Geometric decomposition of entropy production rate:
Wisdom from optimal transport

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In a non-equilibrium steady state, the entropy production rate is generally characterized by the cycle flows and the cycle forces. In steady state thermodynamics, the entropy production rate (EPR) for a non-stationary state can be decomposed into two contributions, namely the excess (nonadiabatic) EPR and the housekeeping (adiabatic) EPR, where the housekeeping EPR is characterized by the cycle flows and the cycle forces in a steady state. The decomposition, originally introduced by Hatano and Sasa, is based on an existence of a unique stable steady state. Thus, a decomposition of the entropy production rate for the chemical rate equation is nontrivial because multiple and unstable steady states generally exist.

Recently, we introduced another decomposition of the EPR, namely the geometric decomposition, for the overdamped Langevin equation from the viewpoint of geometric structure in optimal transport theory [1]. Though this geometric decomposition is mathematically equivalent to the decomposition introduced by Maes and Netočnỳ for the overdamped Langevin system, we reformulated this decomposition under a somewhat different concept based on the Benamou-Brenier formula in optimal transport theory [2, 5]. In our approach, we consider cycle flows that do not affect the time evolution for a non-stationary state. These cycle flows provide an orthogonal decomposition of the EPR into the housekeeping EPR and the excess EPR geometrically, where the housekeeping EPR is defined as a contribution of the dissipation by the cycle flows.

Because this geometric decomposition is not based on the steady state, we can generalize this geometric decomposition for the chemical rate equation in several ways [3, 4]. In this talk, we would like to explain two generalizations of the geometric decomposition for the chemical rate equation (or the Markov jump process) and discuss its applications to thermodynamic trade-off relations.

References

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