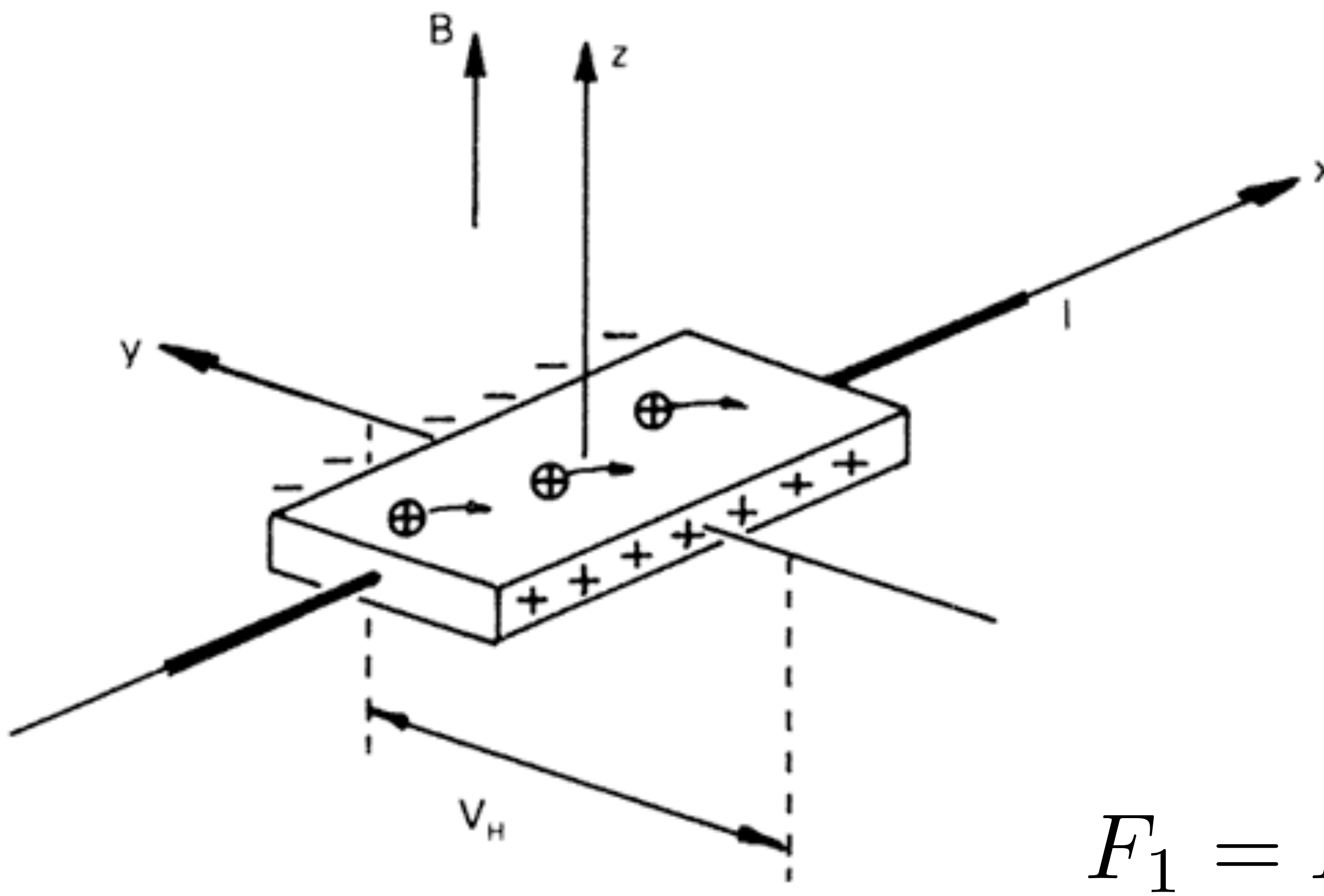
Conference on Fractionalization and Emergent Gauge Fields in Quantum Matter,
Dec. 4th-8th, 2023

Theory of conformal Hilbert spaces and the fractionalization of anyons in fractional quantum Hall systems

Yang Bo
Nanyang Technological University Singapore

Ha Quang Trung and Bo Yang, Phys. Rev. Lett. 127, 046402 (2021).
Yuzhu Wang and Bo Yang, Nat. Commun. 14, 2317 (2023)
Bo Yang, arXiv:2307.06361.

The 2D Hall Bar



Two-dimensional electron gas systems (2DEGs)

$$F_2 = e\vec{v} \times \vec{B} = evB_z$$

$$F_1 = \frac{eV_H}{w}$$

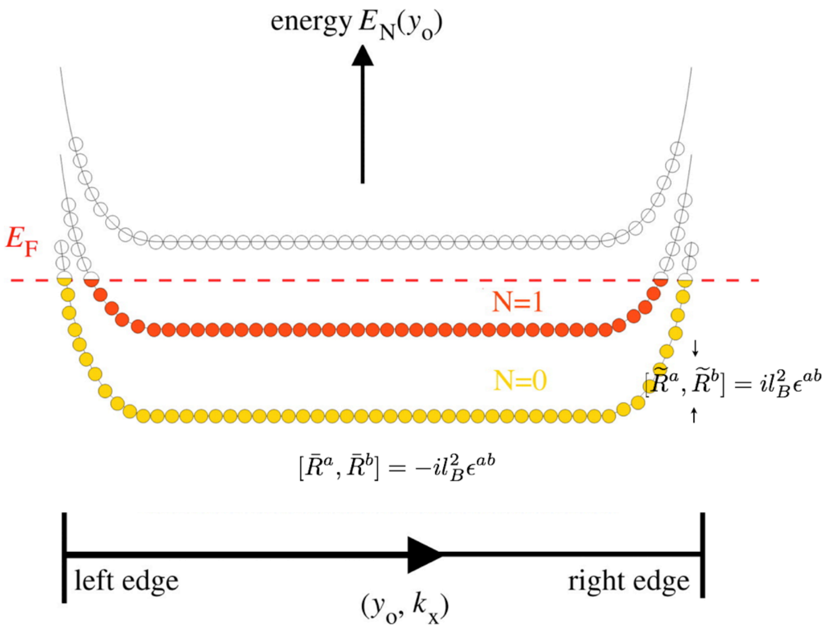
$$F_1 = F_2 \rightarrow R_H = \frac{E_y}{j_x B_z} = \frac{1}{ne}$$

Hall coefficient

charge density

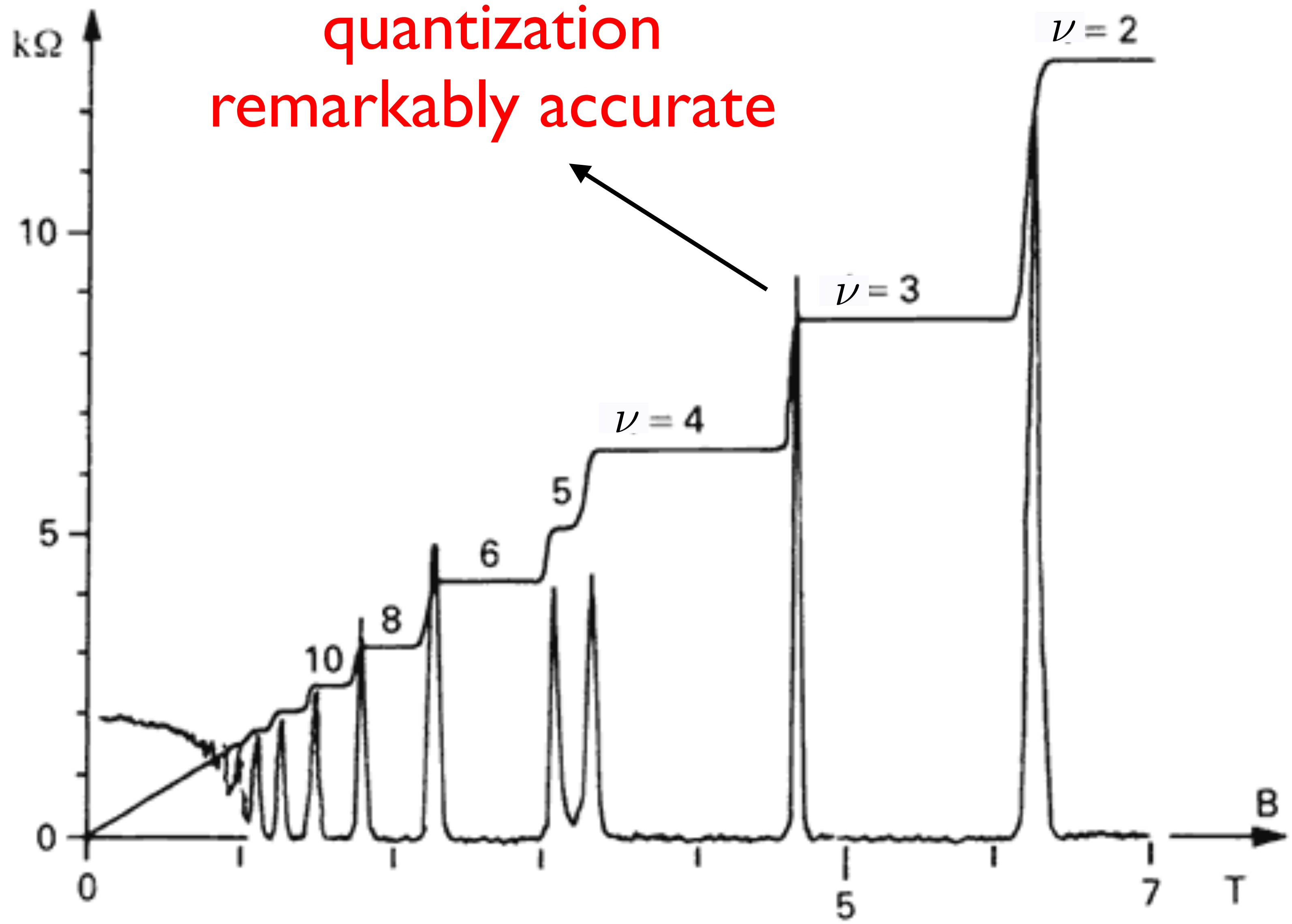
The Integer Quantum Hall Effect

$$\sigma_{xy} = \nu \frac{e^2}{h}$$



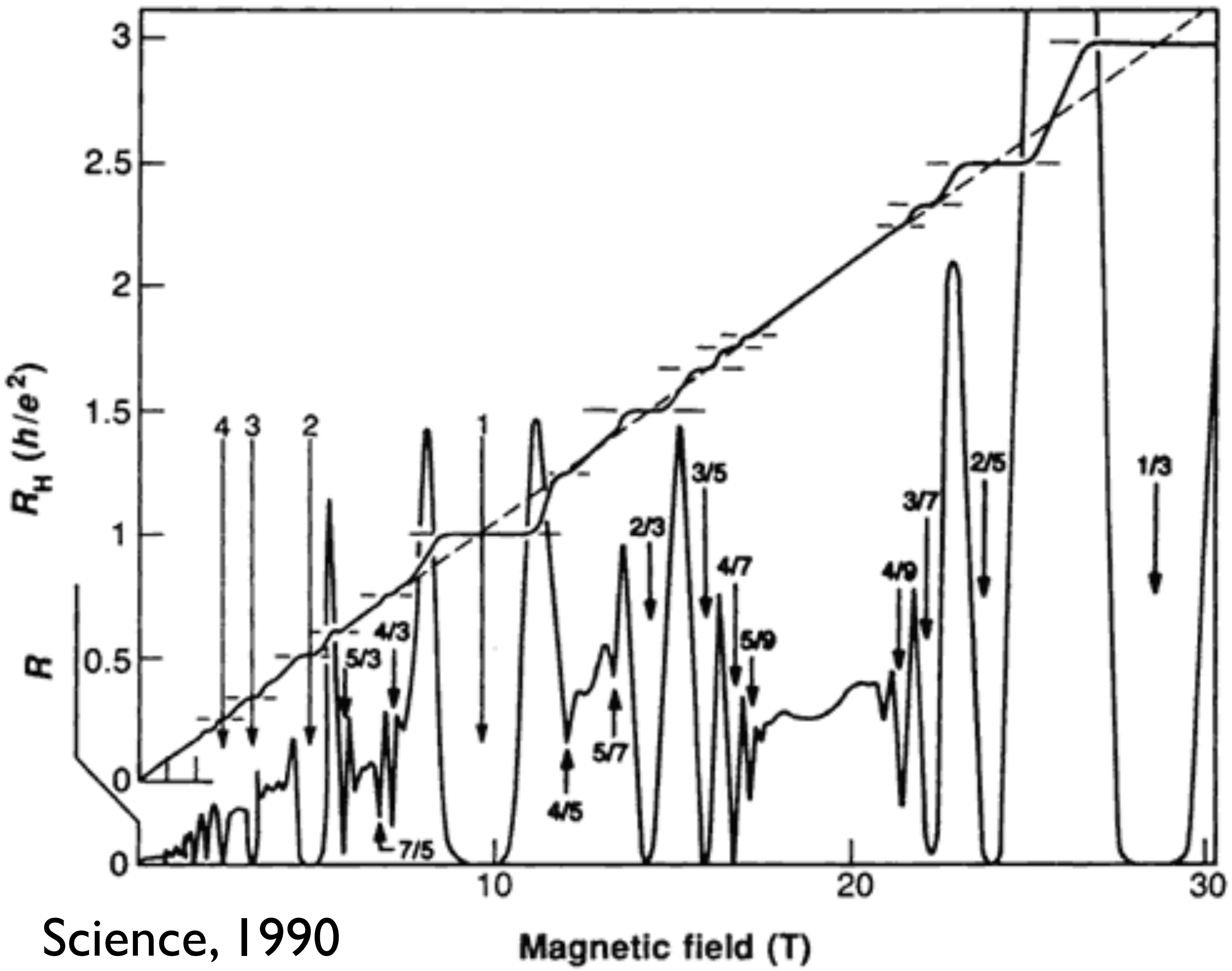
- Ground state **gap**
- Anderson localisation due to **disorder**
- **Topological** "obstruction" of Anderson localisation

quantization
remarkably accurate



IQHE is used as a standard in defining resistance

Fractional Quantum Hall Effect: strongly interacting systems with topological order



Science, 1990

excitations with fractionalized charge

anyon statistics

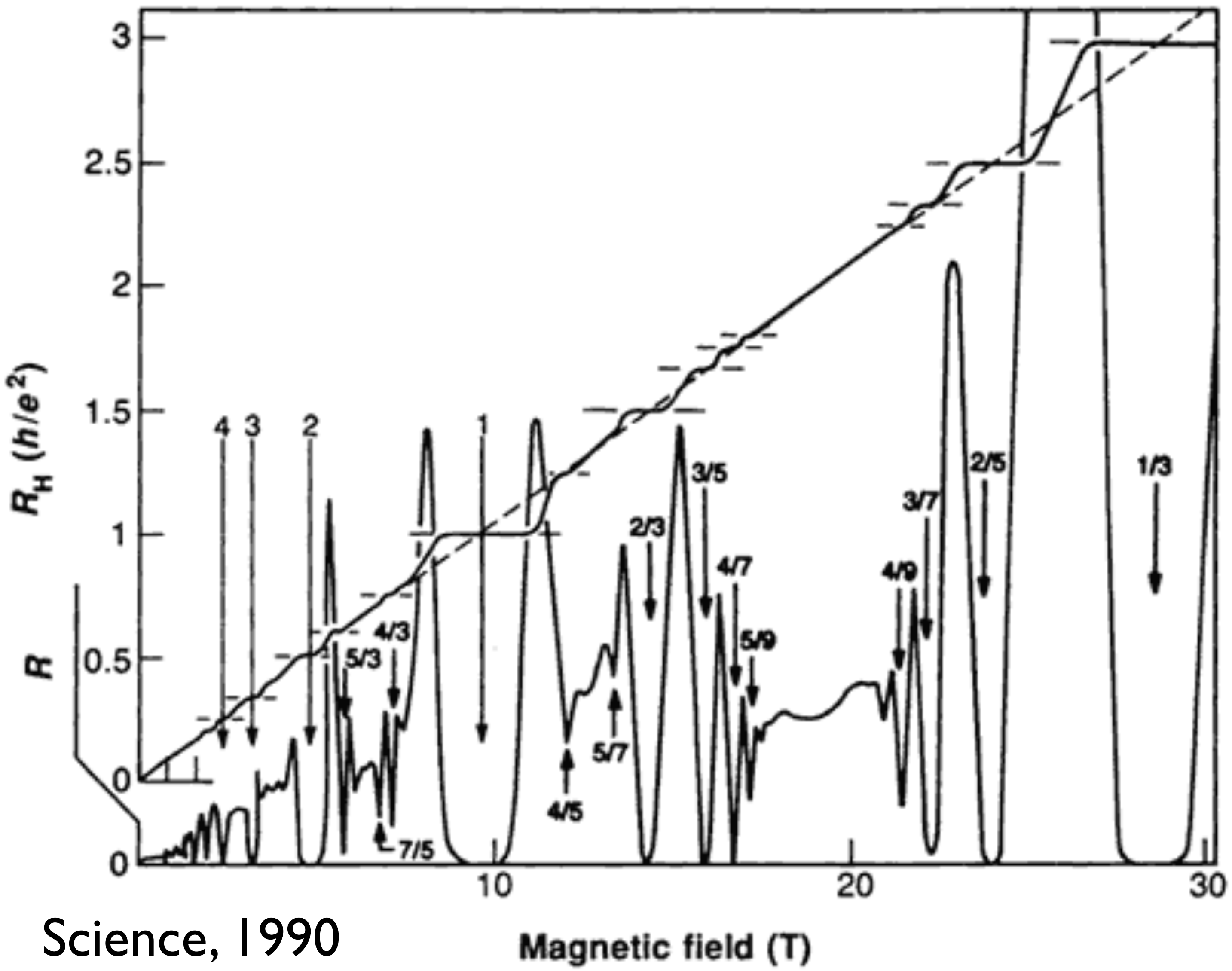
non-Abelian statistics and braiding

entanglement entropy/ spectrum

↑

Topological quantum computer, with robust storing and manipulation of quantum information

Fractional Quantum Hall Effect: strongly interacting systems with topological order



Science, 1990

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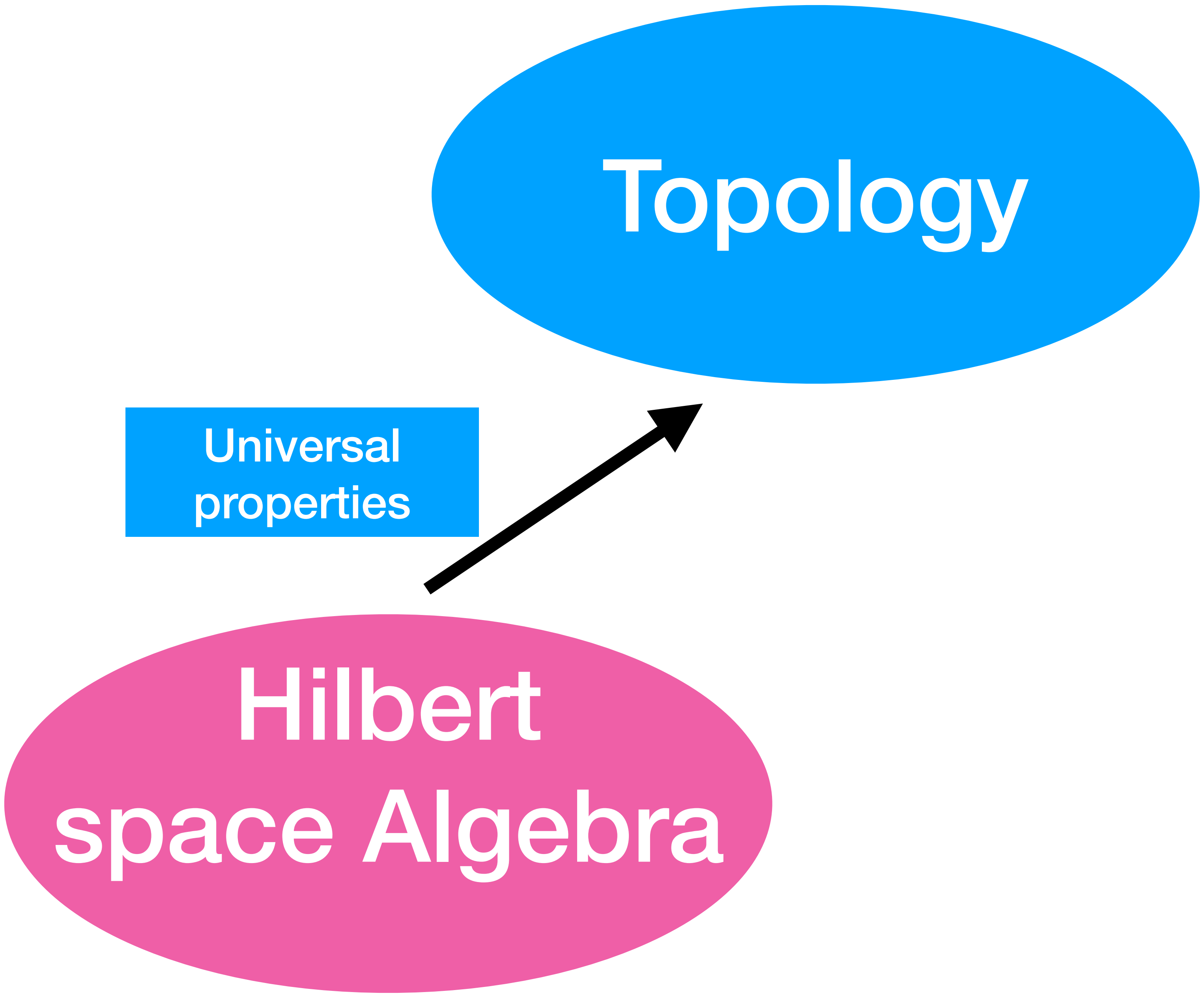
Topological quantum computer, with robust storing and manipulation of quantum information

IQHE: **single particle physics**

FQHE: **many-body physics**

The elementary degrees of freedom in a single Landau level



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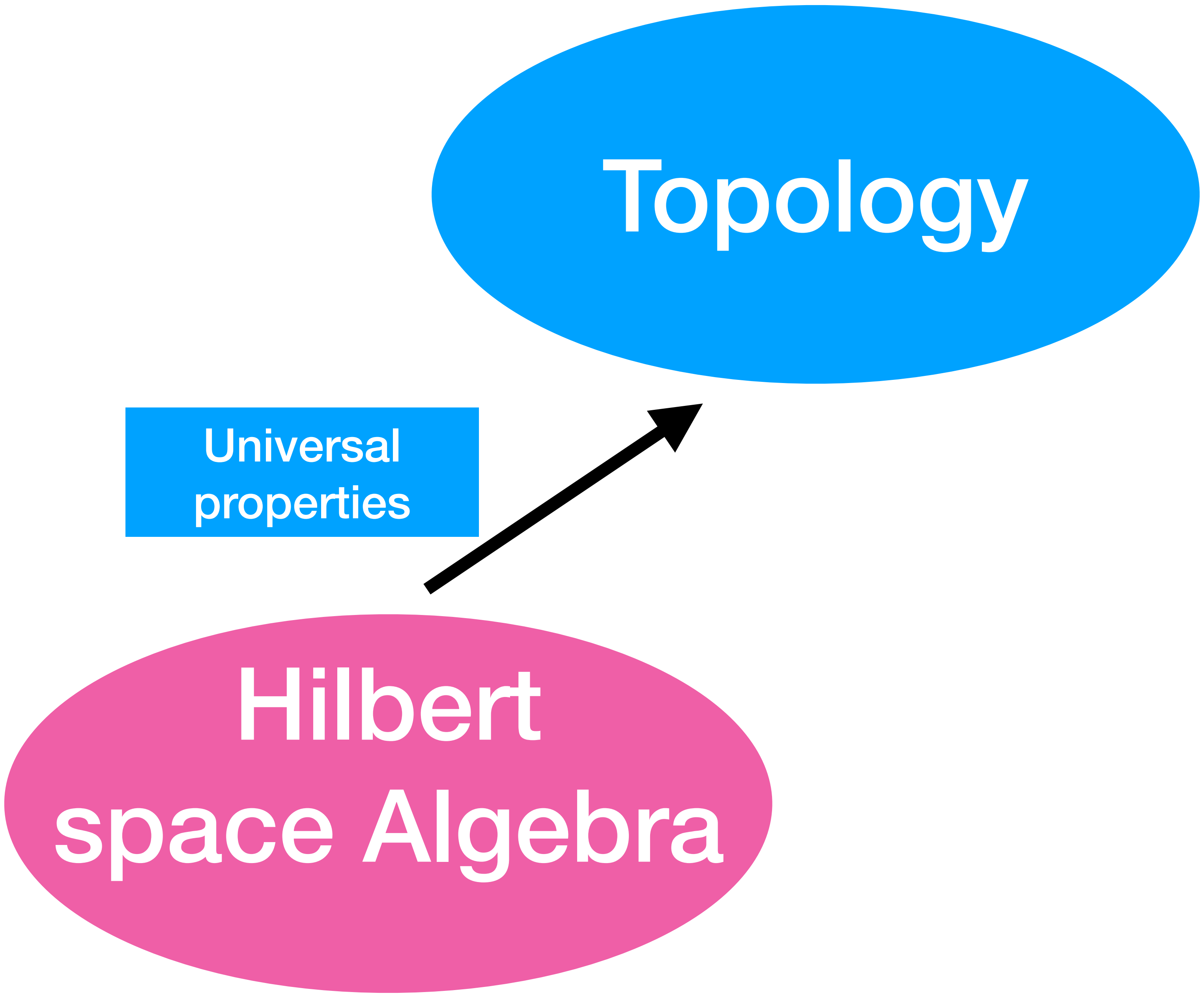

Electrons

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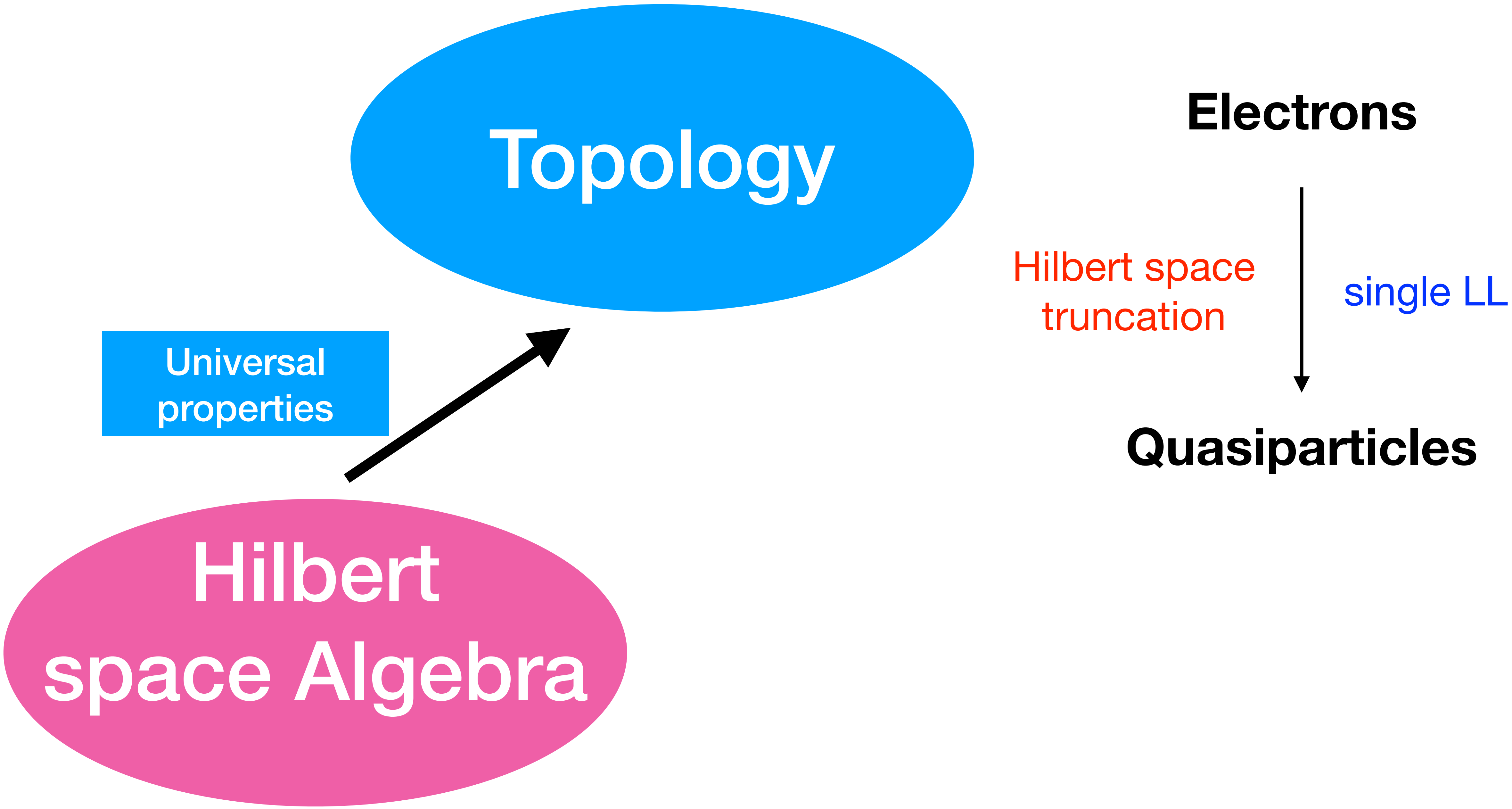


The elementary degrees of freedom in a single Landau level

Topology

Universal properties

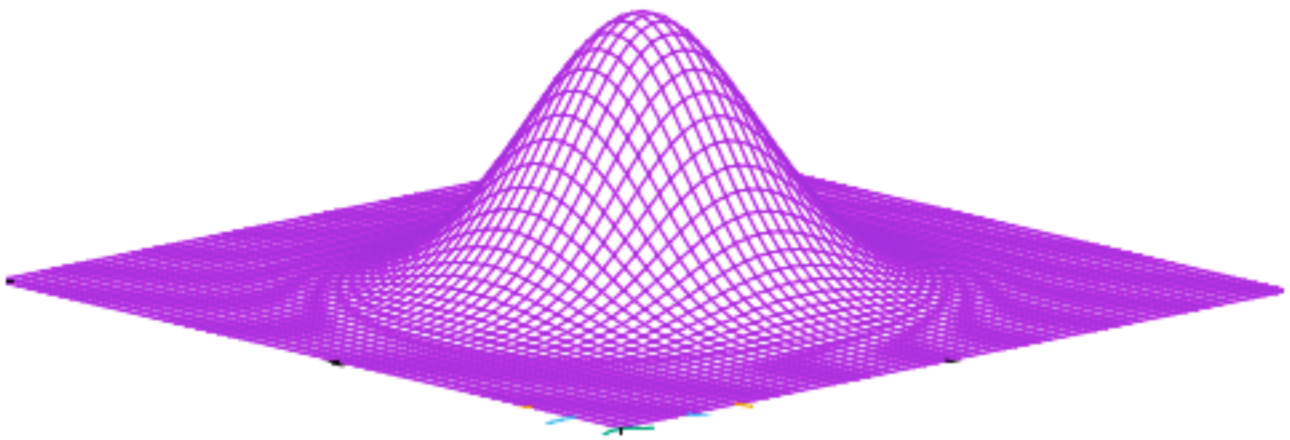
Hilbert space Algebra

Electrons

Hilbert space truncation

single LL

Quasiparticles



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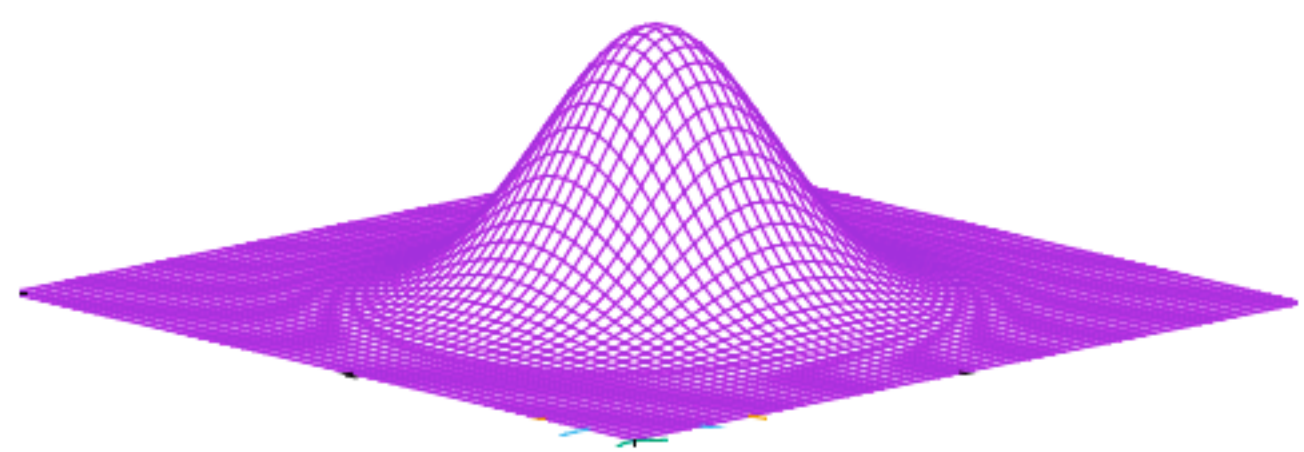
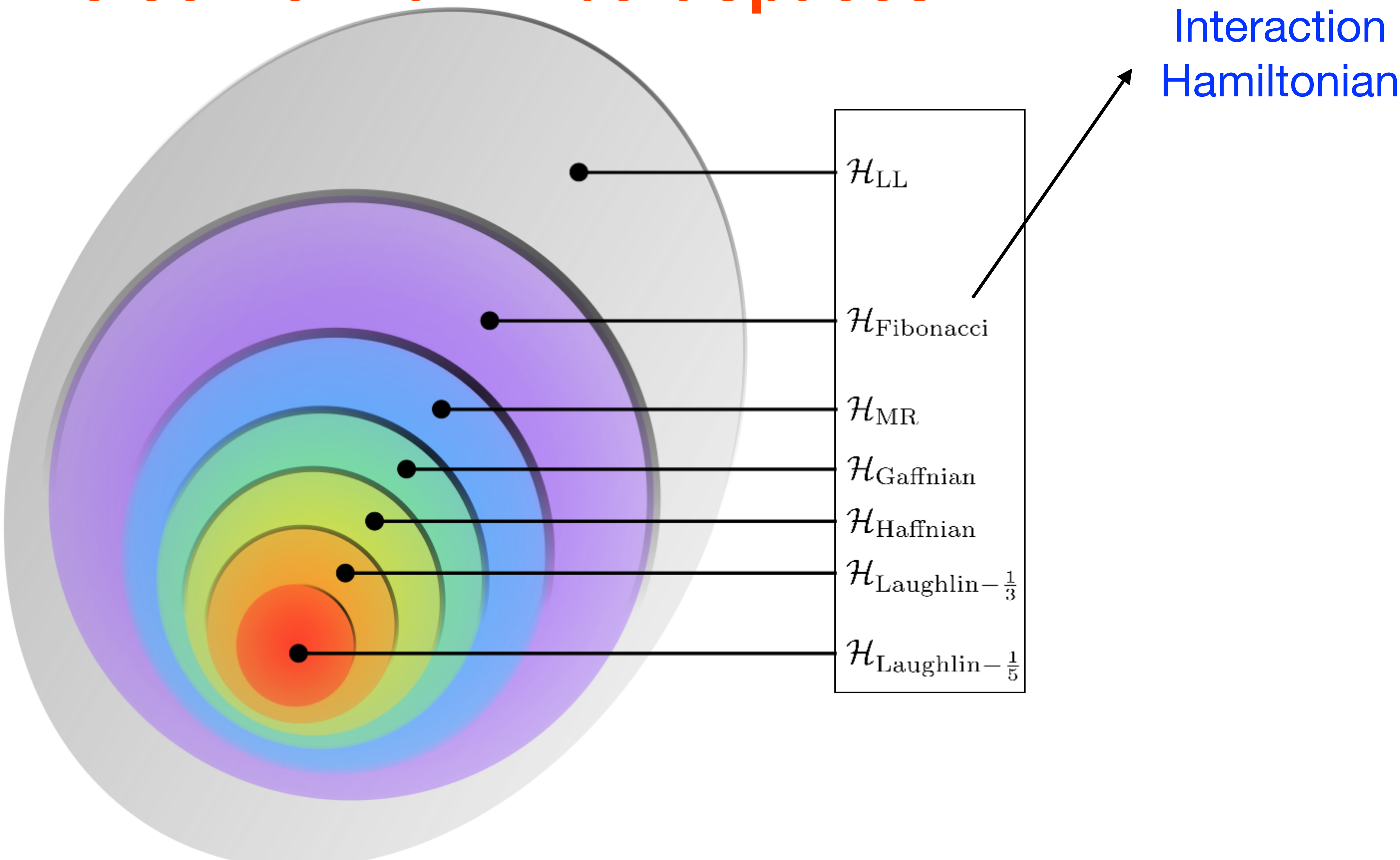


Elementary particles for IQHE

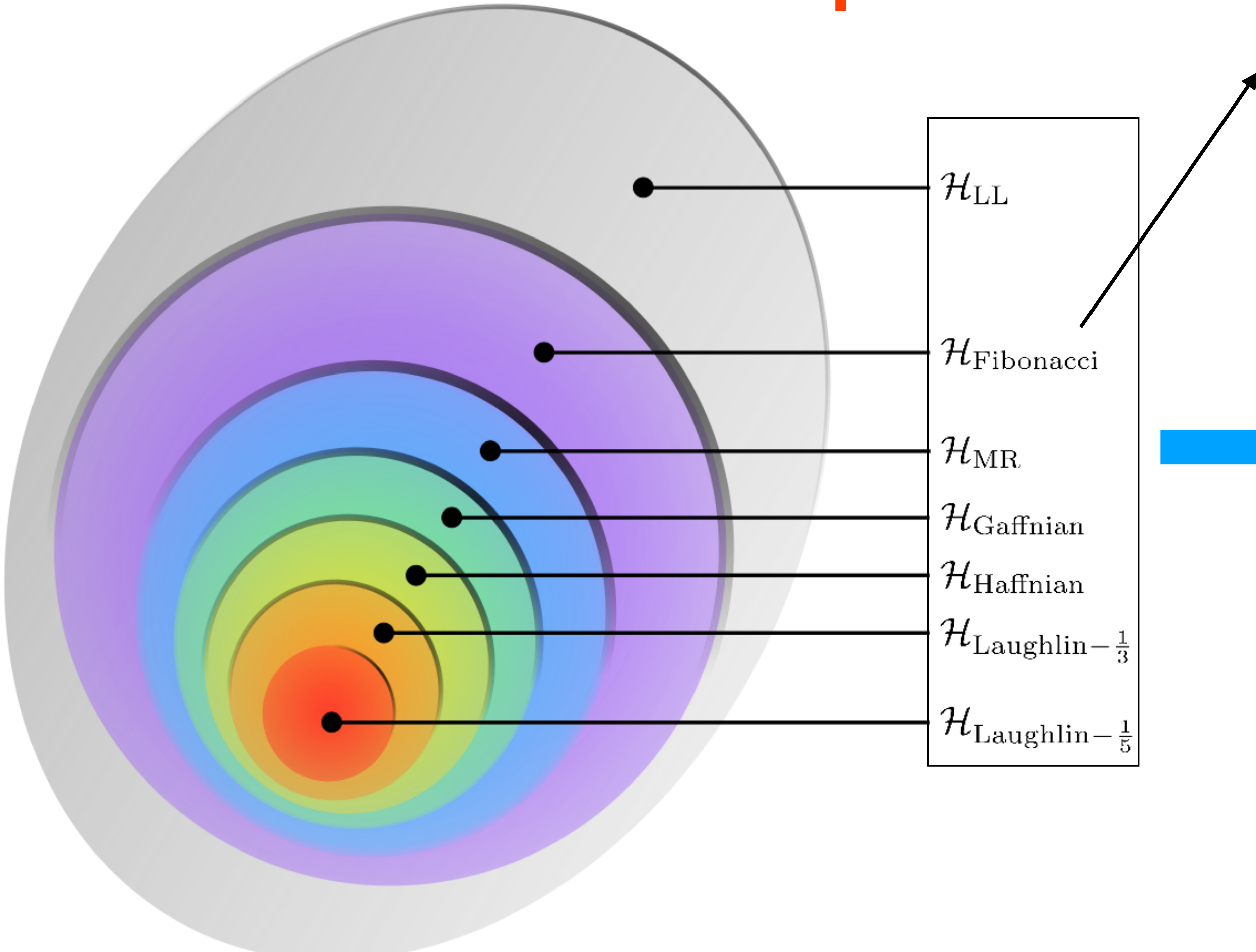
The conformal Hilbert spaces



Null spaces of the interactions

S.H. Simon, E.H. Rezayi and N.R. Cooper, PRB 75, 075318 (2007)
N.Read, PRB 79, 245304 (2009)
Bo Yang, PRB 103, 115102 (2021)

The conformal Hilbert spaces



Interaction Hamiltonian

Different types of anyons

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S.H. Simon, E.H. Rezayi and N.R. Cooper, PRB 75, 075318 (2007)
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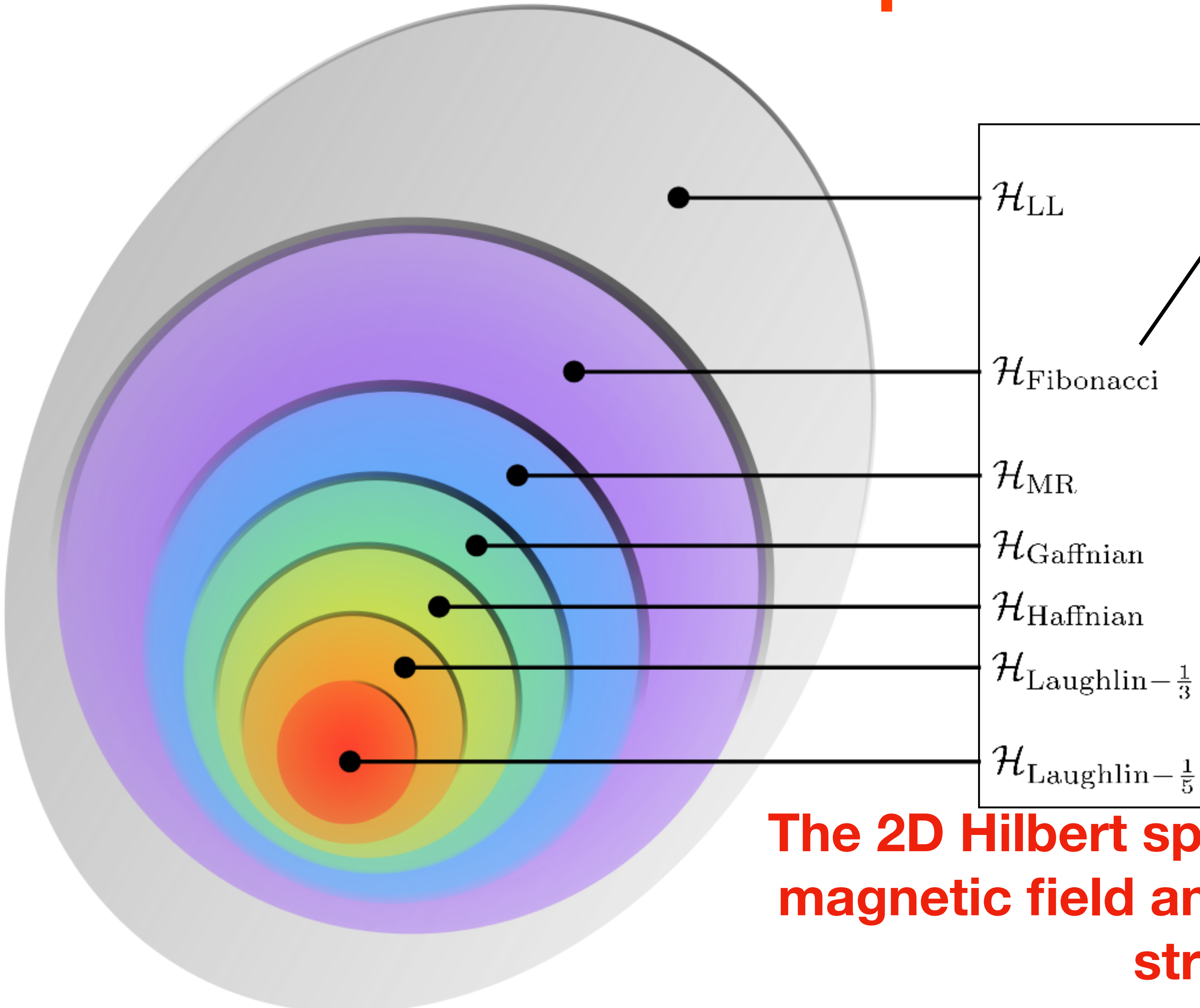


Interaction Hamiltonian

Different types of anyons

The 2D Hilbert space in the presence of magnetic field and interaction is highly structured

Null spaces of the interactions

S.H. Simon, E.H. Rezayi and N.R. Cooper, PRB 75, 075318 (2007)
N.Read, PRB 79, 245304 (2009)
Bo Yang, PRB 103, 115102 (2021)

The conformal Hilbert spaces

edge excitations

Ground state of the 2D quantum fluid

Highest density state

All other states

Entanglement spectrum

One-to-one correspondence

Li and Haldane, PRL 2008

The Virasoro algebra $\left[\hat{\mathcal{L}}_n, \hat{\mathcal{L}}_m\right] = (n-m)\,\hat{\mathcal{L}}_{m+n} + \frac{c}{12} n\left(n^2-1\right)\delta_{m+n,0}$

A holographic description - the bulk-edge correspondence

The elementary degrees of freedom in a conformal subspace

Topology

Universal properties

Hilbert space Algebra

Electrons

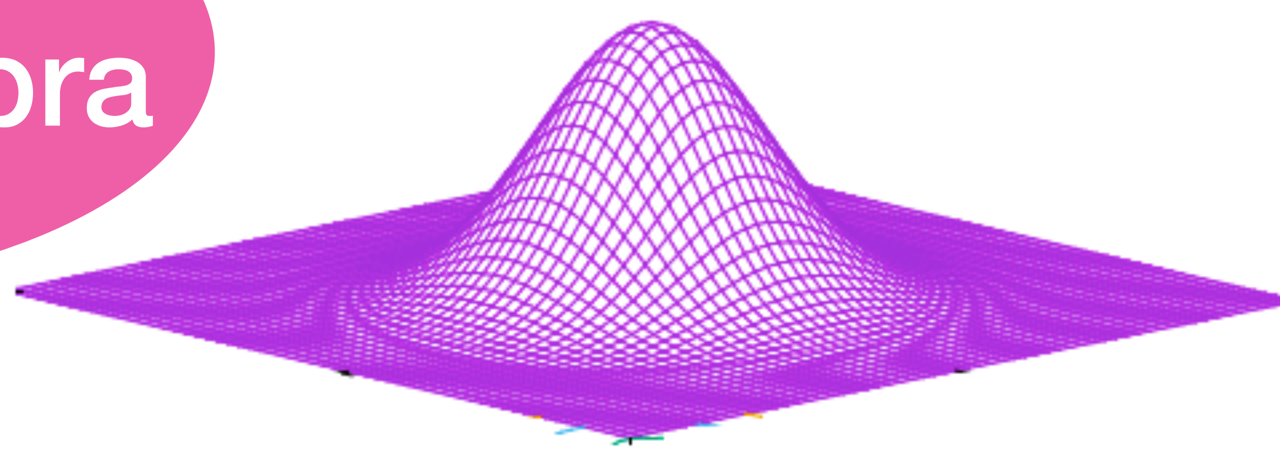
single LL

Quasiparticles

conformal subspace

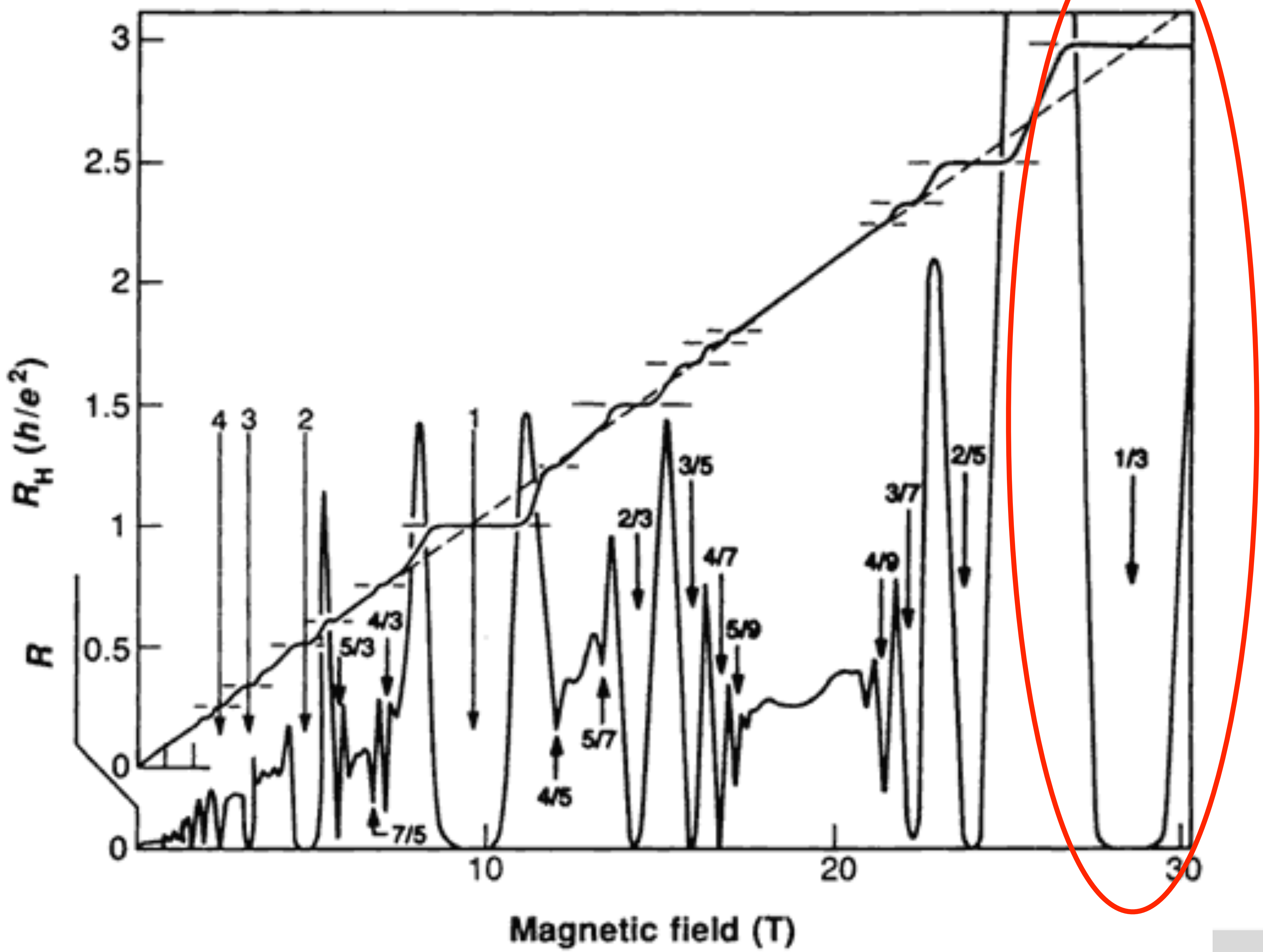
Anyons

fractional charge, fractional/ non-Abelian statistics



Fractionalization of anyons

The Laughlin phase



Laughlin quasiholes/ anyons have charge e/3 and statistical phase of pi/3

The Laughlin phase at 1/3

Laughlin conformal Hilbert space

$$\hat{H} = \hat{V}_1^{\mathrm{2bdy}}$$

artificial model interaction

Ground state and quasiholes $=$ $\mathcal{H}_L$

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Ground state and quasiholes $=$ $\mathcal{H}_L$

$$\hat{H} = \hat{V}_{\text{LLL}}$$

Ground state and quasiholes $\simeq$ $\mathcal{H}_L$

$$\hat{H} = \hat{V}_{\text{SLL}}$$

Ground state and quasiholes $\neq$ $\mathcal{H}_L$

realistic interaction

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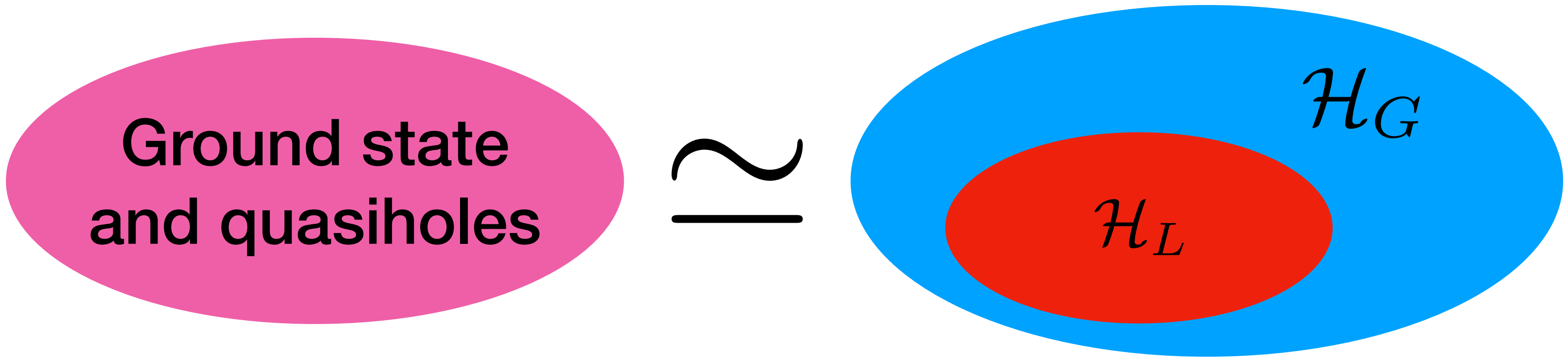
realistic interaction

What can happen to anyons in the presence of realistic Coulomb interaction?

The Laughlin phase at 1/3 in SLL

$$\hat{H} = \hat{V}_{\mathrm{SLL}}$$



Ha Quang Trung and Bo Yang, PRL 127,046402 2021

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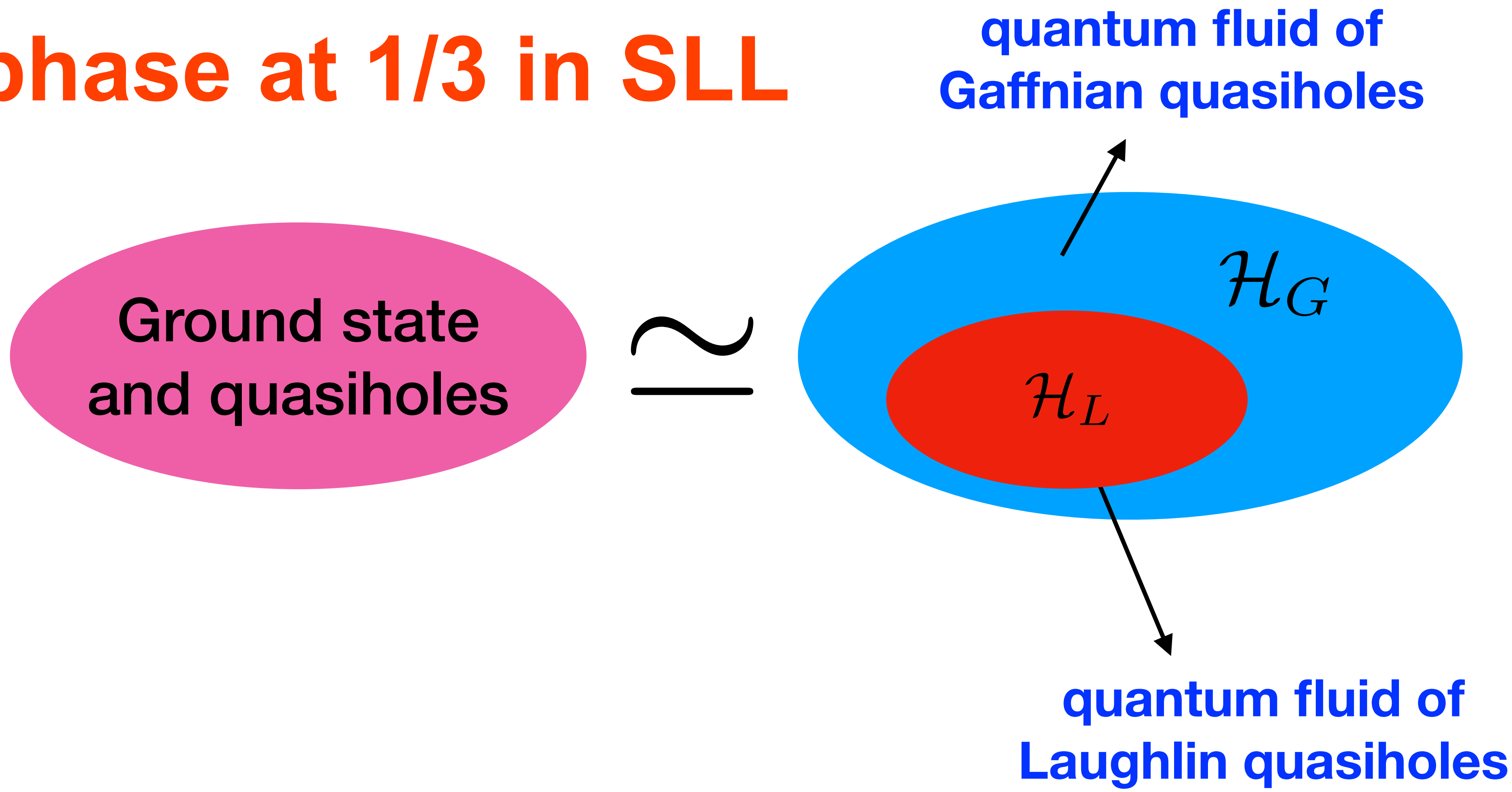

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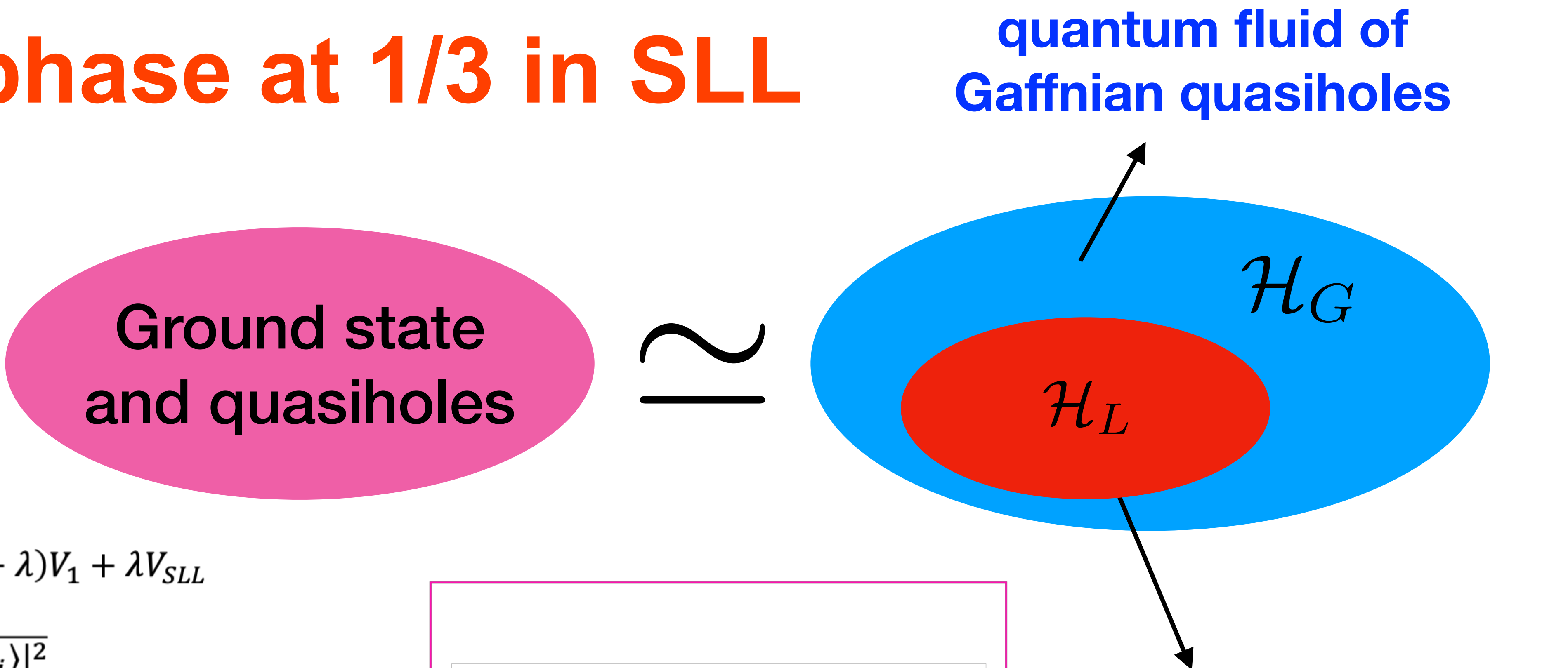
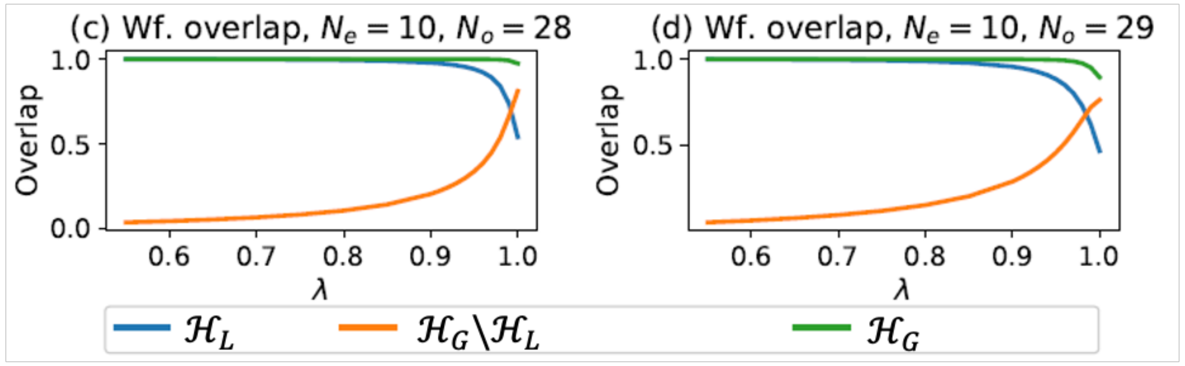


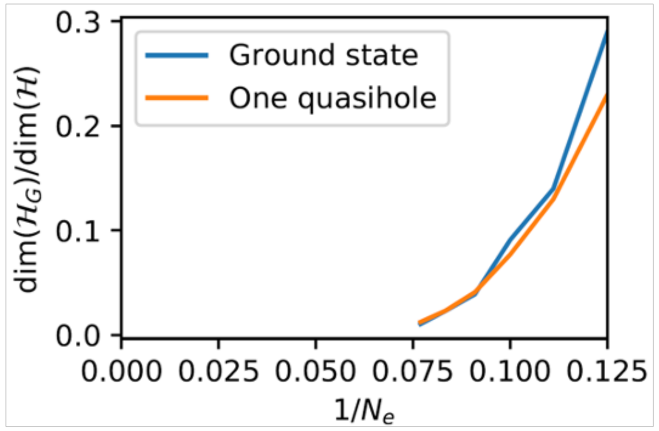
Overlap with ground state of $H(\lambda) = (1-\lambda)V_1 + \lambda V_{SLL}$

- Single-state overlap: $|\langle \psi_0(\lambda)|\psi\rangle|$
- Multiple-state overlap: $\sqrt{\sum_i |\langle \psi_0(\lambda)|\psi_i\rangle|^2}$



Overlap at $\lambda = 1$:

(N_e, N_o)	(9,25)	(9,26)	(10,28)	(10,29)	(11,31)
$\mathcal{O}_L$	0.48	0.45	0.54	0.47	0.70
$\mathcal{O}_G$	0.97	0.97	0.97	0.89	0.97
$\dim(\mathcal{H}_G)/\dim(\mathcal{H})$	0.143	0.135	0.091	0.077	0.039

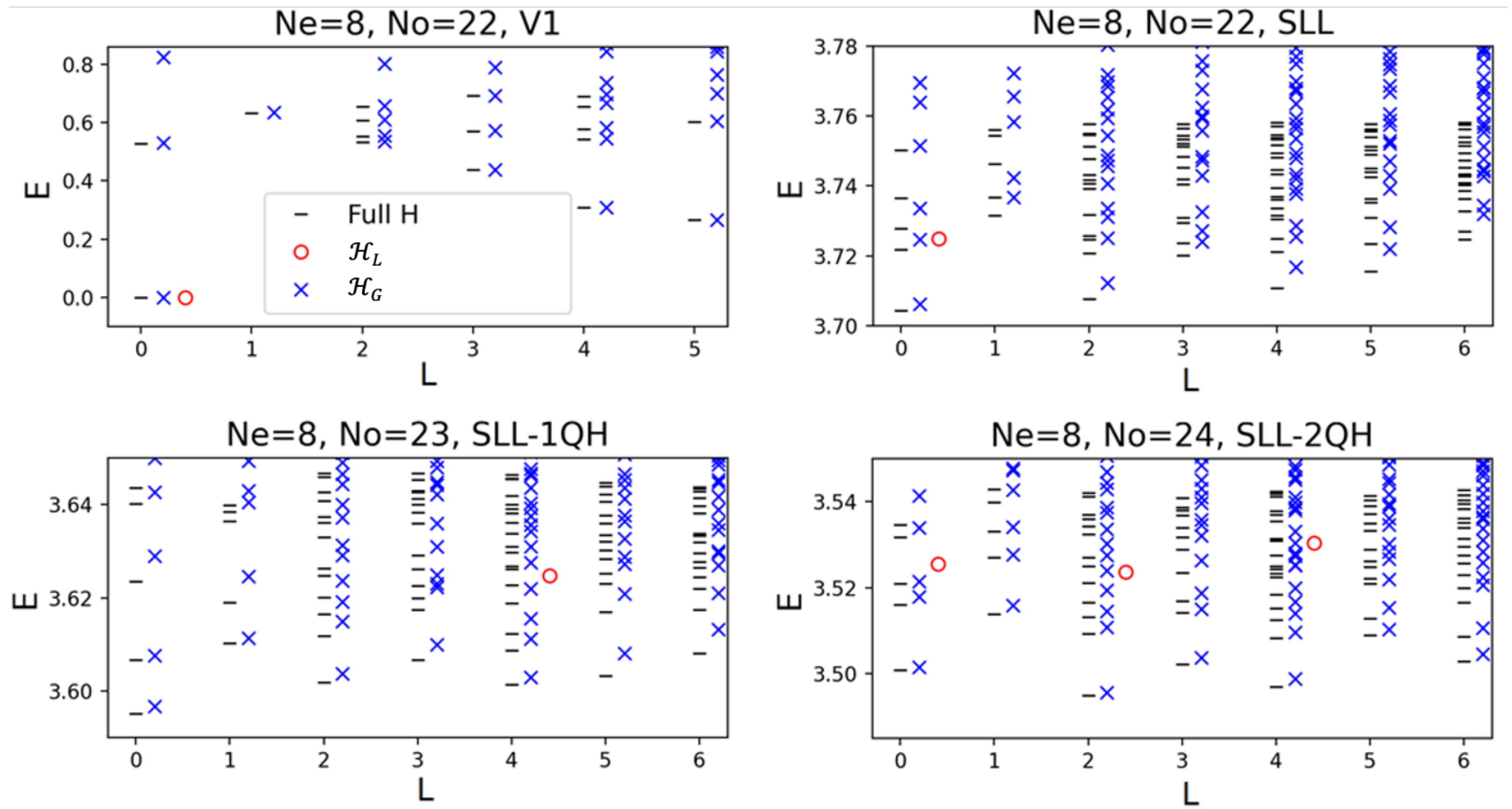


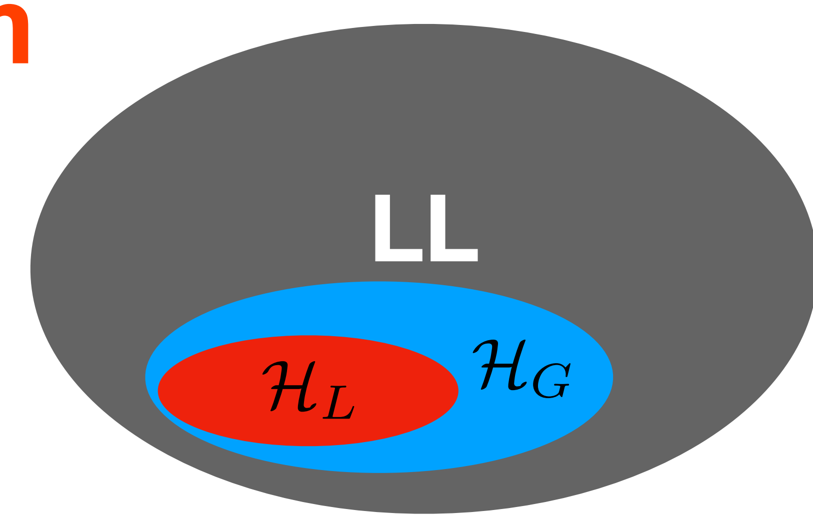
In the limit $N_e \to \infty$, $\mathcal{H}_G$ is a subspace of measure zero

$\mathcal{H}_G$ has high overlap with the true ground state near V_{SLL}

Ha Quang Trung and Bo Yang, PRL 127,046402 2021

The elementary degrees of freedom





The low-lying excitations are quantum fluids of Gaffnian quasiholes

$\mathcal{H}_G$ captures the low-lying energies w.r.t. a wider range of interactions

Ha Quang Trung and Bo Yang, PRL 127,046402 2021

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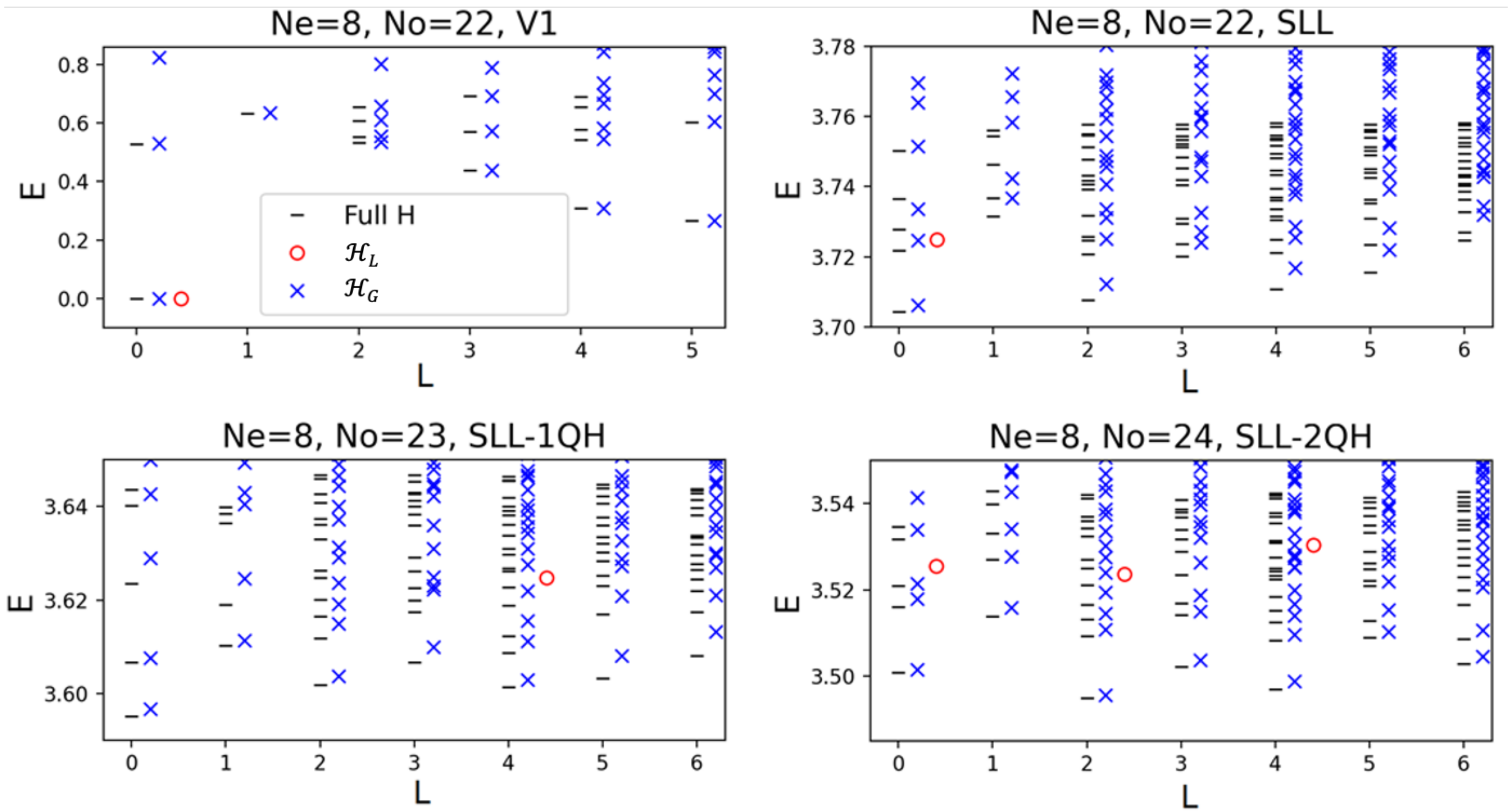

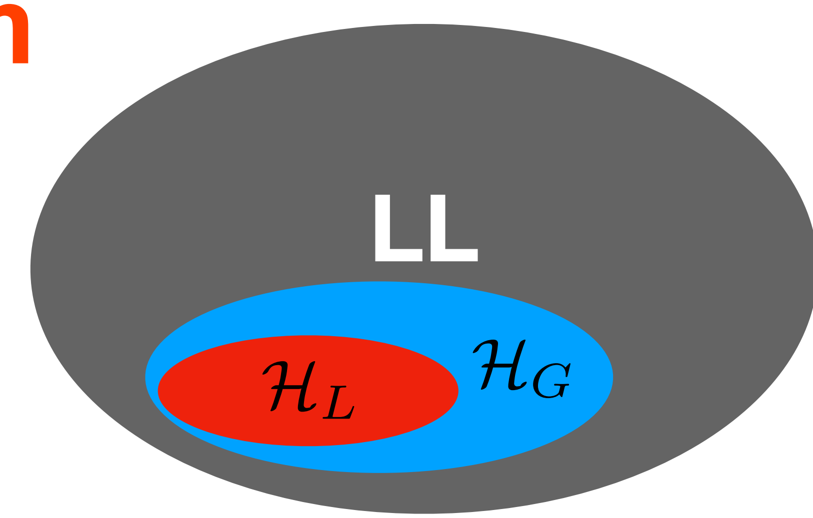

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Ha Quang Trung and Bo Yang, PRL 127,046402 2021

Quantum Hall fluid	**Electrons**	**Gaffnian quasiholes**	**Laughlin quasiholes**
Water	**quarks and leptons**	**hydrogen and oxygen**	**molecules**

Fractionalisation of Laughlin quasiholes

For Jack polynomials, see *Bernevig and Haldane, PRL 100,246802*

For some FQH phases, the conformal Hilbert space is spanned by the Jacks

$$110001100011000\cdots110001100011$$

Highest density state in $\mathcal{H}_G$

$$100100100100\cdots1001001001$$

Highest density state in $\mathcal{H}_L \in \mathcal{H}_G$

$$0100100100100\cdots1001001001$$

One Laughlin quasihole

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$$\nabla$$

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Gaffnian quasiholes

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Laughlin quasihole

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One Laughlin quasihole

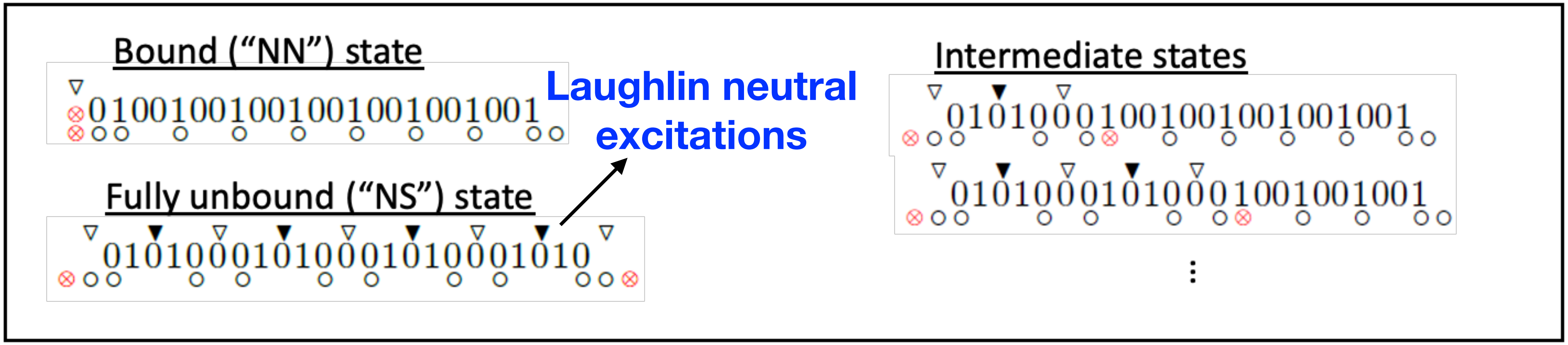
Gaffnian quasiholes

1 Laughlin quasihole = a bound pair of Gaffnian quasiholes

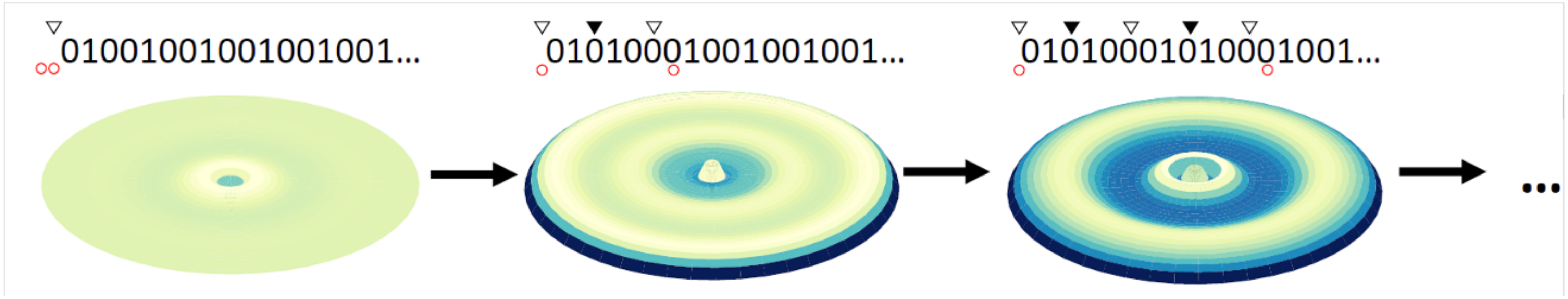
Fractionalisation of Laughlin quasiholes

From root configuration:

- Laughlin quasihole (LQ): fewer than 1 electron every 3 orbitals
- Gaffnian quasihole (GQ): fewer than 2 electrons every 5 orbitals



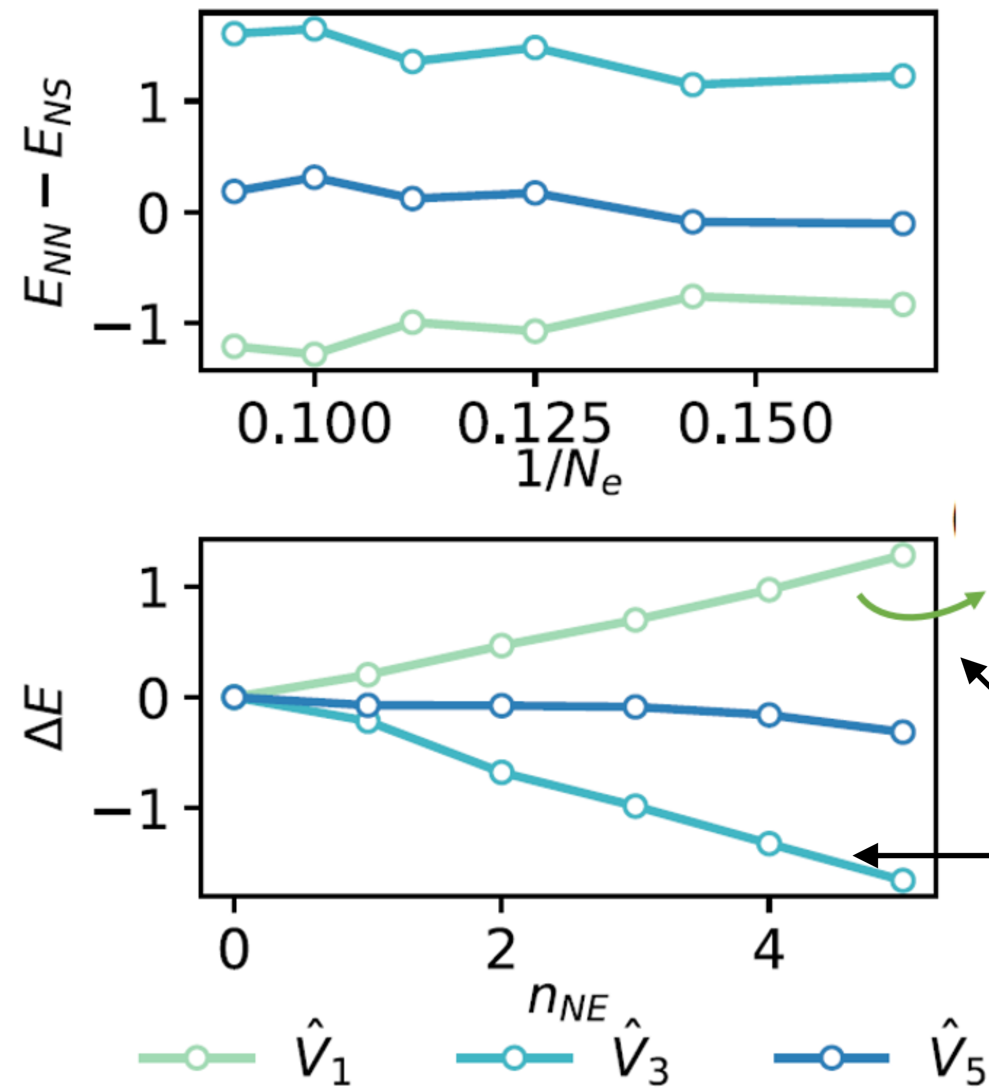
The GQ can be separated by creating neutral excitations

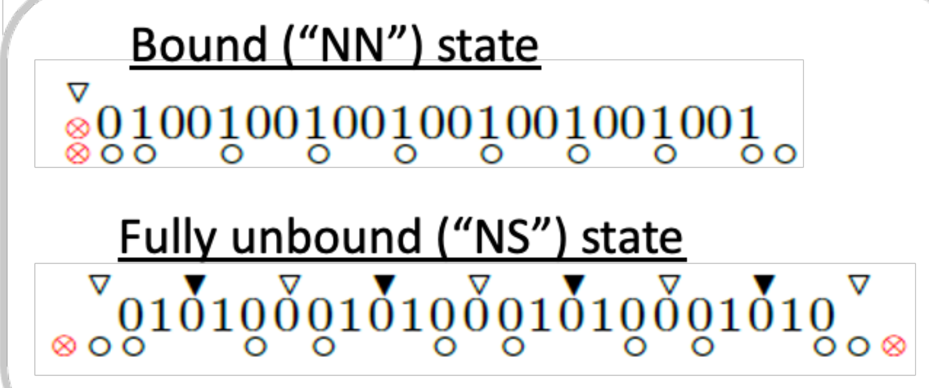


Asymptotic freedom of bound GQs

Energy difference between bound and split GQ states

Energy vs distance between two split GQs





"asymptotic freedom"

V_1 prefers bound GQs but
V_3 prefers unbound GQs

Asymptotic freedom of bound GQs

Laughlin neutral excitations ("gluons")

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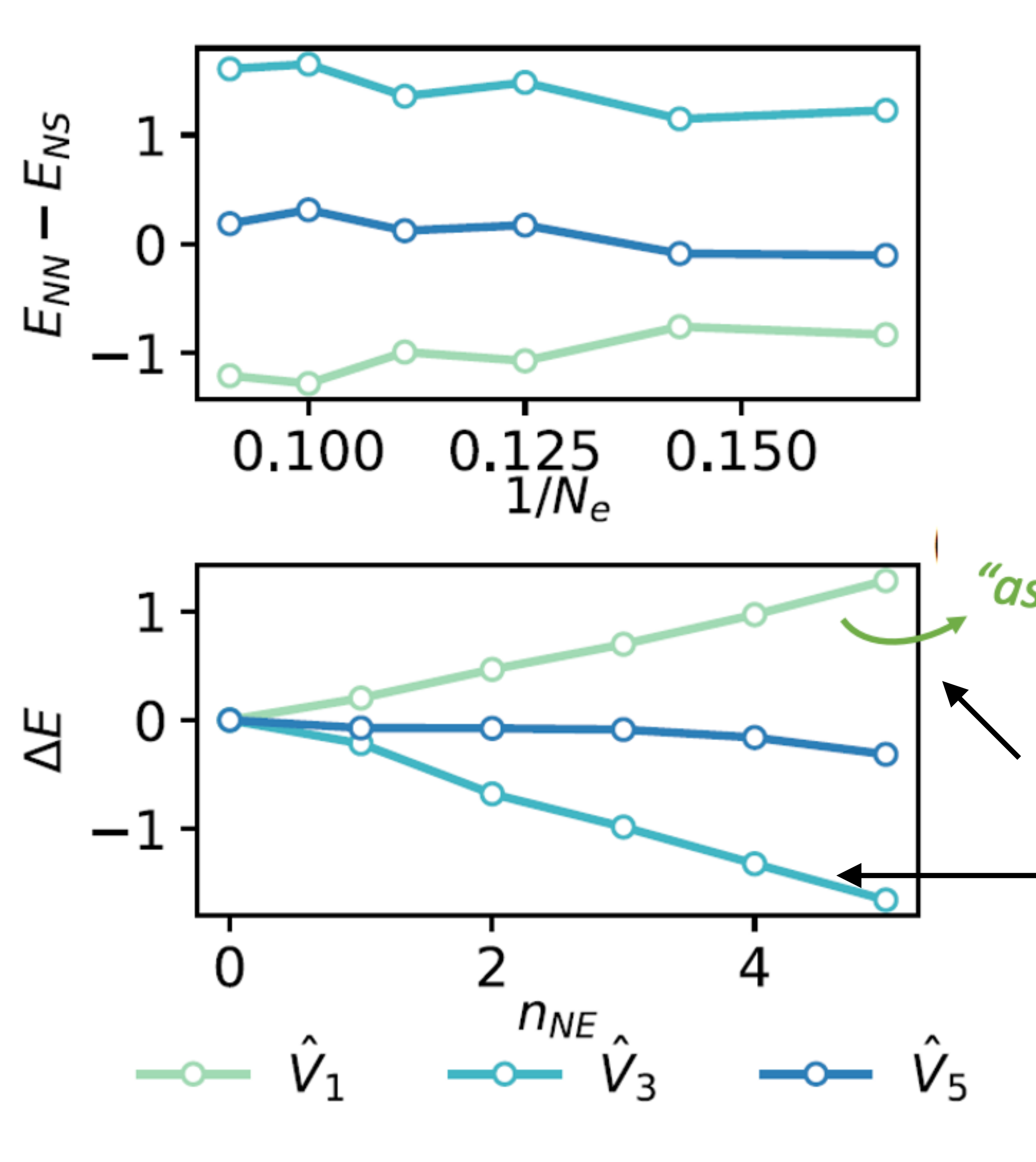
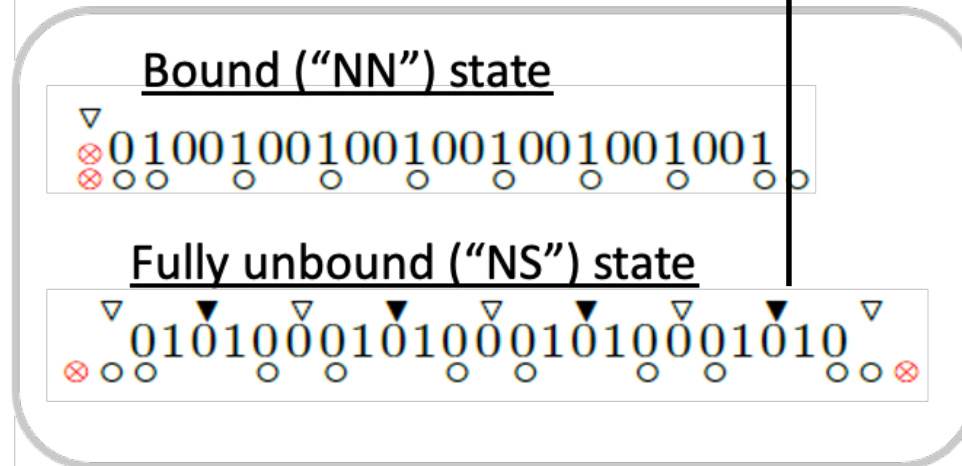
Energy vs distance between two split GQs





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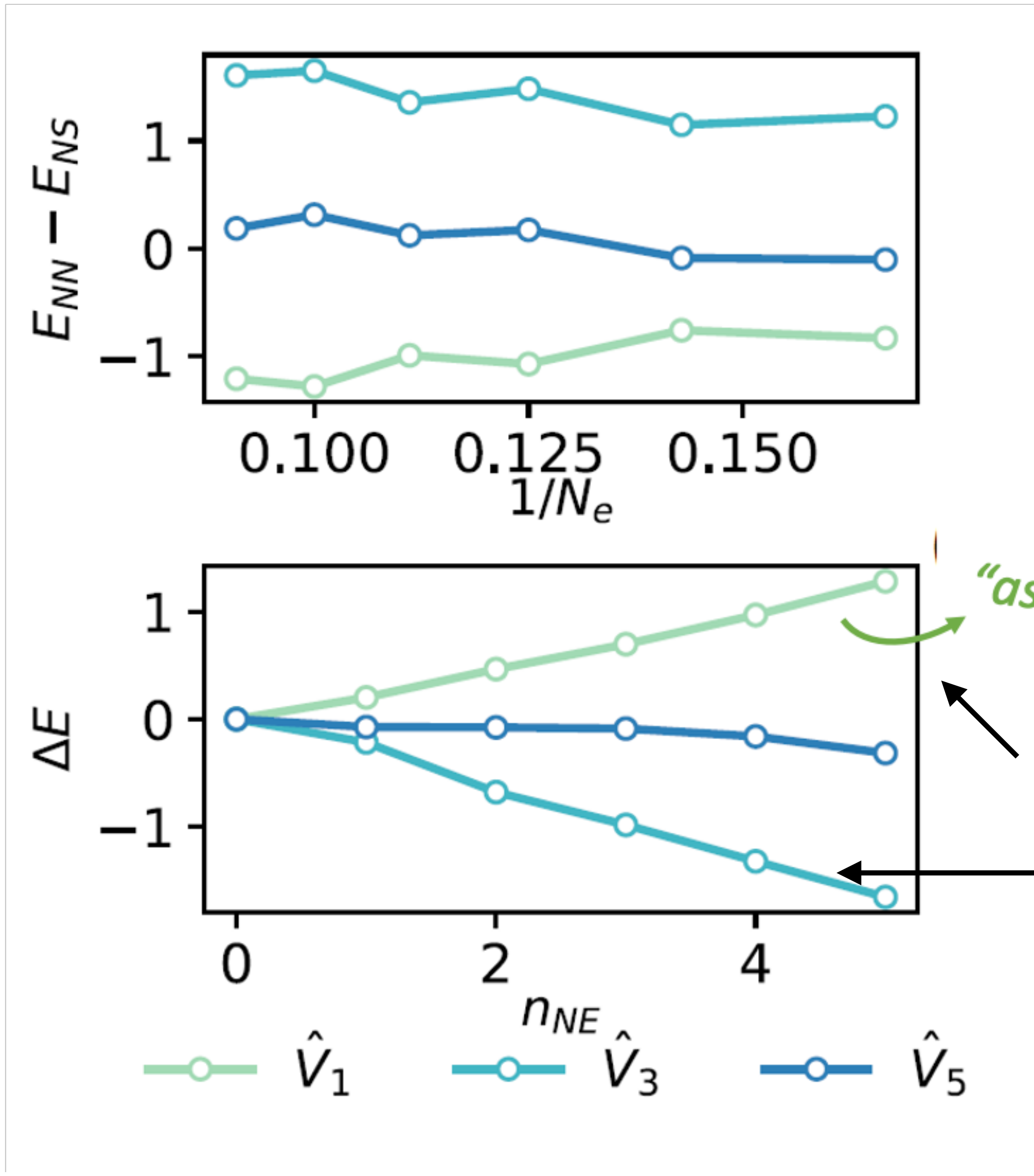

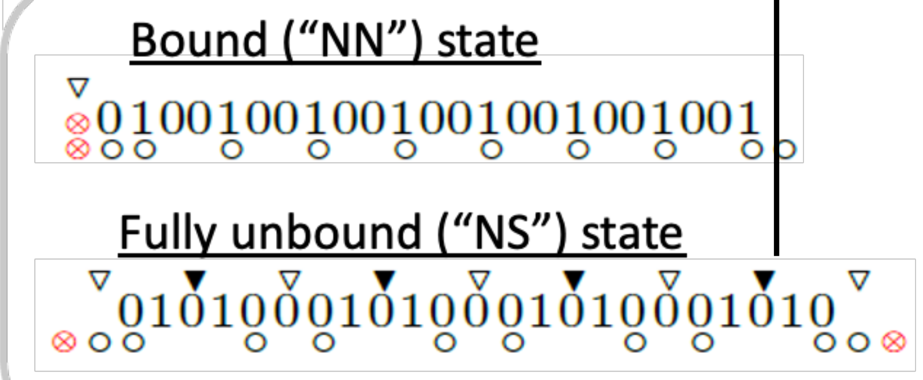

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V_1 prefers bound GQs but
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V1 punishes Laughlin neutral excitations, while V3 favours such excitations energetically

→

Possibility of fractionalisation of Laughlin quasiholes at SLL

Asymptotic freedom of bound GQs

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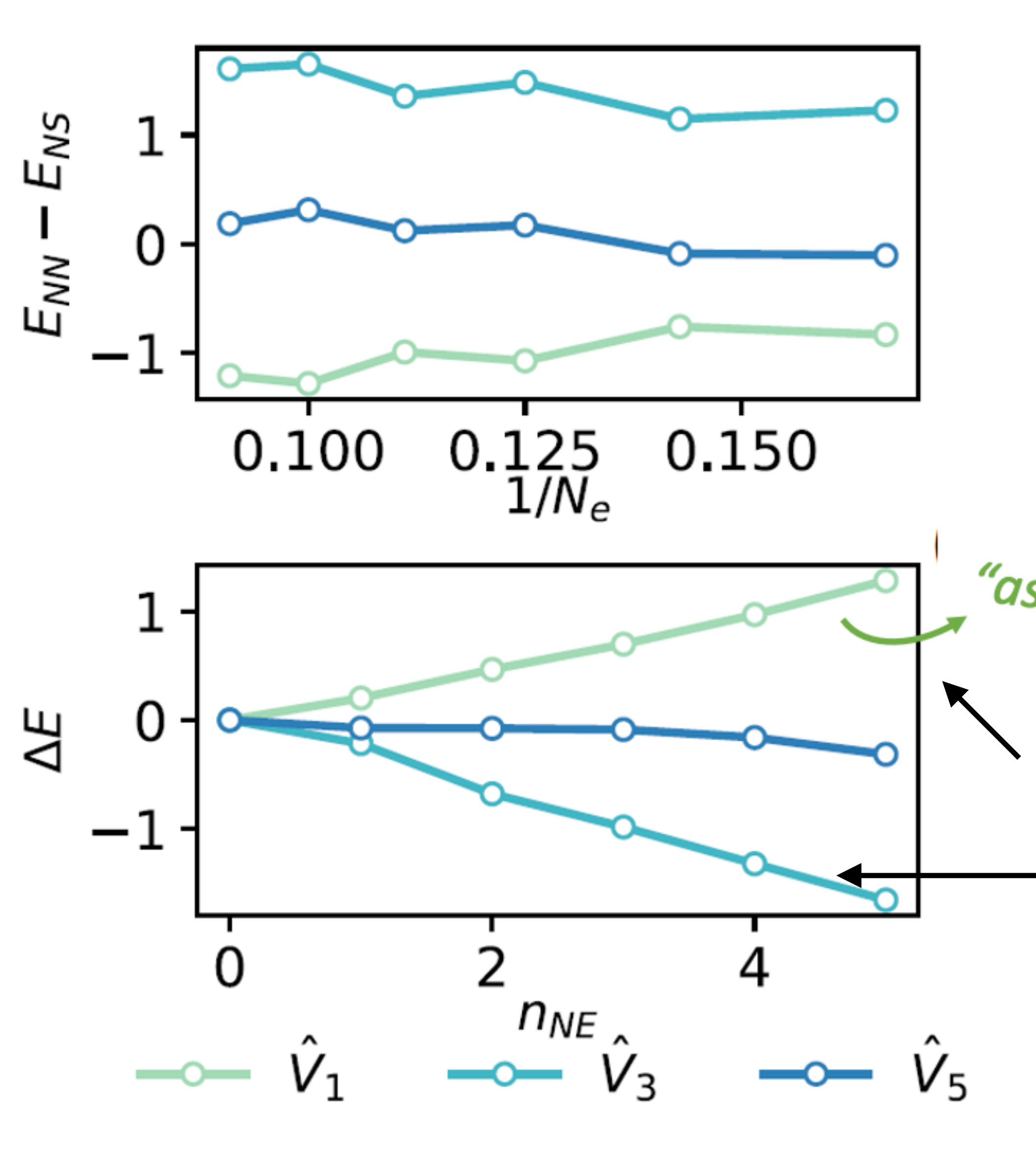

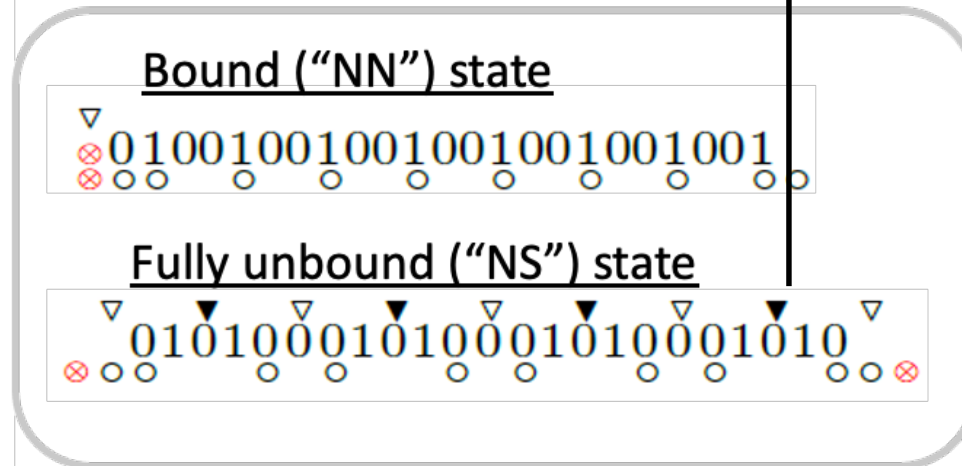

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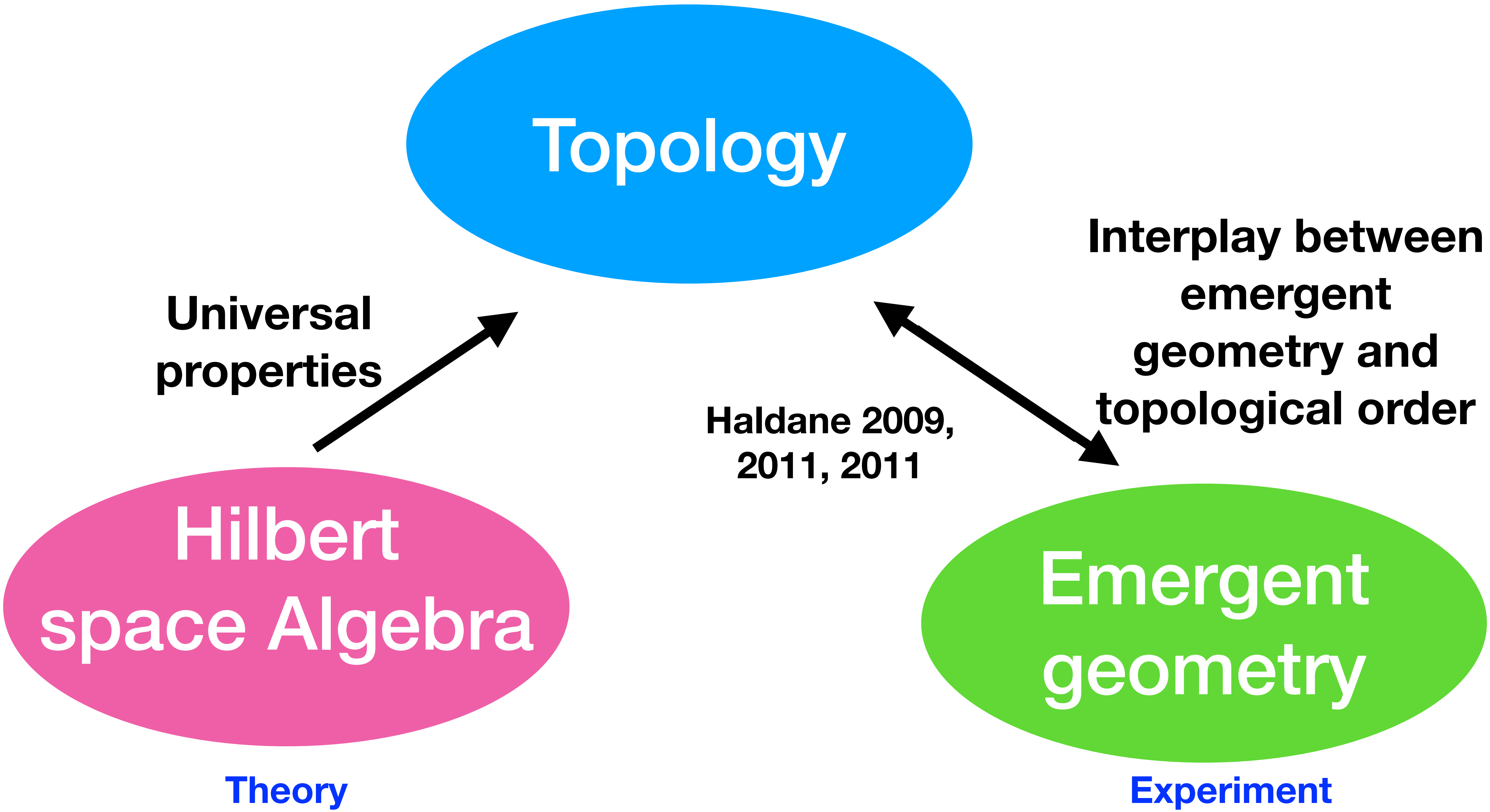
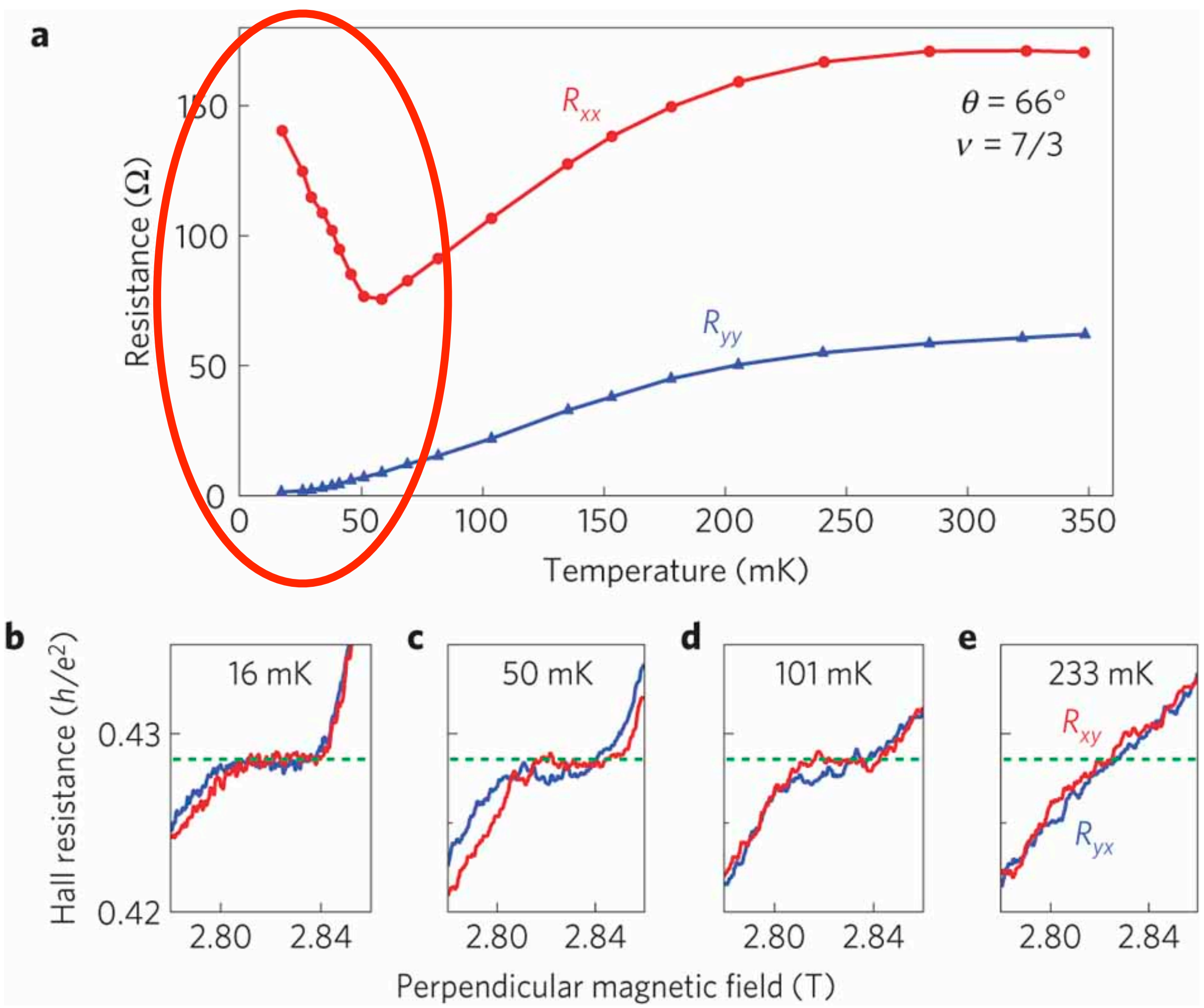
Possibility of fractionalisation of Laughlin quasiholes at SLL

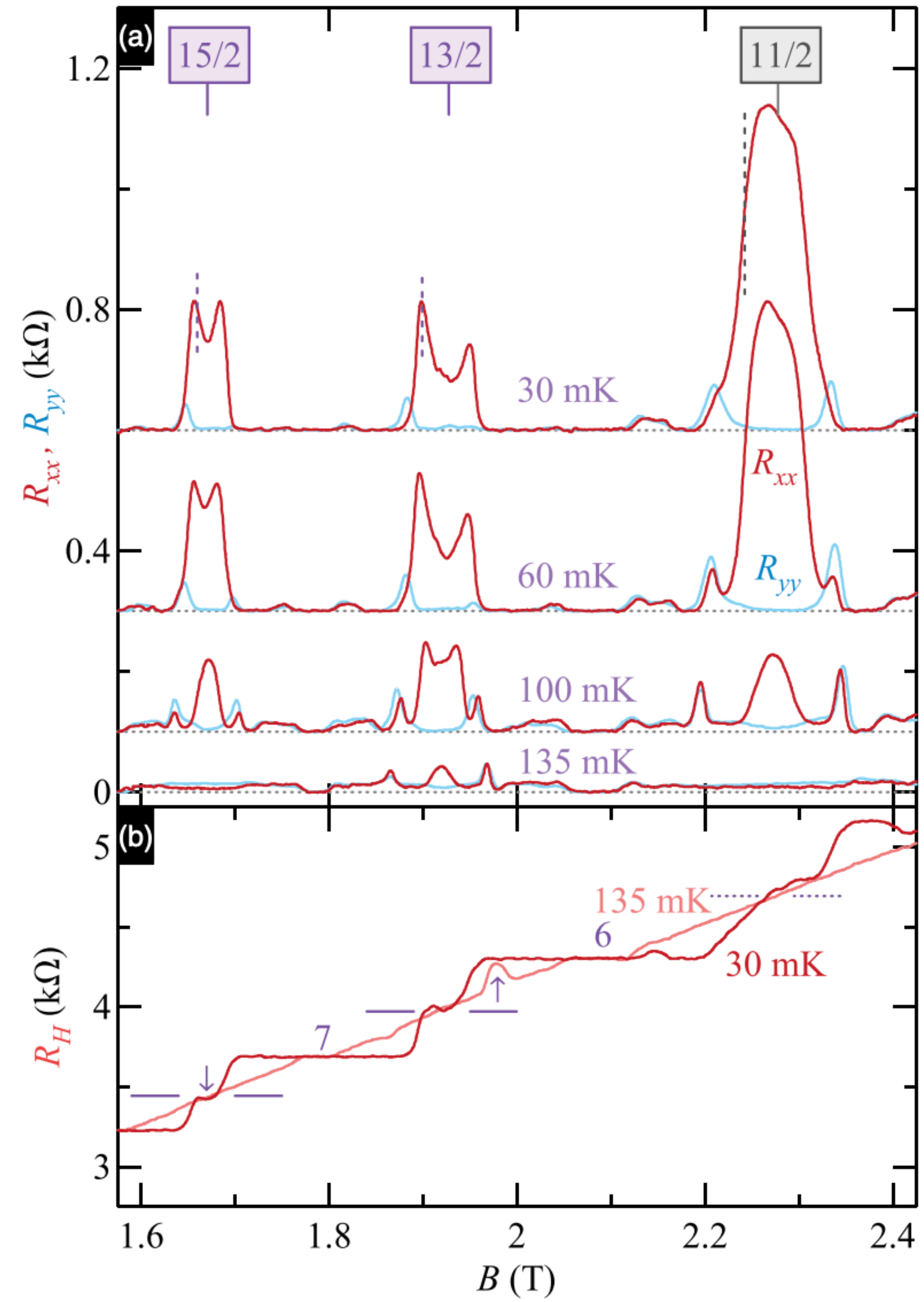
The nematic fractional quantum Hall effect

Dynamics for the robustness of topological indices



The nematic fractional quantum Hall effect





J.Xia et.al, Nature Physics 2011

X. Fu et.al, PRL 2020

L. J. Du, et al, Science Advances 5, eaav3407 (2019).

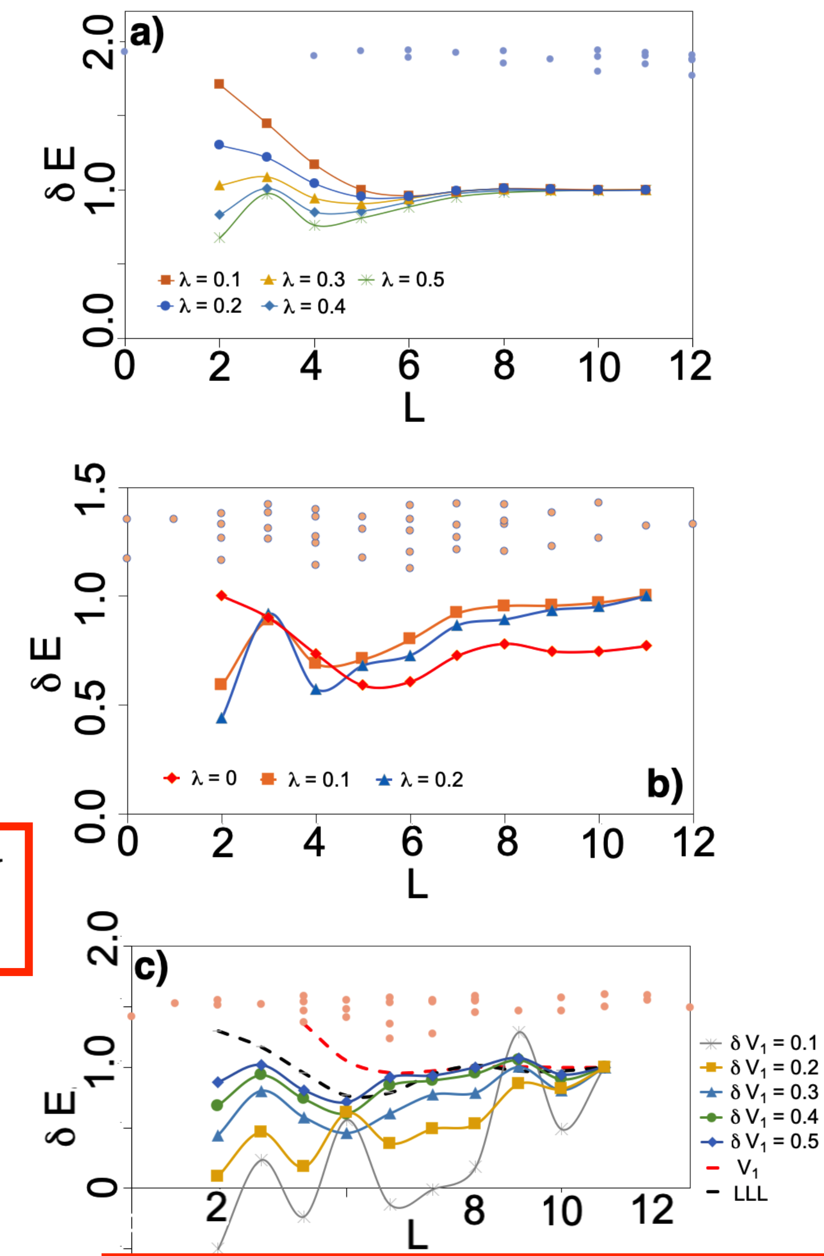
Coexistence of topological order and anisotropic transport

A minimal model for nematic FQHE

$$\lim_{q\to 0}\Delta_{\mathbf{q}}=\frac{1}{256\eta}\Gamma^{mn}_{(1)}c_m d_n+\frac{1}{768\eta}\Gamma^{mn}_{(2)}c_m d_n q^2+O\left(q^4\right)$$

Quadrupole excitations are exact zero energy states of the Haffnian model Hamiltonian

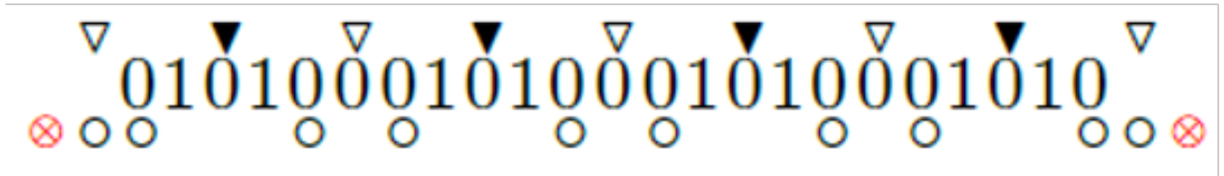
$$\hat{H}\left(\lambda_1,\lambda_2\right)=\hat{H}_{\mathrm{h}}+\lambda_1\hat{V}_1^{\mathrm{2bdy}}+\lambda_2\hat{V}_3^{\mathrm{2bdy}}$$



Dynamics of 2D "Gravitons"

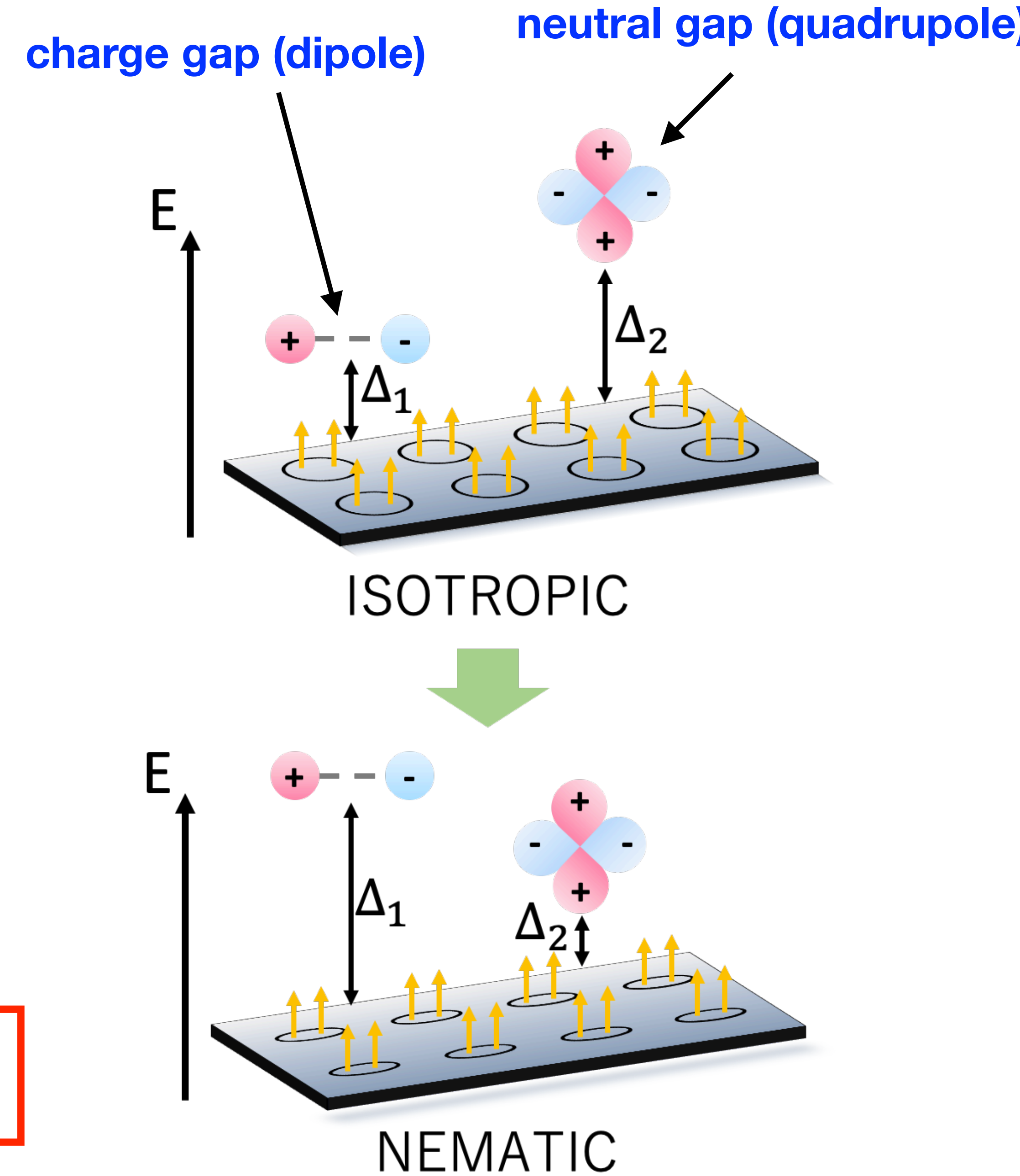
Bo Yang, PR Research, 2, 033362 (2020)
Yuzhu Wang and Bo Yang, PRB 105, 035144 (2022)
D.X. Nguyễn and D.T. Son, PRR 3, 033217 (2021)
D.X. Nguyễn et.al. PRL 2022
A.C. Balram et.al. PRX 2022
Yuzhu Wang and Bo Yang, Nat. Comm. 2023

Quasihole fractionalisation and nematic phase transition are related



Fractionalised Laughlin quasiholes are also exact zero energy states of the Haffnian model Hamiltonian

$$\hat{H}\left(\lambda_1, \lambda_2\right) = \hat{H}_{\mathrm{h}} + \lambda_1 \hat{V}_1^{\mathrm{2bdy}} + \lambda_2 \hat{V}_3^{\mathrm{2bdy}}$$



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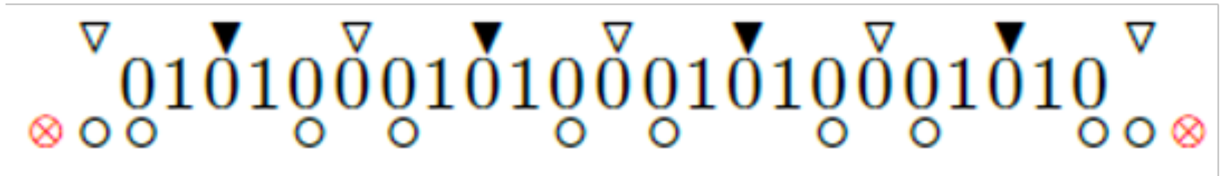
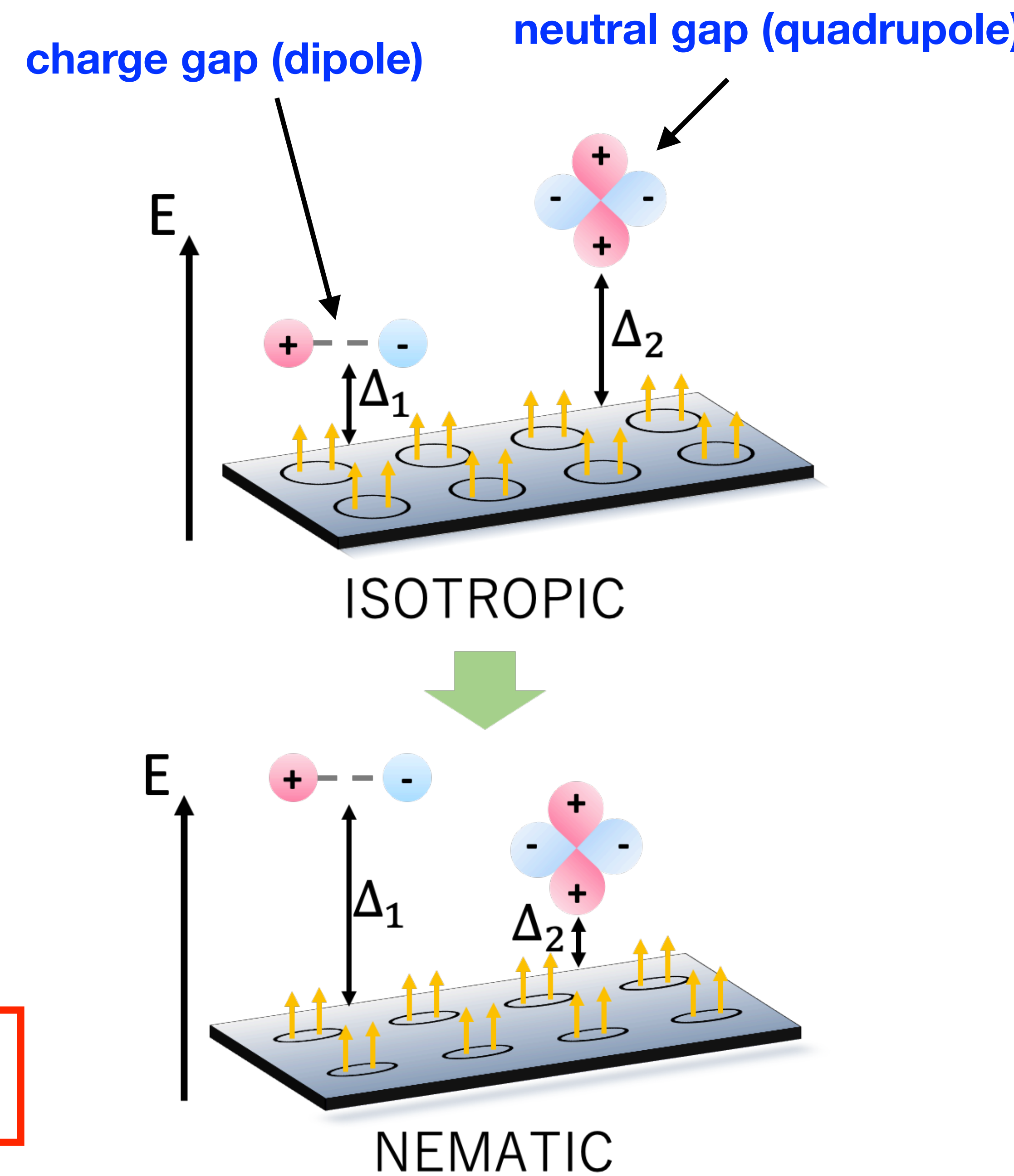


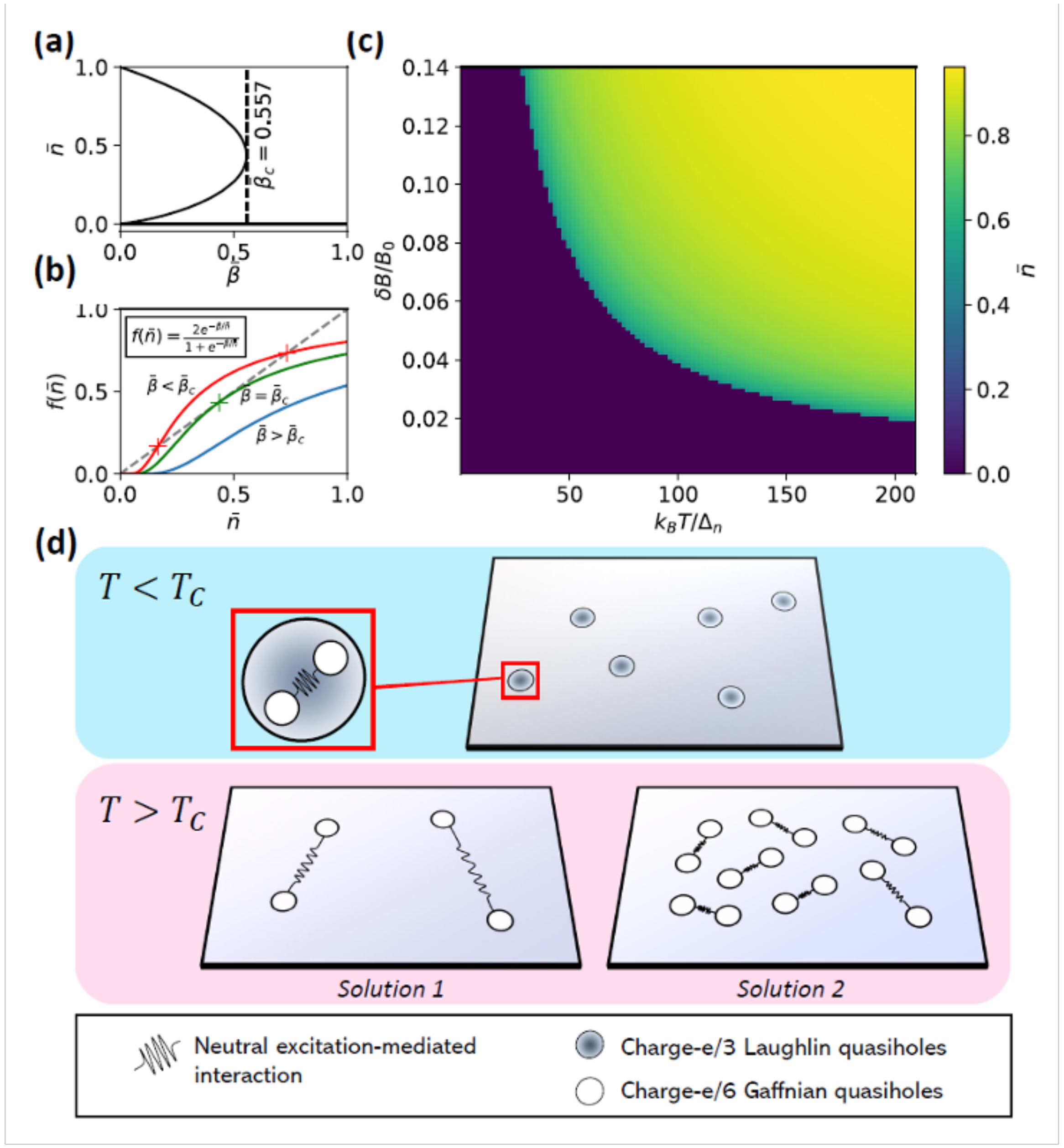
Fractionalised Laughlin quasiholes are also exact zero energy states of the Haffnian model Hamiltonian

$$\hat{H}\left(\lambda_1, \lambda_2\right) = \hat{H}_{\mathrm{h}} + \lambda_1 \hat{V}_1^{\mathrm{2bdy}} + \lambda_2 \hat{V}_3^{\mathrm{2bdy}}$$

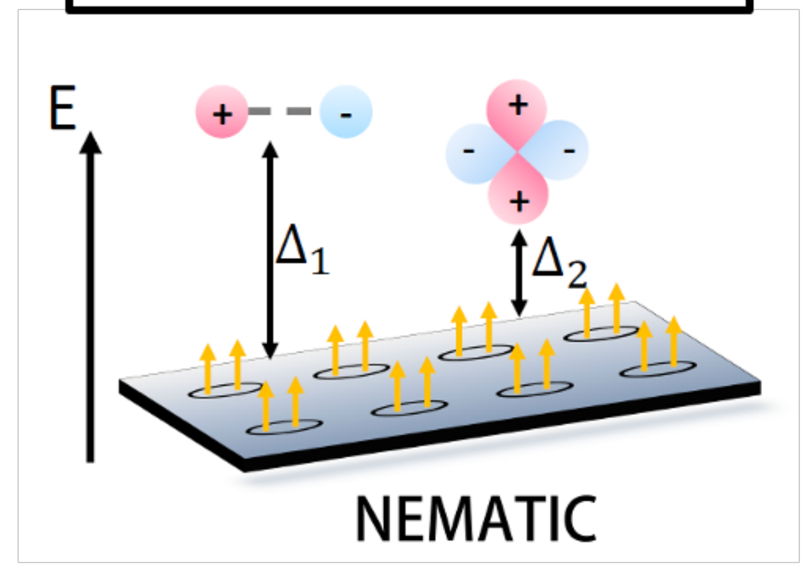


The neutral gap gives the energy cost of quasihole fractionalisation

A BKT-like phase transition



$$T_C \sim \frac{0.381\Delta_2}{k_B}\left(\frac{\delta B}{B_0}\right)^{-1}$$



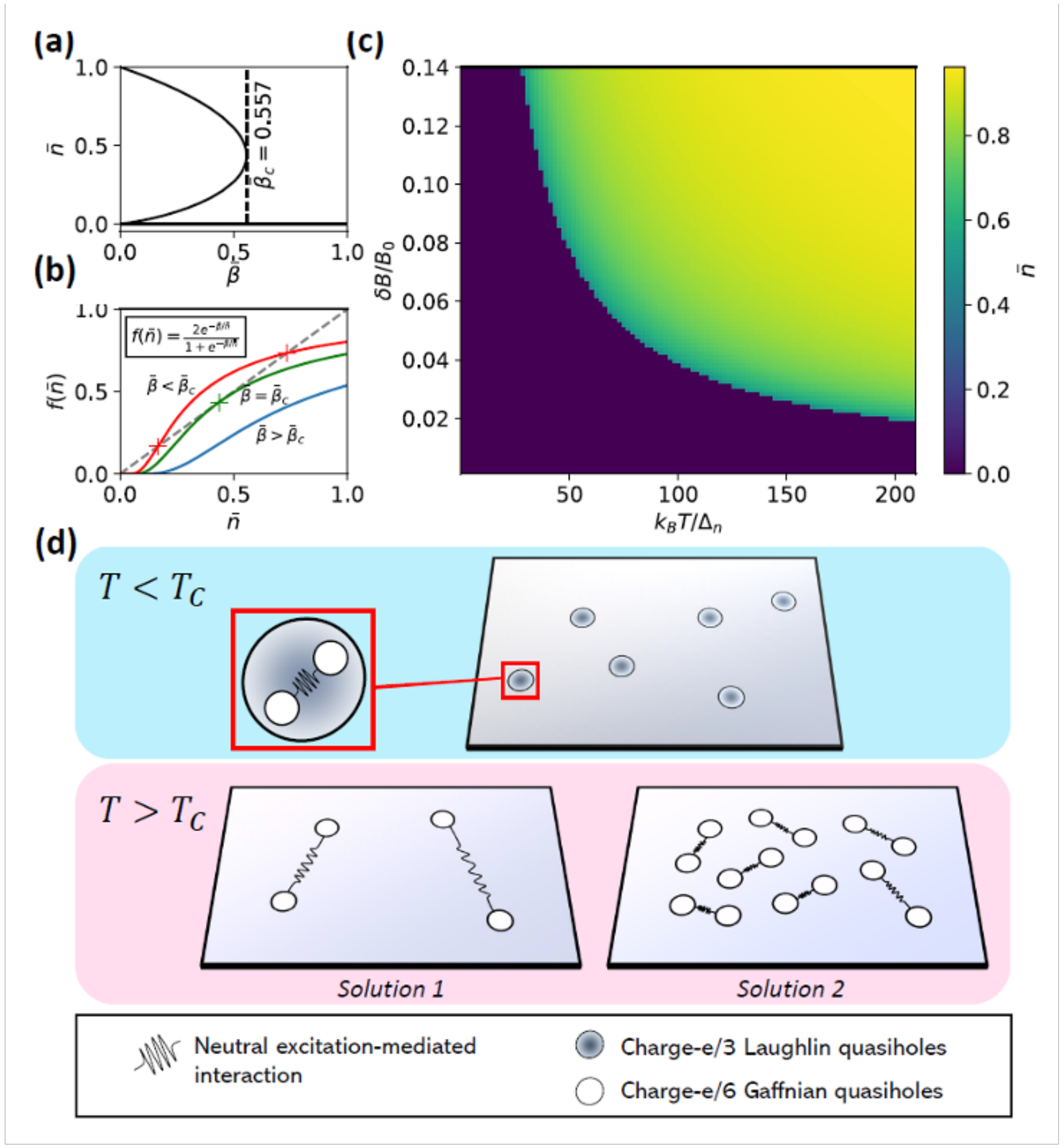
e/6-charges are observable in experiment if

$$T_c < T < \Delta_1/k_B$$

A phase transition of low-lying excitations at the same filling factor

Ha Quang Trung and Bo Yang, PRL 127,046402 2021

A BKT-like phase transition



quadrupole gap

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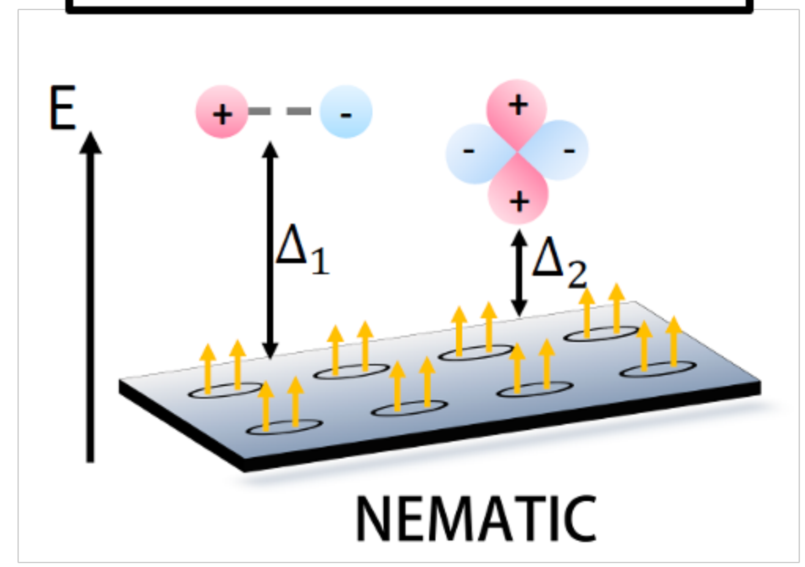

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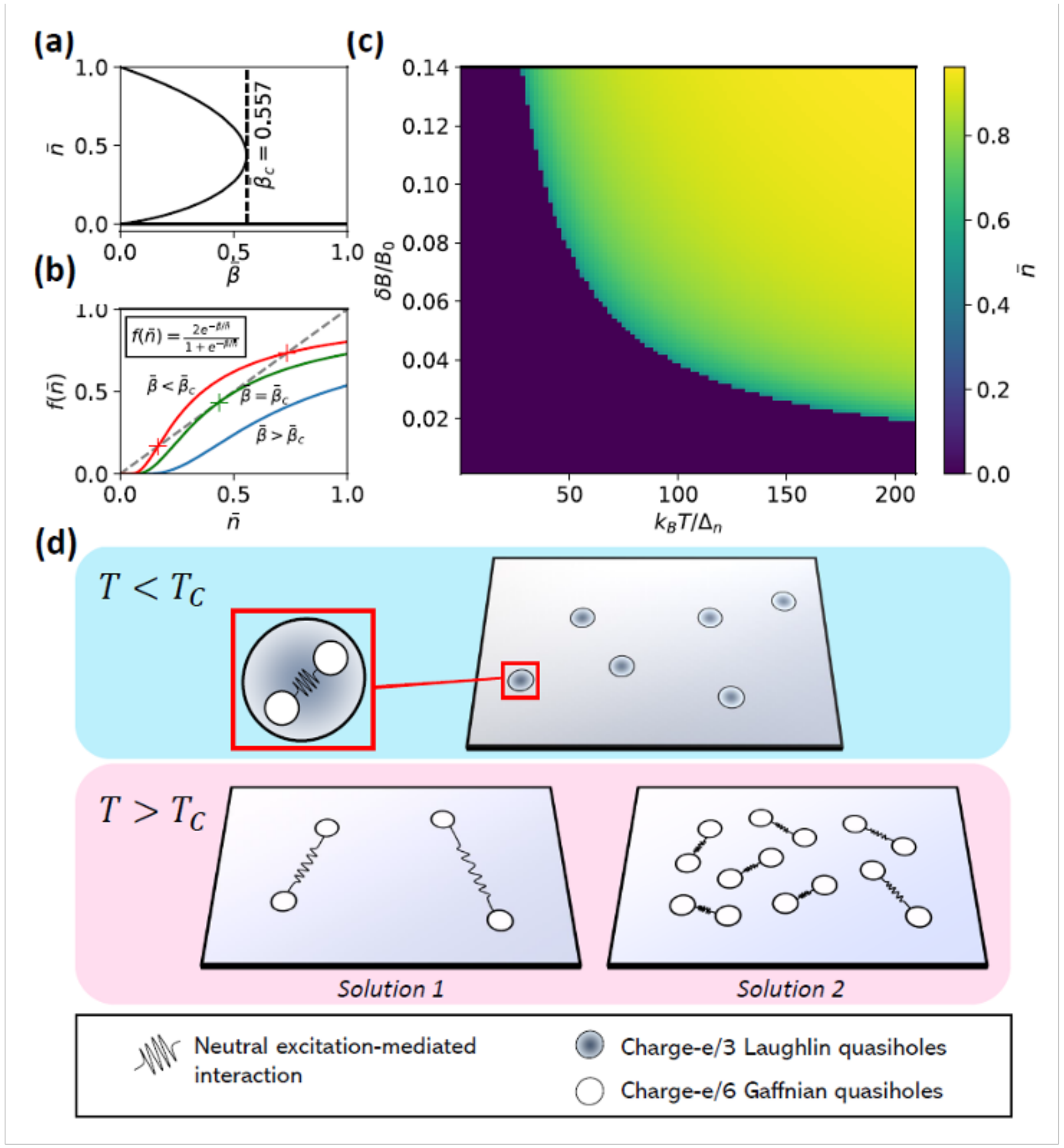
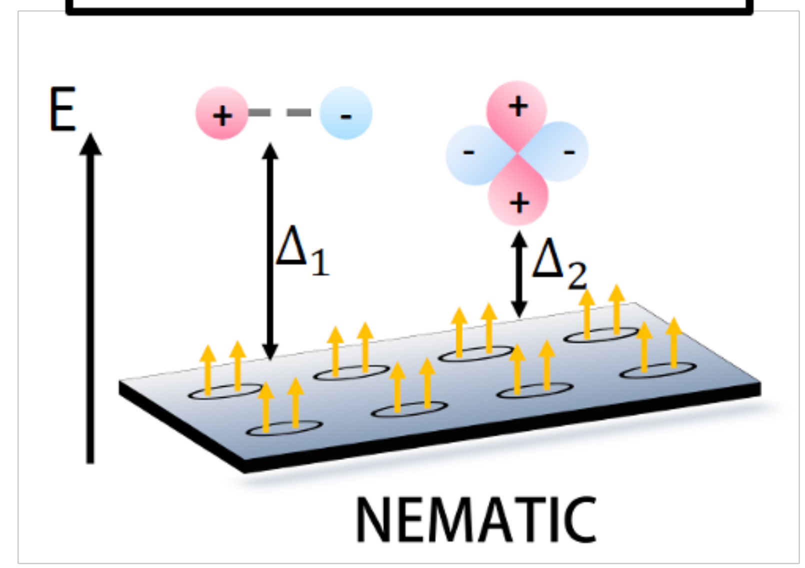


magnetic flux density

quadrupole gap

$$T_C \sim \frac{0.381\Delta_2}{k_B}\left(\frac{\delta B}{B_0}\right)^{-1}$$



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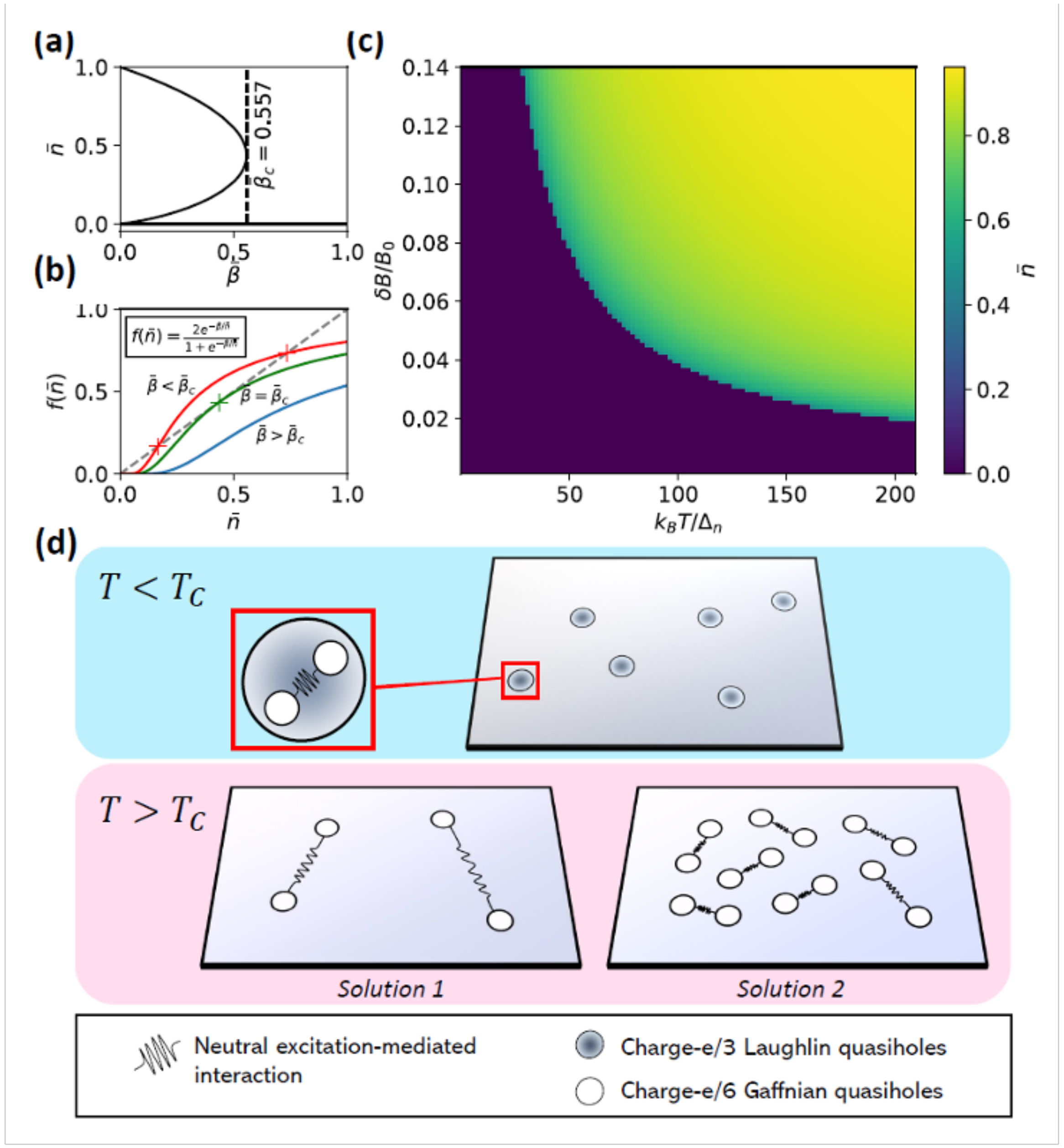
A phase transition of low-lying excitations at the same filling factor

Ha Quang Trung and Bo Yang, PRL 127,046402 2021

A BKT-like phase transition



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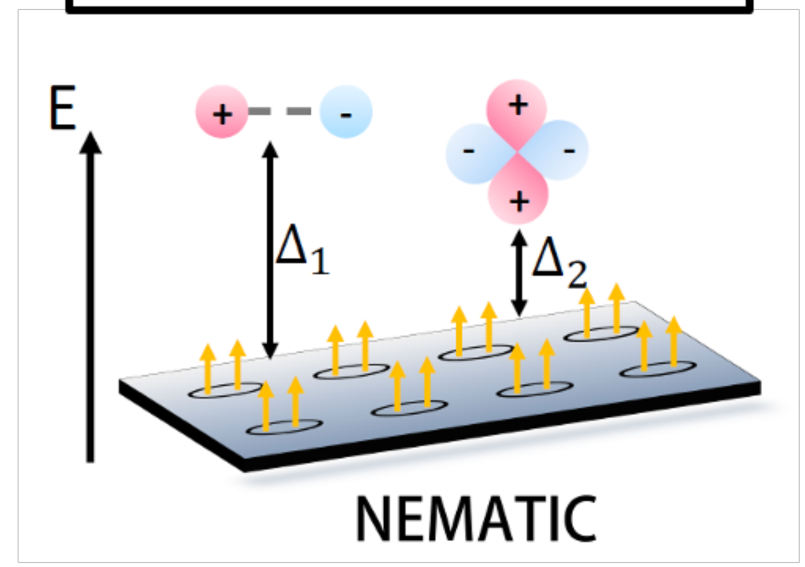


e/6-charges are observable in experiment if

$$T_c < T < \Delta_1/k_B$$

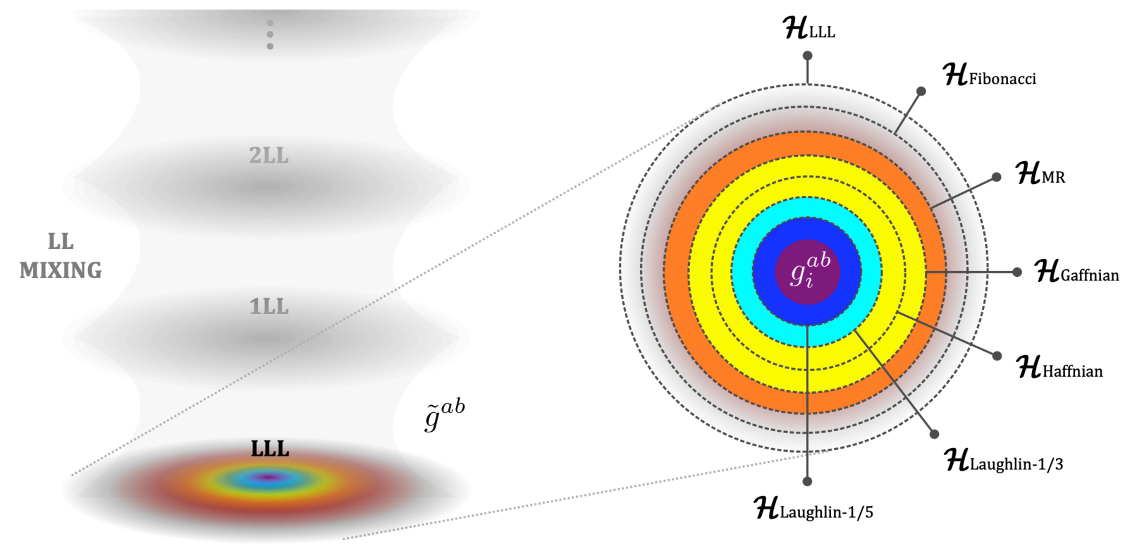
charge gap

A phase transition of low-lying excitations at the same filling factor

Ha Quang Trung and Bo Yang, PRL 127,046402 2021

Multiple gravitons and anyon spin-statistics theorem

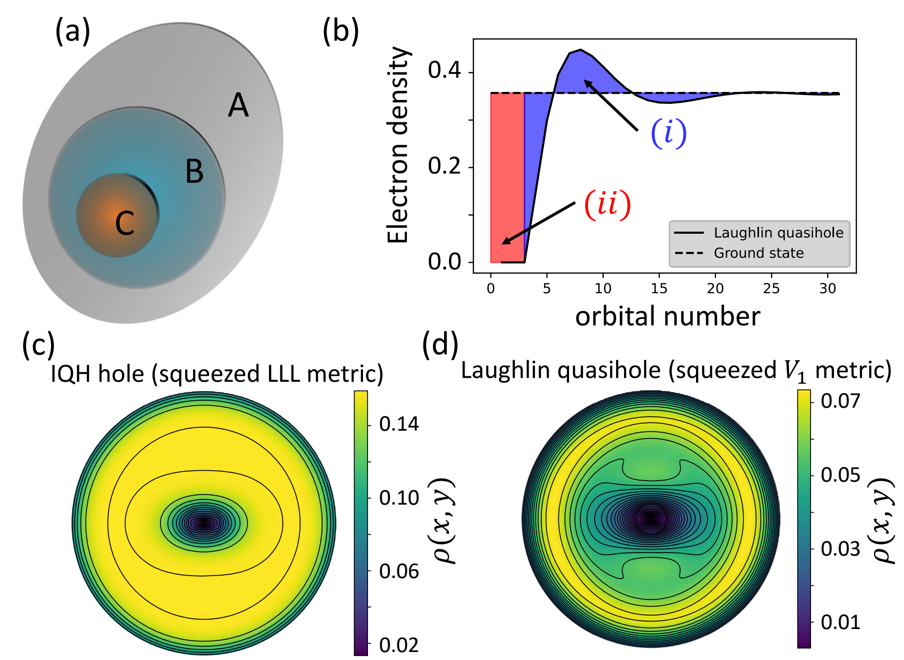
- Certain FQH phases (e.g. with CF particle-hole conjugation) can have multiple graviton excitations, each associated with the metric fluctuation of different CHS



D. X. Nguyen, F. D. M. Haldane, E. H. Rezayi, D. T. Son, K. Yang, PRL 128, 246402
A. C. Balram, Z. Liu, A. Gromov, Z. Papic, PRX 12, 021008

Yuzhu Wang and Bo Yang, Nat. Comm. 14, 2317 (2023)

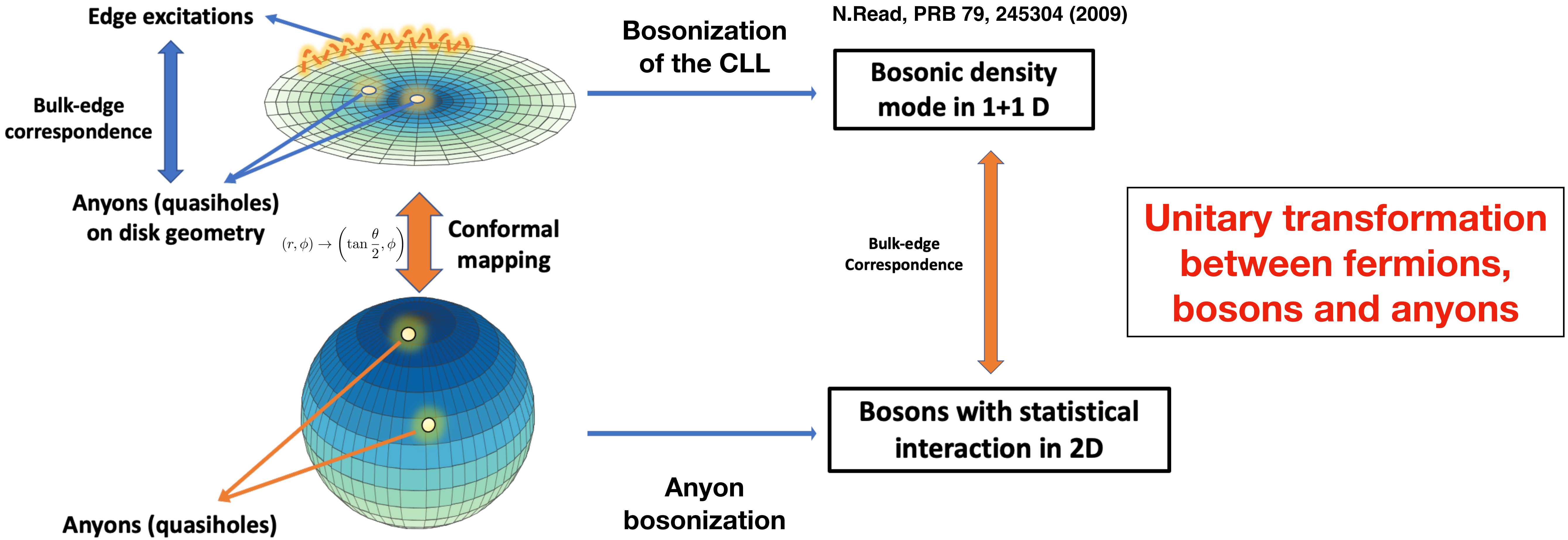
- Spin-statistics theorem for **anyons (with arbitrary internal structure)** in a non-relativistic setting requires a proper definition of anyon intrinsic spin, which is the angular momentum within the proper CHS



T. Einarsson, S. Sondhi, S. Girvin, D. Arovas, Nuclear Physics B 441, 515
T. Comparin, A. Opler, E. Macaluso, A. Biella, A. P. Polychronakos, L. Mazza, Phys. Rev. B 105, 085125

Ha Quang Trung, Yuzhu Wang and Bo Yang, PRB Letter, 107, L201301 (2023)

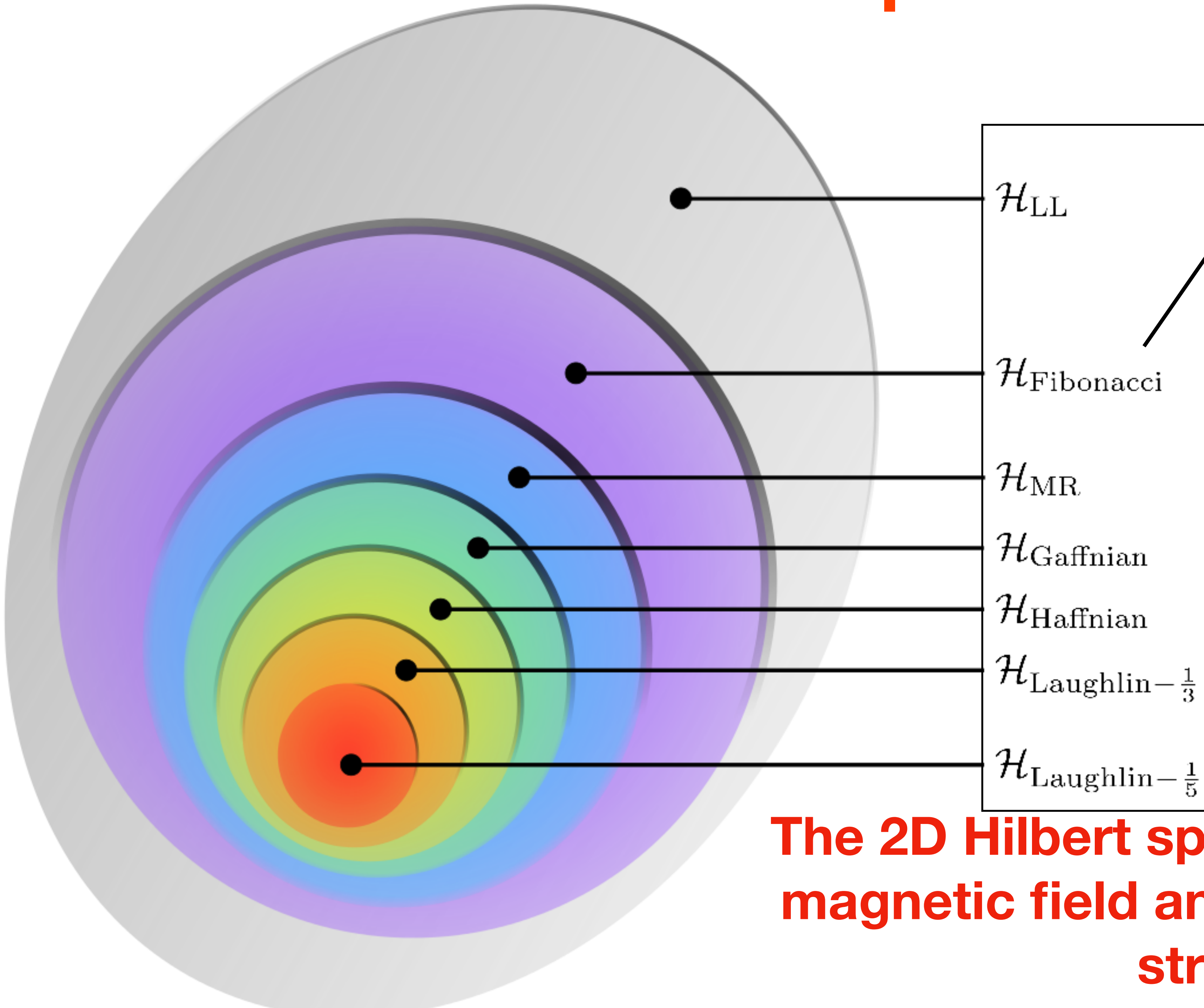
2D Bosonization and Fermionization



Bo Yang, PRL 127, 126406 (2021)

- Microscopic picture of anyons interpreted as composite bosons and composite Fermions
- New family of single-component bosonic FQH states (with their model Hamiltonians)
- Microscopic Hamiltonians capturing statistical interaction

The conformal Hilbert spaces



Interaction Hamiltonian

Different types of anyons

Null spaces of the interactions

The 2D Hilbert space in the presence of magnetic field and interaction is highly structured

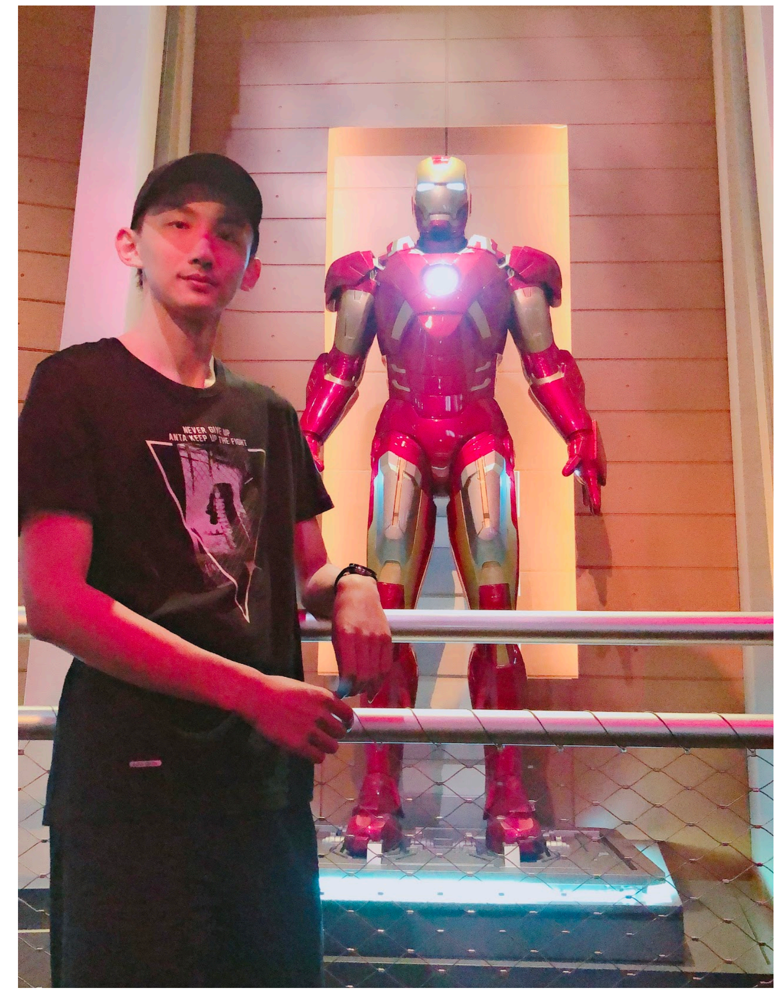
The Fuzzy sphere for 3D CFT: W Zhu, C Han, E Huffman, JS Hofmann, YC He
Physical Review X 13 (2), 021009

Students:

Ha Quang Trung

Ha Quang Trung and Bo Yang, PRL 127,046402 2021

Ha Quang Trung, Yuzhu Wang and Bo Yang, PRB Letter, 107, L201301 (2023)



Wang Yuzhu

Yuzhu Wang and Bo Yang, Nat. Comm. 14, 2317 (2023)

Yuzhu Wang and Bo Yang, Phys. Rev. B 105, 035144 (2022)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College of Science
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OXFORD INSTRUMENTS
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Max Planck Institute, Stuttgart, Germany)
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(Nobel Laureate in Physics,
Princeton University, USA)
Bertrand Halperin
(Wolf Prize in Physics,
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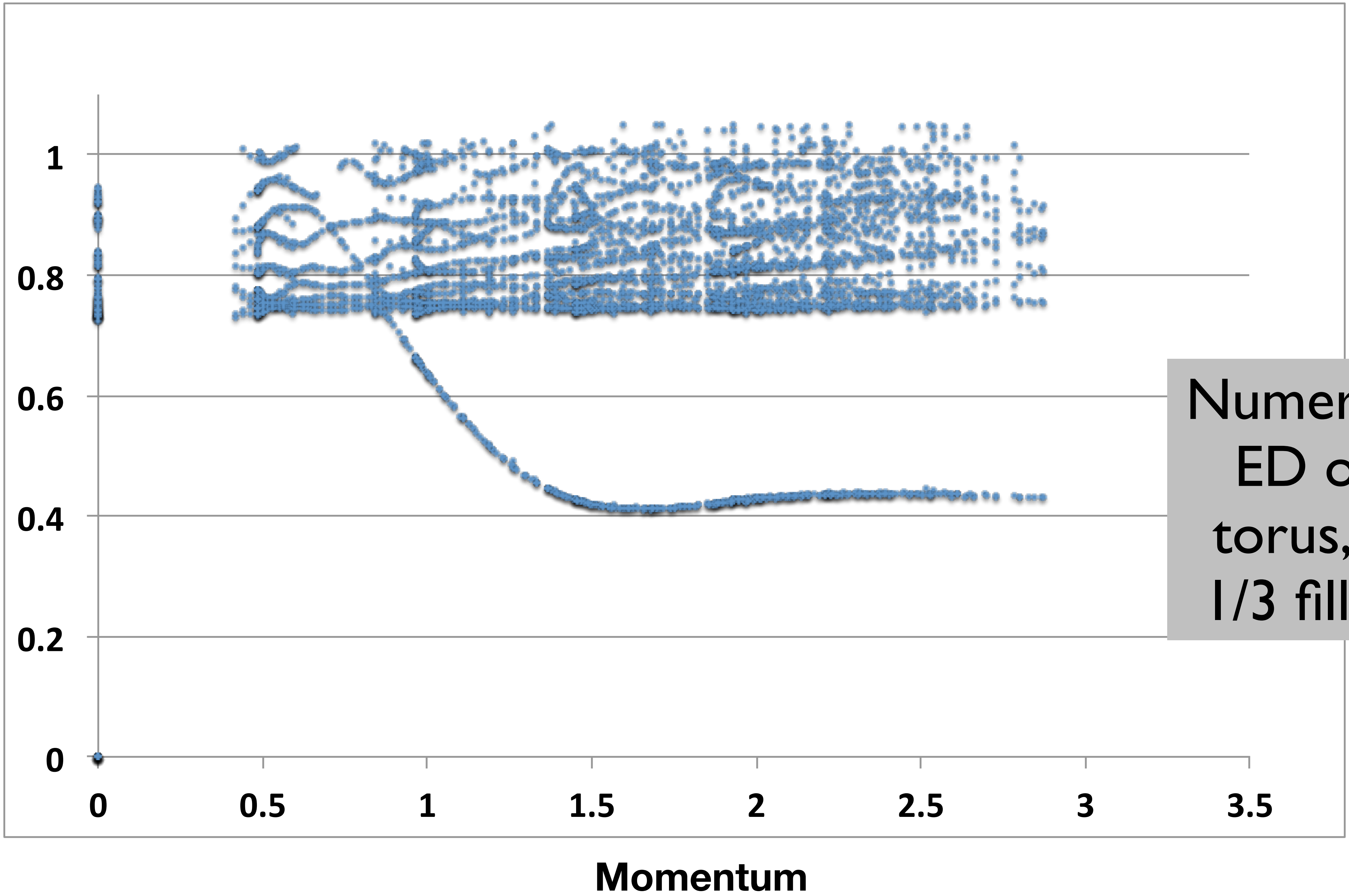
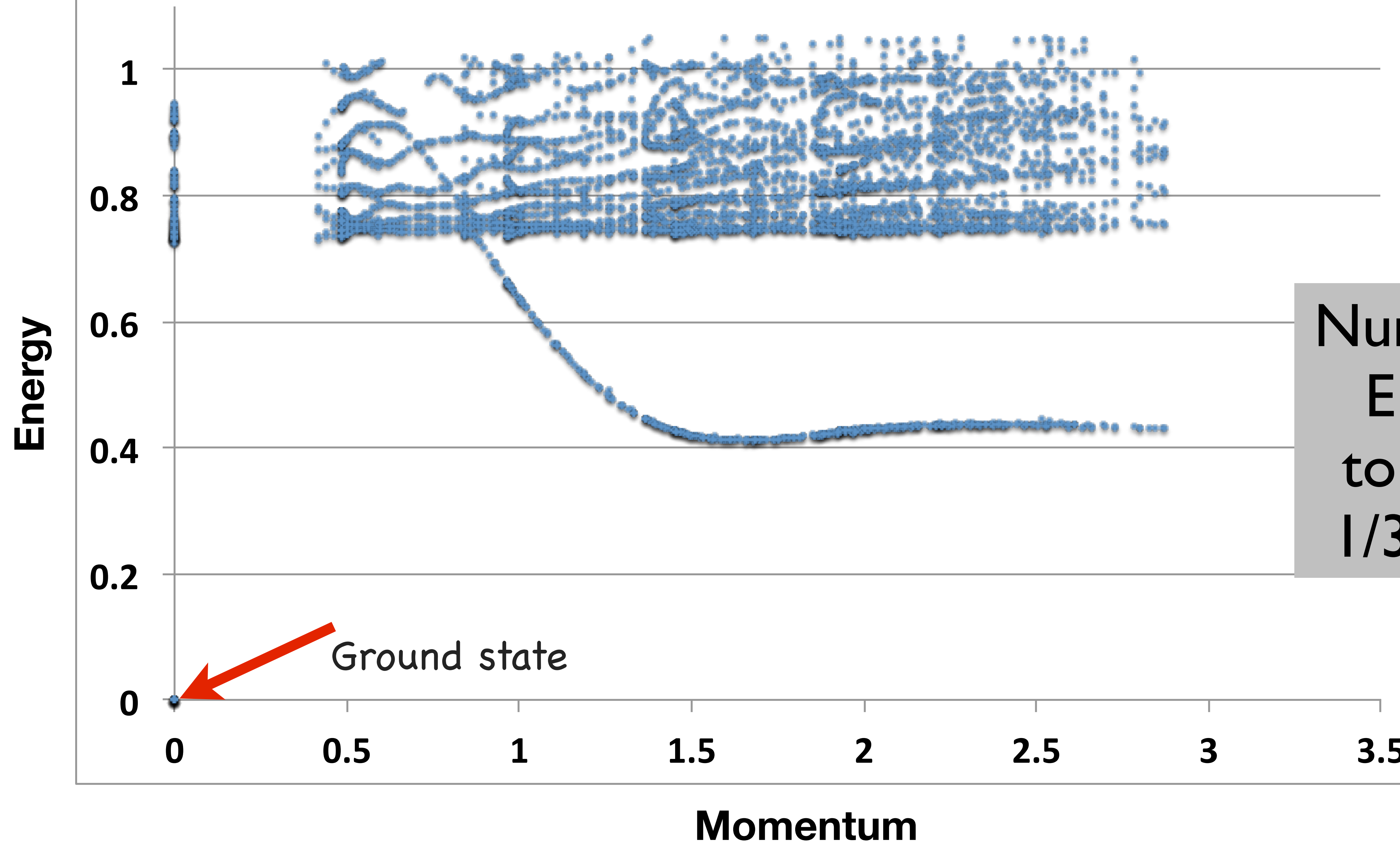
January 3-5, 2024
School of Physical and Mathematical Sciences,
Nanyang Technological University, Singapore

A theoretical question



Numerical ED on torus, at 1/3 filling

A theoretical question



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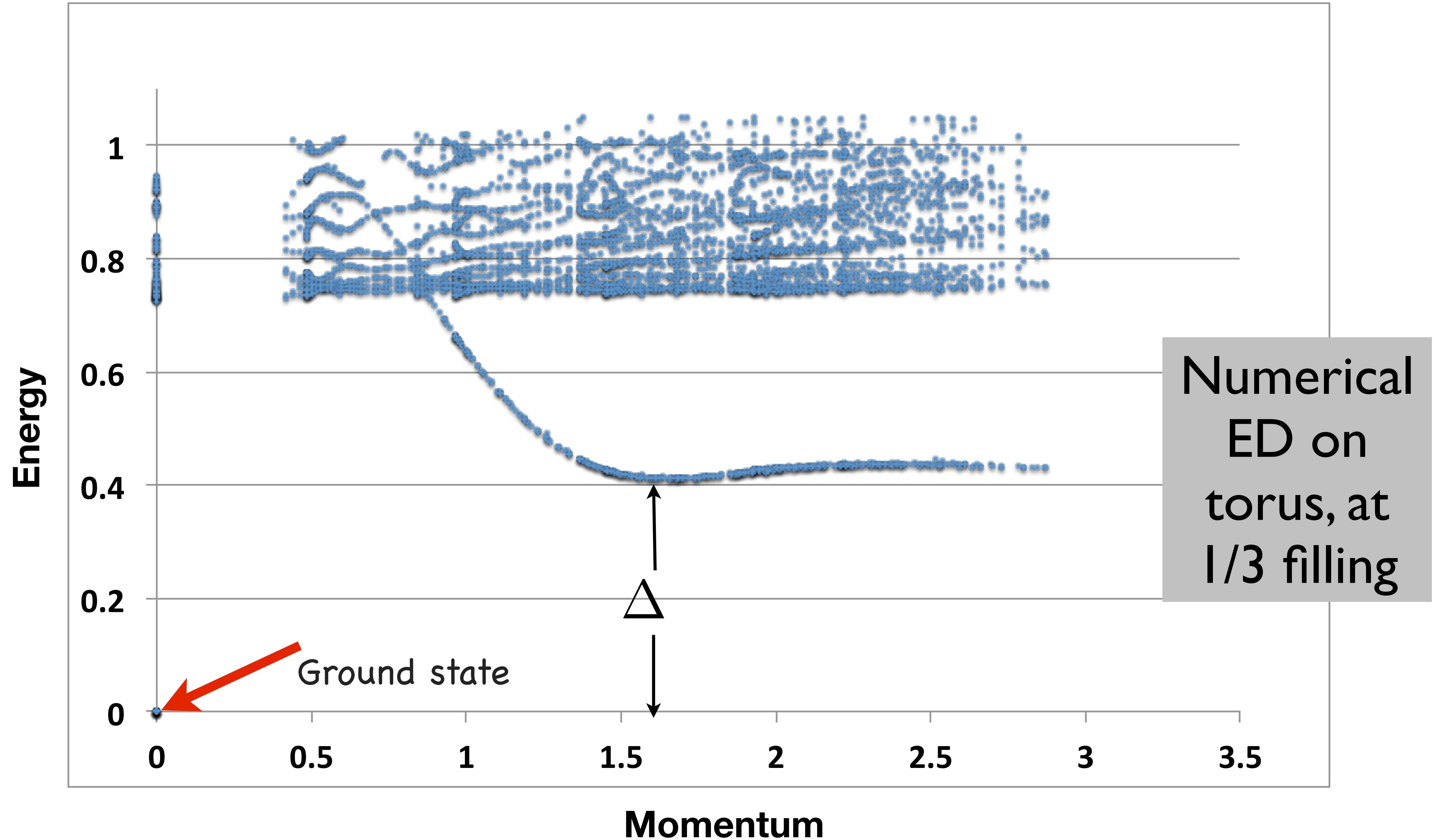
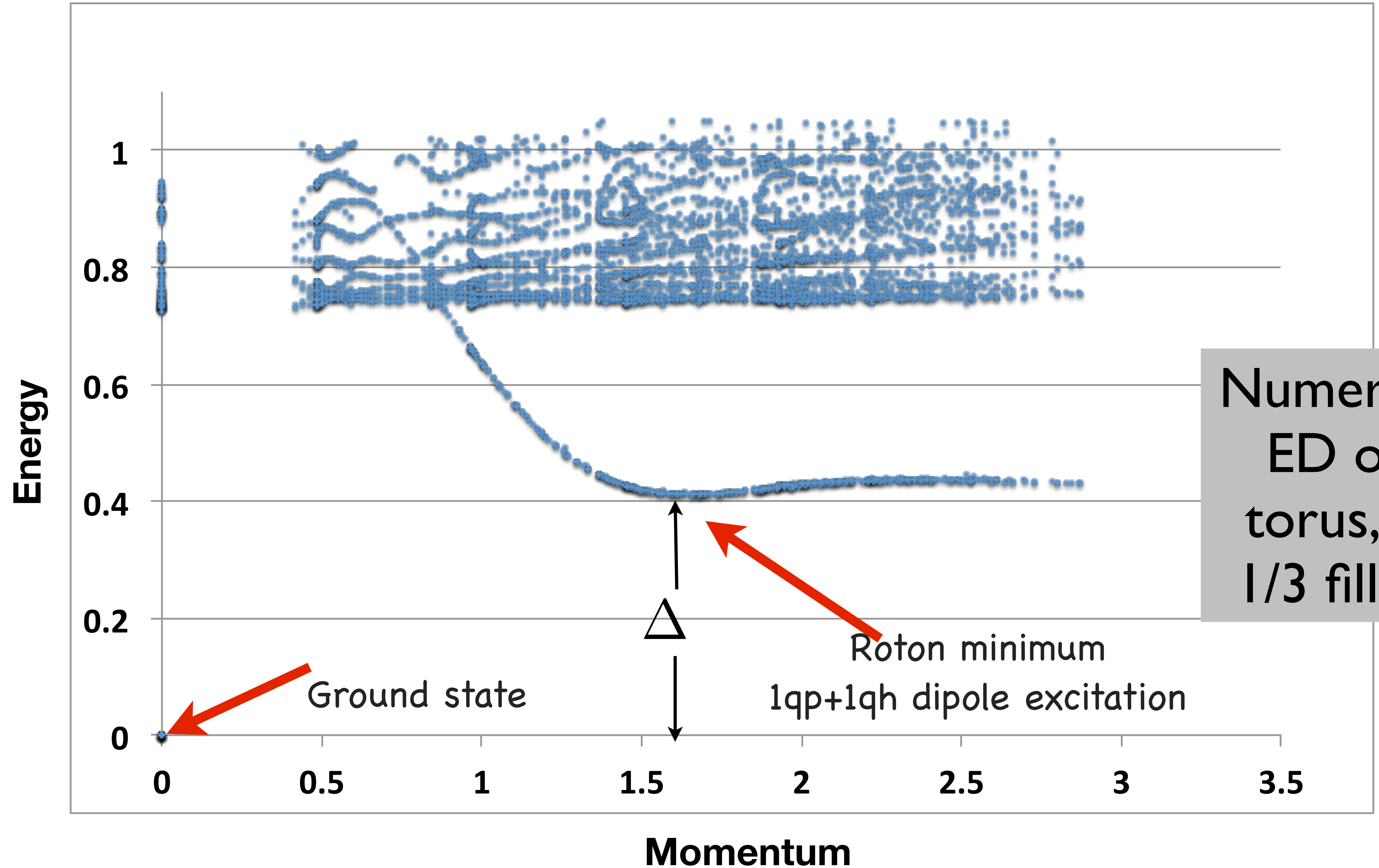


A theoretical question



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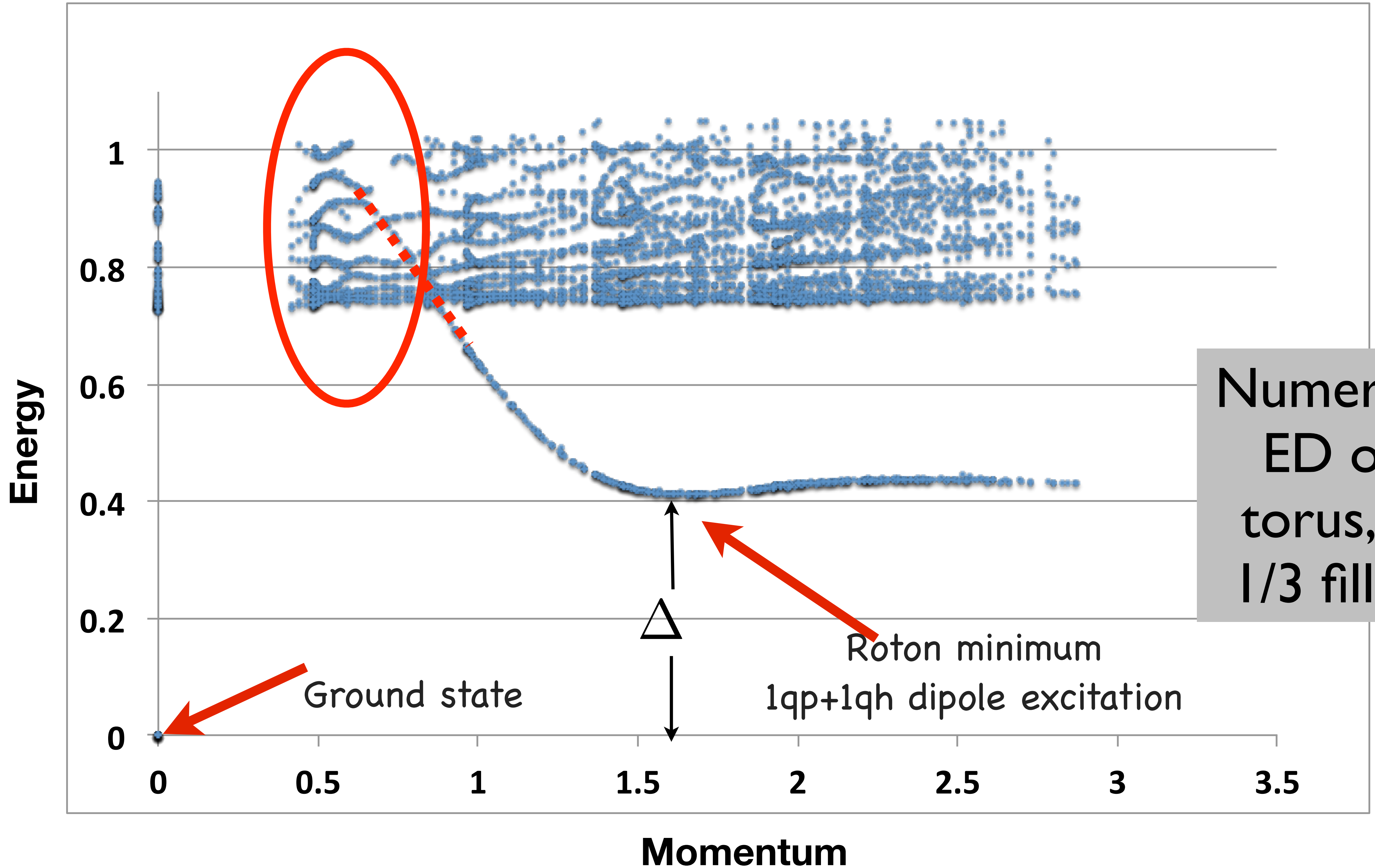

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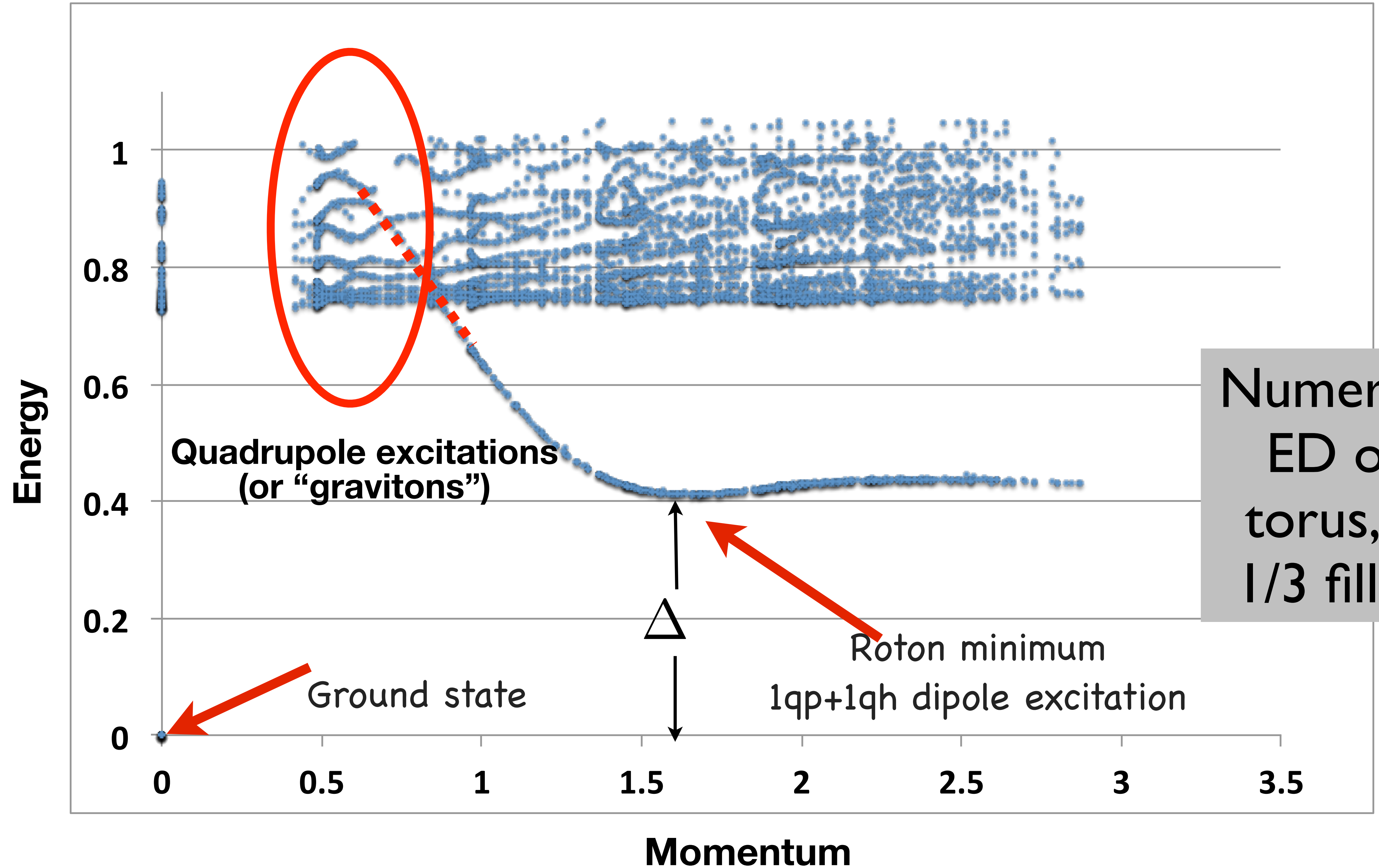

Numerical ED on torus, at 1/3 filling

A theoretical question

Charge gap related to the creation energy of a single quasielectron or quasihole

- **Quadrupole excitations** and **dipole excitations** are different types of FQH excitations
- The nematic FQH results from the softening of the quadrupole excitations
- How tunable are the two energy scales in the FQH topological phase?

M. Mulligan et.al PRB 82, 085120, PRB 84, 195124
J. Maciejko, B. Hsu, S.A. Kivelson, Y. Park and S.L. Sondhi, Phys. Rev. B. 88, 125137 (2013).
Y. You, G.Y. Cho and E. Fradkin, Phys. Rev. X. 4, 041050 (2014).
K. Lee, J. Shao, E.-A. Kim, F.D.M. Haldane and E.H. Rezayi, Phys. Rev. Lett. 121, 147601 (2018).
N. Regnault et al, PRB 96, 035150 (2017)

The generalisation of the conformal Hilbert space formalism

(Can we realise model Hamiltonians with Coulomb based interaction?)

Bo Yang, arXiv:2307.06361

Scale-free electron-electron interaction

$$\hat{H}_\alpha = \sum_i \frac{1}{2m\ell_B^2} g_{ab} \tilde{R}_i^a \tilde{R}_i^b + \int d^2 r_1 d^2 r_2 V_\alpha \left(r_1 - r_2 \right) \hat{\rho}_{r_1} \hat{\rho}_{r_2}$$

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$$[\tilde{R}^a, \tilde{R}^b] = i\ell_B^2 \epsilon^{ab}$$

Scale-free electron-electron interaction

$$\hat{\rho}_r = \sum_i \delta^2(r - r_i)$$

$$\hat{H}_\alpha = \sum_i \frac{1}{2m\ell_B^2} g_{ab} \tilde{R}_i^a \tilde{R}_i^b + \int d^2r_1 d^2r_2 V_\alpha\left(r_1 - r_2\right) \hat{\rho}_{r_1} \hat{\rho}_{r_2}$$

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$$V_\alpha(r) = \frac{1}{r^\alpha}$$ **(scale-free interaction)**

Scale-free electron-electron interaction

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$$[\tilde{R}^a, \tilde{R}^b] = i\ell_B^2 \epsilon^{ab}$$

$$V_\alpha(r) = \frac{1}{r^\alpha}$$ **(scale-free interaction)**

In the LLL:

$$\tilde{V}_\alpha(q) \sim q^{\alpha-2} e^{-\frac{1}{2}q^2} = \sum_{i=0}^{\infty} c_{\alpha,i} V_i(q)$$

Haldane pseudopotentials

Scale-free electron-electron interaction

$$\hat{\rho}_r = \sum_i \delta^2(r - r_i)$$

$$\hat{H}_\alpha = \sum_i \frac{1}{2m\ell_B^2} g_{ab}\tilde{R}_i^a \tilde{R}_i^b + \int d^2r_1 d^2r_2 V_\alpha(r_1 - r_2)\hat{\rho}_{r_1}\hat{\rho}_{r_2}$$

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Haldane pseudopotentials

Only valid for alpha smaller than 2!

Scale-free electron-electron interaction

$$\hat{\rho}_r = \sum_i \delta^2(r - r_i)$$

$$\hat{H}_\alpha = \sum_i \frac{1}{2m} g^{ab} \hat{\pi}_{i,a} \hat{\pi}_{i,b} + \int d^2 r_1 d^2 r_2 V_\alpha (r_1 - r_2) \hat{\rho}_{r_1} \hat{\rho}_{r_2}$$

$$[\hat{\pi}_a, \hat{\pi}_b] = i \ell_B^{-2} \epsilon_{ab}$$

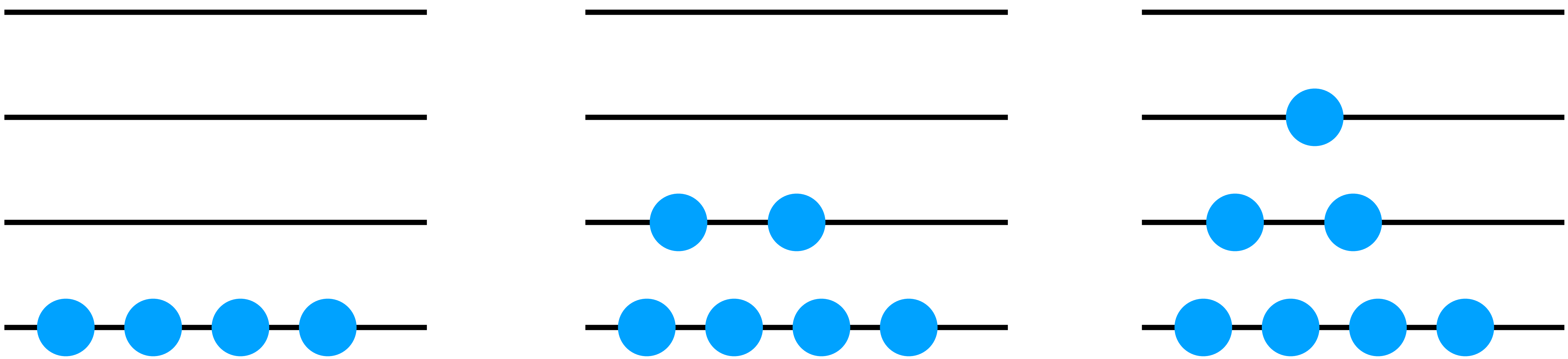
$$V_\alpha(r) = \frac{1}{r^\alpha}$$ **(scale-free interaction)**

In the LLL: $\tilde{V}_\alpha(q) \sim \sum_i c_{i,\alpha} V_i$ $\quad c_{\alpha \to 2k, i \leq k-1} \to \infty$

Haldane pseudopotentials

Strong LL mixing despite large cyclotron gap

$$\text{For } \alpha = 2, \ell_B \to 0, \tilde{V}_\alpha \sim +\infty V_0 \text{ or } +\infty \delta^2(z_i - z_j)$$



$\nu \leq 1/2$

Laughlin phase with quasiholes

$1/2 < \nu \leq 2/3$

Laughlin quasielectrons/Jain quasiholes and Jain 2/3 phase

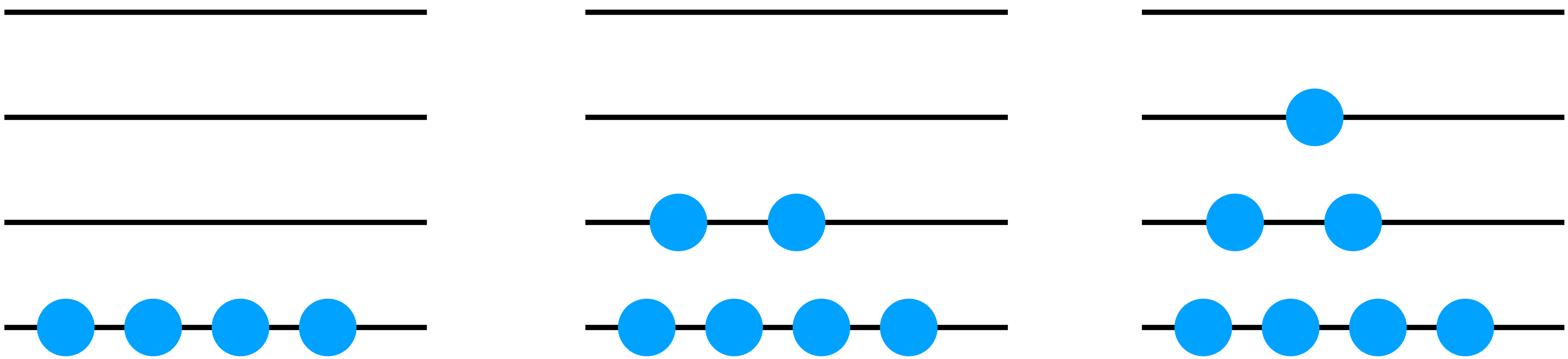
$2/3 < \nu$

Non-abelian parton state

- (Part of) CF theories are exact in a higher dimensional universe
- Unprojected CF states are natural physical states
- Coulomb interaction is a special case of scale free interaction

Strong LL mixing despite large magnetic field

$$\text{For } \alpha = 2, \ell_B \to 0, \tilde{V}_\alpha \sim +\infty V_0 \ \text{ or } \ +\infty \delta^2(z_i - z_j)$$



$\nu \leq 1/2$

Laughlin phase with quasiholes

$1/2 < \nu \leq 2/3$

Laughlin quasielectrons/Jain quasiholes and Jain 2/3 phase

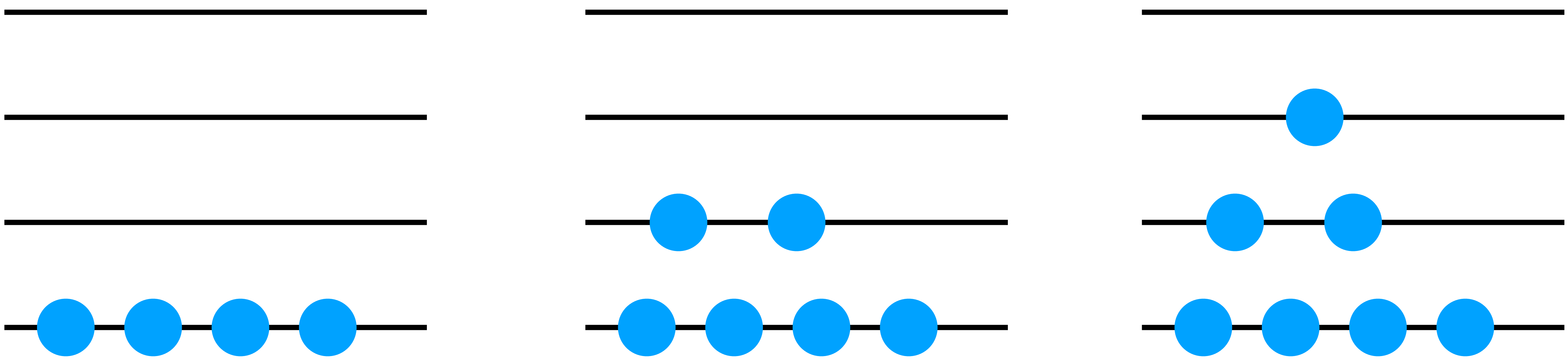
$2/3 < \nu$

Non-abelian parton state

Density-dependent LL mixing that is analytically tractable

Multiple LL with infinite magnetic field

$$\text{For } \alpha = 2, \ell_B \to 0, \tilde{V}_\alpha \sim +\infty V_0 \ \text{ or } \ +\infty\delta^2(z_i - z_j)$$



$\nu \leq 1/2$ — Laughlin phase with quasiholes

$1/2 < \nu \leq 2/3$ — Laughlin quasielectrons/Jain quasiholes and Jain 2/3 phase

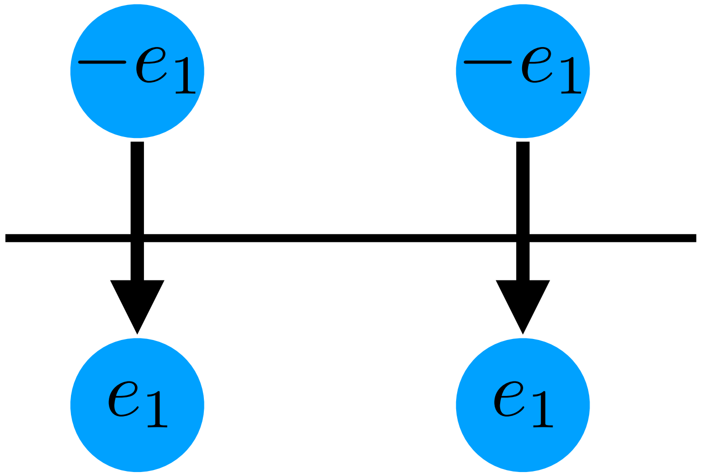
$2/3 < \nu$ — Non-abelian parton state

- **Generalisation of conformal Hilbert spaces to multiple LLs**
- **Non-Abelian topological phases from 2-body interaction**

M. Tanhayi Ahari, S. Bandyopadhyay, Z. Nussinov, A. Seidel, G. Ortiz, Scipost Physics, 2023

Experimental realization?

If only we live in a 4+1 dimensional spacetime!



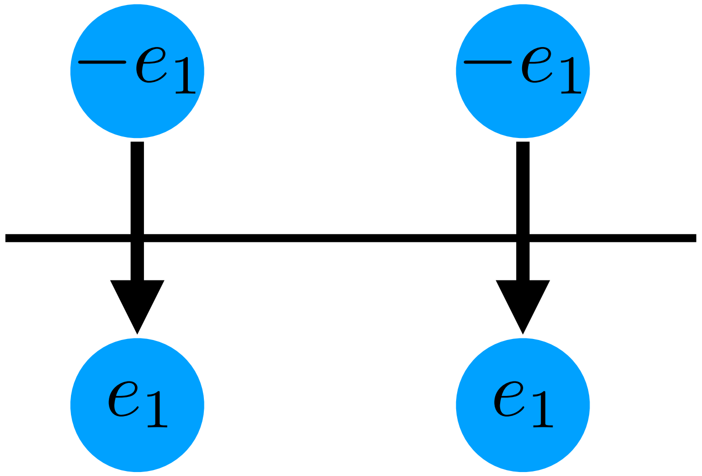
Dipole-dipole interaction:

$$V(r) \sim \frac{1}{r} - \frac{1}{\sqrt{r^2 + d^2}}$$

Short-range interaction is still 1/r

Experimental realization?

If only we live in a 4+1 dimensional spacetime!

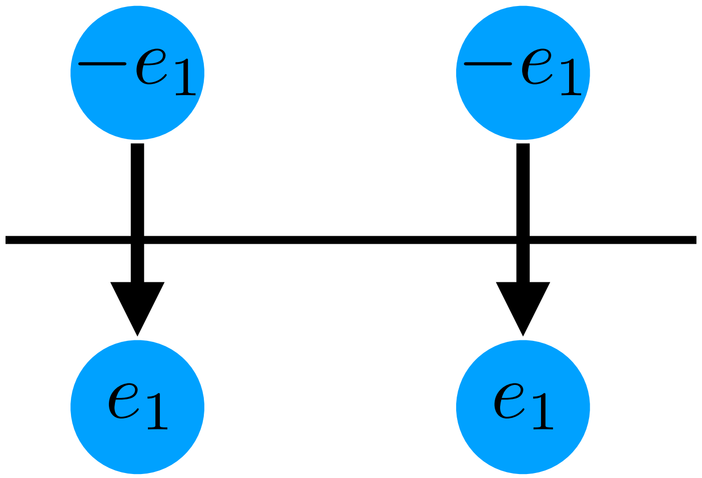


Dipole-dipole interaction:

$$V(r) \sim \frac{1}{r} - \frac{1}{\sqrt{r^2 + d^2}}$$

For $r > \ell_B > d$, we have $V(r) \sim 1/r^3$

Experimental realization?



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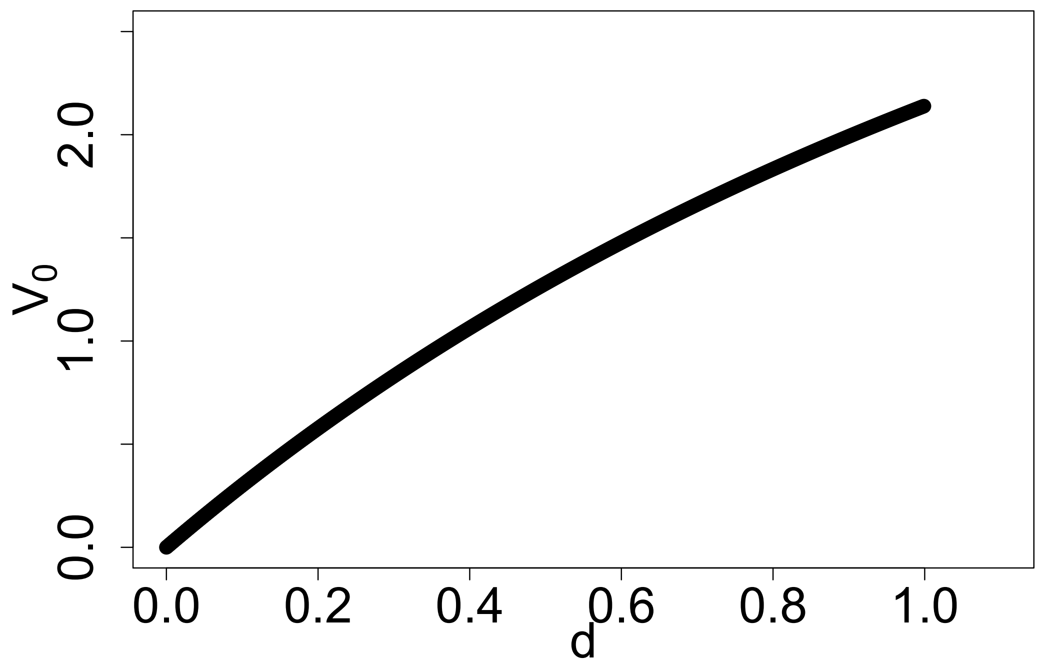
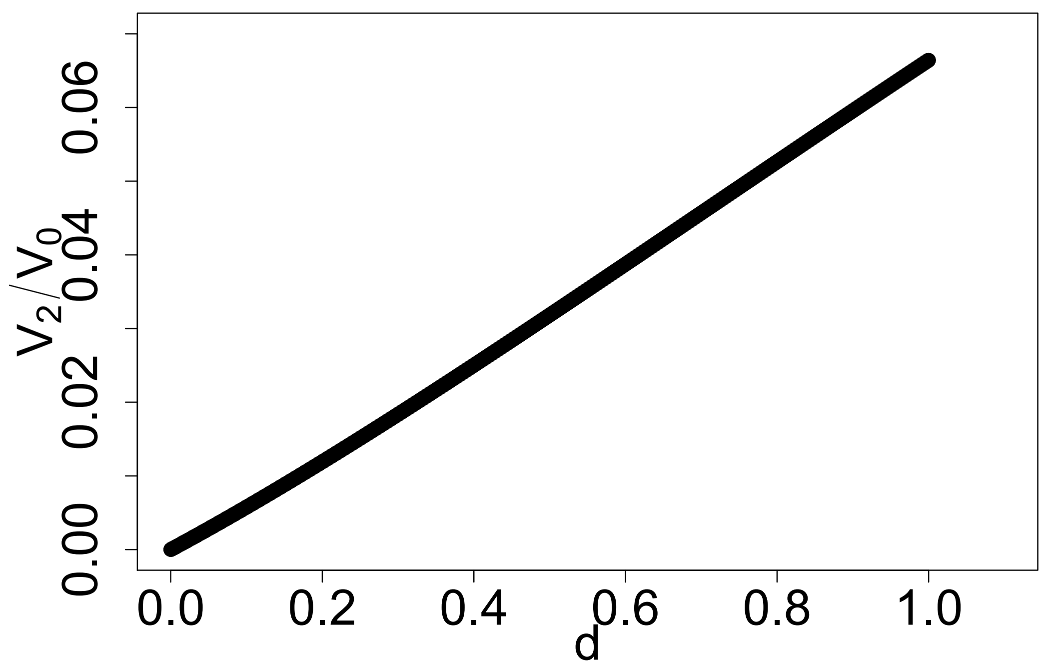
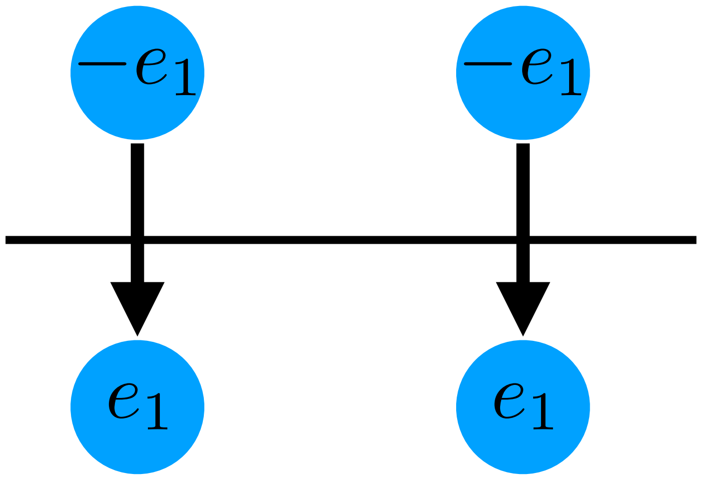


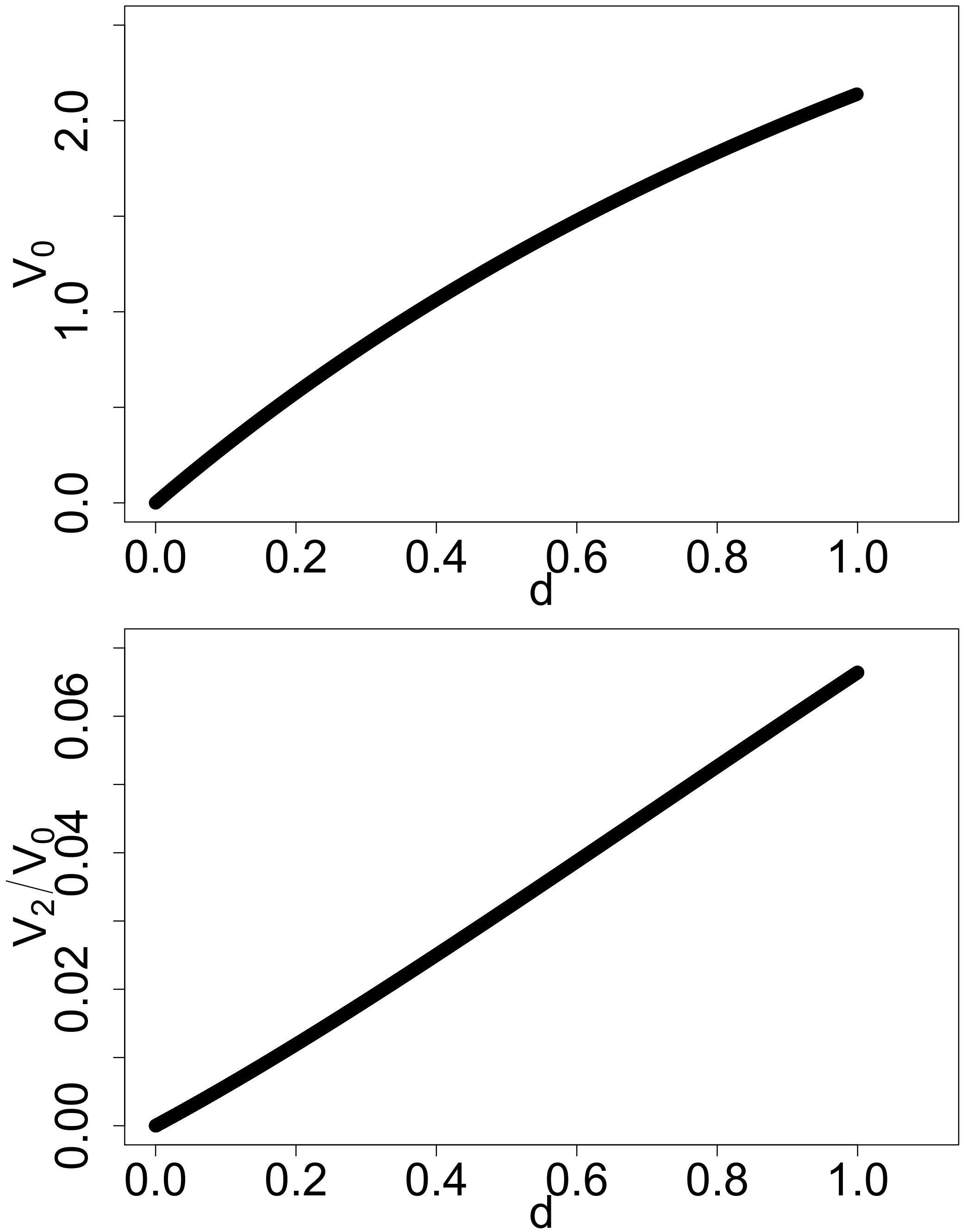


We can make V_0 arbitrarily dominant

Experimental realization?



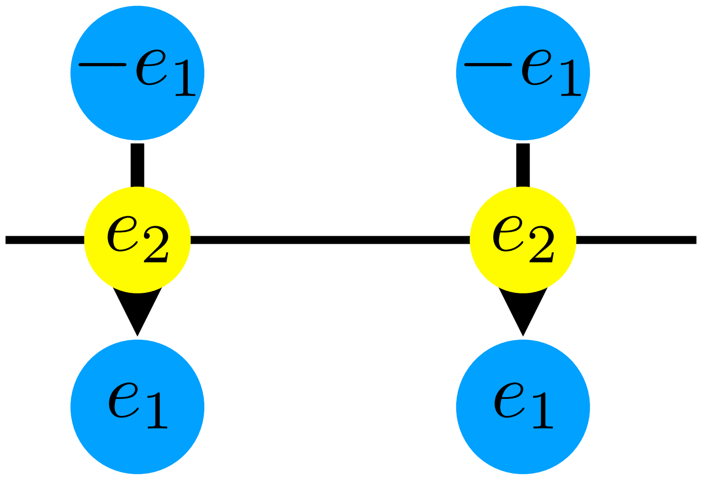
Dipole-dipole interaction:



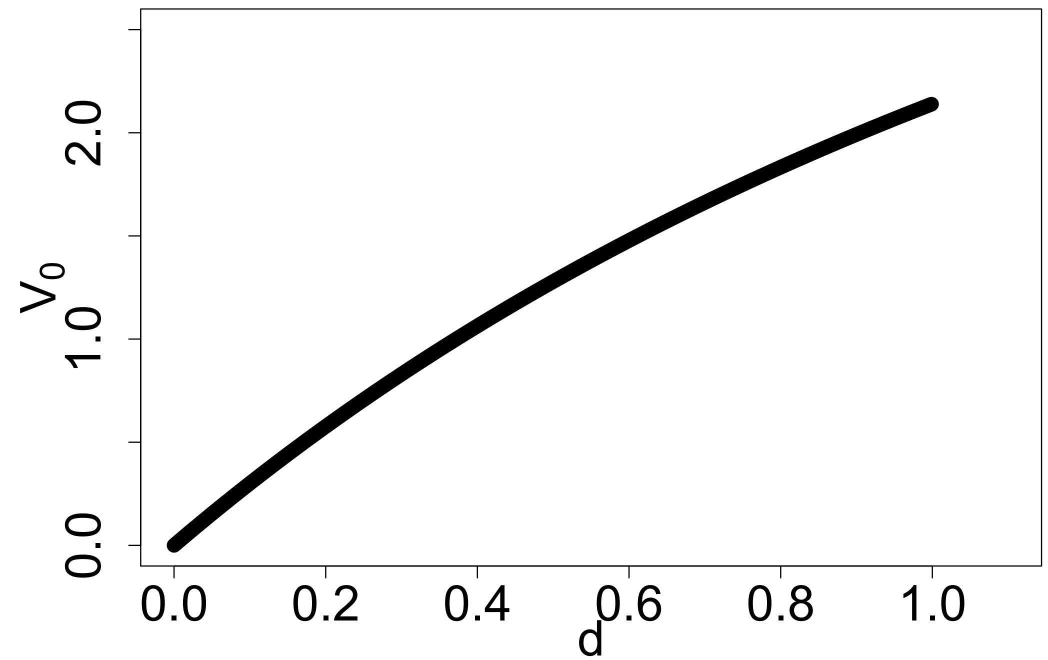
But we still need:

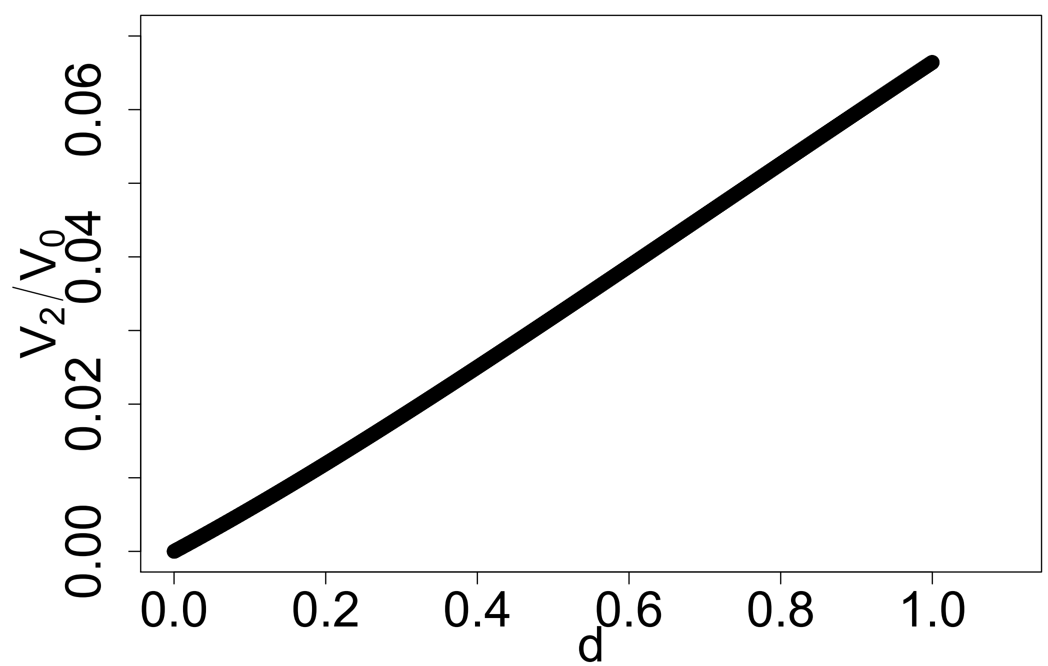
$$V_0 \gg \hbar\omega_c \gg V_{i>0}$$

Experimental realization?



Charged bosons with a dipole





The goal:

$$e_1^2/\ell_B$$

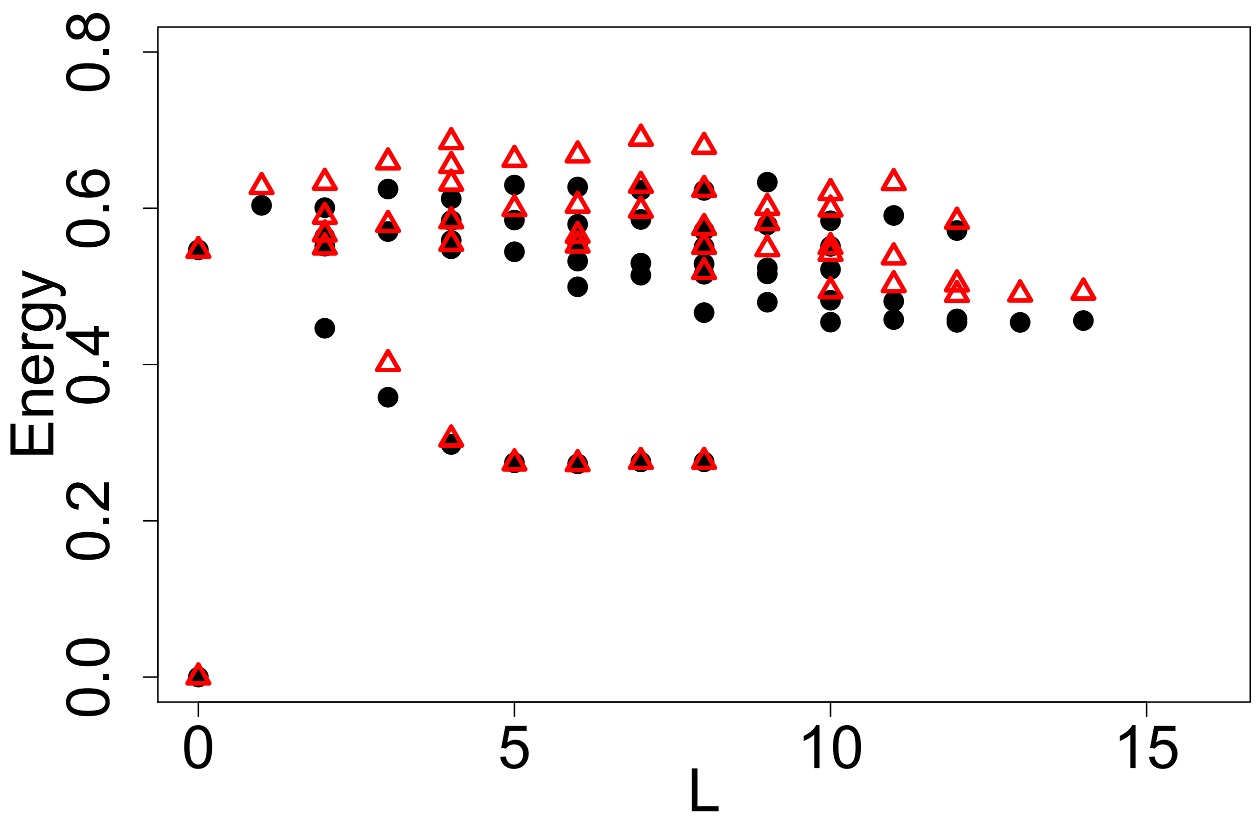
$$V_0 \gg \hbar\omega_c \gg V_{i>0}$$

$$\frac{1}{m\ell_B^2}$$

A final theoretical touch

$$\mathcal{H}_1 = \hat{V}_0 \text{ within the LLL}$$

$$\mathcal{H}_2 = \sum_i \frac{1}{2m} g^{ab} \hat{\pi}_a \hat{\pi}_b \text{ with the TK Ham null space}$$



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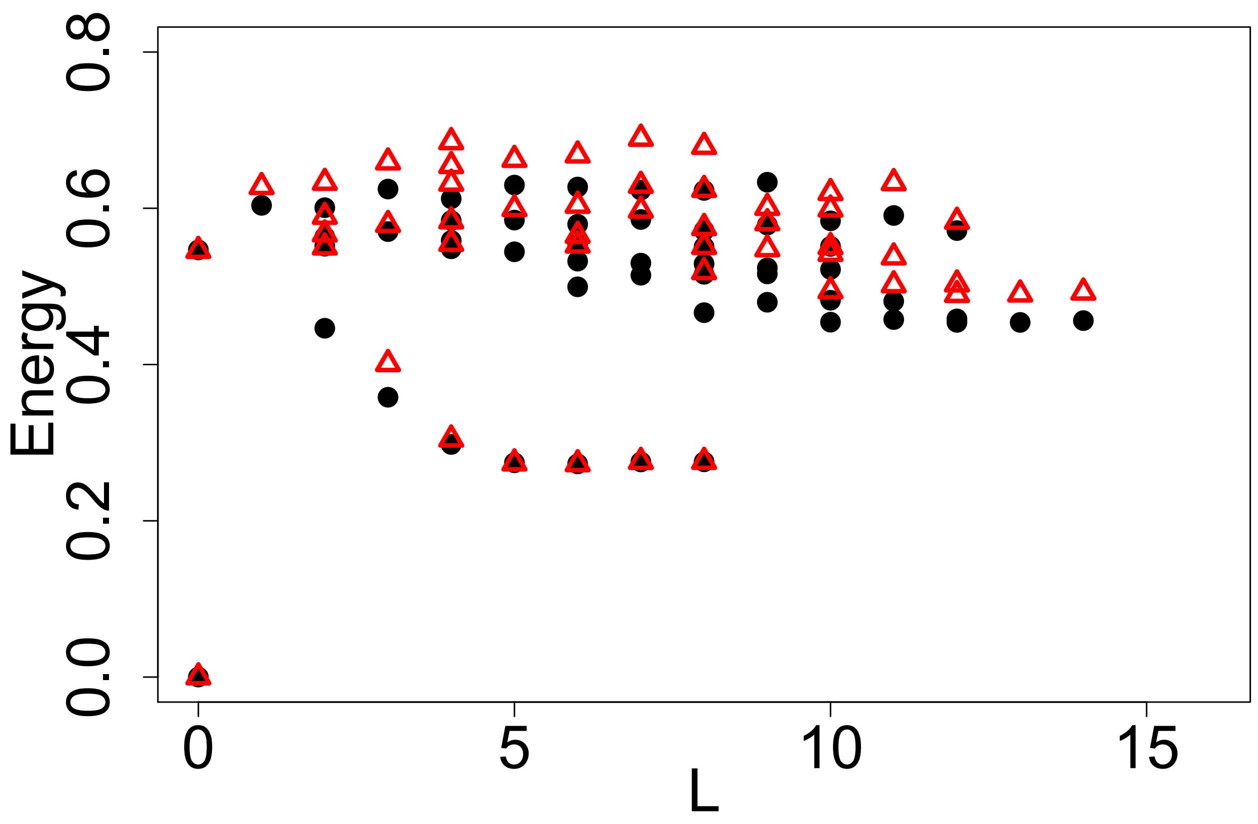
Duality between strongly/weakly interacting systems

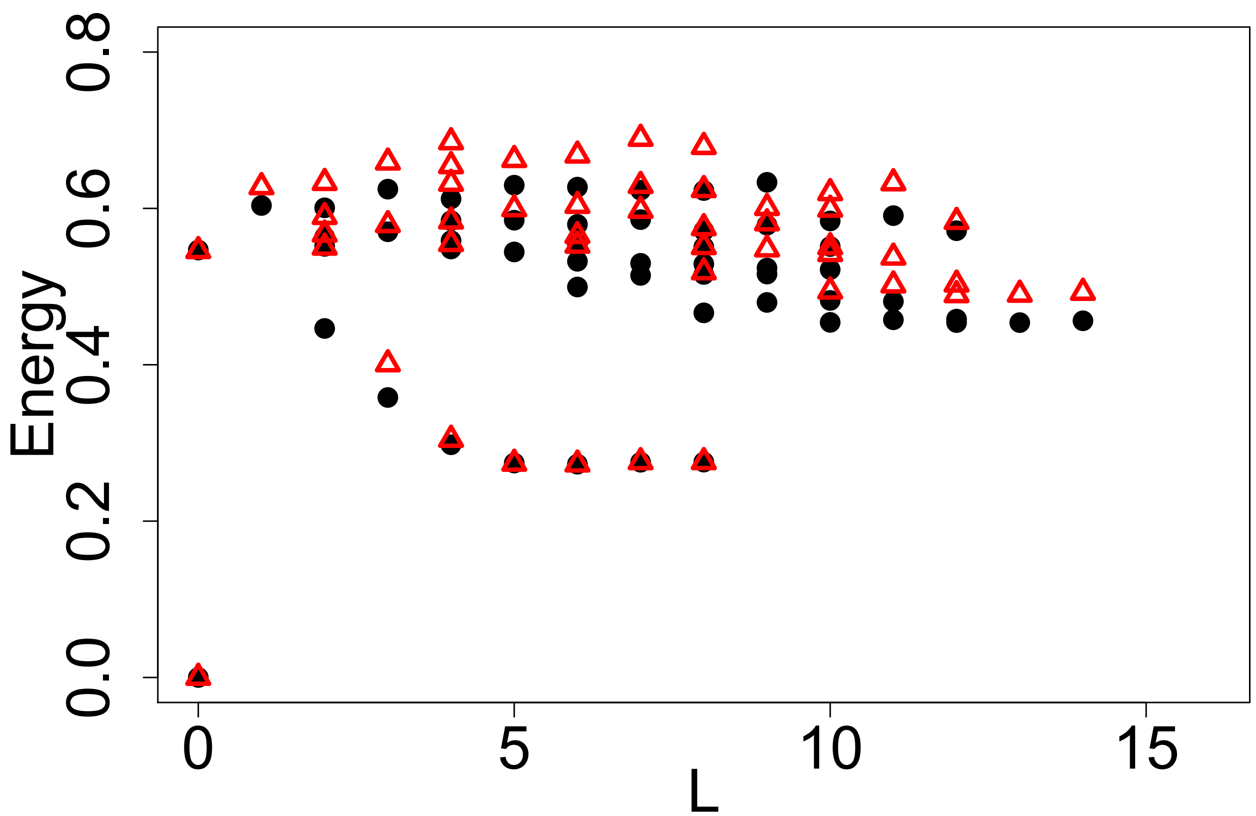


A final theoretical touch

$$\alpha = 2 - \epsilon \longleftarrow \mathcal{H}_1 = \hat{V}_0 \text{ within the LLL}$$

$$\alpha = 2 \longleftarrow \mathcal{H}_2 = \sum_i \frac{1}{2m} g^{ab} \hat{\pi}_a \hat{\pi}_b \text{ with the TK Ham null space}$$

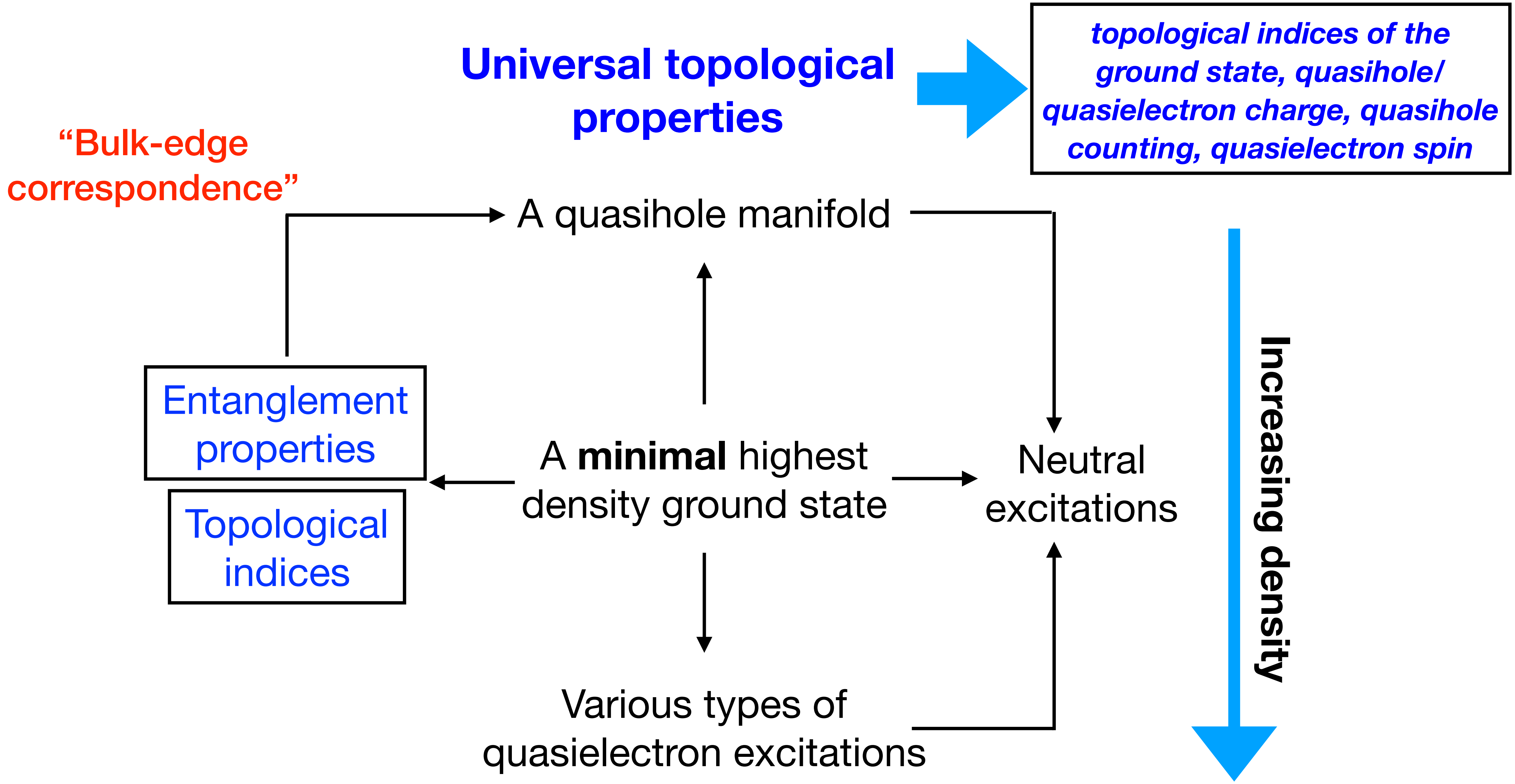
Duality between strongly/weakly interacting systems



Summary

- The formalism of conformal Hilbert spaces could be useful for understanding both the universal features and the dynamics of anyons in FQH systems
- It inherits very rich mathematical/algebraic structure and there is still a lack of good analytical and numerical tools
- There could be a chance of experimentally realising model Hamiltonians for generalised CHS

All topological properties determined by Hilbert space algebra



All universal properties should be determined without involving the (model) Hamiltonian

All energy scales to zero or infinity