Quantum algorithms for quasiparticles

Leon Balents, KITP, UCSB

Fractional and emergent gauge fields in quantum matter, Trieste

Collaborators

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Scientific Developments: A Vision

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2009, 2020?

…some people say "Quantum computers are a nice game that theorists will keep playing for a while, until they get tired. But there are optimists. I am one of those optimists for no logical reason, it is simply a gut feeling."

…"Will there be quantum computers of reasonable size 10 years from now? As I said earlier, I have no logical arguments or vision arising from deep insights…"

Quantum science

- $$:
  - Microsoft Quantum: probably > $300M per year.
  - Many others!

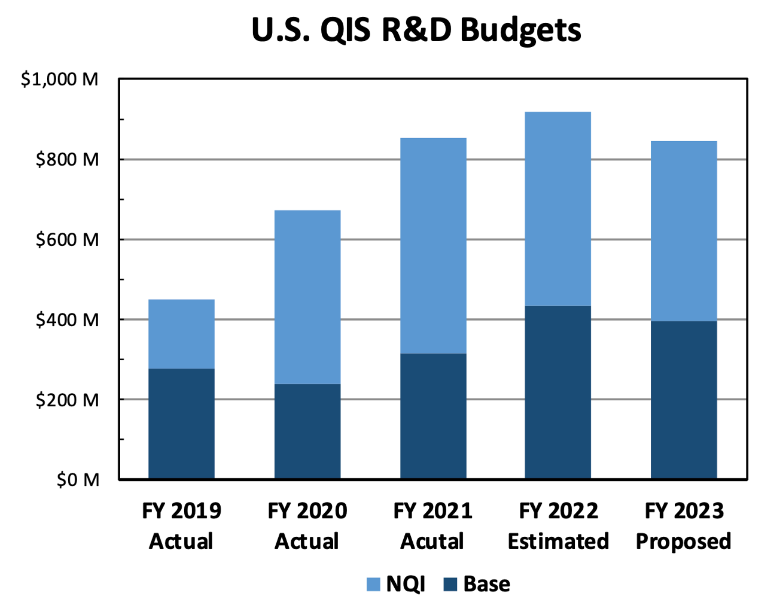
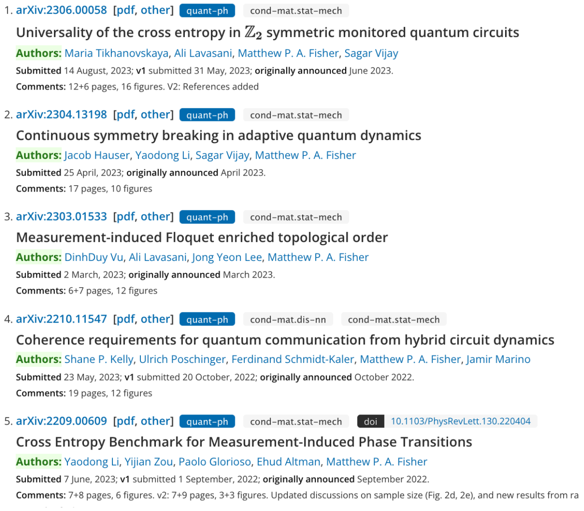


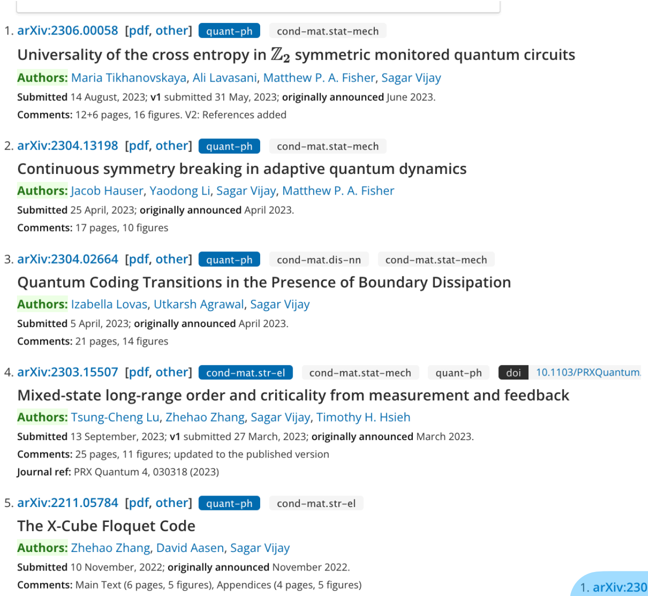
Figure 2.1: U.S. R&D budgets for QIS since the inception of the NQI Act. The bar heights represent the total budget for each fiscal year (FY 2019 – FY 2021 actual expenditures, FY 2022 estimated expenditures, and FY 2023 requested budgets). The portion of each bar marked "NQI" identifies funding allocated for NQI Act-authorized activities; this additional funding is on top of the budgets for baseline QIS R&D activities.

Quantum science

- People:
    - Compare arXiv “new” listings:
        - 172 CM vs 131 Quantum
- Experimentalists going to private sector
- Theorists mass movement to QI:
    - Let’s look at UCSB faculty



1. arXiv:2306.00058 [pdf, other] quant-ph cond-mat.stat-mech
   Universality of the cross entropy in $\mathbb{Z}_2$ symmetric monitored quantum circuits
   Authors: Maria Tikhanovskaya, Ali Lavasani, Matthew P. A. Fisher, Sagar Vijay
   Submitted 14 August, 2023; **v1** submitted 31 May, 2023; **originally announced** June 2023.
   Comments: 12+6 pages, 16 figures. V2: References added
2. arXiv:2304.13198 [pdf, other] quant-ph cond-mat.stat-mech
   Continuous symmetry breaking in adaptive quantum dynamics
   Authors: Jacob Hauser, Yaodong Li, Sagar Vijay, Matthew P. A. Fisher
   Submitted 25 April, 2023; **originally announced** April 2023.
   Comments: 17 pages, 10 figures
3. arXiv:2303.01533 [pdf, other] quant-ph cond-mat.stat-mech
   Measurement-induced Floquet enriched topological order
   Authors: DinhDuy Vu, Ali Lavasani, Jong Yeon Lee, Matthew P. A. Fisher
   Submitted 2 March, 2023; **originally announced** March 2023.
   Comments: 6+7 pages, 12 figures
4. arXiv:2210.11547 [pdf, other] quant-ph cond-mat.dis-nn cond-mat.stat-mech
   Coherence requirements for quantum communication from hybrid circuit dynamics
   Authors: Shane P. Kelly, Ulrich Poschinger, Ferdinand Schmidt-Kaler, Matthew P. A. Fisher, Jamir Marino
   Submitted 23 May, 2023; **v1** submitted 20 October, 2022; **originally announced** October 2022.
   Comments: 19 pages, 12 figures
5. arXiv:2209.00609 [pdf, other] quant-ph cond-mat.stat-mech doi 10.1103/PhysRevLett.130.220404
   Cross Entropy Benchmark for Measurement-Induced Phase Transitions
   Authors: Yaodong Li, Yijian Zou, Paolo Glorioso, Ehud Altman, Matthew P. A. Fisher
   Submitted 7 June, 2023; **v1** submitted 1 September, 2022; **originally announced** September 2022.
   Comments: 7+8 pages, 6 figures. v2: 7+9 pages, 3+3 figures. Updated discussions on sample size (Fig. 2d, 2e), and new results from ra



1. arXiv:2306.00058 [pdf, other] quant-ph cond-mat.stat-mech
   Universality of the cross entropy in $\mathbb{Z}_2$ symmetric monitored quantum circuits
   Authors: Maria Tikhanovskaya, Ali Lavasani, Matthew P. A. Fisher, Sagar Vijay
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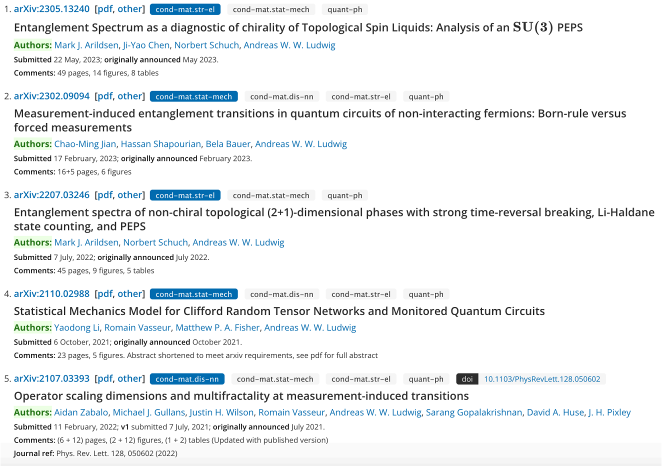
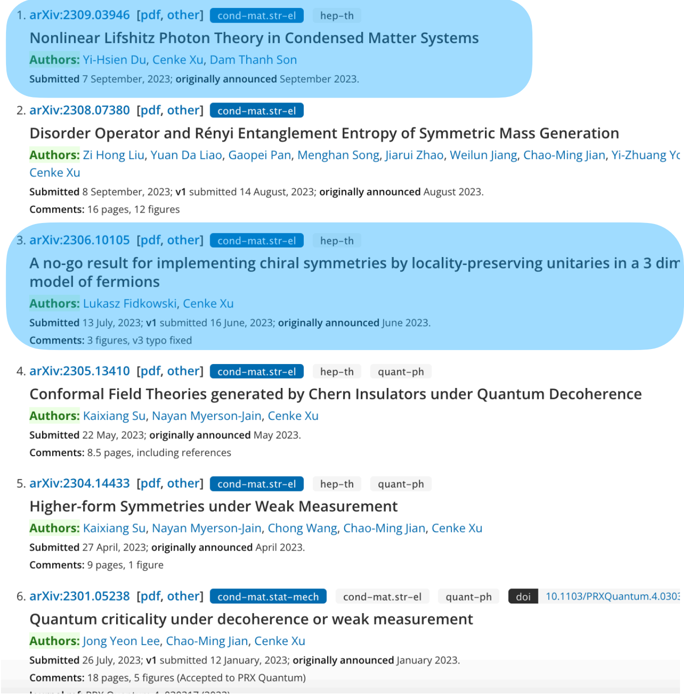
   Comments: 12+6 pages, 16 figures. V2: References added
2. arXiv:2304.13198 [pdf, other] quant-ph cond-mat.stat-mech
   Continuous symmetry breaking in adaptive quantum dynamics
   Authors: Jacob Hauser, Yaodong Li, Sagar Vijay, Matthew P. A. Fisher
   Submitted 25 April, 2023; **originally announced** April 2023.
   Comments: 17 pages, 10 figures
3. arXiv:2304.02664 [pdf, other] quant-ph cond-mat.dis-nn cond-mat.stat-mech
   Quantum Coding Transitions in the Presence of Boundary Dissipation
   Authors: Izabella Lovas, Utkarsh Agrawal, Sagar Vijay
   Submitted 5 April, 2023; **originally announced** April 2023.
   Comments: 21 pages, 14 figures
4. arXiv:2303.15507 [pdf, other] cond-mat.str-el cond-mat.stat-mech quant-ph doi 10.1103/PRXQuantum
   Mixed-state long-range order and criticality from measurement and feedback
   Authors: Tsung-Cheng Lu, Zhehao Zhang, Sagar Vijay, Timothy H. Hsieh
   Submitted 13 September, 2023; **v1** submitted 27 March, 2023; **originally announced** March 2023.
   Comments: 25 pages, 11 figures; updated to the published version
   Journal ref: PRX Quantum 4, 030318 (2023)
5. arXiv:2211.05784 [pdf, other] quant-ph cond-mat.str-el
   The X-Cube Floquet Code
   Authors: Zhehao Zhang, David Aasen, Sagar Vijay
   Submitted 10 November, 2022; **originally announced** November 2022.
   Comments: Main Text (6 pages, 5 figures), Appendices (4 pages, 5 figures)



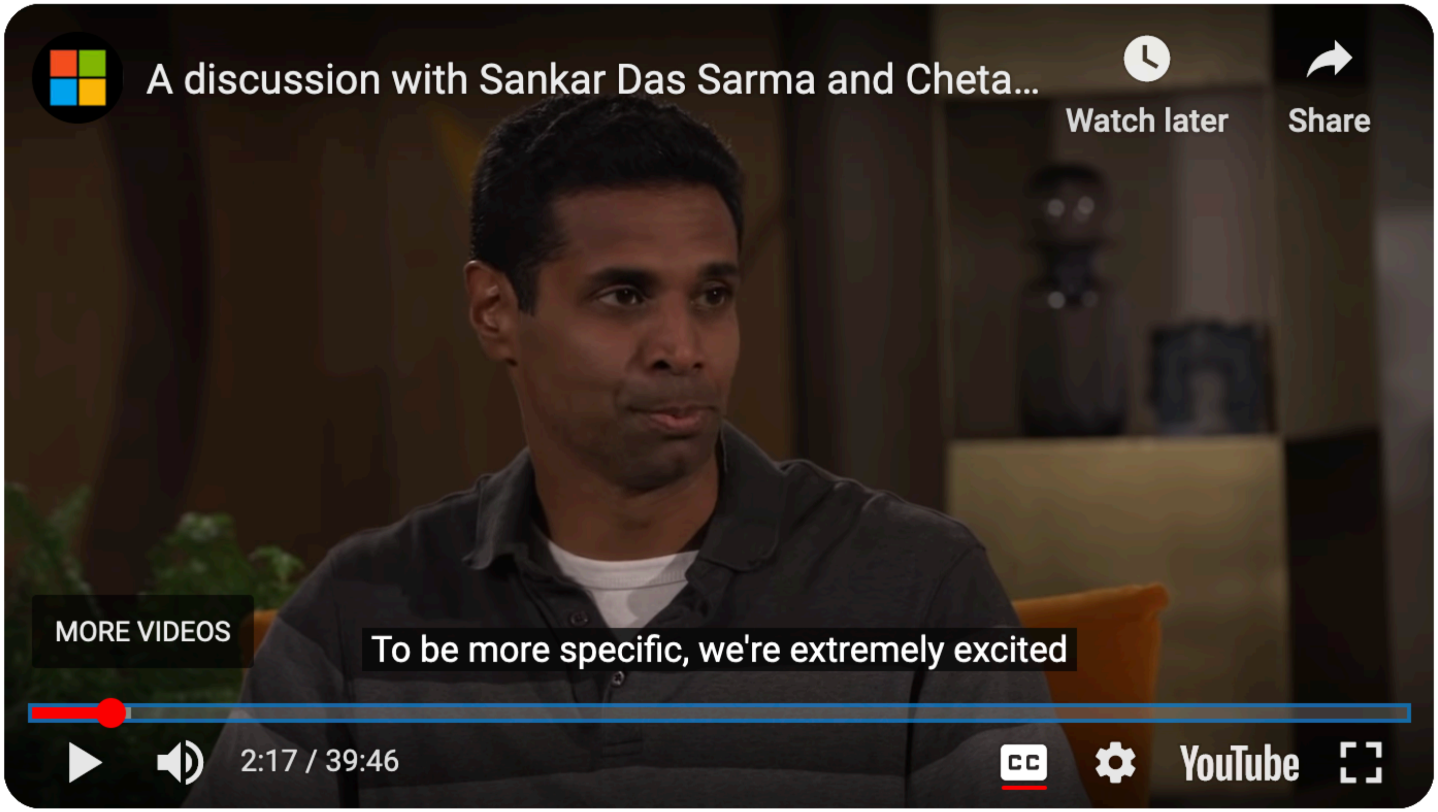
1. arXiv:2305.13240 [pdf, other] cond-mat.str-el cond-mat.stat-mech quant-ph
   Entanglement Spectrum as a diagnostic of chirality of Topological Spin Liquids: Analysis of an $\mathrm{SU}(3)$ PEPS
   Authors: Mark J. Arildsen, Ji-Yao Chen, Norbert Schuch, Andreas W. W. Ludwig
   Submitted 22 May, 2023; **originally announced** May 2023.
   Comments: 49 pages, 14 figures, 8 tables
2. arXiv:2302.09094 [pdf, other] cond-mat.stat-mech cond-mat.dis-nn cond-mat.str-el quant-ph
   Measurement-induced entanglement transitions in quantum circuits of non-interacting fermions: Born-rule versus forced measurements
   Authors: Chao-Ming Jian, Hassan Shapourian, Bela Bauer, Andreas W. W. Ludwig
   Submitted 17 February, 2023; **originally announced** February 2023.
   Comments: 16+5 pages, 6 figures
3. arXiv:2207.03246 [pdf, other] cond-mat.str-el cond-mat.stat-mech quant-ph
   Entanglement spectra of non-chiral topological (2+1)-dimensional phases with strong time-reversal breaking, Li-Haldane state counting, and PEPS
   Authors: Mark J. Arildsen, Norbert Schuch, Andreas W. W. Ludwig
   Submitted 7 July, 2022; **originally announced** July 2022.
   Comments: 45 pages, 9 figures, 5 tables
4. arXiv:2110.02988 [pdf, other] cond-mat.stat-mech cond-mat.dis-nn cond-mat.str-el quant-ph
   Statistical Mechanics Model for Clifford Random Tensor Networks and Monitored Quantum Circuits
   Authors: Yaodong Li, Romain Vasseur, Matthew P. A. Fisher, Andreas W. W. Ludwig
   Submitted 6 October, 2021; **originally announced** October 2021.
   Comments: 23 pages, 5 figures. Abstract shortened to meet arxiv requirements, see pdf for full abstract
5. arXiv:2107.03393 [pdf, other] cond-mat.dis-nn cond-mat.stat-mech cond-mat.str-el quant-ph doi 10.1103/PhysRevLett.128.050602
   Operator scaling dimensions and multifractality at measurement-induced transitions
   Authors: Aidan Zabalo, Michael J. Gullans, Justin H. Wilson, Romain Vasseur, Andreas W. W. Ludwig, Sarang Gopalakrishnan, David A. Huse, J. H. Pixley
   Submitted 11 February, 2022; **v1** submitted 7 July, 2021; **originally announced** July 2021.
   Comments: (6 + 12) pages, (2 + 12) figures, (1 + 2) tables (Updated with published version)
   Journal ref: Phys. Rev. Lett. 128, 050602 (2022)

Not QI



1. arXiv:2309.03946 [pdf, other] cond-mat.str-el hep-th
   Nonlinear Lifshitz Photon Theory in Condensed Matter Systems
   Authors: Yi-Hsien Du, Cenke Xu, Dam Thanh Son
   Submitted 7 September, 2023; **originally announced** September 2023.
2. arXiv:2308.07380 [pdf, other] cond-mat.str-el
   Disorder Operator and Rényi Entanglement Entropy of Symmetric Mass Generation
   Authors: Zi Hong Liu, Yuan Da Liao, Gaopei Pan, Menghan Song, Jiarui Zhao, Weilun Jiang, Chao-Ming Jian, Yi-Zhuang Y
   Cenke Xu
   Submitted 8 September, 2023; **v1** submitted 14 August, 2023; **originally announced** August 2023.
   Comments: 16 pages, 12 figures
3. arXiv:2306.10105 [pdf, other] cond-mat.str-el hep-th
   A no-go result for implementing chiral symmetries by locality-preserving unitaries in a 3 dim
   model of fermions
   Authors: Lukasz Fidkowski, Cenke Xu
   Submitted 13 July, 2023; **v1** submitted 16 June, 2023; **originally announced** June 2023.
   Comments: 3 figures, v3 typo fixed
4. arXiv:2305.13410 [pdf, other] cond-mat.str-el hep-th quant-ph
   Conformal Field Theories generated by Chern Insulators under Quantum Decoherence
   Authors: Kaixiang Su, Nayan Myerson-Jain, Cenke Xu
   Submitted 22 May, 2023; **originally announced** May 2023.
   Comments: 8.5 pages, including references
5. arXiv:2304.14433 [pdf, other] cond-mat.str-el hep-th quant-ph
   Higher-form Symmetries under Weak Measurement
   Authors: Kaixiang Su, Nayan Myerson-Jain, Chong Wang, Chao-Ming Jian, Cenke Xu
   Submitted 27 April, 2023; **originally announced** April 2023.
   Comments: 9 pages, 1 figure
6. arXiv:2301.05238 [pdf, other] cond-mat.stat-mech cond-mat.str-el quant-ph doi 10.1103/PRXQuantum.4.030
   Quantum criticality under decoherence or weak measurement
   Authors: Jong Yeon Lee, Chao-Ming Jian, Cenke Xu
   Submitted 26 July, 2023; **v1** submitted 12 January, 2023; **originally announced** January 2023.
   Comments: 18 pages, 5 figures (Accepted to PRX Quantum)

What is it good for?



To be more specific, we're extremely excited

What is it good for?


A discussion with Sankar Das Sarma and Cheta...

Particularly to design a catalyst.

What is it good for?

for nitrogen fixation, for producing fertilizers

What is it good for?

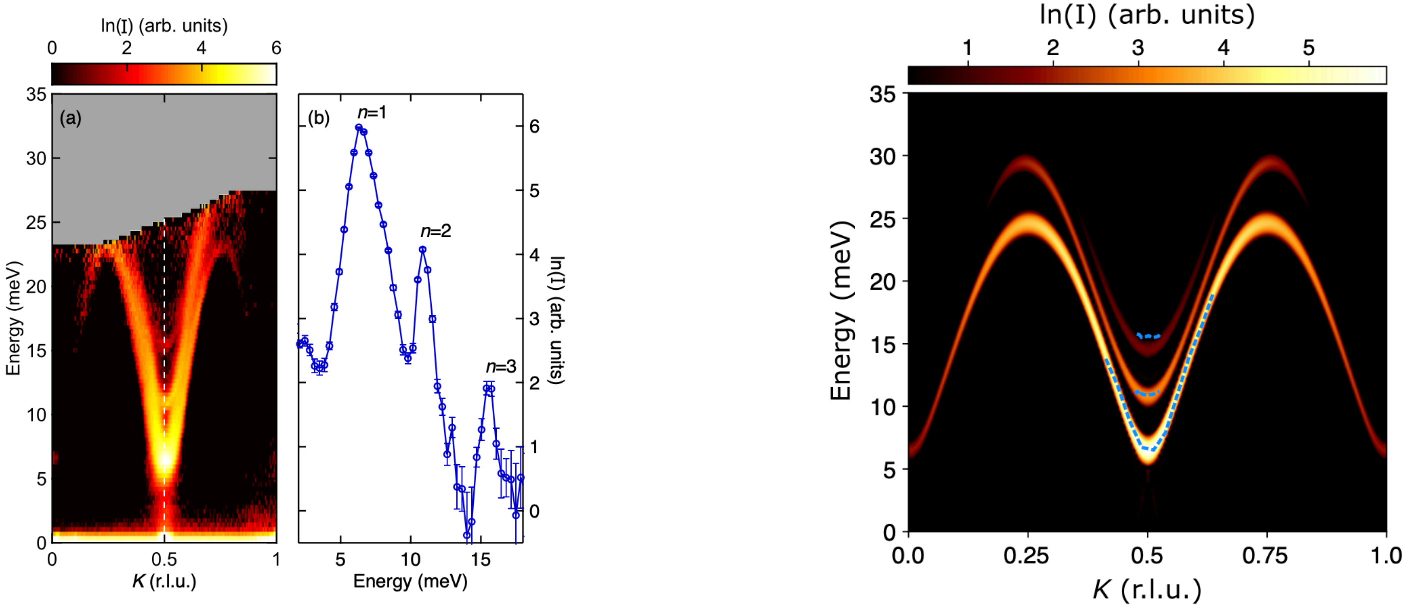
A discussion with Sankar Das Sarma and Cheta...

that could really help humanity.

Application to quantum materials?

- Try to *apply* quantum algorithms to actual quantum problems

- For example: how would we obtain S(k,ω) on a quantum computer?

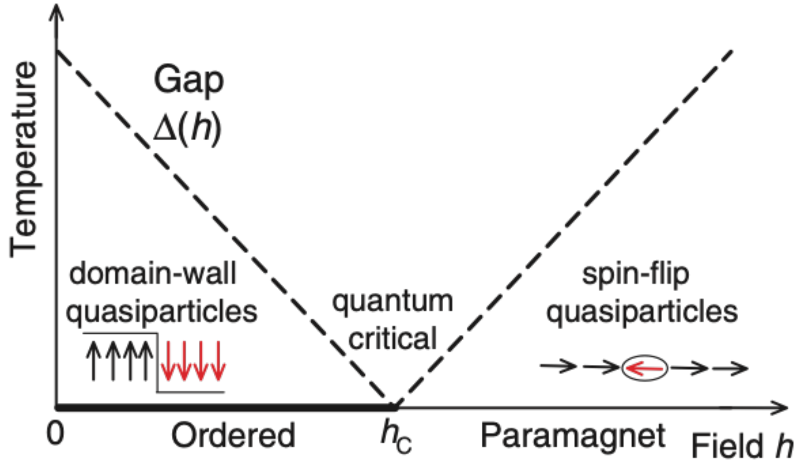


R. Dally *et al*, PRL (2020).

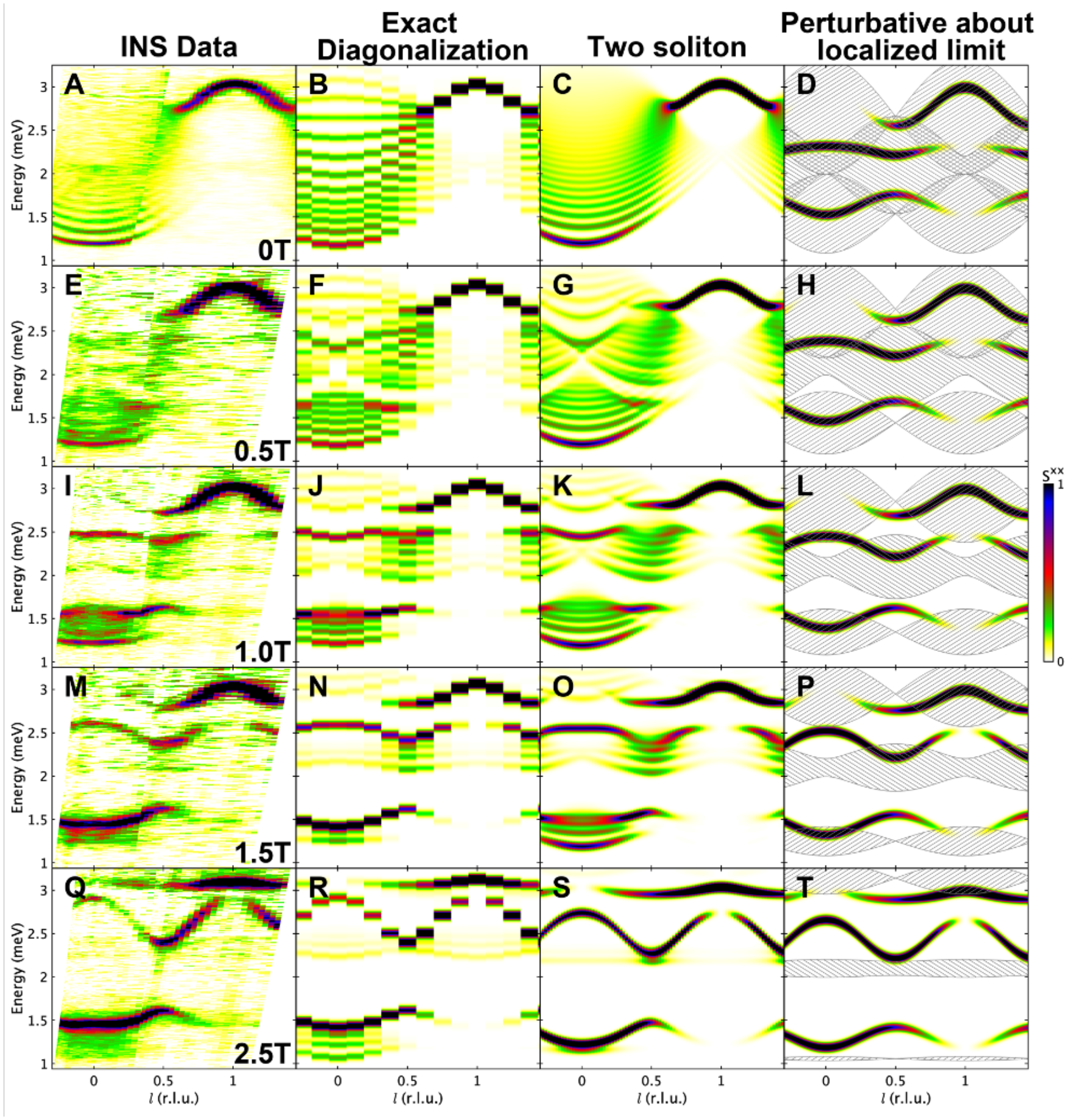
Quantum Ising chain

$$H = \sum_i - J_z S_i^z S_{i+1}^z - h_y S_i^y$$

Solitons = the simplest example of non-local excitations



Quantum theory done classically



$CoNb_2O_6$

- Qualitative and quantitative understanding of complex spectrum based on strongly interacting flat band solitons

Leonie Woodland
Oxford

Radu Coldea
Oxford

Izabella Lovas
KITP

L. Woodland *et al*, PRB 2023

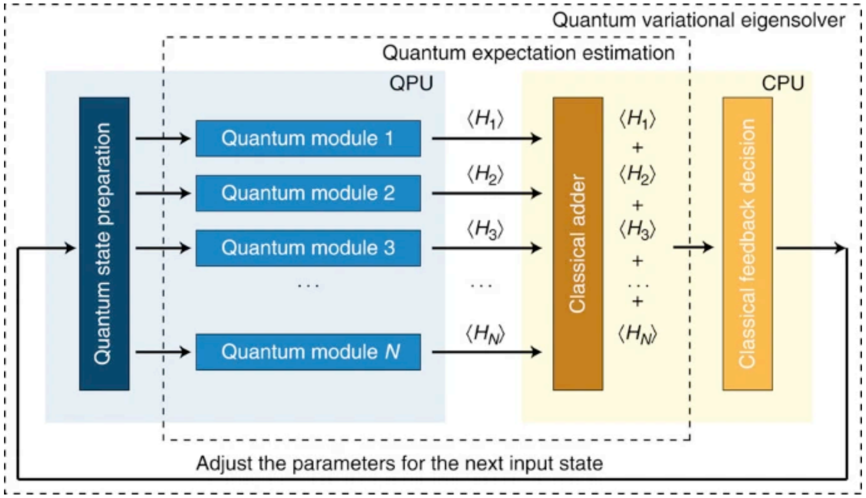
Possible quantum approaches

- Direct time evolution? Q: Isn’t that what a quantum computer is good at?
  - A: Maybe a special purpose simulator, but a digital quantum computer like google machines can’t. They apply controlled 1 and 2 qubit gates
  - You can Trotterize but this introduces substantial errors that can only be improved by scaling to many gates.
- Instead we will try to use a variational approach to obtain eigenstates.

VQE

- Variational quantum eigensolver:

Peruzzo *et al*, 2014



quantum circuit

$$|\Psi(\{\theta_i\})\rangle = U(\{\theta_i\})|\Psi_0\rangle$$

$$E_{\mathrm{var}} = \langle\Psi|H|\Psi\rangle \geq E_0$$

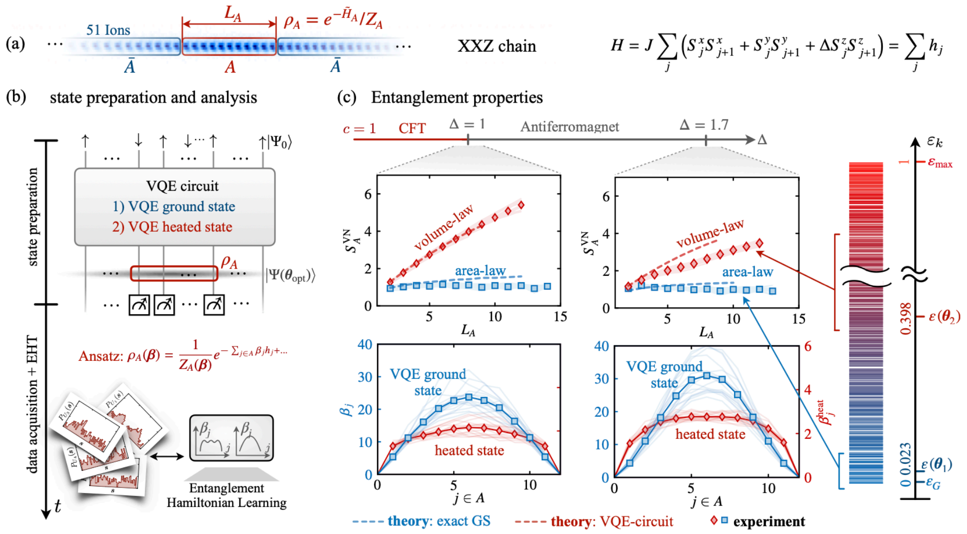
measure

↓

Ground state

Exploring Large-Scale Entanglement in Quantum Simulation

Manoj K. Joshi,[1,2,*] Christian Kokail,[1,3,*] Rick van Bijnen,[1,3,*] Florian Kranzl,[1,2]
Torsten V. Zache,[1,3] Rainer Blatt,[1,2] Christian F. Roos,[1,2] and Peter Zoller[1,3]



PHYSICAL REVIEW B 105, 094409 (2022)

Probing ground-state properties of the kagome antiferromagnetic Heisenberg model using the variational quantum eigensolver

Jan Lukas Bosse[1,2,*] and Ashley Montanaro[2,1,†]
[1]School of Mathematics, University of Bristol, Bristol, BS8 1QU, United Kingdom
[2]Phasecraft Ltd, Bristol, BS1 5DD, United Kingdom

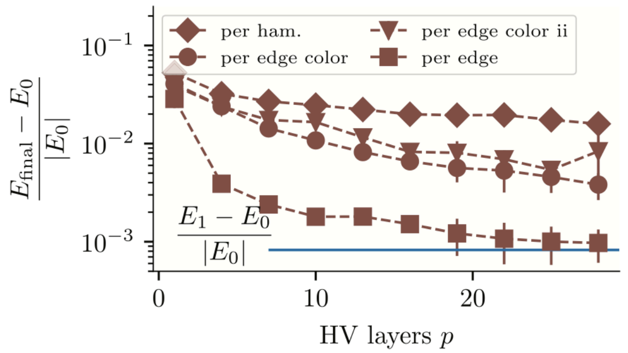


FIG. 12. Scaling of the relative energy error as a function of p for the 3×8 lattice with different ansatz circuits. Results are shown for three runs per data point and with the initial parameters chosen uniformly random within $[0, \frac{1}{p}]$. The error bars reflect the standard deviation between the different runs.

PHYSICAL REVIEW B 106, 214429 (2022)

Variational quantum eigensolver for the Heisenberg antiferromagnet on the kagome lattice

Joris Kattemölle[1,2,3] and Jasper van Wezel[1,2]

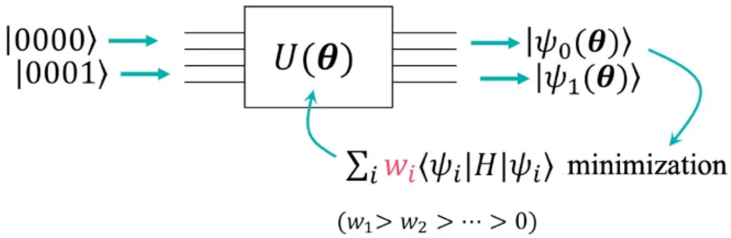
SS VQE

- Subspace Search VQE: for excited states

K. Nakanishi *et al*, 2019

$$|\Psi_n(\{\theta_i\})\rangle = U(\{\theta_i\})|\Psi_{n,0}\rangle$$

Choose N
orthogonal
initial states



$$\langle\Psi_{n'}|\Psi_n\rangle = \langle\Psi_{n',0}|\Psi_{n,0}\rangle$$

$$E_{\text{var}} = \sum_n w_n\langle\Psi_n|H|\Psi_n\rangle$$

$w_n > 0$

Just repeat the VQE *with the same circuit* on N initial orthogonal states and minimize (weighted) energy sum.

Elementary excitations

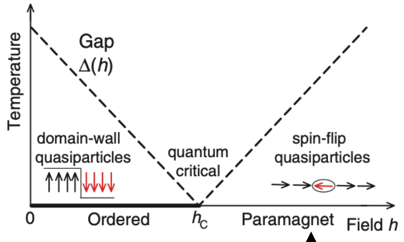
- Transverse field Ising chain

$$H_T = -J\sum_{i=1}^{L} Z_i Z_{i+1} - h\sum_{i=1}^{L} X_i$$

- Excitations at J<<h:

$$|\, -_i \rangle = |\, + + \cdots -_i + \cdots + \rangle$$

$$|k\rangle = \frac{1}{\sqrt{N}}\sum_i e^{ikx_i}|\, -_i \rangle$$



Momentum eigenstates

Exact energy $\epsilon_k = 2\sqrt{h^2 + J^2 - 2hJ\cos k}$

? Can we get this from (SS) VQE?

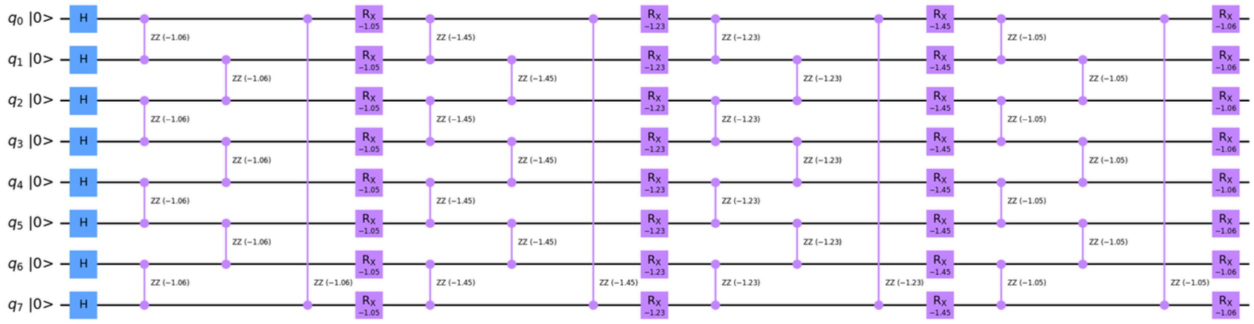
VQE for Ising chain

- Natural circuit: preserve translational symmetry

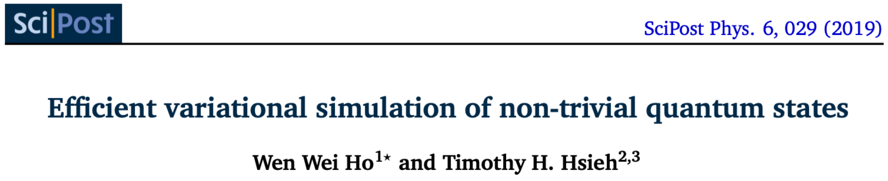
$$H_T = -J\underbrace{\sum_{i=1}^{L} Z_i Z_{i+1}}_{H_1} - h\underbrace{\sum_{i=1}^{L} X_i}_{H_2}$$

$$|\psi_P(\gamma,\beta)\rangle = e^{-i\beta_p H_1} e^{-i\gamma_p H_2} \cdots e^{-i\beta_1 H_1} e^{-i\gamma_1 H_2} \, |\psi_1\rangle$$

Example circuit



VQE for Ising chain



Sci|Post SciPost Phys. 6, 029 (2019)

Efficient variational simulation of non-trivial quantum states

Wen Wei Ho[1⋆] and Timothy H. Hsieh[2,3]

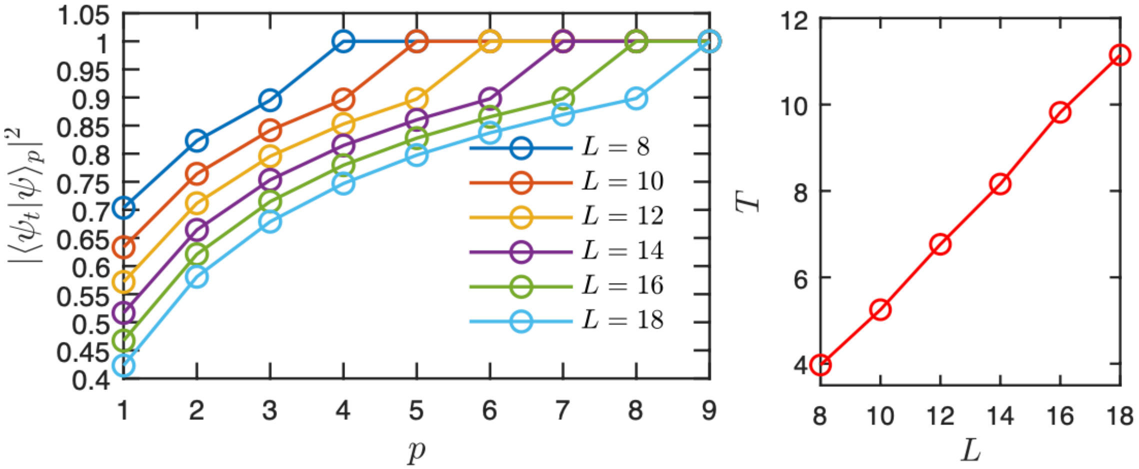


Figure 4: Preparation of critical state. (Left) Many-body overlap $|\langle\psi_t|\psi\rangle_p|^2$ of the prepared state with the target ground state of (9) found by exact diagonalization. Ones sees perfect fidelity for $p \geq L/2$. (Right) Total minimum time $T = \min_{(\gamma,\beta)}\left[\sum_i^{p=L/2}(\gamma_i + \beta_i)\right]$ required for the VQCS to produce the critical state with perfect fidelity using $\text{VQCS}_{p=L/2}$. One sees a linear trend $T \sim L$.

VQE for excited states?

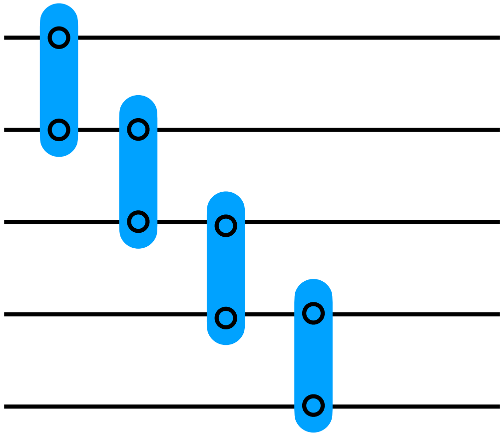
$$|-_i\rangle = |++\cdots-_i+\cdots+\rangle$$

$$|k\rangle = \frac{1}{\sqrt{N}}\sum_i e^{ikx_i}|-_i\rangle$$

For J/h << 1

$$\epsilon_k = 2\sqrt{h^2+J^2-2hJ\cos k}$$

Issue: *translation operator T* cannot be generated with a finite depth circuit (depth proportional to L).



c.f D. Gross *et al*, 2012

Amount of translation is a "topological index" for 1d quantum cellular automata

VQE Attempt 1

- Let's not worry about it and just initialize a momentum state.

$$|\psi_0(k)\rangle = \frac{1}{\sqrt{N}} \sum_i e^{ikx_i} |-_i\rangle$$

- Generate

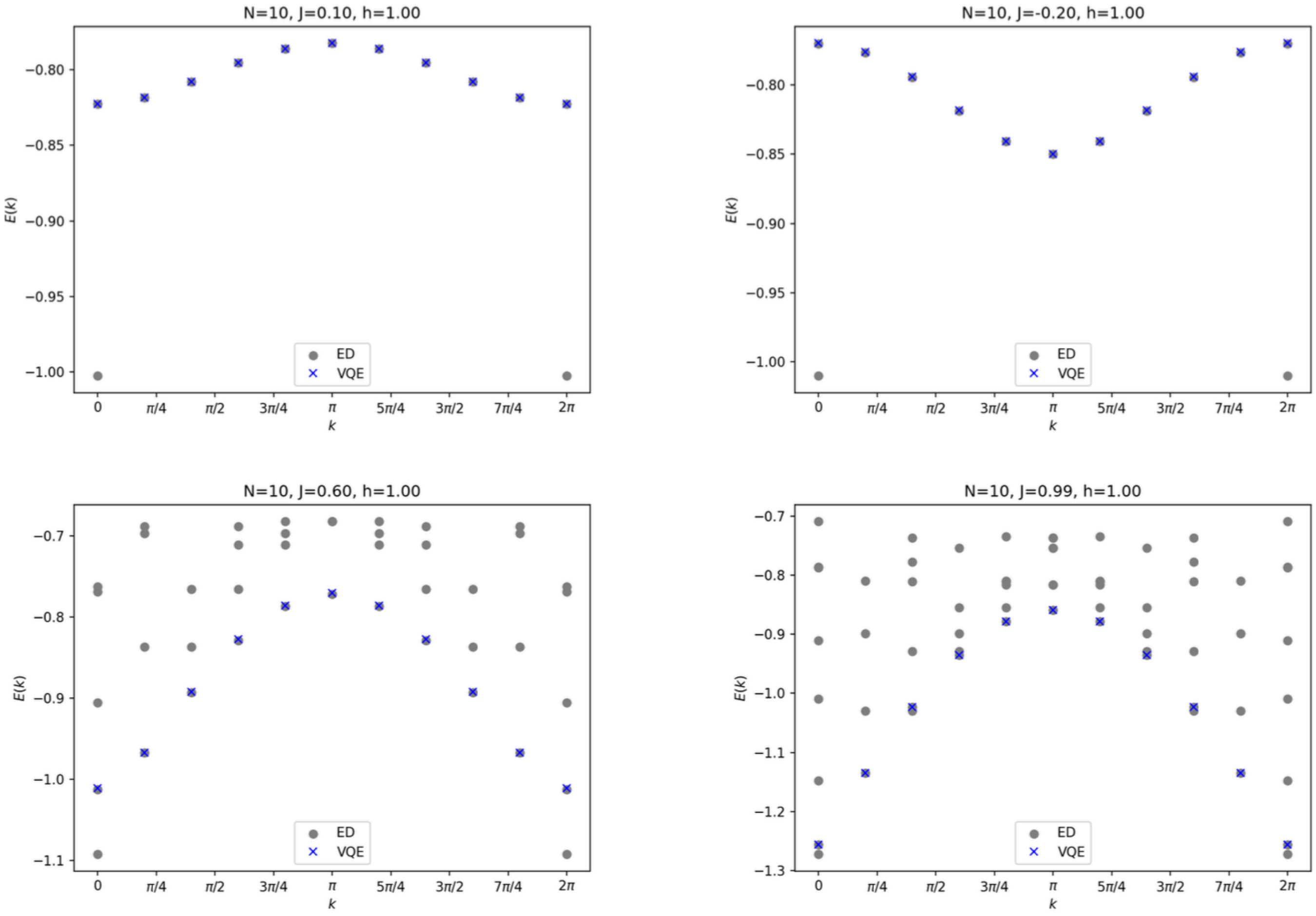
$$|\psi(k)\rangle = U(\{\beta_i\})|\psi_0(k)\rangle$$

- Momentum conservation helps: k is conserved as is $P = \otimes_i X_i$

VQE Attempt 1

Simulations with QISkit

Works!



VQE attempt 2

- Make the system generate k state
  - Trick 1: Parity conservation $P = \otimes_i X_i$.

Ground state P=+1, k=0 $|GS\rangle = U_+|++\cdots+\rangle$

Excited state P=$(-1)^N$, k=0 $|k=0\rangle = U_-|--\cdots-\rangle$

Generates quasiparticle state if N odd!

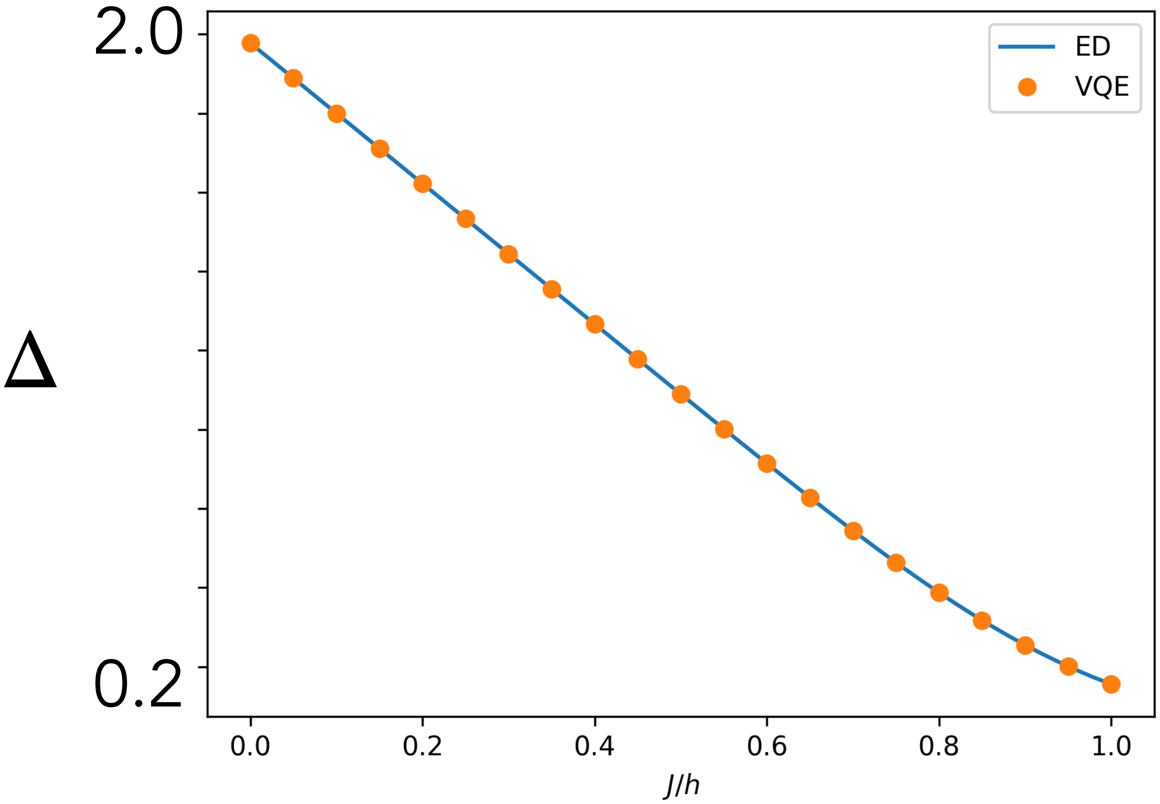
- In general this U depends on J/h (and is non-trivial even for J/h=0).

VQE attempt 2

- It works!

$$|GS\rangle = U_+ |+ + \cdots + \rangle$$

$$|k=0\rangle = U_- |- - \cdots - \rangle$$



VQE attempt 2

- Generate other k values?

- Trick 2: for *ideal* single spin-flip state, can *change* k via local unitary

$$|k\rangle_0 = U_k |k=0\rangle_0$$

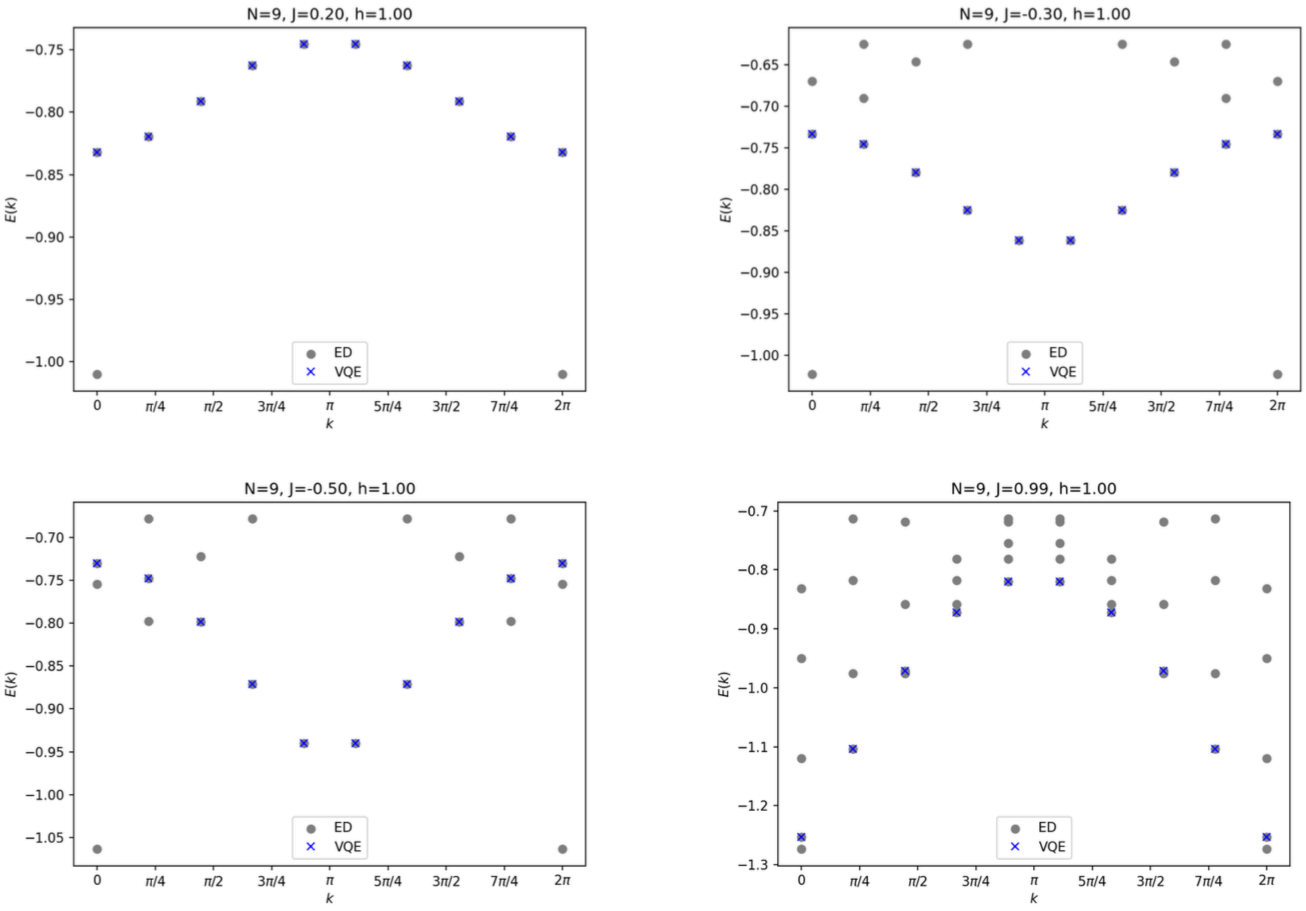
$$|k=0\rangle_0 = \frac{1}{\sqrt{N}} \sum_i |-_i\rangle$$

$$U_k = \prod_j e^{ikx_j(\frac{1}{2}-\frac{X_j}{2})}$$

  - So we have a protocol

$$|k\rangle = U_{\text{int}} |k\rangle_0 = U_{\text{int}}\, U_k\, U_-^0 |- - \cdots -\rangle$$

VQE attempt 2



N=9, J=0.20, h=1.00

N=9, J=-0.30, h=1.00

N=9, J=-0.50, h=1.00

N=9, J=0.99, h=1.00

This also works!

VQE attempt 3

- Can we work in *real* space instead of k space?
- What if we initialize to a *localized* excitation?

$$|x=0\rangle_0 = |++\cdots -_{x=0} + \cdots + \rangle = Z_0 \prod_i |+\rangle_i$$

- Evolved state

$$|x=0\rangle = U[\{\beta_i\}]\,|x=0\rangle_0$$

- Since U is translationally invariant and parity conserving, we have

$$|x=0\rangle = \frac{1}{\sqrt{N}} \sum_k U|k\rangle_0 = \frac{1}{\sqrt{N}} \sum_k |k\rangle$$

VQE attempt 3

- Variational energy

$$\langle x = 0 | H | x = 0 \rangle = \frac{1}{N} \sum_{k,k'} \langle k' | H | k \rangle = \frac{1}{N} \sum_{k} \langle k | H | k \rangle$$

  - ⋆ Minimum is reached only if it is reached for each k state individually!

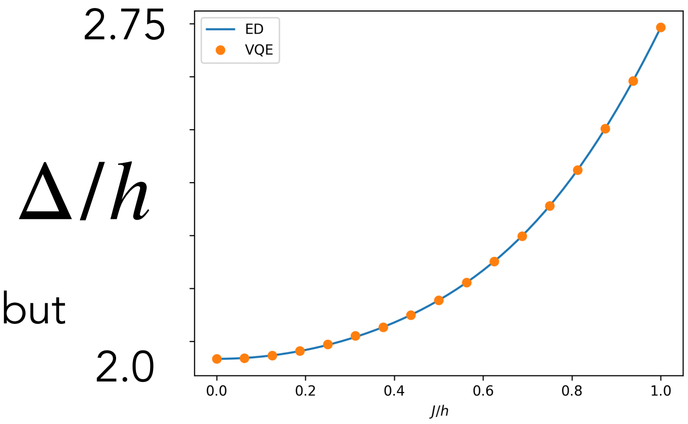
- Quantum parallelism! Just running VQE on this single state encodes the entire band of excited states!

VQE attempt 3

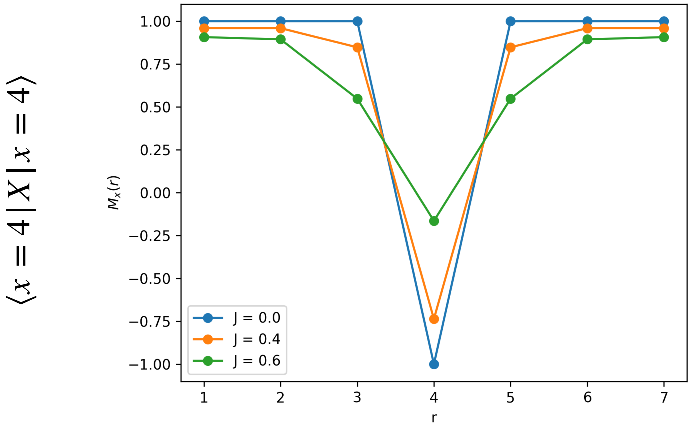
- Variational energy of this state gives the mean energy of the band

$$E_{x=0} - E_{GS} = \frac{1}{N}\sum_k \epsilon_k$$

With some work we can extract the entire band, but we’re still trying to make it efficient



- We can also look at the state itself



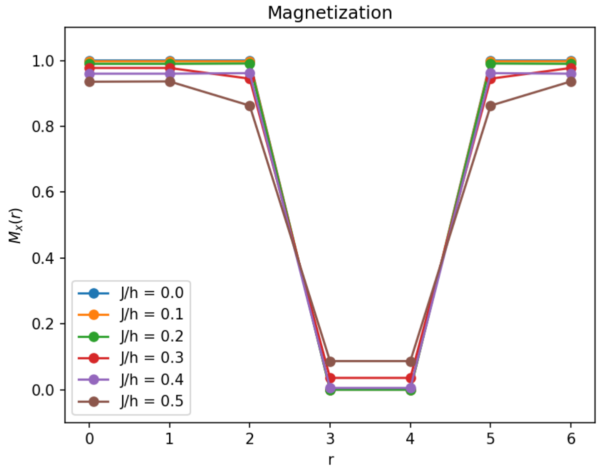
- Physically, we are generating the interacting analog of a Wannier state.

“The quasiparticle”

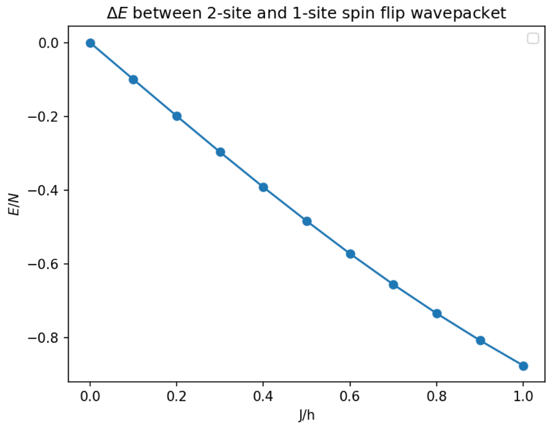
VQE attempt 3

One can investigate many more aspects of the quasiparticle, since we have its wavefunction. For example, we can ask what is the probability distribution of the number of spin flips?

Can also extract "bandwidth".

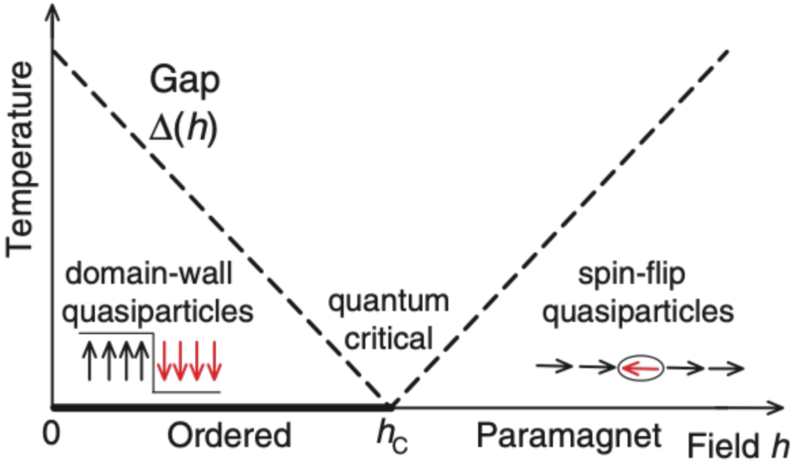


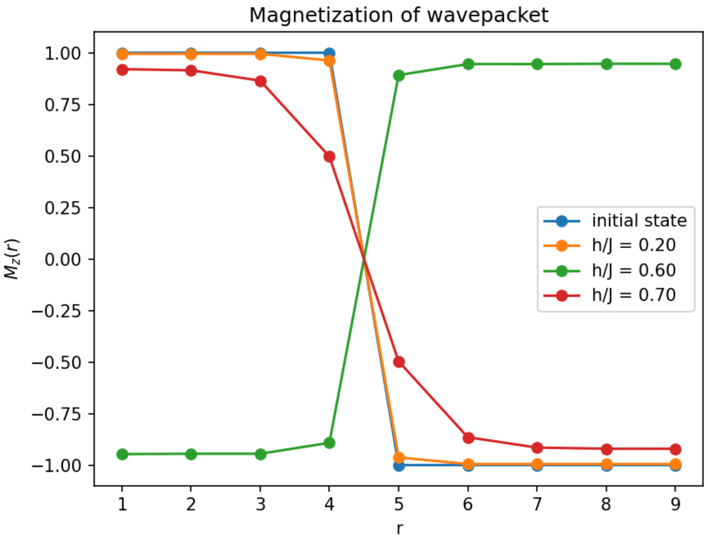
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle + |x+1\rangle\right)$$



$$\langle\psi|H|\psi\rangle - \langle x|H|x\rangle = \int\frac{dk}{2\pi}\,\epsilon(k)\cos k\,.$$

Domain wall QPs





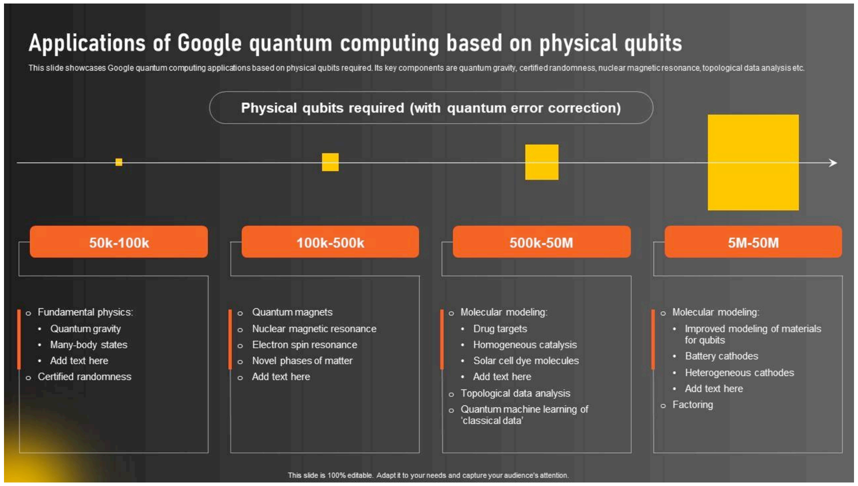
Slightly modified
algorithm produces
localized soliton

Conclusions

- VQE can generate an “exact” (in the sense of variational convergence) localized quasiparticle state
- This procedure benefits from quantum parallelism and unitarity to encode the entire band of quasiparticle states in the localized state
- Extensions to many other quasiparticles seem possible.
- Q: is there a classical algorithm that can do the same, i.e. directly produce the maximally localized exact quasiparticle state?

Is a QC useful for quantum materials?

I'm still not sure about this



Applications of Google quantum computing based on physical qubits

This slide showcases Google quantum computing applications based on physical qubits required. Its key components are quantum gravity, certified randomness, nuclear magnetic resonance, topological data analysis etc.

Physical qubits required (with quantum error correction)

50k-100k

- Fundamental physics:
  - Quantum gravity
  - Many-body states
  - Add text here
- Certified randomness

100k-500k

- Quantum magnets
- Nuclear magnetic resonance
- Electron spin resonance
- Novel phases of matter
- Add text here

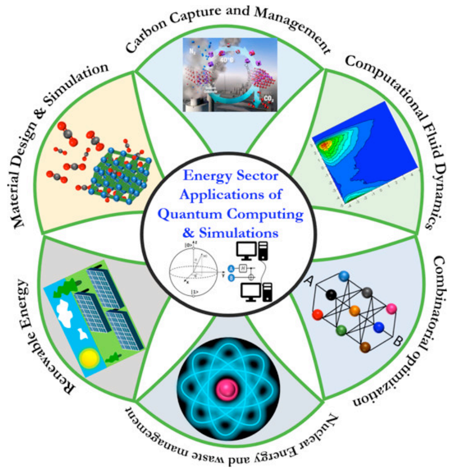
500k-50M

- Molecular modeling:
  - Drug targets
  - Homogeneous catalysis
  - Solar cell dye molecules
  - Add text here
- Topological data analysis
- Quantum machine learning of 'classical data'

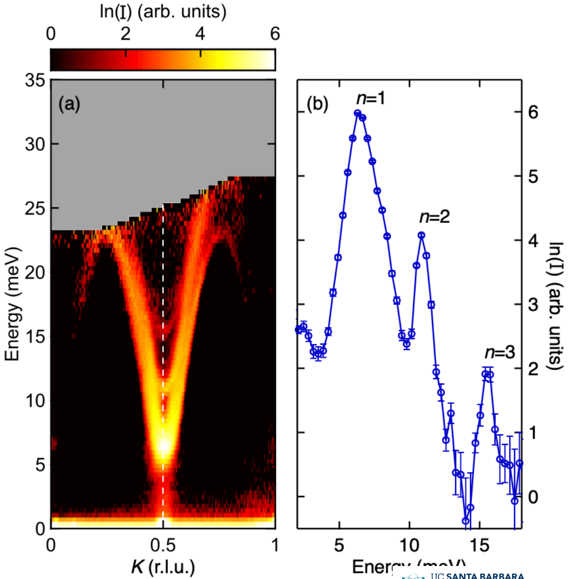
5M-50M

- Molecular modeling:
  - Improved modeling of materials for qubits
  - Battery cathodes
  - Heterogeneous cathodes
  - Add text here
- Factoring

This slide is 100% editable. Adapt it to your needs and capture your audience's attention.



But maybe this.



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Simons Collaboration on Ultra-Quantum Matter