonalization studies indicate an Ising transition at λ_c=1.12 for w=0.1

Multi-leg ladder



For the 3 leg ladder, we assume PBC along both the transverse and the longitudinal directions.

The maximal number of Rydberg excitations on even rungs leads to an excitation density $n=1/(2l_0)=1/6$



$$\lambda_c = \Delta(\ell_0 - 1)/(\ell_0 + 1).$$

The higher λ phase breaks Z_l symmetry. For l=3 The transition belongs to 3 state Potts universality class: z=1, ν=5/6 , and η = 4/15.

Realization of a class of critical points stabilized by order-by-disorder mechanism