Kitaev Model: few exact and non-exact results

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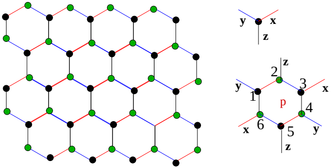
Plan of the Talk

- Kitaev Model-some exact results
- Kitaev model in the presence of other interactions
  - Phy. Rev. Lett 98 (24), 247201(2007)
  - Phy. Rev. B 84 (15), 155121(2011)
  - JPA-Math and Theo 45 (33), 335304(2012)
  - Phy. Rev. B 79 (2), 024426(2009)
  - Phy. Rev. B 90 (10), 104424(2012)
  - arXiv:2306.14839

Kitaev Model



Honeycomb Lattice : Quantum Spin 1/2 at each site

Conserved Quantity :

$$B_p = \sigma_1^y \sigma_2^z \sigma_3^x \sigma_4^y \sigma_5^z \sigma_6^x$$

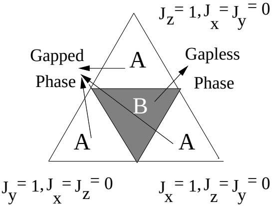
$$H = -\sum_{\langle i,j \rangle_x} J_x \sigma_i^x \sigma_j^x - \sum_{\langle i,j \rangle_y} J_y \sigma_i^y \sigma_j^y - \sum_{\langle i,j \rangle_z} J_z \sigma_i^z \sigma_j^z$$

$$\sigma^x = ic^x c; \quad \sigma^y = ic^y c; \quad \sigma^z = ic^z c$$

$$H = \sum_{\langle i,j \rangle_x} J_x u_{i,j}^x i c_i^a c_j^b + \sum_{\langle i,j \rangle_y} J_y u_{i,j}^y i c_i^a c_j^b + \sum_{\langle i,j \rangle_z} J_z u_{i,j}^z i c_i^a c_j^b$$

Solution continued

Dispersion relation: $E = \pm|J_x e^{ik_x} + J_y e^{ik_y} + J_z|$



Phase Diagram

The phase 'B' acquires a gap in the presence of Magnetic field.

A. Kitaev, Ann. Phy.321 2(2006)

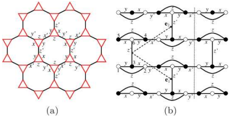
A. Kitaev, Ann. Phy.303 2(2003)

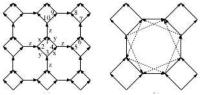
- Study in classical limit – Diptiman Sen, R Shankar and G Baskaran, Phys. B **78**, 115116(2008)
- Order by Singularity in Kitaev clusters by Sarvesh Srinivasan, Subhankar K G. Baskaran and R. Ganesh Phys. Rev. Res. **2**, 023212 (2020)
- Sheikh Moonsun Pervez and SM, arXiv:2306.14839

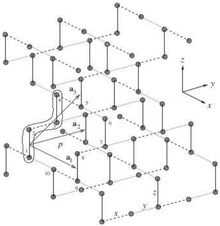
Some speciality of Kitaev model

- Jordan-Wigner transformation
- Explicit eigenstates of conserved quantities
- Exact calculation of spin-spin correlation function
- Four-fold degeneracies in thermodynamic limit
- Abelian nature of quasi particles in $J_z >> J_x, J_y$ limit
- The model can be generalized to any dimension with lattice co-ordination number three.

Kitaev model in other 2d and 3d lattices







- Yao and Kivelson, PRL, 2007
  Exact chiral spin-liquid with Non-Abelian anyonsi decorated triangle lattice

- Tikhonov and Feigel'man, PRL, 2010 Quantum spin metal states

- Baskaran, Santhosh and Shankar, arXiv, 2009 Spin liquid states with Fermi surface in square octagon lattice

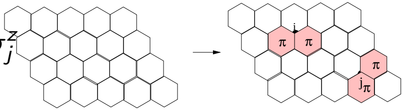
- Mandal and Surendran, PRB, 2009 3D Kitaev model with gapless contour

Static and Dynamical correlation Function

$$\sigma_i^a = ic(\chi_{\langle ij\rangle_a} + \chi^\dagger_{\langle ij\rangle_a}) \longrightarrow ic\hat{\pi}_{1\langle ij\rangle_a}\hat{\pi}_{2\langle ij\rangle_a}$$

$$\langle\psi|\sigma_i^y\sigma_j^z|\psi\rangle$$

$\sigma_i^y\sigma_j^z$



- $\langle S_i^\alpha S_{i+\beta}^\gamma\rangle$ is nonzero iff $\alpha = \beta = \gamma$. Long range string correlation exists
- G Baskaran, S Mandal, R Shankar, PRL, 98 (24), 247201
- All energy scale fractionalizations, even true for small systems.
- $\langle\sigma_i^a(t)\sigma_j^a(0)\rangle = \langle\mathcal{M}_\mathcal{G}|e^{iH[\mathcal{G}^{i\alpha}]t}ic_i(0)e^{-iH[\mathcal{G}^{i\alpha}]t}(-1)c_j(0)|\mathcal{M}_\mathcal{G}\rangle$
- $= \langle\mathcal{M}_\mathcal{G}|ic_i(t)T\left(e^{-2J_\alpha\int_0^t u_{\langle ij\rangle_\alpha}c_i(\tau)c_j(\tau)d\tau}\right)u_{\langle ij\rangle_\alpha}c_j(0)|\mathcal{M}_\mathcal{G}\rangle$

Dynamical Correlation Function

- Long range correlation develops under external perturbations

- PRL 106, 067203(2011) Tikhonov, Fiegel'man and Kitaev

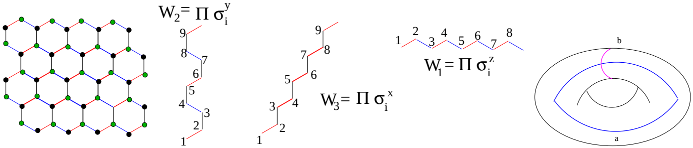
$$g(t,\mathrm{r}) = \frac{16}{\pi^2}\left(\frac{h_z}{h_0}\right)^2 \frac{(r^2-3(Jt)^2)\cos^2(2\pi/3\vec{(x)}\cdot\vec{r})-x^2}{(r^2-3(Jt)^2)^3},$$

- Mandal, Bhattacharya, Sengupta, Shankar and Baskaran, PRB **84**, 155121(2011),

$$\langle\sigma_r^\alpha(t)\sigma_0^\alpha(0)\rangle \sim \lambda^n(r^2-t^2)^{-np/2+1}$$

- A. V. Lunkin, K. S. Tikhonov and M. V. Feigelman, JETP Lett. v.103, p.117(2016)(arXiv:1610.04849), JPCS 128(2019), 130-137 (arXiv: 1710.04138)

Wilson loop for Kitaev model

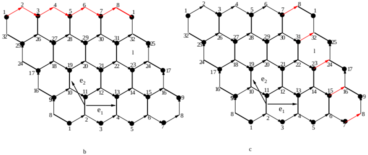


- Number of B_p = Number of dimer = $\frac{1}{2}\times$ Numbers of spin
- $\prod B_p = 1$, any two of W_1, W_2, W_3 are independent.
- Energy of the system depends on the configurations of B_p only.
- Thus W_i $i = 1, 2$ points out 4-fold degeneracy.

Kitaev model on torus

- Introduce, V_1 and V_2 with $\{V_1, V_2\} = 0, \quad \{V_i, W_i\} = 0, \quad [V_i, W_j]_{i \neq j}$
- Construct, $T_1 = V_1$, $T_2 = V_2$, $T_3 = W_1 V_1$, $T_4 = W_2 V_2$
- $\{T_i, T_j\} = 2\delta_{ij}$ (4-dimensional Cliford Algebra)
- $V_1|w_1, w_2\rangle = |-w_1, w_2\rangle, \qquad V_2|w_1, w_2\rangle = |w_1, -w_2\rangle$
- $V_i \rightarrow$ generator of large gauge transformations.
- $V_1 H V_1^{-1} = H + \delta H_1, \qquad V_2 H V_2^{-1} = H + \delta H_2$
  $\delta H_i \rightarrow$ Hamiltonian of a single 1D chain of the 2D system.

continued...



$\bullet V_1 = \sigma_1^z \sigma_3^z \sigma_5^z \sigma_7^z, \quad V_2 = \sigma_1^y \sigma_{16}^y \sigma_{17}^y \sigma_{32}^y \prod_{i=2}^{7} \sigma_i^z \prod_{i=18}^{23} \sigma_i^z$, $\boxed{[V_i, B_p] = 0}$

$\star$ In fermionic representation H and H' are different only in boundary (periodic anti-periodic) condition in matter field.

$\bullet H|\mathcal{E}\rangle = E|\mathcal{E}\rangle \quad \underrightarrow{T} \quad H'|\mathcal{E}\rangle' = E|\mathcal{E}\rangle'; \quad |\mathcal{E}\rangle' = T|\mathcal{E}\rangle$

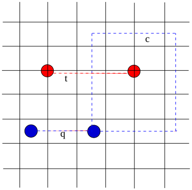
$\bullet$ In thermodynamic limit $|\mathcal{E}\rangle$ and $|\mathcal{E}\rangle'$ becomes eigenfunctions of H only as the corrections due to $\delta H \sim \frac{1}{L} \to 0$ as $L \to \infty$.

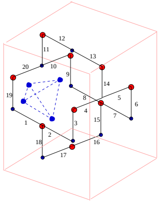
$\star\star$ **Fourfold degeneracy is acheived only in thermodynamic limit**

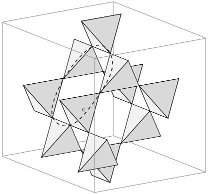
Relevant papers

- S Mandal, R Shankar, G Baskaran Journal of Physics A: Mathematical and Theoretical 45 (33), 335304
- Topological Degeneracy in Kitaev Model, G Kells et. al
  PRL 101, 240404(2008)
- Magnetic translation group, Wen & Niu, PRB, 9377(40), 1990

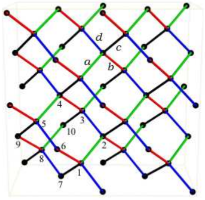
Effective Hamiltonian in $J_z >> J_x, J_y$ limit.







- $A = \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x$
- $B = \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z$

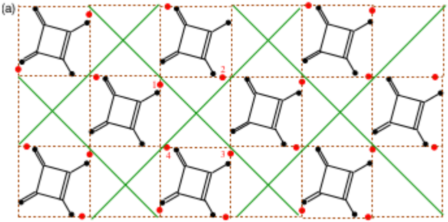


- $B_{p_1} = \sigma_1^x \sigma_2^z \sigma_3^y \sigma_4^x \sigma_5^z \sigma_6^y$
- $B_{p_2} = \sigma_3^z \sigma_4^x \sigma_5^y \sigma_9^z \sigma_8^x \sigma_{10}^y$
- $B_{p_3} = \sigma_5^x \sigma_6^y \sigma_1^z \sigma_7^x \sigma_8^y \sigma_9^z$
- $B_{p_4} = \sigma_1^y \sigma_2^z \sigma_3^x \sigma_{10}^y \sigma_8^z \sigma_7^x$

- Mandal and Surendran, PRB **90**, 104424(2014)

- 8-loop and 6-loop excitations, fractionalizations
- exchange and braiding statistics of loops and membrane excitations
- Toric code limit $\rightarrow$ effective Hilbert space fractionalization
- involves all three pauli matrices, spin-1/2 model with fermionic excitations
- Hamma et. al PRB, 2005, bosonic excitations
- Levin and Wen 2003, Ryu 2009, fermionic excitations with S=3/2
- *We note that Honeycomb Kitaev model came after Toric code model*
- Squaric acid system a platform to generate Toric code Hamiltonian,
  Vikash Vijigiri and Saptarshi Mandal 98, 224425 (2018),
  Bo-Jie Huang and Chyh-Hong Chern, IJMPB, 31, (2017) 1750130

H_2SQ_4 system as realization of Torid code model



- $H = H_0 + H_1 + H_2$
- $H_0 = -J_0 \sum_p \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z - K \sum_i \sigma_i^z$
- $H_1 = J_1 \sum_p (\sigma_1^z \sigma_3^z + \sigma_2^z \sigma_4^z), H_2 = -J_2 \sum_{<AB>} \vec{P}_A \cdot \vec{P}_B$
- Toric code is obtained at fourth order for H_0 with $J_0 >> K$

Generalisations of Kitaev model

- $\alpha - RuCl_3$ Jackeli, G. and Khaliullin, G, PRL **102**, 017205(2009). A_2IrO_3 Singh, Y. et al. PRL **108**, 127203(2012). Li_2IrO_3 (harmonic honeycomb iridiate)Modic, K. A. et al. Nat. Commun. 5, 4203 (2014)

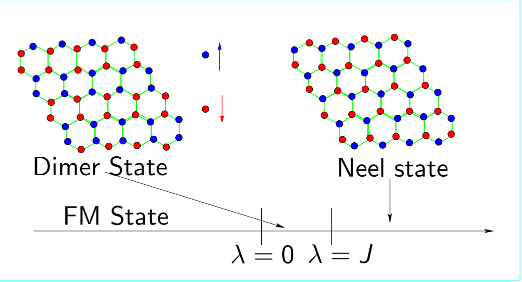
- KSL and magnetic field. Zheng Zhu et al PRB **97**, 241110(R), 2018. Han Li et. al PRB **107**, 115124(2023). Shuang Liang et. al PRB **98**, 104410(2018)

- $J - K - \Gamma$ model. J. G. Rau, et al. PRL **112** 077204(2014), Wei Wang et al. PRB **96**, 115103 (2017), S M. Winter et al. PRB **93**, 214431 (2016), Shi Wang et al. PRB **103**, 054410 (2021).

Dynamics of perturbed Kitaev model

- $H = -\sum_{\langle i,j\rangle_\alpha} J\sigma_i^\alpha \sigma_j^\alpha + \sum_{\langle i,j\rangle_{\rm all}} \lambda \sigma_i^z \sigma_j^z$, $\boxed{[\sigma_i^z \sigma_{i+x,y}^z, B_p] \neq 0}$

- $H = \sum_{\langle i,j\rangle_{\alpha'}} u_{\alpha'} i c_i c_j + \sum_{\langle i,j\rangle_z} i b_i^z b_j^z i c_i c_j + \sum_{\langle i,j\rangle_{all}} i b_i^z b_j^z i c_i c_j$



- We follow appropriate meanfield decomposition for $\sim i b_i c_i i b_j c_j$

Results of meanfield analysis

$$H = \sum_{\langle i,j \rangle_\alpha} J\sigma_i^\alpha \sigma_j^\alpha + \sum_{\langle i,j \rangle_{\text{all}}} \lambda \sigma_i^z \sigma_i^z + \sum_p \kappa B_p$$

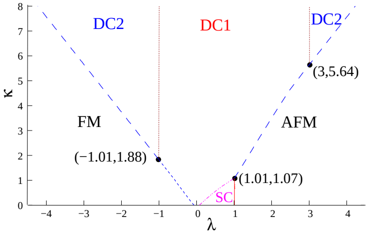
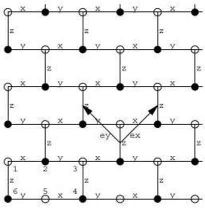


Figure: Meanfield phase diagram

- 10% stability of Kitaev phase.
- 1st order confimement to deconfinement transition.
- Topological phase transition from 'DC1' to 'DC2'.
- O4 formalism for $\kappa >> \lambda$.

On O_4 formalism



- $\xi_{i_n,1(2)} = \sigma^{x(y)}_{i_n 1(2)} T_m, \eta_{i_n,1(2)} = \sigma^{y(x)}_{i_n 1(2)} T_m$
- $H = h^k_{xx} + h^k_{yy} + h^k_{zz} + \lambda H_{ising} + \kappa H_{B_p}$
- $\tilde{H} = h^k_{zz} + \lambda H_{ising} + \kappa H_{B_p}$
- $H_1 = 4 \sum_i (-L^{13}_i L^{24}_i + \kappa L^{24}_i L^{24}_{i+e_x-e_y})$
- $H_2 = 4\lambda \sum_i (L^{12}_i L^{34}_i + L^{12}_i L^{34}_{i+e_x} + L^{12}_i L^{34}_{i+e_y})$

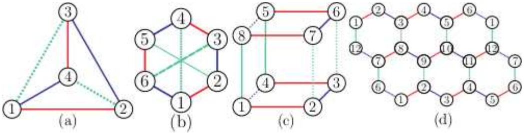
- $L_{12} = \frac{i}{2}\eta_{i1}\xi_{i1}, \quad L^{34} = \frac{i}{2}\eta_{i2}\xi_{i2}, \quad L^{13} = \frac{i}{2}\xi_{i2}\xi_{i1}, \quad L_{24} = \frac{i}{2}\eta_{i2}\eta_{i1},$
  $L^{14} = \frac{i}{2}\eta_{i2}\xi_{i1}, \quad L^{23} = \frac{i}{2}\xi_{i2}\eta_{i1}, \qquad h^k_{xx} \sim i\xi_{i_n}\xi_{i_m}$

- $|\theta\rangle (\prod_i e^{i\frac{\theta}{2}L^{23}_i}) |FM\downarrow\rangle, \langle\theta|H|\theta\rangle = -N(1+\lambda) + \kappa \sin^2\theta + 2\lambda\cos^2\theta$

- $(\kappa - 2\lambda)\sin 2\theta = 0$ triple point for DC1-DC2-AFM

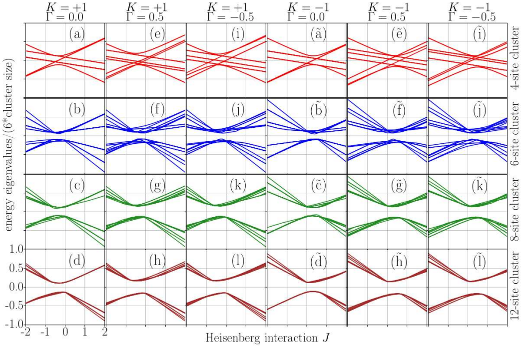
- More in Mandal, Bhattacharya, Sengupta, Shankar and Baskaran, PRB **84**, 155121(2011)

Kitaev-Heisenberg-Gamma model on finite cluster



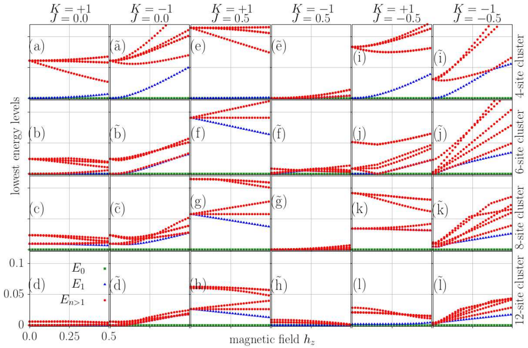
- T. Chowdhuri, A. Rosch, R. Bulla, PRB **101**, 115133(2020), S. Srinivasan, S. Khatua, G. baskaran, R. Ganesh, PRB**2**, 023212(2020)
- Kota Kataoka, PPSP **89**, 114709 (2020) Kitaev spin liquid candidate Os_xCl_3 comprised of Honeycomb Nano domains.
- competition between different magnitude and sign in a single study
- gradual dependency of correlations, magnetization, specific heat susceptibility, low energy gap on cluster sizes
- effect of magnetic field, quantum effect such as quantum speed limit

Eigenvalues plots



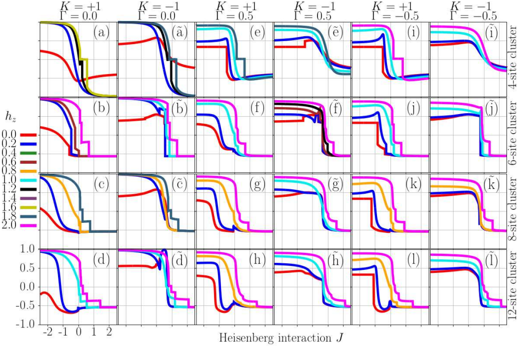
- For four site cluster the gap vanishes for some values of J
- The band width is higher and K and H have same sign
- Continuous tuning of ground state by J

Low energy states with magnetic field



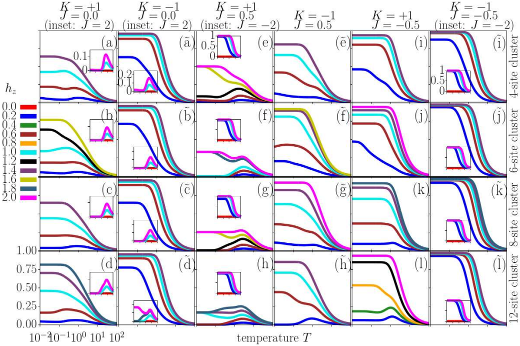
- For $J = 0$, only h_z effects the spectra diffrently for $K = \pm 1$
- For $J = 0.5, K = -1$ eigenvalues do not change much with h_z
- For $J = -0.5, K = -1$ eigenvalues changes a lot with h_z

z-z correlations



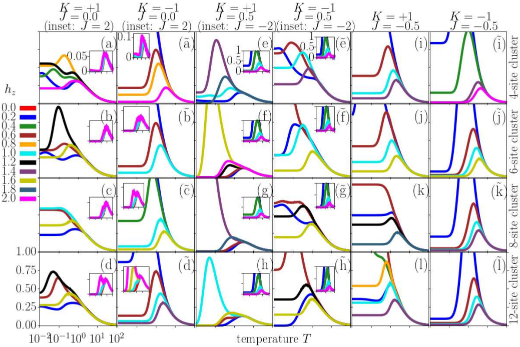
- For FM J, step like behaviour is seen.
- For AFM J, non-monotonicity is observed
- For finite Γ and AFM J saturation magnetization depends on h_z

Magnetization



- $J = 0$ $\mathcal{M}$ gradually increases with h_z as $T \to 0$ for AFM K.
- $J = 0.5$ $\mathcal{M}$ saturates at two different limit as $T \to 0$ for AFM K.
- $J = 0.5$ $\mathcal{M}$ gradually increases with h_z as $T \to 0$ for FM K.
- $J = -0.5$ different response is seen as shown in 5th and 6th column.

Susceptibility



- $J = 0$ χ for FM K shows pronounced maxima at some T.
- $J = 0.5$ $K = -1$, χ is relatively smaller than $K = -1$
- $J = -0.5$ χ is more regular though it reaches different limit for $K = 1$.

Thank you