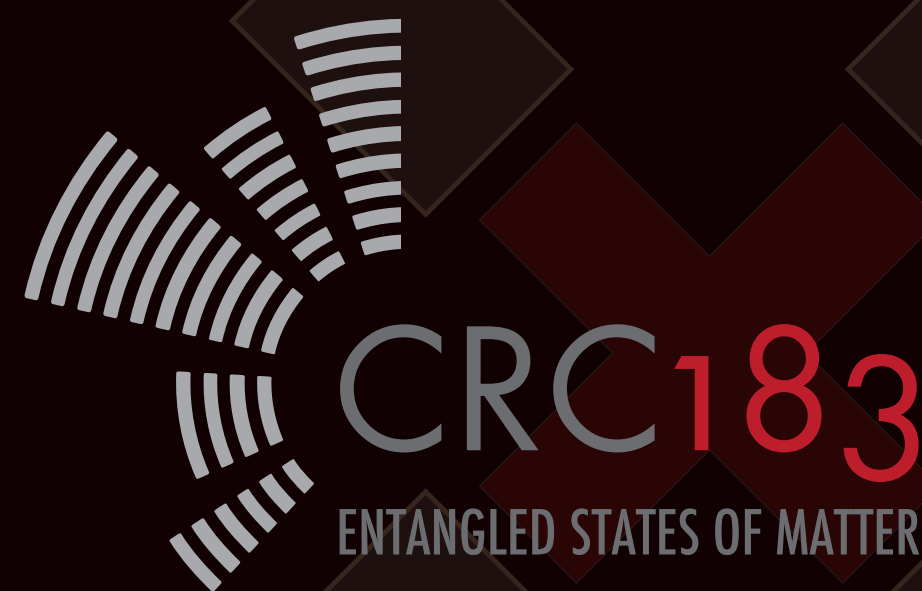


# Qubit Fractionalization and Emergent Majorana Liquids in Quantum Circuits

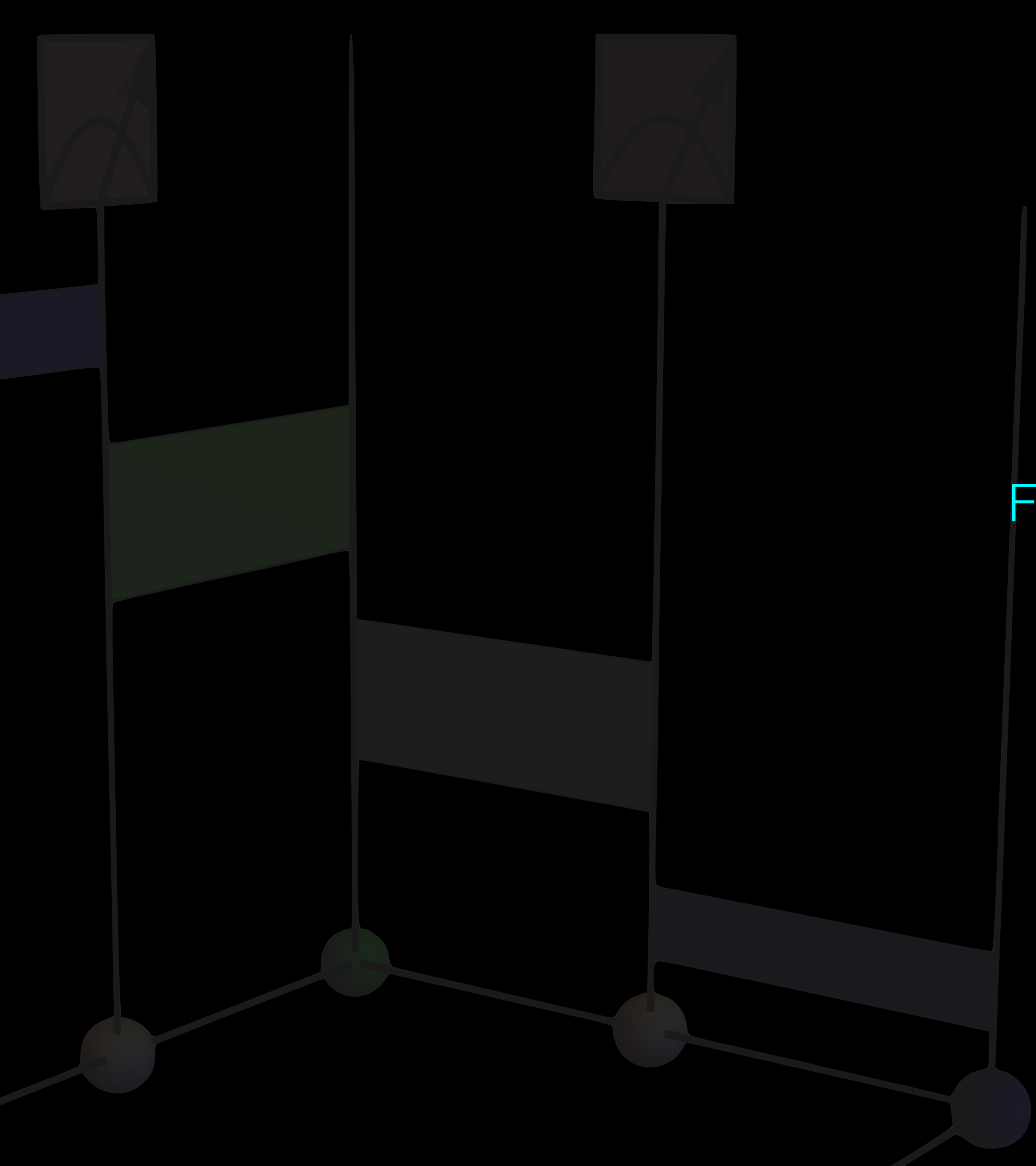


**Simon Trebst**  
University of Cologne



Fractionalization and Emergent Gauge Fields in Quantum Matter

ICTP Trieste, December 2023



Fractionalization & Emergent Gauge Fields  
in **Quantum Matter**



# finite-temperature Kitaev spin liquids

PRL **113**, 197205 (2014)

PHYSICAL REVIEW LETTERS

week ending  
7 NOVEMBER 2014

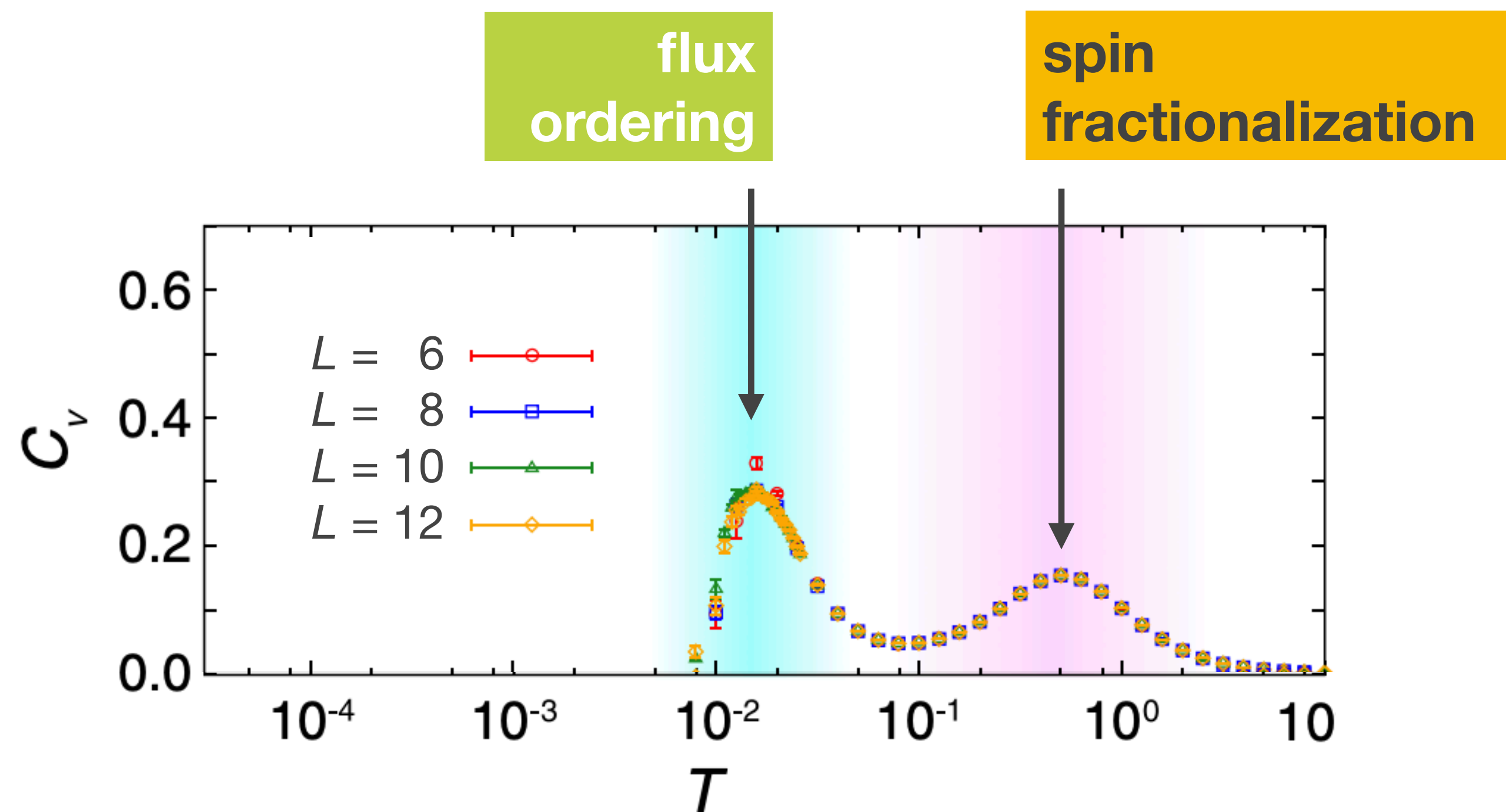
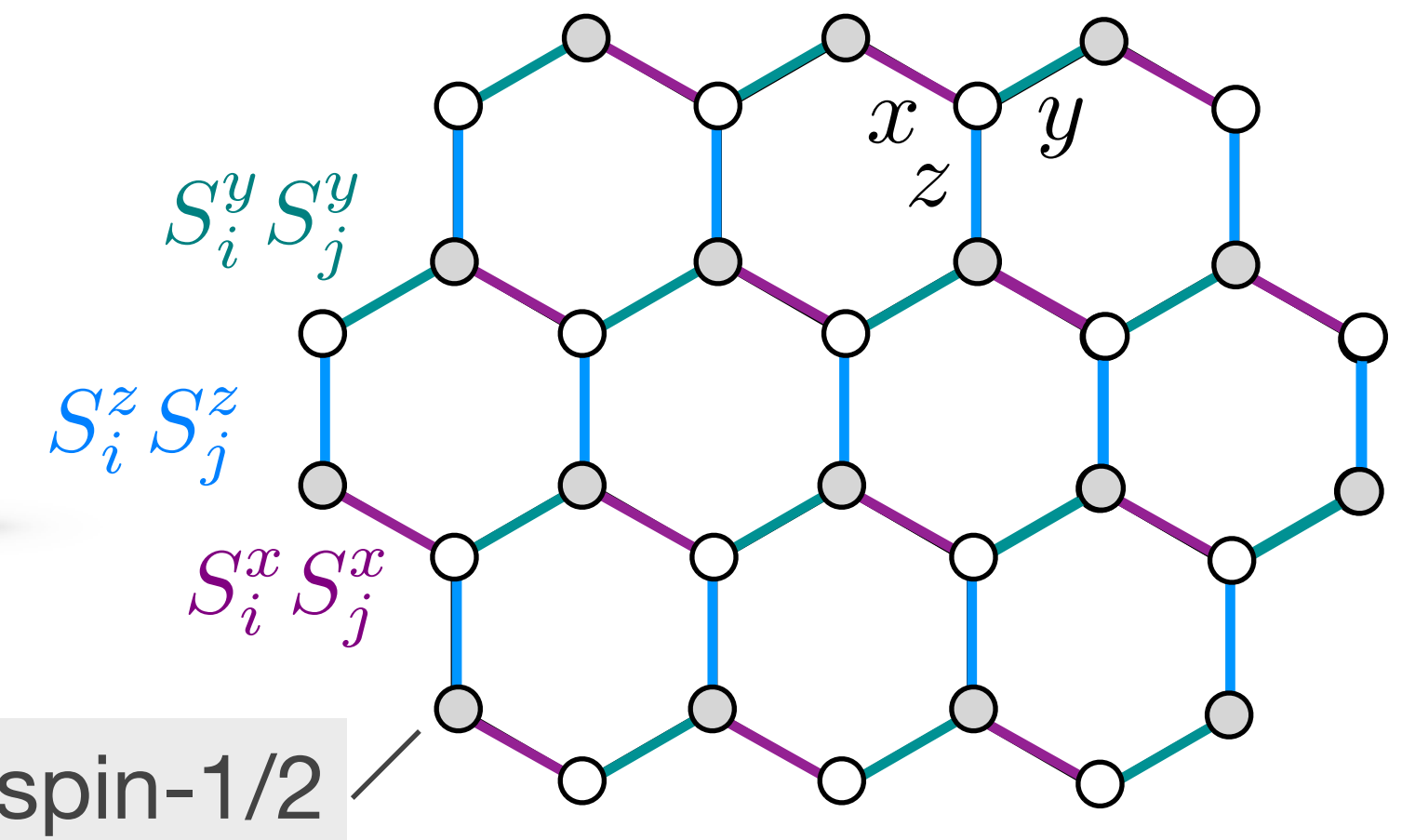
## Vaporization of Kitaev Spin Liquids

Joji Nasu,<sup>1</sup> Masafumi Udagawa,<sup>2</sup> and Yukitoshi Motome<sup>2</sup>

<sup>1</sup>Department of Physics, Tokyo Institute of Technology, Ookayama, 2-12-1, Meguro, Tokyo 152-8551, Japan

<sup>2</sup>Department of Applied Physics, University of Tokyo, Hongo, 7-3-1, Bunkyo, Tokyo 113-8656, Japan

(Received 24 July 2014; revised manuscript received 9 October 2014; published 7 November 2014)



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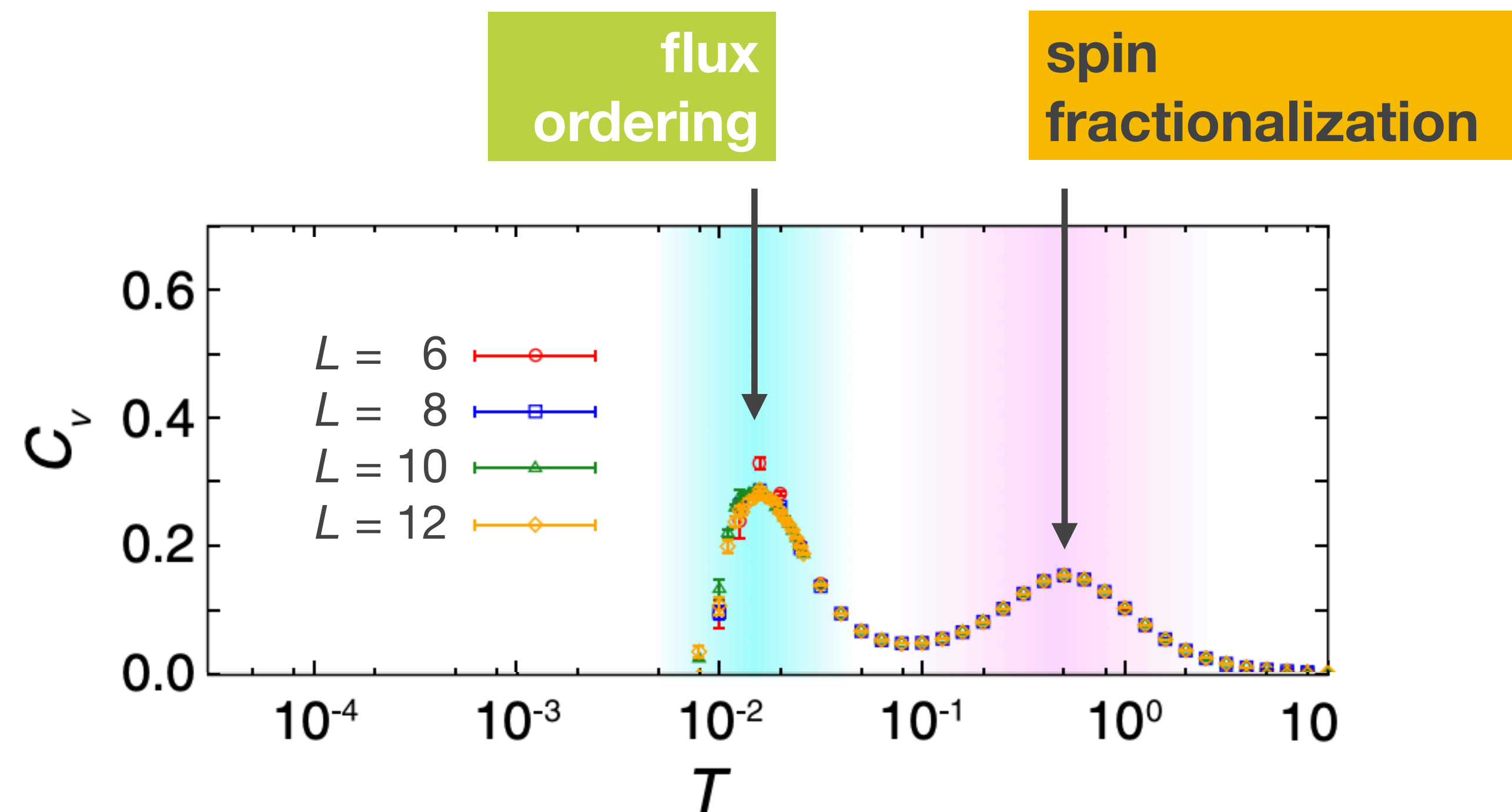
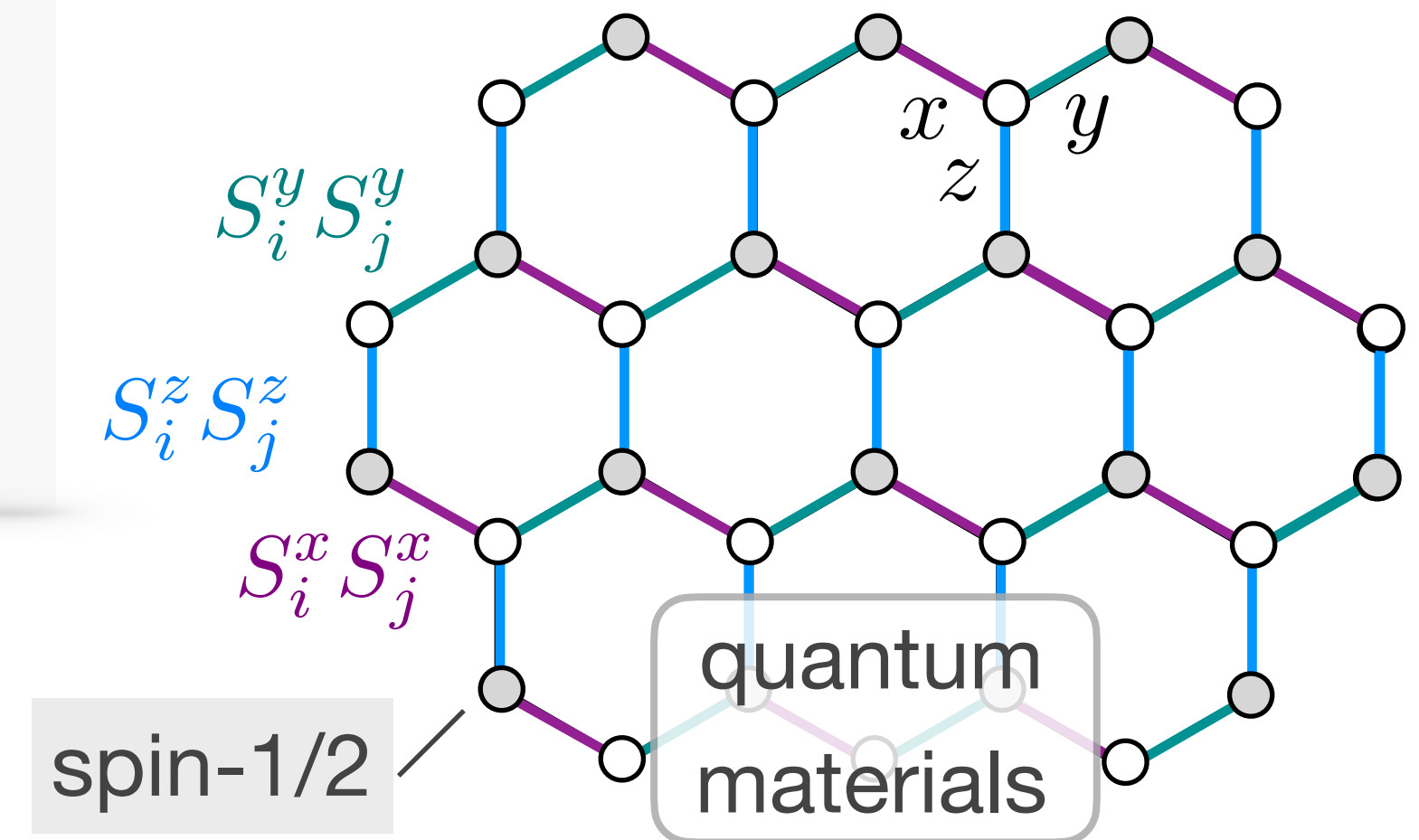
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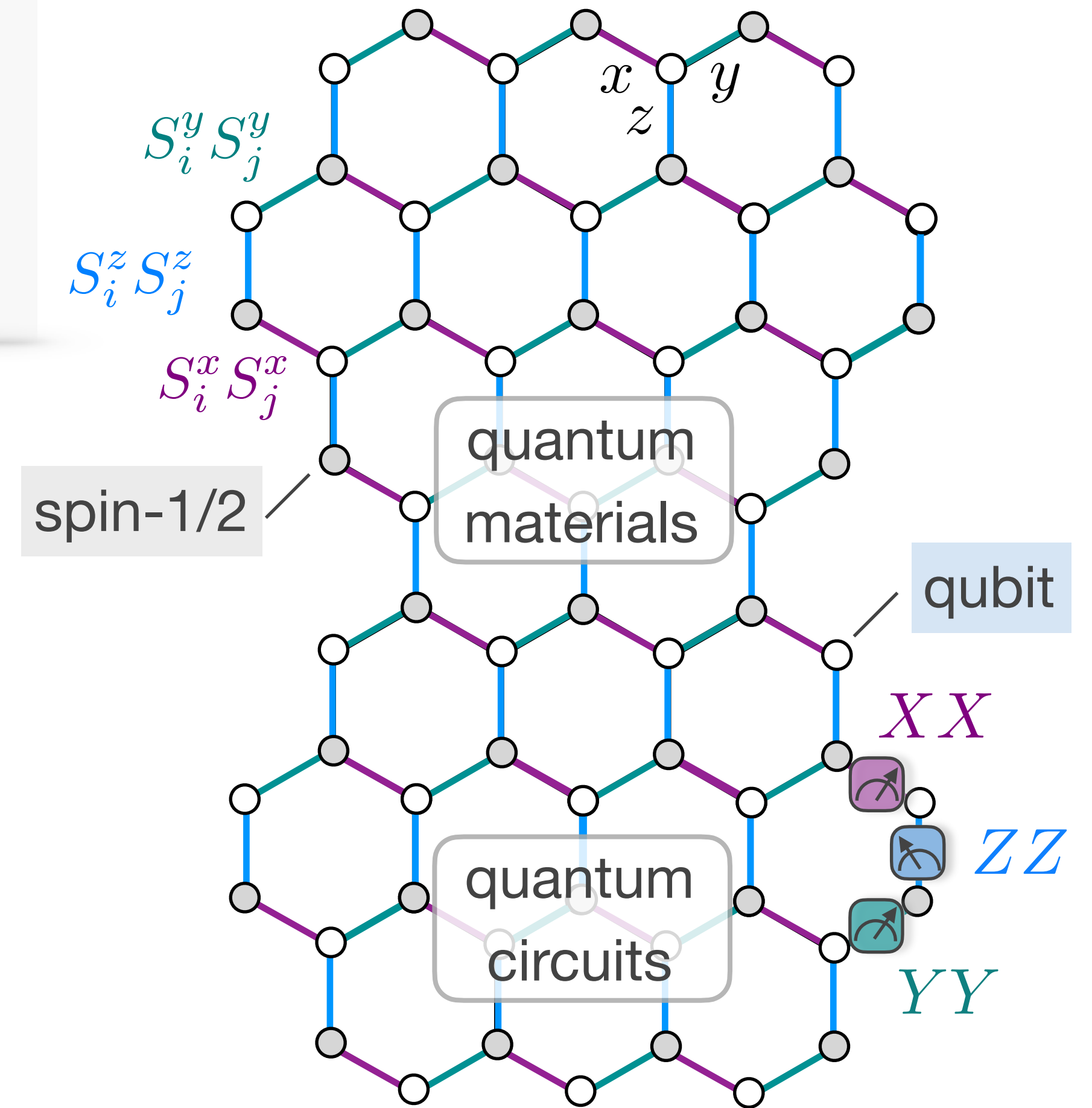
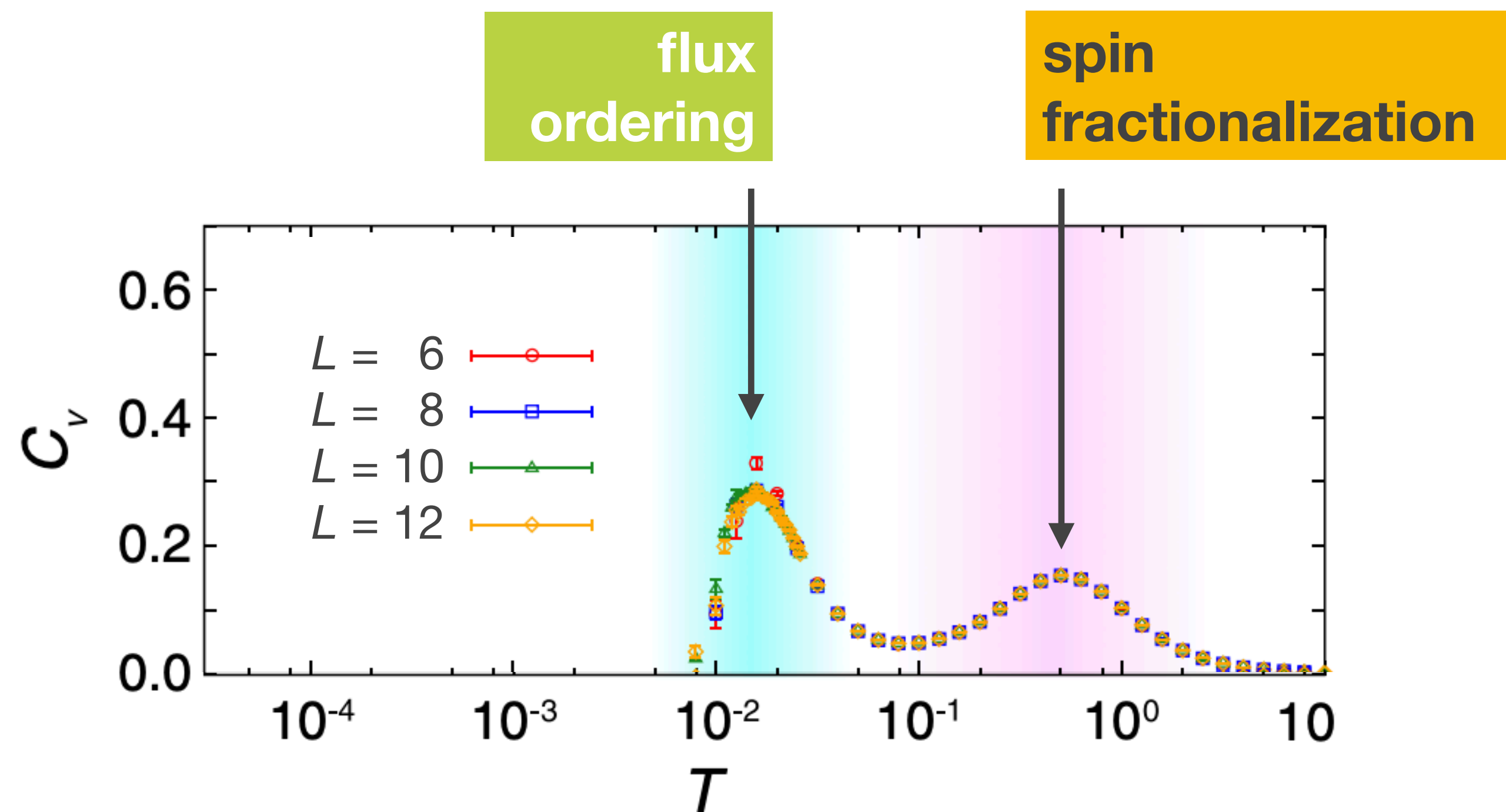
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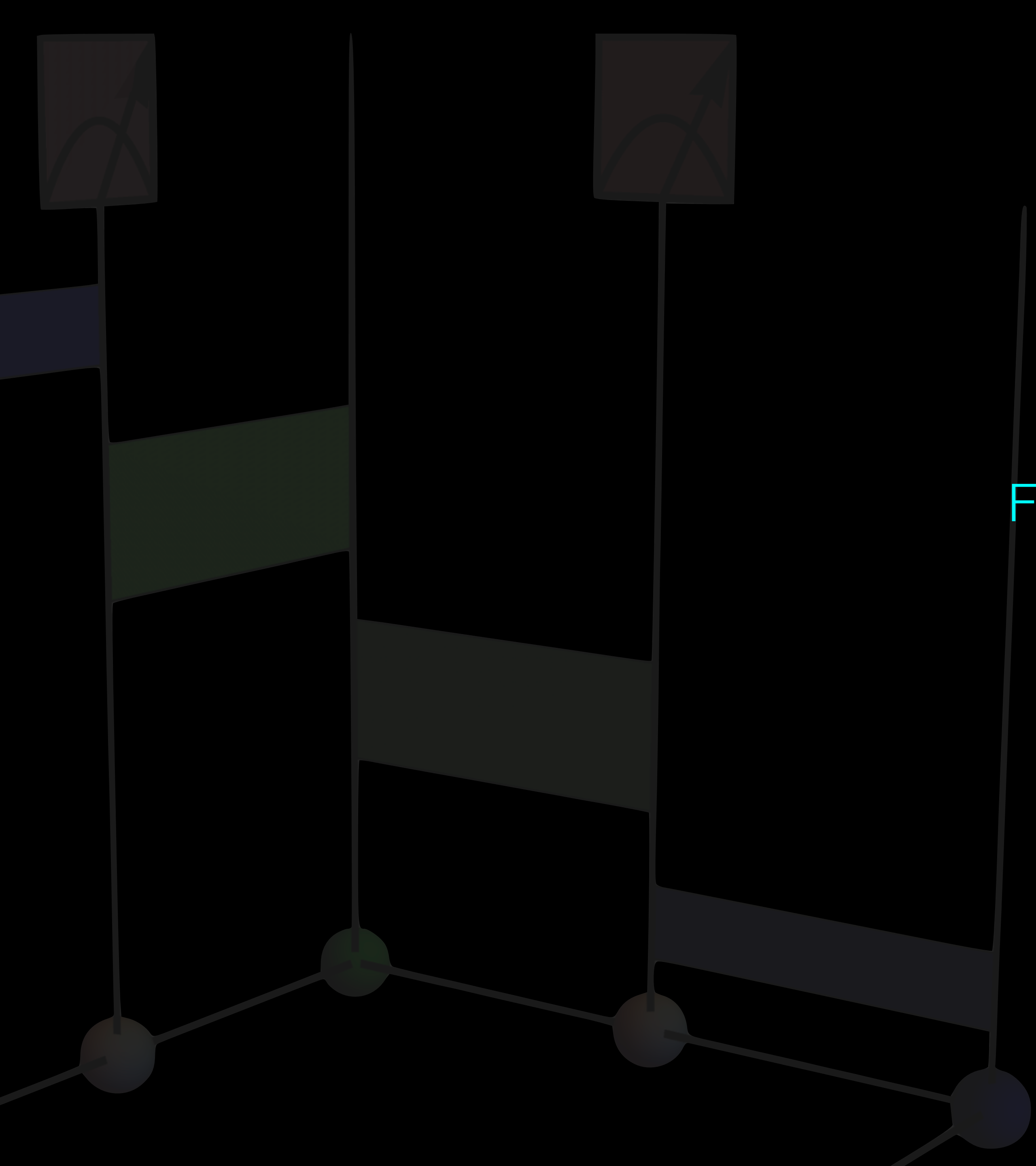
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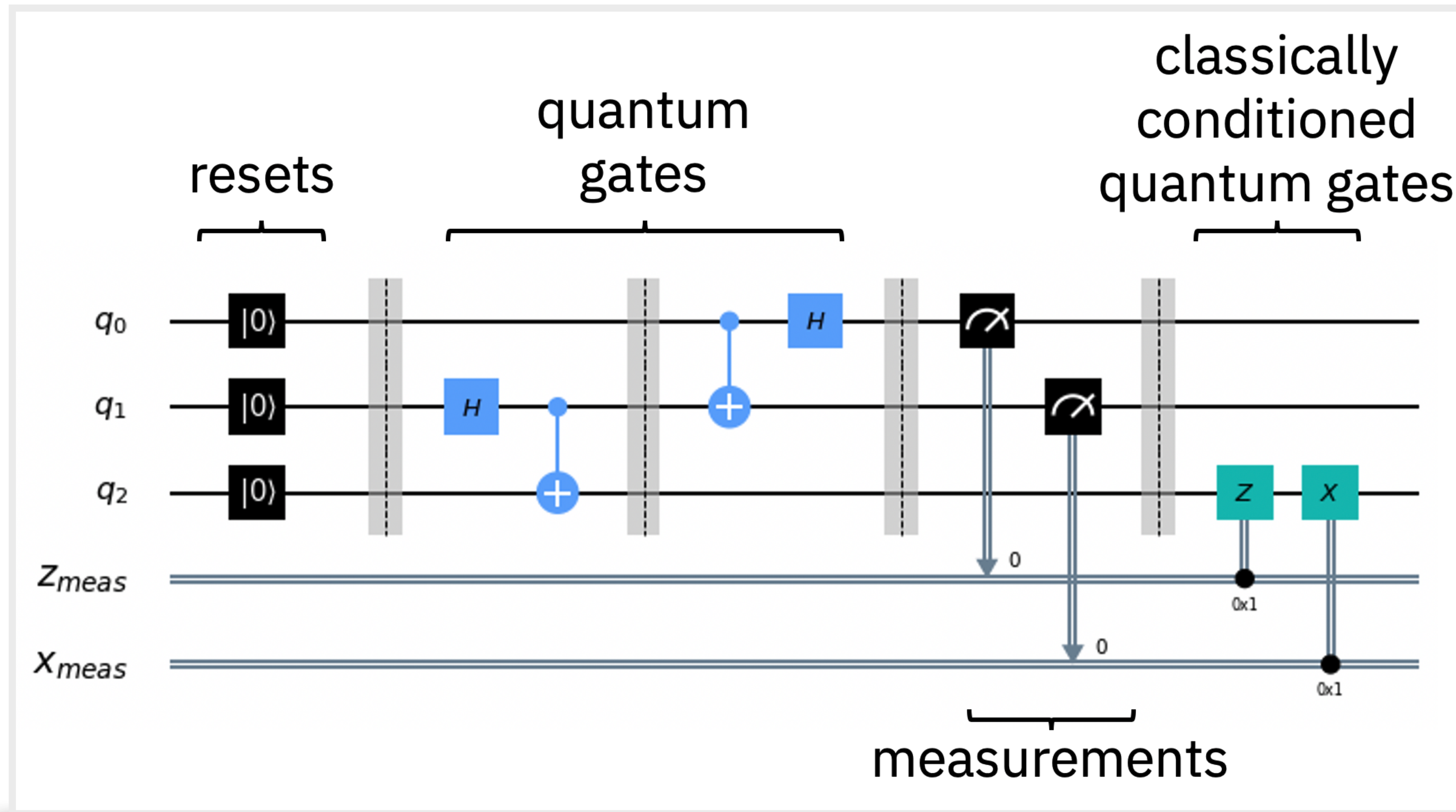




Fractionalization & Emergent Gauge Fields  
in **Quantum Circuits**

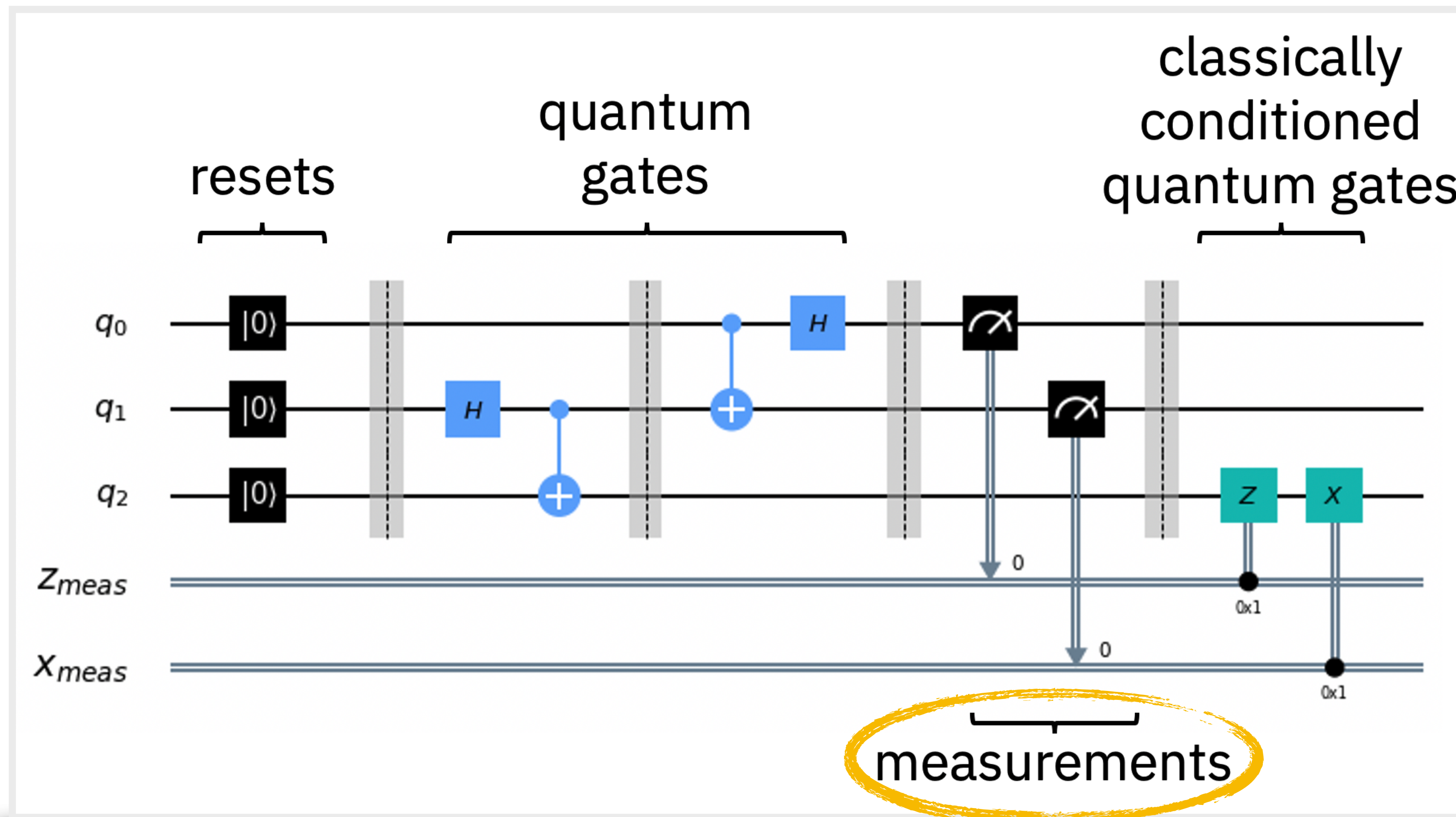


# quantum circuits in a nutshell



Quantum computing in a nutshell, Qiskit documentation / IBM Quantum

# quantum circuits in a nutshell



Quantum computing in a nutshell, Qiskit documentation / IBM Quantum



# quantum measurements



“About your cat, Mr. Schrödinger — I have good news and bad news.”

# quantum measurements



Quantum measurements can

- **extract information**  
from a system

“About your cat, Mr. Schrödinger — I have good news and bad news.”

# quantum measurements



Quantum measurements can

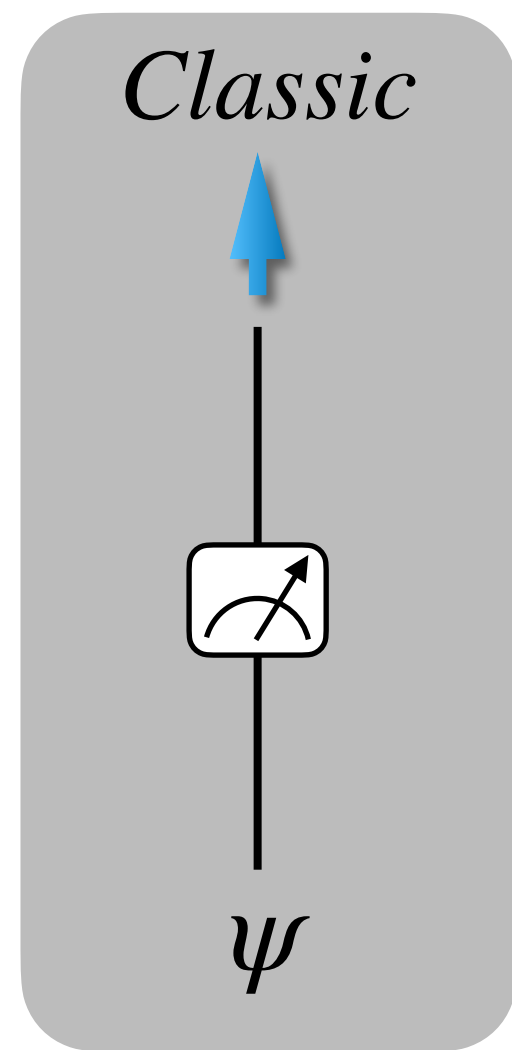
- **extract information** from a system
- **shape entanglement** of a quantum system

“About your cat, Mr. Schrödinger — I have good news and bad news.”

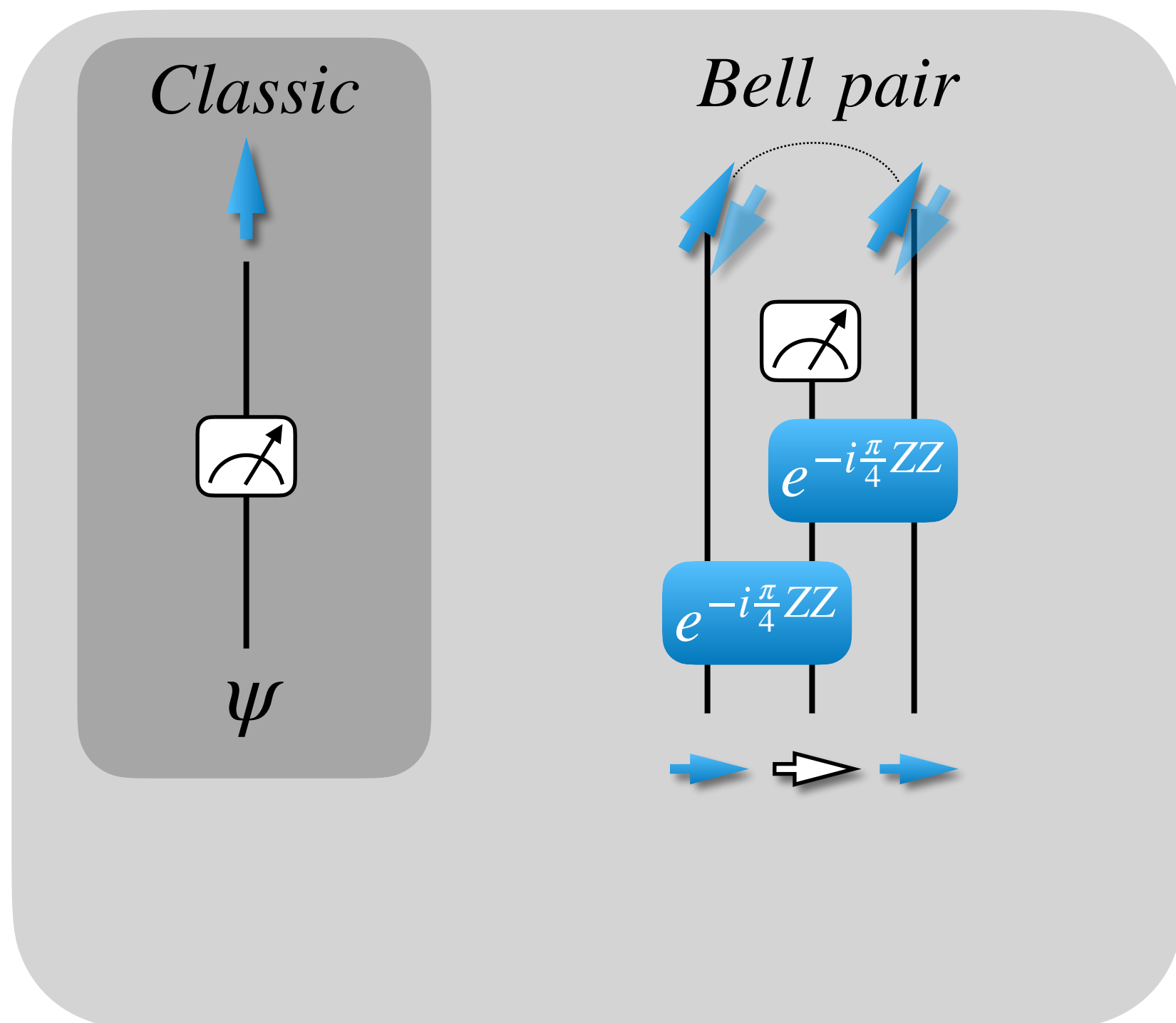
double-faced Janus



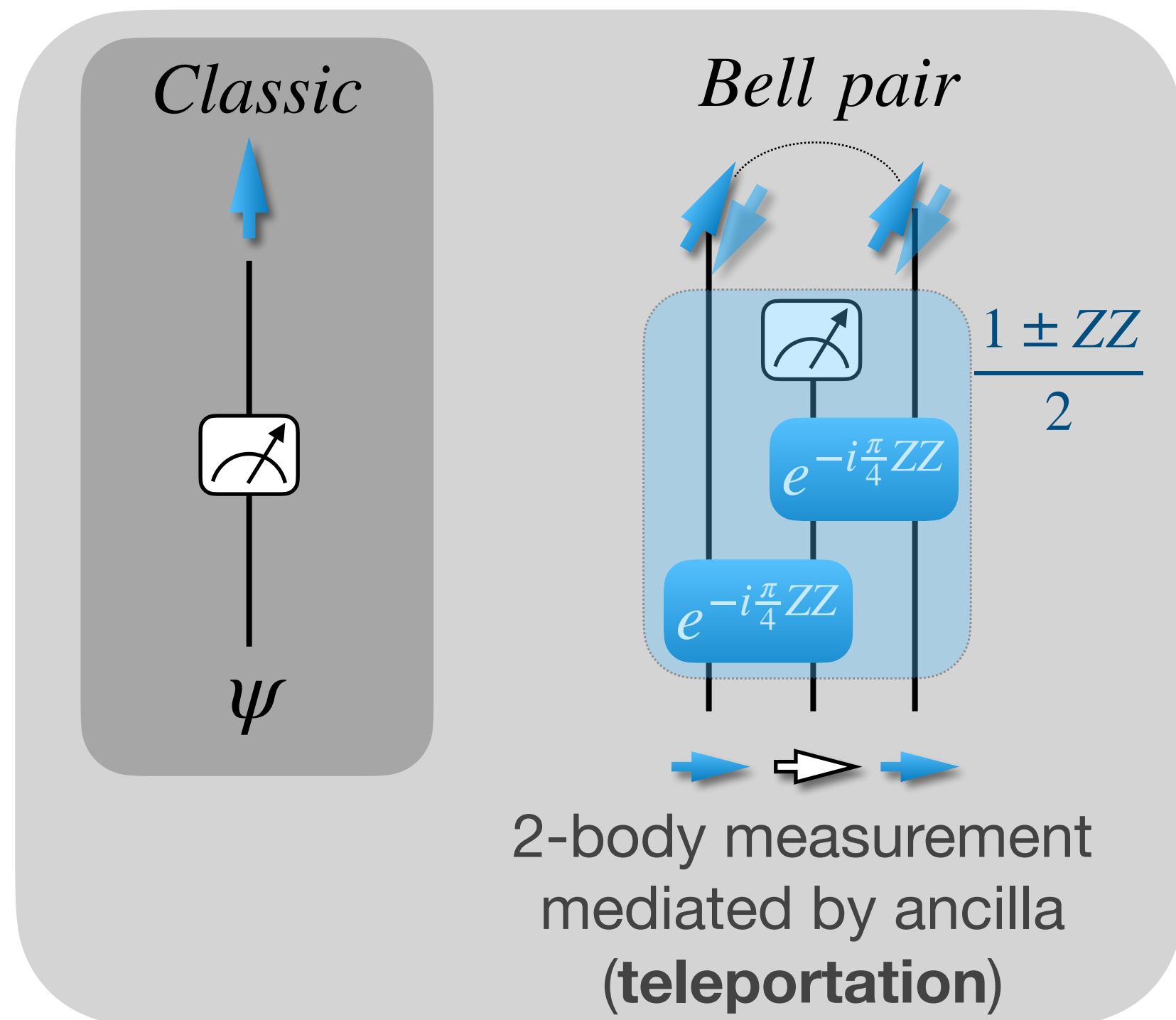
# quantum states from measurements



# quantum states from measurements

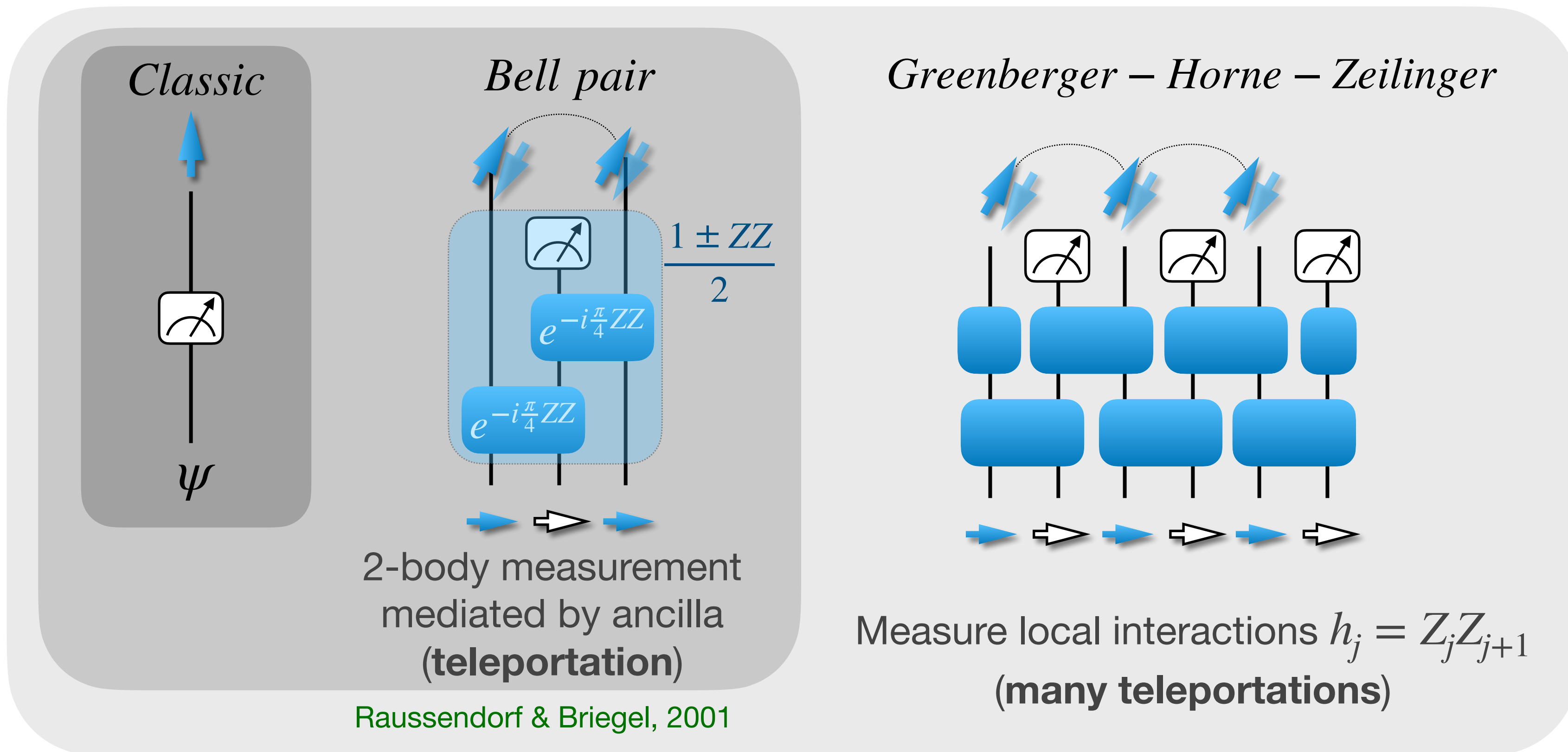


# quantum states from measurements

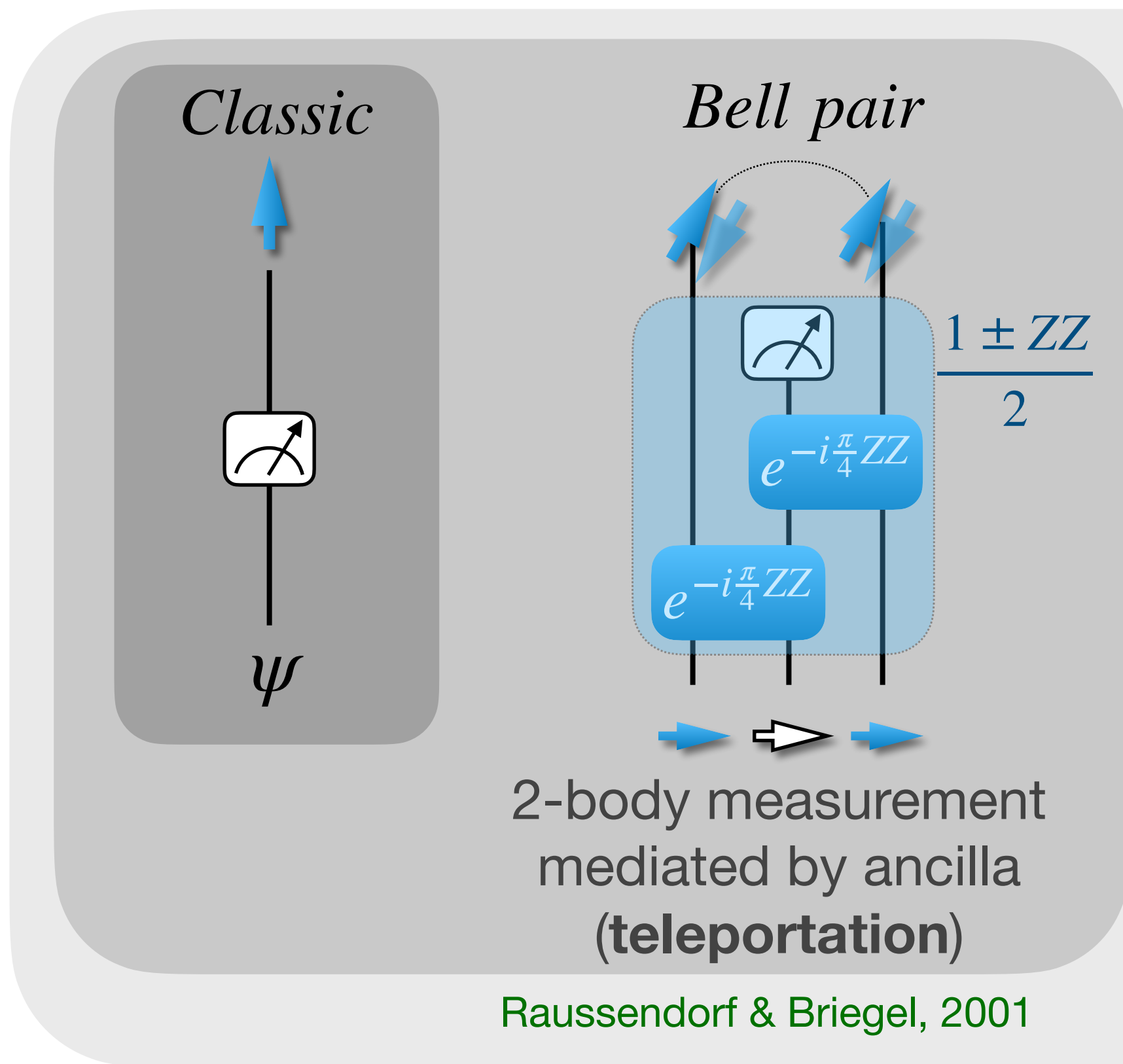


Raussendorf & Briegel, 2001

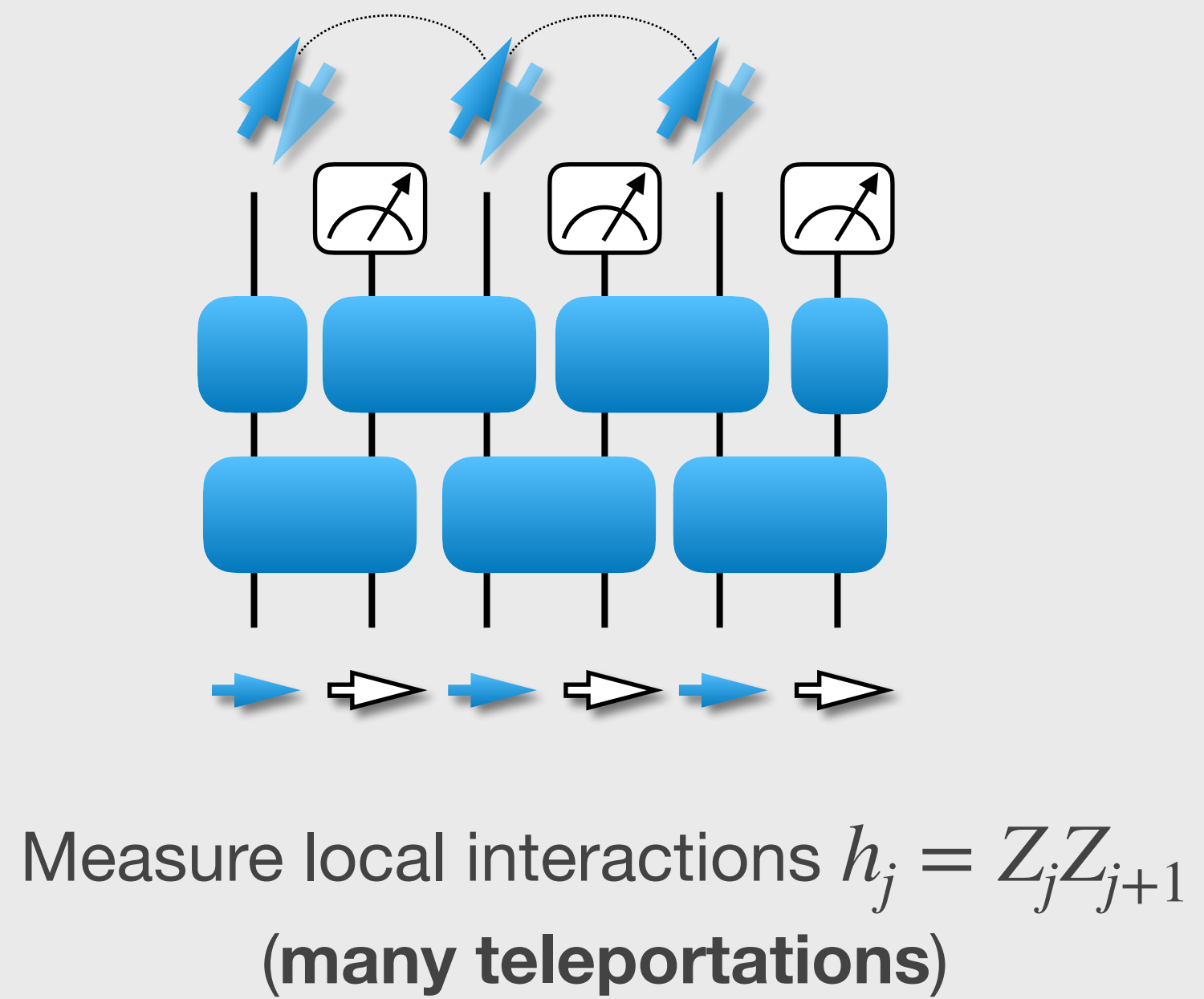
# quantum states from measurements



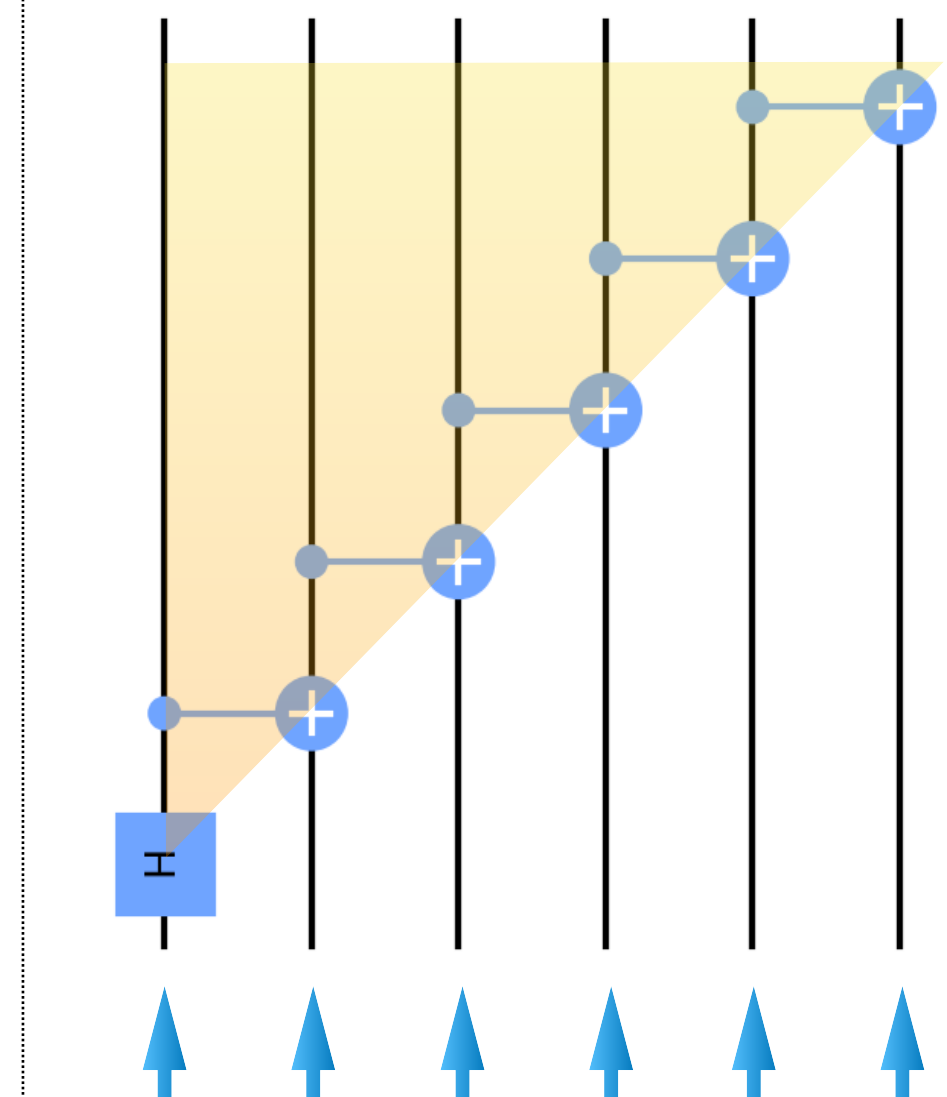
# quantum states from measurements



## Greenberger – Horne – Zeilinger

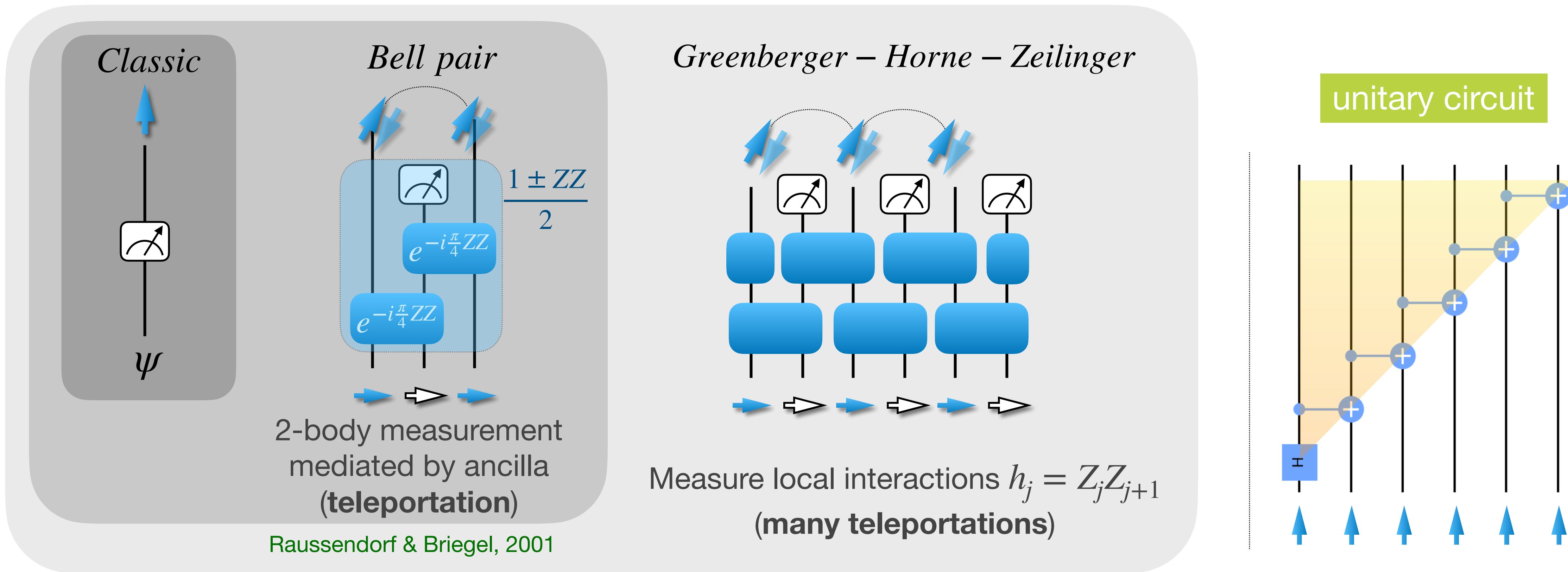


## unitary circuit





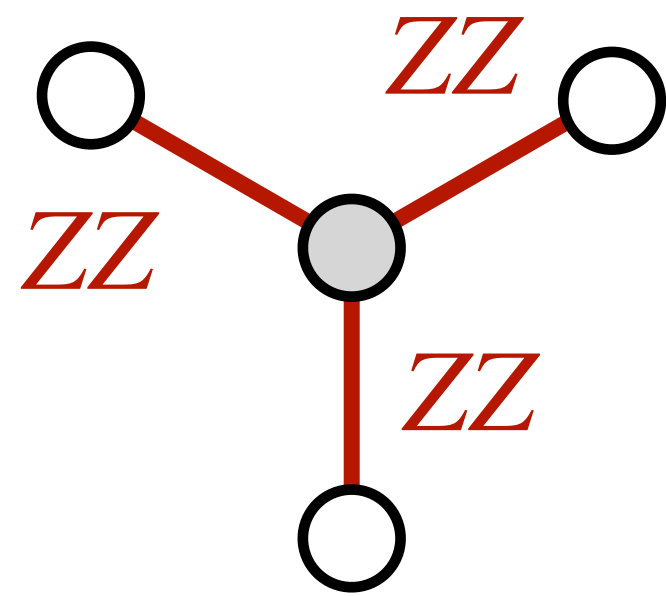
# quantum states from measurements



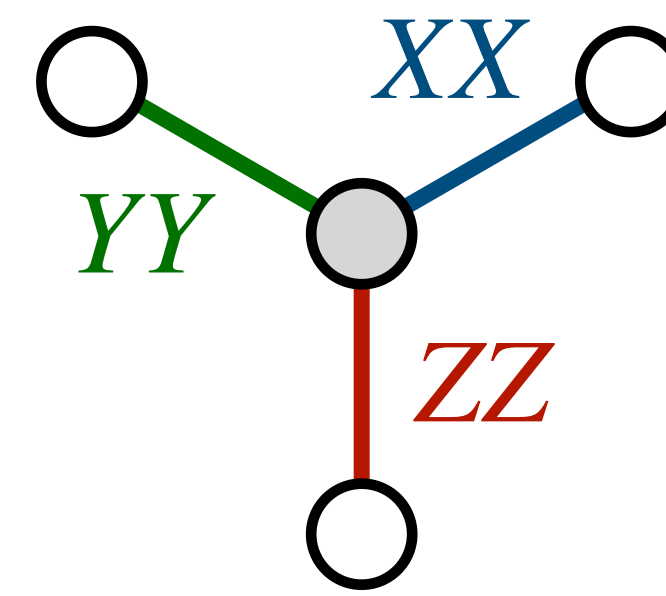
$$t \propto O(1)$$

$$t \propto L$$

# commuting vs non-commuting measurements

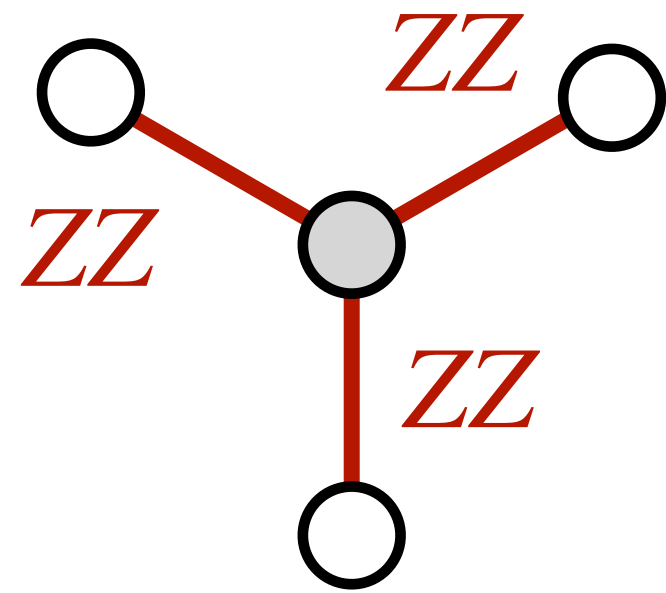


- **commuting**
- parallelized
- no dynamics

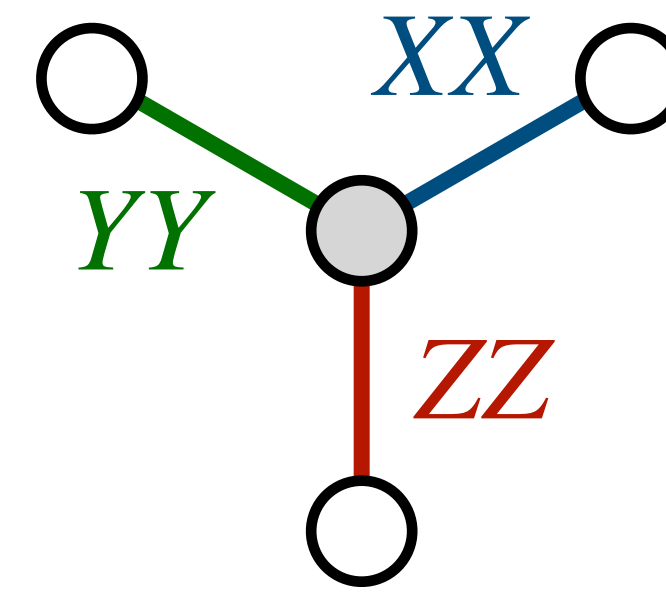


- **non-commuting**
- sequential
- dynamics

# commuting vs non-commuting measurements



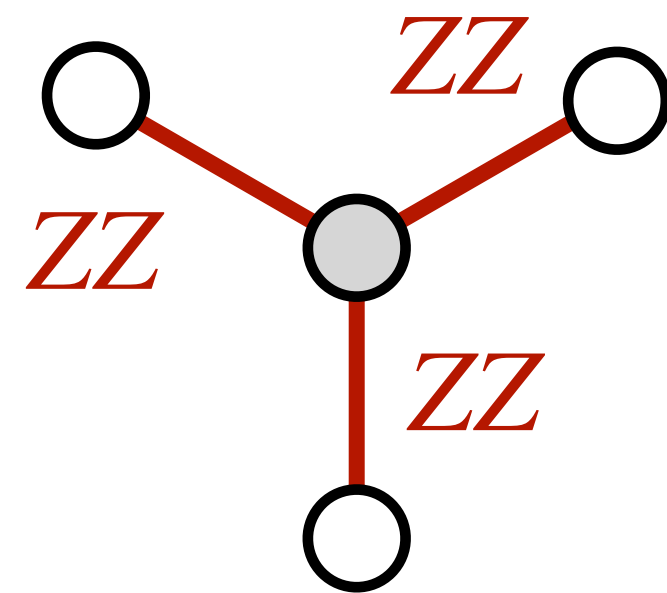
- **commuting**
- parallelized
- no dynamics



## Kitaev spin liquid

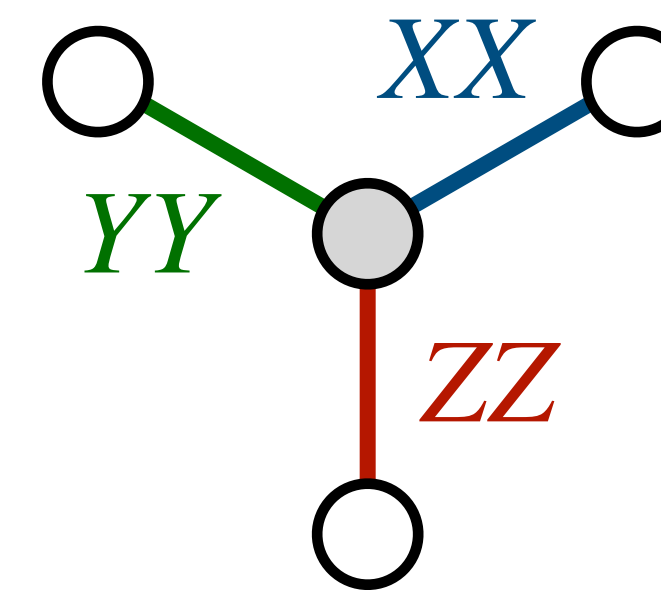
- **non-commuting**
- sequential
- dynamics

# commuting vs non-commuting measurements



## Nishimori's cat

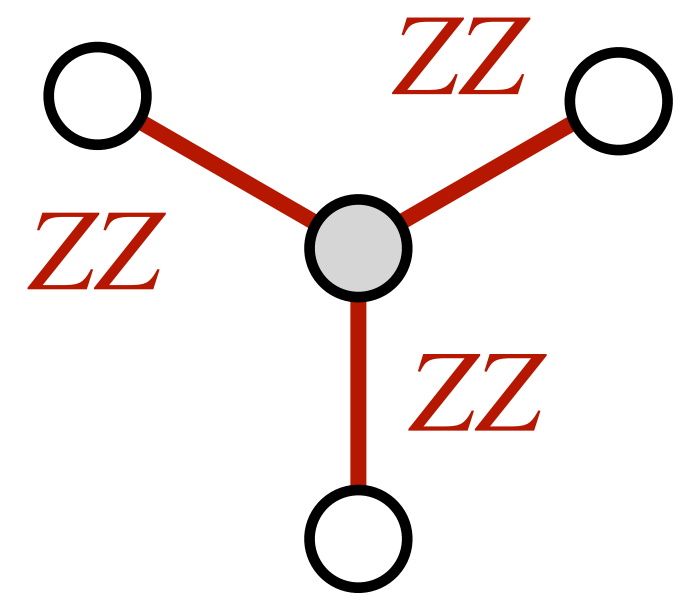
- commuting
- parallelized
- no dynamics



## Kitaev spin liquid

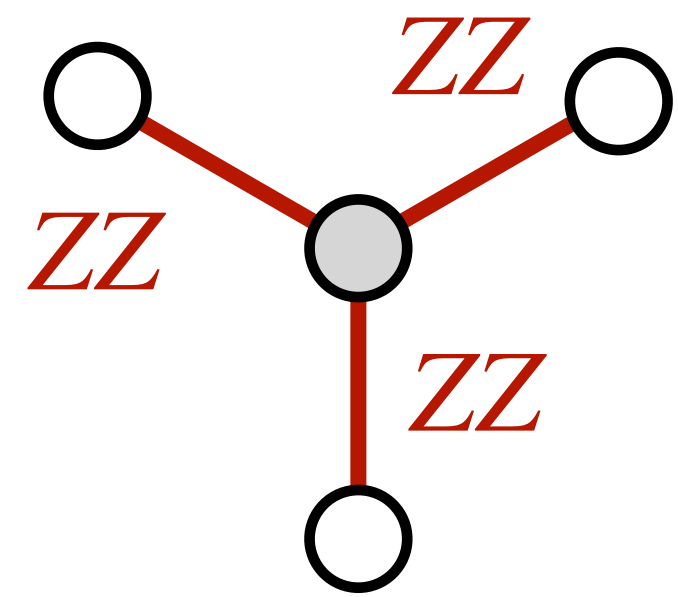
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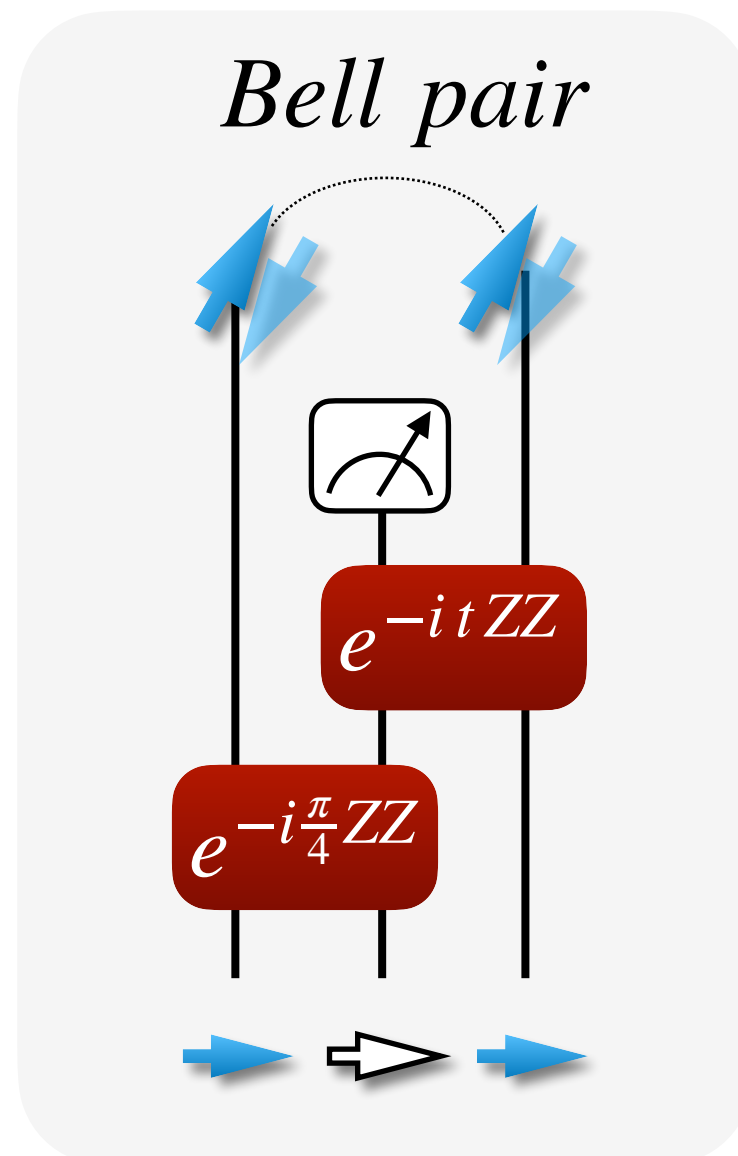


**Nishimori's cat**

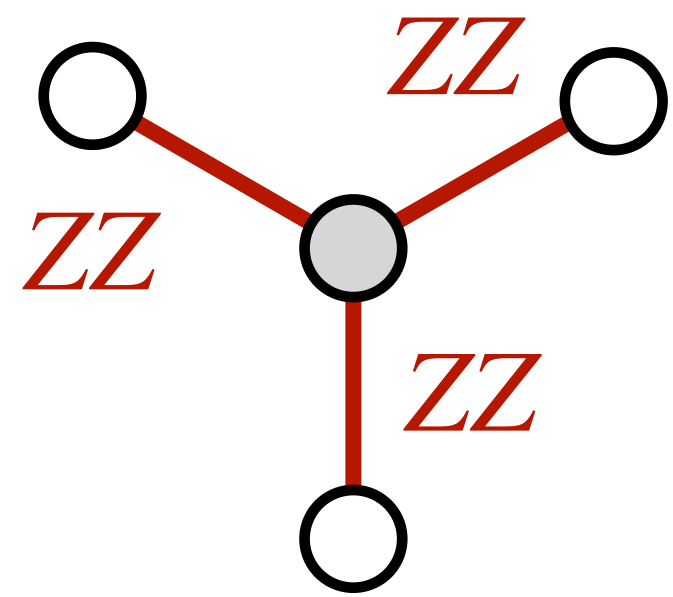
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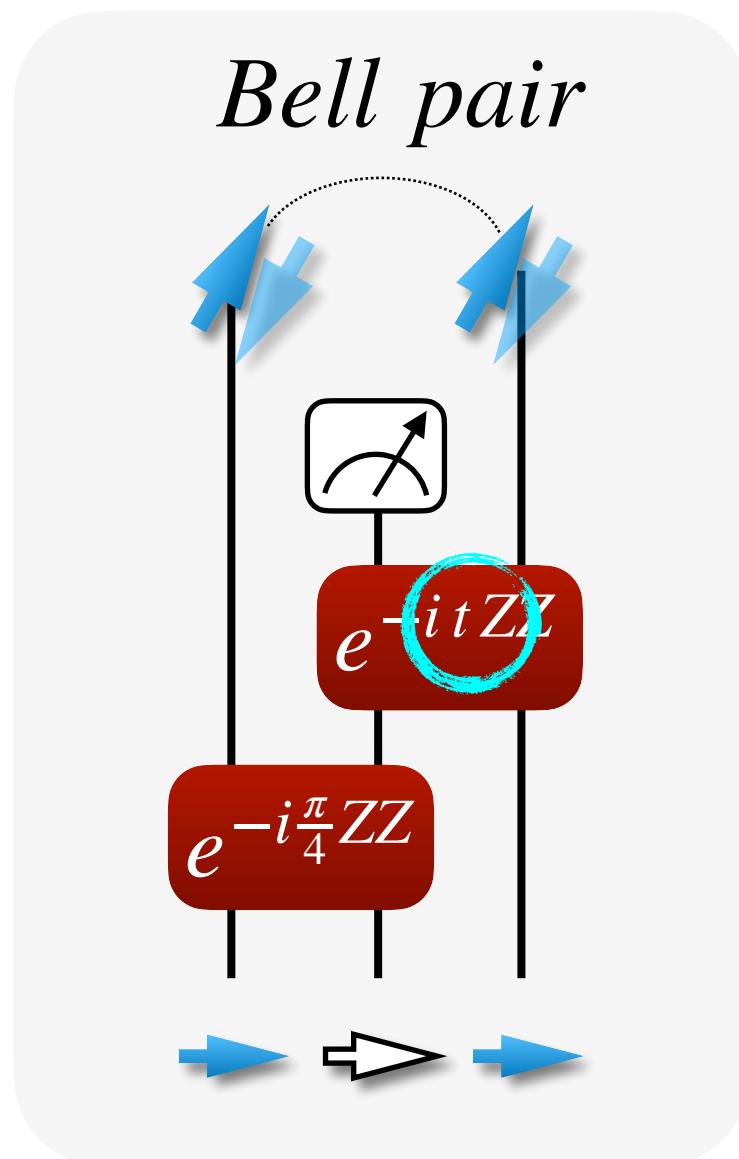
**Nishimori's cat**



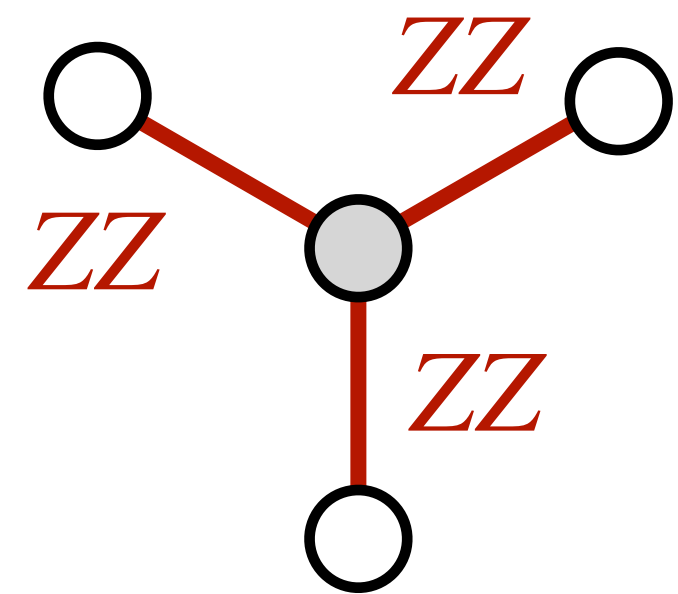
# Nishimori's cat



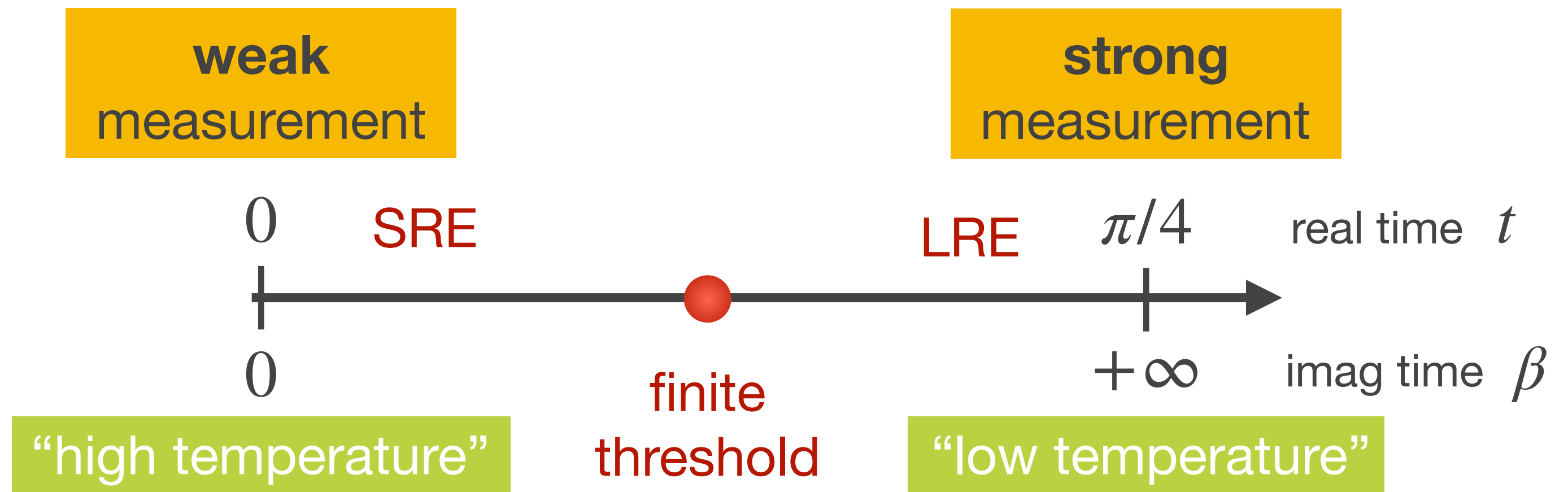
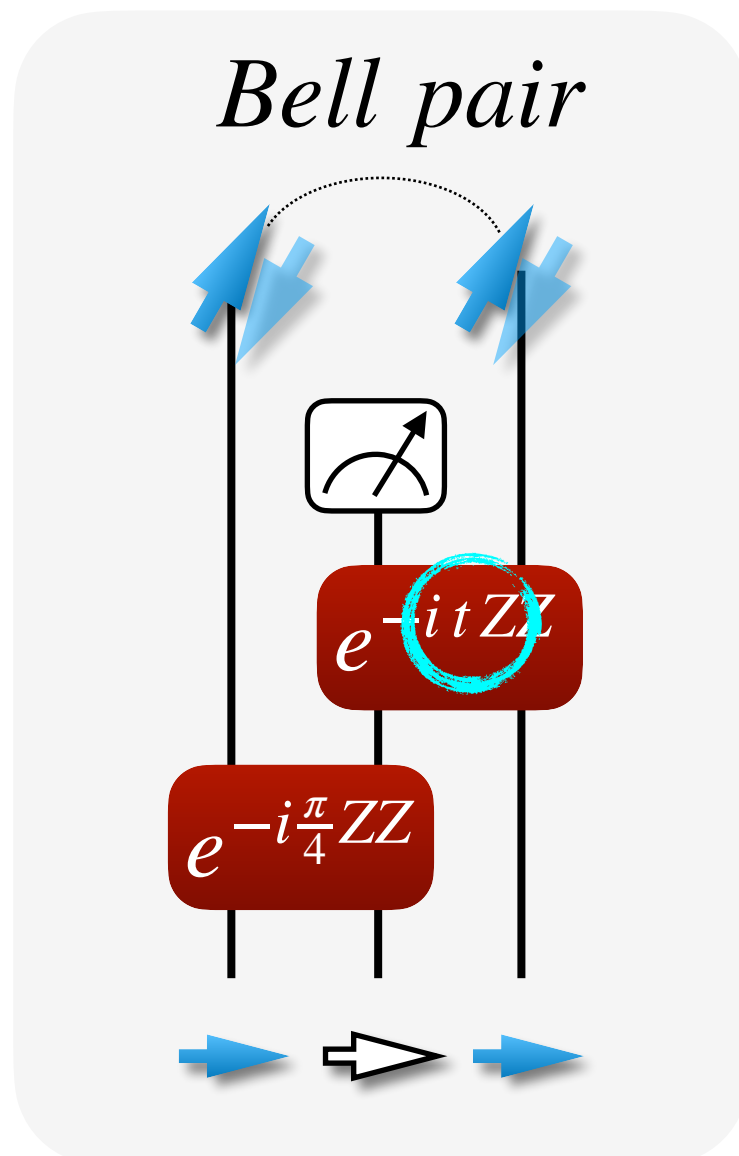
**Nishimori's cat**



# Nishimori's cat

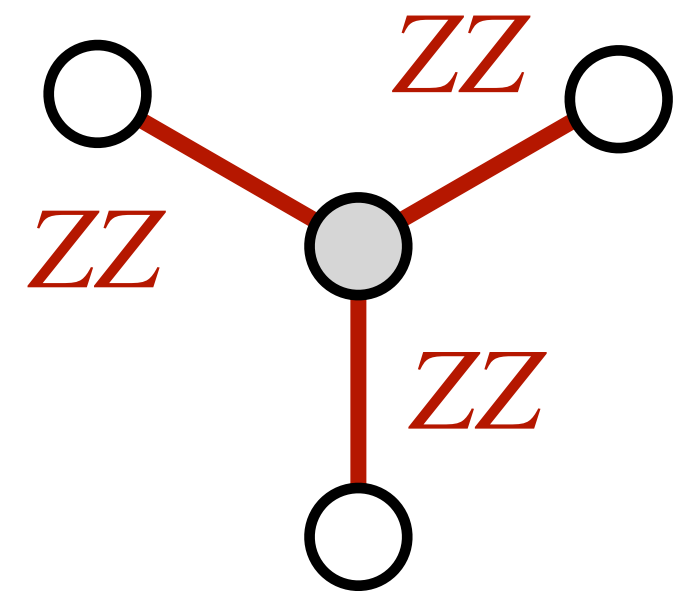


**Nishimori's cat**

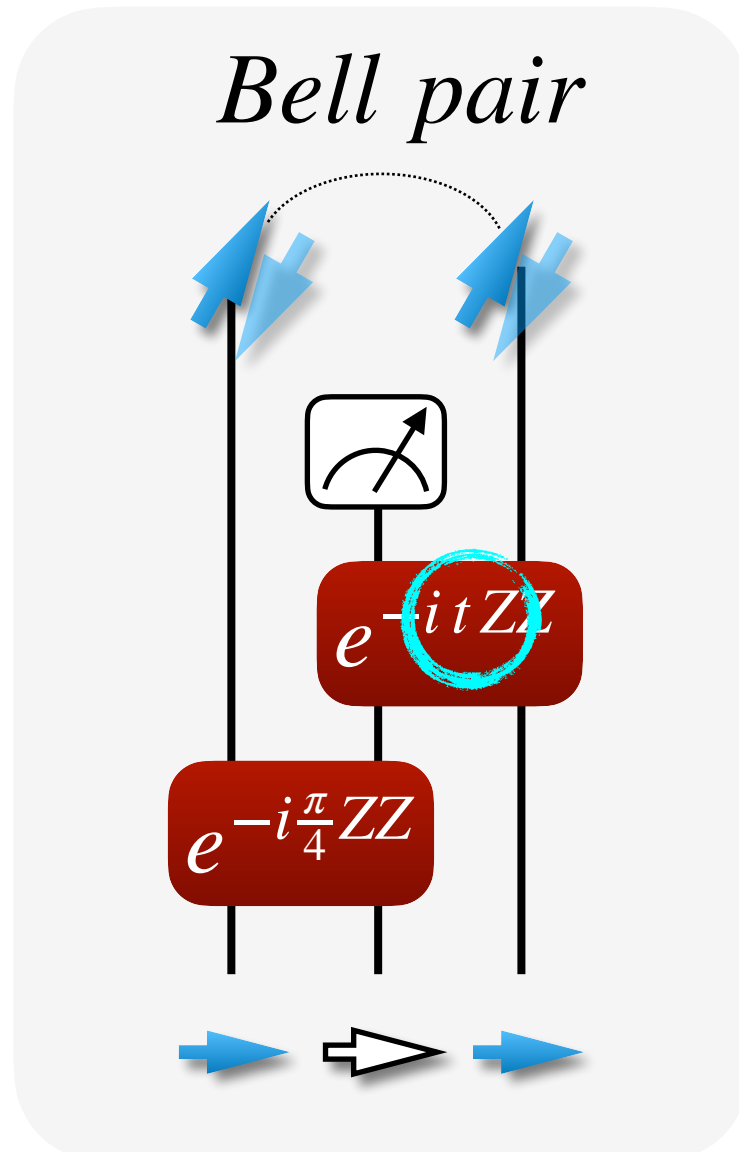
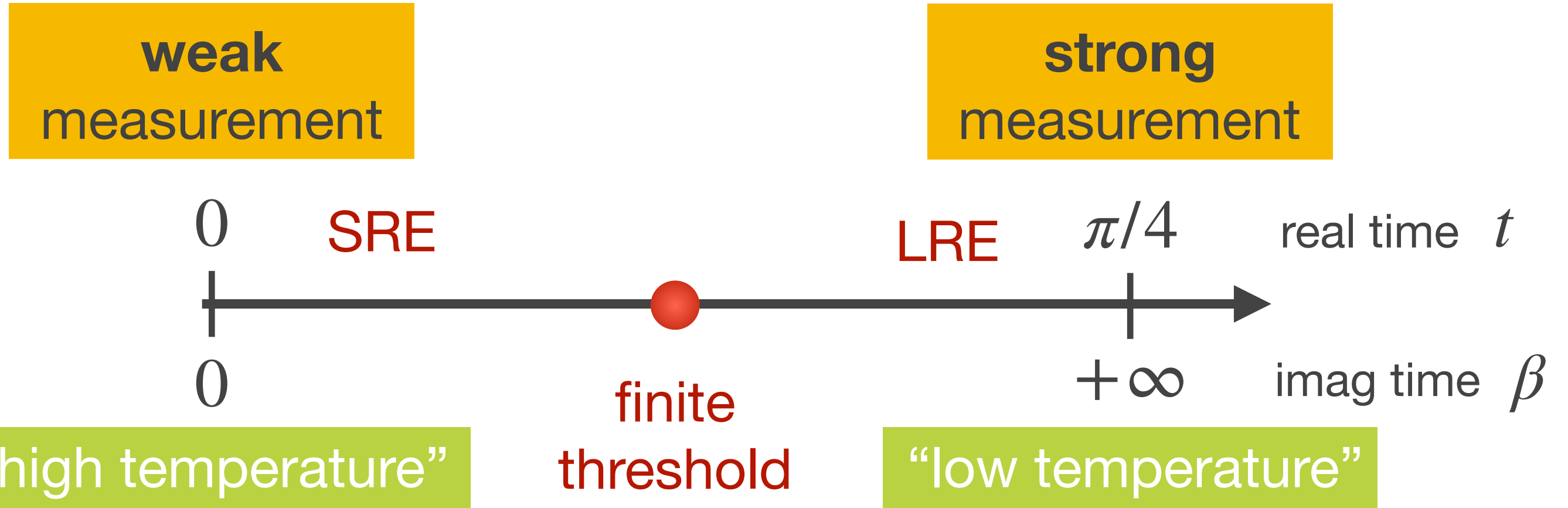




# Nishimori's cat



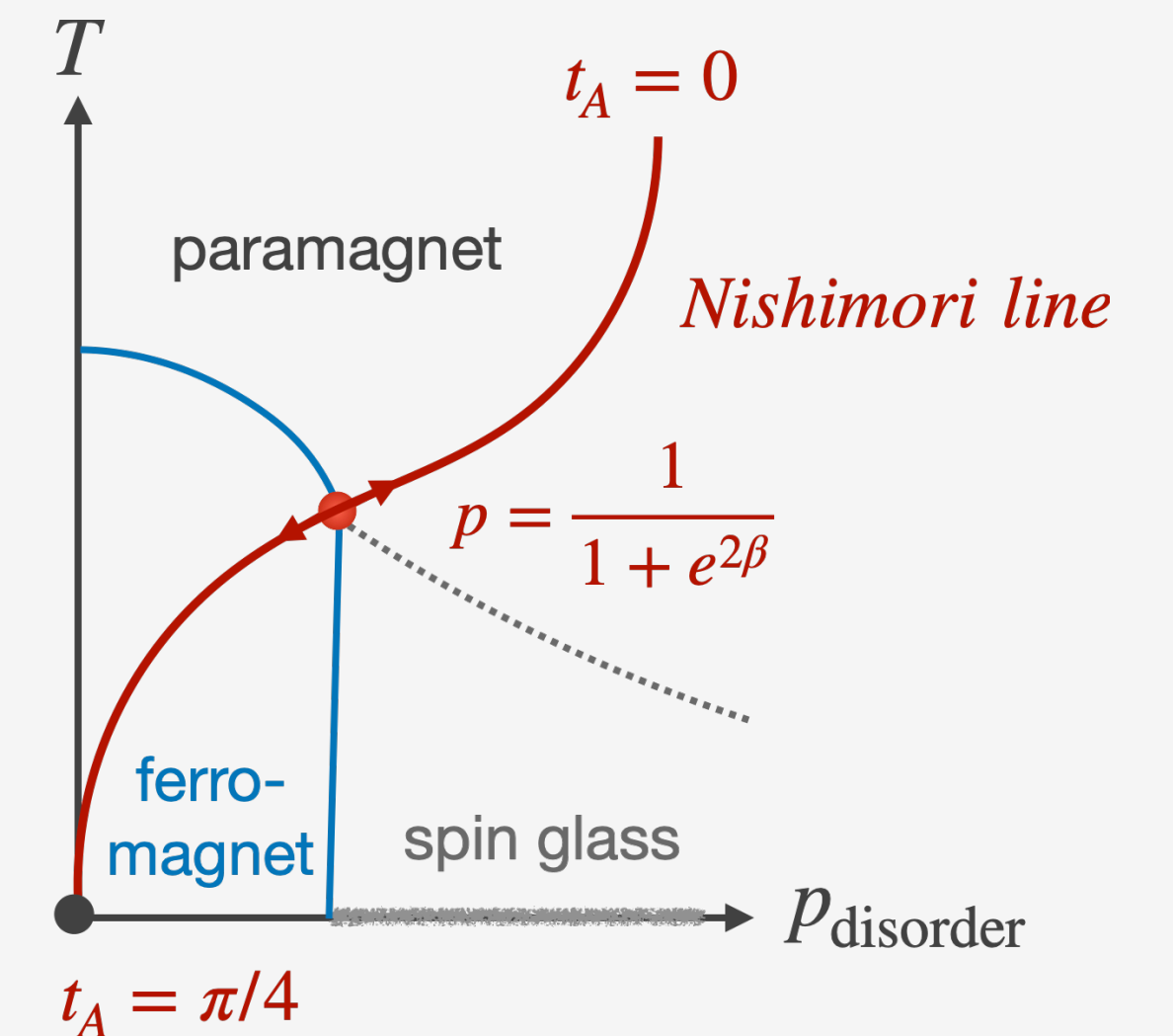
**Nishimori's cat**



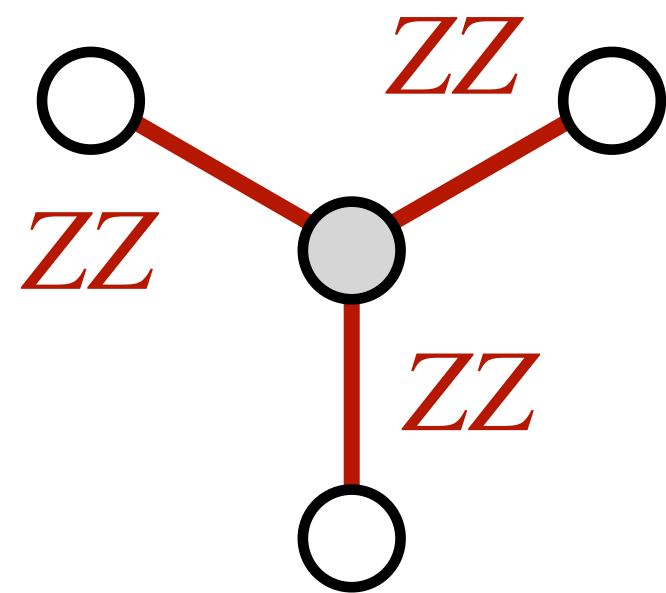
thermal fluctuations and disorder are **locked**

Nishimori (1981)

theory – Phys. Rev. Lett. 131, 200201 (2023)  
 experiment (IBM) – arXiv:2309.02863 (2023)

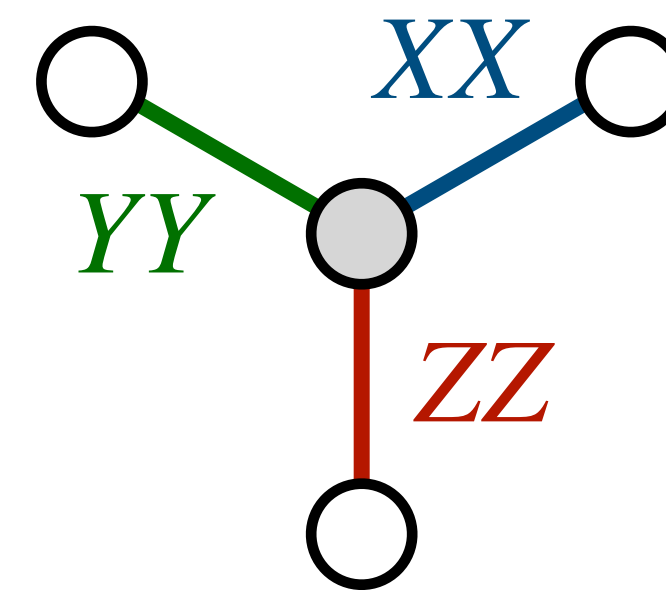


# commuting vs non-commuting measurements



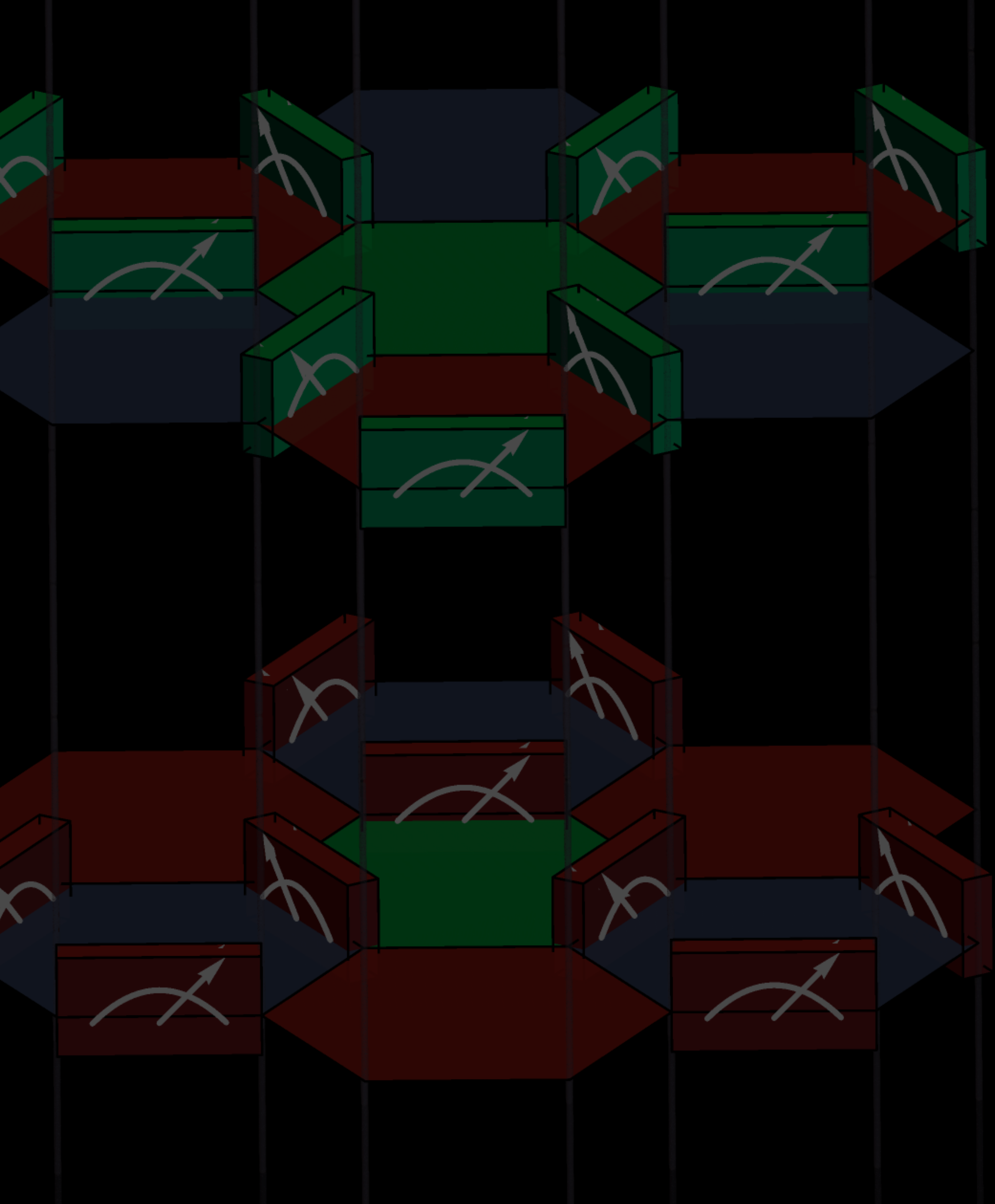
## Nishimori's cat

- commuting
- parallelized
- no dynamics



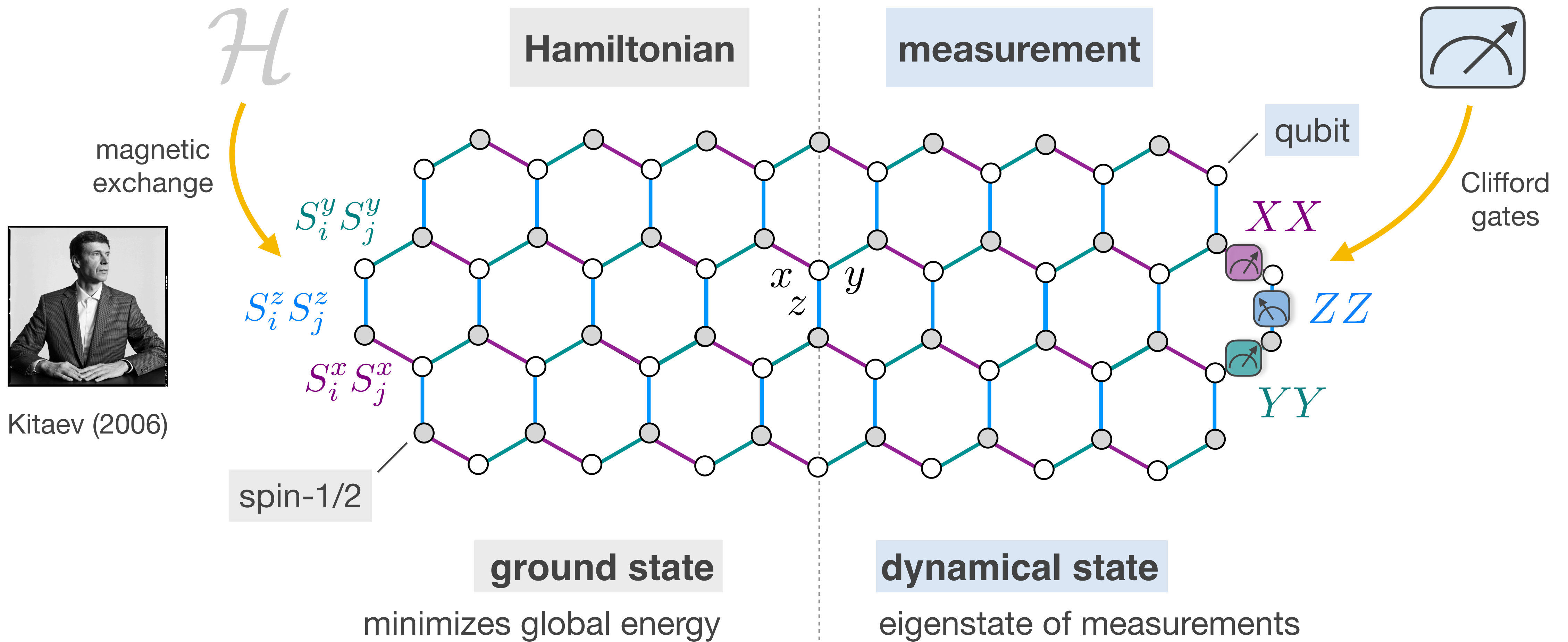
## Kitaev spin liquid

- non-commuting
- sequential
- dynamics

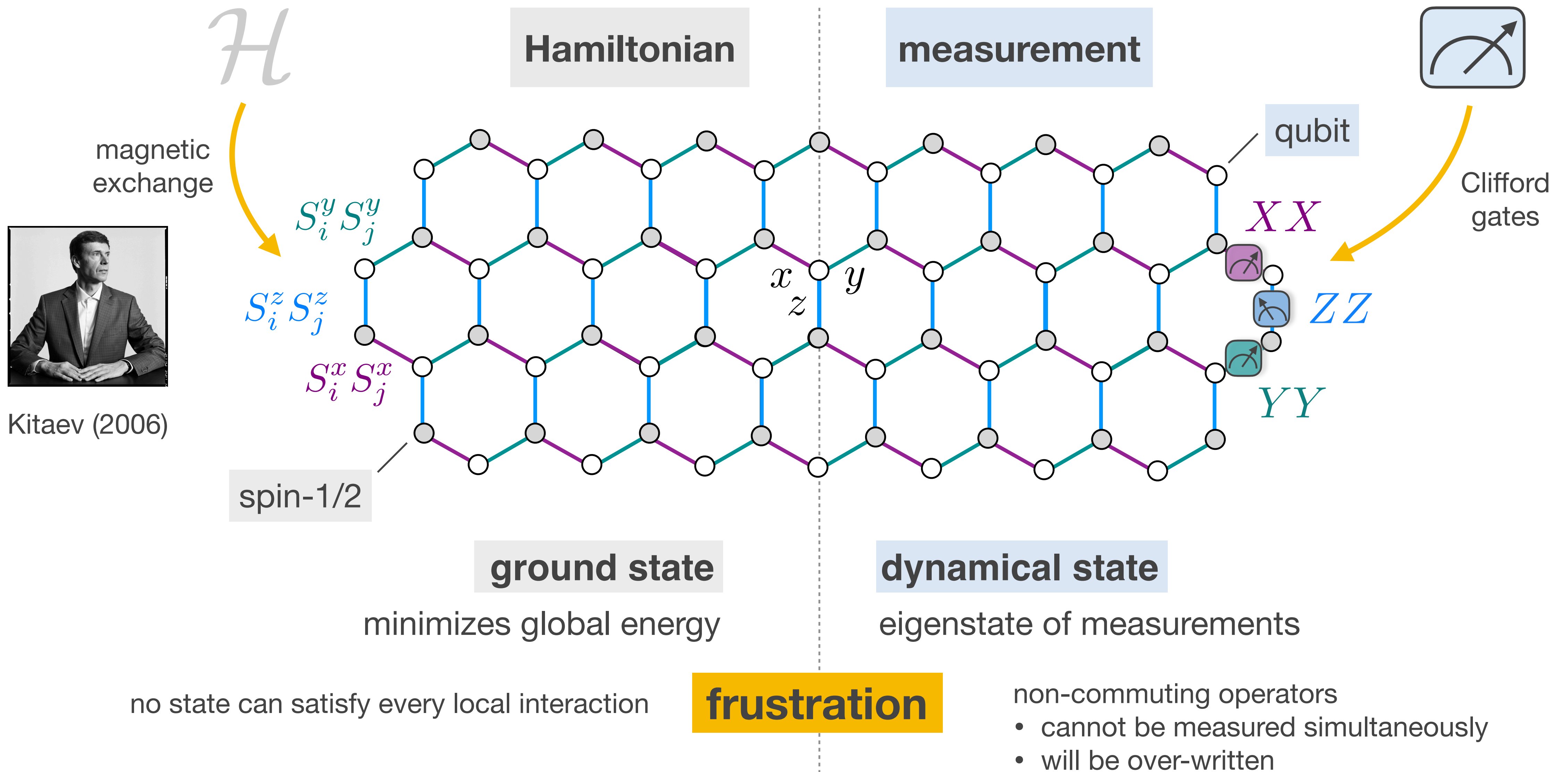


**Floquet  
codes**

# frustration and entanglement



# frustration and entanglement



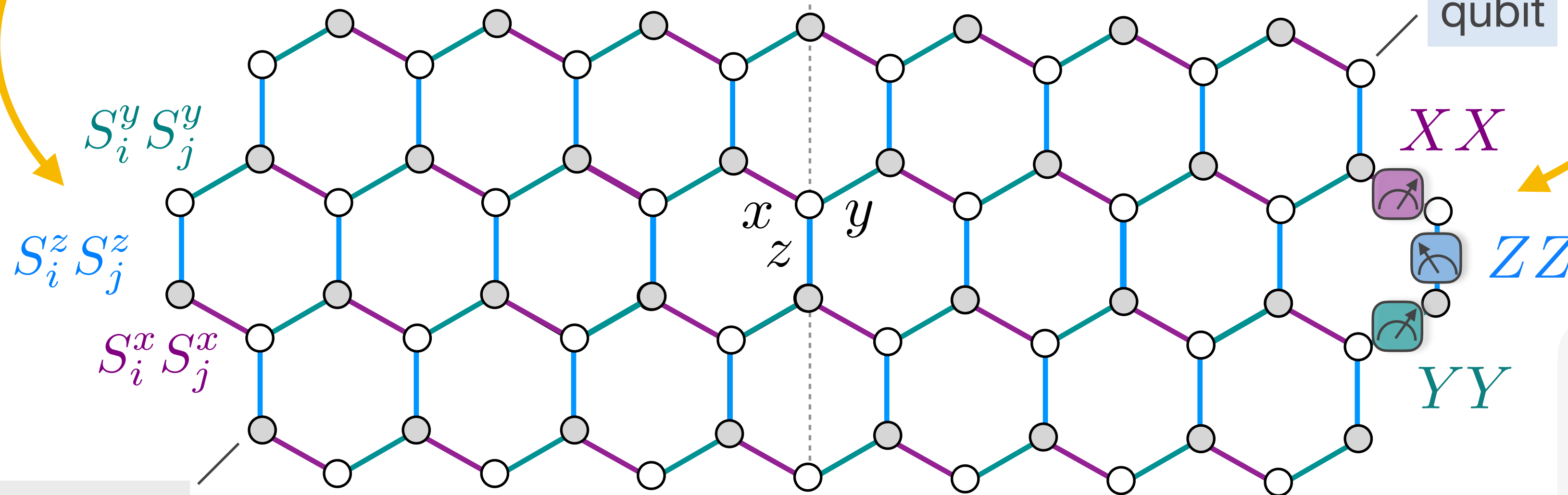
# frustration and entanglement



Kitaev (2006)

$\mathcal{H}$

magnetic exchange



spin-1/2

**Hamiltonian**

**measurement**

**ground state**

**dynamical state**

minimizes global energy

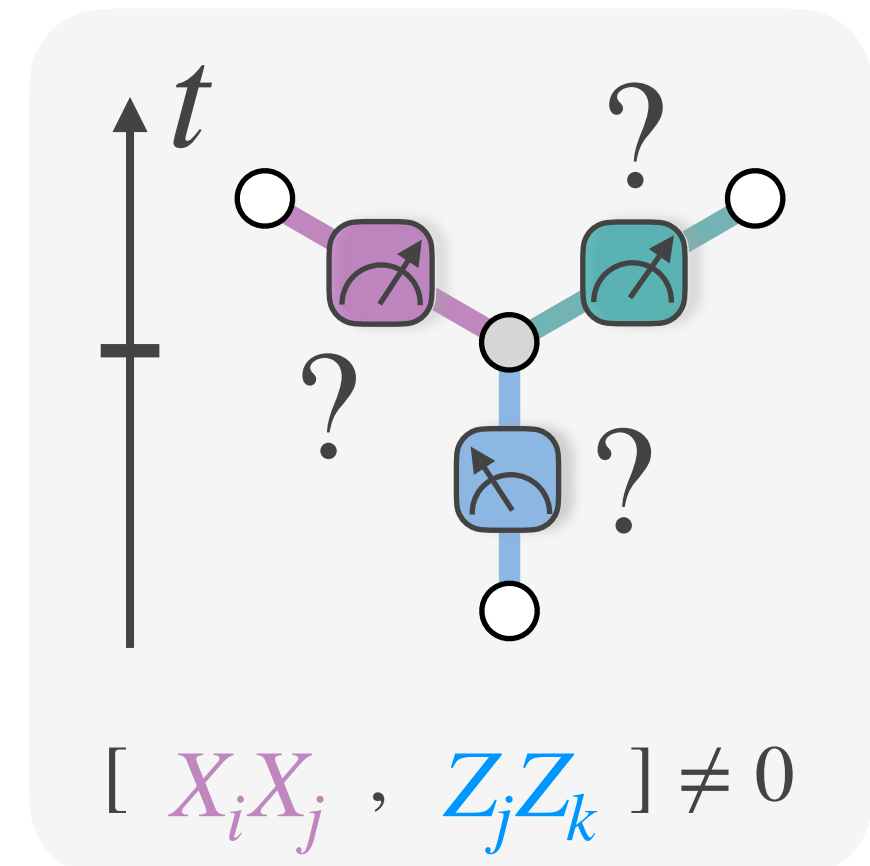
eigenstate of measurements

no state can satisfy every local interaction

**frustration**

non-commuting operators

- cannot be measured simultaneously
- will be over-written

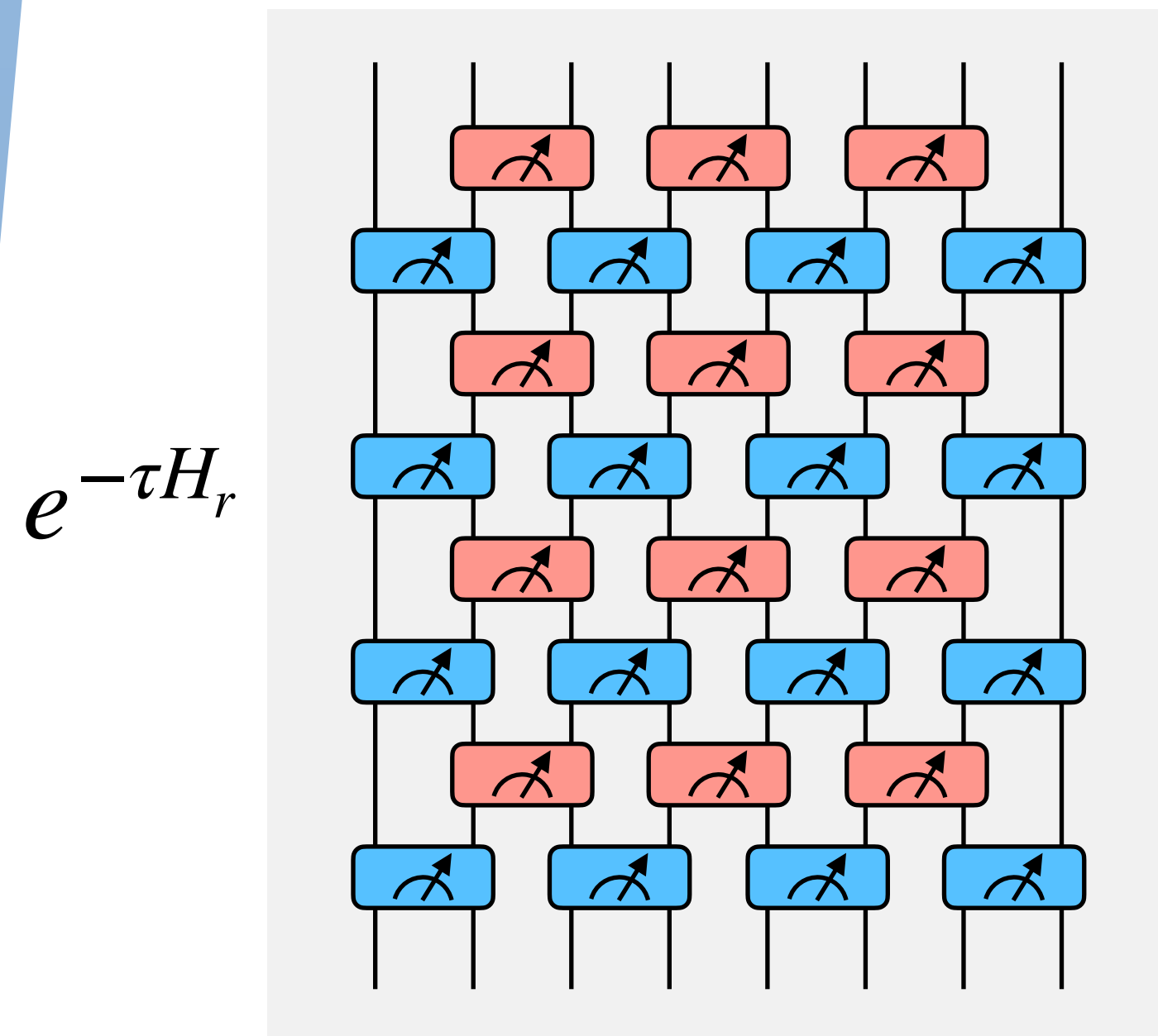


$$[X_i X_j, Z_j Z_k] \neq 0$$

# imaginary time vs. measurement-only

## Hamiltonian ground state

$$e^{-\beta H} |\psi_0\rangle$$



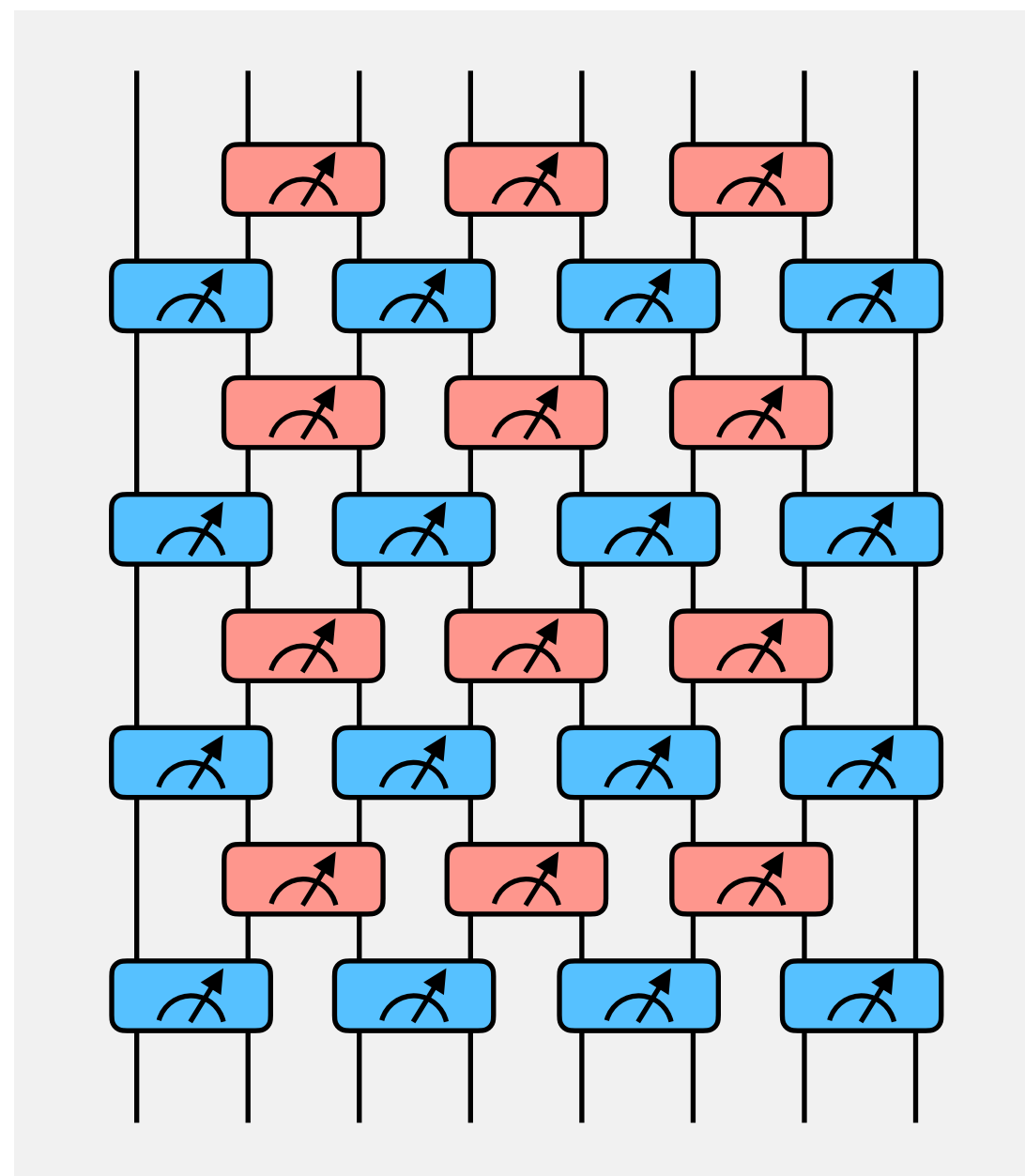
- brickwall circuit
- no disorder
- $\tau \ll 1$

# imaginary time vs. measurement-only

## Hamiltonian ground state

$$e^{-\beta H} |\psi_0\rangle$$

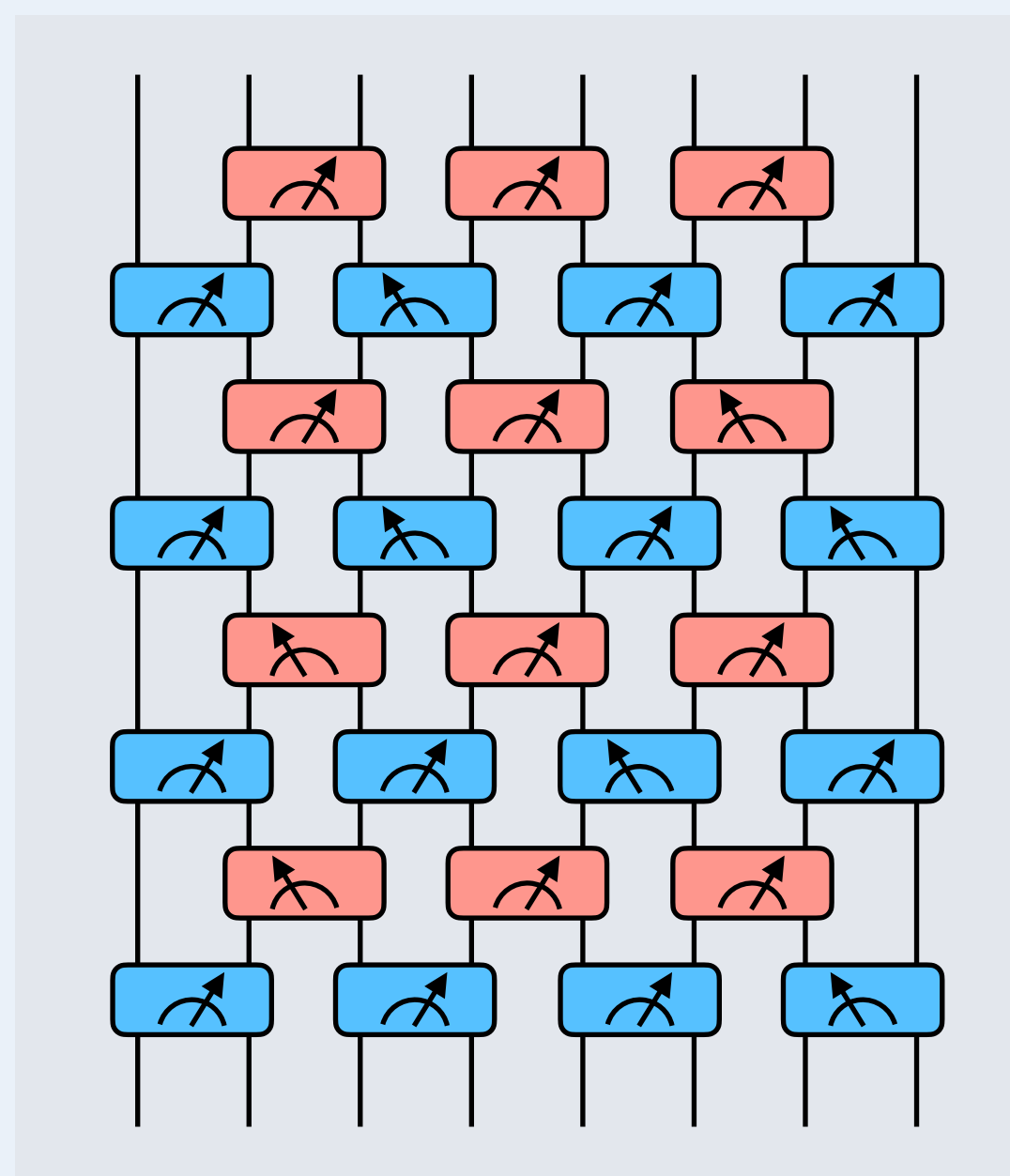
$e^{-\tau H_r}$



- brickwall circuit
- no disorder
- $\tau \ll 1$

## Floquet weak measurement

$$(e^{\mp\tau H_r} \dots e^{\mp\tau H_0}) |\psi_0\rangle$$



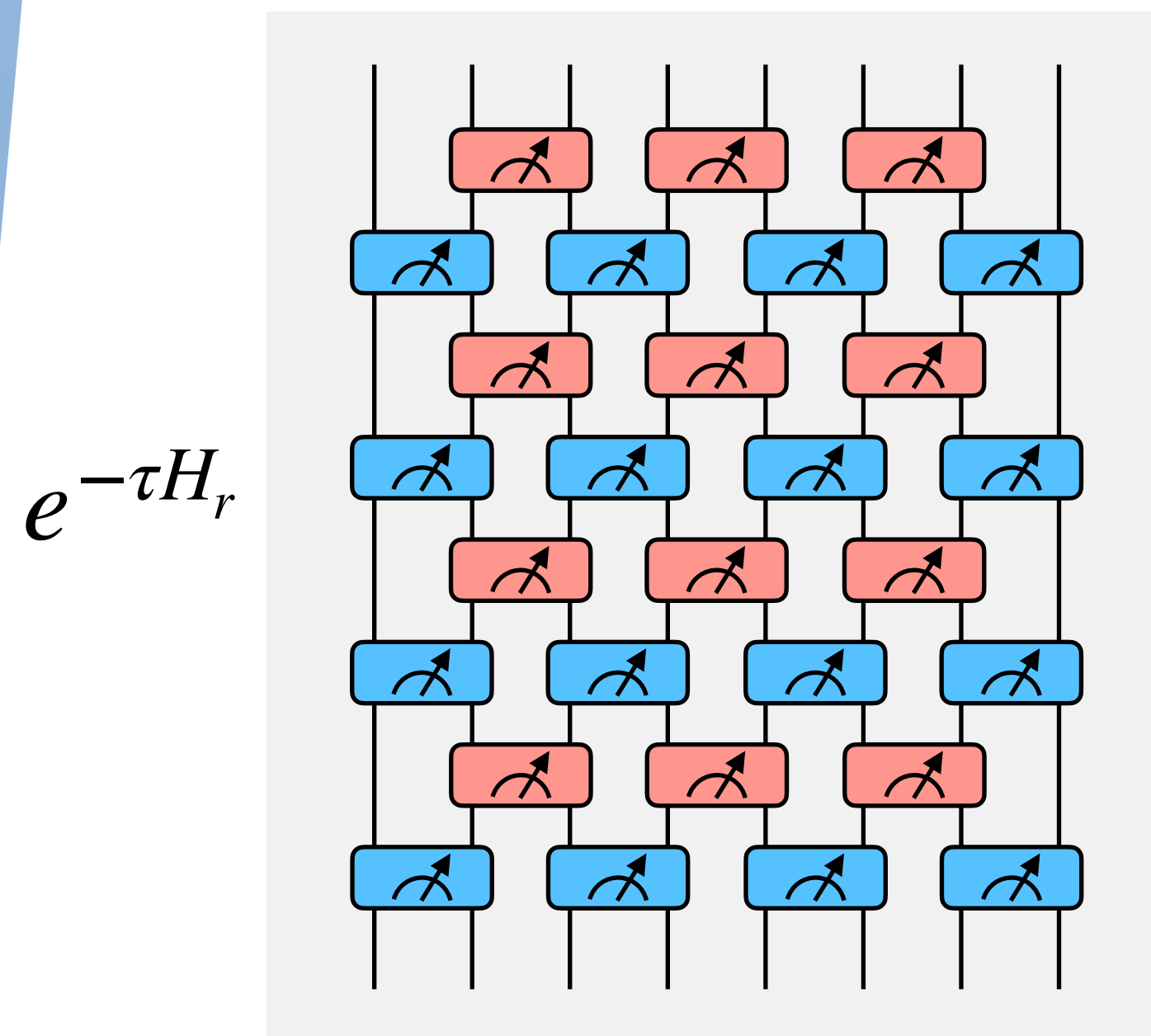
- brickwall circuit
- **Born** disorder
- $\tau \in [0, +\infty)$



# imaginary time vs. measurement-only

## Hamiltonian ground state

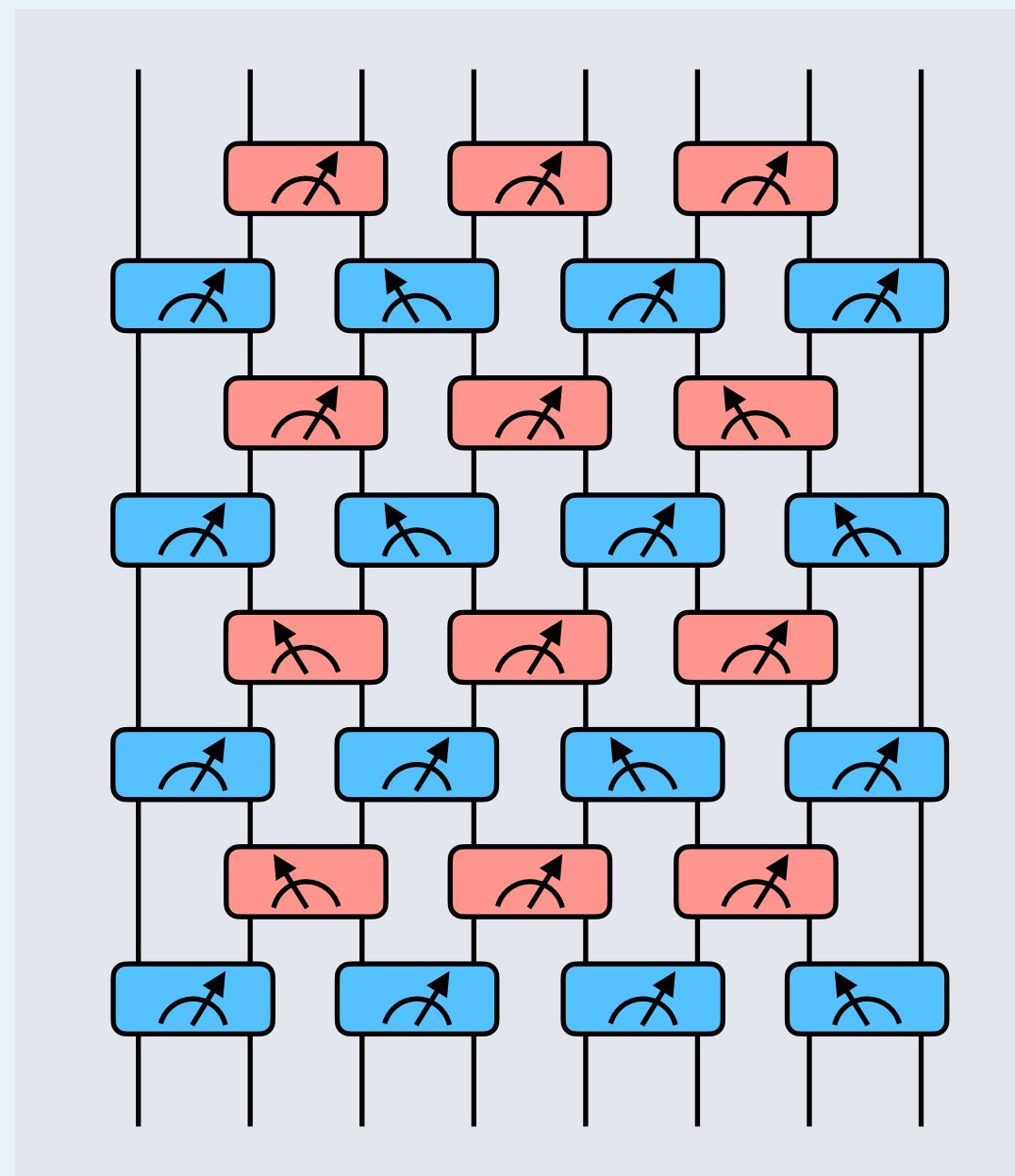
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- brickwall circuit
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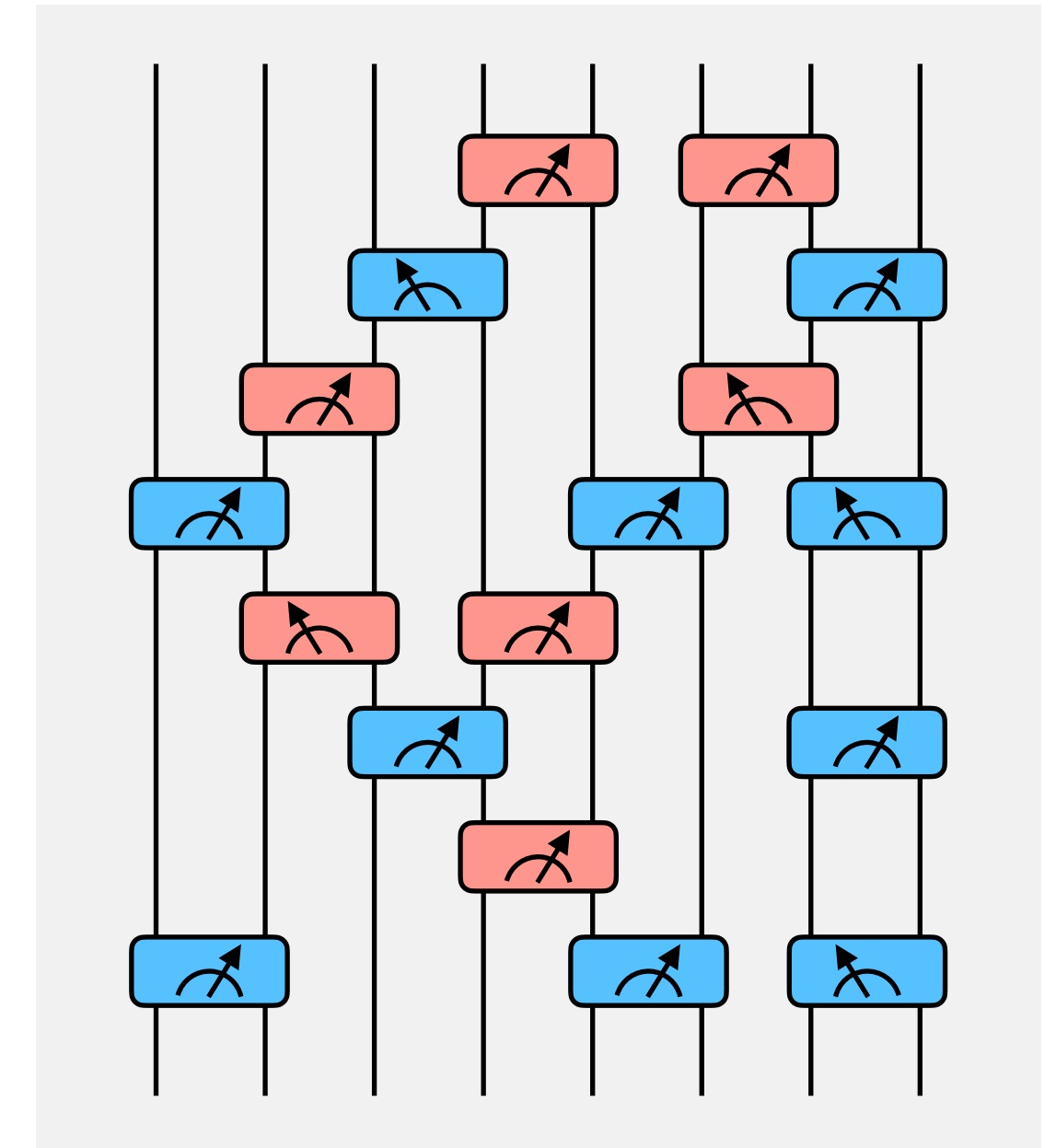
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$$(e^{\mp\tau H_r} \dots e^{\mp\tau H_0}) |\psi_0\rangle$$



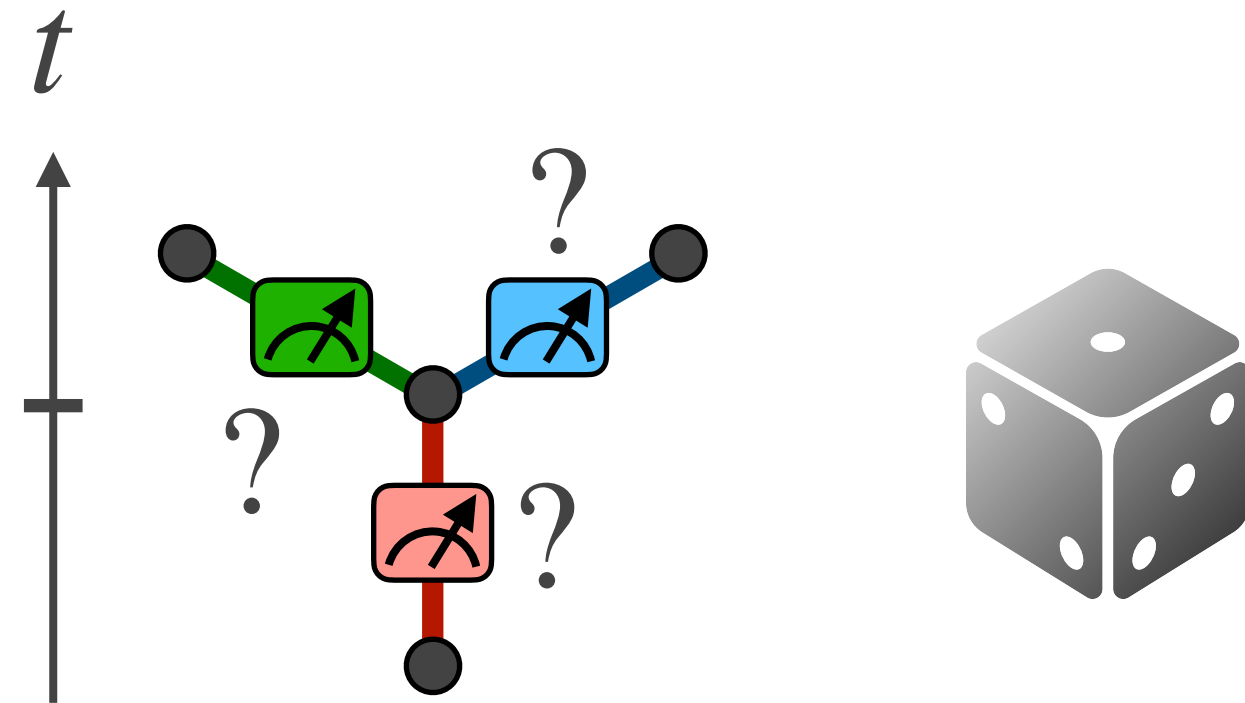
- brickwall circuit
- **Born** disorder
- $\tau \in [0, +\infty)$

## random weak/strong measurement

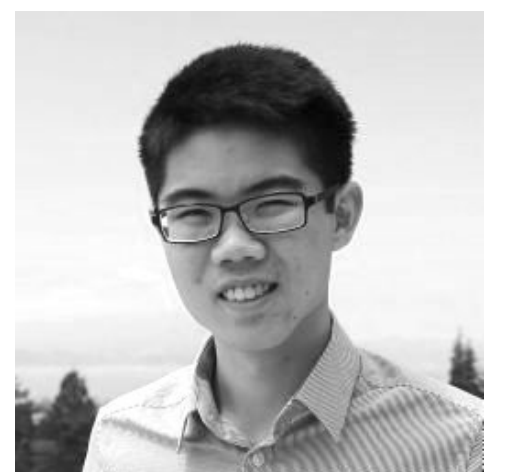


- **stochastic** circuit
- **Born** disorder
- $\tau \in [0, +\infty)$

# random projective Kitaev measurements



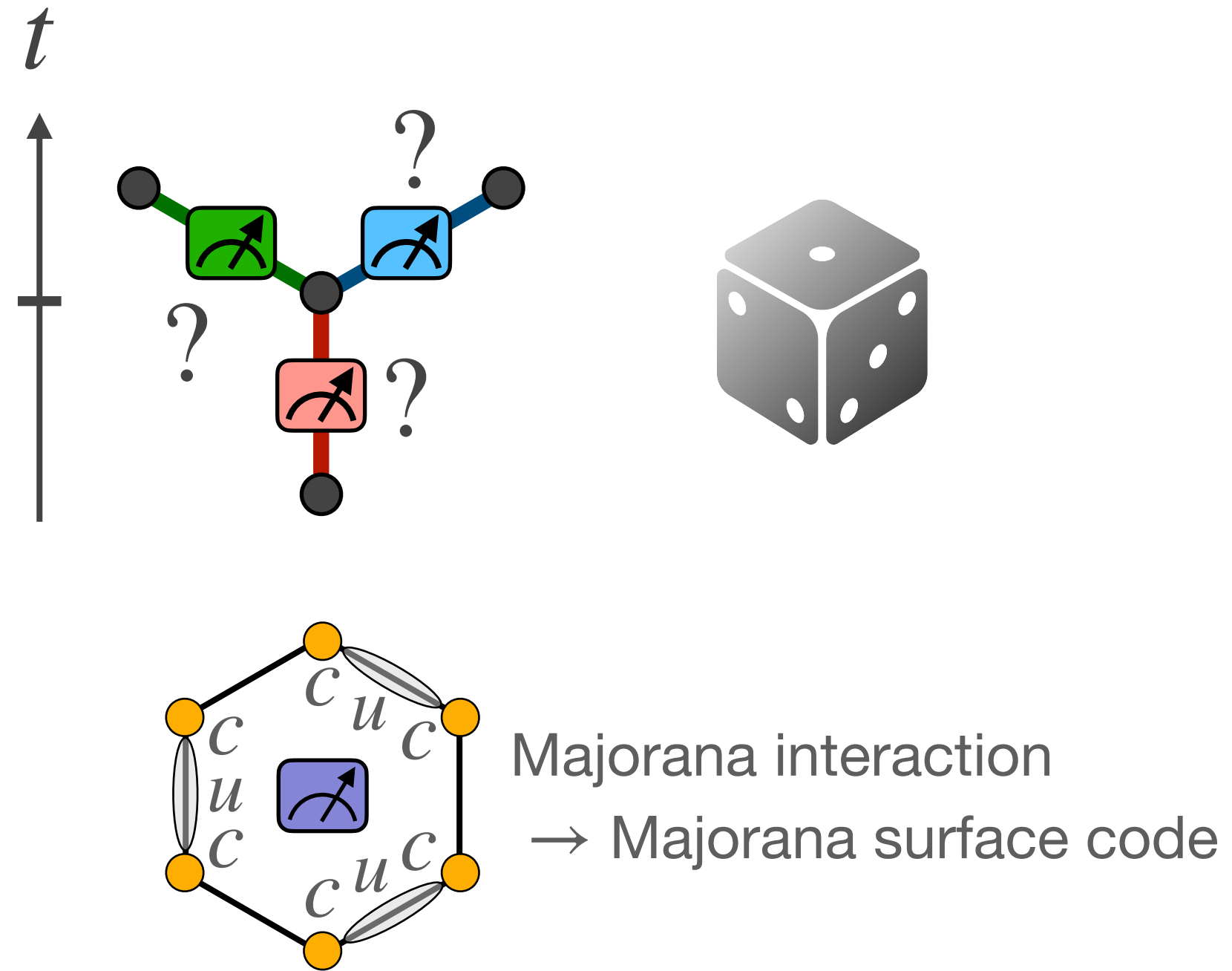
Guo-Yi Zhu



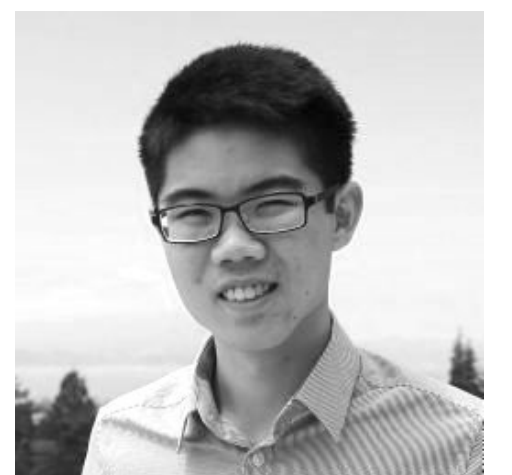
Nathanan  
Tantivasadakarn

Nahum, Skinner 2020; Lavasani, Luo, Vijay 2022; Sriram, Rakovszky, Khemani, Ippoliti 2022; **Zhu, Tantivasadakarn, ST 2023: + Majorana interaction**

# random projective Kitaev measurements

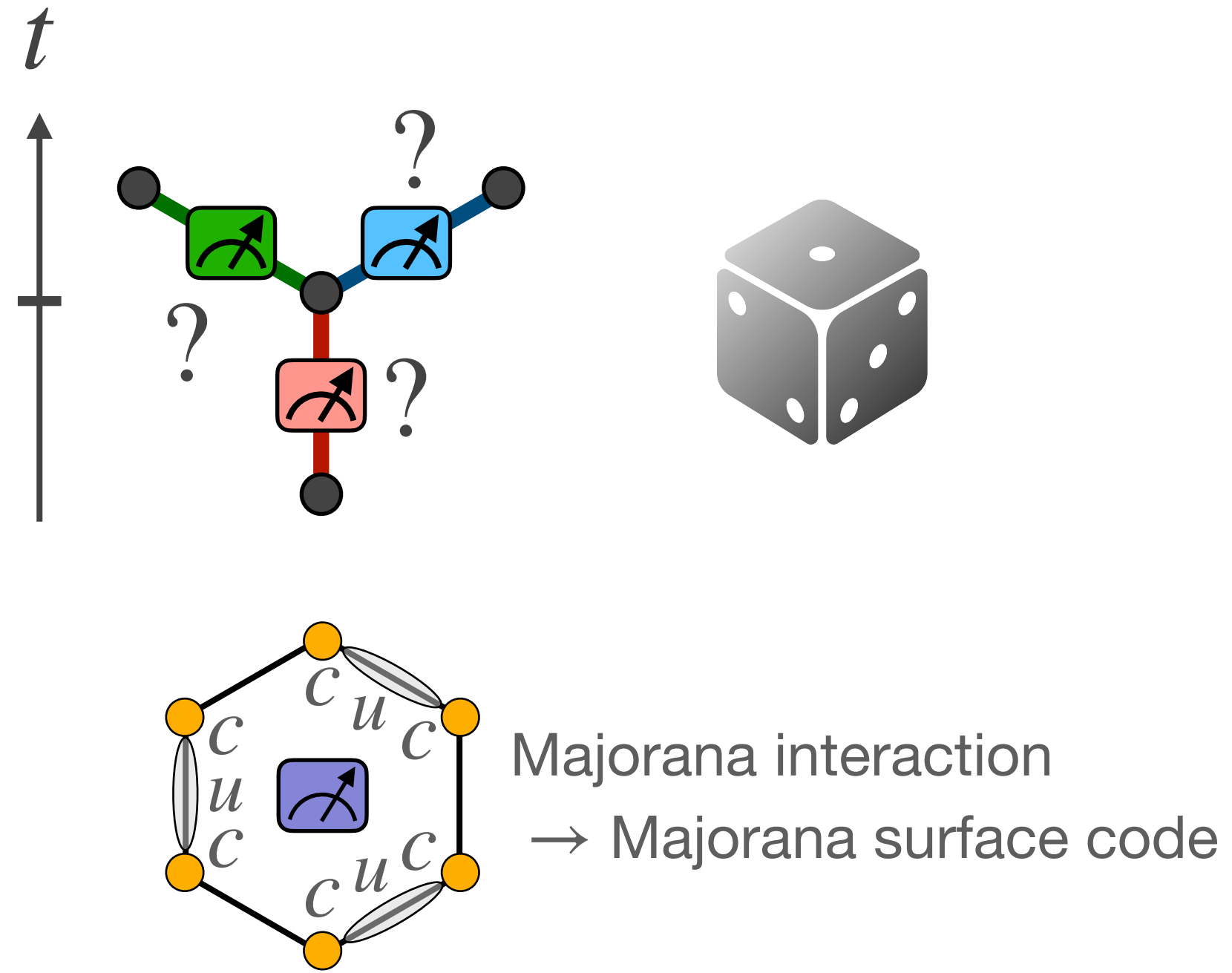


Guo-Yi Zhu

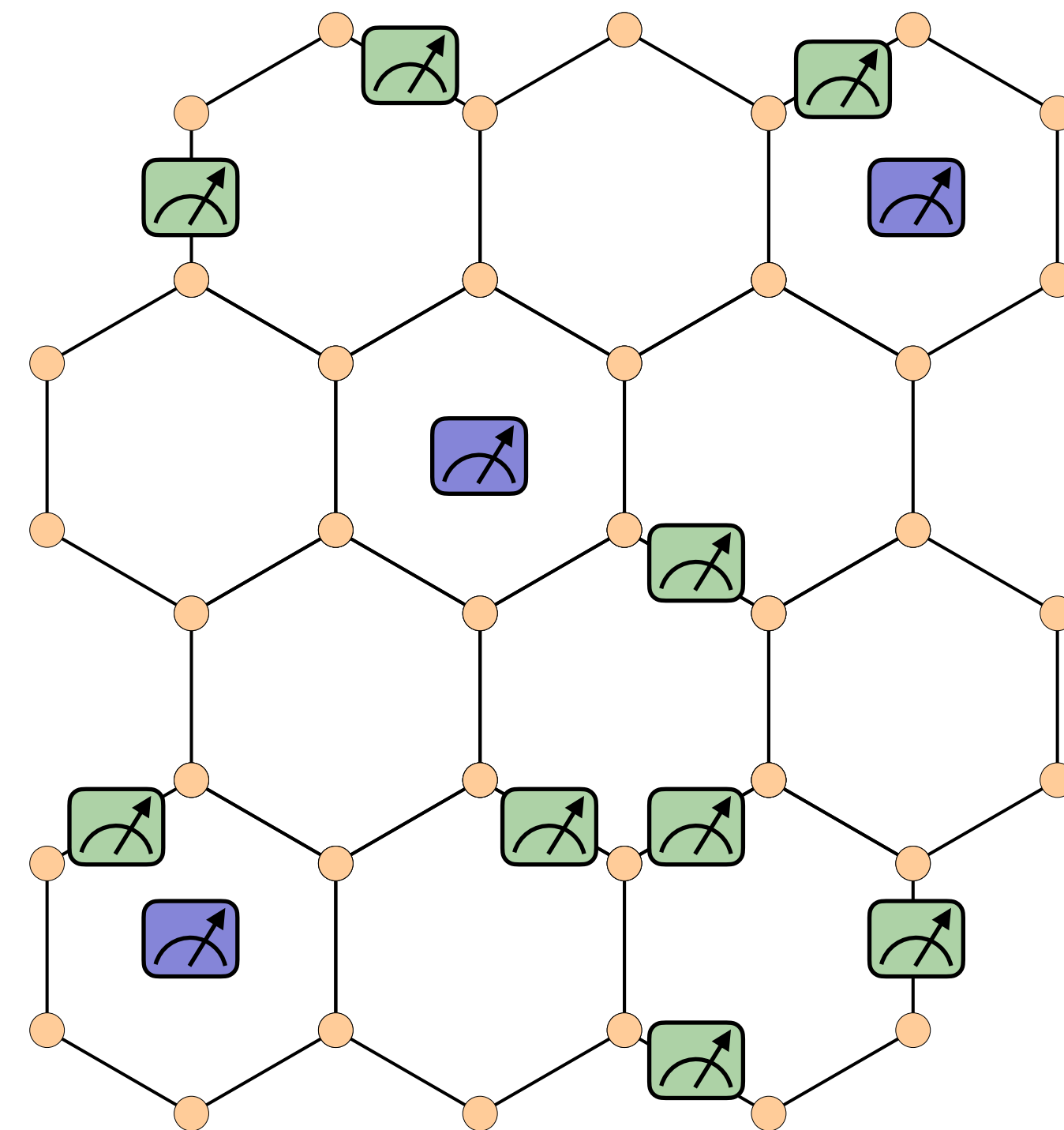


Nathanan  
Tantivasadakarn

# random projective Kitaev measurements



a snapshot: randomly chosen measurements

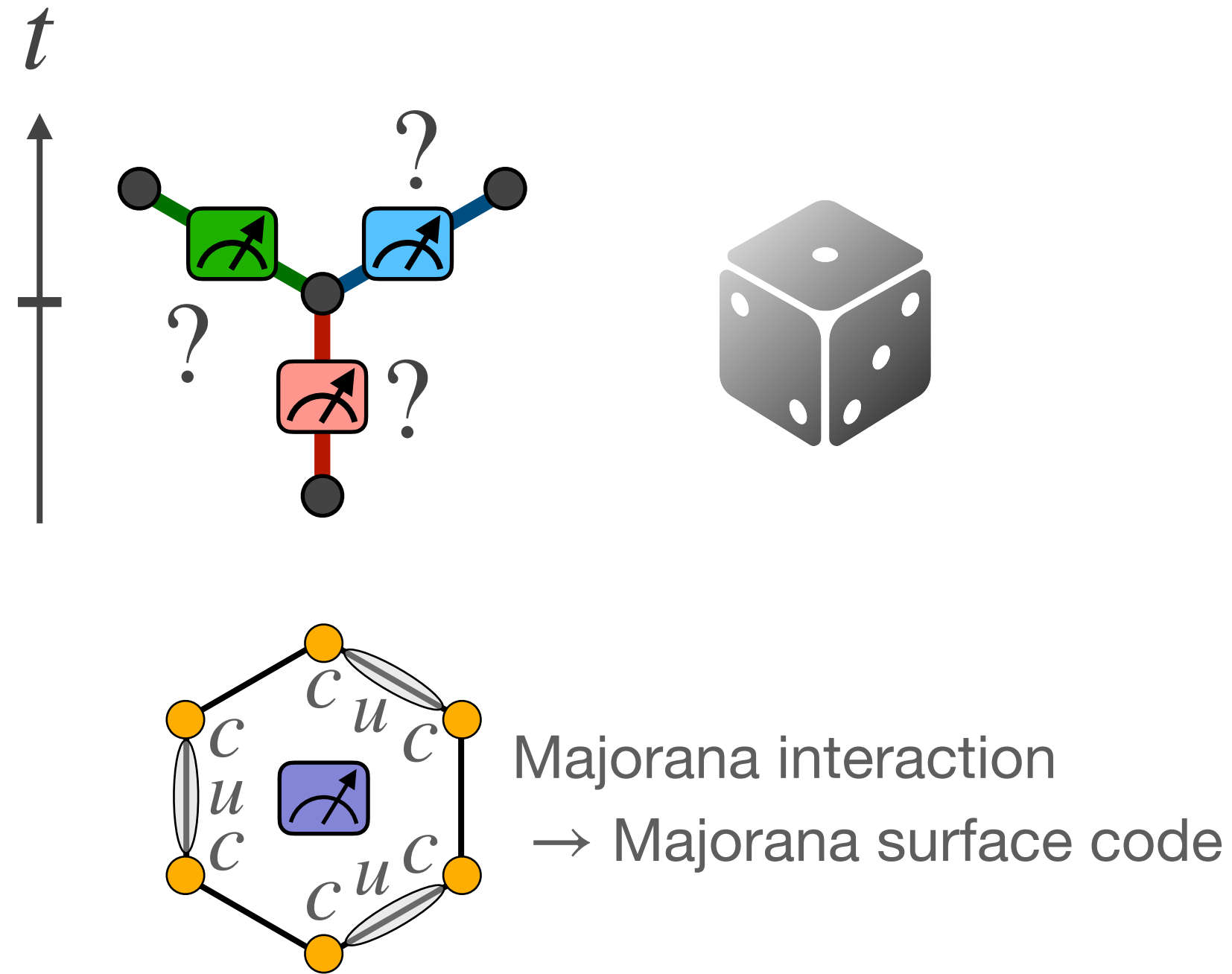


Guo-Yi Zhu



Nathanan  
Tantivasadakarn

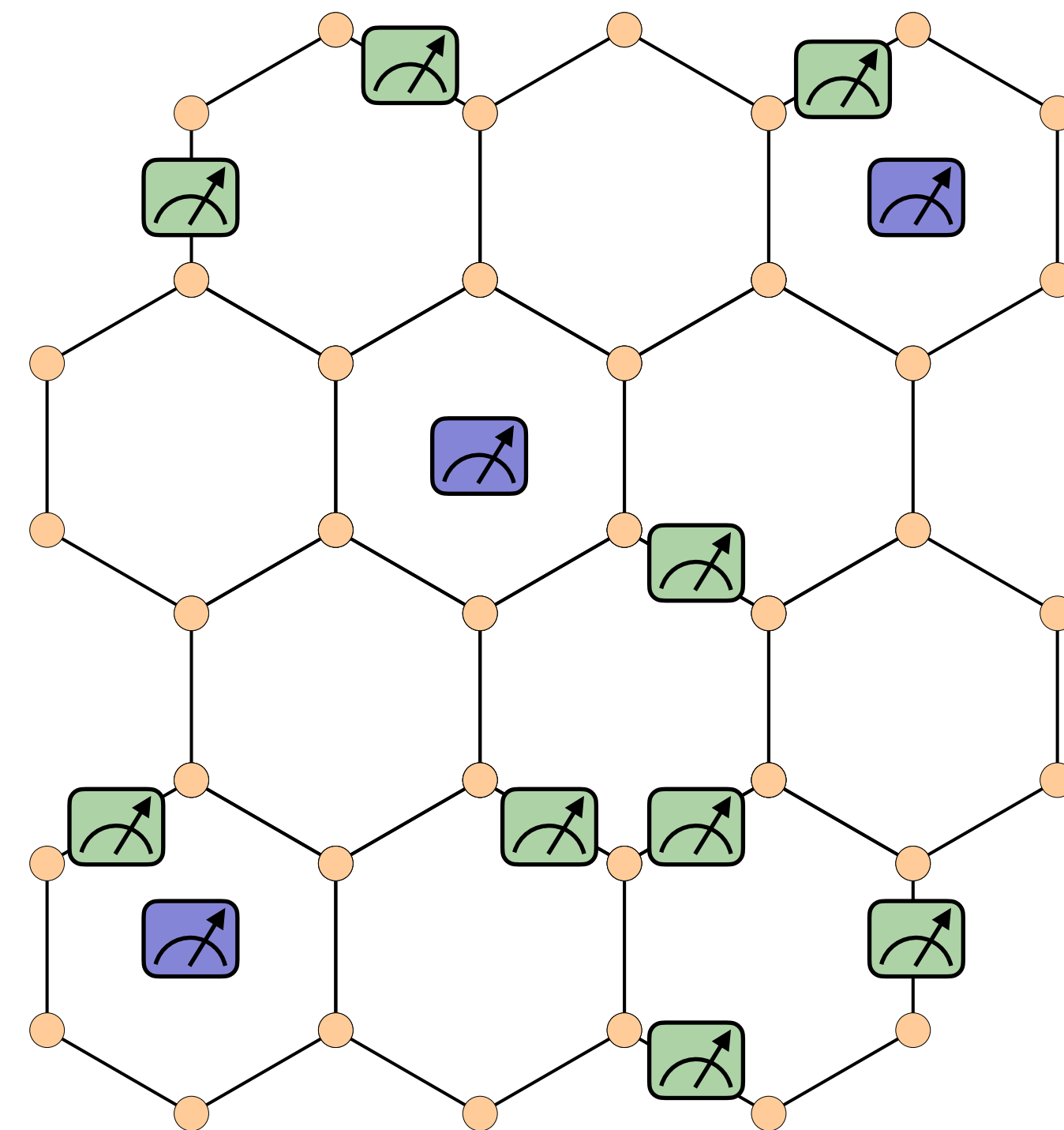
# random projective Kitaev measurements



## Clifford circuit

even **interacting** problem can be simulated in polynomial time (in Heisenberg picture)

a snapshot: randomly chosen measurements

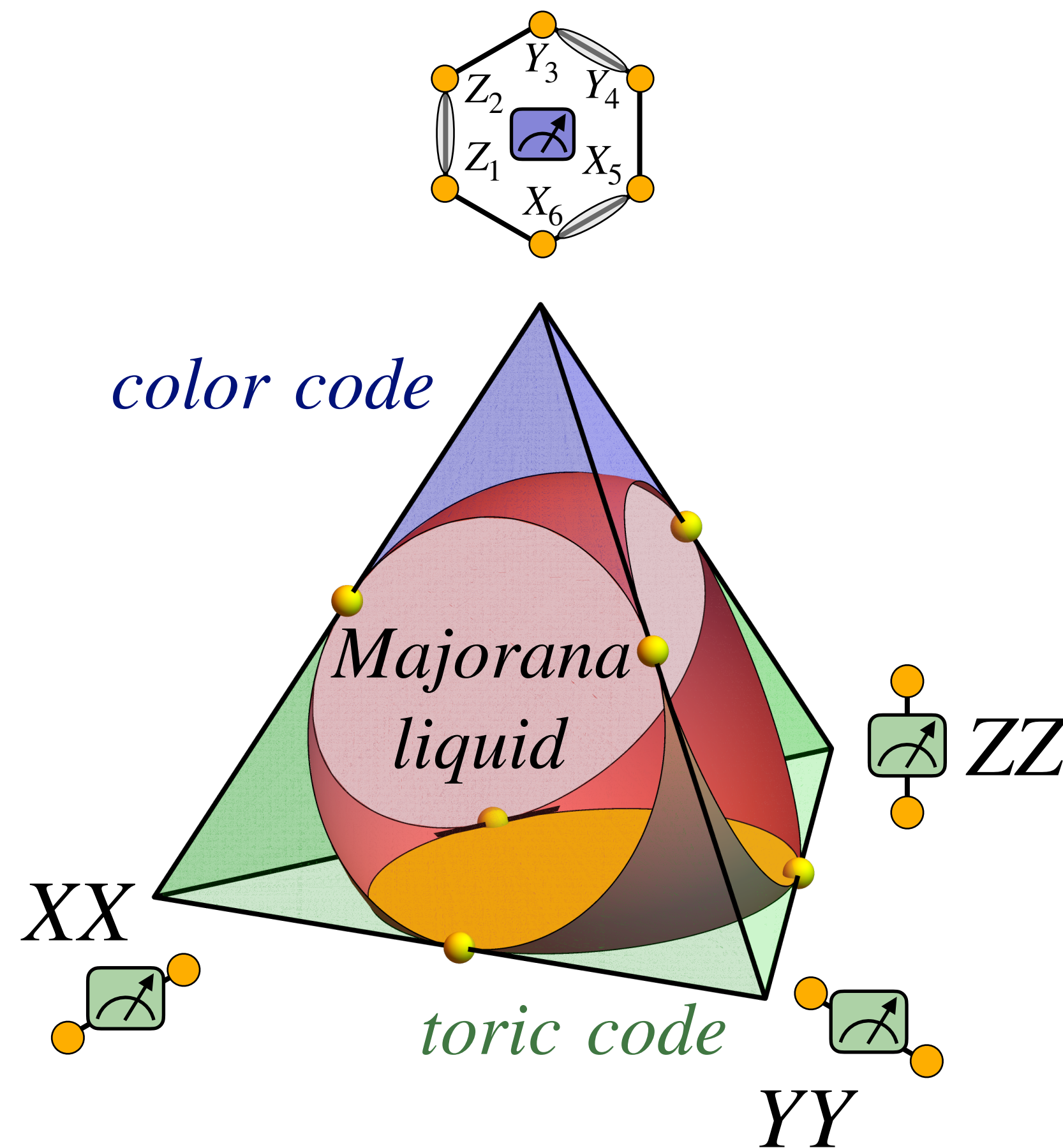


Guo-Yi Zhu



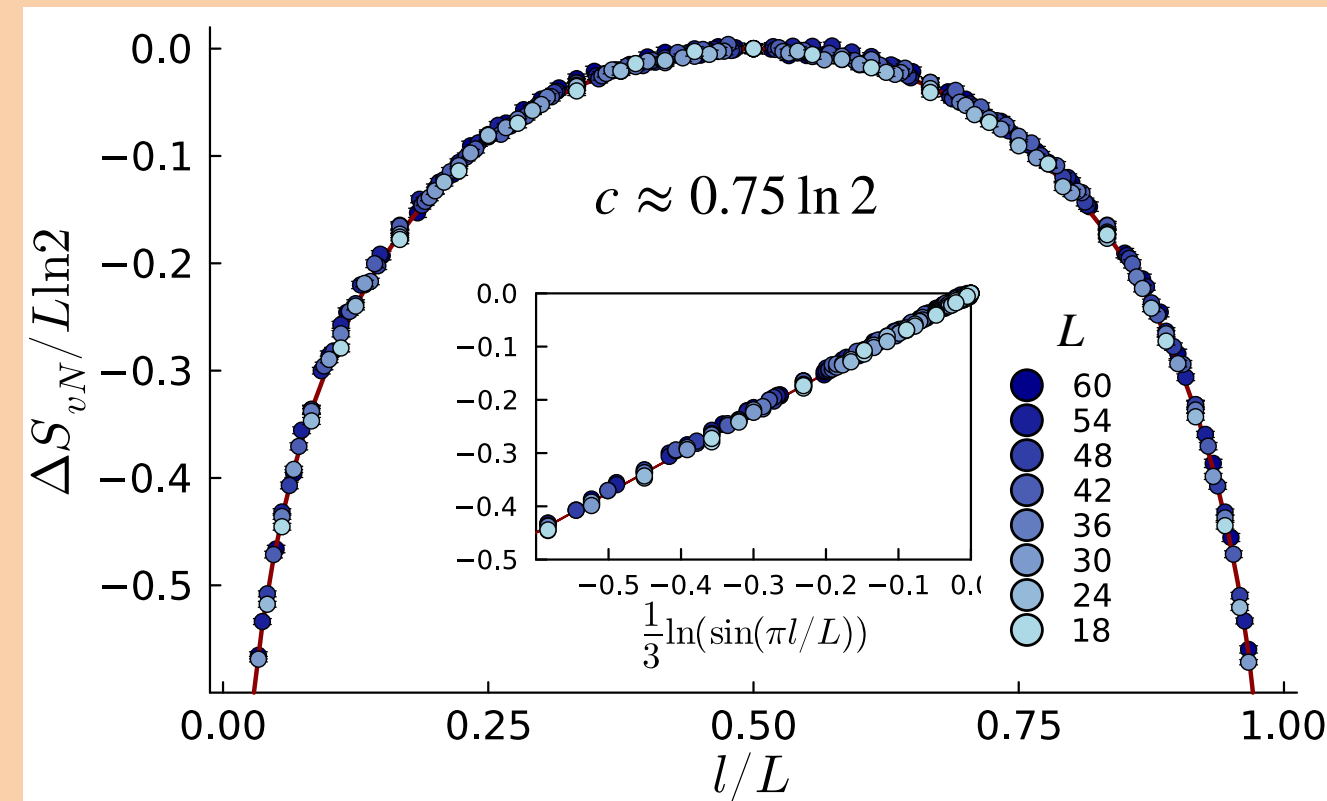
Nathanan  
Tantivasadakarn

# entanglement phase diagram



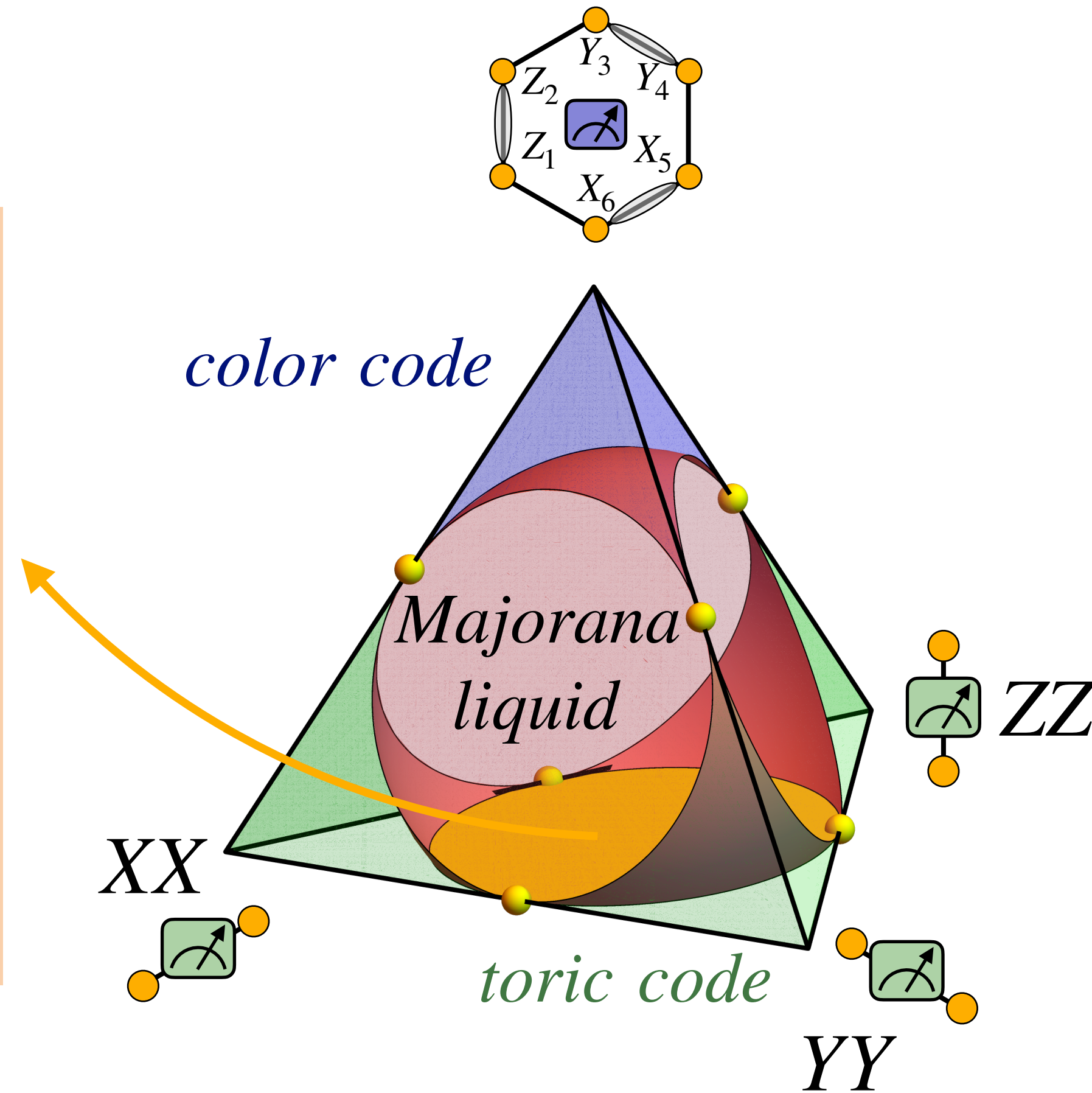
Nahum, Skinner 2020; Lavasani, Luo, Vijay 2023; Sriram, Rakovszky, Khemani, Ippoliti 2023  
Zhu, Tantivasadakarn, ST 2023: + Majorana interaction

# entanglement phase diagram



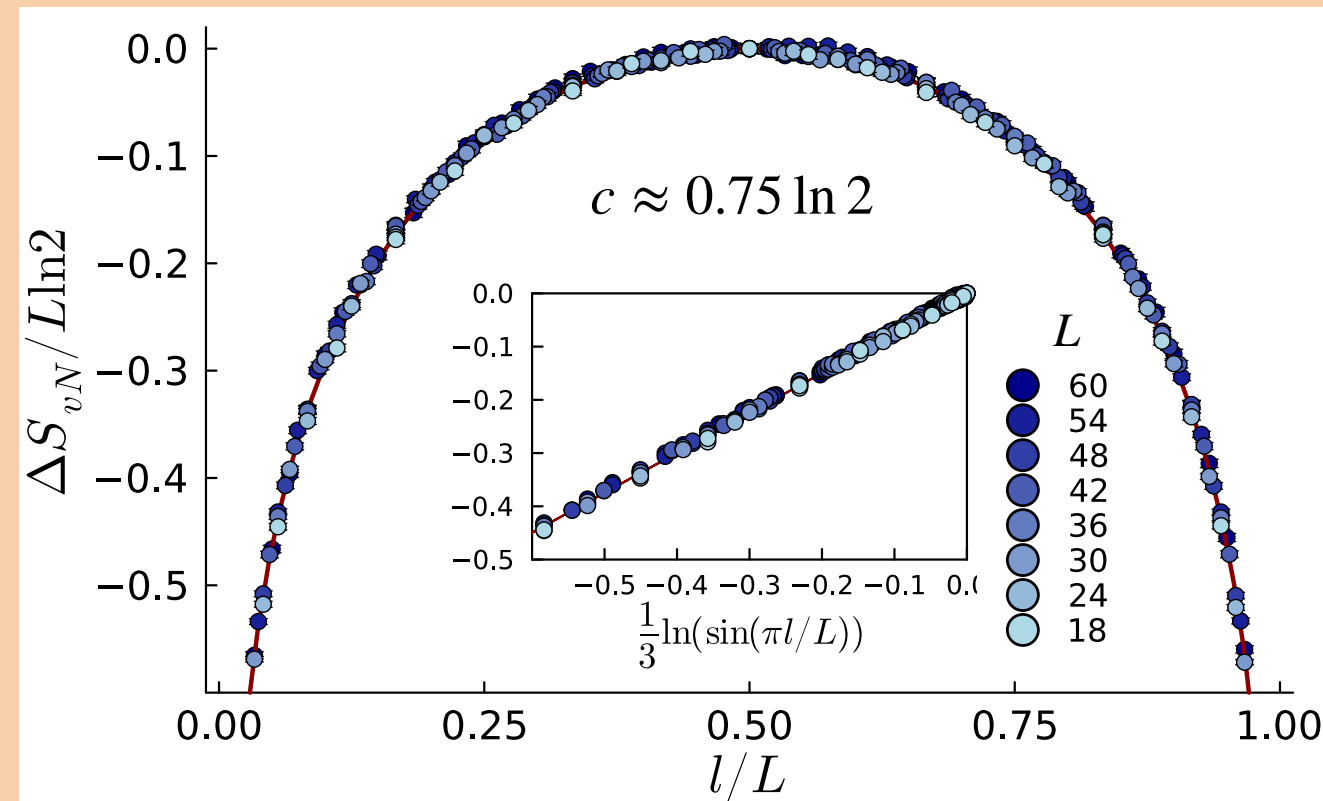
*Fermi-surface scaling*

$$S_{vN}(l, L) = \frac{cL}{3} \ln \left( \frac{L}{\pi} \sin \frac{\pi l}{L} \right) + \dots$$



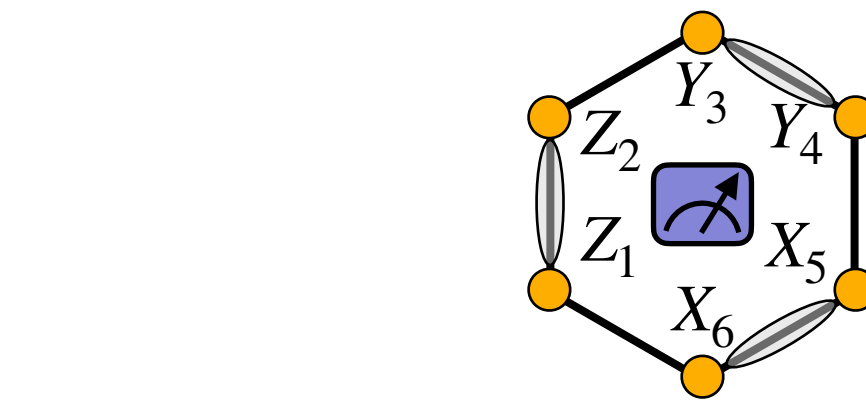
Nahum, Skinner 2020; Lavasani, Luo, Vijay 2023; Sriram, Rakovszky, Khemani, Ippoliti 2023  
**Zhu, Tantivasadakarn, ST 2023: + Majorana interaction**

# entanglement phase diagram

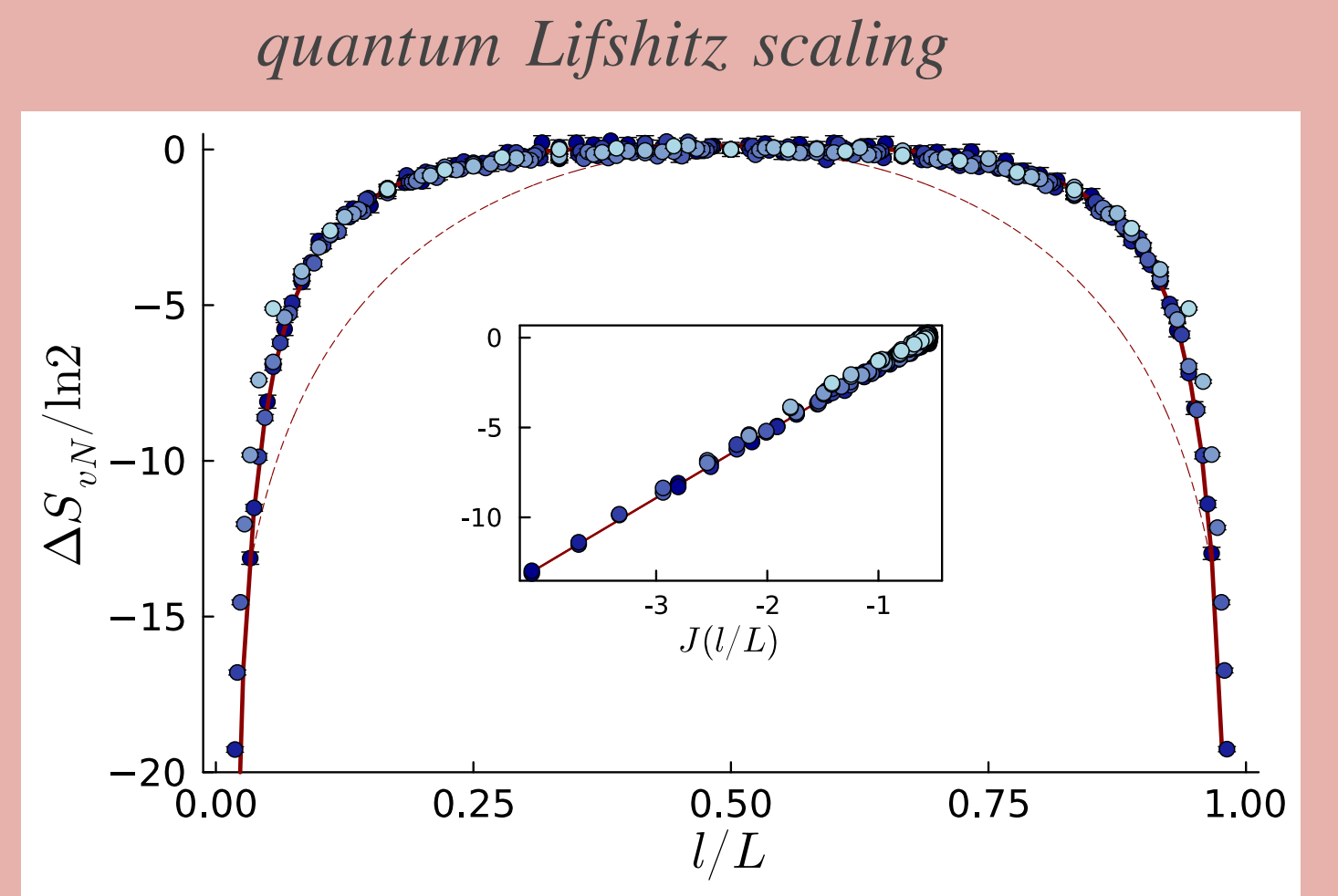
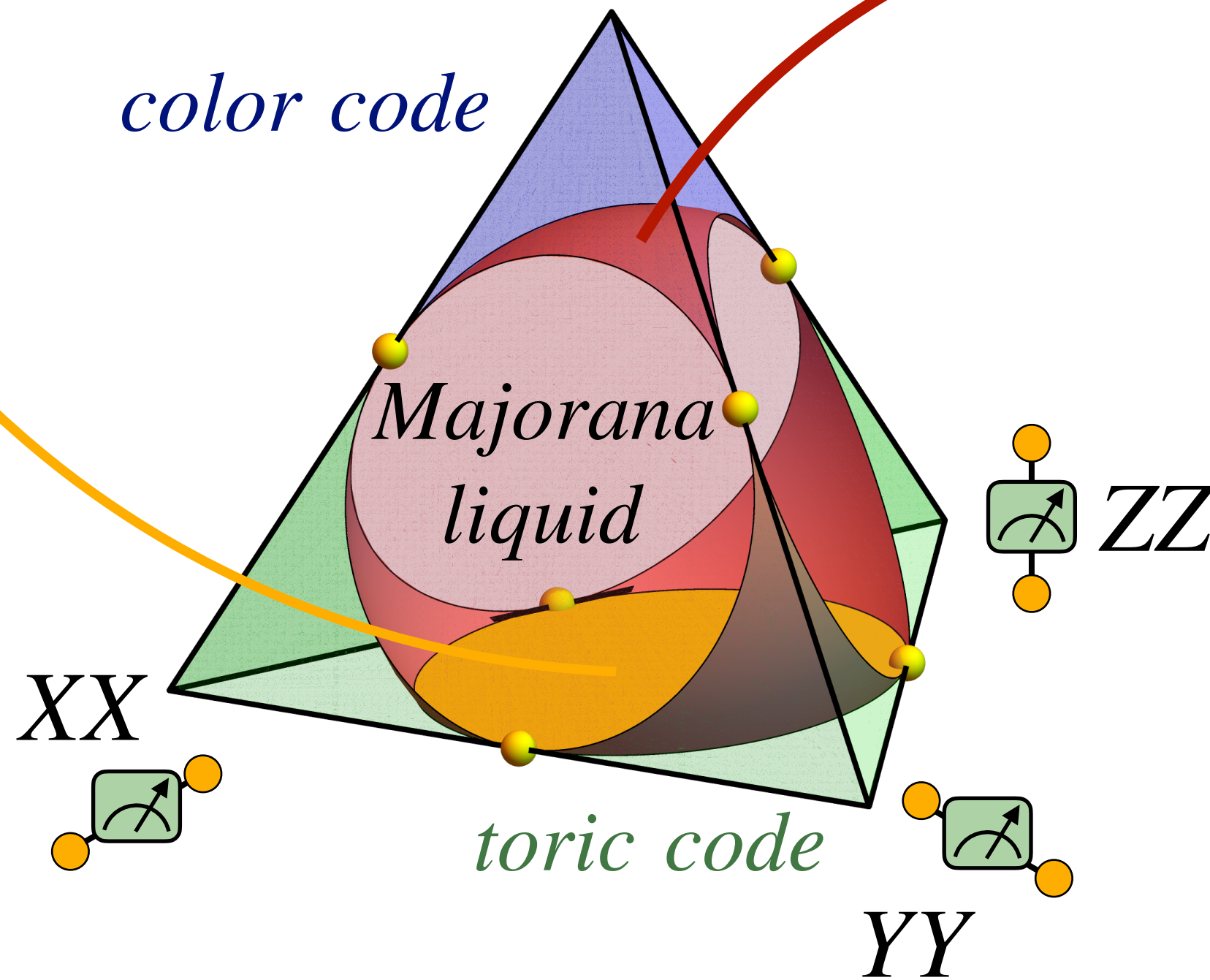


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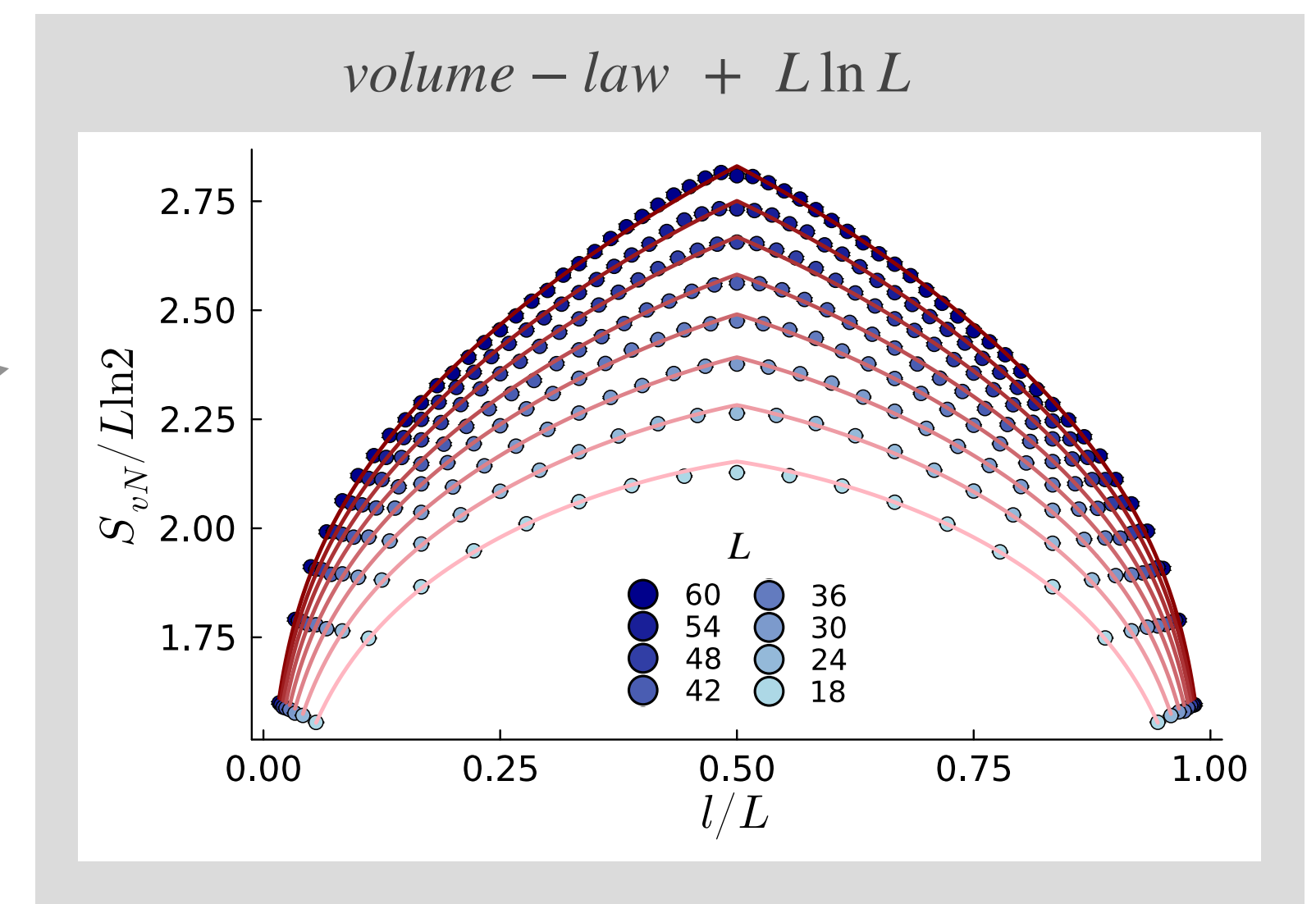
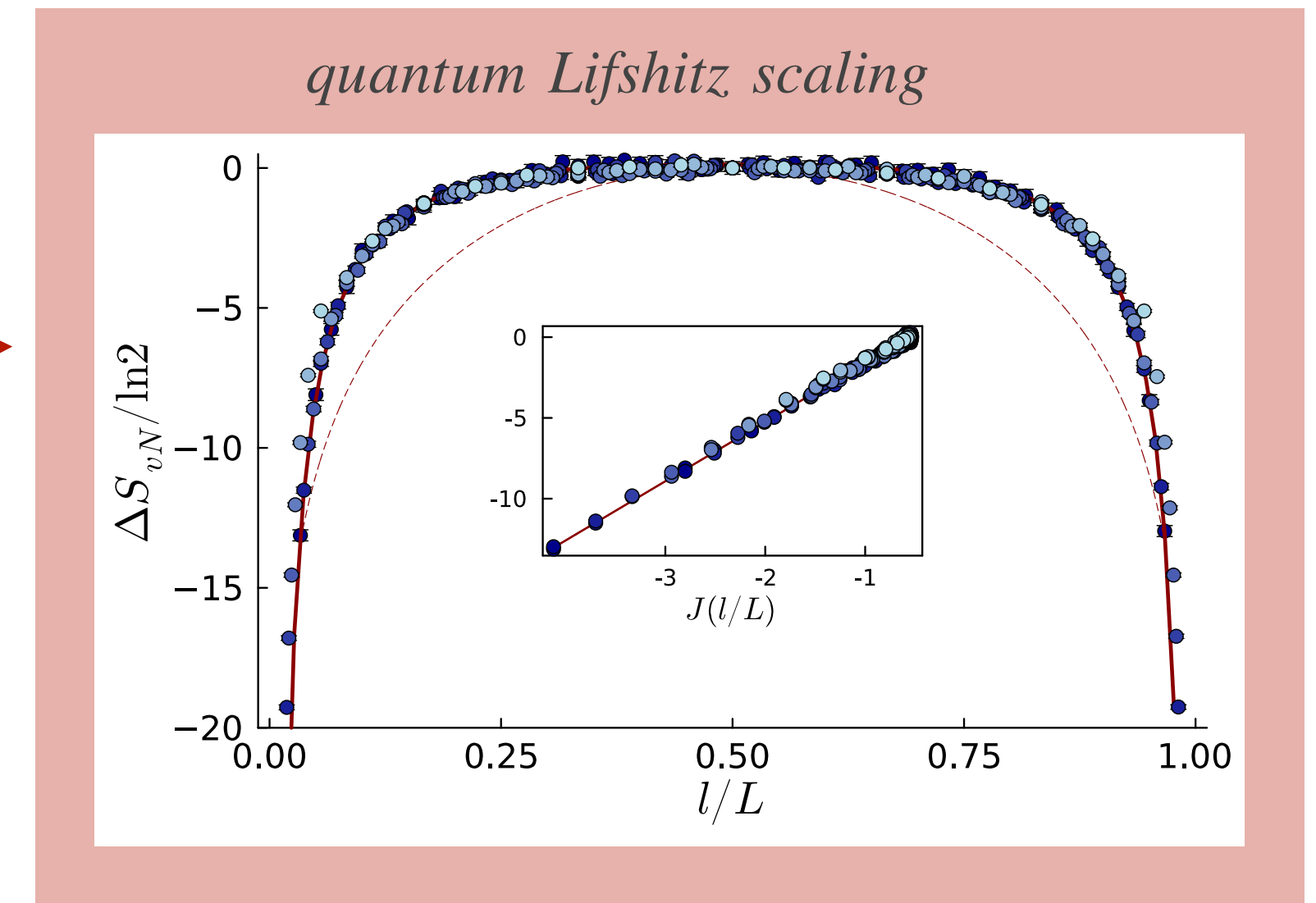
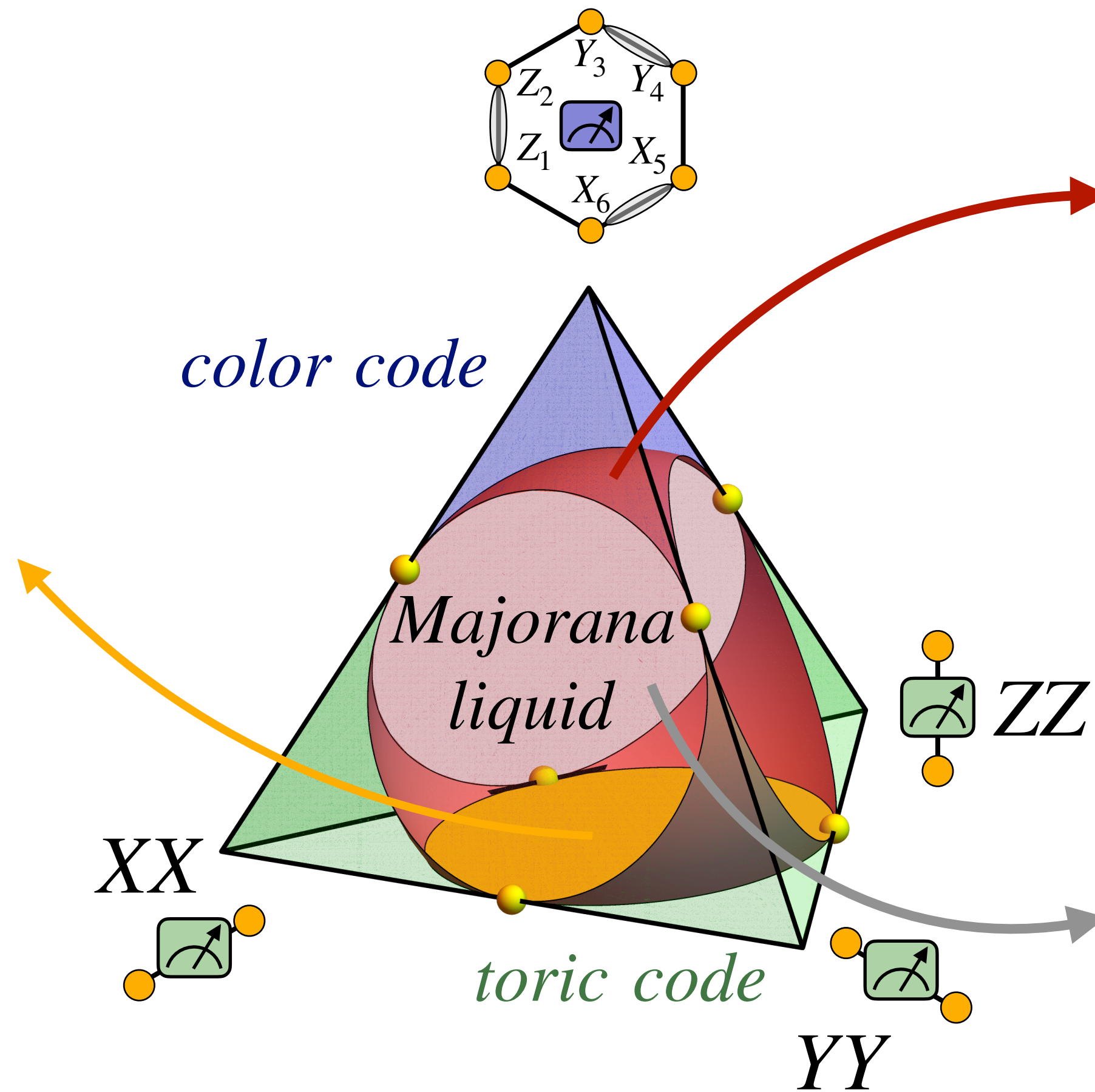
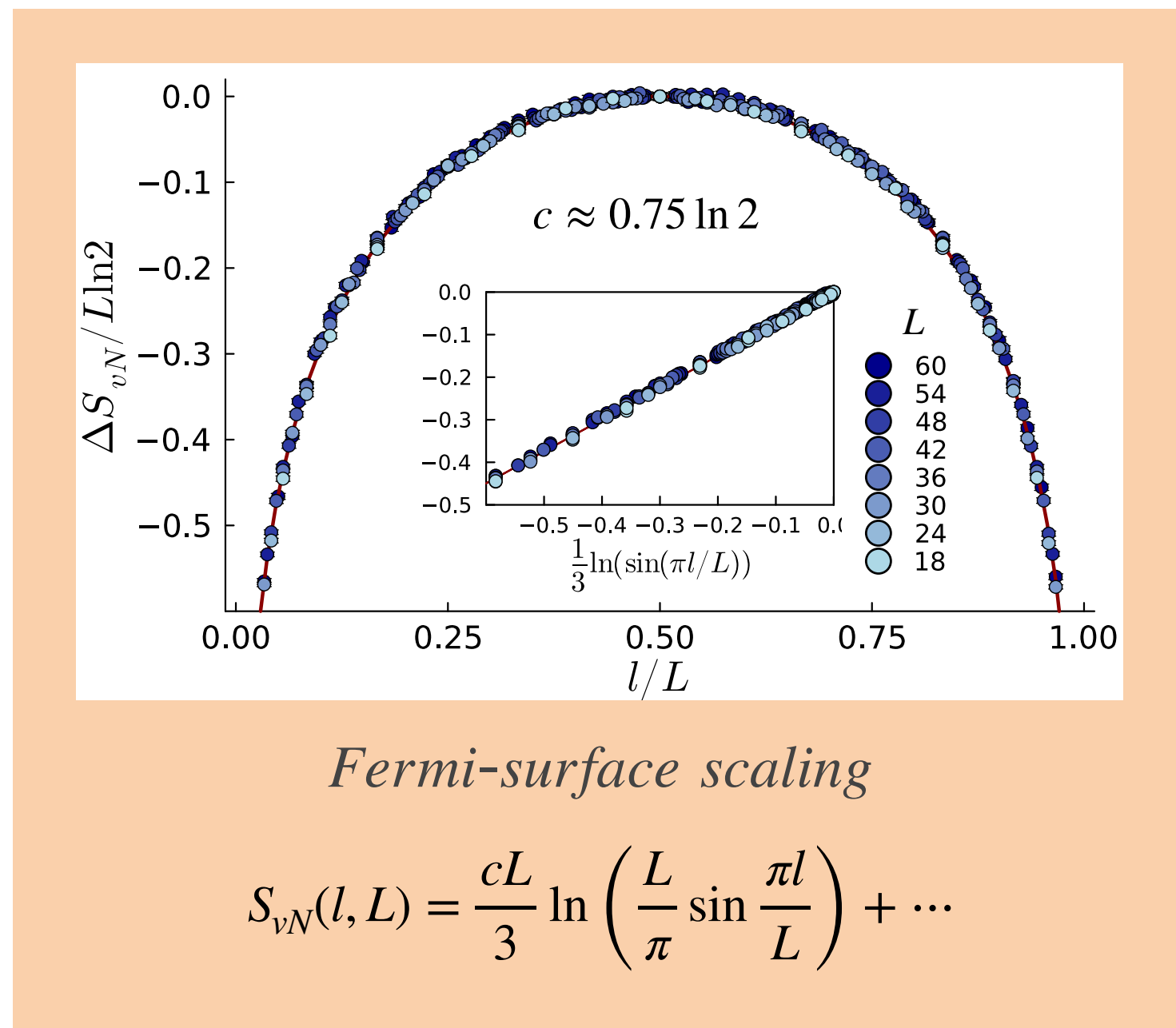
*color code*



Nahum, Skinner 2020; Lavasani, Luo, Vijay 2023; Sriram, Rakovszky, Khemani, Ippoliti 2023  
**Zhu, Tantivasadakarn, ST 2023: + Majorana interaction**

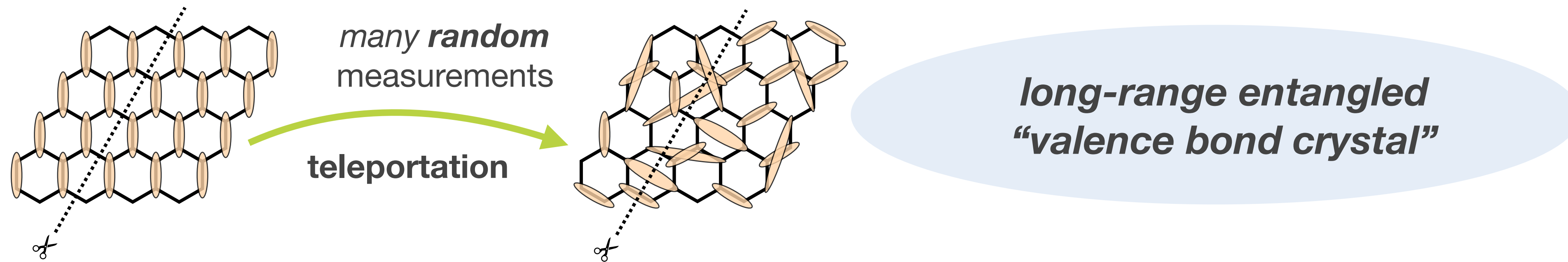


# entanglement phase diagram

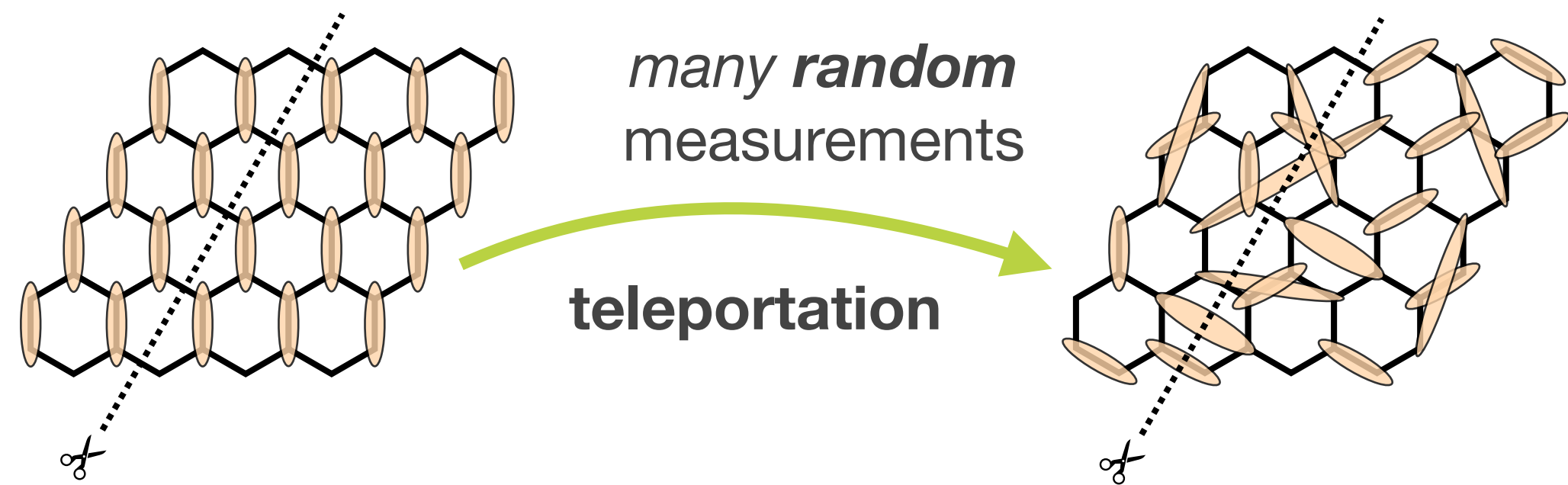


Nahum, Skinner 2020; Lavasani, Luo, Vijay 2023; Sriram, Rakovszky, Khemani, Ippoliti 2023  
 Zhu, Tantivasadakarn, ST 2023: + Majorana interaction

# measurement, teleportation, and beyond



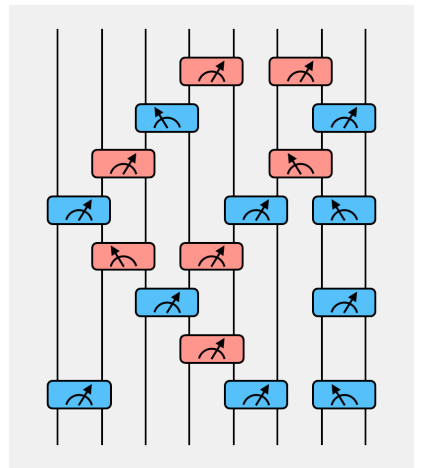
# measurement, teleportation, and beyond



*long-range entangled  
"valence bond crystal"*

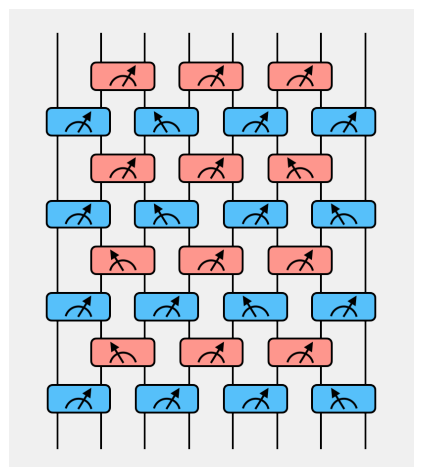
space-time disorder

random

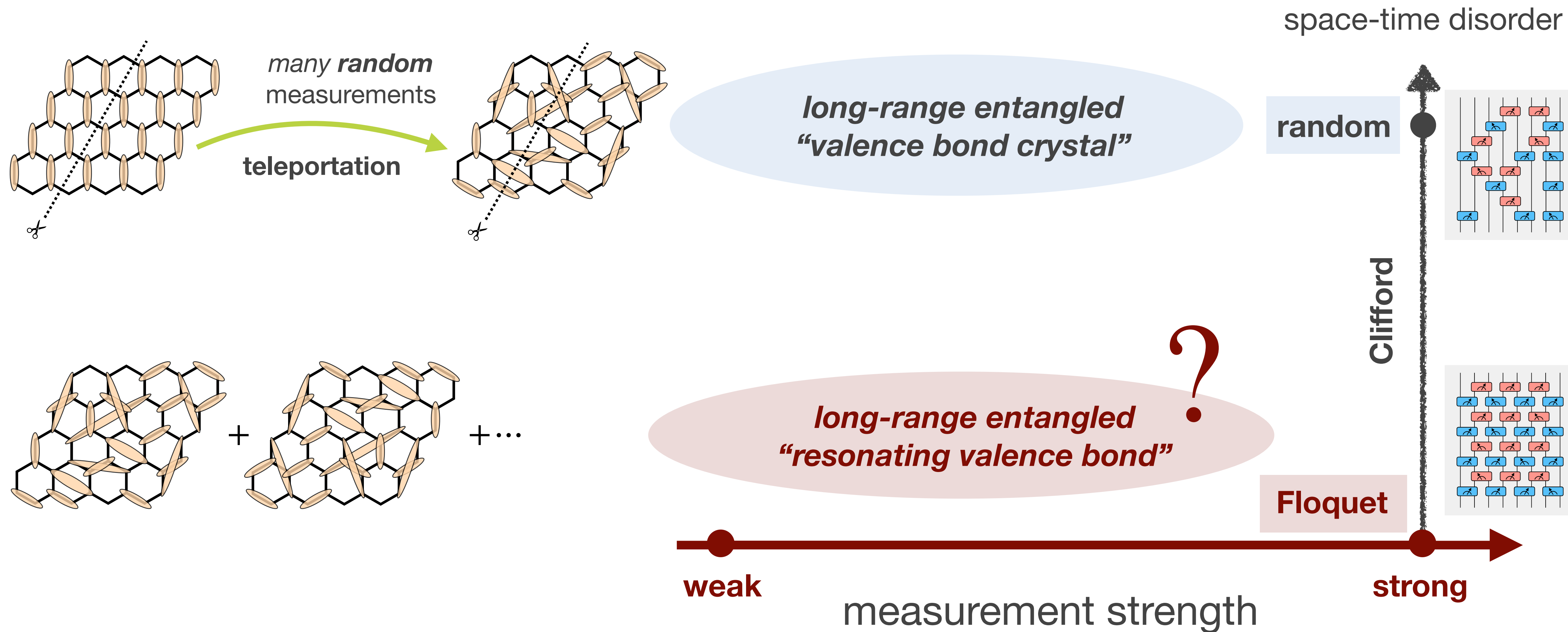


Clifford

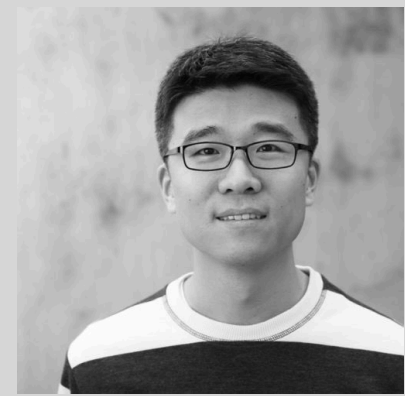
Floquet



# measurement, teleportation, and beyond

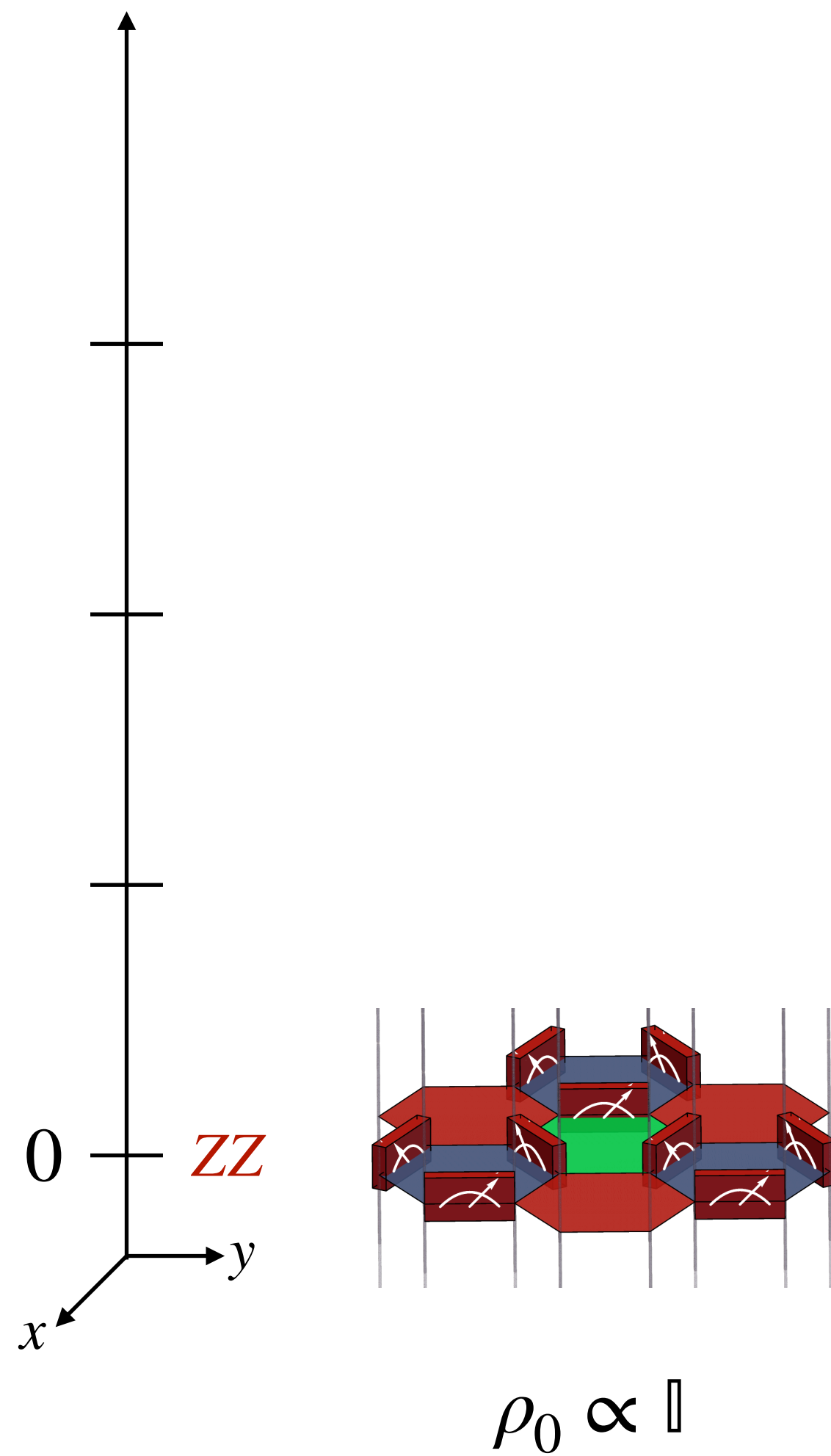


# dynamical protocol

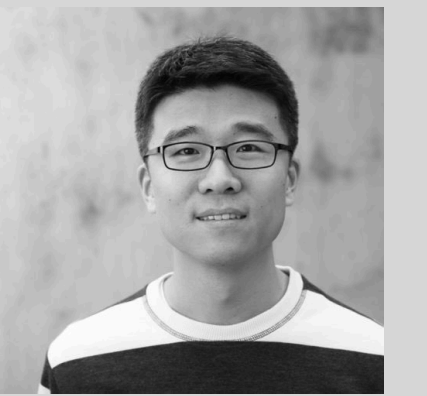


Hastings, Haah (2021)

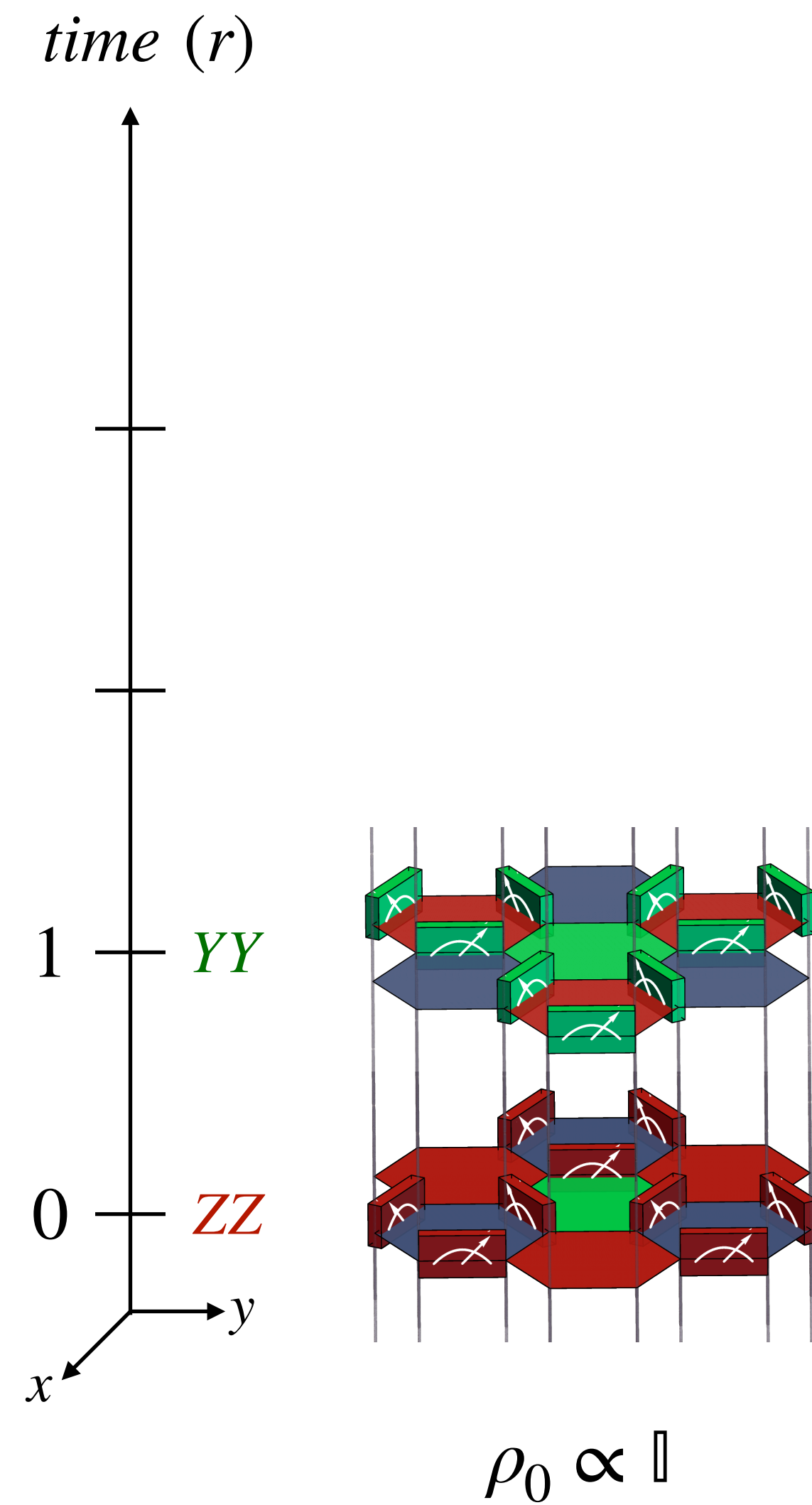
time ( $r$ )



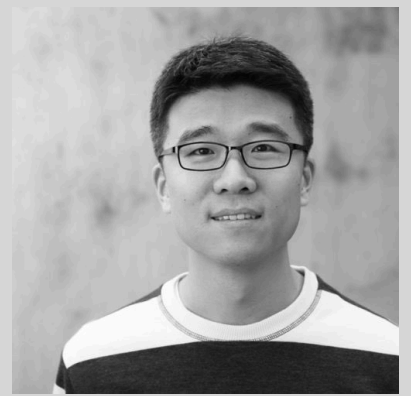
# dynamical protocol



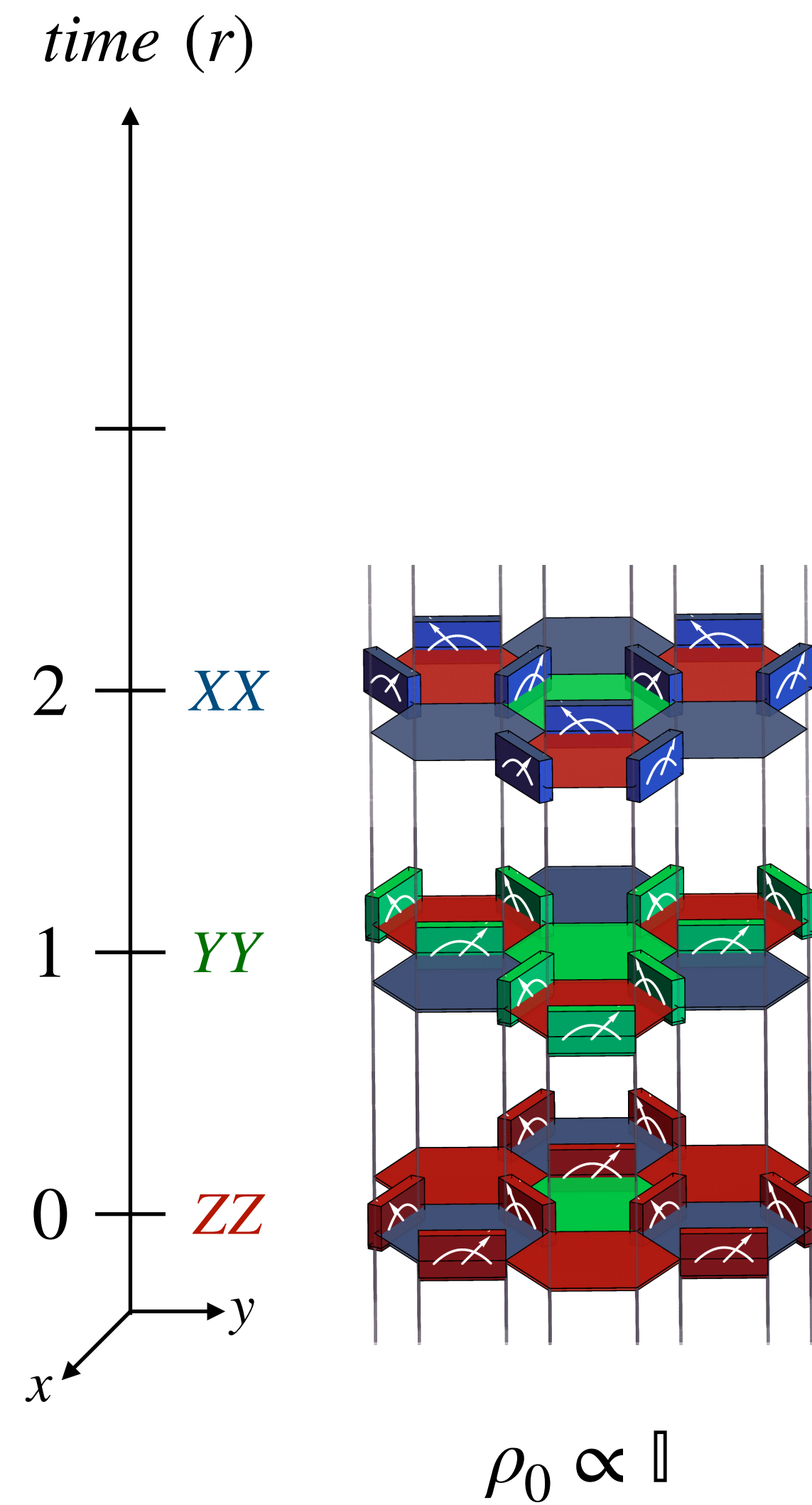
Hastings, Haah (2021)



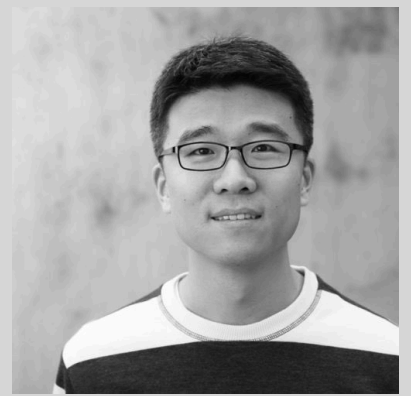
# dynamical protocol



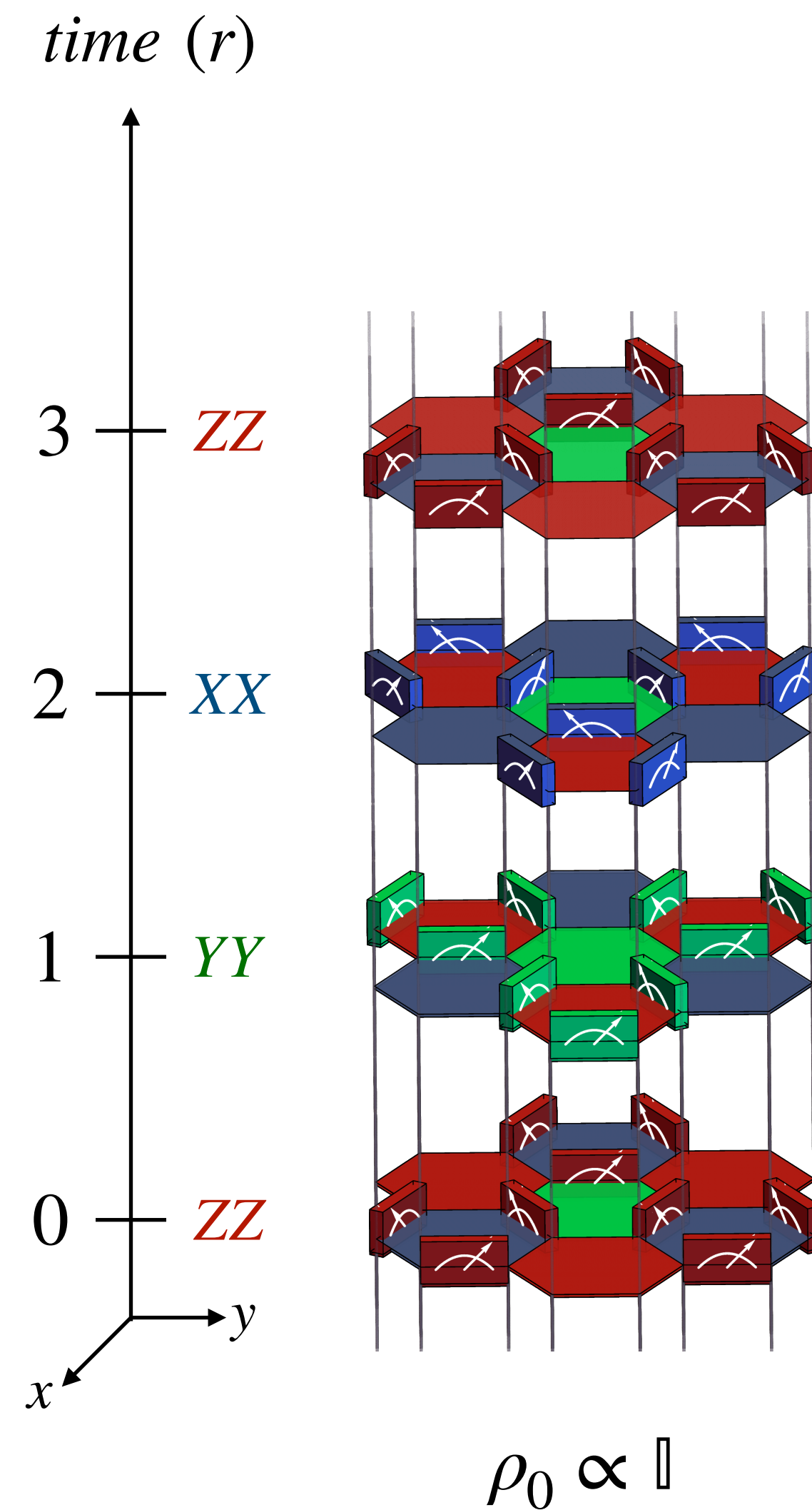
Hastings, Haah (2021)



# dynamical protocol

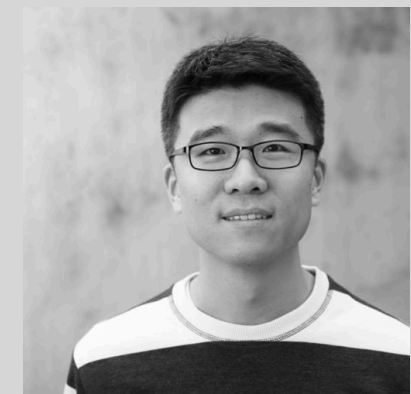


Hastings, Haah (2021)

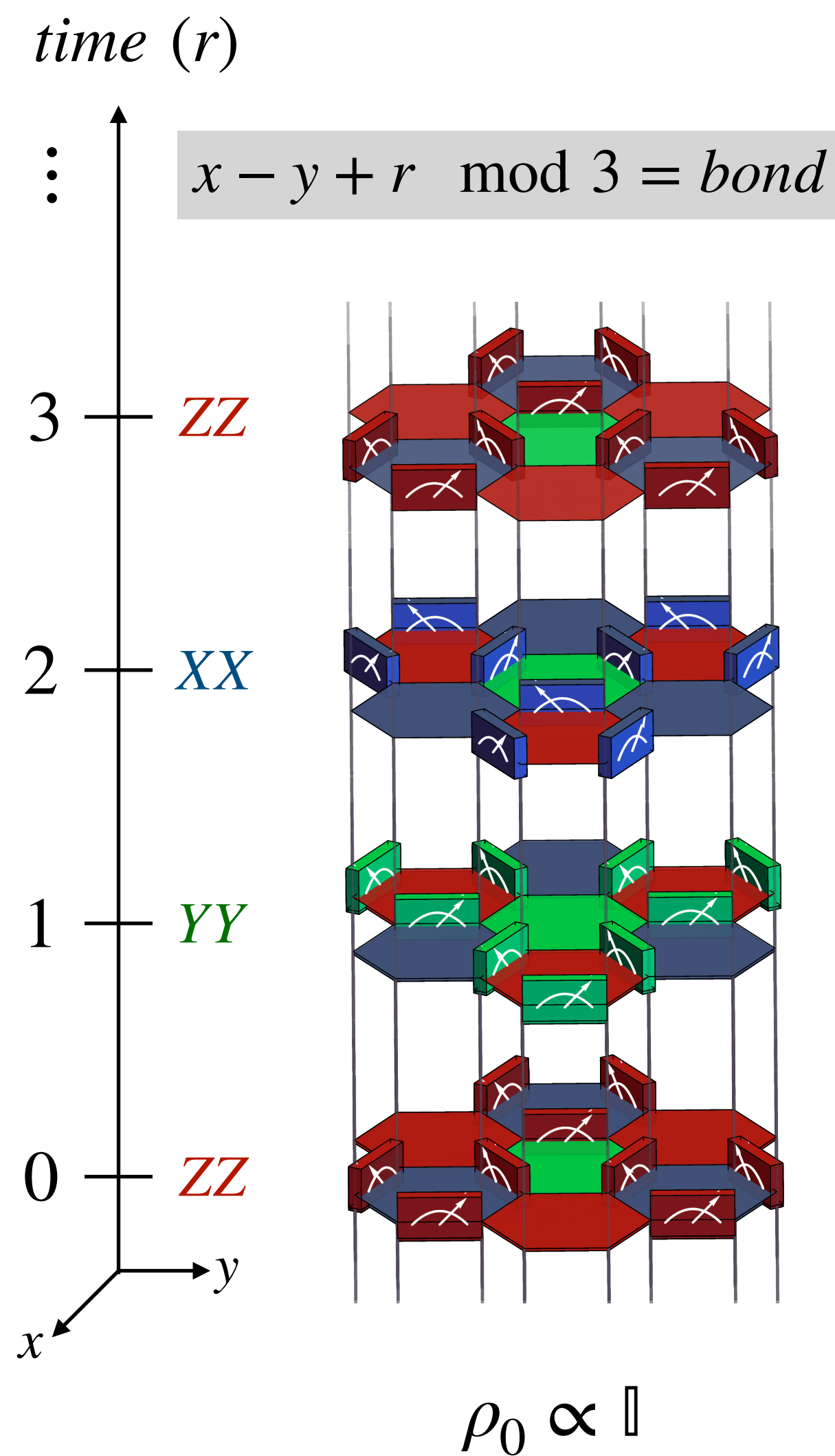




# dynamical protocol



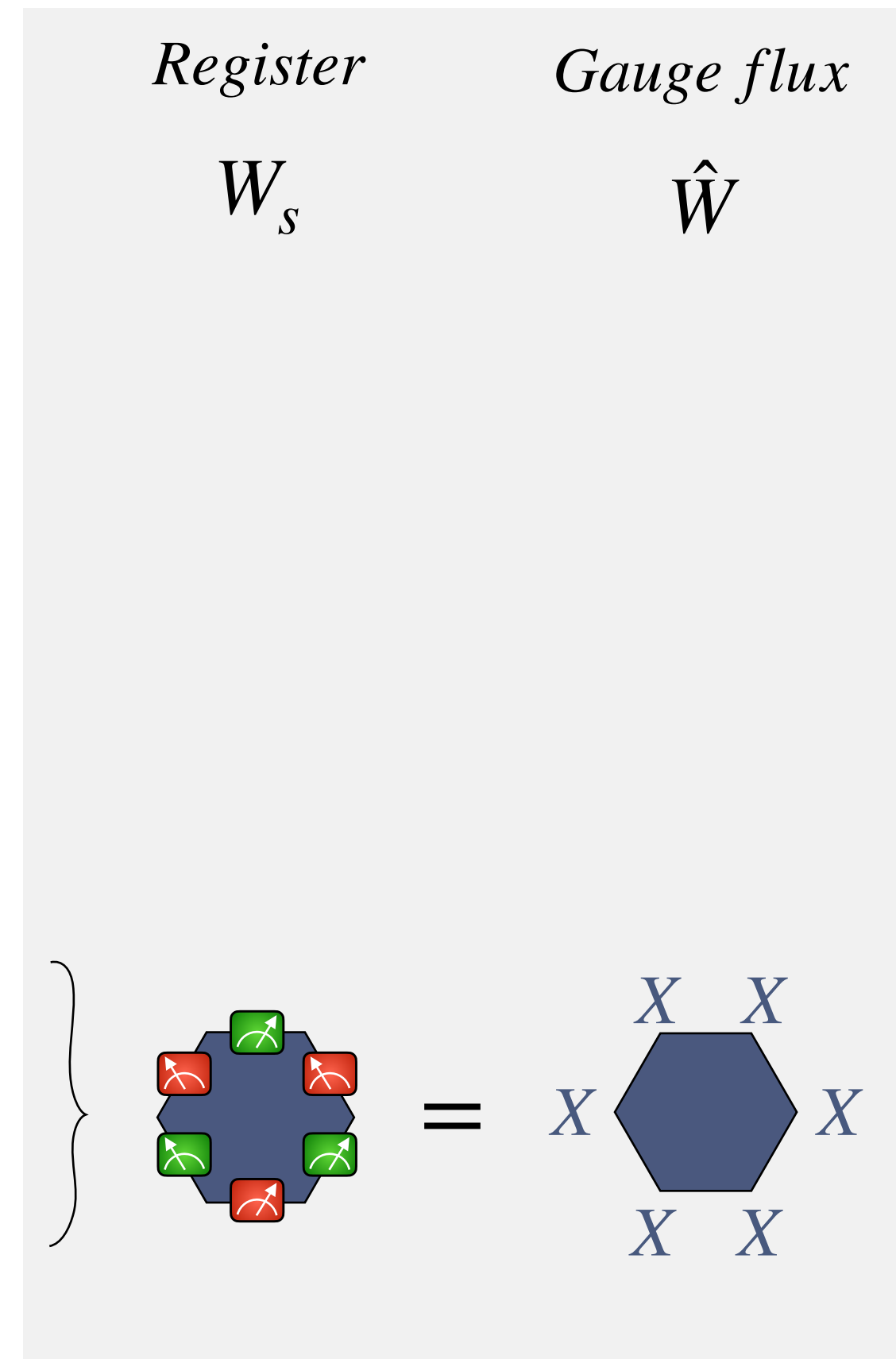
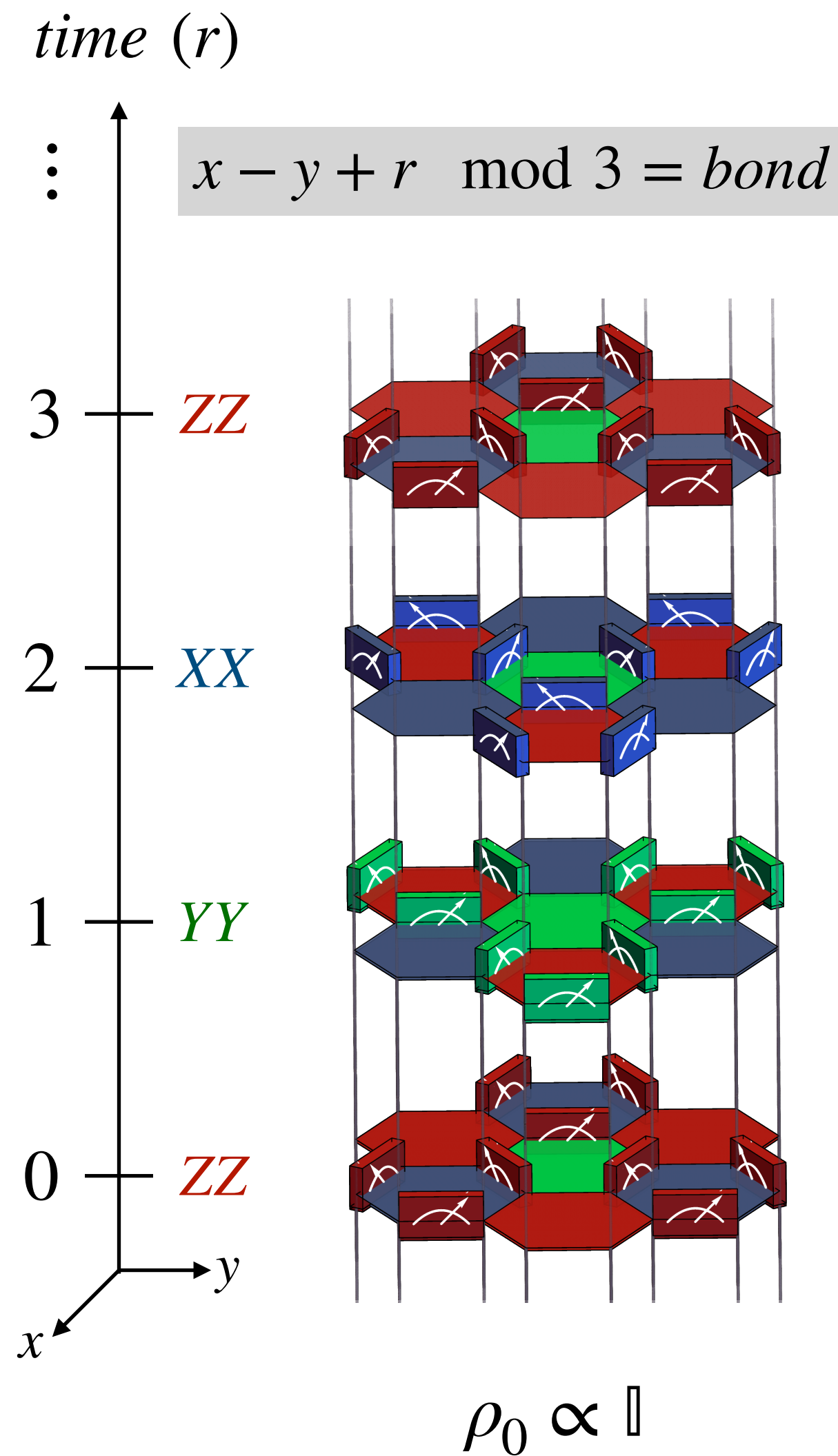
Hastings, Haah (2021)



# dynamical protocol



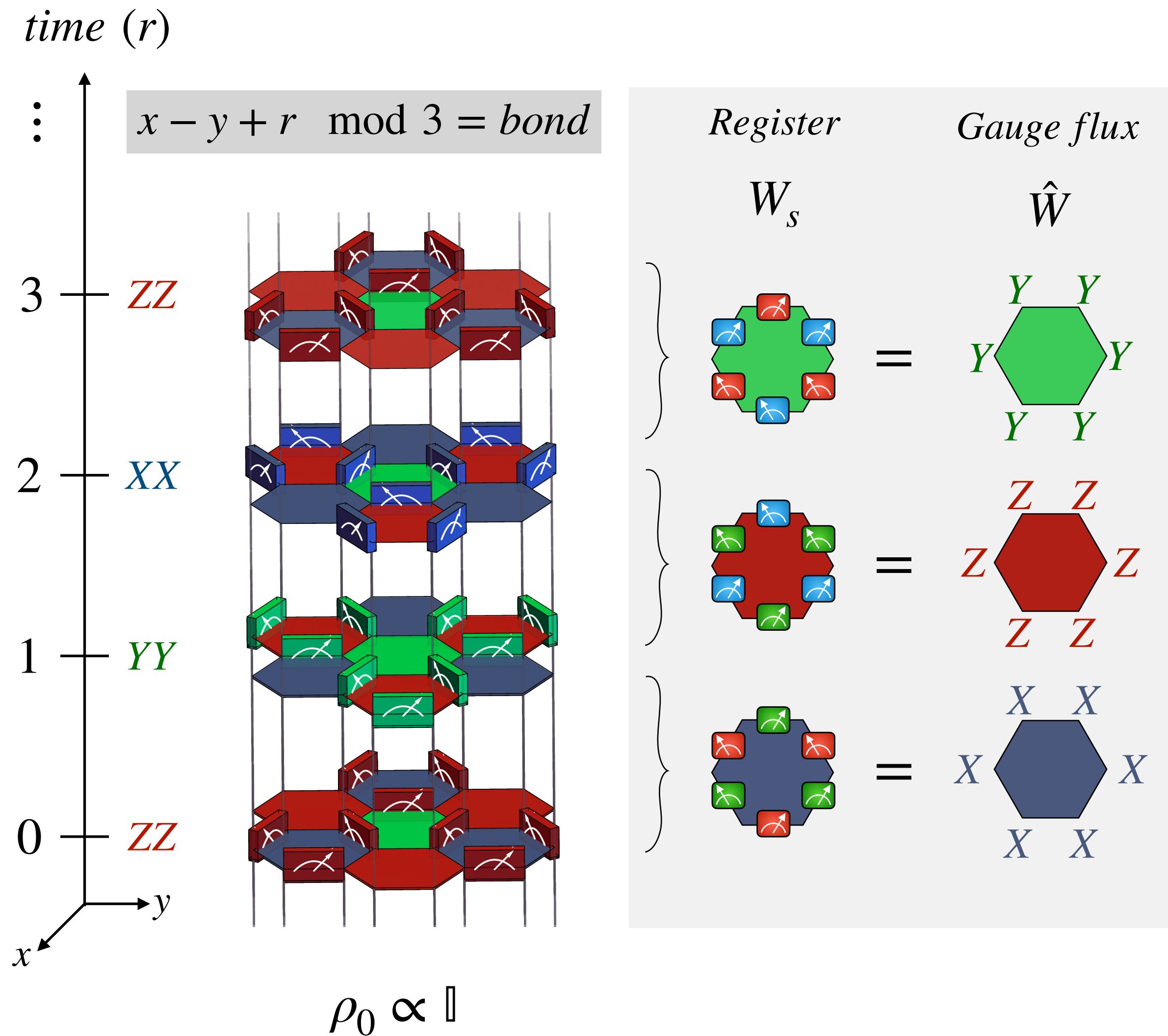
Hastings, Haah (2021)



# dynamical protocol



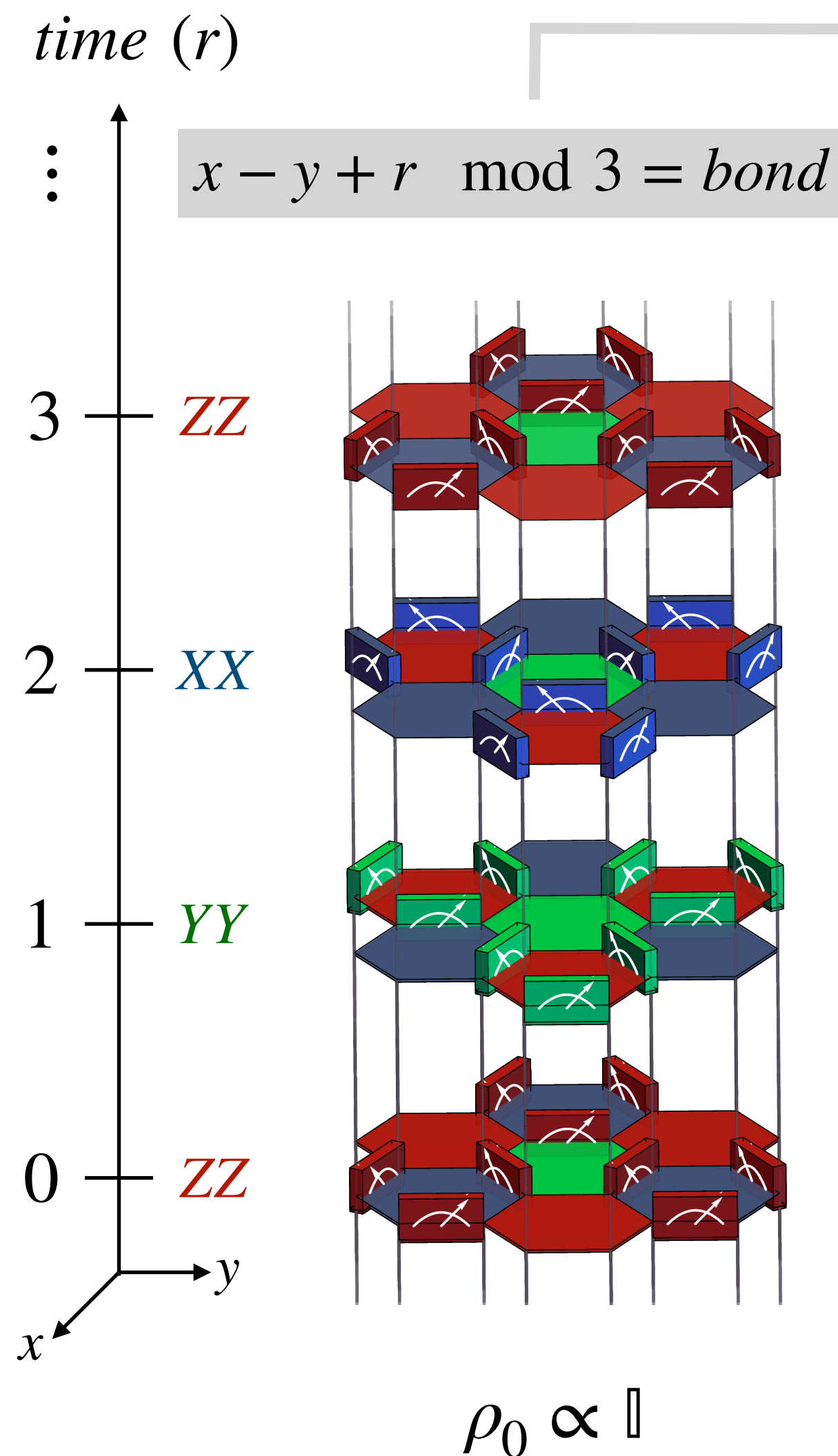
Hastings, Haah (2021)



# dynamical protocol

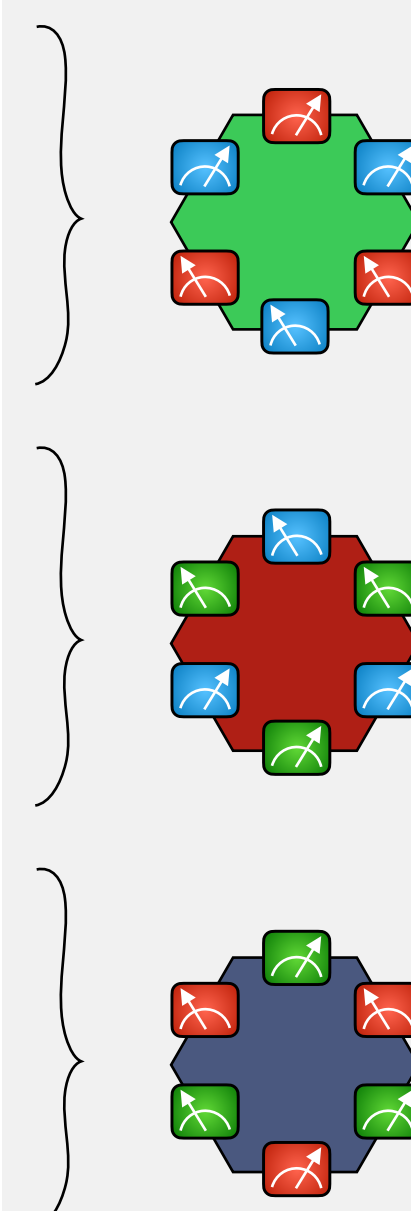


Hastings, Haah (2021)



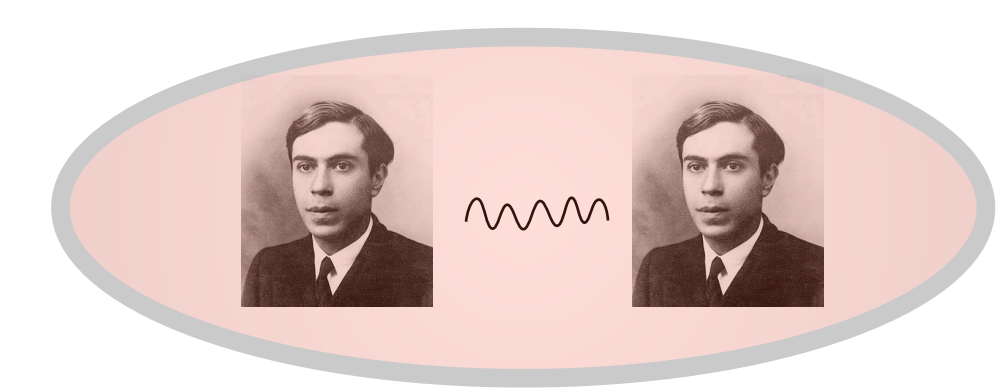
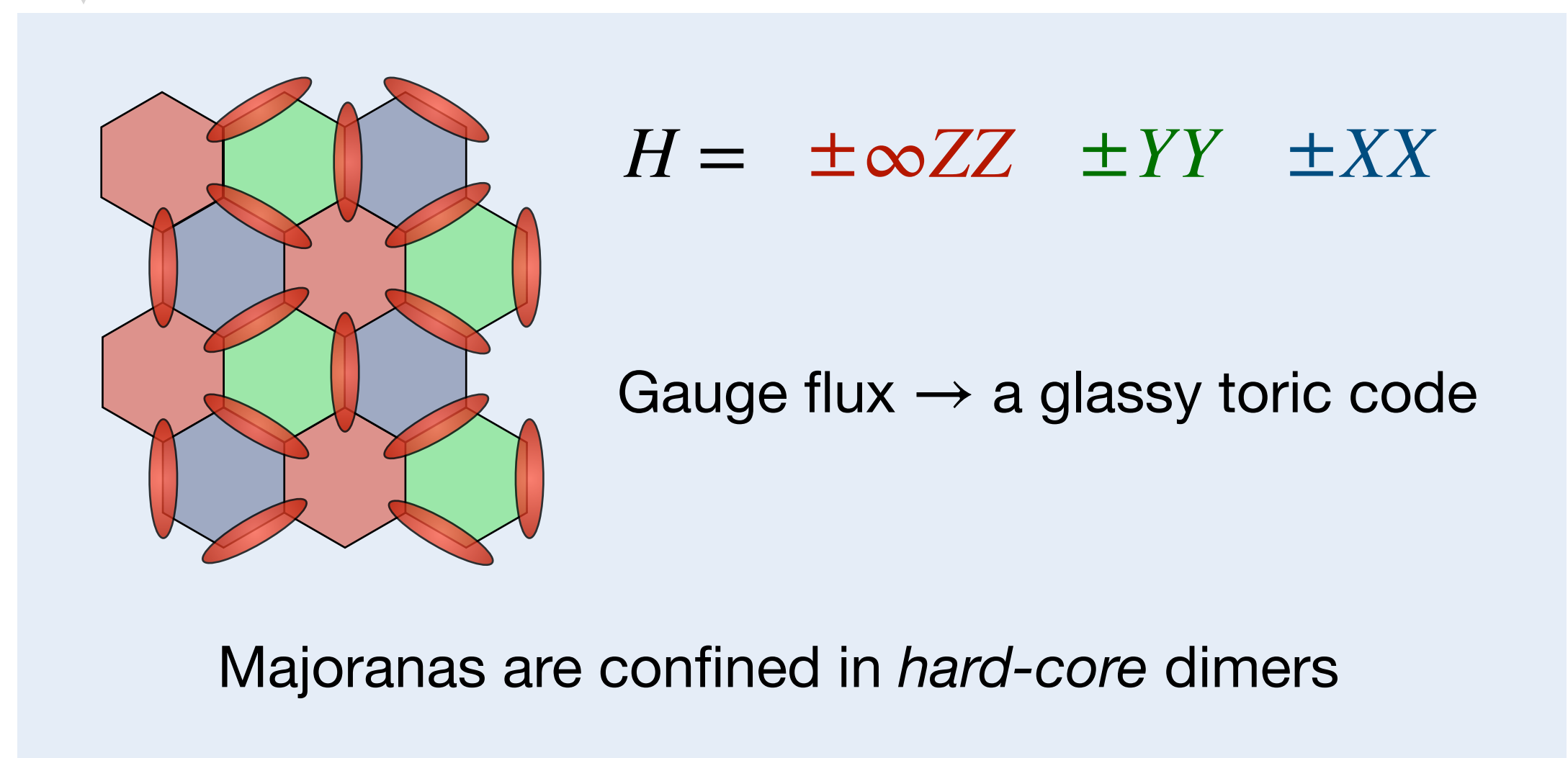
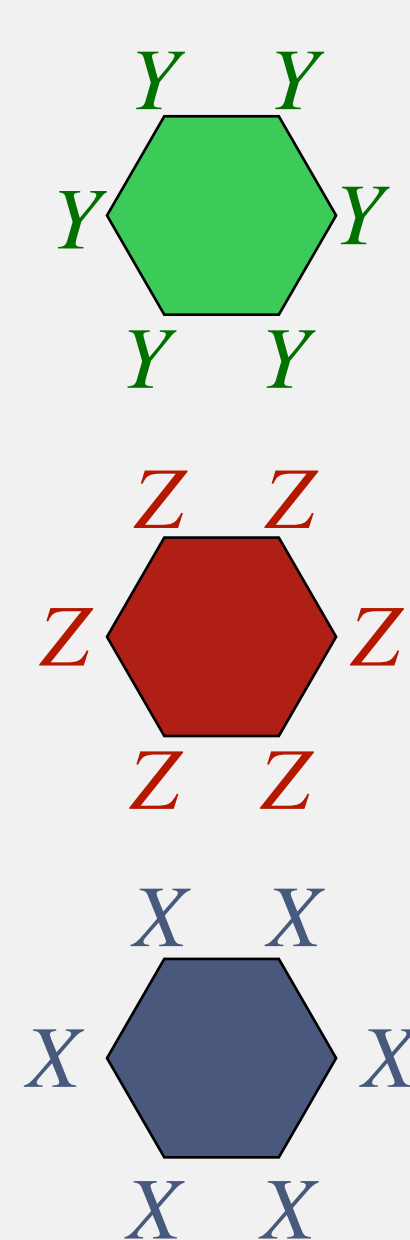
Register

$$W_s$$



Gauge flux

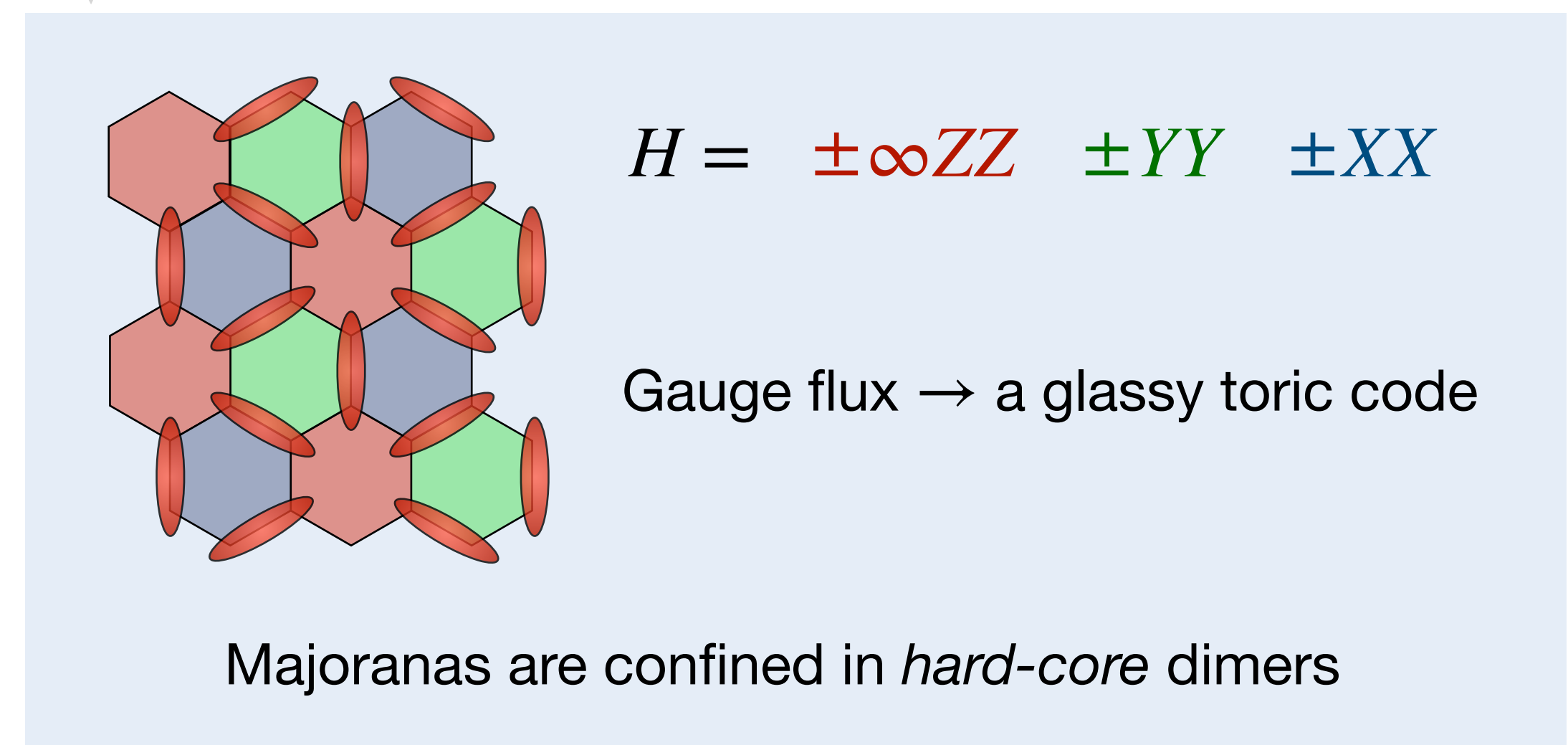
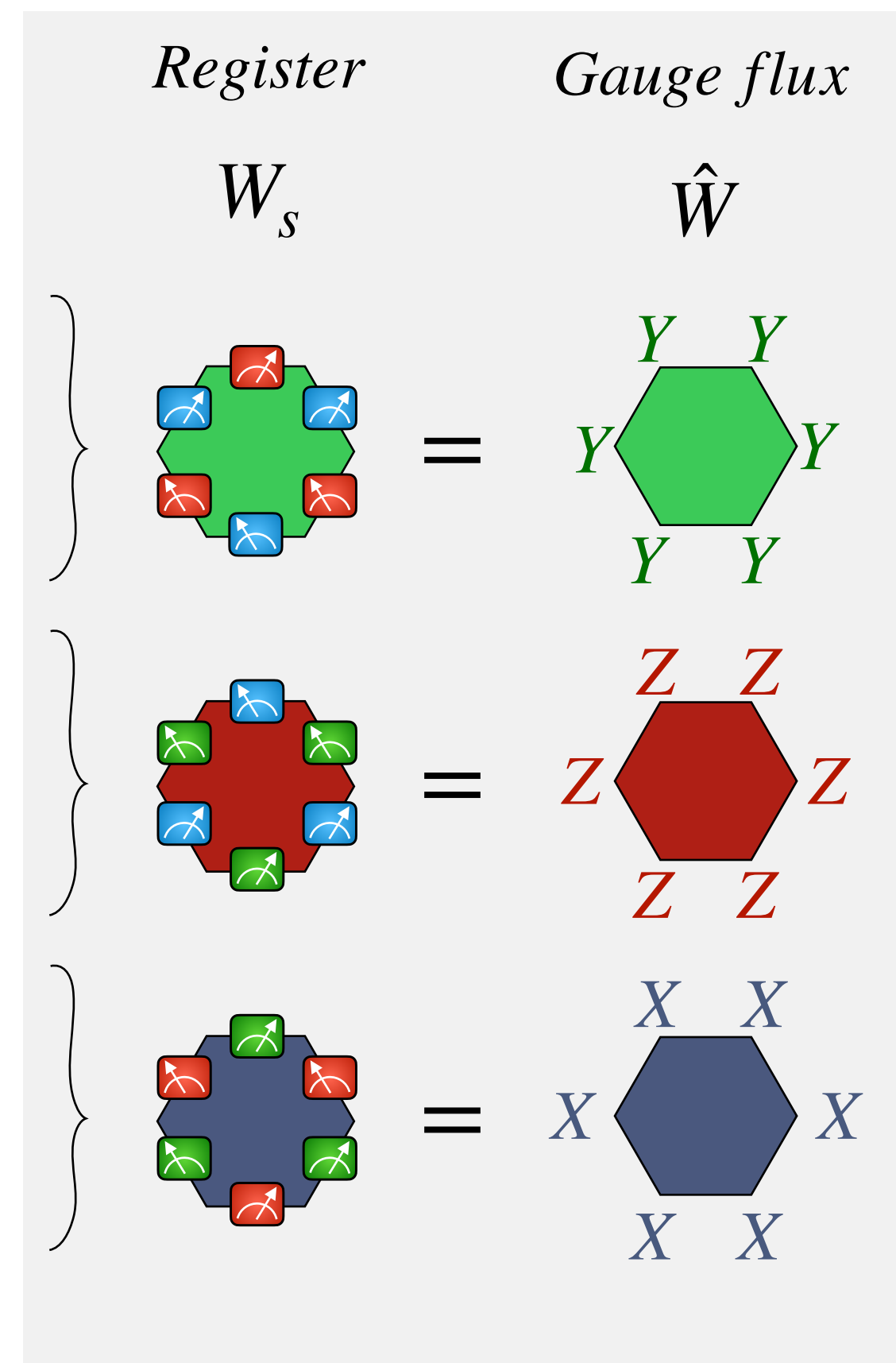
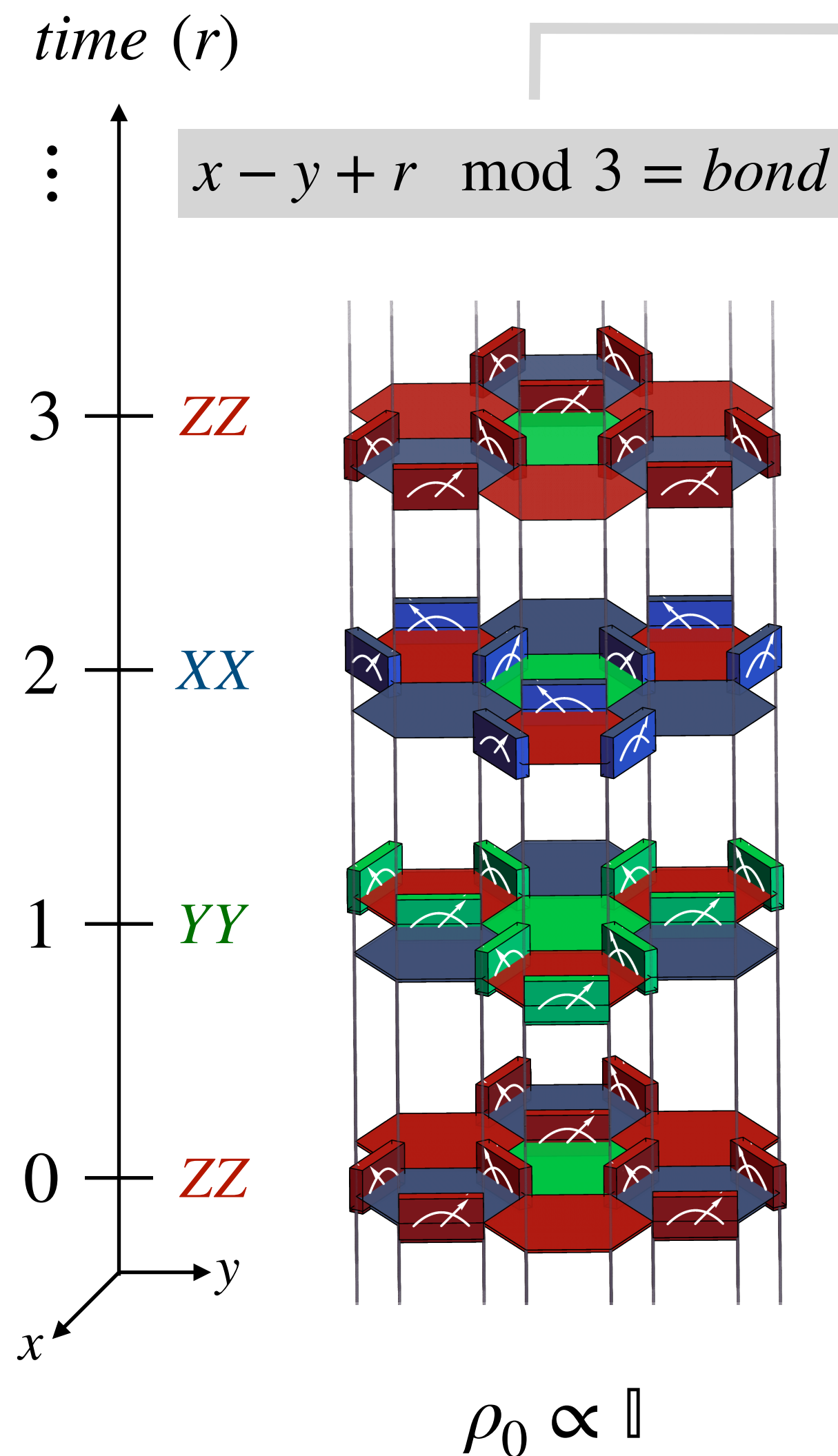
$$\hat{W}$$



# dynamical protocol

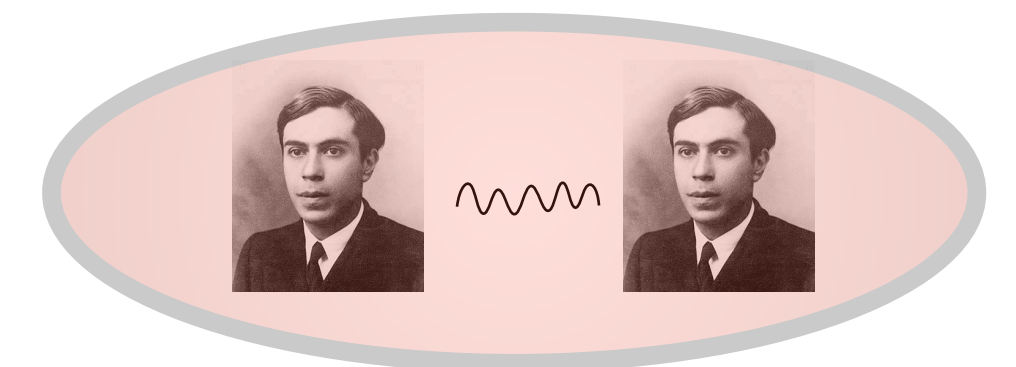


Hastings, Haah (2021)



Questions:

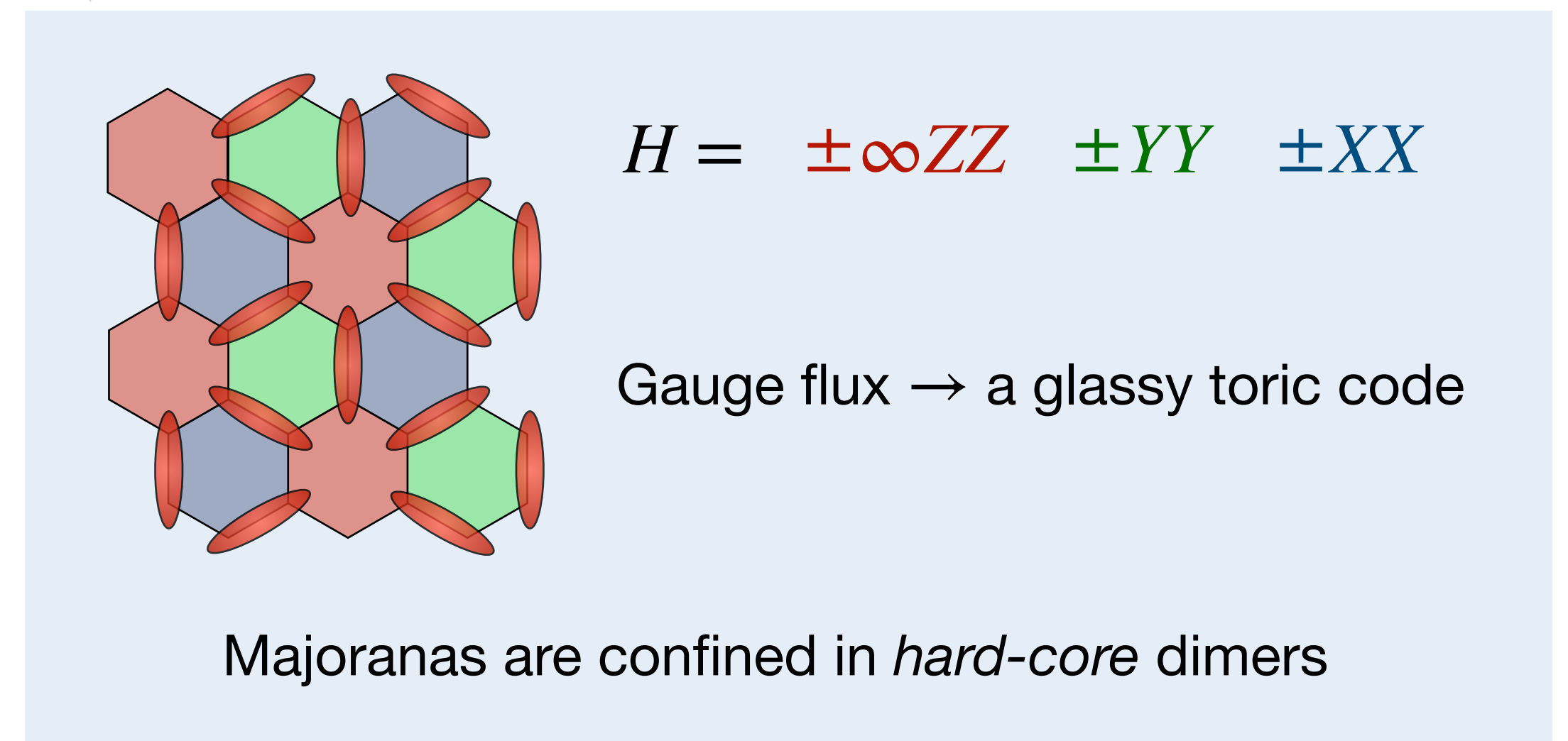
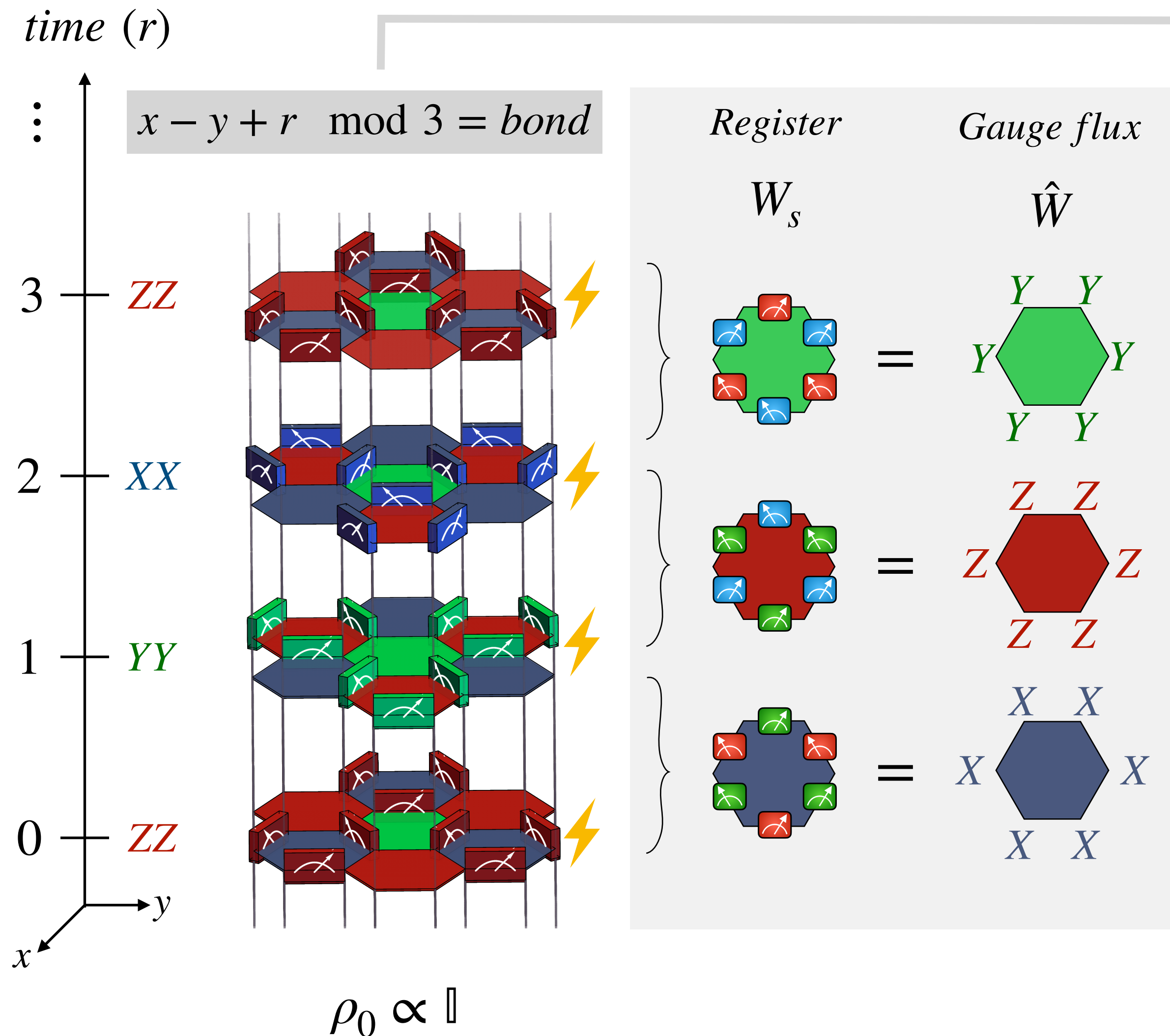
- How to liberate Majorana?
- Stability of the code?



# dynamical protocol

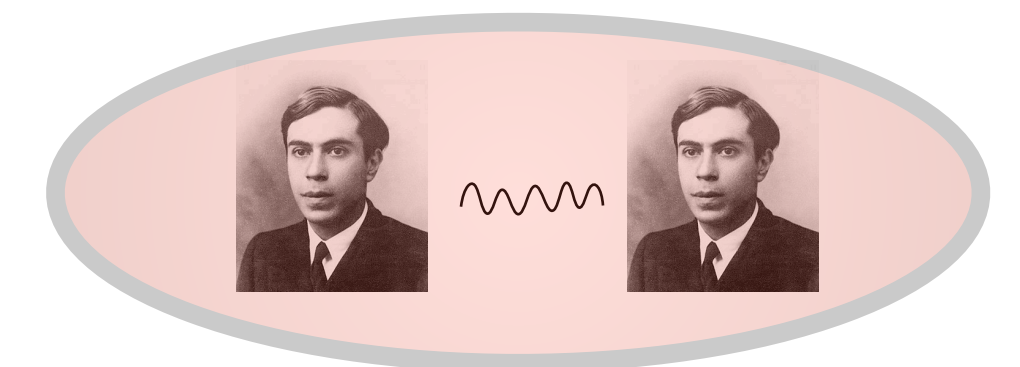


Hastings, Haah (2021)



Questions:

- How to liberate Majorana?
- Stability of the code?



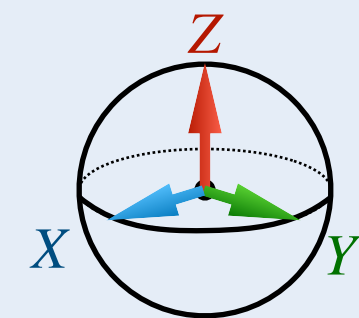
coherent error / weak measurement  $\rightarrow$

soften dimers – a channel for Majorana to escape !

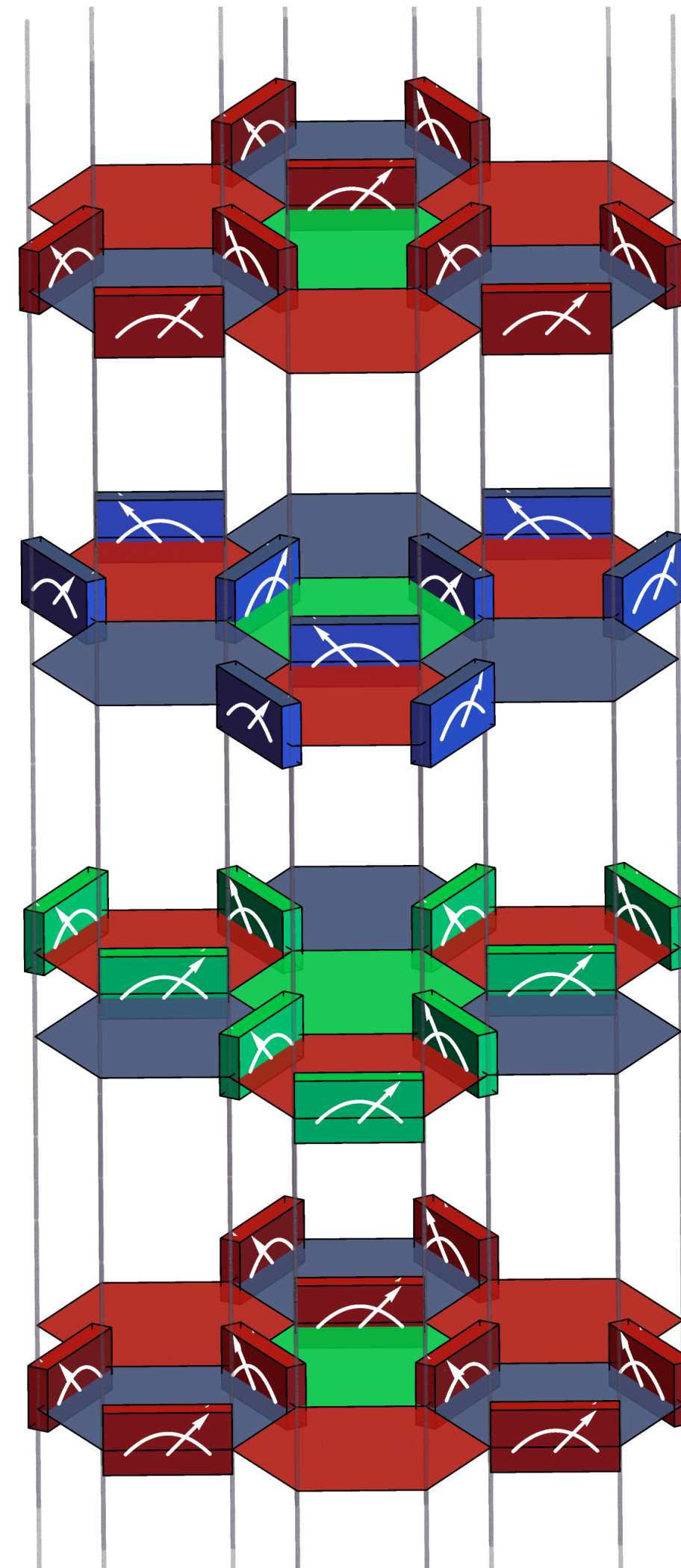
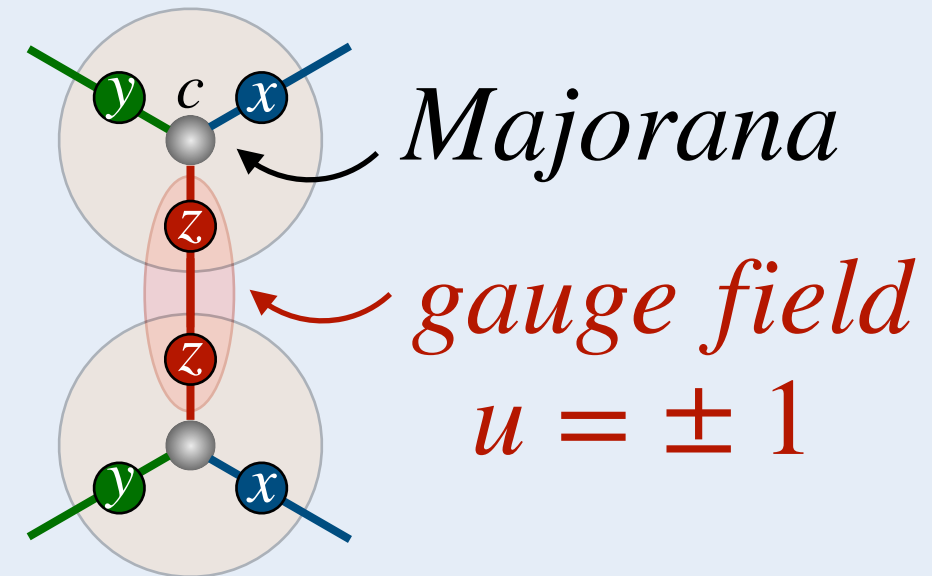
# Majorana, flux pillars, loops



Guo-Yi Zhu



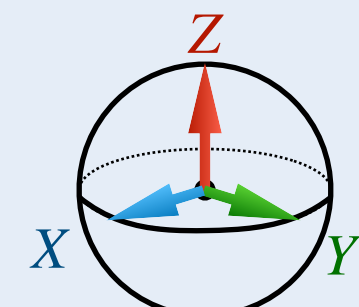
$$\begin{aligned} Z &= ib^z c \\ X &= ib^x c \\ Y &= ib^y c \end{aligned}$$



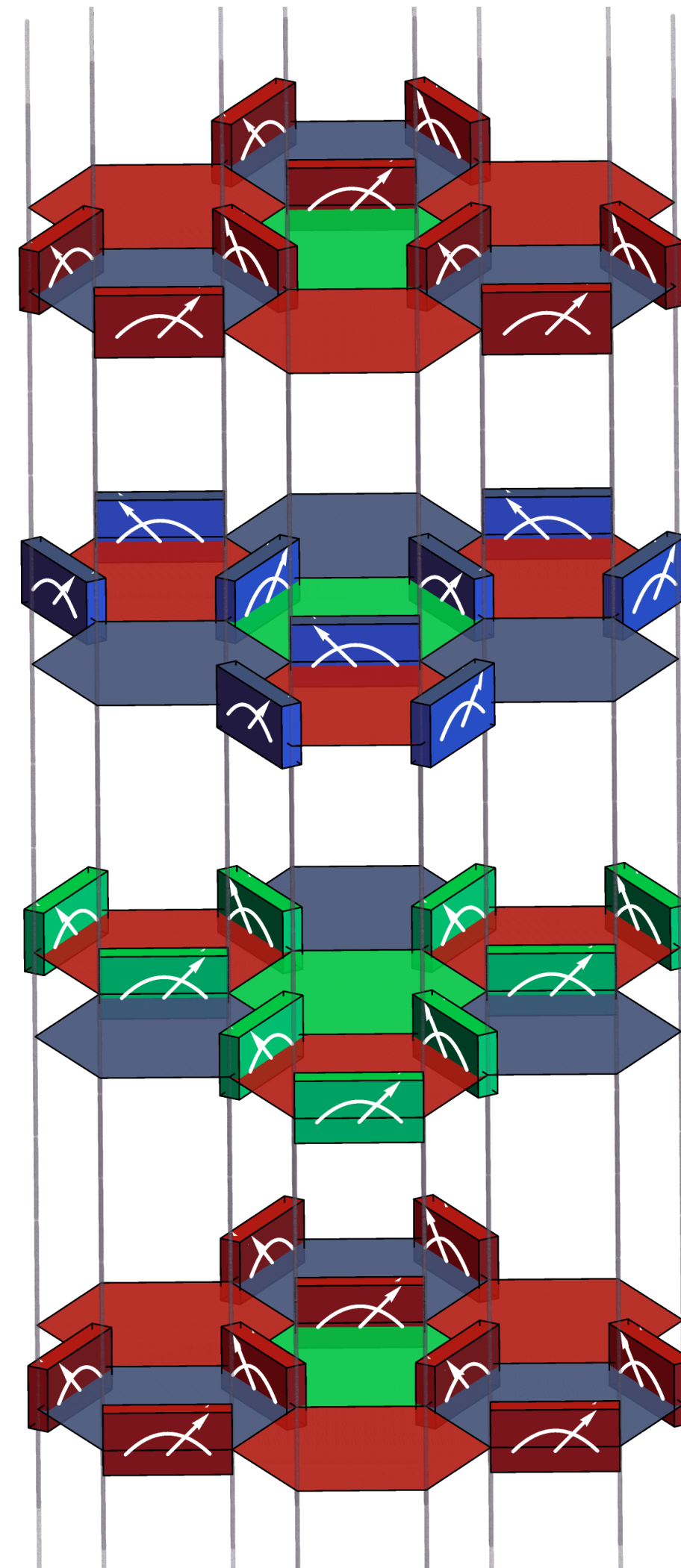
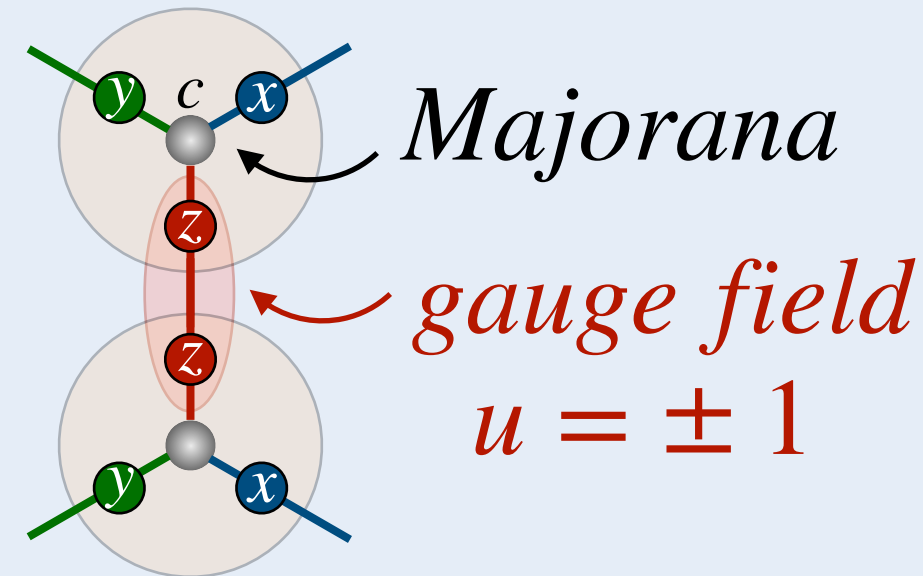
# Majorana, flux pillars, loops



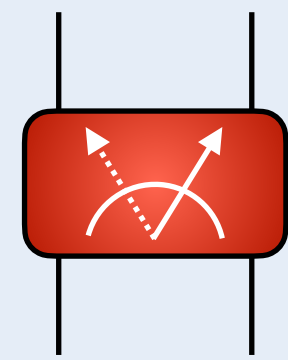
Guo-Yi Zhu



$$\begin{aligned} Z &= ib^z c \\ X &= ib^x c \\ Y &= ib^y c \end{aligned}$$



$$\exp(-\tau s Z Z) \quad (s = \pm 1)$$



$$\exp(-\tau \quad (s u) \quad i c_A c_B)$$

measurement strength  
 $\tau = \tanh^{-1}(\sin(2t))$

net gauge field

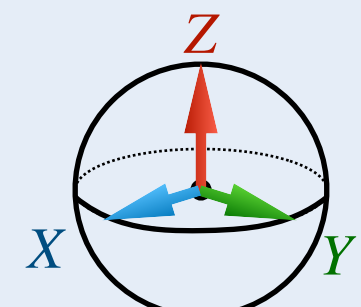
Majorana bilinear



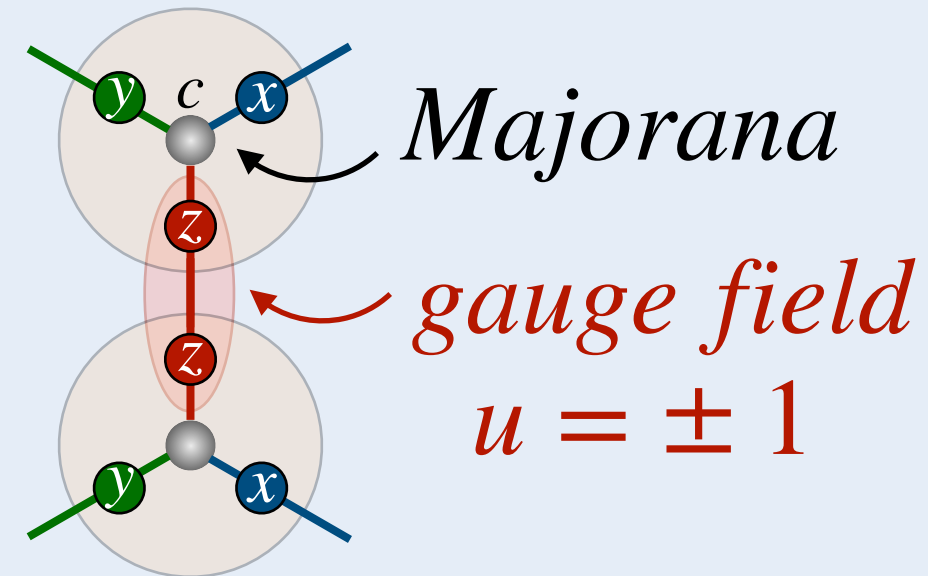
# Majorana, flux pillars, loops



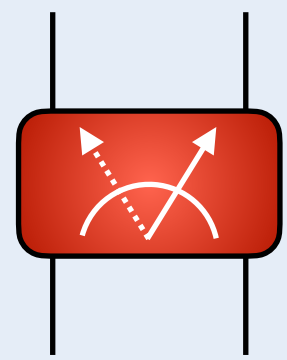
Guo-Yi Zhu



$$\begin{aligned} Z &= ib^z c \\ X &= ib^x c \\ Y &= ib^y c \end{aligned}$$



$$\exp(-\tau s ZZ) \quad (s = \pm 1)$$

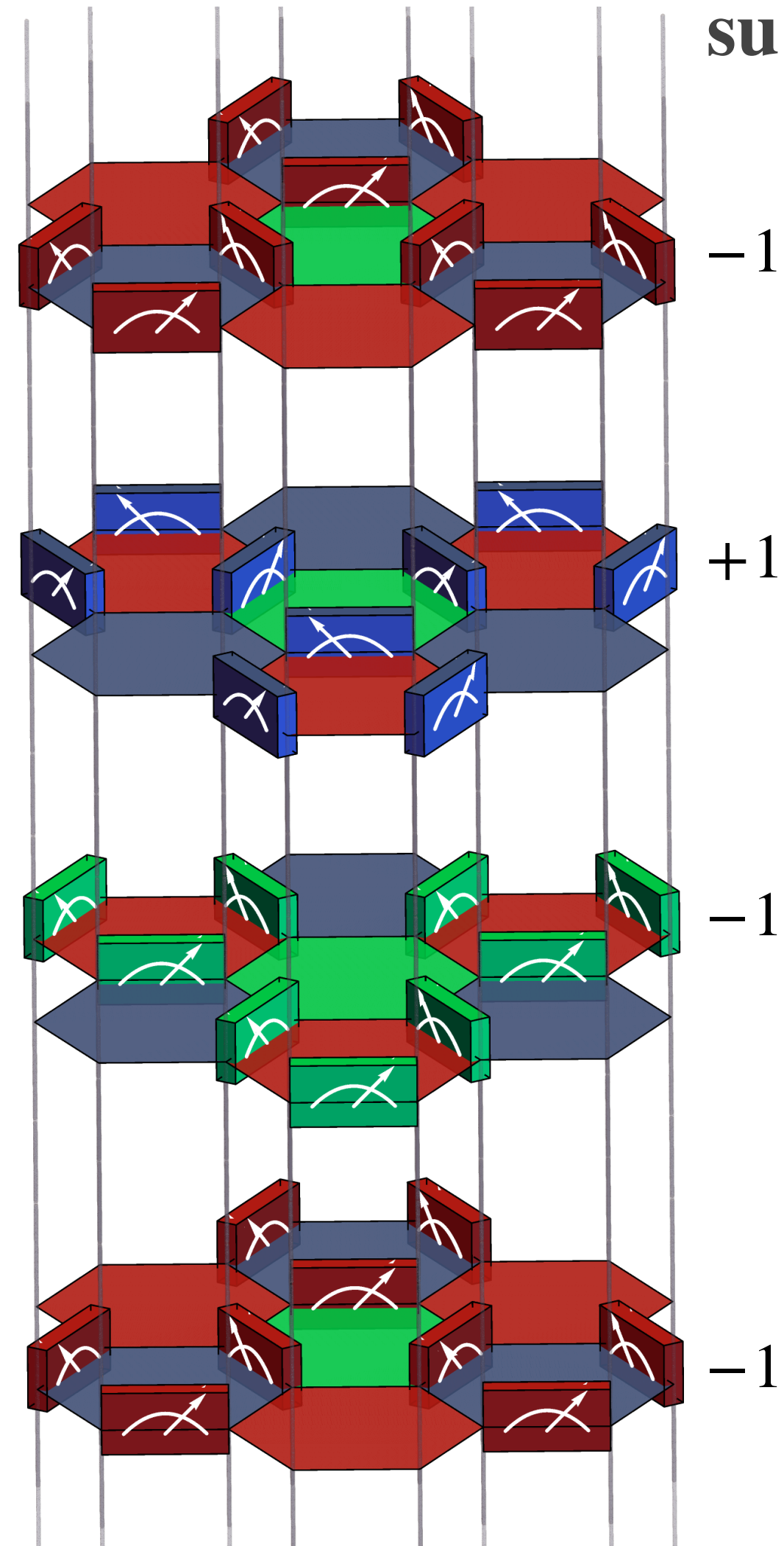


$$\exp(-\tau \quad (su) \quad ic_A c_B)$$

measurement strength  
 $\tau = \tanh^{-1}(\sin(2t))$

net gauge field

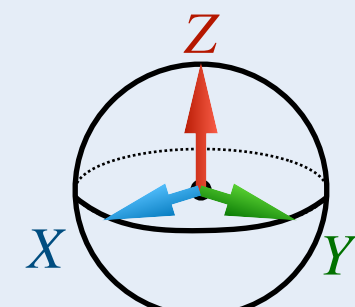
Majorana bilinear



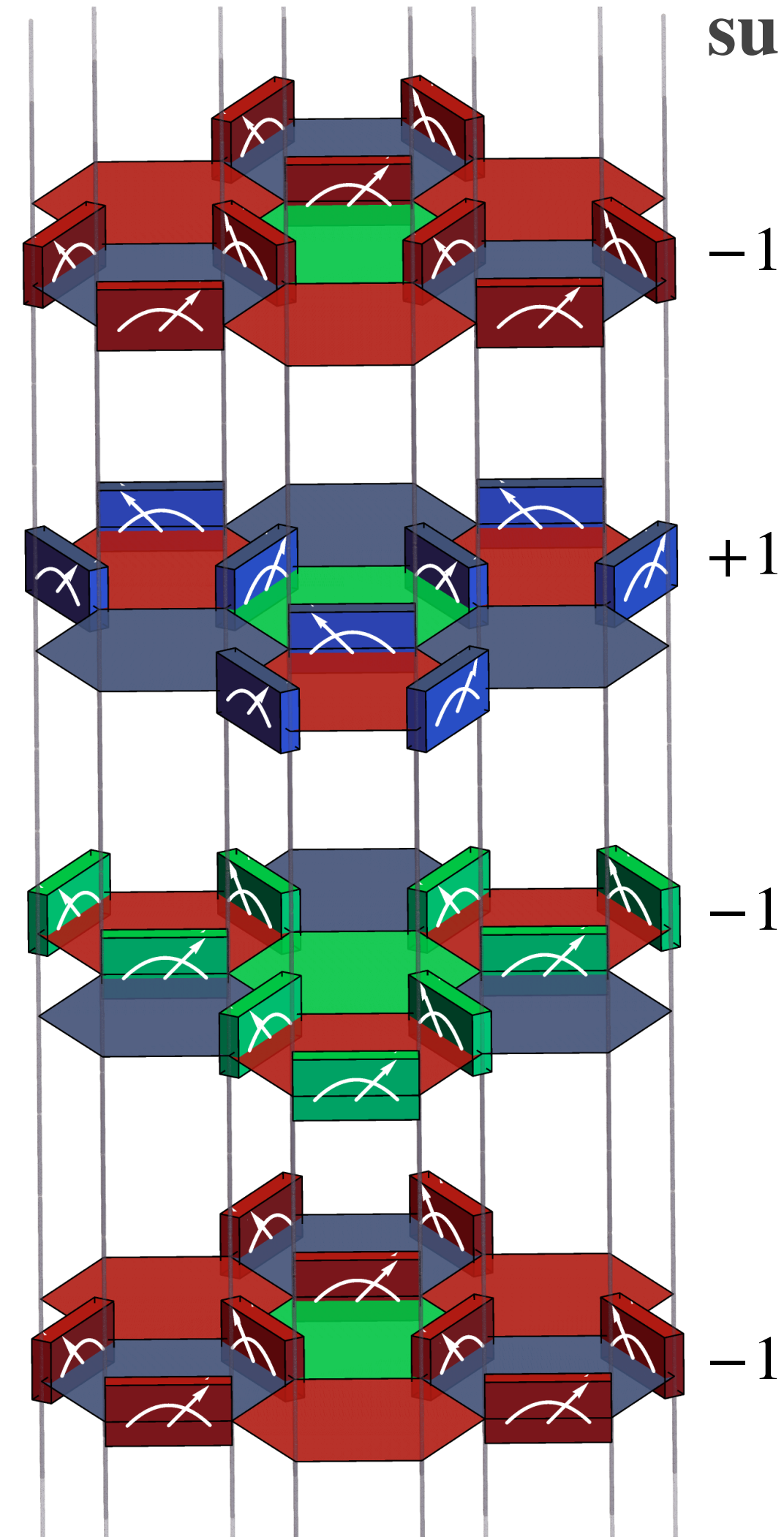
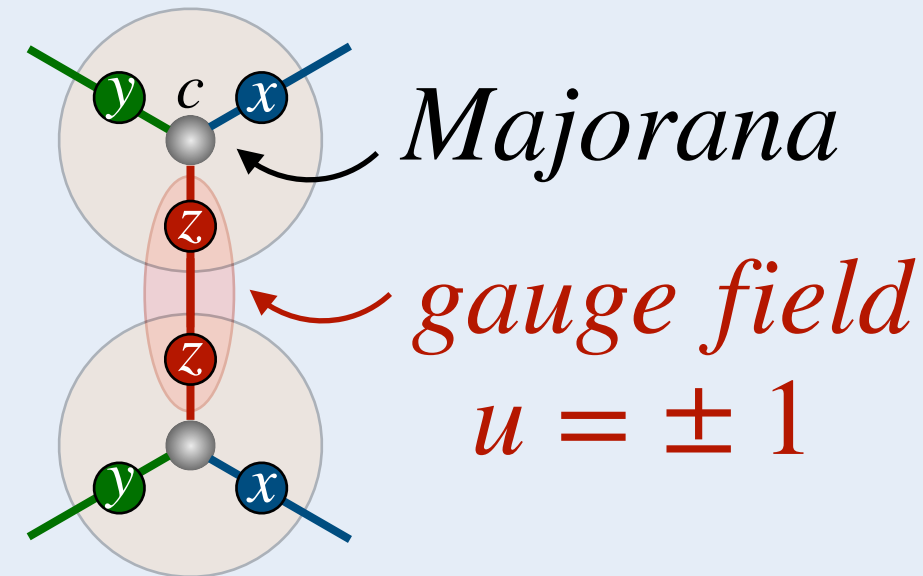
# Majorana, flux pillars, loops



Guo-Yi Zhu

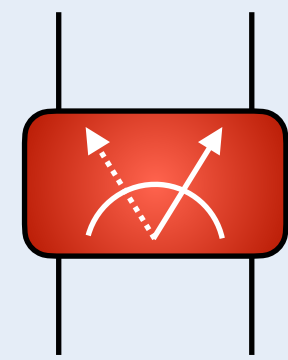


$$\begin{aligned} Z &= ib^z c \\ X &= ib^x c \\ Y &= ib^y c \end{aligned}$$



random Gaussian fermion circuit  
conditioned on  
gauge trajectory su

$$\exp(-\tau s Z Z) \quad (s = \pm 1)$$

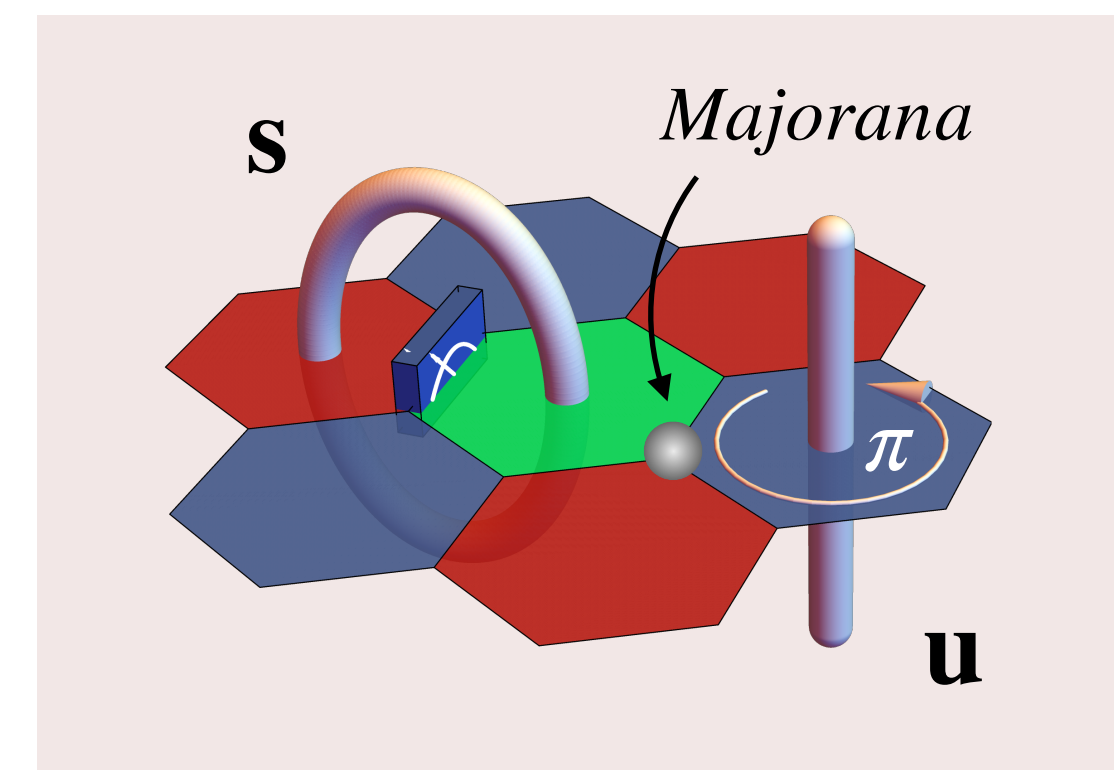


$$\exp(-\tau \quad (su) \quad ic_A c_B)$$

measurement strength  
 $\tau = \tanh^{-1}(\sin(2t))$

net gauge field

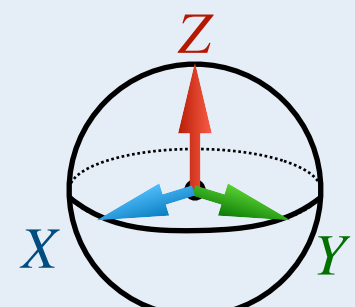
Majorana bilinear



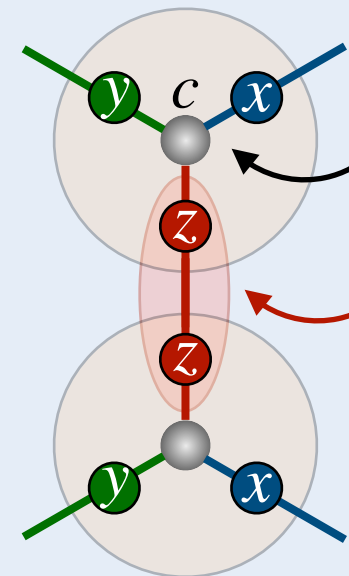
# Majorana, flux pillars, loops



Guo-Yi Zhu

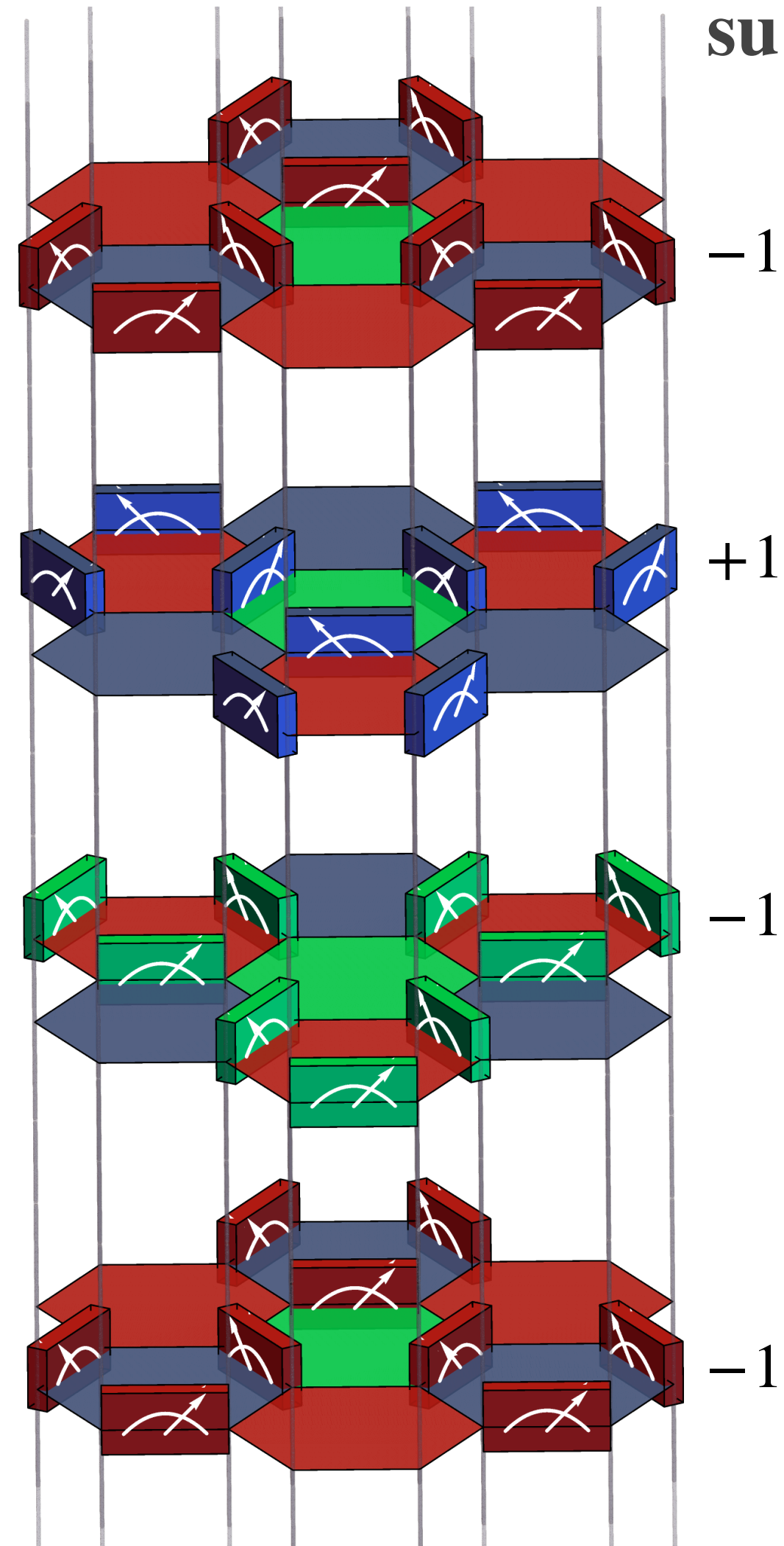


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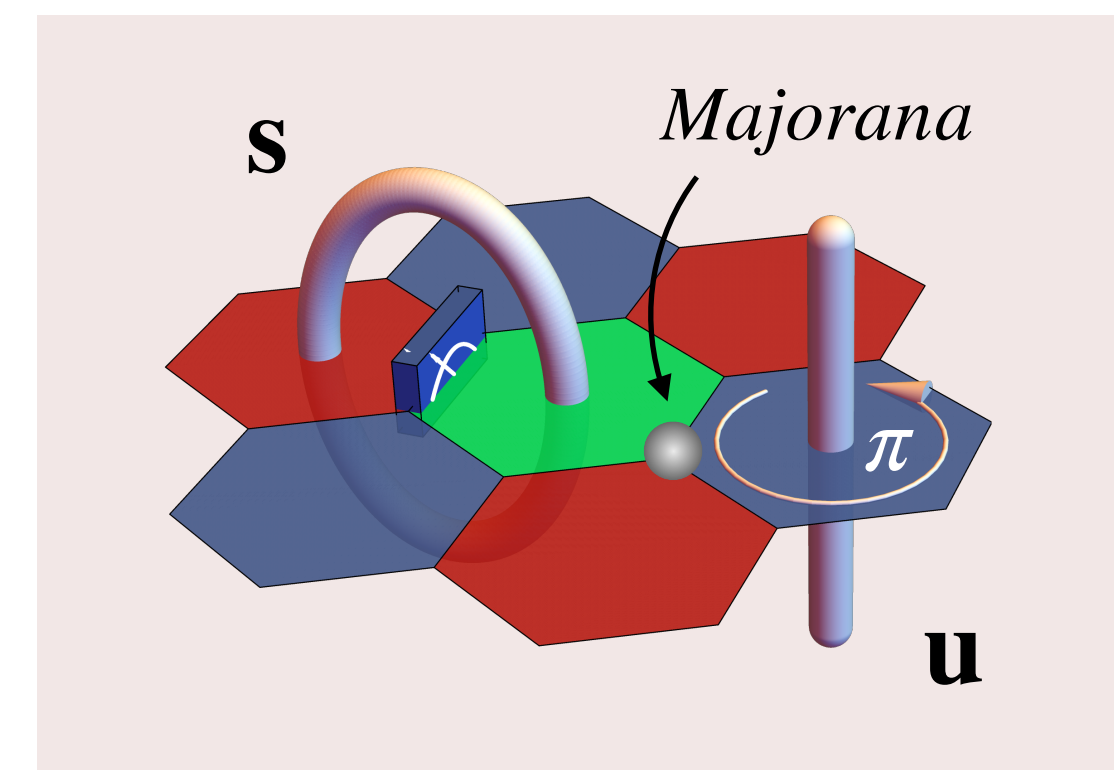


Majorana

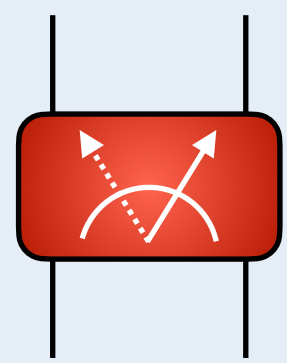
gauge field  
 $u = \pm 1$



random Gaussian fermion circuit  
conditioned on  
gauge trajectory  $su$



$$\exp(-\tau s ZZ) \quad (s = \pm 1)$$



$$\exp(-\tau \quad (su) \quad ic_A c_B)$$

measurement strength  
 $\tau = \tanh^{-1}(\sin(2t))$

net gauge field

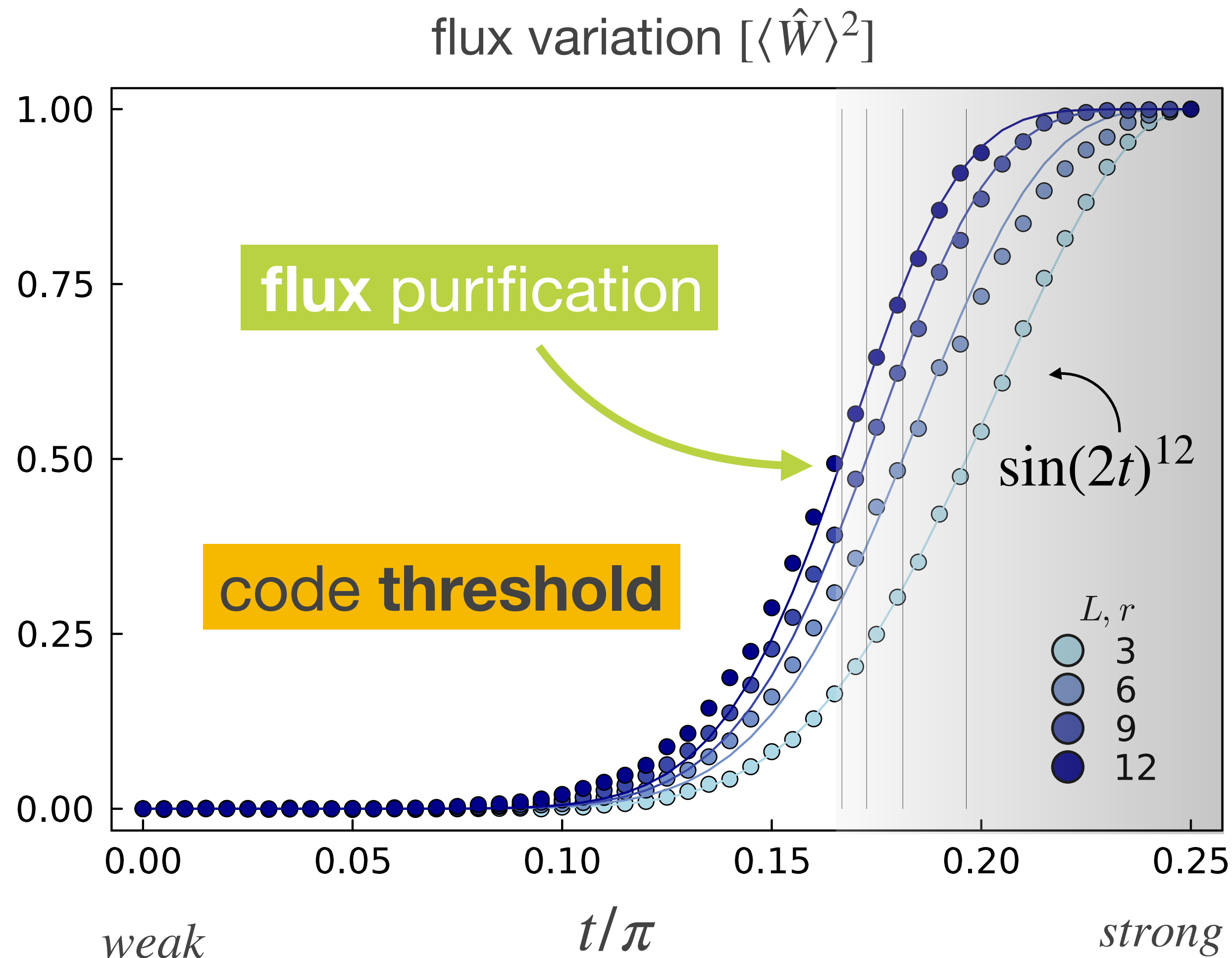
Majorana bilinear

Born probability

=

Majorana partition function

# purification of fluxes



- **flux** expectation value

$$[\langle \hat{W} \rangle] = 0$$

quantum average

measurement average

- **Edwards-Anderson** order parameter

$$[\langle \hat{W} \rangle^2] = \sum_{\mathbf{s}} P(\mathbf{s}) \langle \hat{W} \rangle_{\mathbf{s}}^2 = \sum_q \sum_{\mathbf{s}, \mathbf{u}} \frac{P_{\mathbf{s}} \cdot P_{\mathbf{s}\mathbf{u}}}{P(\mathbf{s})} \left( \prod_{l \in q} u_l \right)$$

- **exponential** purification

$$S_u := -\log_2 \frac{1 + [\langle \hat{W} \rangle^2]}{2} \approx \left( -\log_2 \frac{1 + \sin(2t)^{12}}{2} \right)^{\frac{r+1}{4}}$$

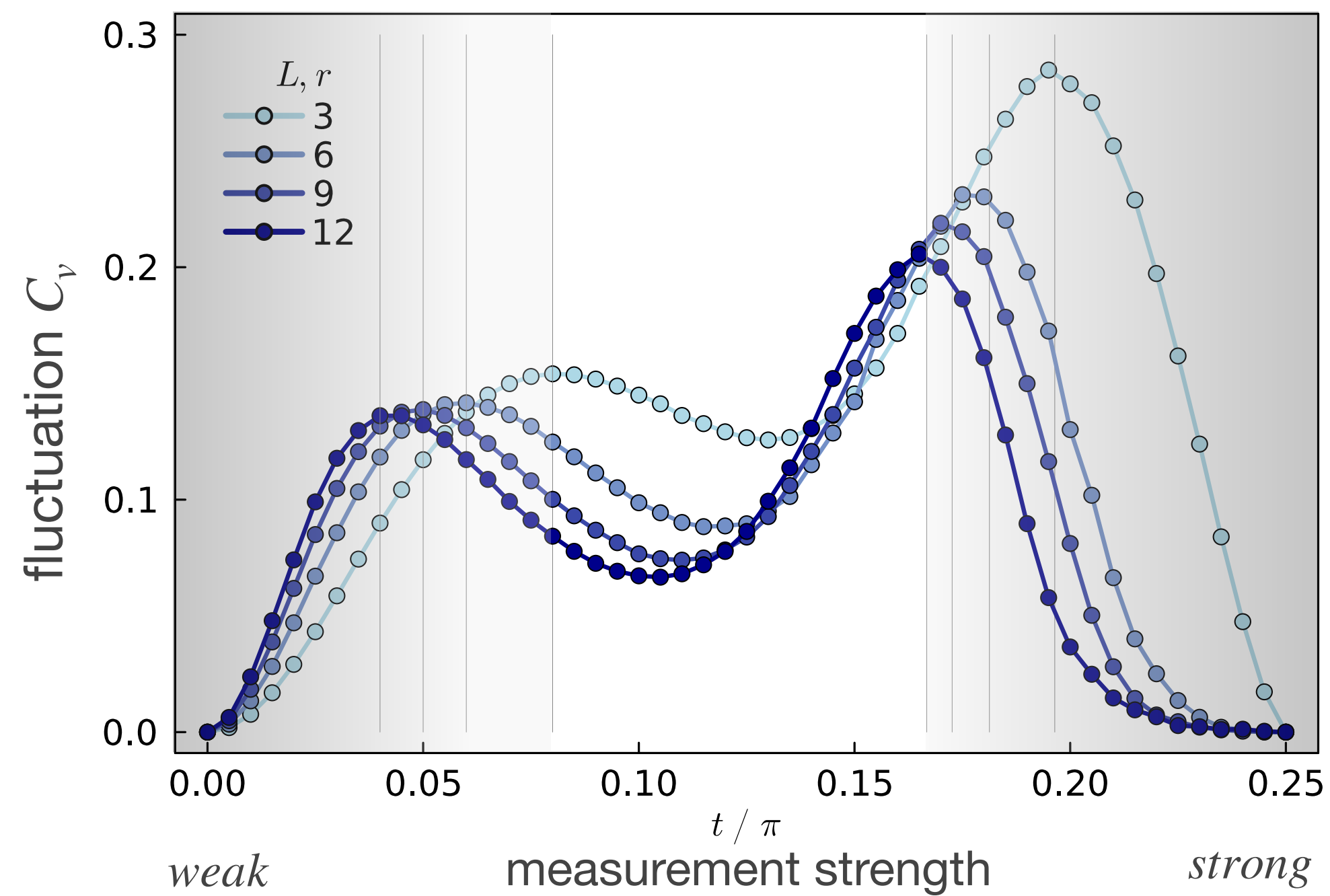
# but there is more – double-peaks

weak measurement-only circuit

circuit depth  
↓

$$\rho_{\text{su}} \propto \exp\left(-\frac{\beta}{4} \mathbf{c} H_{\text{su}} \mathbf{c}\right)$$

↑  
effective Hamiltonian

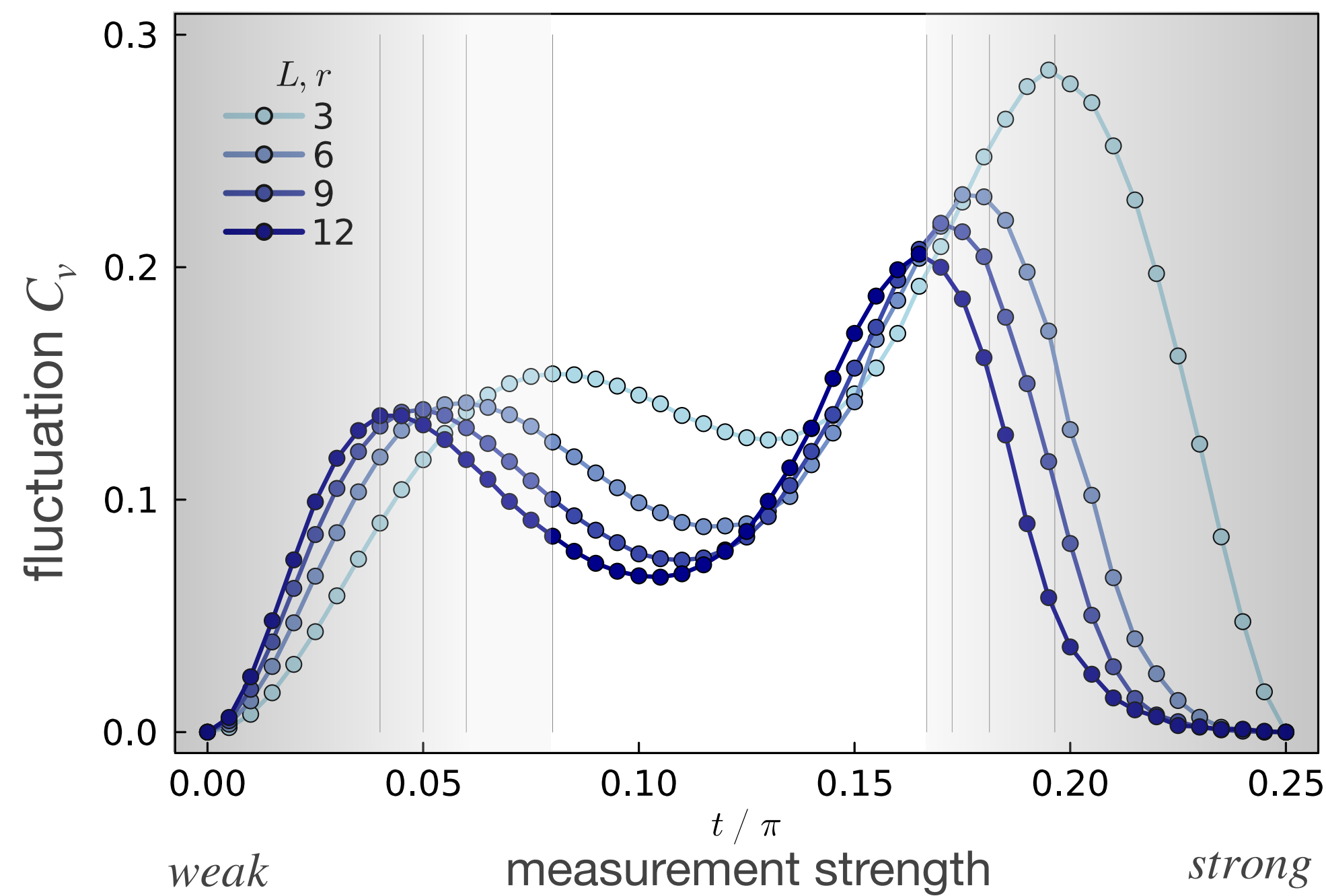


# but there is more – double-peaks

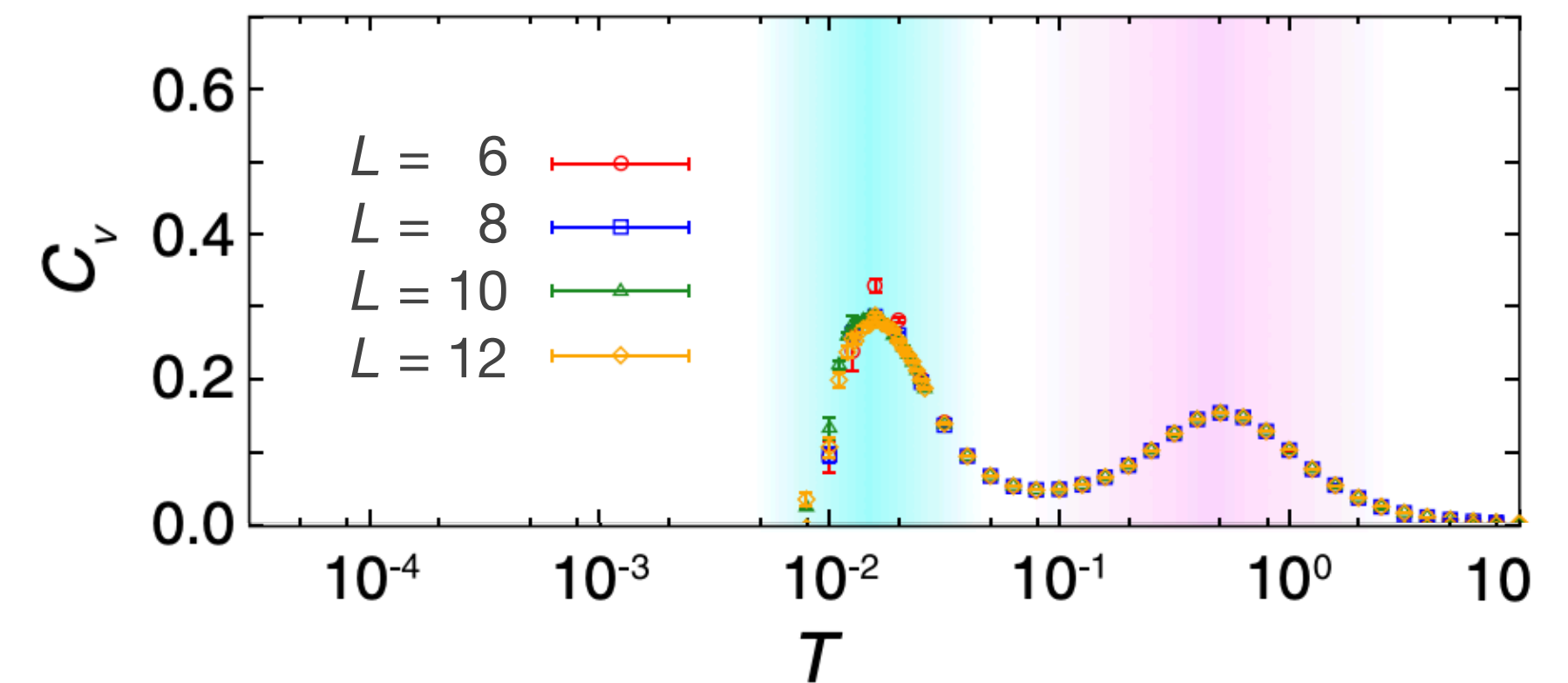
## weak measurement-only circuit

$$\rho_{\text{su}} \propto \exp\left(-\frac{\beta}{4} \mathbf{c} H_{\text{su}} \mathbf{c}\right)$$

circuit depth  
↓  
β  
↑  
effective Hamiltonian



## Hamiltonian at finite temperature



Nasu, Udagawa, Motome, 2014

PRL 113, 197205 (2014)

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7 NOVEMBER 2014

### Vaporization of Kitaev Spin Liquids

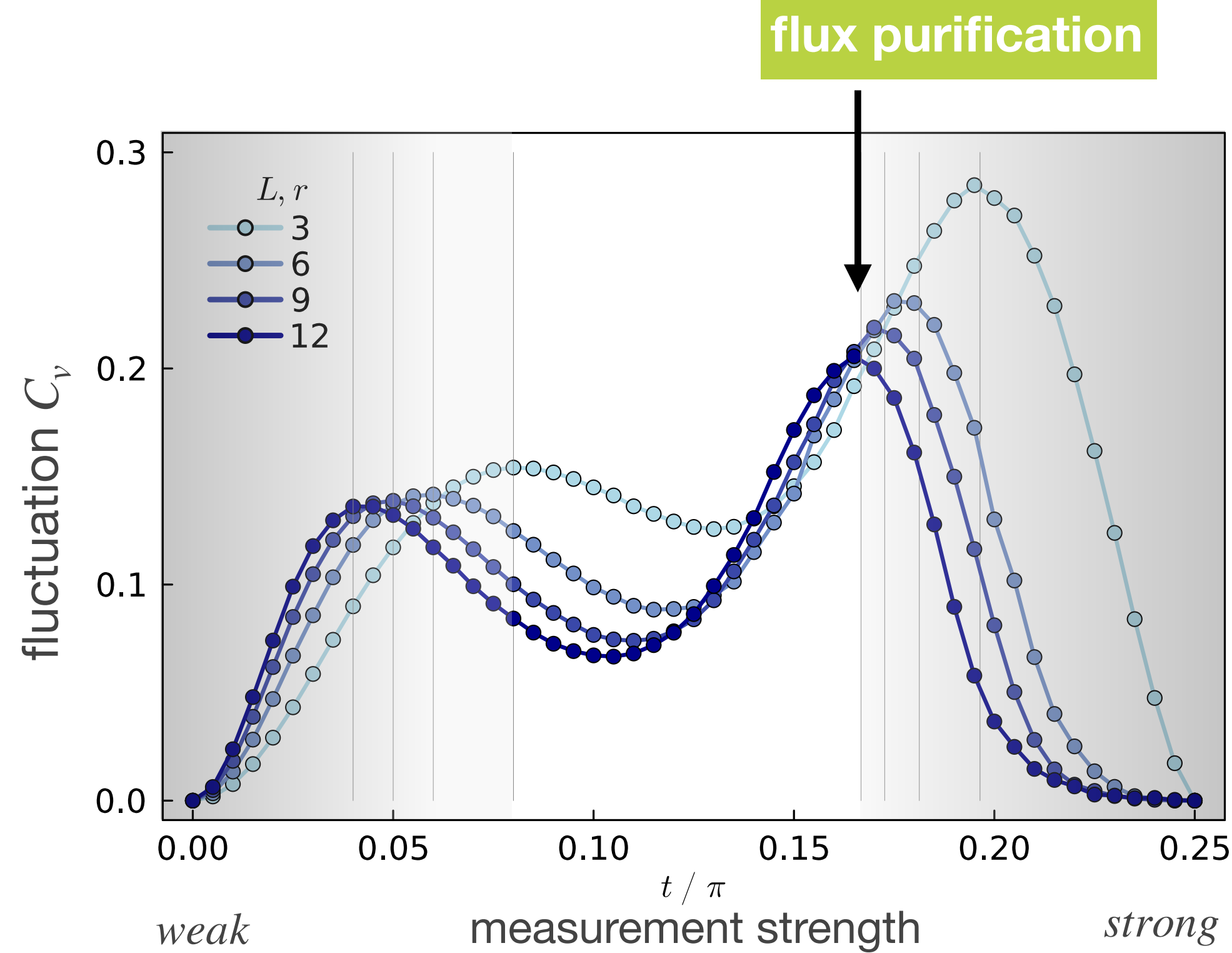
Joji Nasu,<sup>1</sup> Masafumi Udagawa,<sup>2</sup> and Yukitoshi Motome<sup>2</sup>

<sup>1</sup>Department of Physics, Tokyo Institute of Technology, Ookayama, 2-12-1, Meguro, Tokyo 152-8551, Japan

<sup>2</sup>Department of Applied Physics, University of Tokyo, Hongo, 7-3-1, Bunkyo, Tokyo 113-8656, Japan  
(Received 24 July 2014; revised manuscript received 9 October 2014; published 7 November 2014)

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weak measurement-only circuit

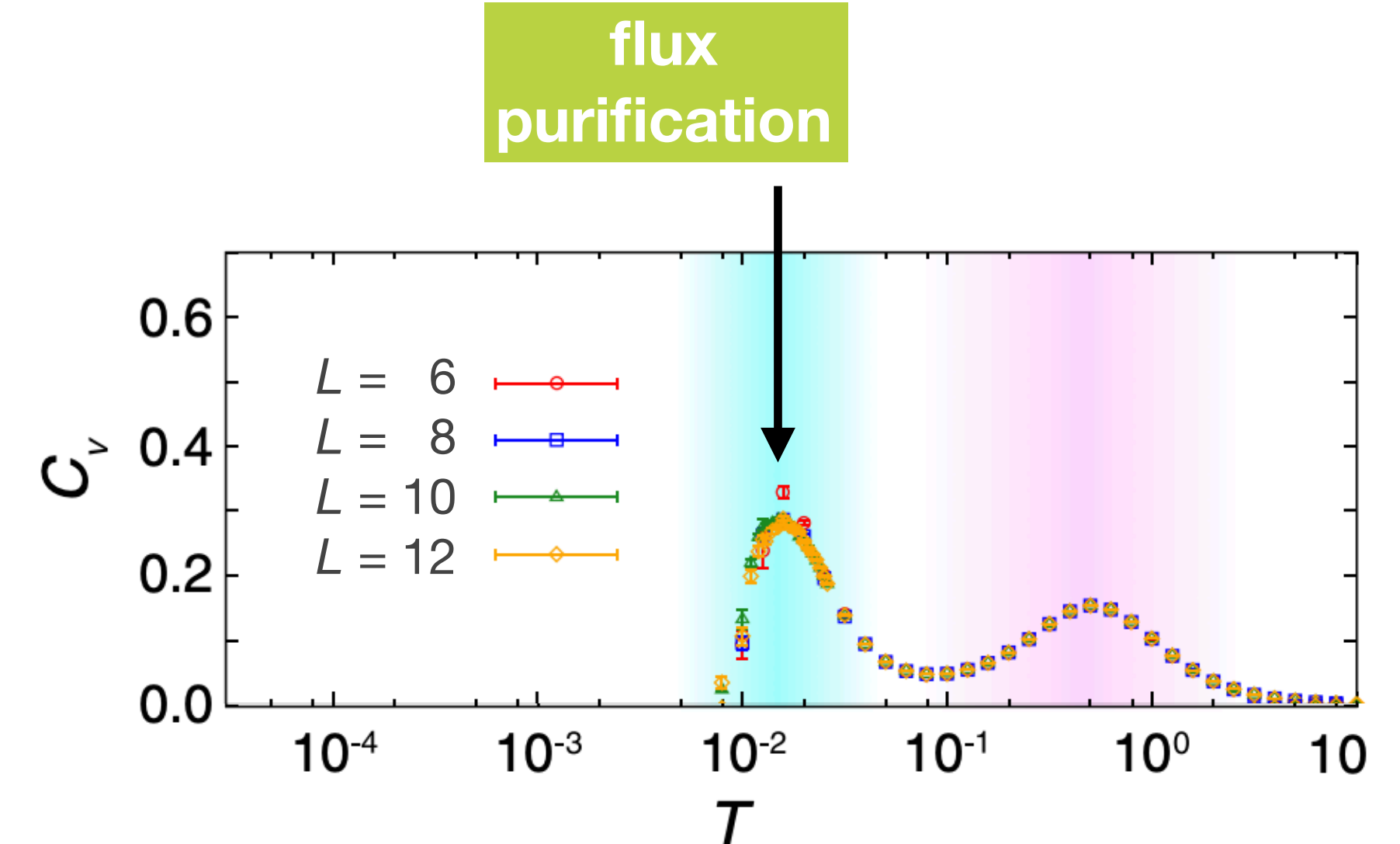


circuit depth

$$\rho_{\text{su}} \propto \exp\left(-\frac{\beta}{4} \mathbf{c} H_{\text{su}} \mathbf{c}\right)$$

effective Hamiltonian

Hamiltonian at finite temperature



Nasu, Udagawa, Motome, 2014

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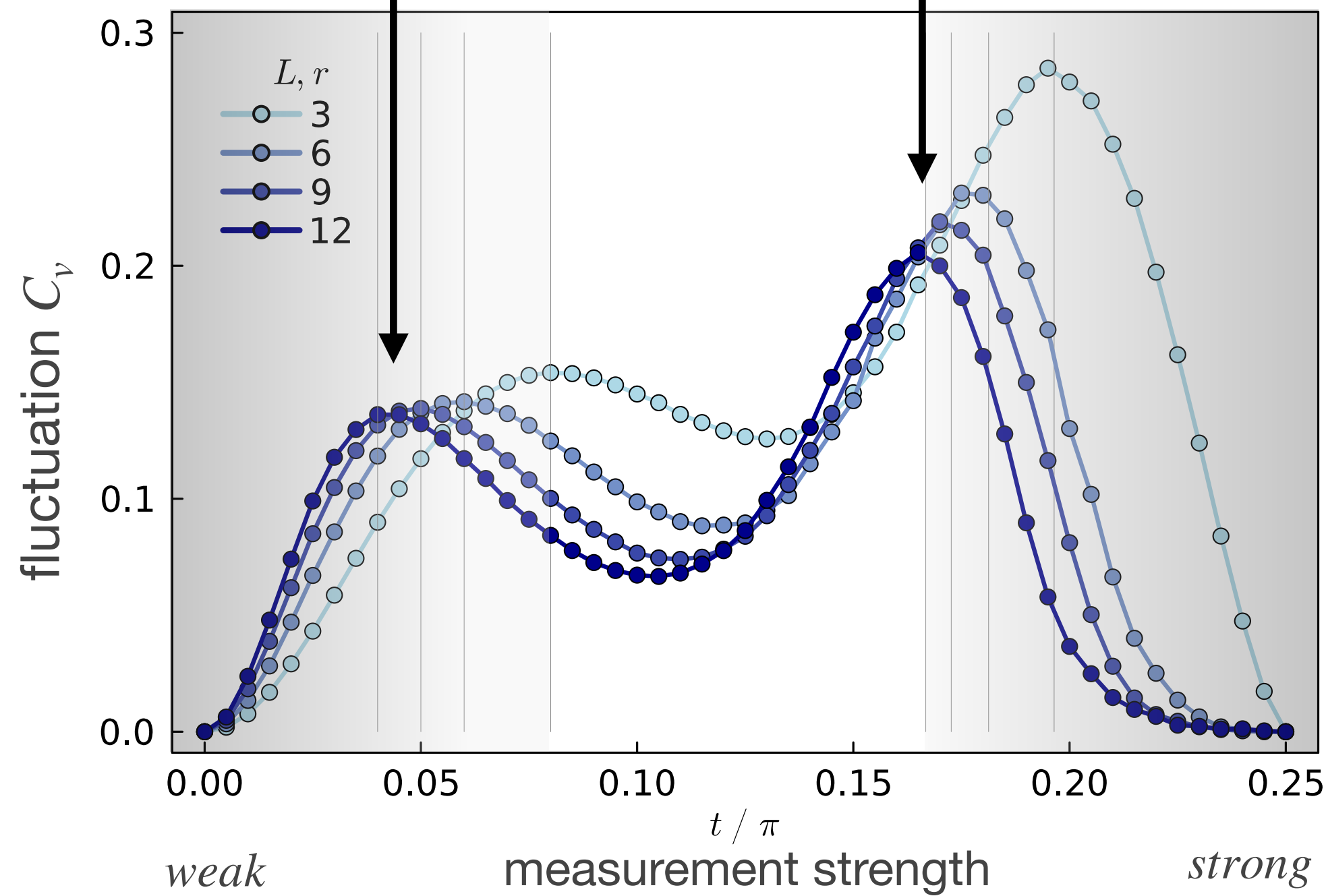
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# but there is more – double-peaks

weak measurement-only circuit

qubit fractionalization

flux purification



circuit depth

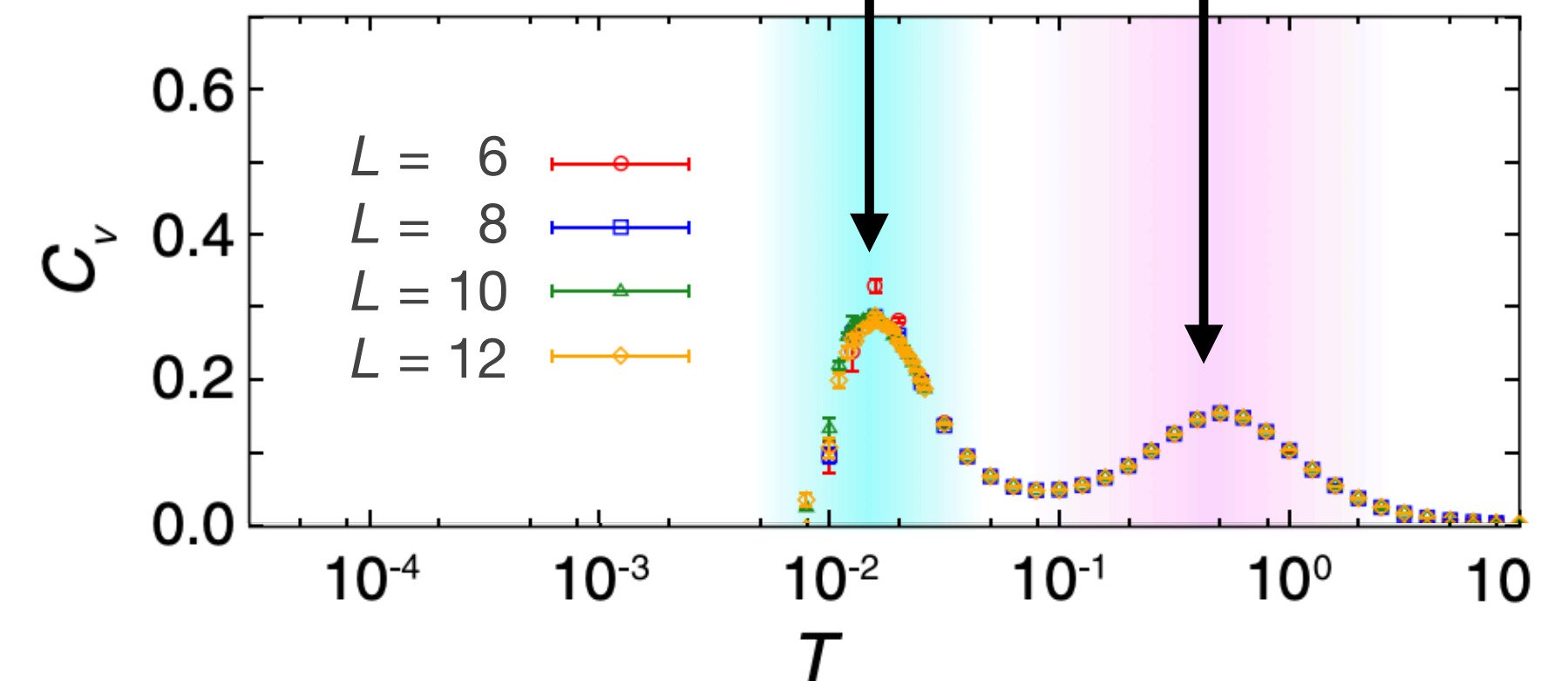
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effective Hamiltonian

Hamiltonian at finite temperature

flux purification

spin fractionalization



Nasu, Udagawa, Motome, 2014

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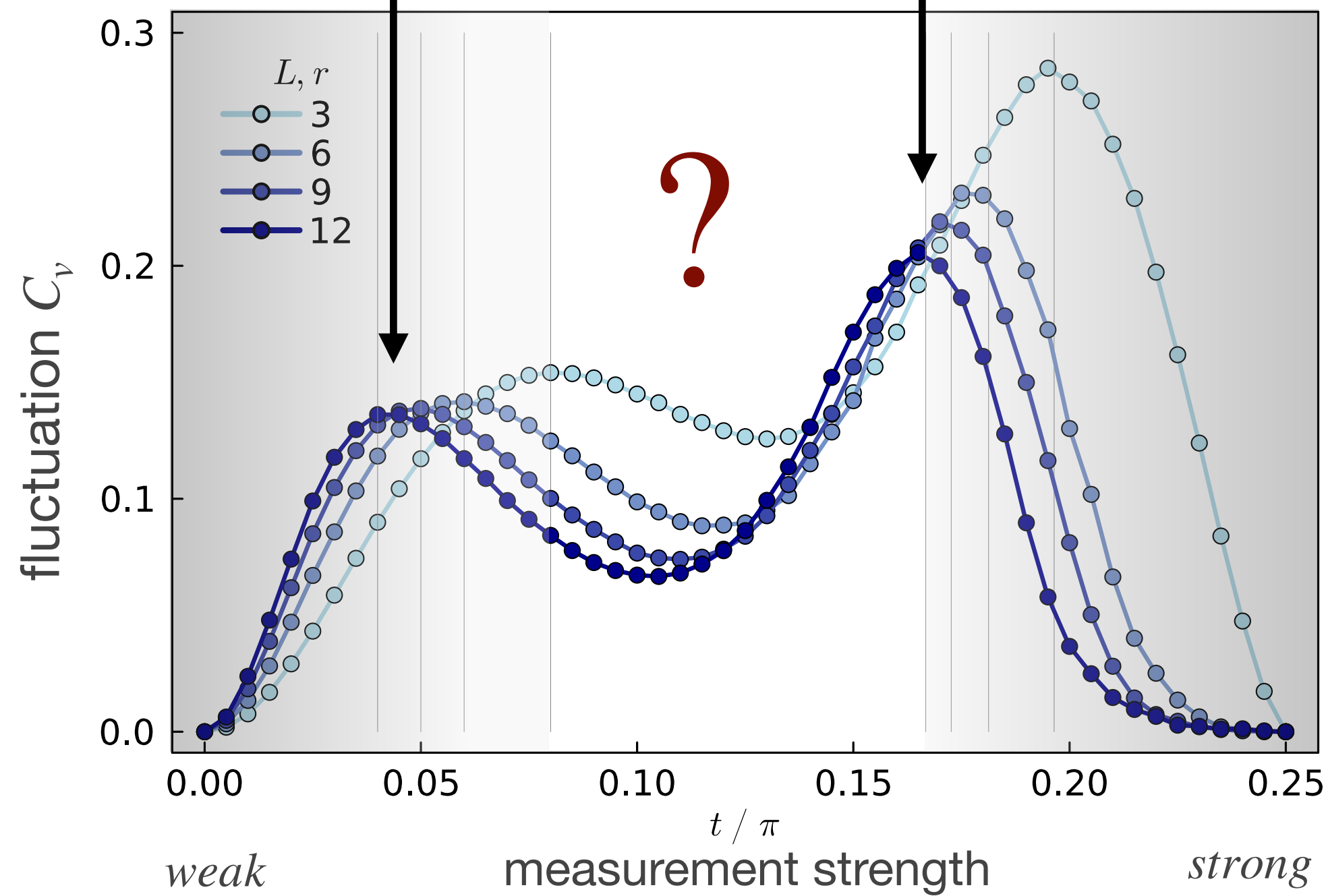


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circuit depth

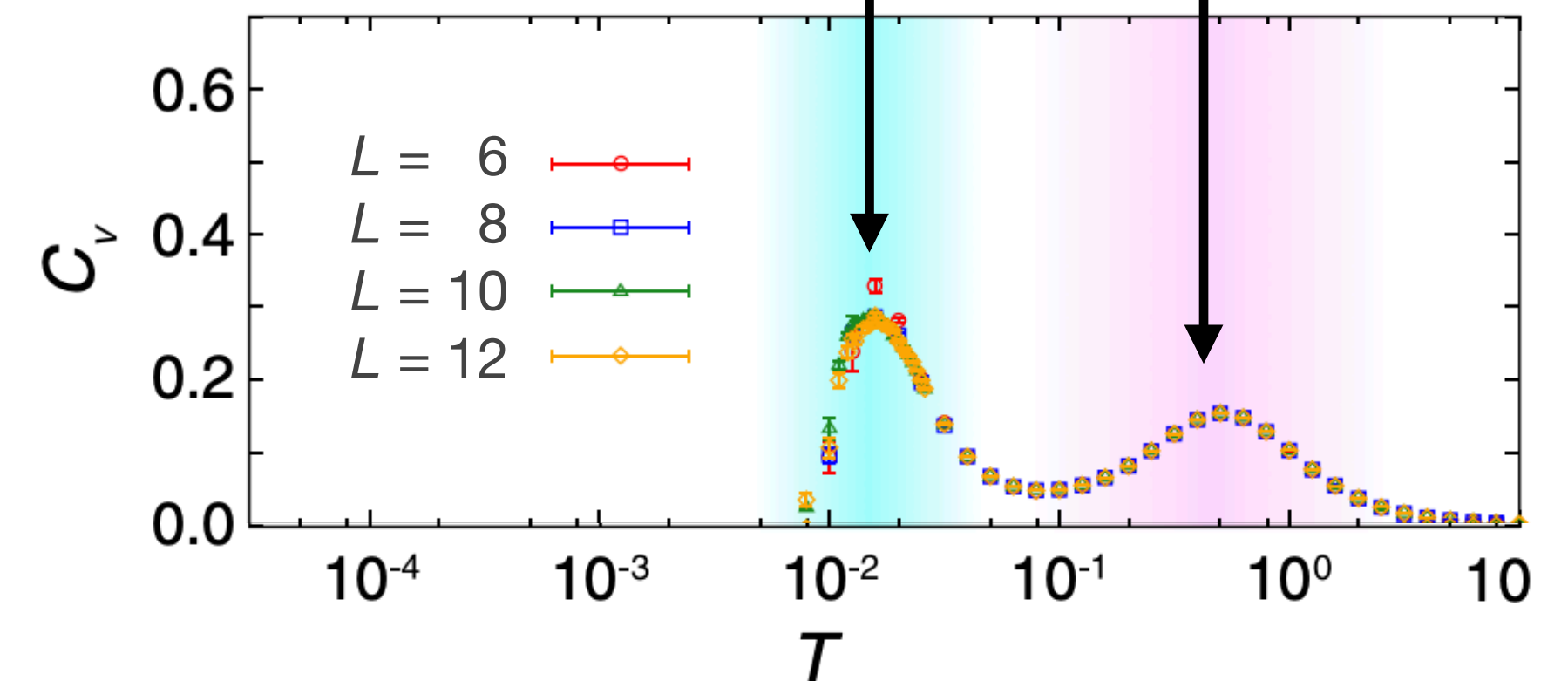
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effective Hamiltonian

Hamiltonian at finite temperature

flux purification

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Nasu, Udagawa, Motome, 2014

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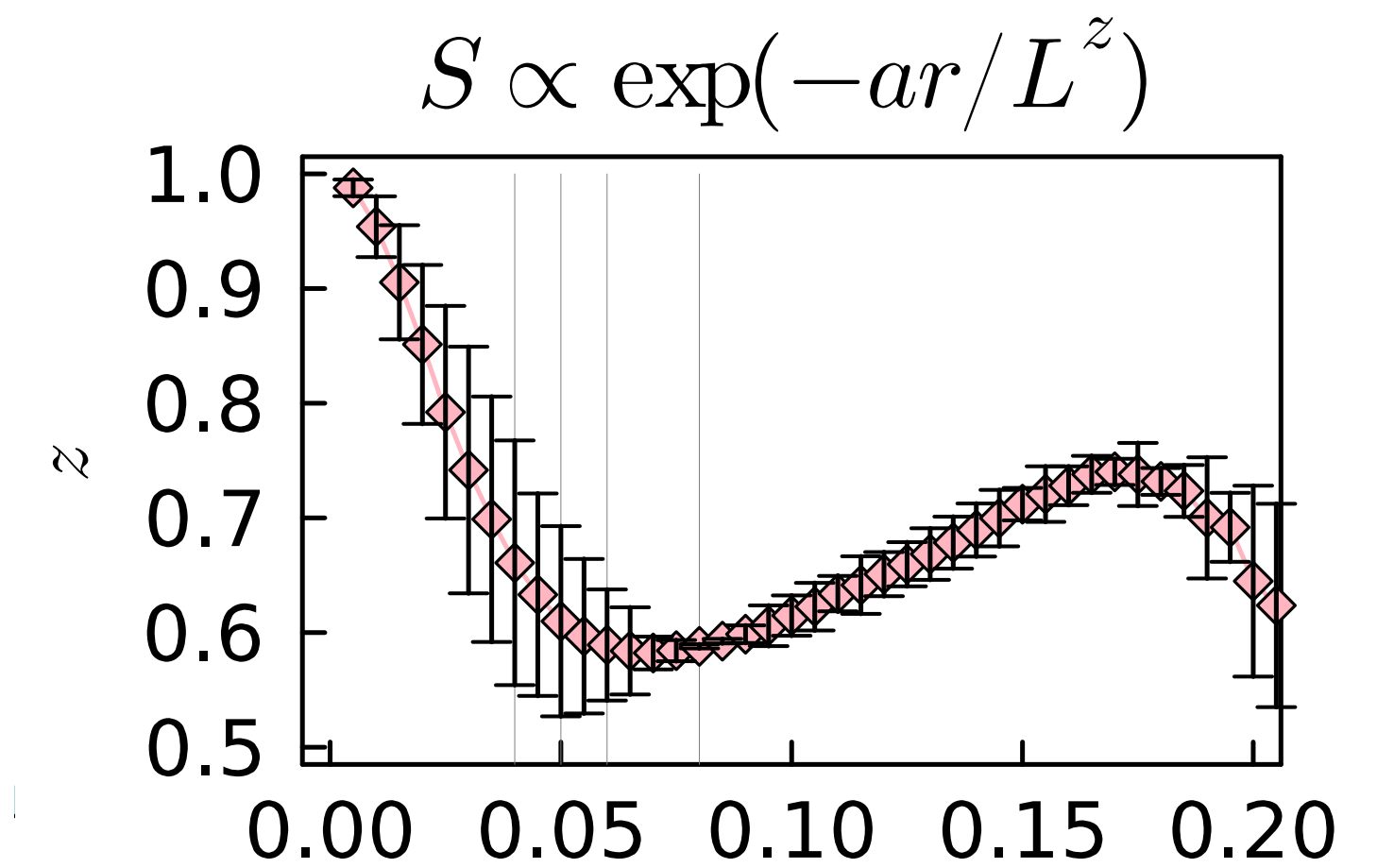
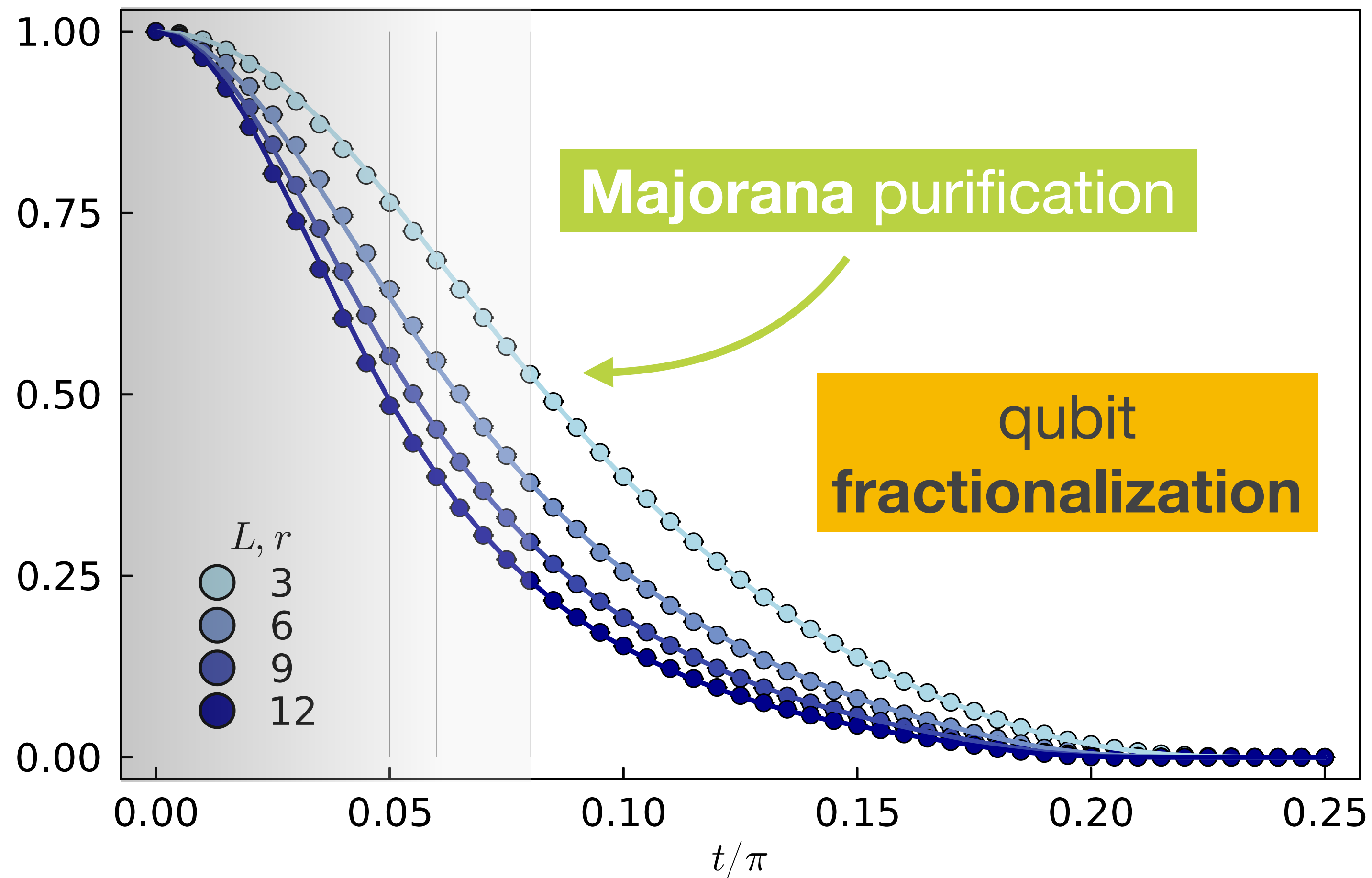
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<sup>2</sup>Department of Applied Physics, University of Tokyo, Hongo, 7-3-1, Bunkyo, Tokyo 113-8656, Japan  
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# purification of Majoranas

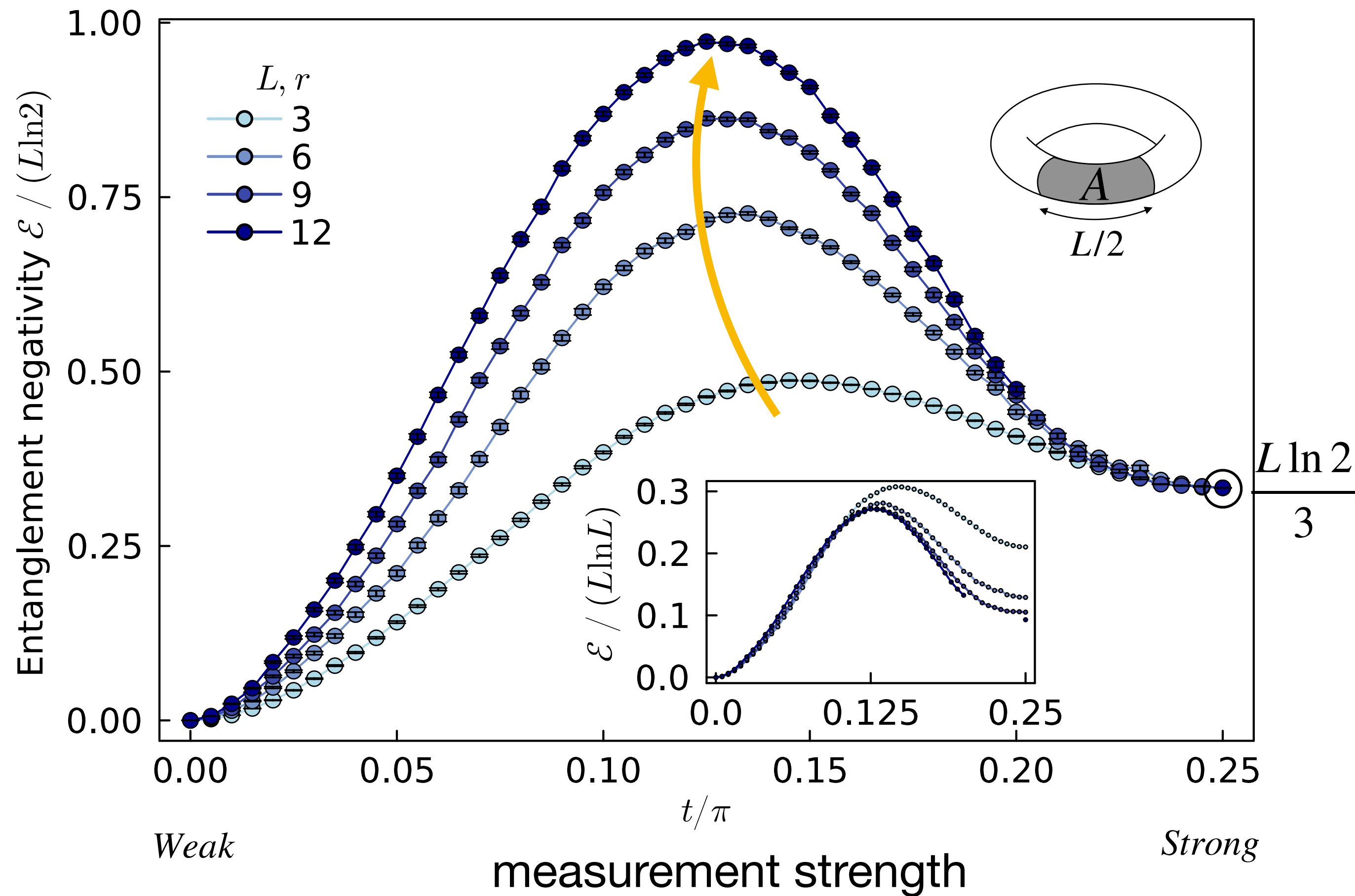
Majorana entropy density [ $\ln 2$ ]

$$S = \beta(E - F)$$



dynamical  
critical exponent

# Majorana liquid



- fermionic **entanglement negativity**
  - definition: response under *partial* time reversal
  - property: **distill out thermal entropy**
  - diagnose: **mixed state entanglement**

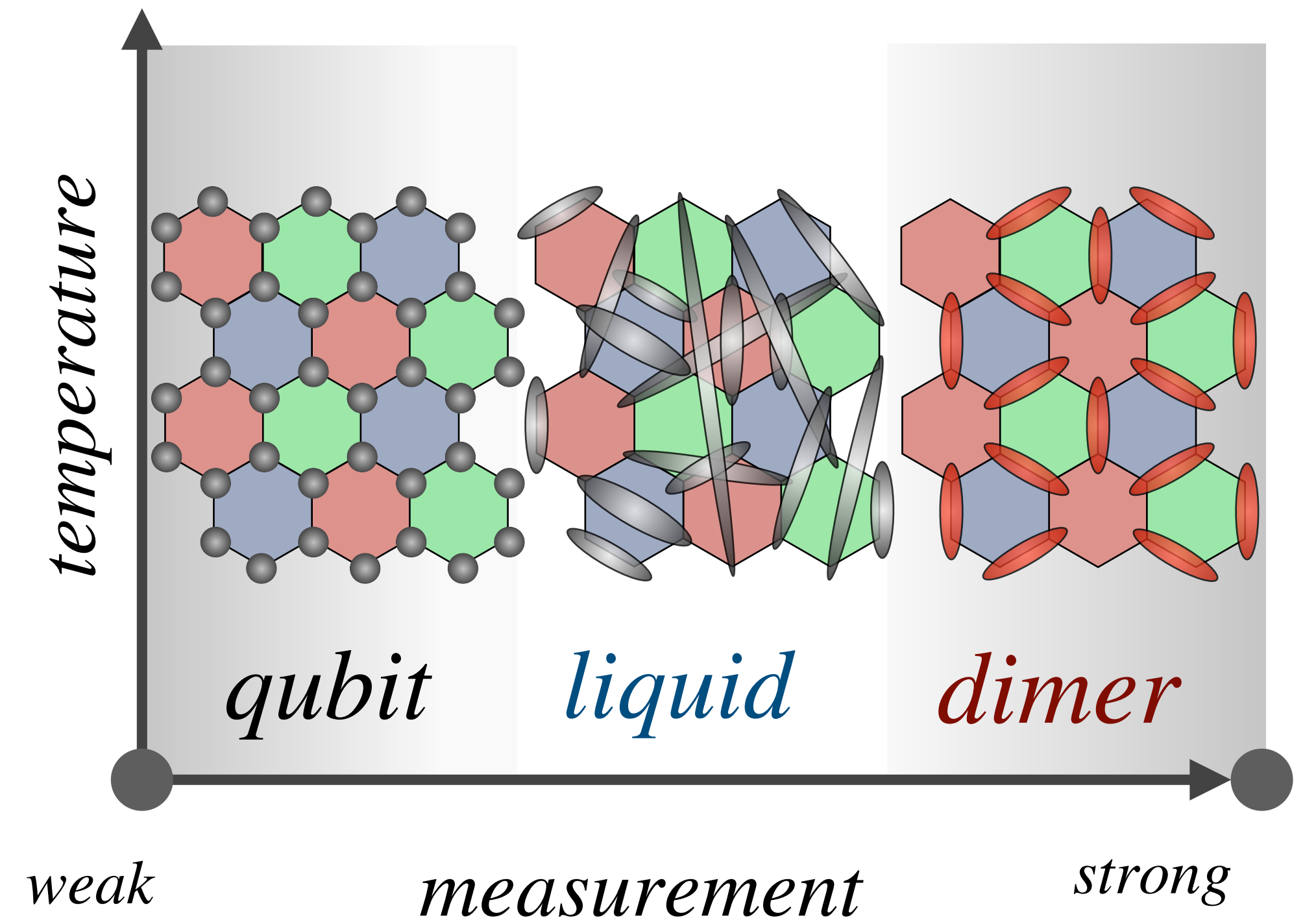
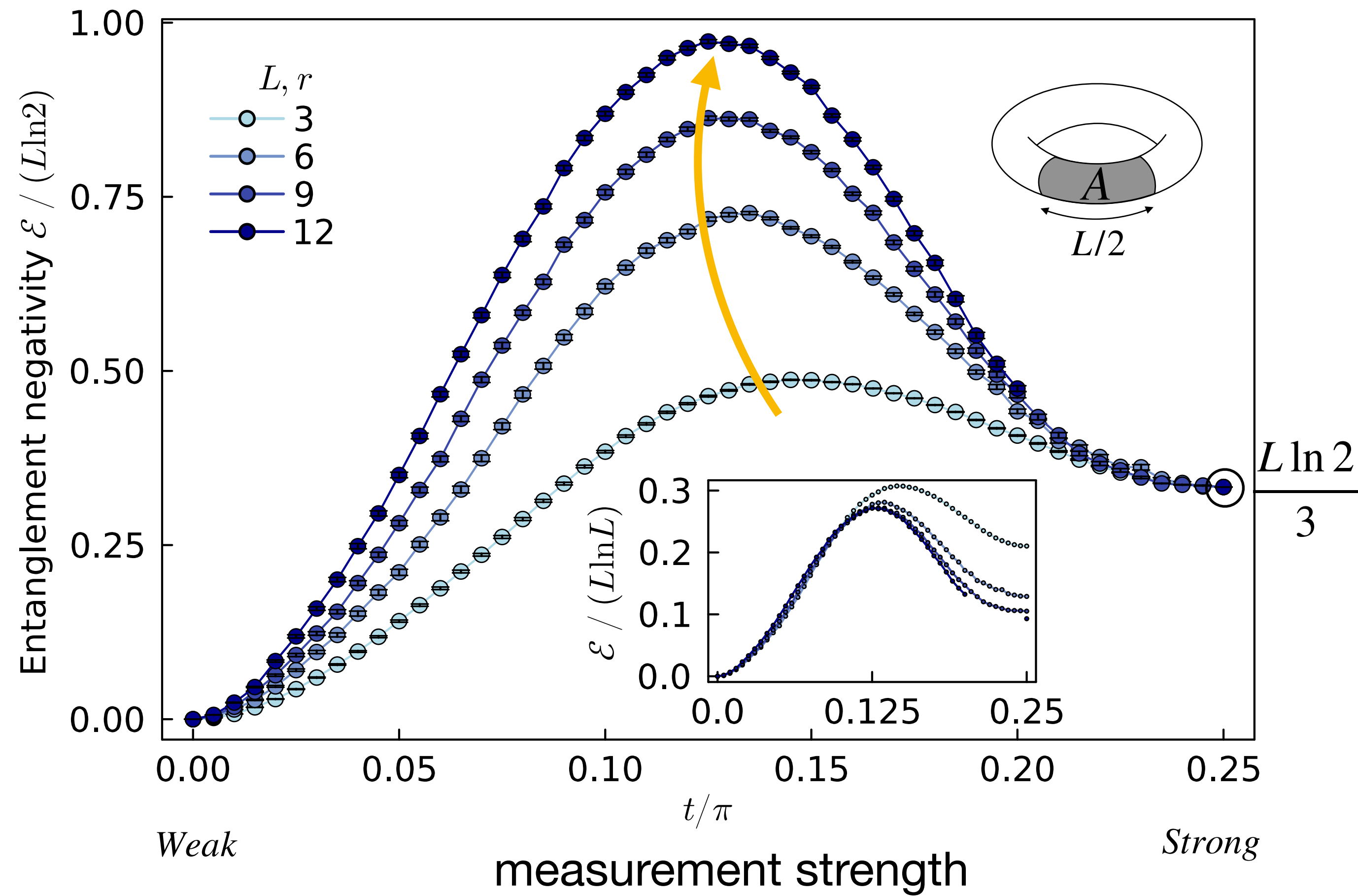
Shapourian, Shiozaki, Ryu, 2017

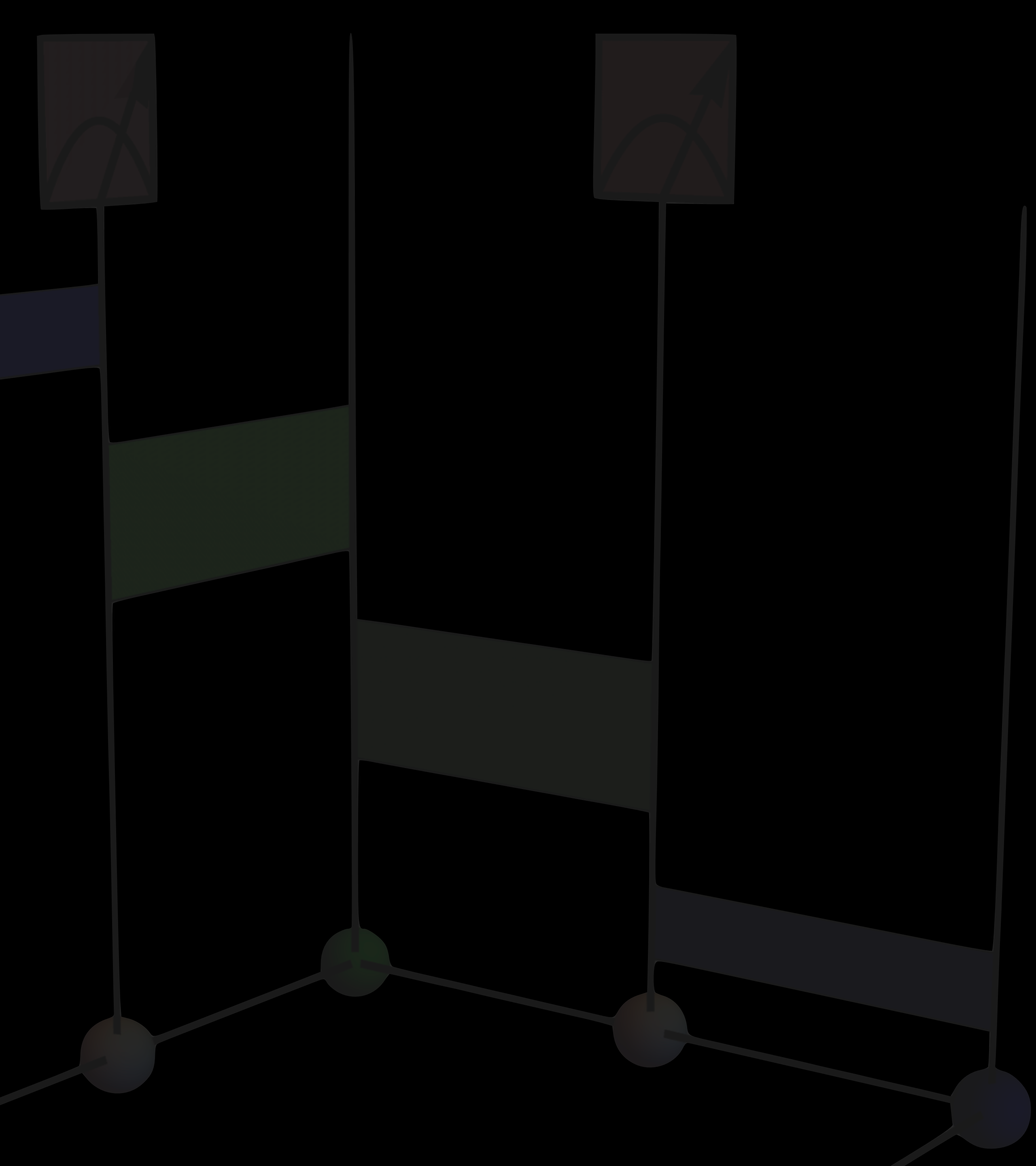
$$\mathcal{E} = \sum_{\text{su}} p_{\text{su}} \cdot \ln || \rho_{\text{su}}^{R_A} ||_1$$

- **entanglement phase transition**  
stable fermion phase with  $L \ln L$  entanglement

Fava, Piroli, Swann, Bernard, Nahum, NLM, 2023

# Majorana liquid





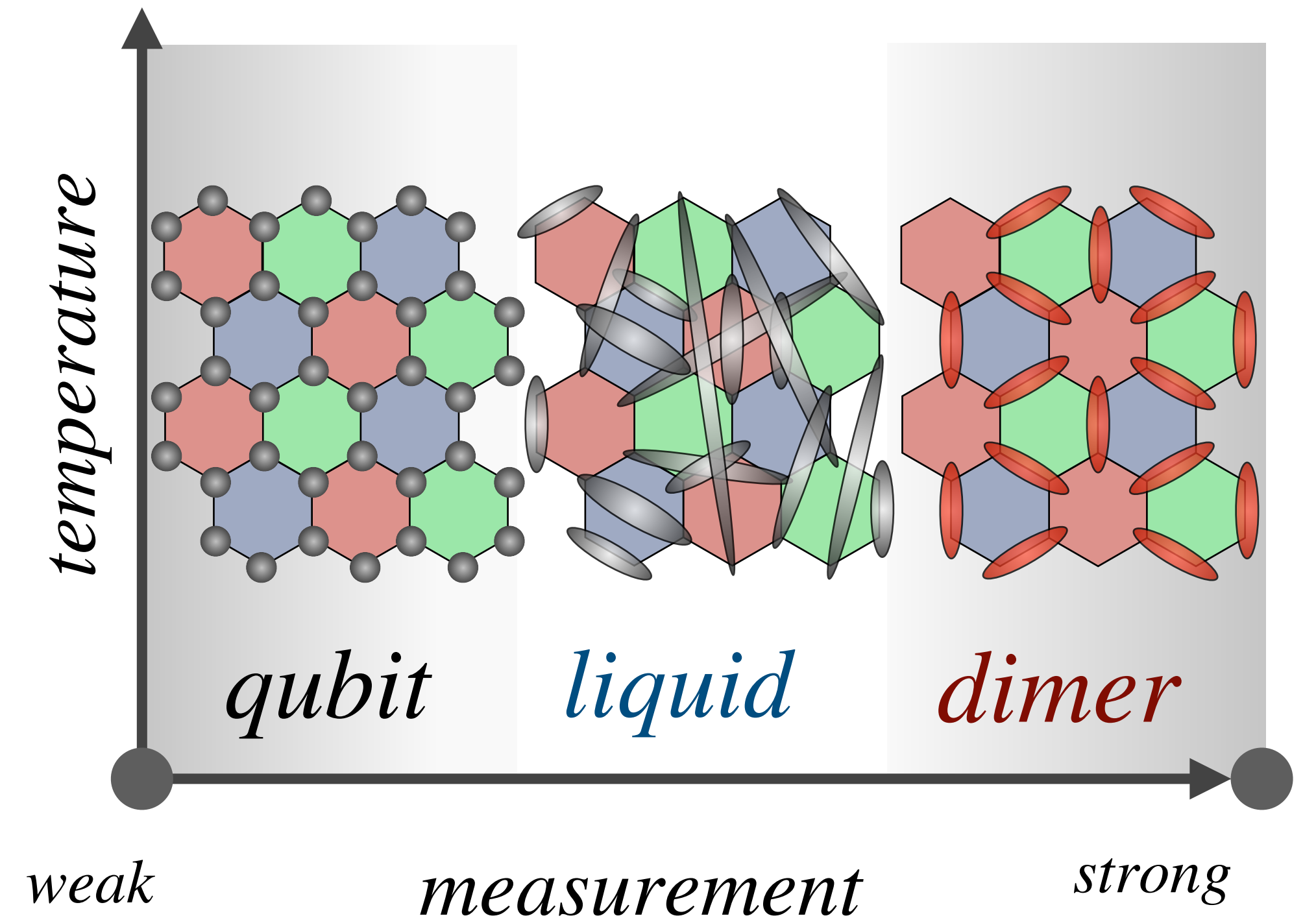
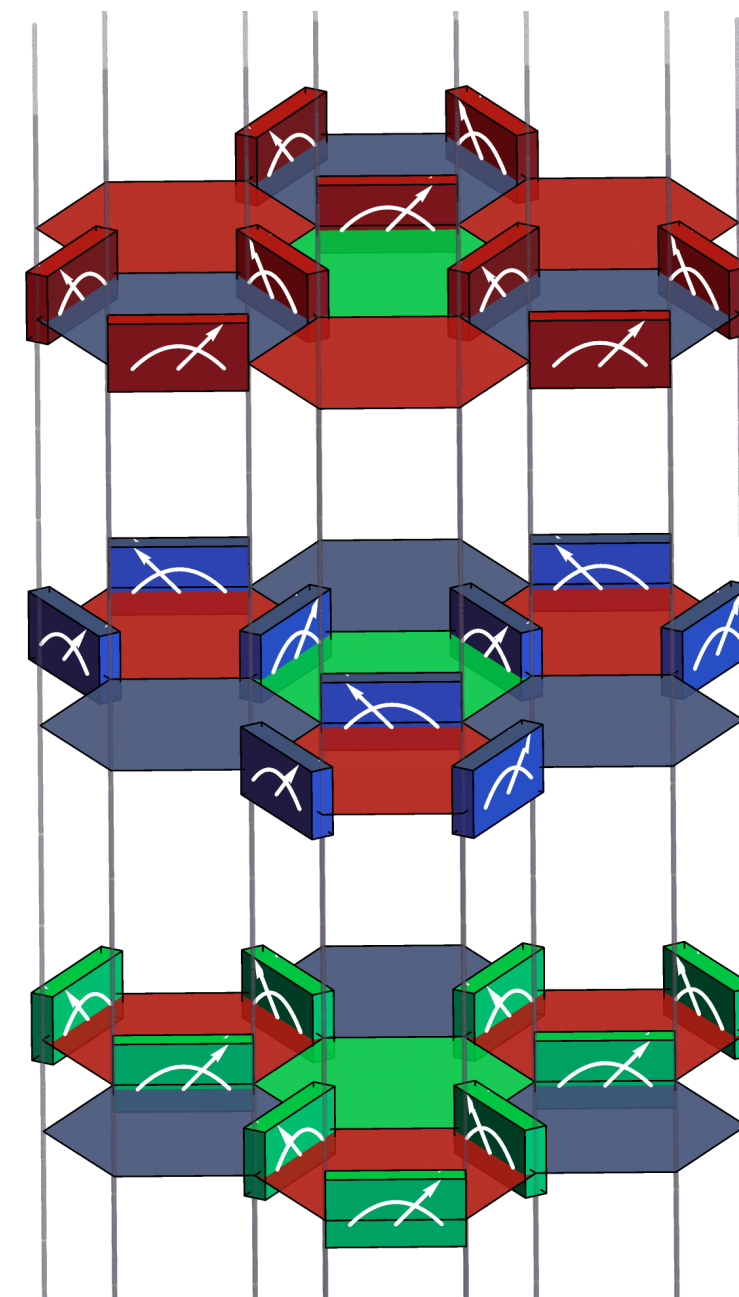
**summary**

# Floquet code — conclusions



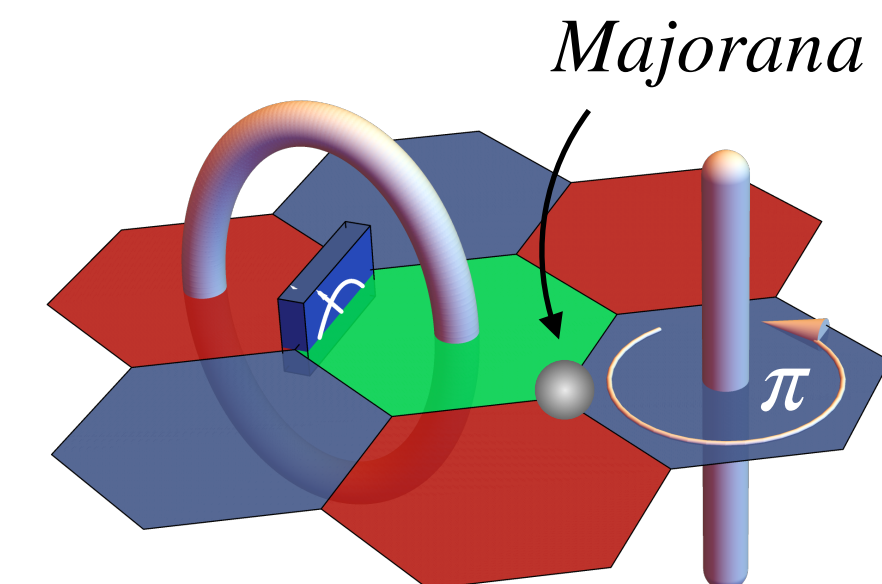
Guo-Yi Zhu

- **frustration** & **qubit fractionalization** by tunable weak measurement
- Floquet code **breakdown** to non-trivial state under coherent error
- **Majoranas** escape confinement and form **long-range entangled liquid**



## Outlook

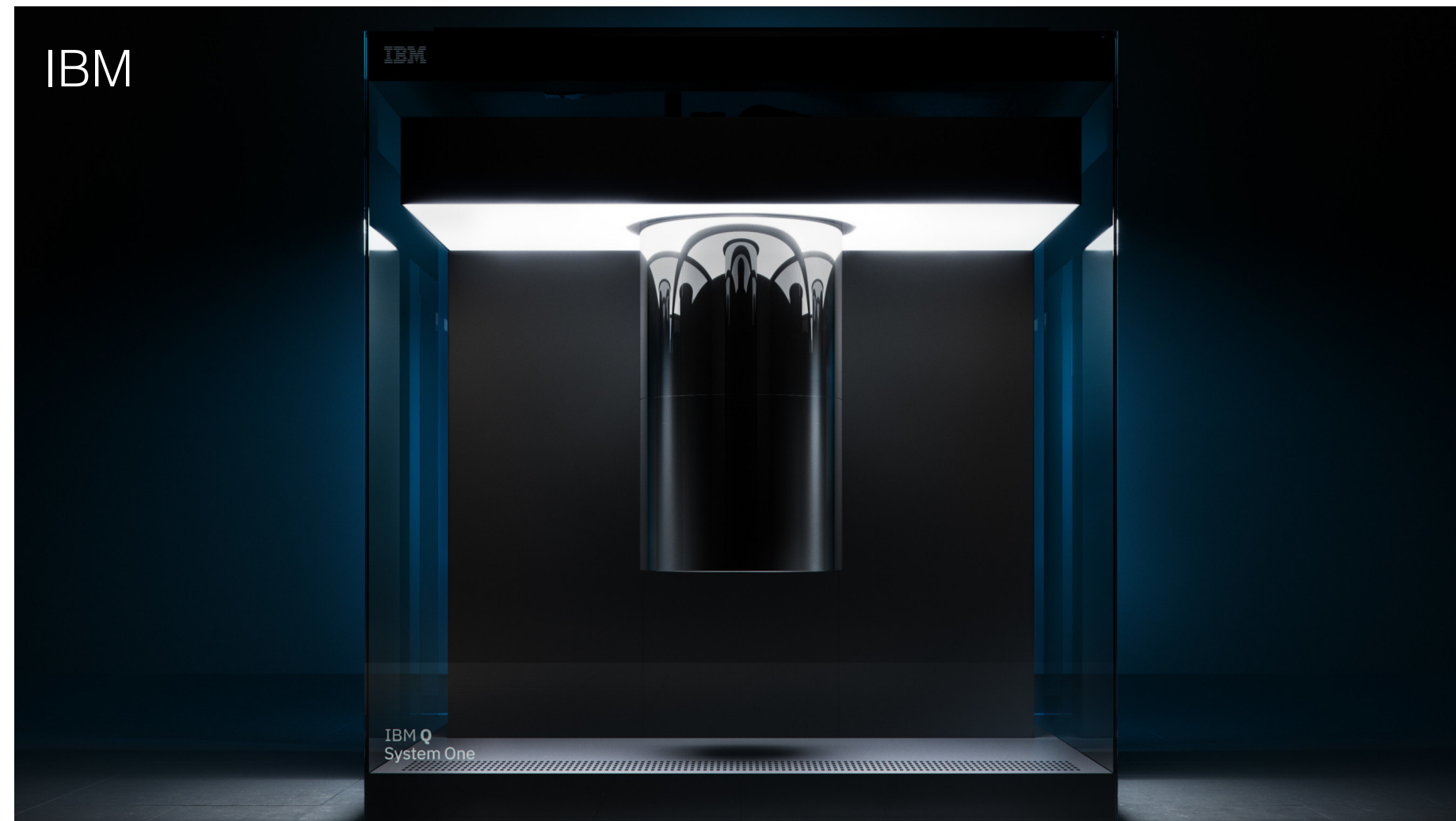
- **Feed-forward** deterministic preparation?
- topological phase transition from a parent **color code** (+ Majorana interaction)?



Guo-Yi Zhu & ST, arXiv: 2311.08450



# IBM quantum cloud devices



NISQ devices built on transmon qubits

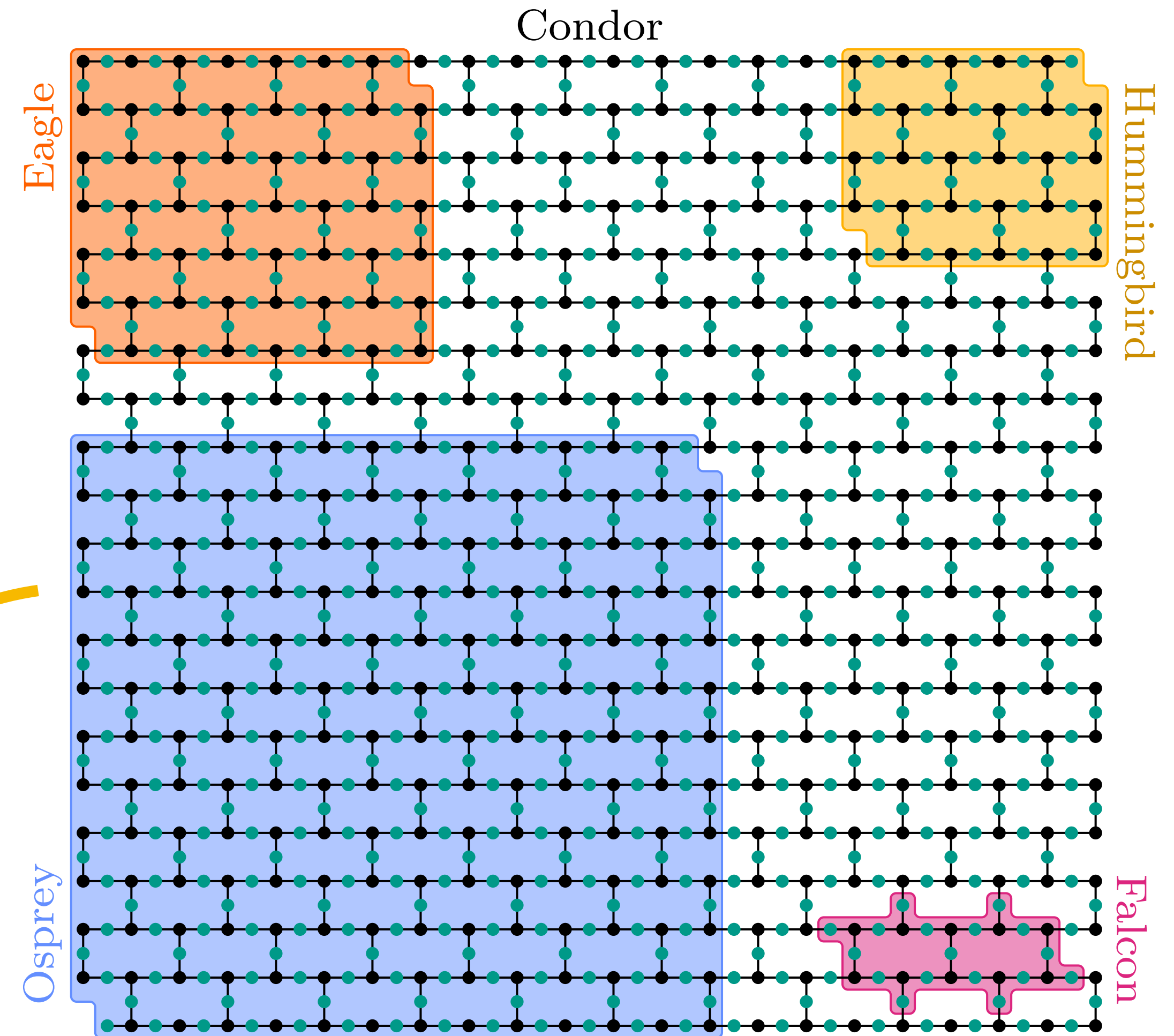
noisy intermediate  
scale quantum  
devices

heavy-hexagon

geometry

+

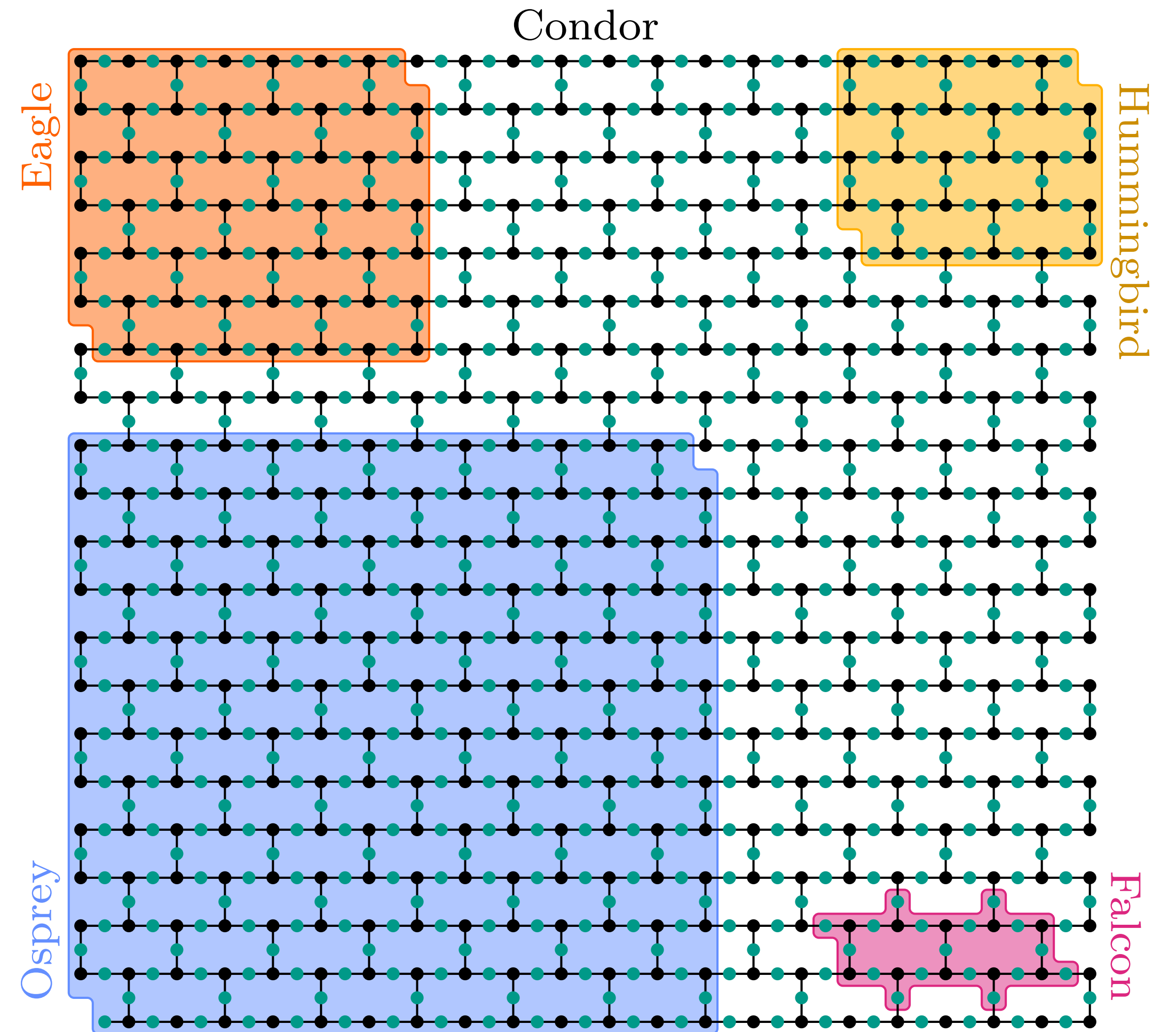
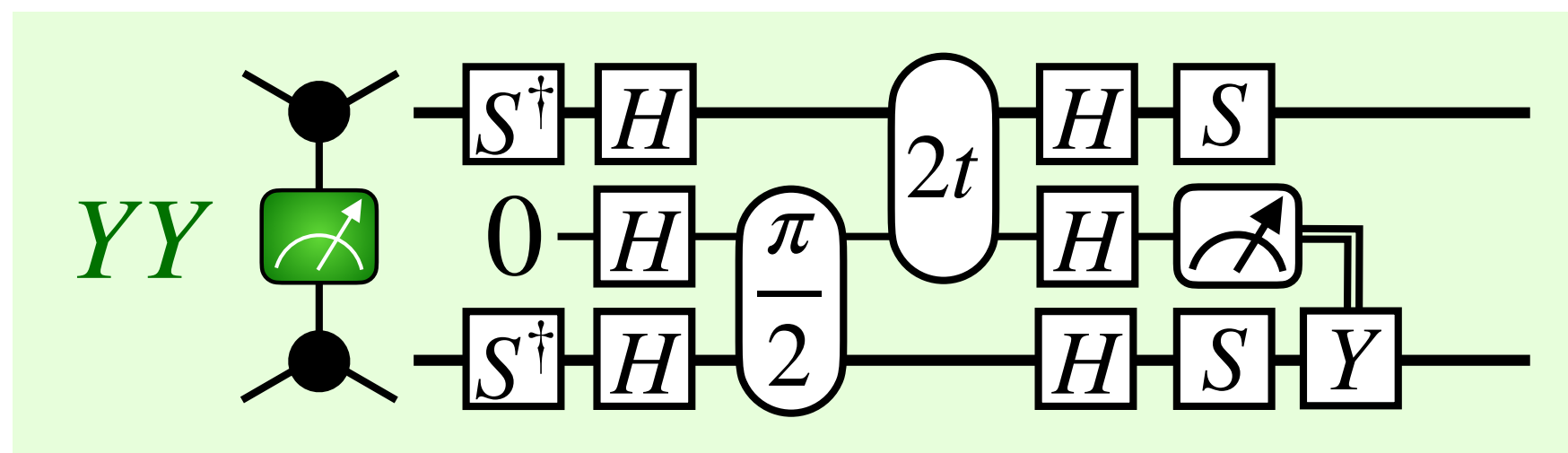
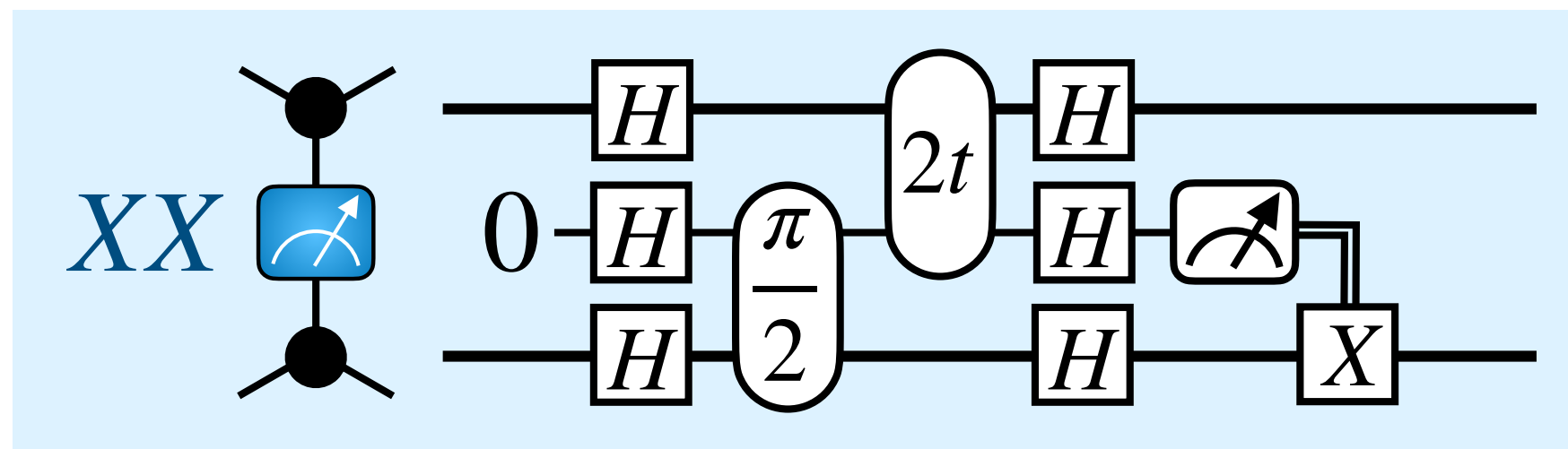
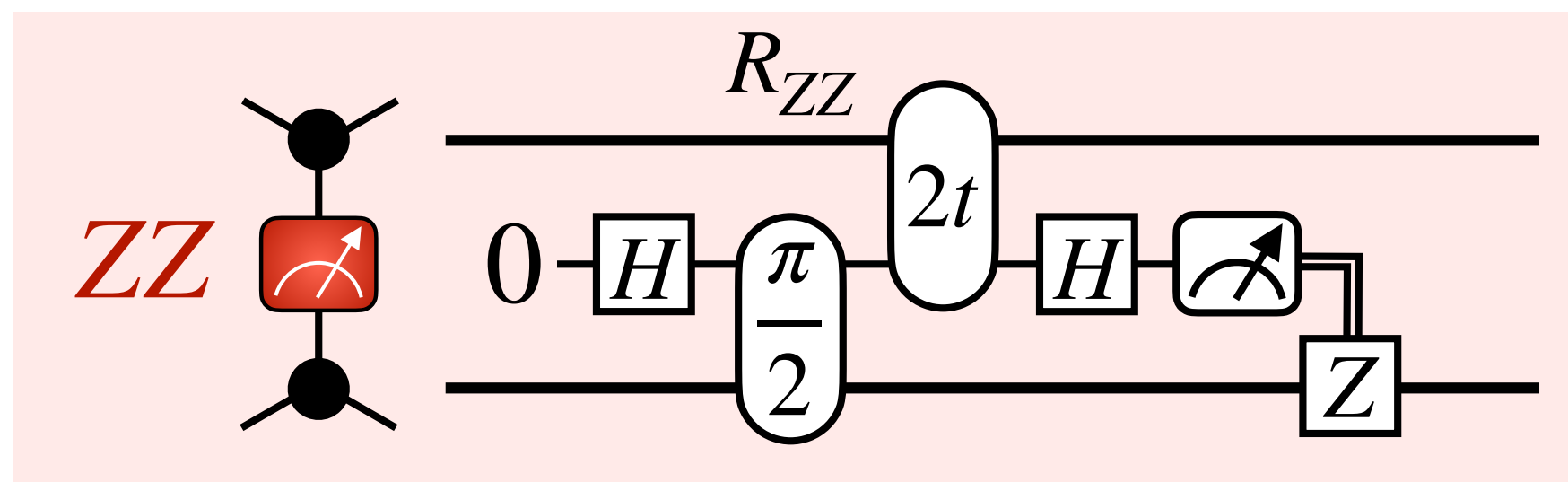
Ising evolution gates





# IBM quantum cloud devices

## parity checks



# two-qubit parity checks

