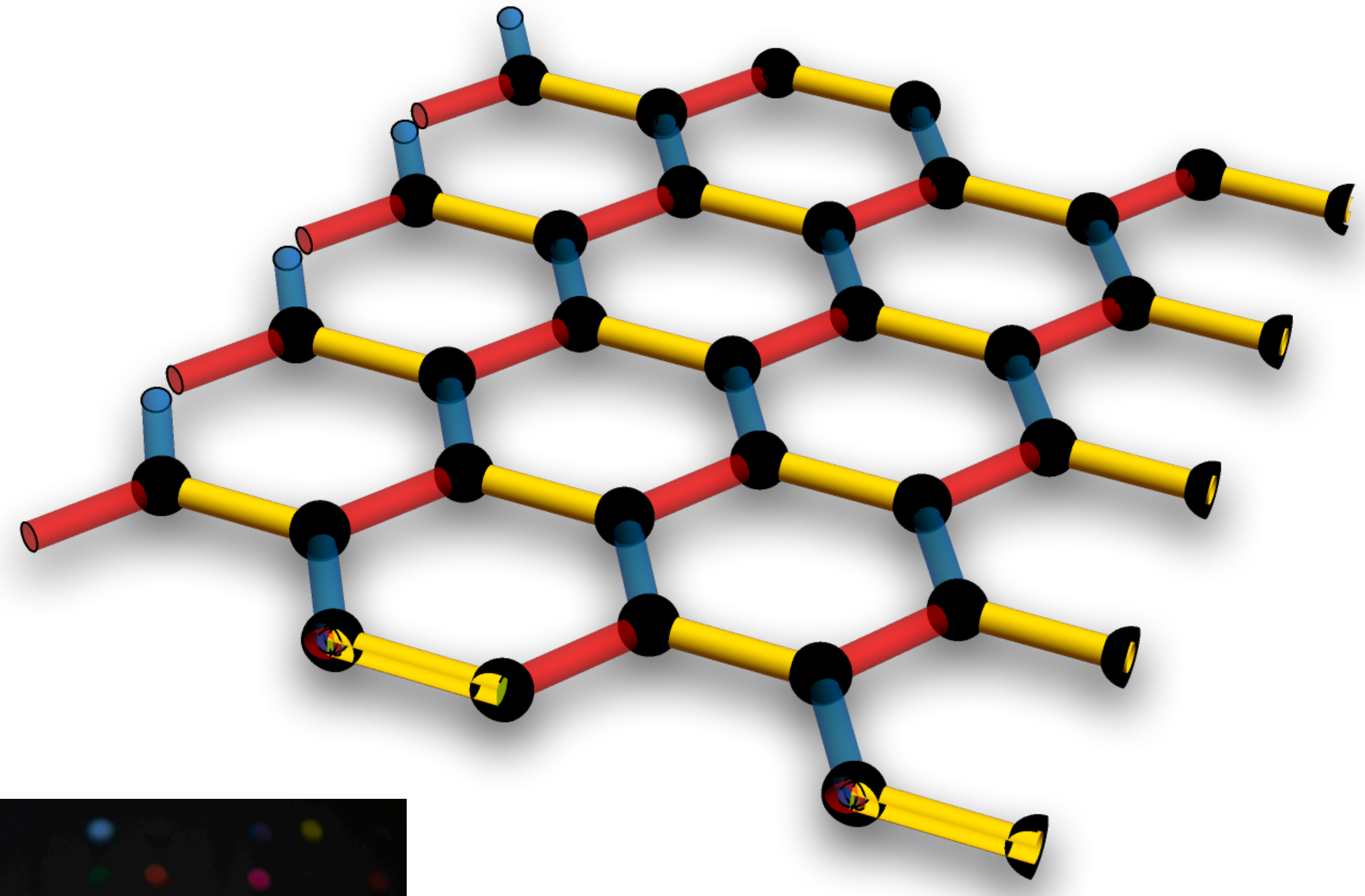
Z2 spin liquids in spin-S Kitaev honeycomb model:

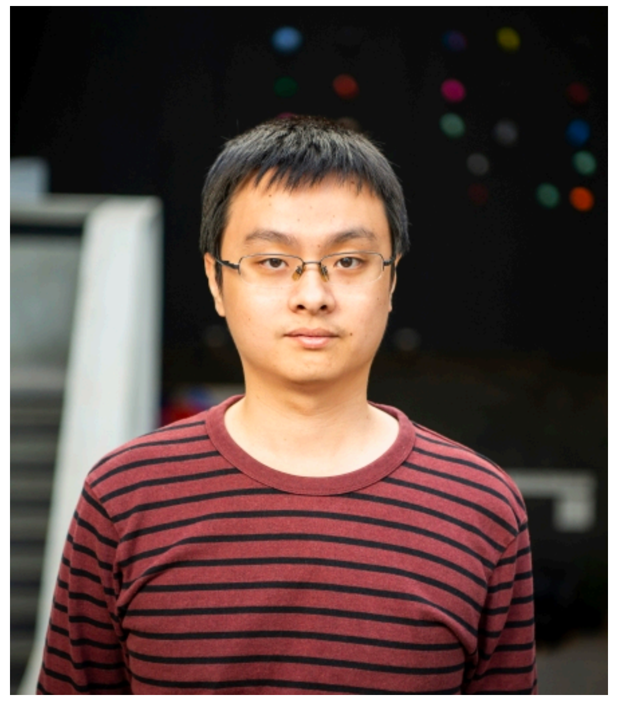
An exact Z2 gauge structure in non-integrable models

PERIMETER INSTITUTE



Han Ma
Perimeter Institute





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Liujun Zou (Perimeter)

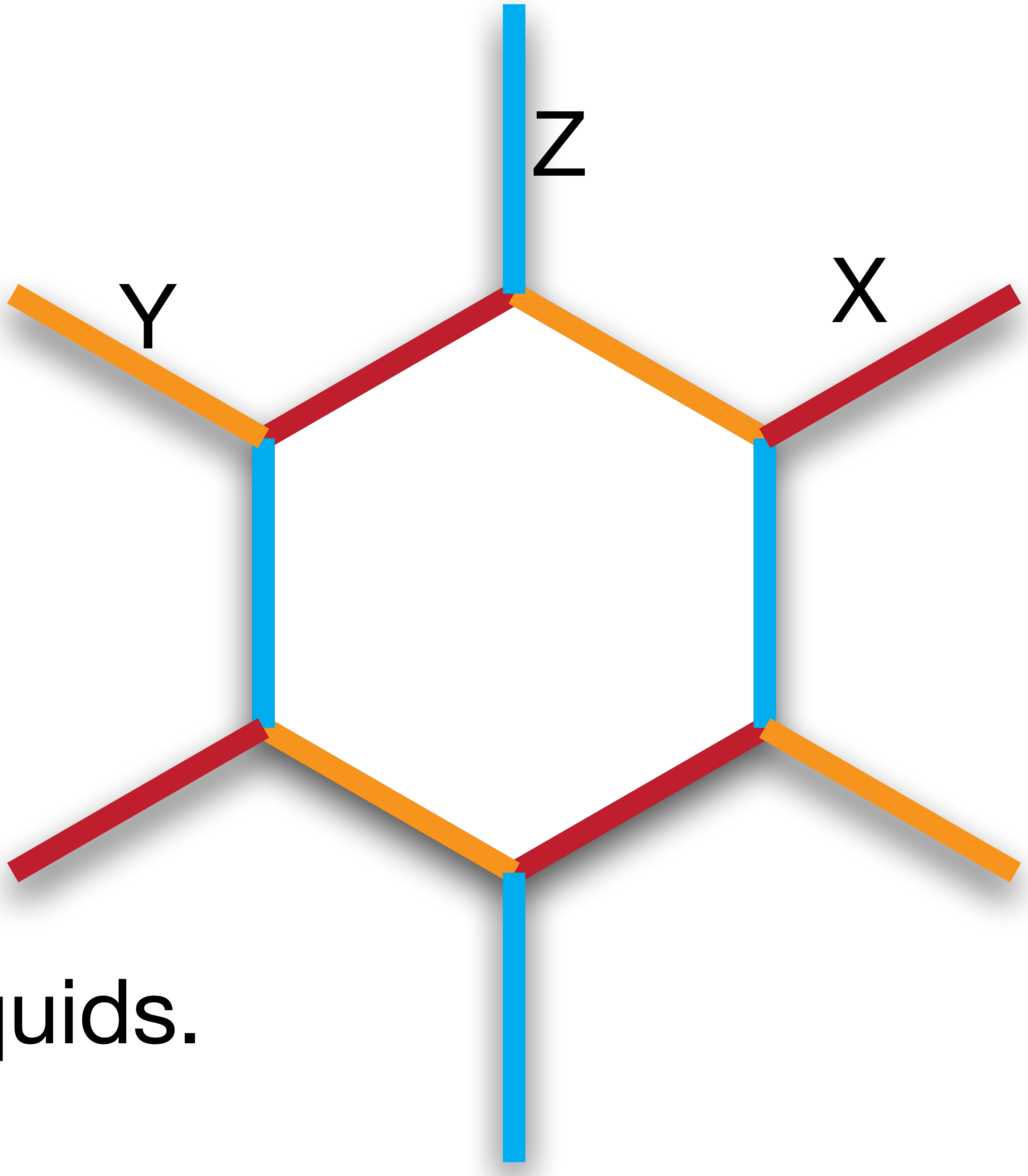
HM, Phys. Rev. Lett. **130**, 156701 (2023) (Editor's Suggestion)
Ruizhi Liu, Ho Tat Lam, **HM**, Liujun Zou, arXiv:2310.16839 (2023)

Fractionalization and Emergent Gauge Fields in Quantum Matter, ICTP, Trieste, Dec.7, 2023

Kitaev honeycomb model

$$H = -\sum_{\mu} J_{\mu} \sum_{\langle ij \rangle \in \mu} S_i^{\mu} S_j^{\mu}$$

Kitaev (2006)



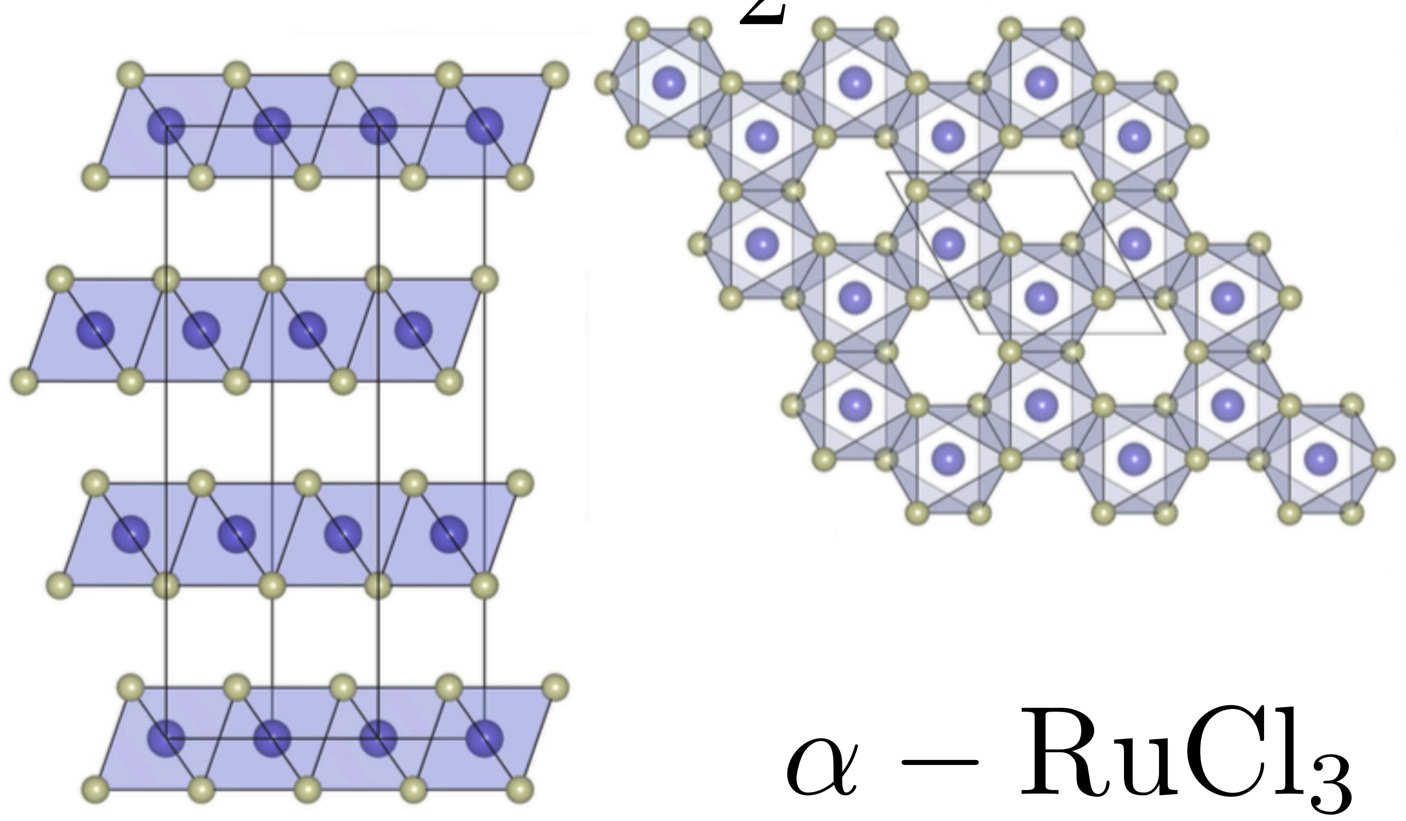
- We study the higher spin version of it for possible spin liquids.
- Motivation I: It has spin liquid phases.
- Motivation II: Candidate materials have been proposed.

Baskaran, Sen, Shankar (2008)
Lee, Kawashima, Kim (2020)
Lee, Suzuki, Kim, Kawashima (2021)
Chen, Genzor, Kim, and Kao (2022)
Hickey, Berke, Stavropoulos, Kee and Trebst (2020)
Dong and Sheng (2020)
……

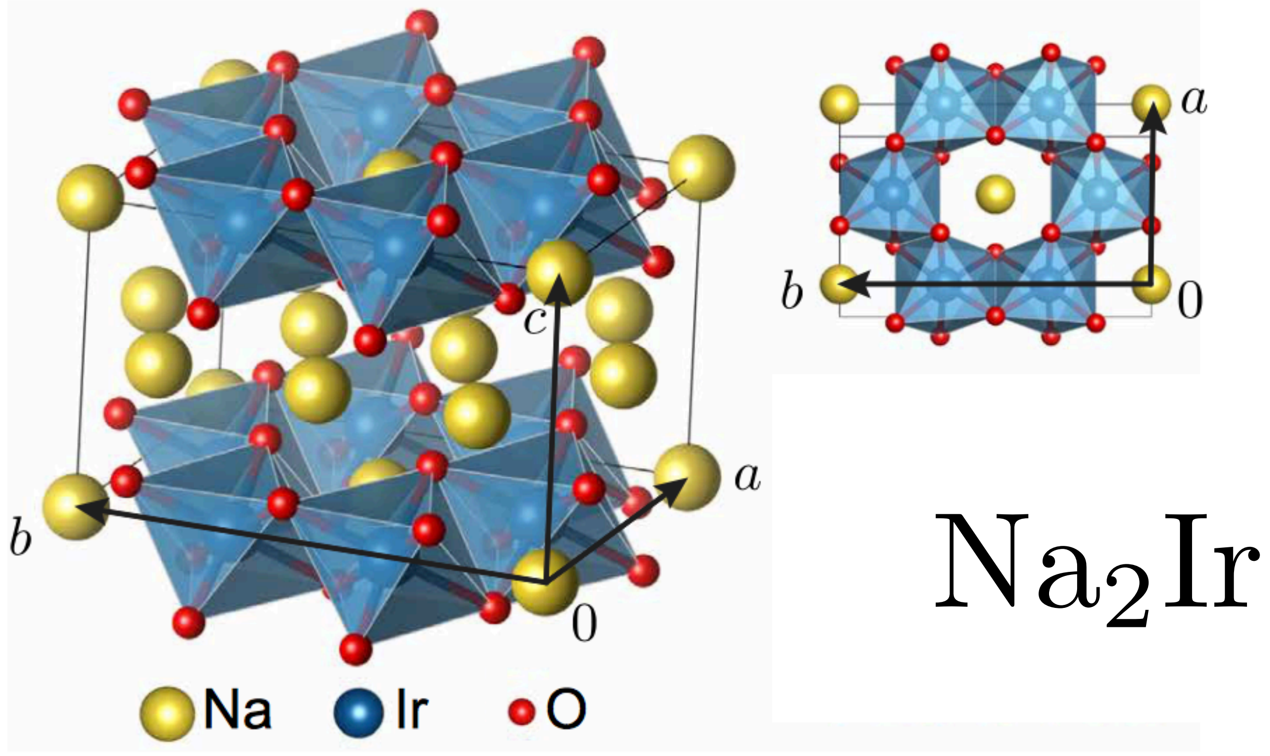
Candidate Kitaev Materials

Jackli, Khaliullin (2009)
Trebst, Hickey (2022)

$$S = \frac{1}{2}$$

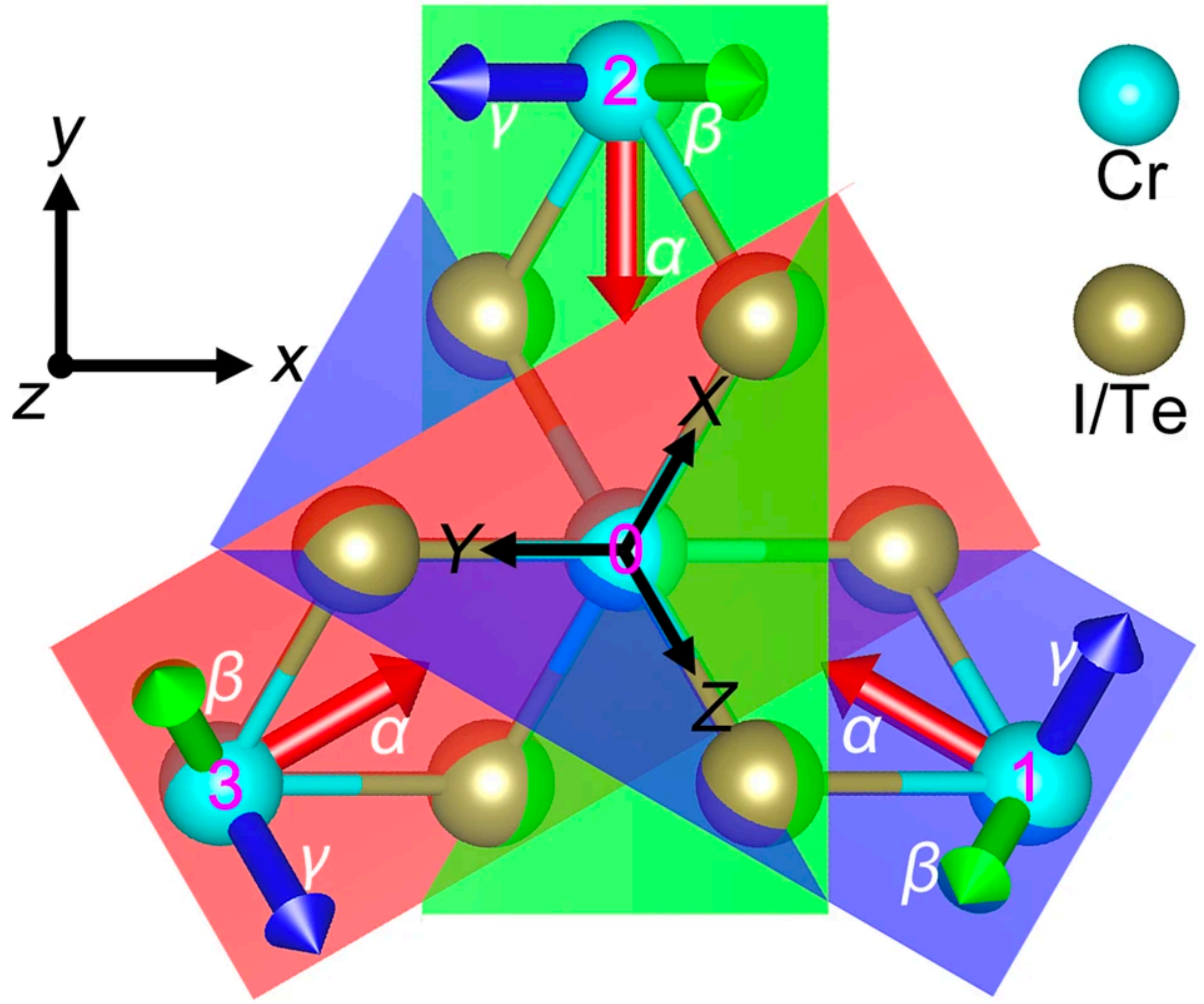


$\alpha - \mathrm{RuCl_3}$



$\mathrm{Na_2IrO_3}$

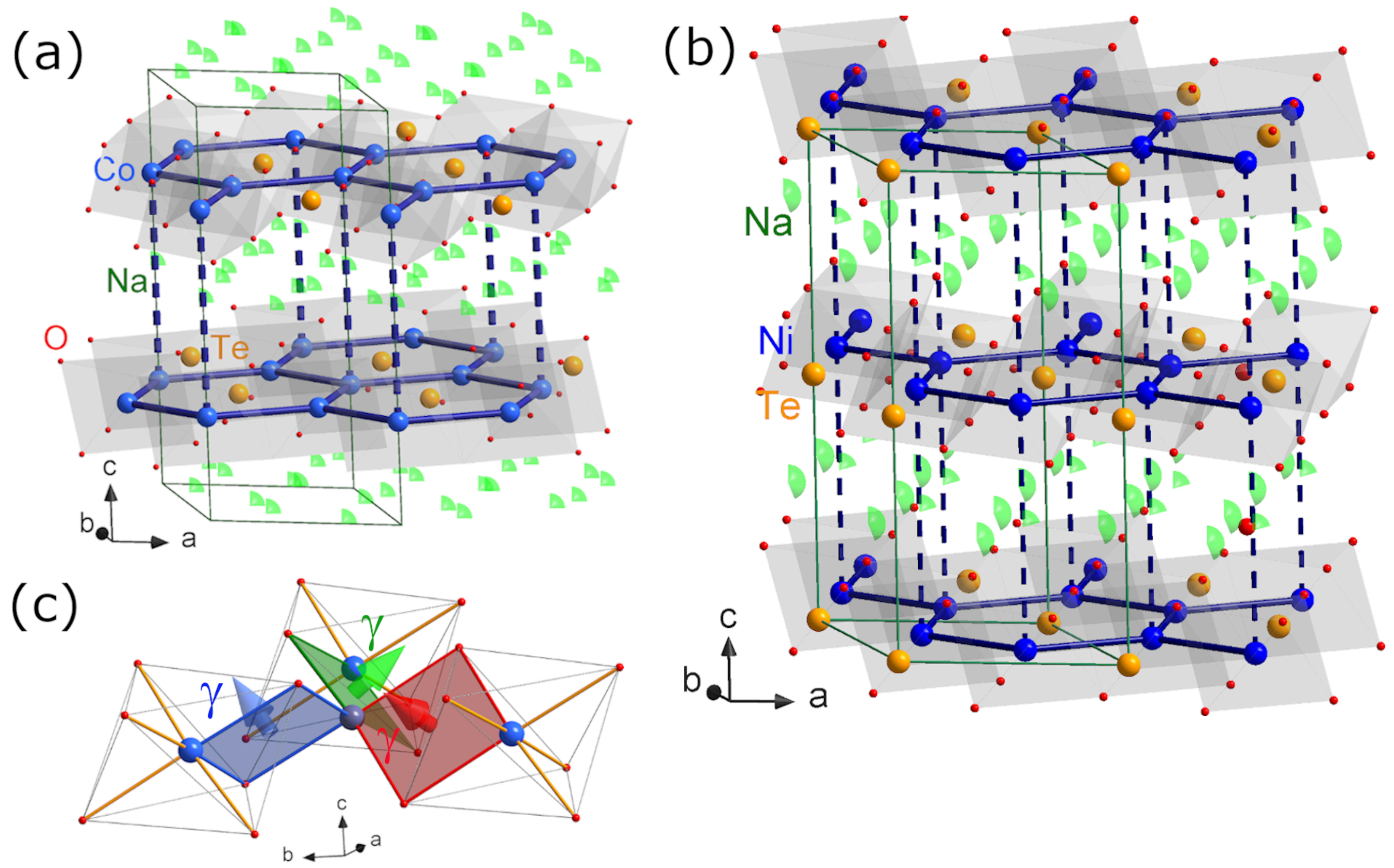
honeycomb iridates



$$S = \frac{3}{2}$$

$\mathrm{CrI_3}$
$\mathrm{CrGeTe_3}$

Xu, Feng, Xiang, Bellaiche (2018)
Stavropoulos, Pereira, and Kee (2019)
Samarakoon, Chen, Zhou, Garlea (2021)
……



$S = 1$

$\mathrm{Na_2Ni_2TeO_6}$

Kitaev honeycomb model

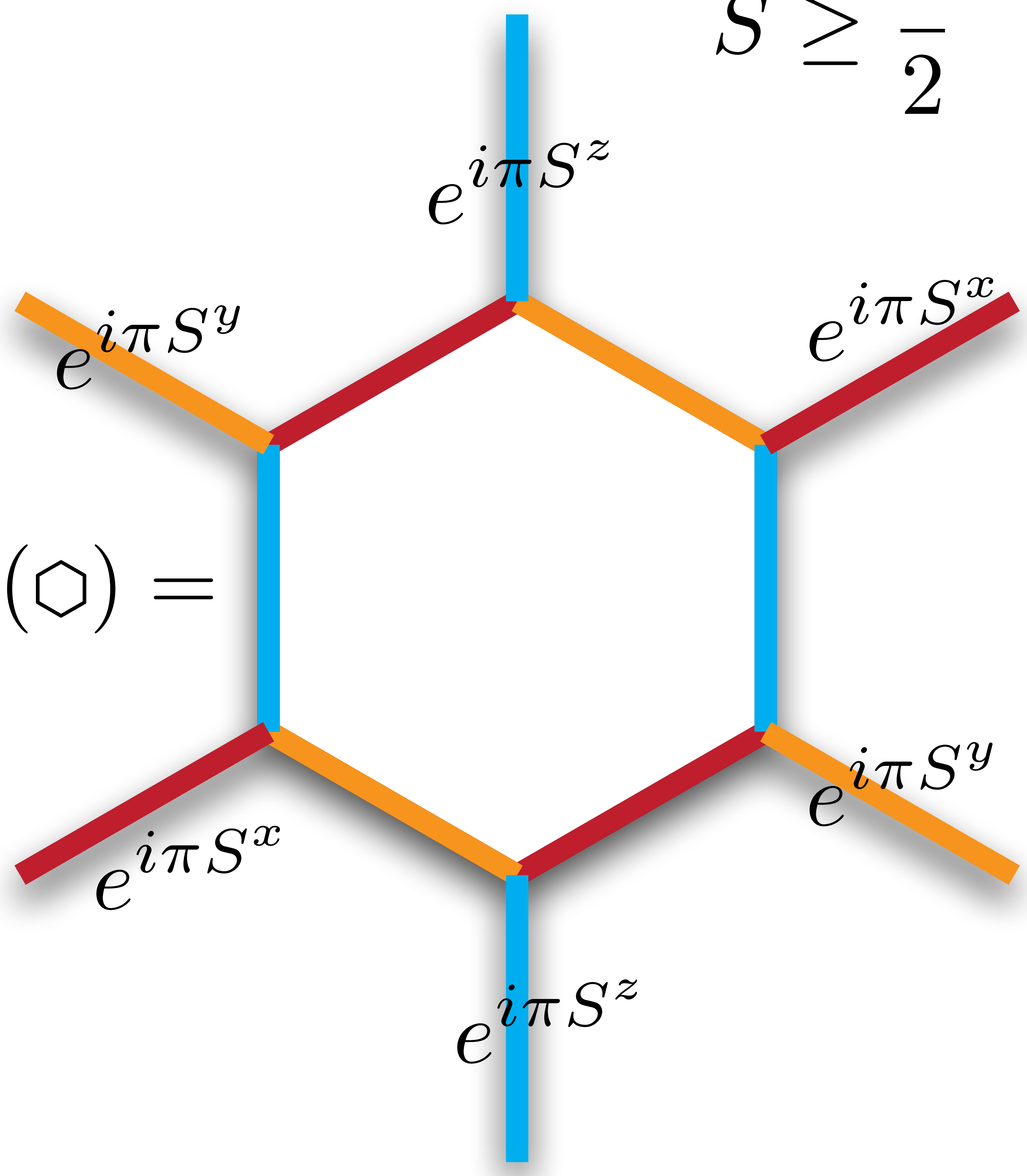
$$H = -\sum_{\mu} J_{\mu} \sum_{\langle ij \rangle \in \mu} S_i^{\mu} S_j^{\mu}$$

- Motivation III:
  there are extensive local conserved quantities/commuting operators.

1. The model of spin-1/2 is solvable
2. Higher spin model is ***not*** solvable
   due to larger Hilbert space.
3. $W_p(\hexagon)$ is $\mathbb{Z}_2$ gauge flux in spin-1/2 model
4. Can we understand the $W_p(\hexagon)$ the same way
   in higher spin model?

$$S \geq \frac{1}{2}$$

$$W_p(\hexagon) =$$

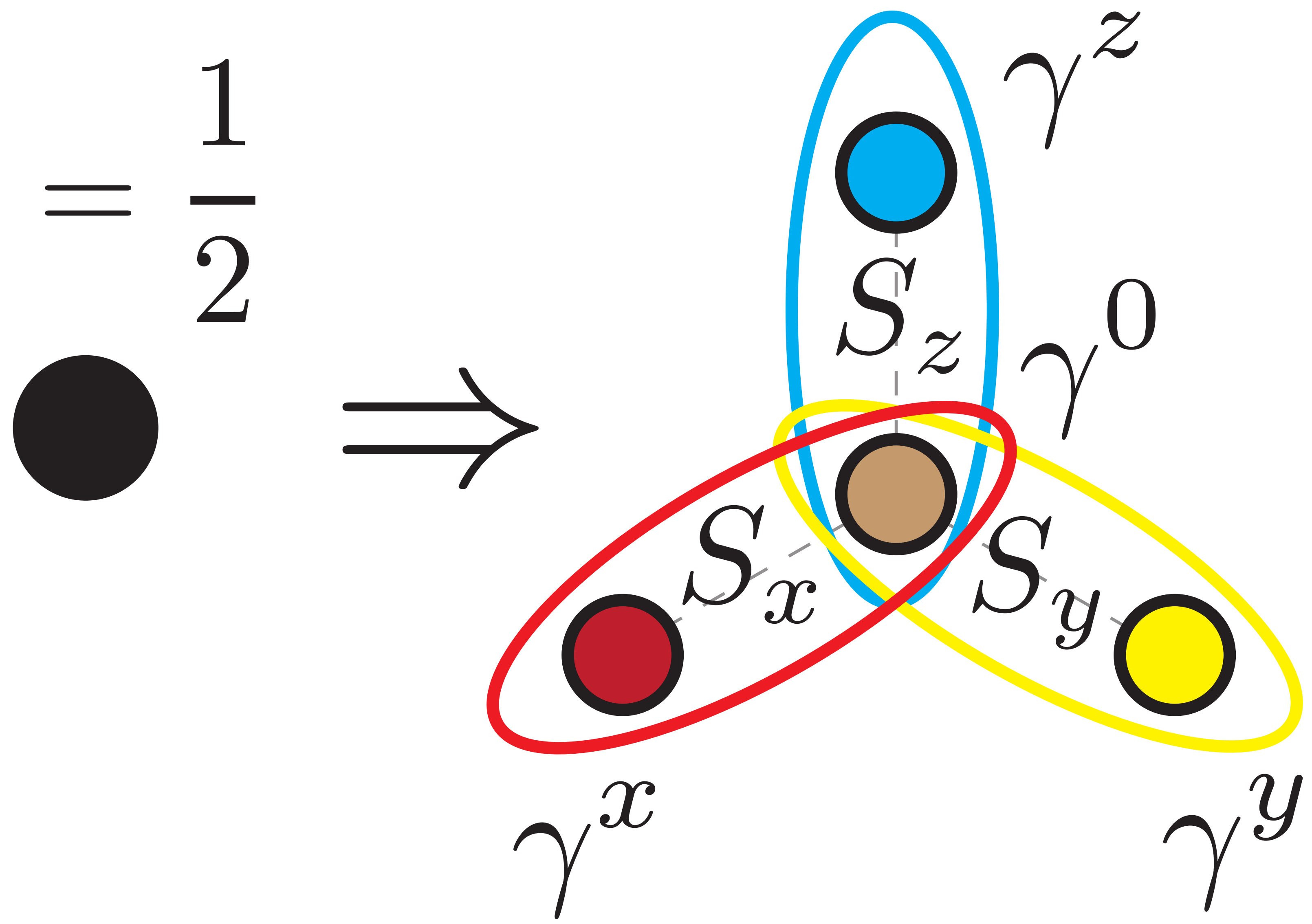


Spin-1/2 Kitaev model

Review the spin-1/2 Kitaev model and its flux operators

Parton construction — spin-1/2

$$S = \frac{1}{2}$$



$$\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2\delta_{\alpha\beta} \qquad \alpha, \beta = 0, \mu$$

$$2S^\mu = i\gamma^0 \gamma^\mu \qquad \mu = x, y, z$$

Local constraint: $\gamma^0 \gamma^x \gamma^y \gamma^z = 1$

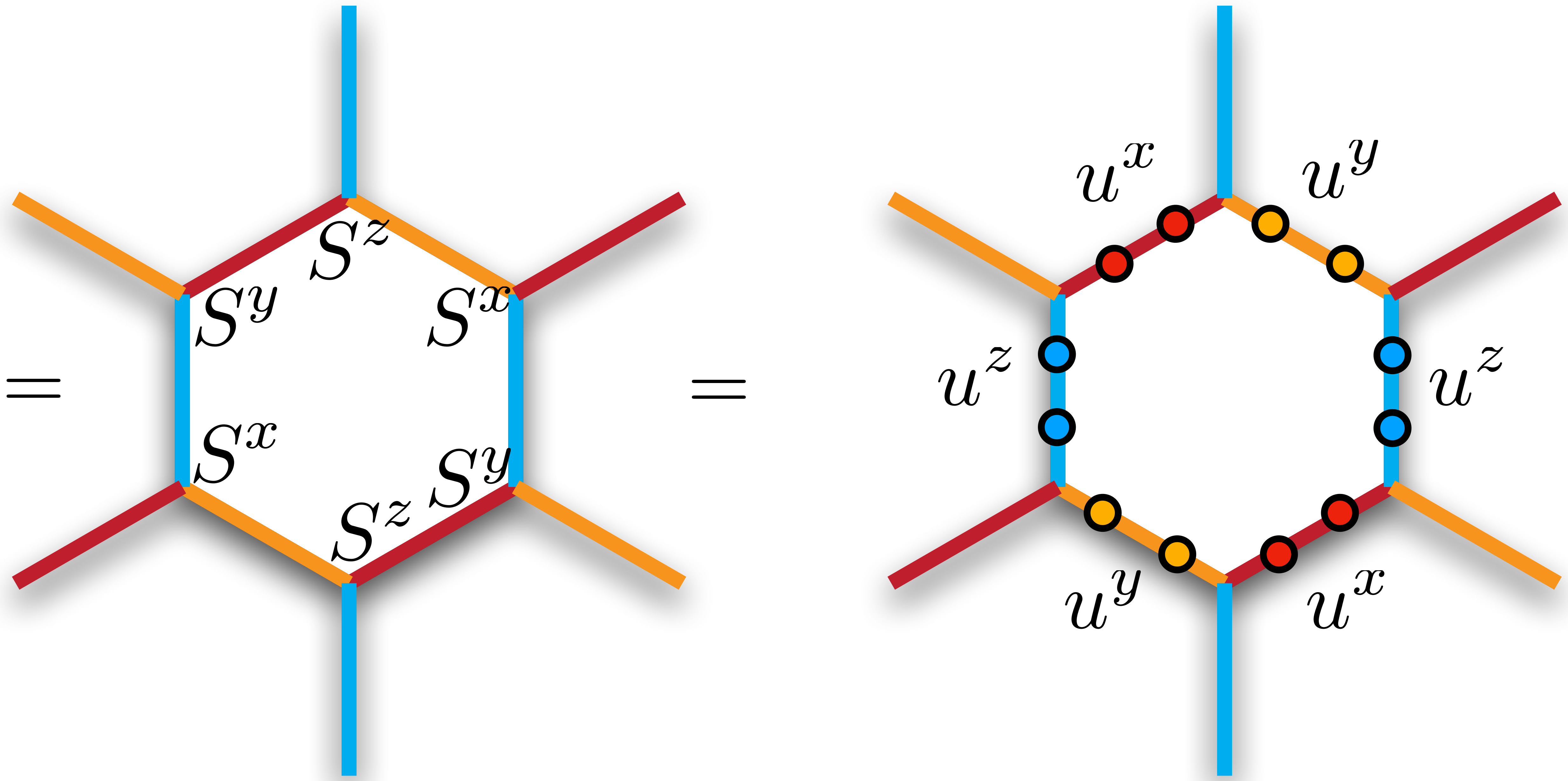
$\mathbb{Z}_2$ gauge redundancy: $\gamma^{0,\mu} \to -\gamma^{0,\mu}$

Parton construction — spin-1/2

- Hamiltonian: $H = -\sum_{\mu} J_{\mu} \sum_{\langle ij \rangle \in \mu} S_i^{\mu} S_j^{\mu} = -\sum_{\mu} J_{\mu} \sum_{\langle ij \rangle \in \mu} \gamma_i^0 \gamma_i^{\mu} \gamma_j^{\mu} \gamma_j^0$

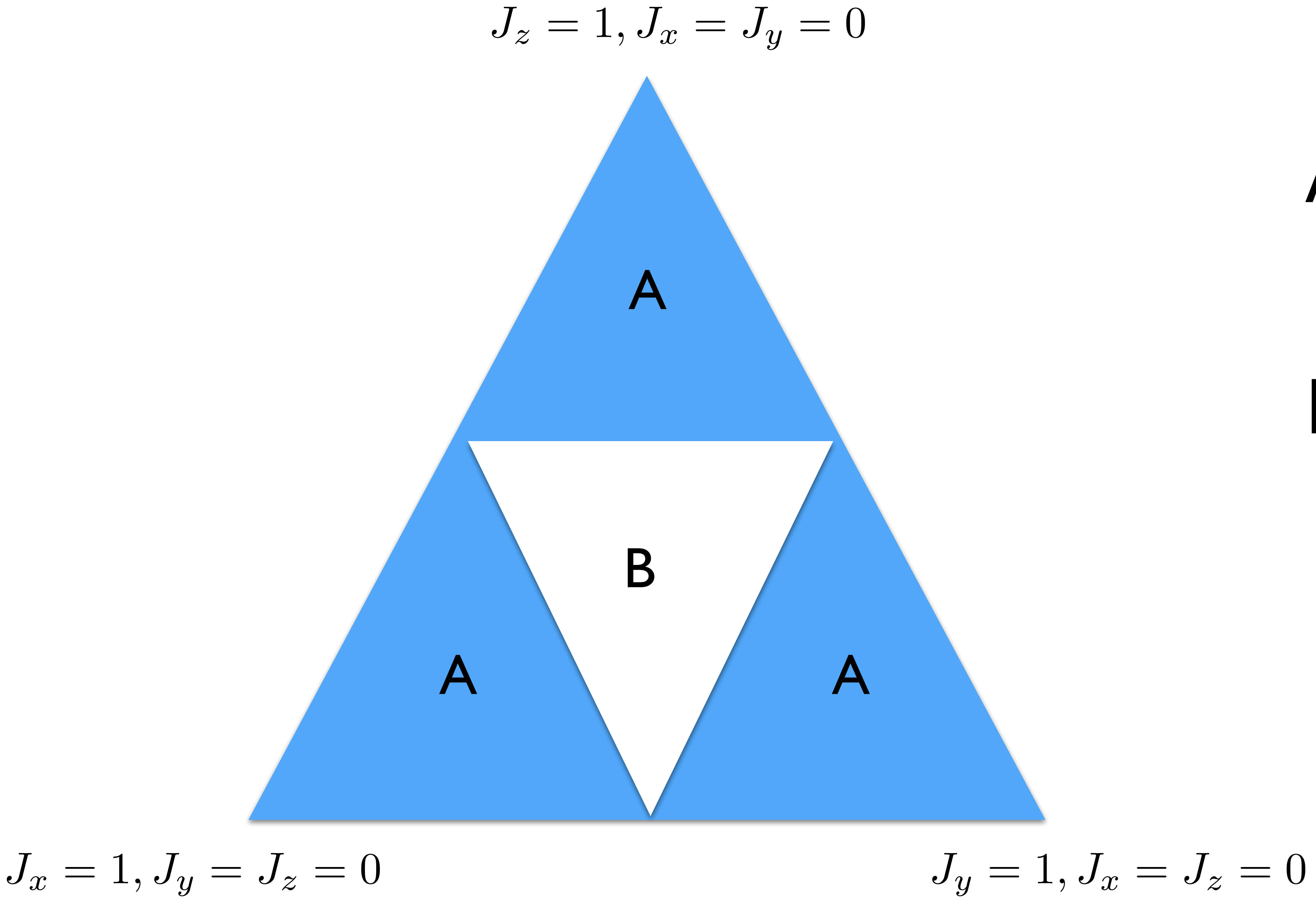
- $\mathbb{Z}_2$ gauge field: $u_{ij}^{\mu} = i\gamma_i^{\mu}\gamma_j^{\mu}$ commutes with Hamiltonian

- $\mathbb{Z}_2$ Flux operator: $W_p(\hexagon) =$



Phase diagram of spin-1/2 Kitaev model

$$H = i \sum_{\mu} J_{\mu} \sum_{\langle ij \rangle \in \mu} \gamma_i^0 u_{ij}^{\mu} \gamma_j^0$$



Phases are determined by the physics of γ^0

A: Majorana fermion γ^0 is gapped.
This phase is a $\mathbb{Z}_2$ topological order.

B: Majorana fermion γ^0 is gapless and
is coupled to a $\mathbb{Z}_2$ gauge field.

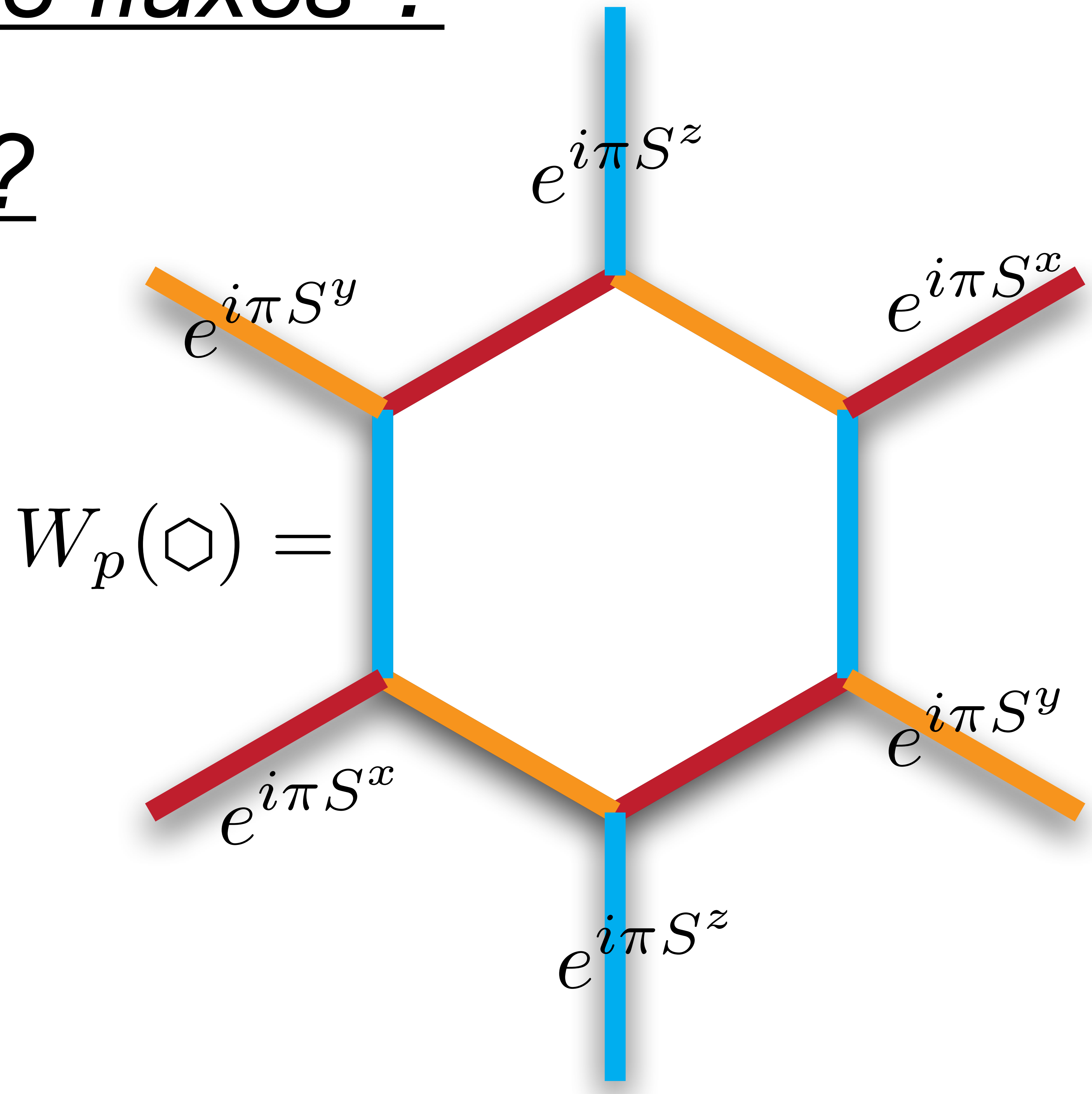
Kitaev (2006)

Spin-1/2 -> Higher spin Kitaev model

Are the conserved quantities also $\mathbb{Z}_2$ gauge fluxes ?

Can we see this from Parton construction?

What are the possible phases?



Parton construction — higher spin

HM, Phys. Rev. Lett. 130, 156701 (2023)

$$\gamma_a^\alpha \gamma_b^\beta + \gamma_b^\beta \gamma_a^\alpha = 2\delta_{\alpha\beta}\delta_{ab}$$

$$\alpha, \beta = 0, \mu \qquad a, b = 1, \ldots, 2S$$

$$2S^\mu = i \sum_{a=1}^{2S} (\gamma_a^0 \gamma_a^\mu) = i \left[\gamma_1^0, \gamma_2^0, \ldots, \gamma_{2S}^0\right] \mathbb{1} \begin{bmatrix} \gamma_1^\mu \\ \gamma_2^\mu \\ \vdots \\ \gamma_{2S}^\mu \end{bmatrix}$$

Local constraints:

$$\sum_\mu \left(\sum_{a=1}^{2S} \gamma_a^0 \gamma_a^\mu\right)^2 = S(S+1)$$

$$\gamma_a^0 \gamma_a^x \gamma_a^y \gamma_a^z = 1$$

O(2S) gauge redundancy: $U^T U = \mathbb{1}$

---

1. $S^\mu = \sum_{a=1}^{2S} S_a^\mu$ Local constraint: $\sum_\mu \left(\sum_{a=1}^{2S} S_a^\mu\right)^2 = S(S+1)$

2. $2S_a^\mu = i\gamma_a^0 \gamma_a^\mu$ Local constraints: $\gamma_a^0 \gamma_a^x \gamma_a^y \gamma_a^z = 1$

Giant Parton operators

SO(2S) singlet

$$\Gamma^{0,\mu} = (-1)^{\frac{S(2S-1)}{2}} \prod_{a=1}^{2S} \gamma_a^{0,\mu} = \frac{(-1)^{\frac{S(2S-1)}{2}}}{(2S)!} \epsilon_{a_1,a_2,\ldots,a_{2S}} \gamma_{a_1}^{0,\mu} \gamma_{a_2}^{0,\mu} \ldots \gamma_{a_{2S}}^{0,\mu}$$

rotation among Majorana fermions

O(2S) transformation: $\Gamma^{0,\mu} = (-1)^{\frac{S(2S-1)}{2}} \prod_{a}^{2S} \left[\sum_{b=1}^{2S} \mathcal{U}_{ab} \tilde{\gamma}_b^{0,\mu} \right]$

$$= (-1)^{\frac{S(2S-1)}{2}} \sum_{\sigma \in S_{2S}} \mathrm{sgn}(\sigma) (\mathcal{U}_{1\sigma_1} \ldots \mathcal{U}_{2S\sigma_{2S}}) \prod_{b=1}^{2S} \tilde{\gamma}_b^{0,\mu}$$

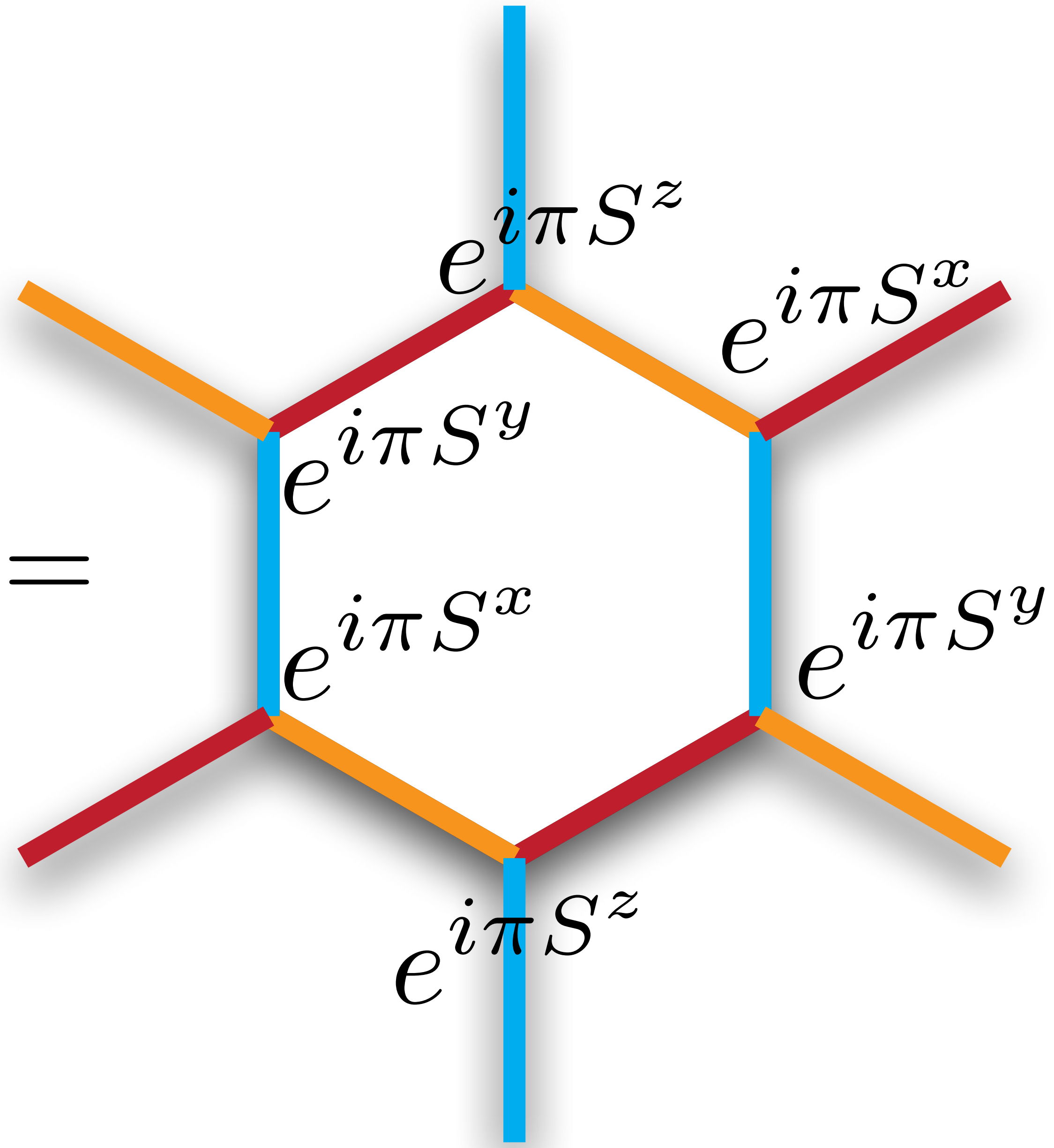
$$= \det(\mathcal{U}) \tilde{\Gamma}^{0,\mu} = \pm \tilde{\Gamma}^{0,\mu}$$

carrying improper $\mathbb{Z}_2$ charge

Higher spin Kitaev model

- Hamiltonian: $H = -\sum_{\mu} J_\mu \sum_{\langle ij \rangle \in \mu} S_i^\mu S_j^\mu = -\sum_{\mu} J_\mu \sum_{\langle ij \rangle \in \mu} \sum_{a,b=1}^{2S} \gamma_{a,i}^0 \gamma_{a,i}^\mu \gamma_{b,j}^\mu \gamma_{b,j}^0$

- $\mathbb{Z}_2$ Flux operator: $W_p(\hexagon) =$



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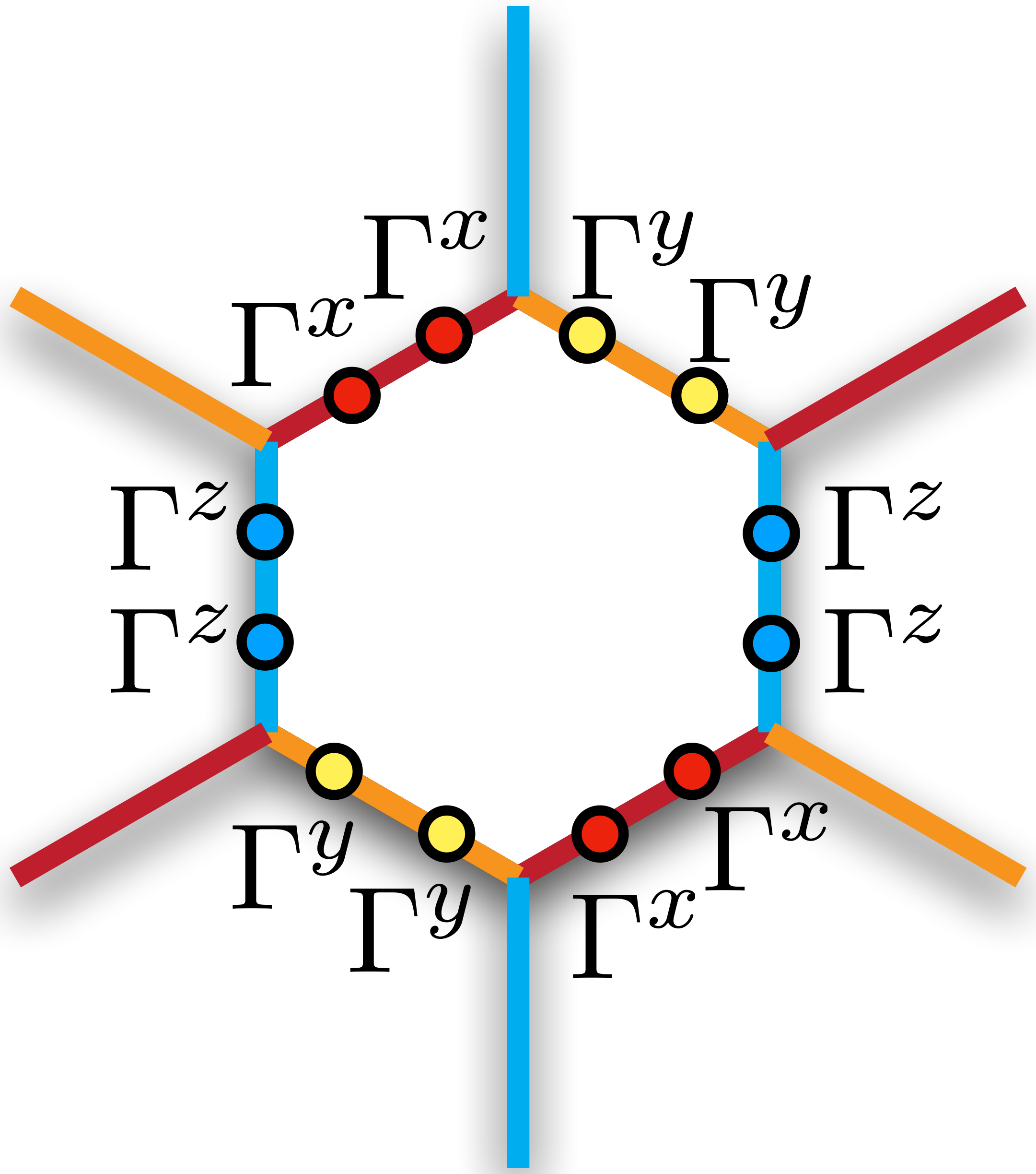
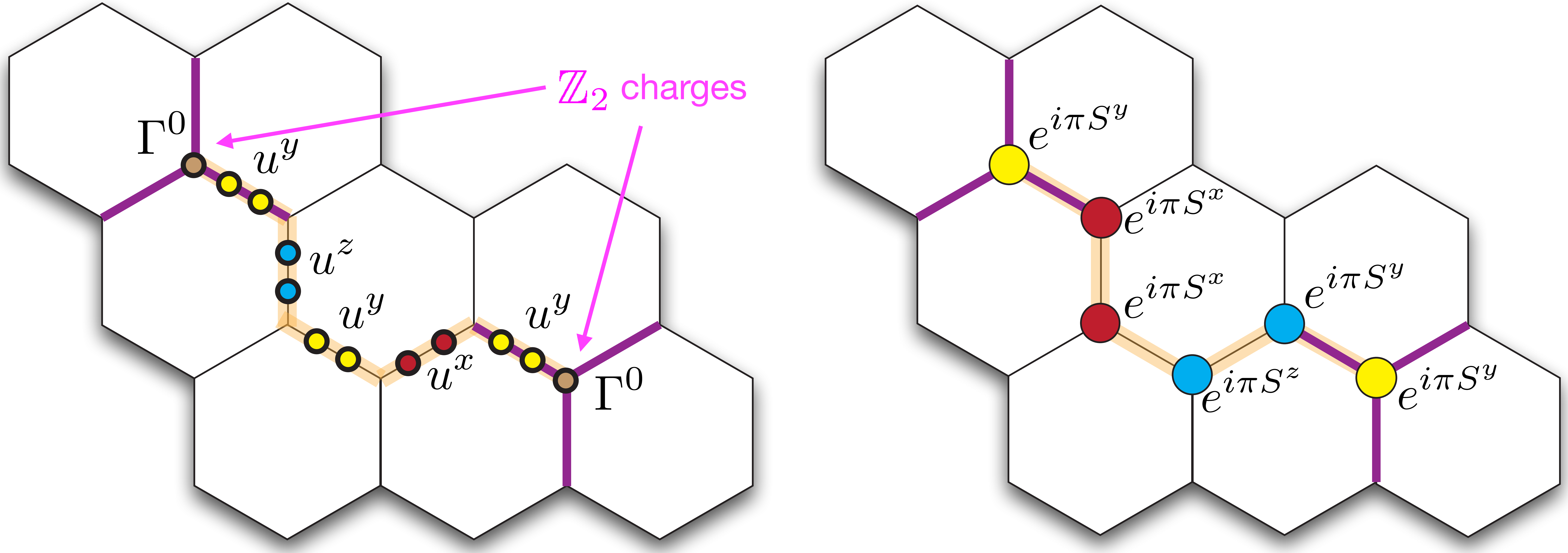


- $\mathbb{Z}_2$ gauge field: $u_{ij}^\mu = i\Gamma_i^\mu \Gamma_j^\mu$ commutes with Hamiltonian

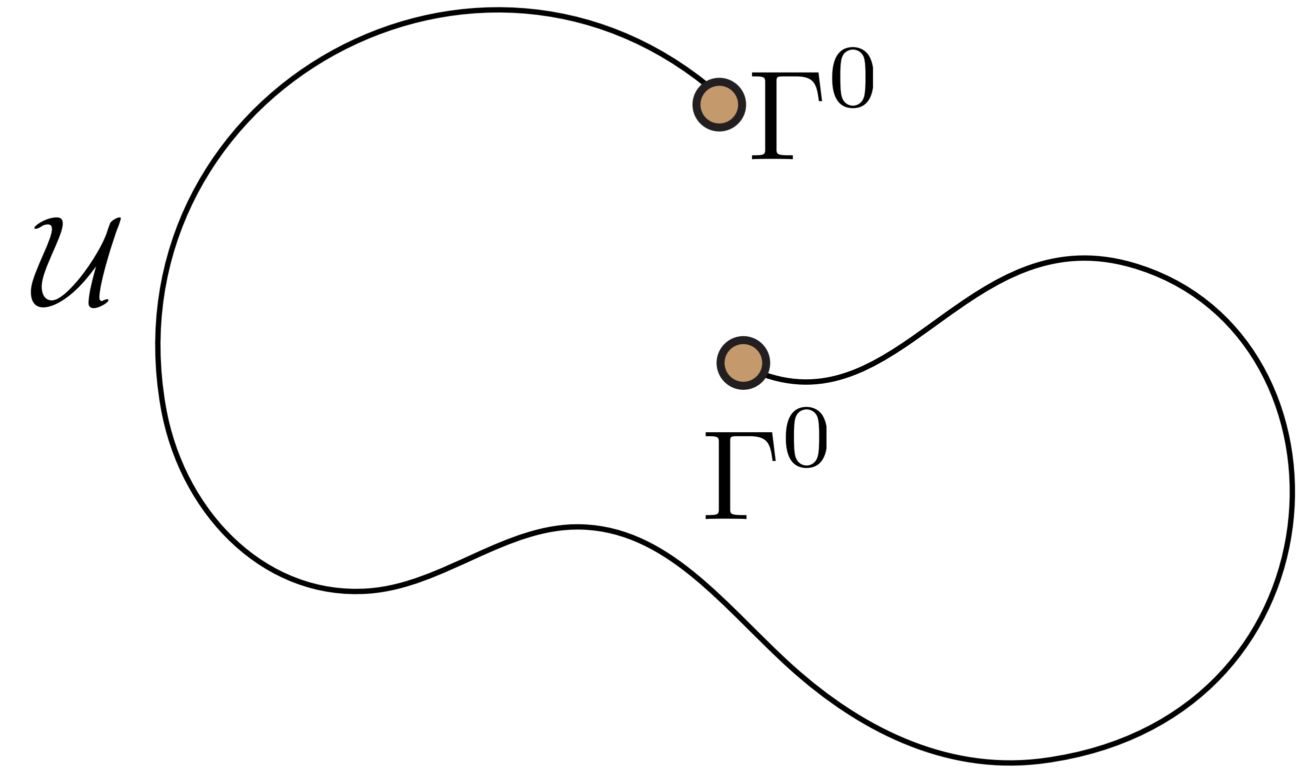
Giant string operator

- The $\mathbb{Z}_2$ charge is attached to a tensionless string operator.
- The string operator commutes with Hamiltonian except the two end points.



Giant string operator

- The tensionless string operator $\mathcal{U}$ indicates that the charges Γ^0 are either deconfined or condensed.



- condensed charges $H(\mathcal{U}|\mathrm{gs}\rangle) = E_0|\mathrm{gs}\rangle$

- deconfined charges $H(\mathcal{U}|\mathrm{gs}\rangle) = (E_0 + 2\delta E)|\mathrm{gs} + 2\Gamma^0\rangle$

$\delta E \sim \mathcal{O}(1)$ and is independent of the length of string

Giant charge

- $\mathbb{Z}_2$ gauge charge: $\Gamma^0 = \frac{(-1)^{\frac{S(2S-1)}{2}}}{(2S)!}\epsilon_{a_1,a_2,\ldots,a_{2S}}\gamma^0_{a_1}\gamma^0_{a_2}\cdots\gamma^0_{a_{2S}}$

  - It is a SO(2S) singlet.
  - It carries the charge of improper $\mathbb{Z}_2$ of O(2S).

  - It is a boson when S is an integer.
  - It is a fermion when S is a half-integer.

Fate of Giant charge

If the string operators $\mathcal{U}$ for charge Γ^0 are tensionless.

- Fermionic charges Γ^0

They can only be deconfined.

- Bosonic charges Γ^0

They can be condensed or deconfined depending on dynamics.

Fate of Giant charge

HM, Phys. Rev. Lett. 130, 156701 (2023)

If the string operators $\mathcal{U}$ for charge Γ^0 are tensionless.

- Fermionic charges Γ^0

They can only be deconfined.

Half-integer spin systems are always in gapped/ gapless deconfined phases

- Bosonic charges Γ^0

Integer spin systems can be in gapped spin liquid phases or trivial phases.

They can be condensed or deconfined depending on dynamics.

Fate of giant charge in higher spin Kitaev model

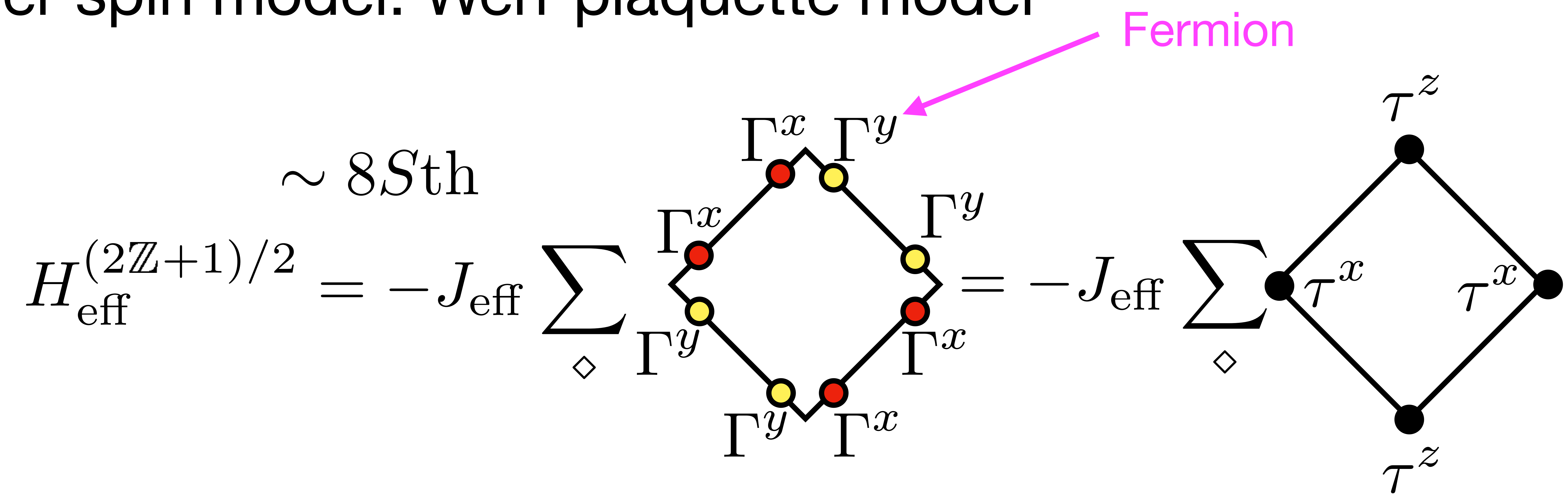
The fate of the giant partons in the anisotropic limit

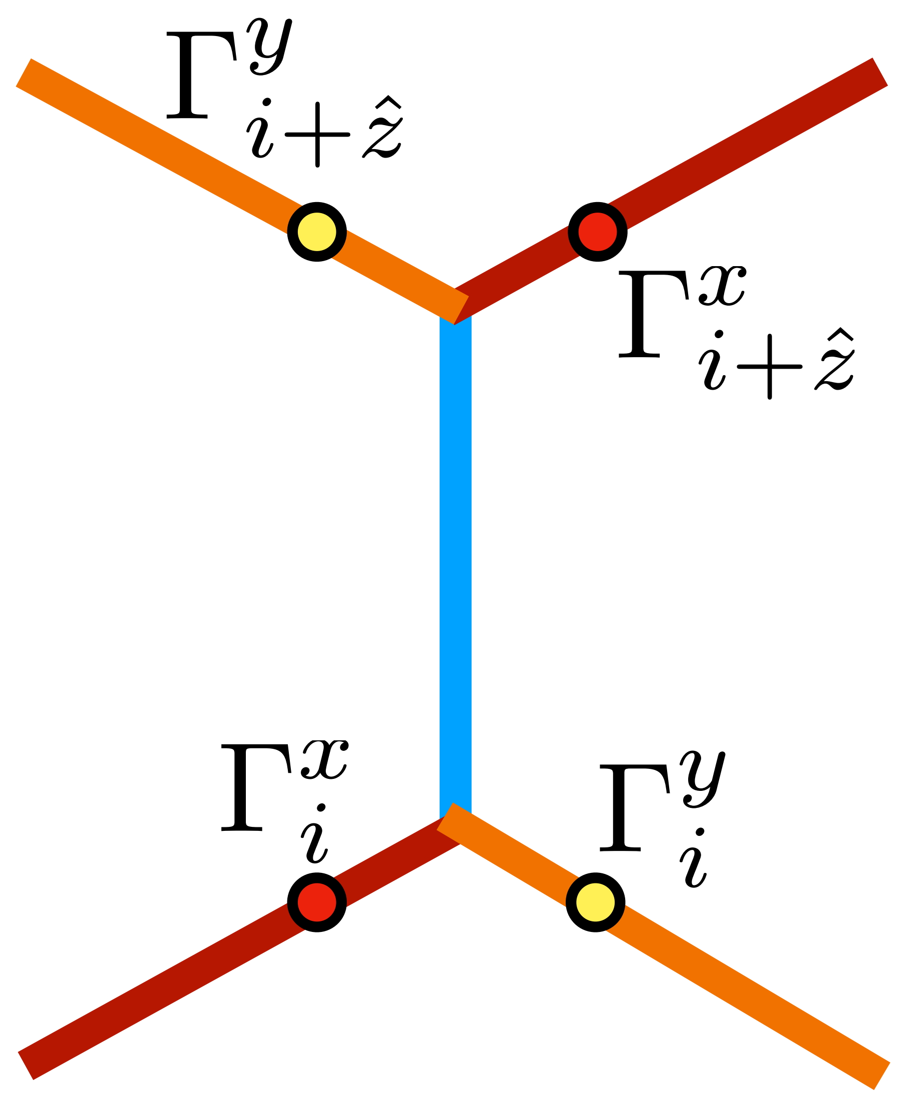
Higher spin Kitaev model

In the anisotropic limit $J_z \gg J_x \sim J_y$

Lee, Suzuki, Kim, Kawashima (2021)

Half-integer spin model: Wen-plaquette model





The effective Hamiltonian is the same as the half spin model.

Higher spin Kitaev model

In the anisotropic limit $J_z \gg J_x \sim J_y$

Lee, Suzuki, Kim, Kawashima (2021)

Half-integer spin model: Wen-plaquette model

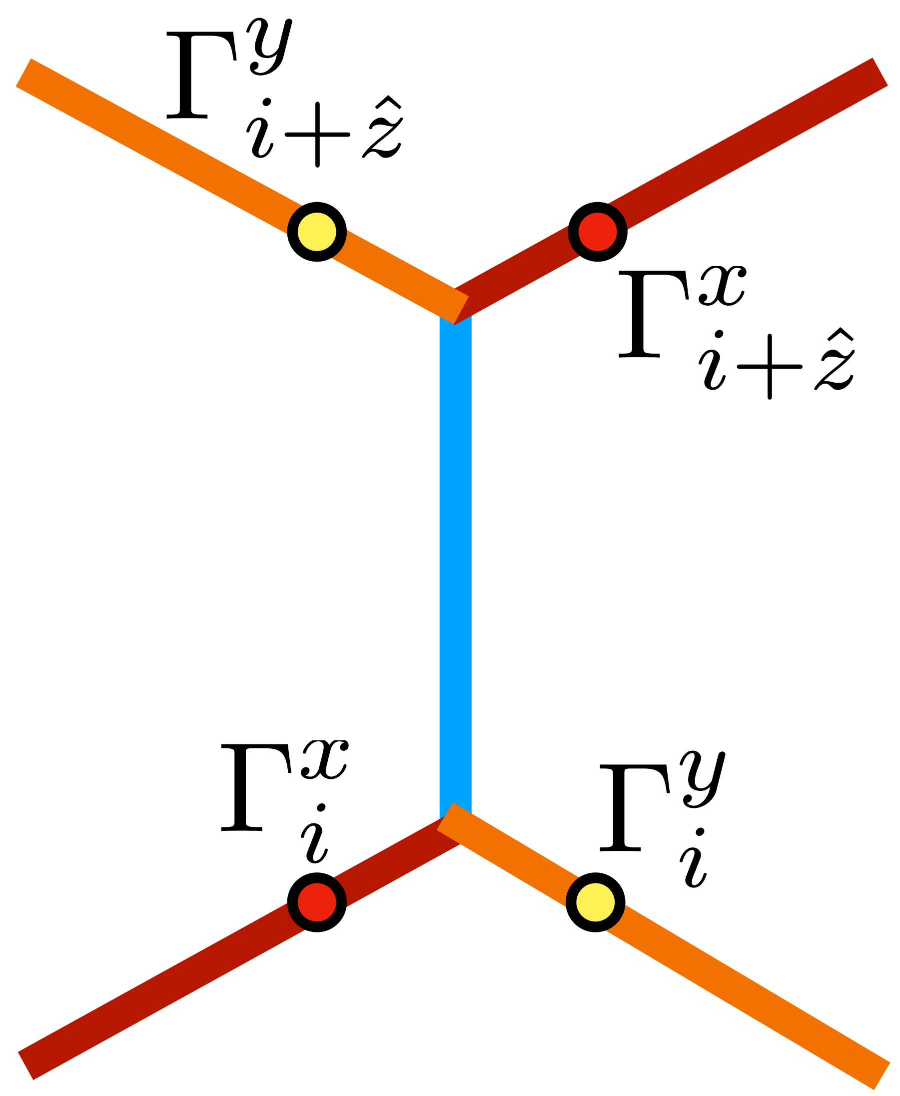
Ground state is the $\mathbb{Z}_2$ topological order.

Excitations:

$$\Gamma_i^{0,x,y,z} \sim \varepsilon, e \times m$$

$$i\Gamma_i^x \Gamma_i^y \sim e \times e$$

$$i\Gamma_i^x \Gamma_{i+\hat{z}}^y \sim m \times m$$



Higher spin Kitaev model

In the anisotropic limit $J_z \gg J_x \sim J_y$

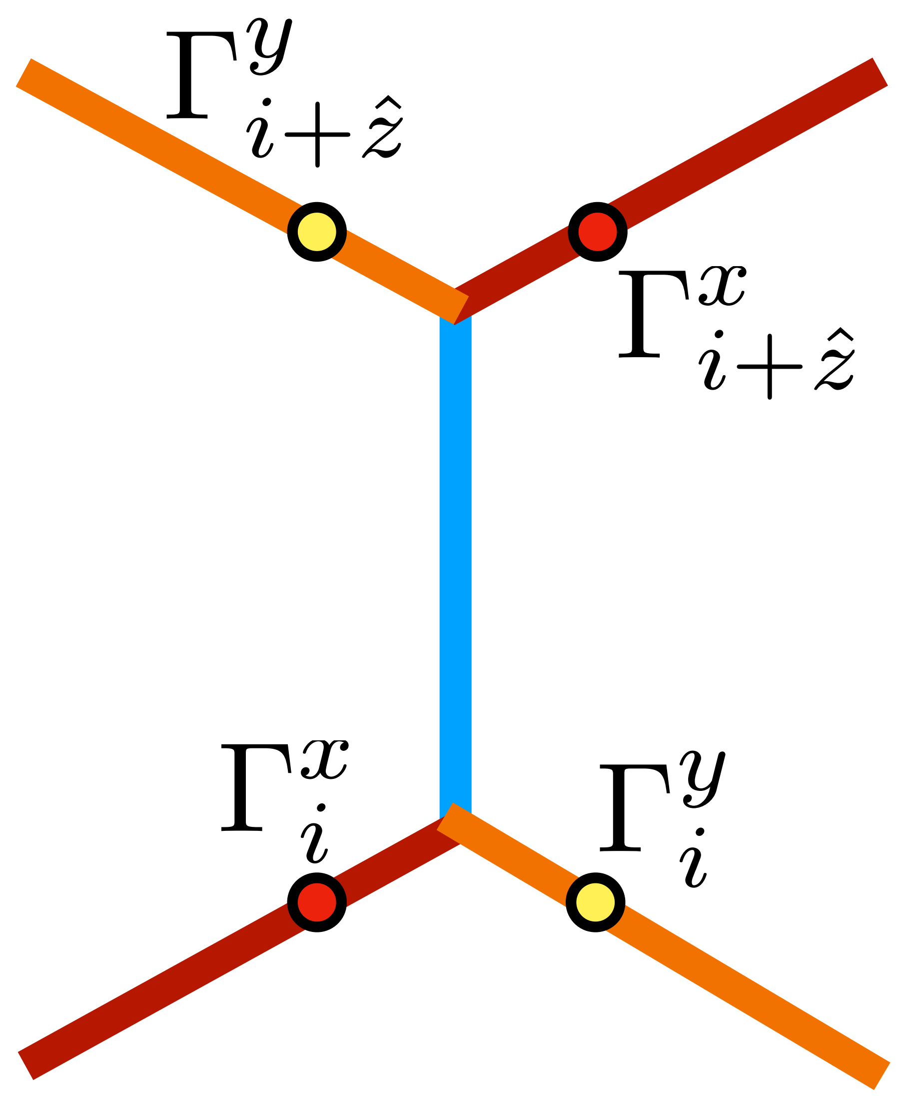
Lee, Suzuki, Kim, Kawashima (2021)

Integer spin model: trivial

Boson

$$\sim 4S\text{th}$$

$$H^{\mathbb{Z}}_{\text{eff}} = -J_{\text{eff}} \sum_i \Gamma_i^x \Gamma_{i+\hat{z}}^y = -J_{\text{eff}} \sum_{\ell \in z} \tau_\ell^x$$

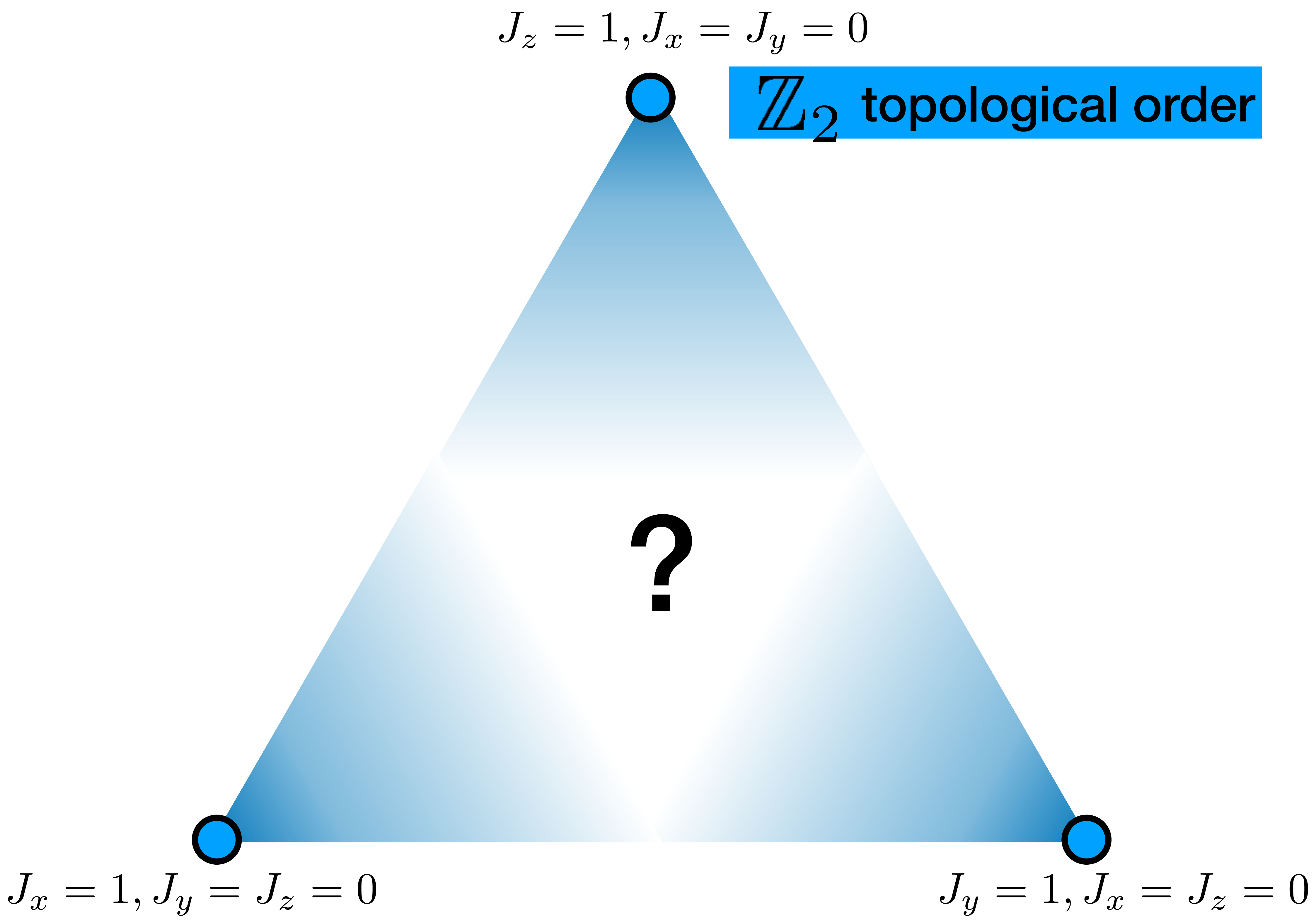


The effective Hamiltonian is trivial.

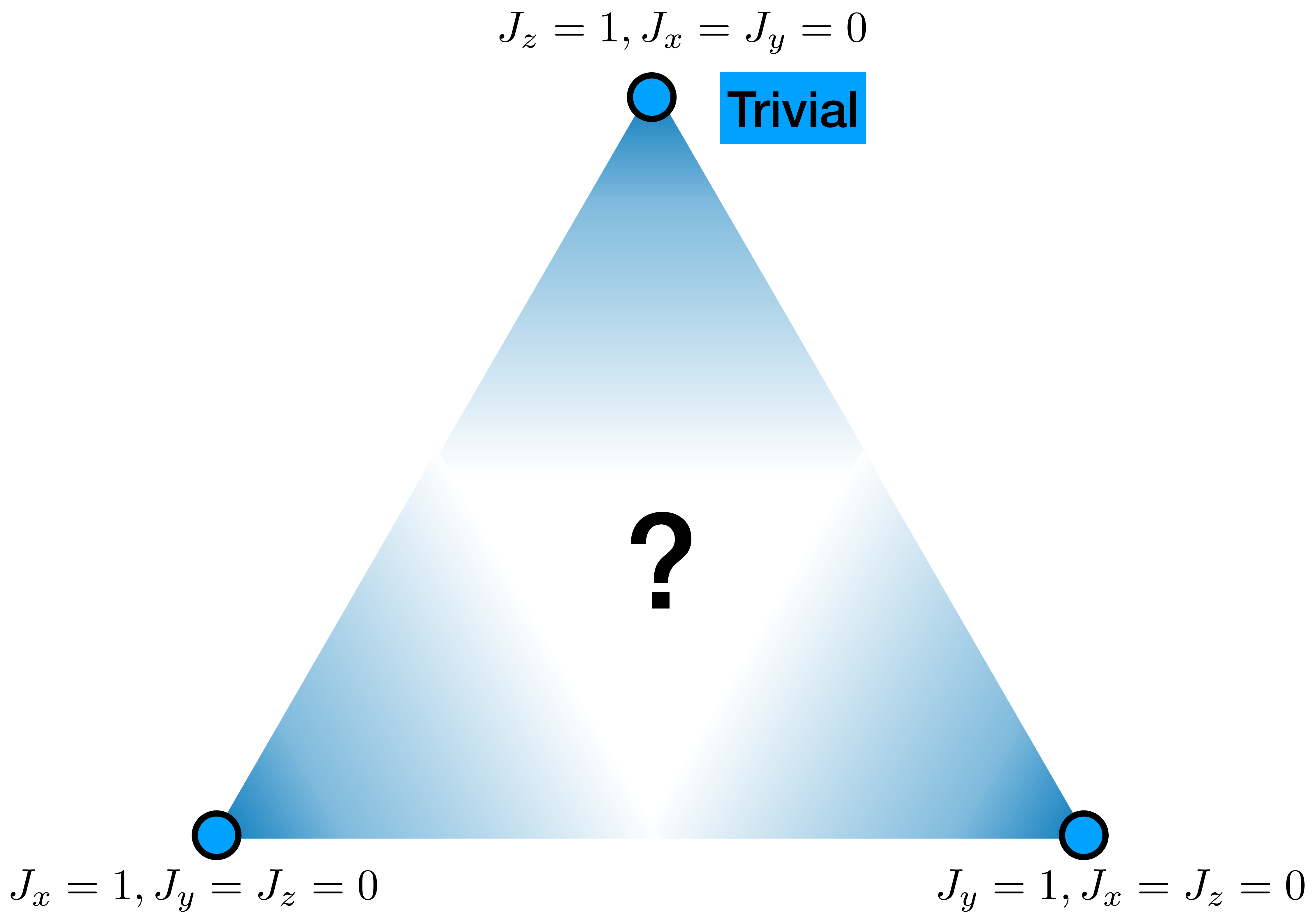
Ground state is the trivial paramagnet with $\Gamma_i^x = \Gamma_i^y = \Gamma_{i+\hat{z}}^x = \Gamma_{i+\hat{z}}^y$

Any string operator is 1. Since $H(\Gamma^0|\text{gs}\rangle) = E_0\ (\Gamma^0|\text{gs}\rangle)$, the boson Γ^0 is condensed.

Higher spin Kitaev model



Half-integer spin



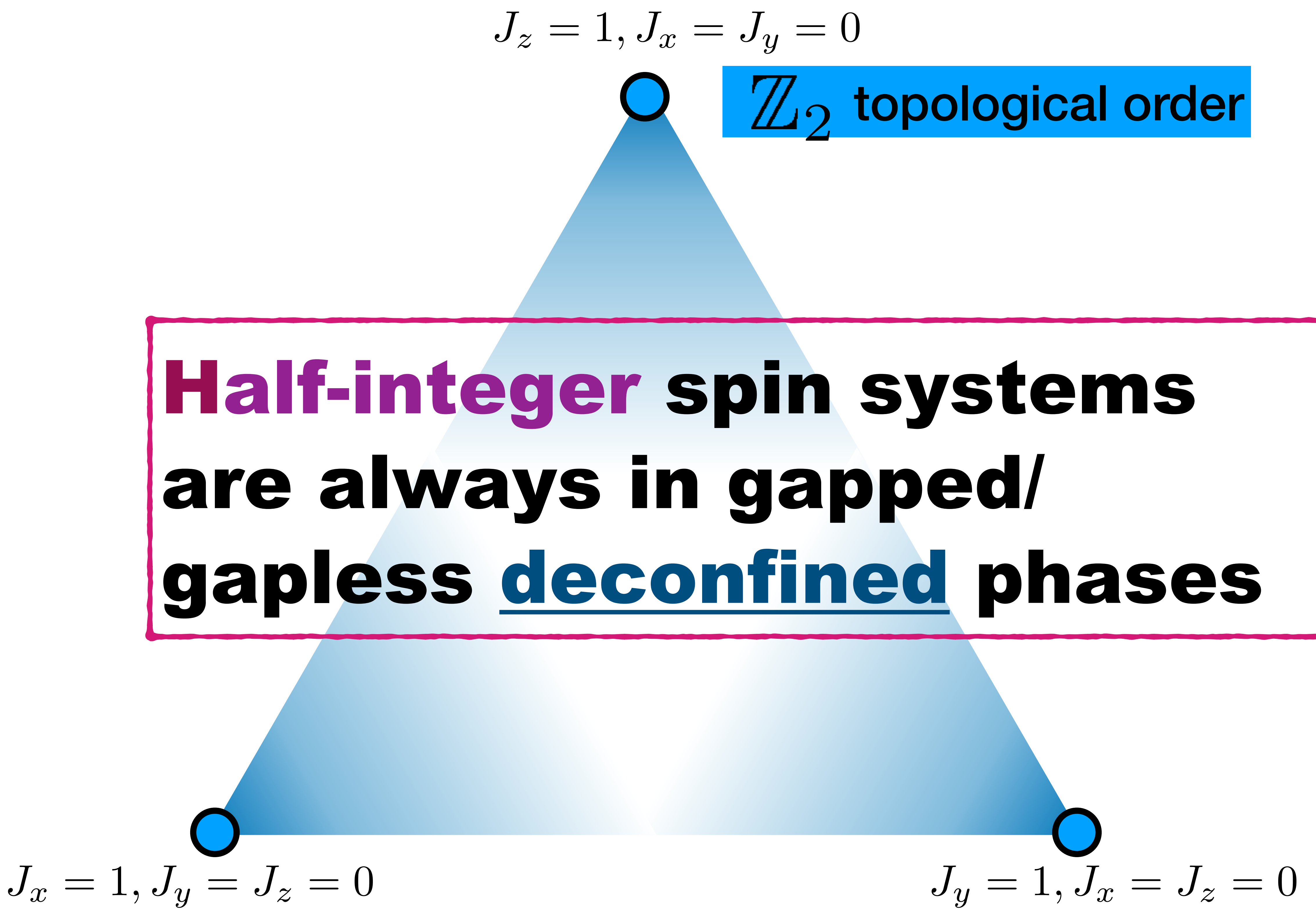
Integer spin

Higher spin Kitaev model

General claim for the isotropic model ?

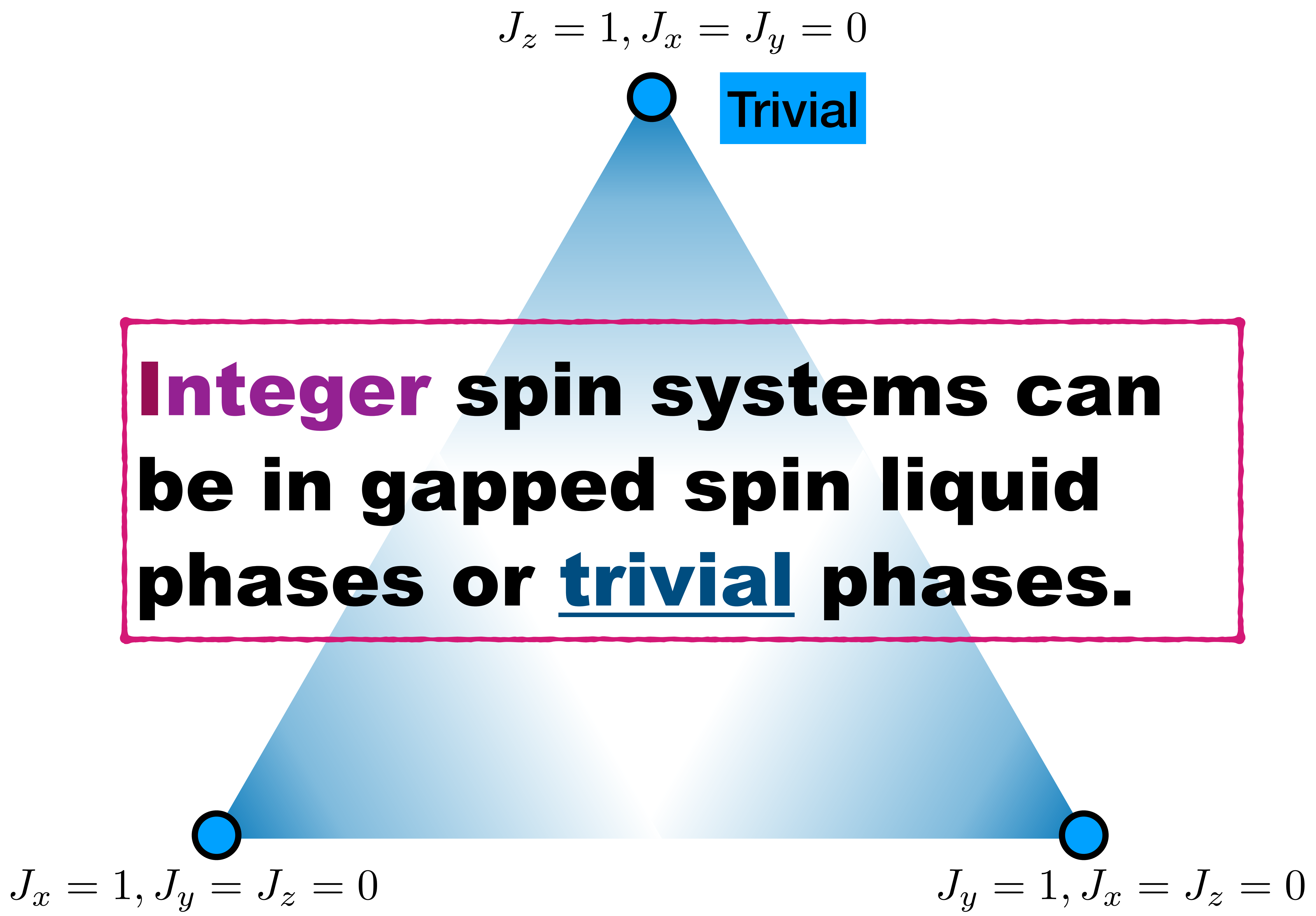
Higher spin Kitaev model

HM, Phys. Rev. Lett. 130, 156701 (2023)



Half-integer spin systems are always in gapped/gapless deconfined phases

Half-integer spin



Integer spin systems can be in gapped spin liquid phases or trivial phases.

Integer spin

Even-odd effect

HM, Phys. Rev. Lett. 130, 156701 (2023)

Haldane chain Haldane (1983)

Lieb-Schultz-Mattis (LSM) theorem: In a spin system with translation and spin rotation symmetry, half-integer spin per unit cell does not admit a gapped symmetric ground state lacking fractionalized excitations. Half-spin per site on Honeycomb lattice is anomaly-free.

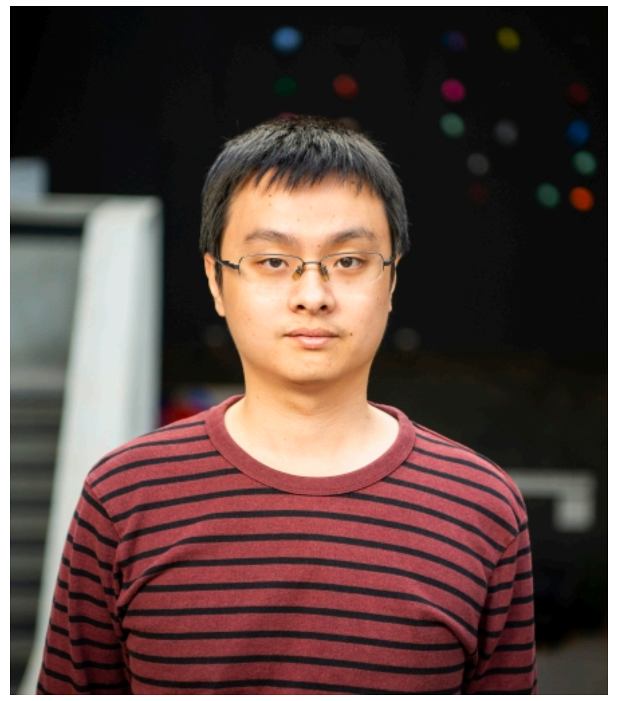
Our system?

Lieb, Schultz, and Mattis (1961)
Affleck and Lieb, (1986)
Oshikawa (2000)
Hastings (2004)
……

Oshikawa, "Oddness in the spin-S Kitaev honeycomb model," Journal Club for Condensed Matter Physics (2023)

Ruizhi Liu, Ho Tat Lam, **HM**, Liujun Zou, arXiv:2310.16839 (2023)

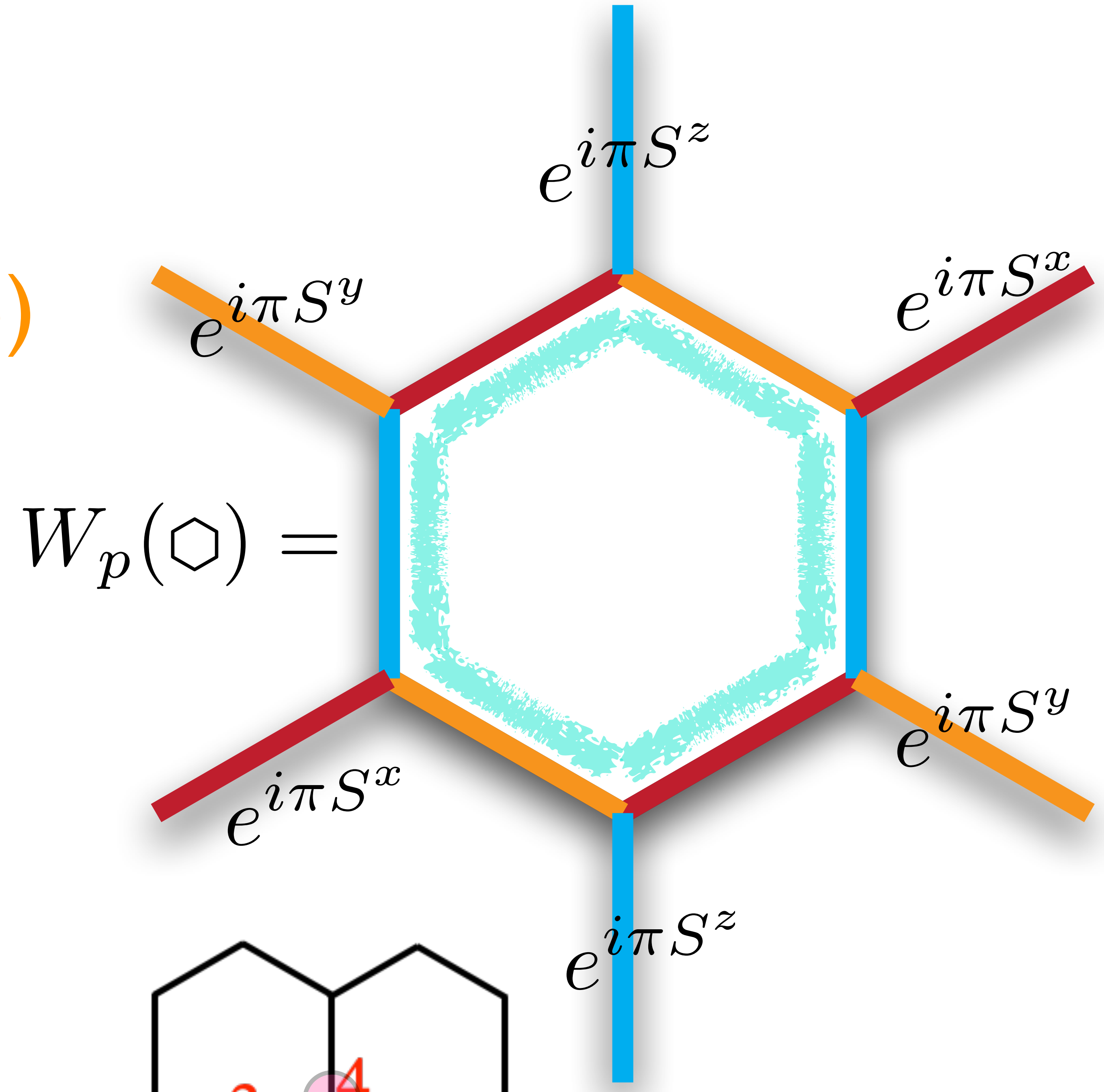




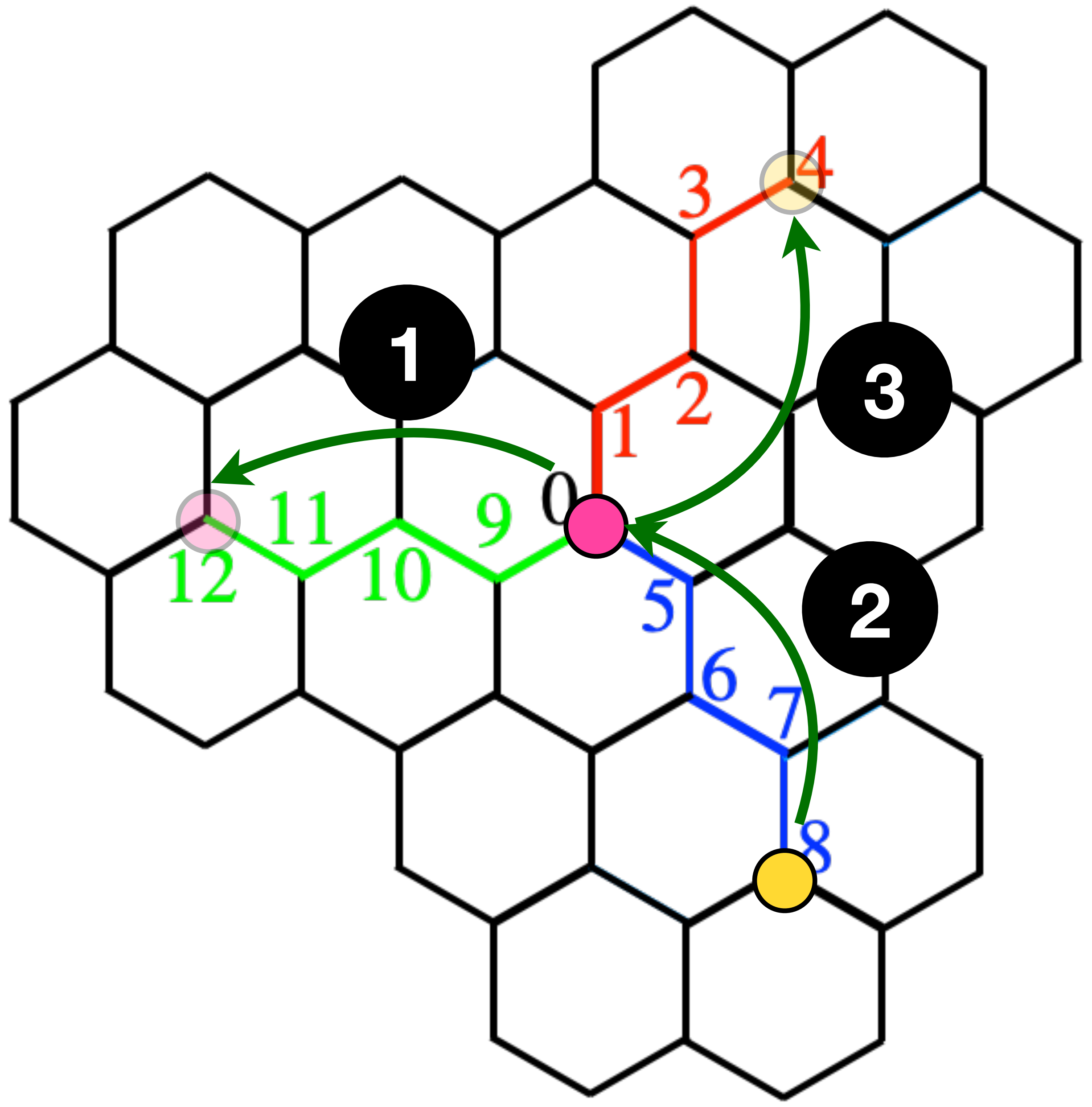
Even-odd effect

Ruizhi Liu, Ho Tat Lam, **HM**, Liujun Zou, arXiv:2310.16839 (2023)

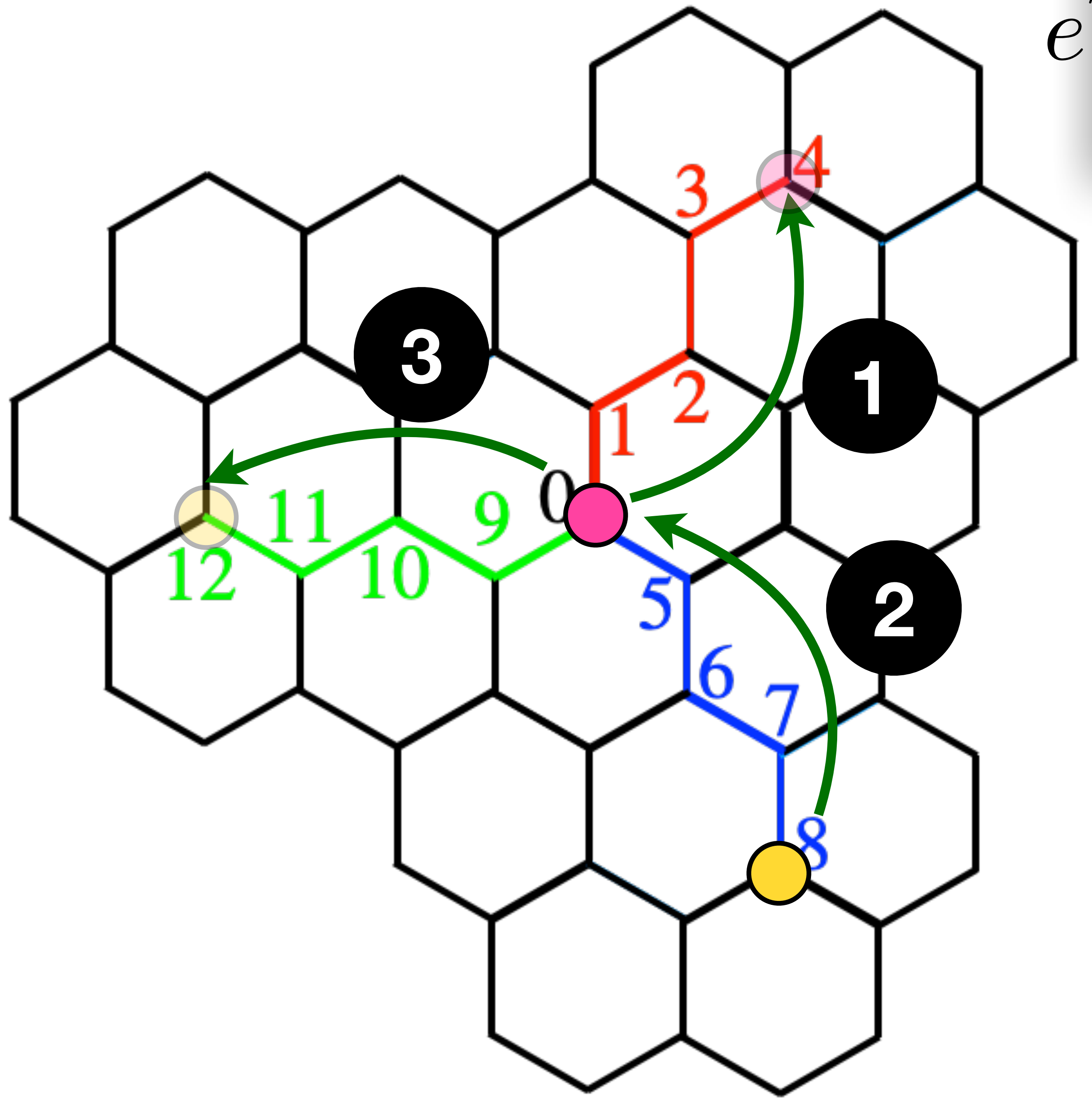
- Exact Z2 1-form symmetry: generated by



- Anomalous or not: check the statistics of its end points (symmetry defect)



$$= (-1)^{2S}$$



X.G Wen (2019)
Levin, Wen (2003)
Kawagoe, Levin, (2020)
……

- The exact Z2 1-form symmetry is anomalous in half-integer spin systems

Summary

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Ruizhi Liu, Ho Tat Lam, **HM**, Liujun Zou, arXiv:2310.16839 (2023)

- In the higher spin Kitaev model, local commuting operators are Z2 fluxes.
- The Z2 charge is a Majorana fermion in the half-integer spin model. The system always has deconfined excitations and is expected to form a spin liquid phase.
- The Z2 charge is a boson in the integer spin model. The system can be trivial when the bosons condense.
- The fundamental reason for this even-odd effect is the exact 1-form symmetry being anomalous in half-integer spin systems.

Open questions

- Is the full SO(2S) confined in the isotropic limit?
- Are bosonic charges condensed in the isotropic model?
- Given the Z2 1-form symmetry is anomalous, what is the extra effect of lattice symmetries?
- How does the solvability depend on the number of local conserved quantities and the dimension of Hilbert space?
- …….

Thank you for your attention!