## Nonequilibrium thermodynamics of uncertain stochastic processes

Jan Korbel<sup>1,2</sup> and David H. Wolpert<sup>3, 2, 4, 5, 6</sup>

<sup>1</sup>Section for Science of Complex Systems, CeMSIIS, Medical University of Vienna, Spitalgasse 23, 1090 Vienna, Austria\*

<sup>2</sup> Complexity Science Hub Vienna, Josefstädter Strasse 39, 1080 Vienna, Austria

<sup>3</sup>Santa Fe Institute, Santa Fe, NM, USA<sup>†</sup>

<sup>4</sup>Arizona State University, Tempe, AZ, USA

<sup>5</sup>International Center for Theoretical Physics, Trieste, Italy

<sup>6</sup>Albert Einstein Institute for Advanced Study, New York, USA

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Stochastic thermodynamics is formulated under the assumption of perfect knowledge of all thermodynamic parameters. However, in any real-world experiment, there is non-zero uncertainty about the precise value of temperatures, chemical potentials, energy spectrum, etc. Here we investigate how this uncertainty modifies the theorems of stochastic thermodynamics. We consider two scenarios: in the (called *effective*) scenario we fix the (unknown, randomly generated) experimental apparatus and then repeatedly observe (stochastic) trajectories of the system for that fixed apparatus. In contrast, in a (called *phenomenological*) scenario the (unknown) apparatus is re-generated for each trajectory. We derive expressions for thermodynamic quantities in both scenarios. We also discuss the physical interpretation of effective (scenario) entropy production (EP), derive the effective mismatch cost, and provide a numerical analysis of the effective thermodynamics of a quantum dot implementing bit erasure with uncertain temperature. We then analyze the protocol for moving between two state distributions that maximize effective work extraction. Next, we investigate the effective thermodynamic value of information, focusing on the case where there is a delay between the initialization of the system and the start of the protocol. Finally, we derive the detailed and integrated fluctuation theorems (FTs) for the phenomenological EP. In particular, we show how the phenomenological FTs account for the fact that the longer a trajectory runs, the more information it provides concerning the precise experimental apparatus, and therefore the less EP it generates. Reference: https://arxiv.org/abs/2210.05249

<sup>\*</sup> jan.korbel@meduniwien.ac.at