Ludwig Hruza: How Free Probability appears in mesoscopic quantum systems

Probability theory is the basic mathematical framework of statistical mechanics and deals with commuting random variables. In the case where the random variables do not commute – for example large random matrices - there is an analogous theory: Free probability theory. Its philosophy is to replace the concept of “independence” by “freeness”, a concept that includes both, the notion of probabilistic independence and algebraic independence. Curiously, this free probability theory becomes relevant when studying transport in mesoscopic quantum systems. In these systems of intermediate size, particles conserve their ability to (quantum mechanically) interfere over large distances, but the microscopic interaction between particles is complex enough (i.e. non-integrable) to produce an effective noise which leads to diffusive transport.
After a purely mathematical introduction to free probability theory, concentrating on its combinatorial flavour in terms of non-crossing partitions, I will introduce a toy model that captures relevant properties of mesoscopic quantum systems, the Quantum Symmetric Simple Exclusion Process (QSSEP). I will explain why fluctuations of quantum coherences in this model (i.e. off-diagonal entries in the density matrix) can be interpreted as the so-called free cumulants in free probability theory. If time permits, I can discuss how free probability helps to find exact results for the entanglement structure in this model.

References:
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