Towards a unified description of quantum Hall effects

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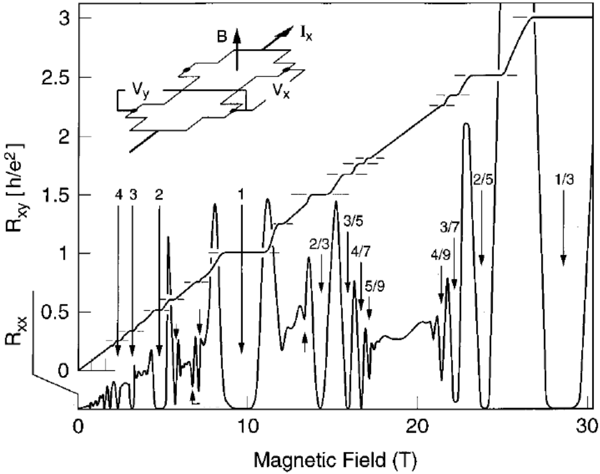
Ajit C. Balram, SciPost Phys. **10**, 083 (2021)

Plan of the talk

- Fractional quantum Hall effect (FQHE) in the Lowest Landau level (LLL): composite fermion (CF) theory
- FQHE states in the second LL and a description of them in terms of partons
- Conclusion and outlook

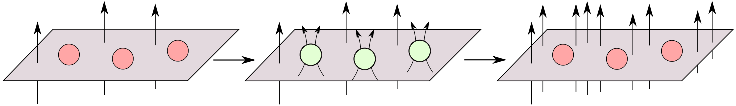
FQHE in the LLL predominantly occurs at $\nu = n/(2pn \pm 1)$



J. P. Eisenstein and H. L. Stormer, Science **248**, 4962, 1510-1516 (1990)

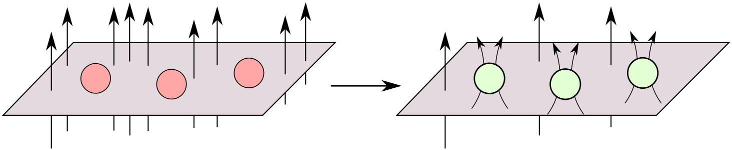
FQHE as IQHE of composite fermions

A composite fermion (CF) is a bound state of an electron and even number of vortices/flux quanta.



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE as IQHE of composite fermions

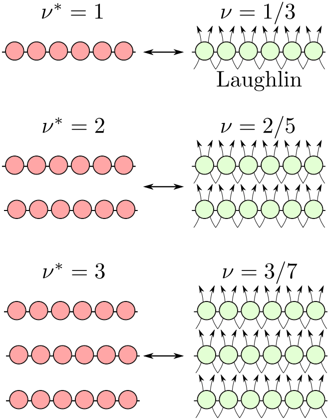


$$B^* = B - 2p\rho\phi_0, \quad \phi_0 = hc/e$$

$$\nu = \frac{\rho\phi_0}{B}, \quad \nu^* = \frac{\rho\phi_0}{|B^*|}, \quad \nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE ground states are analogous to IQHE ones



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE wave functions can be built from IQHE ones

- Jain wave functions at $\nu = n/(2pn \pm 1)$:

$$\Psi^{\rm CF}_{\nu=\frac{n}{2pn\pm1}} = \mathcal{P}_{\rm LLL}\left(\Phi_{\pm n}\prod_{i<j}(z_i - z_j)^{2p}\right).$$

(dropped Gaussian factor for ease of notation)

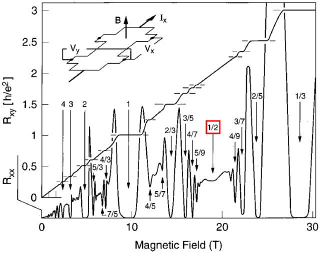
Φ_n wave function of n filled LLs.

$\mathcal{P}_{\rm LLL}$ implements lowest Landau level projection.

- no adjustable parameters in these wave functions
- wave functions can be evaluated for large system sizes

J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989)

Mystery of the $\nu = 1/2$ state



- composite fermions absorb all of the magnetic flux: $B^* = 0$

  Halperin, Lee and Read, Phys. Rev. B **47**, 7312 (1993)

- In zero effective magnetic field CFs form a Fermi sea

Overlaps of CF states with LLL Coulomb ground states

overlaps obtained from direct projected states

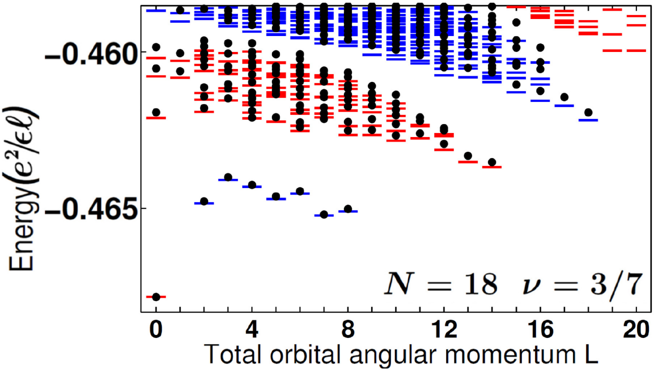
ν	N	Hilbert space dimension	$\lvert\langle\Psi^{0\mathrm{LL}}\lvert\Psi^{\mathrm{CF}}\rangle\rvert$
1/3	15	2×10^9	0.9876 (Laughlin)
1/5	11	4×10^8	0.9413 (Laughlin)
2/5	12	3×10^5	0.9971
3/7	12	6×10^4	0.9988
2/9	10	1×10^7	0.9744

$\lvert\Psi^{0\mathrm{LL}}\rangle$ is obtained by brute-force exact diagonalization in the spherical geometry

Ajit C. Balram and A. Wójs, Phys. Rev. Research **2**, 032035(R) (2020)
B. Yang and Ajit C. Balram, New J. Phys. **23**, 013001 (2021)

Ajit C. Balram, SciPost Phys. **10**, 083 (2021)

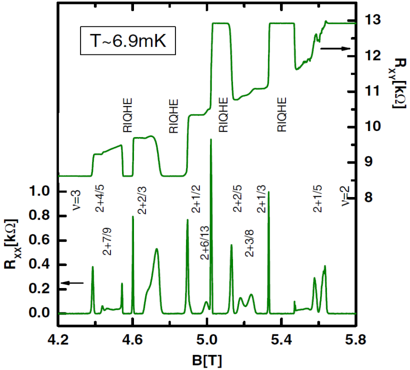
CF theory is extremely accurate in the lowest Landau level



dashes are obtained by brute-force exact diagonalization
$\sim 10^6$ states at each total orbital angular momentum L

Ajit C. Balram, A. Wójs and J. K. Jain, Phys. Rev. B **88**, 205312 (2013)

FQH states in the second Landau level



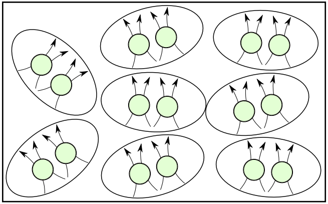
- appearance of *even* denominator fractions
- 6/13 appears "out of order"

A. Kumar *et al.* Phys. Rev. Lett. **105**, 246808 (2010)

Candidate states for $\nu = 5/2$: Pfaffian

$$\Psi^{\mathrm{MR}}_{\nu=1/2} = \mathrm{Pf}\left[\frac{1}{z_i - z_j}\right] \prod_{i<j} (z_i - z_j)^2$$

G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991)



p-wave paired state of composite fermions

N. Read and Dmitry Green, Phys. Rev. B **61**, 10267 (2000)

N	Hilbert space dimension	$\lvert\langle \Psi^{1\mathrm{LL}} \vert \Psi^{\mathrm{MR}}_{\nu=1/2} \rangle\rvert$
20	4×10^8	0.6736

Ajit C. Balram and A. Wójs, Phys. Rev. Research **2**, 032035(R) (2020)

Candidate states for $\nu = 5/2$: anti-Pfaffian

- anti-Pfaffian is the particle-hole conjugate of Pfaffian

$$\Psi^{\rm aPf}_{\nu=1/2} = \mathcal{P}_{ph}\left(\text{Pf}\left[\frac{1}{z_i - z_j}\right]\prod_{i<j}(z_i - z_j)^2\right)$$

M. Levin *et al.*, Phys. Rev. Lett. **99**, 236806 (2007), S. S. Lee *et al.*, Phys. Rev. Lett. **99**, 236807 (2007)

- construction extremely difficult to implement numerically
- recent numerics suggest anti-Pfaffian is favored in the presence of LL mixing

E. H. Rezayi, Phys. Rev. Lett. **119**, 026801 (2017)

Candidate states in the second Landau level

- 1/2: likely a *p*-wave paired state of CFs
- 1/3 and 2/3: likely analogous to the LLL states

  Ajit C. Balram *et al.*, Phys. Rev. Lett. **110**, 186801 (2013)

- 2/5: aRR3 or a Bonderson-Slingerland state

  N. Read and E. H. Rezayi, Phys. Rev. B **59**, 8084 (1999)

  Parsa Bonderson and J. K. Slingerland, Phys. Rev. B **78**, 125323 (2008)

- 3/8: Bonderson-Slingerland state

  J. A. Hutasoit *et al.*, Phys. Rev. B **95**, 125302 (2017)

- 6/13: Levin-Halperin state

  M. Levin and B. I. Halperin, Phys. Rev. B **79**, 205301 (2009)

Can we find a unified description of the second LL FQHE?

Yes.
In terms of “parton” states.

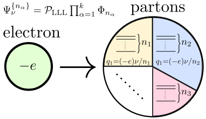
Parton states: product of fermionic states

- break each electron into fictitious partons, place partons into IQH (or any) fermionic states, fuse the partons back to recover the electron

$$\Psi_{\nu}^{\{n_\alpha\}} = \mathcal{P}_{\rm LLL} \prod_{\alpha=1}^{k} \Phi_{n_\alpha}(\{z_i\})$$

- k is odd for fermions

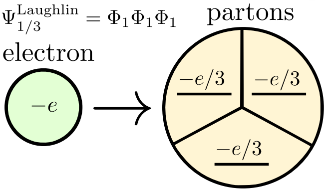
$$q_\alpha = (-e)\frac{\nu}{n_\alpha}, \quad \nu^{-1} = \sum_{\alpha=1}^{k} n_\alpha^{-1}, \quad \mathcal{S} = \sum_{\alpha=1}^{k} n_\alpha$$



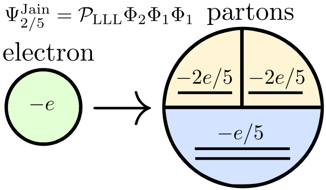
J. K. Jain, Phys. Rev. B **40**, 8079 (1989)

Laughlin and Jain states are parton states

- Laughlin state is a "$111\cdots$" parton state $\left[\Phi_1 \equiv \prod_{i<j}(z_i - z_j)\right]$



- Jain/CF states are "$n11\cdots$" parton states



J. K. Jain, Phys. Rev. B **40**, 8079 (1989)

Parton sequence for the second Landau level

A parton sequence (with its hole-conjugate states) captures almost all the observed FQH states in the second LL

$$\Psi^{[\bar{2}1]^p\bar{n}1^2}_{\frac{2n}{(p+4)n-2}} = \mathcal{P}_{\rm LLL}[\Phi_{-2}\Phi_1]^p\ \Phi_{-n}\Phi_1^2 \sim \left[\frac{\Psi^{\rm Jain}_{2/3}}{\Phi_1}\right]^p \Psi^{\rm Jain}_{n/(2n-1)}$$

- $p=1$: primary sequence $n=1,2,3,\cdots \rightarrow 2/3, 1/2, 6/13, \cdots$
- $p=2$: secondary sequence $n=1,2,3,\cdots \rightarrow 1/2, 2/5, 3/8, \cdots$
- Predicts FQHE at $\nu = 2+4/9$ and $2+4/11$

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018) and Phys. Rev. B **99**, 241108 (2019)

Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)

The "$\bar{n}\bar{2}1^3$" ansatz

$$\Psi^{\bar{n}\bar{2}1^3}_{\nu=2n/(5n-2)} = \mathcal{P}_{\rm LLL}[\Phi_n^*][\Phi_2^*]\Phi_1^3 \sim \frac{\Psi^{\rm Jain}_{n/(2n-1)}\Psi^{\rm Jain}_{2/3}}{\Phi_1}$$

- $n=1 \implies \nu=2/3$: standard composite fermion state
- $n=2 \implies \nu=1/2$: parton state in the anti-Pfaffian phase
- $n=3 \implies \nu=6/13$: a new candidate state

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)

The "$\bar{2}^2 1^3$" ansatz ∼ anti-Pfaffian

$$\Psi^{\bar{2}^2 1^3}_{\nu=1/2} = \mathcal{P}_{\rm LLL}[\Phi_2^*][\Phi_2^*]\Phi_1^3 \sim \frac{[\Psi^{\rm Jain}_{2/3}]^2}{\Phi_1}$$

- state occurs at a shift $\mathcal{S}=-1$: same as the anti-Pfaffian shift
- better than anti-Pfaffian for second LL Coulomb

N	$2l$	$\lvert\langle\Psi^{\rm 1LL}_{1/2}\lvert\Psi^{\bar{2}^2 1^3}_{1/2}\rangle\rvert^2$	$\lvert\langle\Psi^{\bar{2}^2 1^3}_{1/2}\lvert\Psi^{\rm aPf}_{1/2}\rangle\rvert^2$	$\lvert\langle\Psi^{\rm 1LL}_{1/2}\lvert\Psi^{\rm aPf}_{1/2}\rangle\rvert^2$
8	17	0.862	0.908	0.702
10	21	0.774	0.881	0.671
12	25	0.614	0.861	0.481

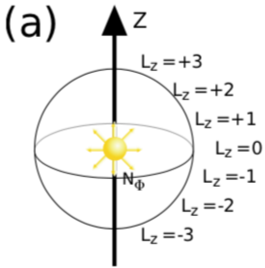
Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

Ajit C. Balram, Phys. Rev. B **105**, L121406 (2022)

Entanglement spectrum

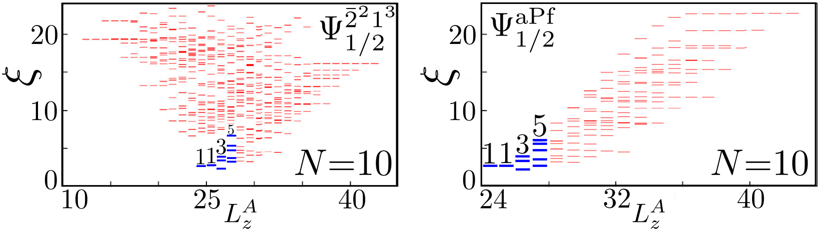
- Logarithm of the eigenvalues of the reduced density matrix
- Counting of low-lying entanglement levels: carries topological fingerprint of the state (Li-Haldane conjecture)
- can be evaluated from just the ground state wave function

related to edge excitations (bulk-edge correspondence)



H. Li and F. D. M. Haldane, Phys. Rev. Lett. **101**, 010504 (2008)

Entanglement spectrum of the $\bar{2}^2 1^3$ state



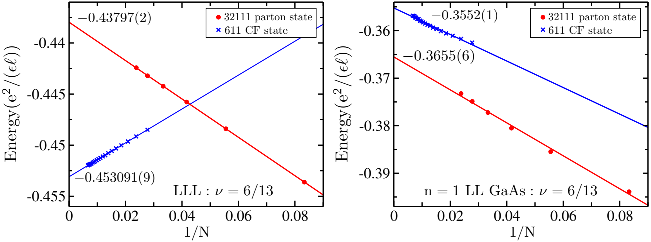
counting: $1, 1, 3, 5, \cdots$

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

"$\bar{3}\bar{2}1^3$" is topologically different from the 6/13 Jain state

$$\Psi^{\bar{3}\bar{2}1^3}_{\nu=6/13} = \mathcal{P}_{\rm LLL}[\Phi_3^*][\Phi_2^*]\Phi_1^3 \sim \frac{[\Psi^{\rm Jain}_{3/5}][\Psi^{\rm Jain}_{2/3}]}{\Phi_1}$$

- occurs at $\mathcal{S}=-2$: topologically different from 6/13 Jain state
- different thermal Hall conductance from the 6/13 Jain state
- energetically better than the 6/13 Jain state in the second LL



Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)

What makes our parton states special?

- Composite fermion ($n11\cdots$ parton) states capture the most prominent LLL plateaus
  $\rightarrow$ placing partons into $\nu = 1$ states, i.e., $\Phi_1 = \prod_{i<j}(z_i - z_j)$ builds good correlations in the many-body state
- Simplest generalization $\rightarrow$ $nm11\cdots$ where $m = 2$ or $m = -2$
- Comes down to energetics: for the second LL interaction our sequence of parton states appear most plausible
- Open problem: for a given interaction which parton state(s) is likely to be stabilized

Outlook

- Parton theory can potentially also capture delicate states observed in the LLL that are *not* part of the Jain sequence.

  Rakesh K. Dora and Ajit C. Balram, Phys. Rev. B **105**, L241403 (2022)

  Ajit C. Balram, Phys. Rev. B **103**, 155103 (2021)

  Ajit C. Balram and A. Wójs, Phys. Rev. Research **3**, 033087 (2021)

- Very high-energy excitations in the Jain states that are not described by composite fermions but lend themselves to a description in terms of partons.

  Ajit C. Balram, Zhao Liu, Andrey Gromov, and Zlatko Papić, Phys. Rev. X **12**, 021008 (2022)

Almost all fractional quantum Hall states can be described as products of integer quantum Hall states.

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