Quantum Effects on Unconventional Pinch Point Singularities

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Freie Universität Berlin, Helmholtz Zentrum Berlin

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Collaborators

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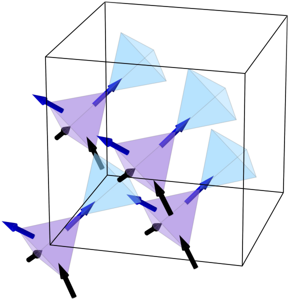
LMU Munich/ U Tübingen

Benedikt Schneider
LMU Munich

N. Niggemann, Y. Iqbal, and JR, PRL 130, 196601 (2023)

Introduction: Emergent electrodynamics in spin ice



2-in-2-out spin ice rule for low-energy states in pyrochlore Ising model.

$H = J \sum_{ij} S_i^z S_j^z$

Corresponds to effective Gauss law $\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = 0$ when identifying $\boldsymbol{S} \leftrightarrow \boldsymbol{E}$.

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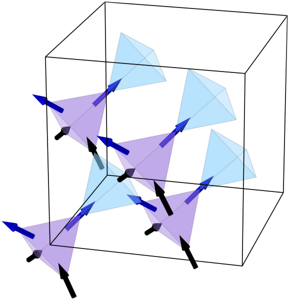

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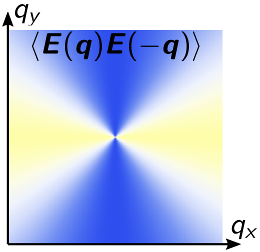

Consequence of $\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = 0$:

Sharp pinch-points in correlator $\langle \boldsymbol{E}(\boldsymbol{q})\boldsymbol{E}(-\boldsymbol{q})\rangle$.

Dipolar correlations!

S. V. Isakov, K. Gregor, R. Moessner, S. L. Sondhi, PRL 93, 167204 (2004)

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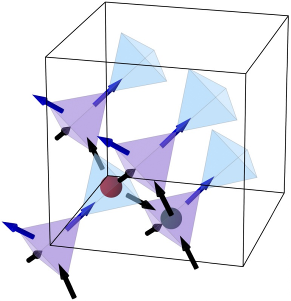

Violations of 2-in-2-out rule mean $\boldsymbol{\nabla} \cdot \boldsymbol{E}(\boldsymbol{r}) = \rho \neq 0$.

$\rho =$ emergent charge or spinon (conserved and gapped)

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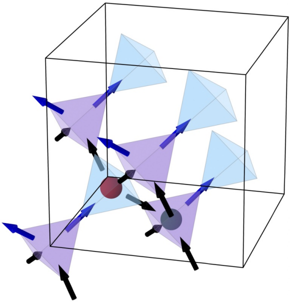

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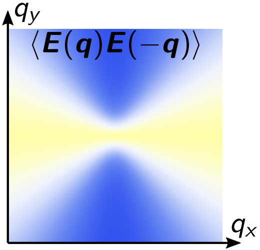

Consequence of $\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) \neq 0$:

Broadened pinch points, exponential correlations.

Effective electrostatics theory!

Introduction: Emergent electrodynamics in spin ice

Include quantum fluctuations, e.g., by changing Ising interactions to XXZ interactions $H = J\sum_{ij}\left[S_i^z S_j^z + \delta(S_i^x S_j^x + S_i^y S_j^y)\right]$:

QSL described by U(1) gauge theory, quantum electrodynamics for $\delta \ll 1$!

Emergent photon quasiparticle with $\omega(\boldsymbol{q}) = c|\boldsymbol{q}|$.

D. A. Huse, W. Krauth, R. Moessner, S. L. Sondhi, PRL 91, 167004 (2003)
M. Hermele, M. P. A. Fisher, and L. Balents, PRB 69, 064404 (2004)

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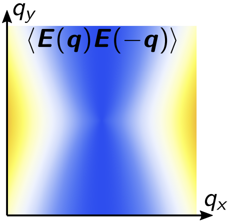
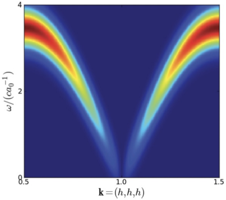


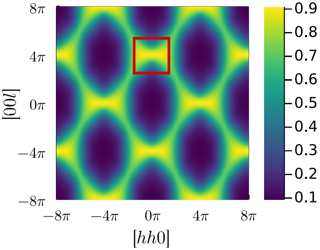
Consequence for correlator $\langle \boldsymbol{E}(\boldsymbol{q})\boldsymbol{E}(-\boldsymbol{q})\rangle$:

Extra factor $\omega(\boldsymbol{q})$ at $T = 0$, suppresses pinch point!



O. Benton, O. Sikora, N. Shannon, PRB 86, 075154 (2012)

Heisenberg limit of nearest neighbor pyrochlore model



$H = J\sum_{ij}\left[S_i^z S_j^z + \delta(S_i^x S_j^x + S_i^y S_j^y)\right]$
with $\delta = 1$

Broadened pinch points from PMFRG.
$[H_{\text{tetra}_1}, H_{\text{tetra}_2}] \neq 0$

No indications of effective electrodynamics!

N. Niggemann, JR, B. Sbierski, SciPost Phys. 12, 156 (2022)

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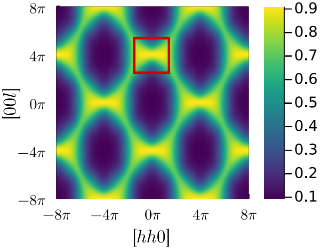

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Indications for lattice symmetry breaking. (Valence bond solid?)

I. Hagymási, R. Schäfer, R. Moessner, D. J. Luitz, PRL 126, 117204 (2021)
N. Astrakhantsev et al., PRX 11, 041021 (2021)
M. Hering, V. Noculak, F. Ferrari, Y. Iqbal, JR, PRB 105, 054426 (2022)
R. Schäfer, B. Placke, O. Benton, R. Moessner, PRL 131, 096702 (2023)

Or a quantum spin liquid?

R. Pohle, Y. Yamaji, M. Imada, arXiv:2311.11561 (2023)

Generalization: Tensor gauge theories

Generalize vector Gauss law $\partial_\mu E^\mu = \rho$ to matrix Gauss law $\partial_\mu \partial_\nu E^{\mu\nu} = \rho$.

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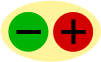
Dipole moment  is responsible for mobility of charges:

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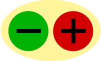
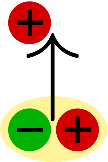
Dipole moment  is responsible for mobility of charges:



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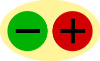
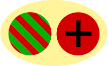
Dipole moment  is responsible for mobility of charges:



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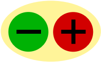
Dipole moment  is responsible for mobility of charges:

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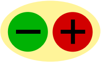
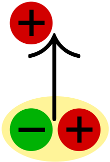
Dipole moment  is responsible for mobility of charges:



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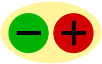
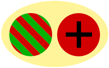
Dipole moment  is responsible for mobility of charges:



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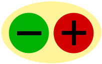
Dipole moment  is responsible for mobility of charges:

Dipole moment conserved $\longrightarrow$ charges are immobile
$\Longrightarrow$ New type of kinetically constrained quasiparticle: **Fracton**

R. Nandkishore, M. Hermele, Annu. Rev. Condens. Mat. Phys., 10, 295 (2019)

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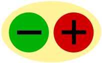
Dipole moment  is responsible for mobility of charges:

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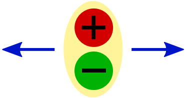
Composite particles can still move along 1D lines (lineons):



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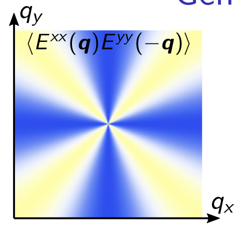
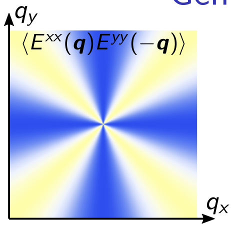


Consequence of Gauss-law $\partial_\mu \partial_\nu E^{\mu\nu} = 0$:

Four-fold pinch points in $\langle E^{xx}(\boldsymbol{q})E^{yy}(-\boldsymbol{q})\rangle$.

A. Prem et al., Phys. Rev. B 98, 165140 (2018)

Generalization: Tensor gauge theories



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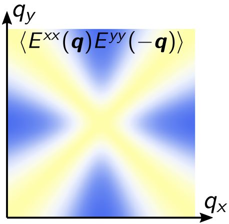

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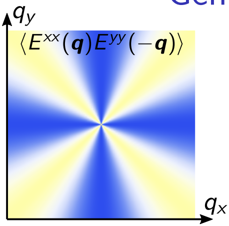

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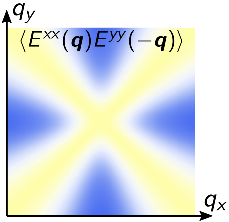

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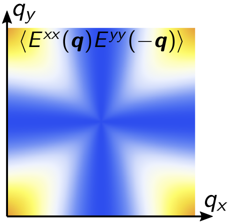


Quantum fluctuations leads to photon excitations with $\omega(\boldsymbol{q}) = c|\boldsymbol{q}|$.

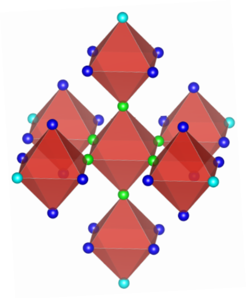
$\Longrightarrow$ Rank-2 U(1) quantum electrodynamics.

Extra factor $\omega(\boldsymbol{q})$ in correlator.

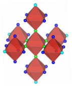
Spin model realizing higher-rank gauge theories

Only classical spin models known that realize the classical electrostatics part (Gauss law) of higher-rank U(1) gauge theories.

Example: classical octochlore model:



$$H = \frac{J}{2} \sum \left(\sum_{i \in {\color{green}\bullet}} \mathbf{S}_i + \alpha \sum_{i \in {\color{blue}\bullet}} \mathbf{S}_i + \beta \sum_{i \in {\color{cyan}\bullet}} \mathbf{S}_i \right)^2$$

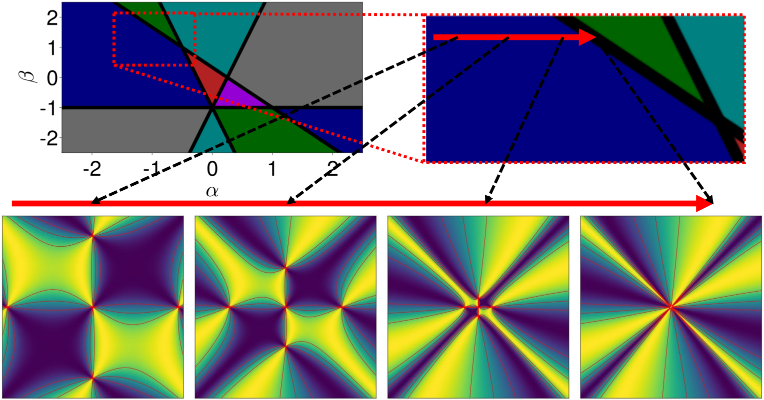


O. Benton and R. Moessner, PRL 127, 107202 (2021)

For other models, see also:

H. Yan. O. Benton, L. D. C. Jaubert, and N. Shannon, PRL 124, 127203 (2020)
N. Davier, F. G. Albarracín, H. D. Rosales, P. Pujol, PRB 108, 054408 (2023)
H. Yan, O. Benton, A. H. Nevidomskyy, R. Moessner, arXiv:2305.19189 (2023)

Multifold pinch points in classical octochlore model

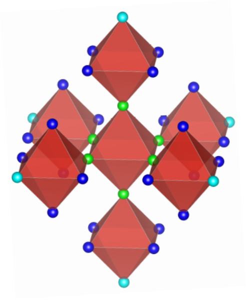


Spin structure factor $\langle \boldsymbol{S}(\boldsymbol{q})\boldsymbol{S}(-\boldsymbol{q})\rangle \sim$ electric field correlator.

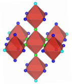
Rank-3 gauge theory from merging 2-fold pinch points at phase boundary (in self-consistent Gauss approximation).

O. Benton and R. Moessner, PRL 127, 107202 (2021)

Our work: Investigate quantum octochlore model



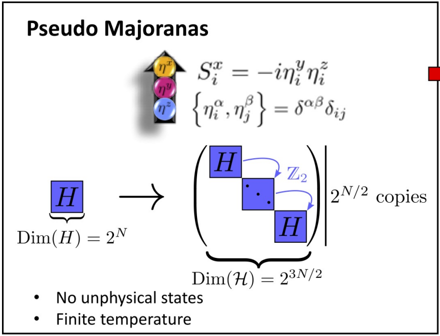
$$H = \frac{J}{2} \sum_{\text{(octochlore lattice)}} \left(\sum_{i \in \bullet} \boldsymbol{S}_i + \alpha \sum_{i \in \bullet} \boldsymbol{S}_i + \beta \sum_{i \in \bullet} \boldsymbol{S}_i \right)^2$$

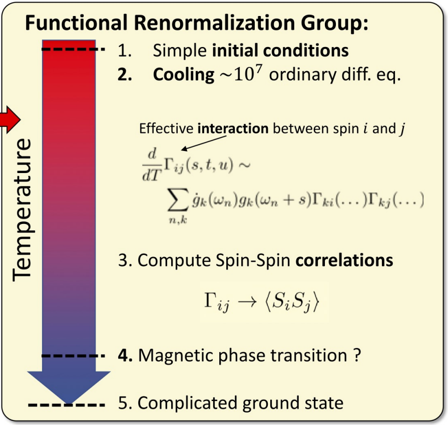


$\boldsymbol{S}_i$ = spin-1/2 operator, Heisenberg interactions.

Do pinch points survive? Spin liquid ground state?

Pseudo Majorana functional renormalization group





Majorana representation: A. M. Tsvelik, PRL 69, 2142 (1992)

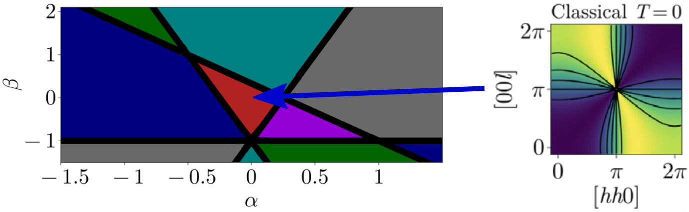
Applications: N. Niggemann, B. Sbierski, and JR, PRB 103, 104431 (2021)
N. Niggemann, JR, B. Sbierski, SciPost Phys. 12, 156 (2022)

Review article: T. Müller, D. Kiese, N. Niggemann, B. Sbierski, JR, S. Trebst, R. Thomale, Y. Iqbal, arXiv:2307.10359 (2023)

Octochlore model: Two-fold pinch points

Quantum model non-magnetic in entire α-β parameter space!

Two-fold pinch points at $\alpha = \beta = 0$ near $\boldsymbol{q} = (\pi, \pi, \pi)$



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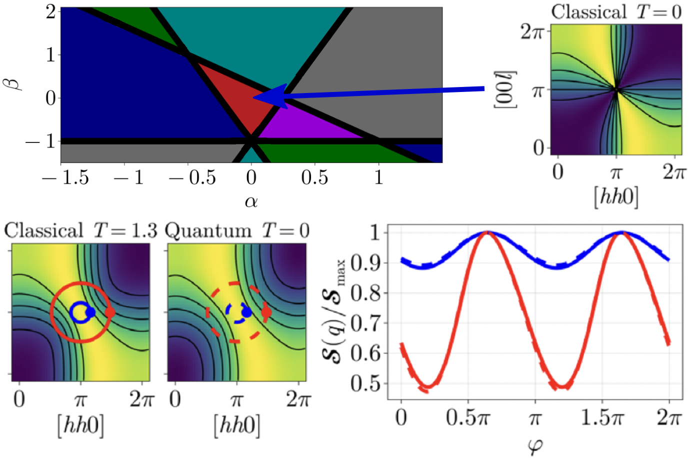
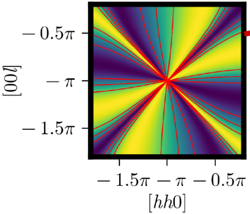
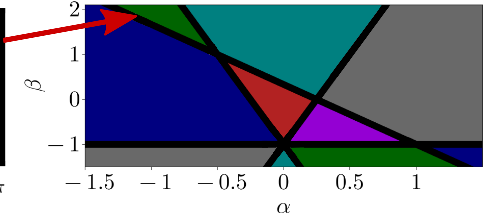


Featureless broadening due quantum fluctuations similar to thermal fluctuations. Signal at pinch points strong!

Octochlore model: Multifold pinch points

Pinch points with six lobes of strong intensity in the hhl-plane at $\boldsymbol{q} = (\pi, \pi, \pi)$ along phase boundary α-β-parameter space.





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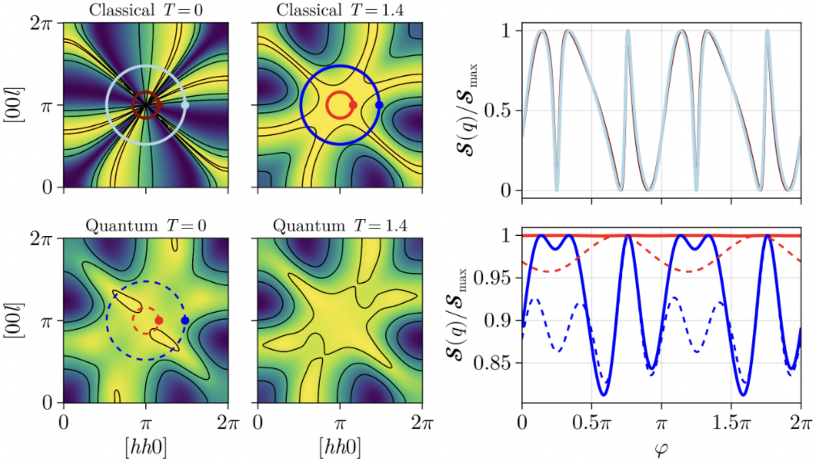

- Significant broadening of pinch point under quantum fluctuations.
- Soft peaks at incommensurate $\boldsymbol{q}$ for $T = 0$ in quantum case.
- Thermal and quantum fluctuations act very differently.
- No signatures of photons!

Summary

- Unconventional pinch points in octochlore model are very fragile under quantum fluctuations $\rightarrow$ No signatures of higher-rank gauge theories (different type of spin liquid?)
- Unconventional pinch points are more drastically modified by quantum fluctuations than standard two-fold pinch points.
- Thermal and quantum fluctuations act very differently on unconventional pinch-points.
- **Next:** Investigate *XXZ* models!

Thank you for your attention!

Check out poster #60 by Nils Niggemann for more details!