Tim Cohen - ICTP Lectures 2023

The Standard Model of Particle Physics

Who am I?

What is the Standard Model (SM)?

All known particles and their interactions

Unpack the name:

- "Standard": It is so extraordinary, no name could do it justice $\Rightarrow$ use a dull word
- "Model": SM is an Effective Field Theory
- "Particle Physics": Quantum mechanics (states live in Hilbert space + Symmetries are unitary transformations) + special relativity ($\text{locality} + E = mc^2$) $\Rightarrow$ unique framework

Quantum Field Theory $\Rightarrow$ Microscopic description in terms of "fundamental particles" described by states in Fock space (Hilbert space w/ any number of particles)

Purpose of quantum fields $\hat{\varphi}(x)$ is to make locality manifest.

- Central tension of QFT:

States transform in infinite dimensional unitary representation of Poincaré group; fields transform in finite dimensional representation.

Ex: Vector field $A^{\mu}(x)$ (four degrees of freedom) for theories of photons $|p, h=\pm\rangle$ (two dofs)

Model Building

Rules of the game

- Specify global symmetry group (+ Poincaré)
- Specify particle/field content (with charges/representations)
- Write all allowed terms in the Lagrangian $\mathcal{L}$ up to mass dimension 4.
- Introduce gauge bosons ("gauge global sym") by adding gauge boson kinetic term $\mathcal{L} \supset -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ and promote $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu$

Dimensional analysis in QFT

"Natural units"

Want to describe processes with

1) Small numbers of particles (quanta) $\Rightarrow \hbar$
2) moving very fast $\Rightarrow c$

$\Rightarrow$ System of units with $\hbar = 1$ and $c = 1$

$[\hbar]$ = Energy $\times$ time $\Rightarrow \hbar = 1 \Rightarrow$ Energy $\sim$ 1/time

$[c]$ = length/time $\Rightarrow c = 1 \Rightarrow$ length $\sim$ time

$\Rightarrow$ Energy $\sim$ 1/length (Typically use GeV $\sim m_{\text{proton}}$)

High energy $\Leftrightarrow$ short distance

"LHC is world's largest microscope"

Apply to QFT: $Z = \int \mathcal{D}\varphi \, \exp\left(i S[\varphi]/\hbar\right)$

$\Rightarrow [S] = 1$. $S = \int d^4x \, \mathcal{L} \Rightarrow [\mathcal{L}] = 4$

$\mathcal{L}_\varphi \supset \frac{1}{2}(\partial_\mu \varphi)^2 \Rightarrow [\varphi] = 1$

$\mathcal{L}_\psi \supset i\bar{\psi}\not{\partial}\psi \Rightarrow [\psi] = 3/2$

$\mathcal{L}_A \supset -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \Rightarrow [A_\mu] = 1$

$\Rightarrow \mathcal{L} \supset a\varphi^3 + \lambda\varphi^4 + \varkappa\varphi^5 \Rightarrow [a] = 1, \ [\lambda] = 0, \ [\varkappa] = -1$

Implications: Scattering cross section

$$\sigma \sim \frac{1}{E^2}\left[\left(\frac{a}{E}\right)^n, \lambda^m, \left(\varkappa E\right)^g, \cdots\right]$$

$\Rightarrow$ φ^3 important at low energy: relevant

φ^4 important at all energy: marginal

φ^5 important at high energy: irrelevant

General lesson: only sensitive to relevant and marginal interactions at low energies. These are the "renormalizable" theories.

Including irrelevant interactions $\Rightarrow$ EFTs.

Chiral fermion couplings

Dirac fermion Ψ can be decomposed into a pair of Weyl fermions ψ_L and ψ_R:

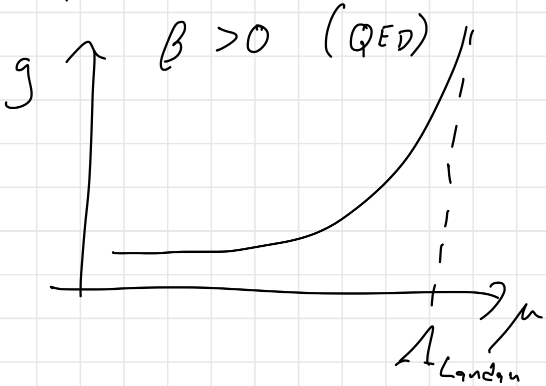
$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad \text{w/} \quad P_L \Psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad P_R \Psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$$

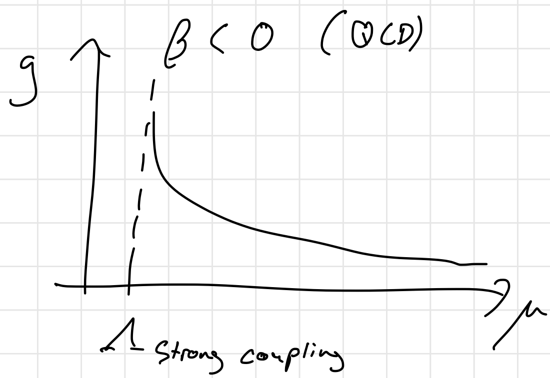
and $P_{L/R} = \frac{1}{2}\left(\mathbb{1} \mp \gamma^5\right)$

The Electroweak gauge bosons $W^{+/-}$ and Z^0 couple to ψ_L and ψ_R differently.

Running Couplings and Dimensional Transmutation

One should interpret coupling "constants" as scale dependent, where the running is determined by the renormalization group: $\mu \frac{dg}{d\mu} = \beta(g)$





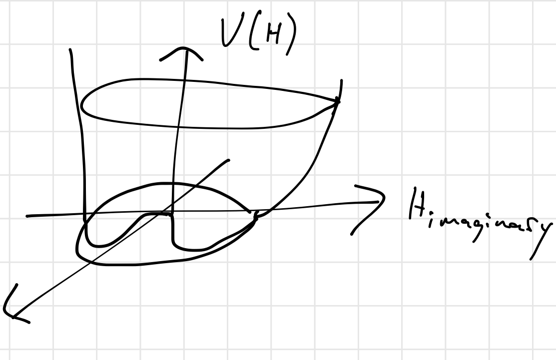
In both cases, a new dimensionful scale emerges.

Spontaneous Symmetry Breaking

Note $\mathcal{L} \supset +\mu^2 |H|^2 - \lambda |H|^4$

$$\Rightarrow V \supset -\mu^2 |H|^2 + \lambda |H|^4$$

$\Rightarrow$



Linear vs non-linear realization of GB

Specialize to complex scalar field theory w/ $U(1)=SO(2)$ symmetry

$$\mathcal{L} = |\partial_\mu \varphi|^2 + \mu^2 |\varphi|^2 - \frac{\lambda}{4}|\varphi|^4$$

has global sym: $\varphi \to e^{i\alpha}\varphi$ when $\langle \varphi \rangle = 0$

Minimize $V(\varphi) \Rightarrow |\varphi_0|^2 = \frac{2\mu^2}{\lambda}$

$\Rightarrow \infty$ family of equavelent vacuua w/ $\langle \Omega_\theta | \varphi | \Omega_\theta \rangle = \sqrt{\frac{2\mu^2}{\lambda}}\, e^{i\theta}$

Pick one w/ v real and positive: $\frac{v}{\sqrt{2}} = \sqrt{\frac{2\mu^2}{\lambda}}$ (Note v vs $v/\sqrt{2}$ convention)

- Linear realization: expand $\varphi = v + \frac{1}{\sqrt{2}}(\varphi_R + i\varphi_I)$

$$\Rightarrow V = \mu^2 \varphi_R^2 + \frac{\sqrt{\lambda \mu^2}}{2}\varphi_R(\varphi_R^2 + \varphi_I^2) + \frac{\lambda}{16}(\varphi_R^2 + \varphi_I^2)^2$$

$\Rightarrow$ Massless $\varphi_I \Rightarrow U(1)$ broken to nothing $\Rightarrow$ 1 Goldstone boson.

$\Rightarrow$ Massive φ_R w/ $m_{\varphi_R} = 2\mu$

- Non-linear realization: expand $\varphi = \frac{1}{\sqrt{2}}(v + \sigma(x))\exp(i\,\pi(x)/F)$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \left(v + \frac{1}{\sqrt{2}}\sigma(x)\right)^2 \frac{1}{F^2}(\partial_\mu \pi)^2 - \left(-\frac{v^2}{4} + \mu^2\sigma^2 + \frac{1}{2}\sqrt{\lambda}\,\mu\sigma^3 + \frac{1}{16}\lambda\sigma^4\right)$$

Setting $F = v \Rightarrow$ Canonical norm for π.

Related to linear version via field redef. Now identify

$M_\pi = 0 \Rightarrow$ Goldstone & $M_\sigma = 2\mu$ is radial mode

Note: seeming paradox with $\varphi_I\varphi_I \to \varphi_I\varphi_I$, see H/W

Non-linear realization makes manifest that $\pi \to \pi + F\theta$ is sym

This is a "shift symmetry". All Goldstones have shift sym.

This is another signature of spontaneous sym breaking

Higgs mechanism (Abelian (ie $U(1)$))

Next, gauge the $U(1)$ symmetry:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + (\partial_\mu \varphi^* - ie A_\mu \varphi)(\partial_\mu \varphi + ie A_\mu \varphi) + \mu^2|\varphi|^2 - \frac{\lambda}{4}|\varphi|^4$$

minimize $V(\varphi) \Rightarrow |\langle\varphi\rangle| = \frac{v}{\sqrt{2}} = \sqrt{\frac{2\mu^2}{\lambda}}$

$$\Rightarrow \varphi(x) = \left(\frac{v + \sigma(x)}{\sqrt{2}}\right) \exp\left(i\pi(x)/v\right) \qquad (\text{set } F = v)$$

$$\Rightarrow \mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \left(\frac{v+\sigma}{\sqrt{2}}\right)^2\left[-i\frac{\partial_\mu \pi}{v} + \frac{\partial_\mu \sigma}{v+\sigma} - ie A_\mu\right]\left[i\frac{\partial_\mu \pi}{v} + \frac{\partial_\mu \sigma}{v+\sigma} + ie A_\mu\right]$$
$$- \left(-\frac{\mu^4}{\lambda} + \mu^2\sigma^2 + \frac{1}{2}\sqrt{\lambda}\,\mu\sigma^3 + \frac{1}{16}\lambda\sigma^4\right)$$

Terms involving only A_μ: $\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2 v^2 A_\mu^2 \Rightarrow m_A = ev$

Note: $m_\sigma = 2\mu$ and $m_\pi = 0$. σ is the "Higgs boson"

Simplify things by taking $m_\sigma \to \infty$ w/ v fixed $\Rightarrow$ σ decouples from low energy physics

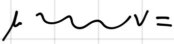
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m_A^2\left(A_\mu + \frac{1}{m_A}\partial_\mu \pi\right)^2$$

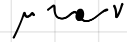
Performing gauge transform

$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x) \quad \text{and} \quad \pi(x) \to \pi(x) - v\alpha$$

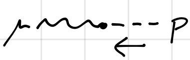
So can pick gauge where $\pi(x) = 0$ "unitary gauge"

More generally, recall that in Lorentz gauge ($\xi = 0$)

 $= \dfrac{-i}{p^2 + i\varepsilon}\left(g^{\mu\nu} - \dfrac{p^\mu p^\nu}{p^2}\right)$

If we treat $\mathcal{L} \supset -\frac{1}{2} m_A^2 A_\mu^2$ as vertex  $= i m_A^2 g^{\mu\nu}$

The π propagator as $= \dfrac{i}{p^2}$ (massless Goldstone)

and cross-term  $= i m_A(-i p_\mu)$

Then 

$$= \frac{-i}{p^2 + i\varepsilon}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + \underbrace{i m_A^2 g^{\mu\nu} + m_A p^\mu \frac{i}{p^2} m_A p^\nu}_{= i m_A^2\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)} + \cdots$$

sum all m_A dep diags

$$\overset{\downarrow}{=} \frac{-i}{p^2 - m_A^2 + i\varepsilon}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)$$

is massive prop in Lorentz gauge

This is how gauge boson eats Goldstone to become massive

Connection to superconductivity!

The SM Lagrangian

Gauge bosons:

B^μ (hypercharge), $W^{a\mu}$ (weak), $G^{a\mu}$ (gluons)

$U(1)_Y$ $\quad SU(2)_L,\ a=1,2,3 \quad SU(3)_C,\ a=1,\dots,8$

$$\mathcal{L} \supset -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \operatorname{tr} W^{\mu\nu} W_{\mu\nu} - \frac{1}{2} \operatorname{tr} G^{\mu\nu} G_{\mu\nu}$$

Higgs boson:

H^a $(a=1,2)$ $SU(2)$ doublet with hypercharge $Y_H = 1/2$

$$\mathcal{L} \supset \left| \left(\partial_\mu - i g W_\mu^a \tau^a - i g' Y_H B_\mu \right) H \right|^2 + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2$$

note sign

Fermions:

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in (\underset{SU(3)_C}{3}, \underset{SU(2)_W}{2}{}_{\underset{U(1)_Y}{1/6}})$$

$$u_R \in (\bar{3}, 1_{2/3}), \quad d_R \in (\bar{3}, 1_{-1/3})$$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \in (1, 2_{-1/2}), \quad e_R \in (1, 1_{-1})$$

$$\mathcal{L} \supset \sum_\psi \left(i \bar{\psi} \gamma^\mu P_L D_\mu \psi + i \bar{\psi} \gamma^\mu P_R D_\mu \psi \right)$$

* No right handed ν's $\Rightarrow$ massless ν

$$D_\mu = \partial_\mu - i g' Y B_\mu - i g W_\mu^a \tau^a - i g_s G_\mu^a t^a$$

depending on the representation / charge and the "Yukawa couplings"

$$\mathcal{L} \supset -y_u Q_L H^c u_R - y_d Q_L H d_R - y_e L_L H e_R$$

$$(H^c)_\alpha \equiv \varepsilon_{\alpha\beta} (H^*)^\beta \qquad \varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}$$

Really there are 3 families, so the Yukawa couplings are matrices.

Note $[g] = [g'] = [g_s] = [y_{u,d,e}] = [\lambda] = 0$

while $[\mu^2] = 2$

$\Rightarrow$ SM only has one dimensionful scale!

But this scale appears in front of $|H|^2$

$\Rightarrow$ No symmetry that is compatible with the Standard Model fields can forbid this term

$\Rightarrow$ Expect it to recieve contributions from all heavy scales. This is the hierarchy problem.

Electroweak Theory

Start with just $SU(2)$ gauge bosons

Algebra $[\tau^a, \tau^b] = i\varepsilon^{abc}\tau^c$

w/ $a = 1, 2, 3$ + $\varepsilon^{123} = 1$, $\varepsilon^{132} = -1$, ...

Generators in fundamental (doublet) rep:

$$\tau^a = \frac{\sigma^a}{2} \text{ w/ } \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

3 generators $\Rightarrow$ 3 gauge fields $W_\mu = W_\mu^a \tau^a$

Explicitly

$$W_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & W_\mu^3 \end{pmatrix}$$

$SU(2)$ gauge transformation $\Omega(x) = \exp(i\alpha^a(x)\tau^a)$

$$\Rightarrow W_\mu \to \Omega W_\mu \Omega^{-1} + i\Omega\partial_\mu\Omega^{-1}$$

Take $\alpha^1 = \alpha^2 = 0$ isolates $U(1)$ subgroup of $SU(2)$

(will be part of $U(1)_{E+M}$)

For α^3 = infinitesimal constant

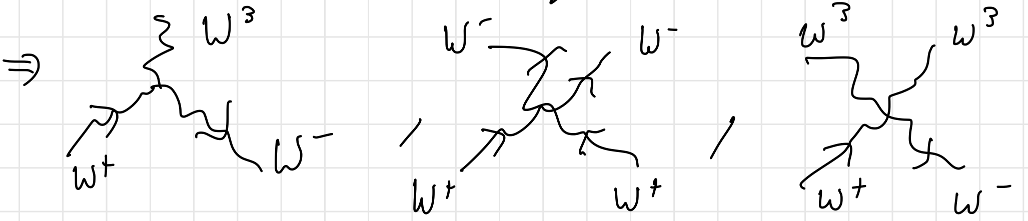
$$\Rightarrow W_\mu \to e^{i\alpha^3\tau^3} W_\mu e^{-i\alpha^3\tau^3} \simeq W_\mu + i\alpha^3[\tau^3, W_\mu]$$

$$= W_\mu + i\alpha^3 \begin{pmatrix} 0 & W_\mu^1 - iW_\mu^2 \\ -(W_\mu^1 + iW_\mu^2) & 0 \end{pmatrix}$$

So $W_\mu^1 \pm i W_\mu^2 = \sqrt{2} W_\mu^\mp$ transforms under this $U(1)$ with charge ∓ 1 while W_μ^3 is neutral

Expected since $U(1)$ generator τ^3 does not commute with other generators.

$\Rightarrow$



Now let's couple these gauge bosons to matter

Introduce Dirac fermion in fundamental rep of $SU(2)$, a "doublet":

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad w/ \; \Psi \to e^{i\alpha^a \tau^a} \Psi$$

Lagrangian w/ global $SU(2)$ symmetry:

$$\mathcal{L} = \bar{\Psi} i \partial\!\!\!/ \Psi - m \bar{\Psi} \Psi$$
$$= \bar{\psi}_1 i \partial\!\!\!/ \psi_1 - m \bar{\psi}_1 \psi_1 + \bar{\psi}_2 i \partial\!\!\!/ \psi_2 - m \bar{\psi}_2 \psi_2$$

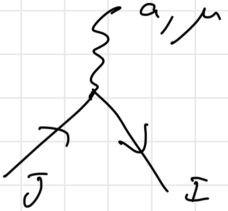
Note $m_1 = m_2 = m$ required by $SU(2)$

Introduce gauge boson interactions $\partial_\mu \to D_\mu$

w/ $D_\mu = \partial_\mu - i g W_\mu^a \tau^a$

$$\Rightarrow \mathcal{L}_{int} = g \bar{\Psi} \gamma^\mu \tau^a \Psi W_\mu^a$$

$$= g (\tau^a)_I{}^J \bar{\Psi}^I \gamma^\mu \Psi_J W_\mu^a$$

$\Rightarrow$ 

$$= i g (\tau^a)_I{}^J \gamma^\mu$$

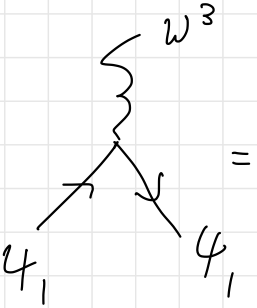
τ non-Abelian "charge"

Rewrite in terms of $W^{+/-}$ + W^3

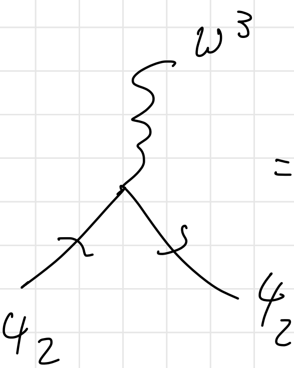
Note $\tau_3 \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} +\Psi_1 \\ -\Psi_2 \end{pmatrix}$

$$\Rightarrow \mathcal{L}_{int} = \frac{g}{2} \bar{\Psi}_1 \cancel{W}^3 \Psi_1 - \frac{g}{2} \bar{\Psi}_2 \cancel{W}^3 \Psi_2$$

$$+ \frac{g}{\sqrt{2}} \bar{\Psi}_1 \cancel{W}^+ \Psi_2 + \frac{g}{\sqrt{2}} \bar{\Psi}_2 \cancel{W}^- \Psi_1$$

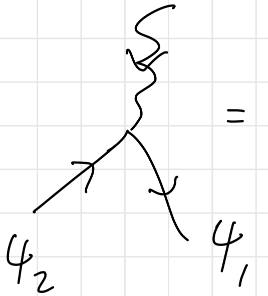
$\Rightarrow$ 

$= \frac{i}{2} g \gamma^\mu$

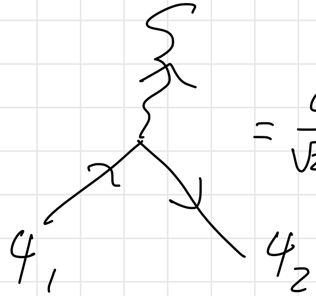


$= -\frac{i}{2} g \gamma^\mu$

These play a role in γ + Z interactions.



$$= \frac{i}{\sqrt{2}} g \gamma^\mu$$



$$= \frac{i}{\sqrt{2}} g \gamma^\mu$$

These are "charged current" interactions.

SM Higgs Mechanism

Now lets extend the model to understand the full $\underset{\text{weak}}{SU(2)_L} \times \underset{\text{Hypercharge}}{U(1)_Y} \to U(1)_{EM}$ structure

Focus on bosonic sector: 3 W_μ^a gauge bosons, one B_μ gauge boson, an $SU(2)$ doublet with $Y = 1/2$ Higgs field $H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$.

Assume Higgs potential takes wine bottle shape

$$V = \lambda\left(|H|^2 - \frac{\mu^2}{2\lambda}\right)^2 \Rightarrow \sqrt{|\langle H_1 \rangle|^2 + |\langle H_2 \rangle|^2} = \frac{\mu}{\sqrt{2\lambda}} = \frac{v}{\sqrt{2}} \neq 0$$

Choose to expand theory around vacuum

$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

Note that the $U(1) \subset SU(2)$ acts non-trivially on $\langle H \rangle$ $\left(\tau^3 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \right)$

However, the combination $Q = \underset{\text{eigenvalue of } \tau^3}{T^3} + \underset{\text{Hypercharge}}{Y}$ gives

$$Q \langle H \rangle = \begin{pmatrix} 1/2 + 1/2 & 0 \\ 0 & 1/2 - 1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

$\Rightarrow$ Identify Q with electric charge such that vacuum is neutral. This corresponds to the unbroken $U(1)_{EM}$

Also, note $Y = 0$ for W^μ, so we have

$Q(W^{+/-}) = \pm 1$ and $Q(W^3) = 0$

The $U(1)_{EM}$ gauge transformation is

$$\Omega_Q = \exp\left(i\alpha^3 \tau^3 + i\,\alpha_Y/2\, \mathbb{1}\right) = \exp(i\alpha_Q Q)$$

$\Rightarrow W_\mu^3 \to W_\mu^3 + \partial_\mu \alpha_Q$ and $B_\mu \to B_\mu + \partial_\mu \alpha_Q$

Therefore, the combination

$Z_\mu = \underset{\text{some constant}}{c}\left(W_\mu^3 - B_\mu\right)$ does not transform under $U(1)_Q$

$\Rightarrow Z_\mu Z^\mu$ is allowed $\Rightarrow$ massive neutral gauge boson

The orthogonal linear combination transforms non-trivially $\Rightarrow$ must be massless.

We can verify this by brute force (in unitary gauge s.t. Goldstone bosons are eaten)

$$H(x) = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$$

$h(x)$ is real scalar field, the "Higgs boson".

The covariant derivative in unitary gauge is

$$D_\mu H = \partial_\mu H - i\left(W_\mu + \tfrac{1}{2} B_\mu \mathbb{1}\right) H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \partial_\mu h \end{pmatrix} - i \frac{v+h}{\sqrt{2}} \begin{pmatrix} \sqrt{2} W^+ \\ B_\mu - W^3_\mu \end{pmatrix}$$

$\frac{1}{c} Z_\mu$

$\Rightarrow \mathcal{L} \supset |D_\mu H|^2 \Rightarrow$ mass for $W^{+/-}$ and Z.

Note that these expressions are valid for

$$\mathcal{L} \supset -\frac{1}{2g^2} \mathrm{Tr}\left[W_{\mu\nu} W^{\mu\nu}\right] - \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu}$$

To determine $W + Z$ mass, need to work with canonical normalization $\Rightarrow$

rescale $W \to gW$ and $B \to g'B$

$\Rightarrow Z = c(gW^3 - g'B)$ and $\mathcal{L} \supset -\frac{1}{4}\left[(W^3_{\mu\nu})^2 + (B_{\mu\nu})^2\right]$

Normalize Z so that it is a rotation in W^3, B space so that it preserves the normalization

$$\Rightarrow Z = \cos\theta_W W^3 - \sin\theta_W B = \frac{1}{\sqrt{g^2+g'^2}}\left(gW^3 - g'B\right)$$

$$\Rightarrow \cos\theta_W = c_W = \frac{g}{\sqrt{g^2+g'^2}} \quad \text{and} \quad \sin\theta_W = s_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

θ_W is "weak mixing angle" (or Weinberg angle)

The orthogonal combination is the photon:

$$A_\mu = c_W B_\mu + s_W W^3_\mu$$

We can also relate the $U(1)_Q$ coupling to g & g' since w/ canonical norm, we have

$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$ under $U(1)_Q$ gauge trans.

and for $U(1)_W$ and $U(1)_Y$ we have

$$W^3_\mu \to W^3_\mu + \frac{1}{g}\partial_\mu \alpha \quad \& \quad B_\mu \to B_\mu + \frac{1}{g'}\partial_\mu \alpha$$

$$\Rightarrow A_\mu = c_w B_\mu + s_w W^3_\mu \to A_\mu + \left(\frac{c_w}{g'} + \frac{s_w}{g}\right)\partial_\mu \alpha$$

$$\Rightarrow \frac{1}{e} = \frac{1}{\sqrt{g^2+g'^2}}\left(\frac{g}{g'} + \frac{g'}{g}\right) = \frac{\sqrt{g^2+g'^2}}{gg'} = \sqrt{\frac{1}{g^2}+\frac{1}{g'^2}}$$

$$\Rightarrow e = \frac{gg'}{\sqrt{g^2+g'^2}}$$

Now we can compute the masses and interactions for the bosonic electroweak sector:

Note $D_\mu H = \begin{pmatrix} 0 \\ \frac{\partial_\mu h}{\sqrt{2}} \end{pmatrix} - i\frac{v+h}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\, g\, W^+_\mu \\ \sqrt{g^2+g'^2}\, Z_\mu \end{pmatrix}$

$\Rightarrow$ Higgs kinetic terms $\Rightarrow$

$$|D_\mu H|^2 = \frac{1}{2}(\partial_\mu h)^2 + \frac{(v+h)^2}{8}\left[2g^2|W|^2 + (g^2+g'^2)Z^2\right]$$

$$\Rightarrow m_W = \frac{1}{2}gv \quad \& \quad m_Z = \frac{1}{2}\sqrt{g^2+g'^2}\,v = \frac{1}{2}\frac{g}{c_w}v = \frac{m_W}{c_w}$$

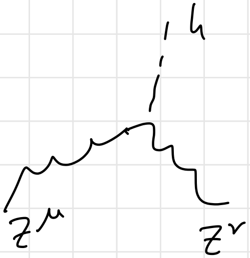
Note, the combination

$$\rho \equiv \frac{m_W^2}{m_Z^2 c_W^2} \underset{\text{tree-level}}{=} 1$$

This is an important precision observable for testing the SM (sensitive to loop effects and BSM physics).

The Higgs kinetic term also gives us interactions:

$$= \frac{i}{2} g^2 v \eta_{\mu\nu} = 2i \frac{m_W^2}{v} \eta_{\mu\nu}$$



$$= \frac{i}{2} (g^2 + g'^2) v \eta_{\mu\nu} = 2i \frac{m_Z^2}{v} \eta_{\mu\nu}$$

$$= \frac{i}{2} g^2 \eta_{\mu\nu}$$

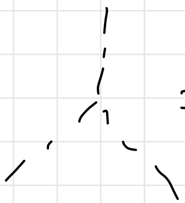
$$= \frac{i}{2} \frac{g^2}{c_W^2} \eta_{\mu\nu}$$

The Higgs potential gives

$$U(H) \overset{\substack{\text{unitary}\\ \text{gauge}}}{=} \frac{1}{2}(2\lambda v^2)\,h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$$

$$\Rightarrow \; m_h = \sqrt{2\lambda}\, v = \sqrt{2}\,\mu$$

and we have Higgs self interactions



$= -6i\lambda v$

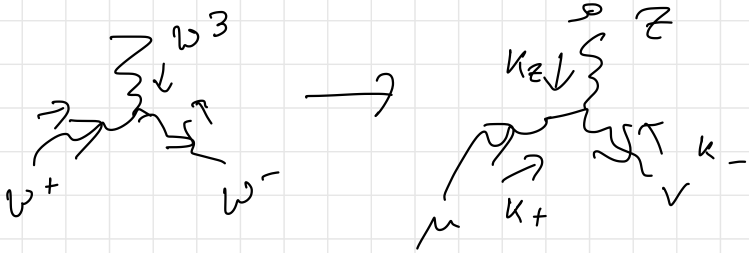
$= -6i\lambda$

We also have gauge boson self interactions:

All gauge self interactions come from $SU(2)$ structure $\Rightarrow$ we can use the calculations for pure $SU(2)$, with the replacement

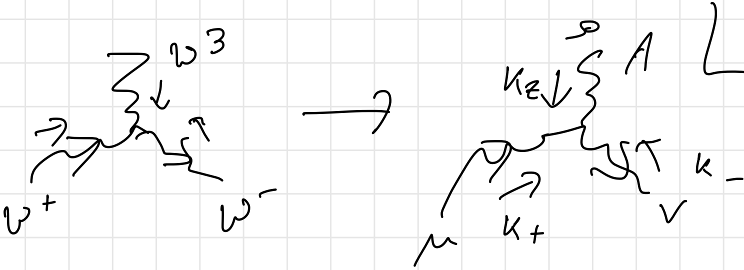
$$W^3 = c_w Z + s_w A \qquad (\text{also } B = c_w A - s_w Z)$$

where A is the photon. $\Rightarrow$



$$= -ig c_w\left[(k_+ - k_-)_\rho\, \eta_{\mu\nu} + (k_- - k_Z)_\mu\, \eta_{\nu\rho} + (k_Z - k_+)_\nu\, \eta_{\mu\rho}\right]$$

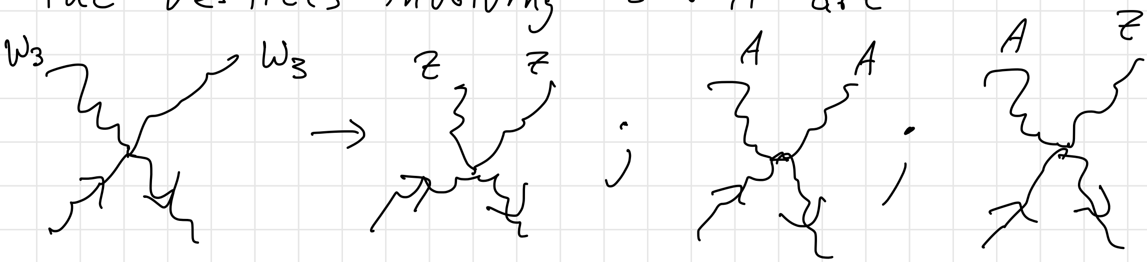
Also, we have:



$= -ig\, s_W [\dots]$

Similarly, the pure $W^{+/-}$ vertices are unchanged

The vertices involving Z & A are



$\sim g^2 c_W^2$; $\sim g^2 s_W^2$; $\sim g\, s_W c_W$

<u>Parameters of bosonic sector of SM</u>

We see that we have 4 parameters:

$$g, g', \mu, \lambda$$

Historically, what was actually measured first were

- e from electromagnitism / QED
- v (really G_F) from weak decays (μ decay)
- s_W from ν-e scattering and Z-pole
- λ Higgs mass

Summary for bosonic sector

$$m_W = \frac{1}{2} g v = c_W m_Z$$

$$m_h = \sqrt{2\lambda}\, v$$

$$s_W = \frac{g'}{\sqrt{g^2+g'^2}} \qquad c_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$e = \frac{g g'}{\sqrt{g^2+g'^2}} = g s_W = g' c_W$$

Measured values:

$m_W \simeq 80$ GeV, $m_Z = 91$ GeV, $m_h = 125$ GeV

$v \simeq 246$ GeV, $s_W^2 \simeq 0.23$, $g \simeq 0.65$,

$g' \simeq 0.35$, $\lambda \simeq 0.13$, $\mu \simeq 88$ GeV

Quarks and Leptons

Now we need to introduce the spin ½ matter fields. There are two types: the quarks (charged under $SU(3)$) and the leptons (neutral under $SU(3)$). There are three copies of each, which are called the "families".

In more detail, a family of <u>leptons</u> consists of 3 Weyl spinors. We will use Dirac notation, $\psi_{L,R}$ with the property

$$\gamma_5 \psi_L = -\psi_L \quad , \quad \gamma_5 \psi_R = +\psi_R$$

The 3 fields are l_L, ν_L, l_R

It is possible that ν_R also exists, but we do not know for sure so we do not typically include it in the SM.

In other words, neutrinos are massless in the SM. Notice we treat left and right differently.

We combine the left handed fields into the "lepton doublet" $L_L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}$

which transforms in the fundamental rep of $SU(2)$. We want l_L to be the electron, so we should choose its hypercharge so that $Q(l_L) = -1$. Using $Q = T^3 + Y$ and $T^3 = -\frac{1}{2}$ for l_L we need $Y(l_L) = -1/2$

This implies $Q(\nu) = \frac{1}{2} - \frac{1}{2} = 0$ so the neutrinos are neutral.

We also need ℓ_R to have $Q(\ell_R) = -1$. Since it is an $SU(2)$ singlet, we have $T_3(\ell_R) = 0 \Rightarrow Y(\ell_R) = -1$

$$\Rightarrow L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \in \underset{\uparrow SU(2)}{2}{}_{\underset{\nwarrow U(1)_Y}{-1/2}} \quad \& \quad \ell_R \in \mathbb{1}_{-1}$$

When we write Dirac spinors, $\ell = \ell_L + \ell_R$, we also write $\nu = \nu_L + \nu_R$. But ν_R does not couple to anything so it plays no physical role.

Quarks

One family of quarks is made up of

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in 2_{1/6} \ \& \ u_R \in \mathbb{1}_{2/3} \ \& \ d_R \in \mathbb{1}_{-1/3}$$

(neglecting $SU(3)$)

Families

The families have identical interaction structure.

Only difference is the masses and mixing effects (we will discuss this later when we introduce the CKM matrix).

The 3 families are:

		I		II		III	
leptons	ν_e	(0)	ν_μ	(0)	ν_τ	(0)
	e	$(511\,keV)$	μ	$(106\,MeV)$	τ	$(1.8\,GeV)$
quarks	u	$(2.4\,MeV)$	c	$(1.3\,GeV)$	t	$(172\,GeV)$
	d	$(4.8\,MeV)$	s	$(104\,MeV)$	b	$(4.2\,GeV)$

Let's work out their couplings to the $W^{+/-}$, Z, A:

Writing
$$W_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}\,W_\mu^+ \\ \sqrt{2}\,W_\mu^- & -W_\mu^3 \end{pmatrix}$$

Then for a generic doublet D_L we have

$$D_\mu D_L = \left(\partial_\mu \mathbb{1} - ig\,W_\mu - ig'\,Y(D_L)\,B_\mu\,\mathbb{1}\right) D_L$$

For a generic singlet S_R we have

$$D_\mu S_R = \left(\partial_\mu - ig'\,Y(S_R)\,B_\mu\right) S_R$$

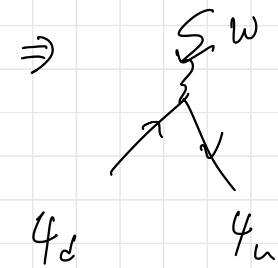
This gives "charged current" interactions for $D_L = \begin{pmatrix} \psi_{u,L} \\ \psi_{d,L} \end{pmatrix}$

$$\mathcal{L} \supset \frac{1}{\sqrt{2}} g \bar{\psi}_{u,L} \gamma^\mu W_\mu^+ \psi_{d,L} + \frac{1}{\sqrt{2}} g \bar{\psi}_{d,L} \gamma^\mu W_\mu^- \psi_{u,L}$$

If we want to express this using $\psi = \psi_L + \psi_R$ we use $\psi_{L/R} = \frac{1 \mp \gamma^5}{2} \psi$

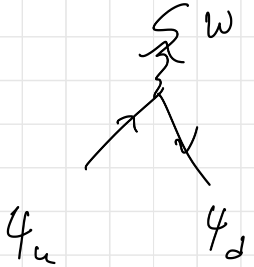
$$\Rightarrow \mathcal{L} = \frac{g}{2\sqrt{2}} \bar{\psi}_u \gamma^\mu (1-\gamma^5) W_\mu^+ \psi_d + \frac{g}{2\sqrt{2}} \bar{\psi}_d \gamma^\mu (1-\gamma^5) W_\mu^- \psi_u$$

$\Rightarrow$



$$= i \frac{g}{2\sqrt{2}} \gamma^\mu (1-\gamma^5)$$

l or d ν or u



$$= i \frac{g}{2\sqrt{2}} \gamma^\mu (1-\gamma^5)$$

Note the universality of the couplings.

We can write this in terms of a "charged current"

$$J_\mu^+ \equiv \sum_{\text{doublets}}' \bar{\psi}_d \gamma_\mu (1-\gamma^5)\psi_u$$

$$= \bar{l}\gamma_\mu(1-\gamma^5)\nu + \bar{d}\gamma_\mu(1-\gamma^5)u + \text{other families}$$

and $J_\mu^- = (J_\mu^+)^\dagger$

$$\Rightarrow \mathcal{L} \supset \frac{g}{2\sqrt{2}} W_\mu^+ J^{-\mu} + \frac{g}{2\sqrt{2}} W_\mu^- J^{+\mu}$$

"Neutral currents" $(Z + A)$ follow from the same logic.

$$\mathcal{L}_{D_L} \supset \sum_{l=u,d} \bar{\psi}_{l,L}\gamma^\mu\left(g T^3(l) W_\mu^3 + g' Y B_\mu\right)\psi_{l,L}$$

Then using $Q = T^3 + Y$

$$\Rightarrow \mathcal{L}_{D_L} \supset \sum_l' \bar{\psi}_{l,L}\gamma^\mu\left[(g W^3 - g' B) T^3(l) + g' Q(l) B_\mu\right]\psi_{l,L}$$

Plug in $W^3 = c_W Z + s_W A$ + $B = c_W A - s_W Z$

$$\Rightarrow \mathcal{L}_{D_L} \supset \sum_l' e\, Q(l)\, \bar{\psi}_{L,l}\gamma^\mu A_\mu \psi_{l,L} = \mathcal{L}_{QED}$$

For the Z, we have

$$\mathcal{L}_{D_L} \supset \frac{g}{c_W} \sum_{\ell} \bar{\psi}_{\ell,L} \gamma^\mu Z_\mu \left[T^3(\ell) - s_W^2 Q(\ell) \right] \psi_{\ell,L}$$

For right handed fields, we have

$$\mathcal{L}_{S_R} = g' B_\mu Y(\psi_R) \bar{\psi}_R \gamma^\mu \psi_R$$

$$= e Q(\psi_R) \bar{\psi}_R \gamma^\mu A_\mu \psi_R - \frac{g}{c_W} s_W^2 Q(\psi_R) \bar{\psi}_R \gamma^\mu Z_\mu \psi_R$$

$\Rightarrow$ Again we get QED.

We can write the coupling to the Z in terms of a neutral current

$$J^0_\mu \equiv \sum_{\substack{\text{all} \\ \text{fermions}}} \bar{\psi}_i \gamma^\mu \left(g_V(i) - g_A(i) \gamma^5 \right) \psi_i$$

where $\mathcal{L} = \frac{g}{c_W} Z_\mu J^{0\mu}$ and we have defined "vector" and "axial" couplings

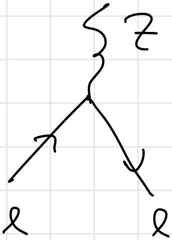
$$g_V(i) = \frac{1}{2} \left[T^3(i) - 2 s_W^2 Q(i) \right]$$

$$g_A(i) = \frac{1}{2} \left[T^3(i) \right]$$

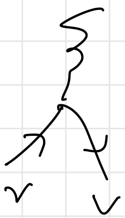
where T^3 denotes the T^3 quantum number for the left handed component of the Dirac field.

Feynman rules:

$= ie Q \gamma^\mu$



$= i \frac{g}{4c_w} \gamma^\mu \left[(-1 + 4 s_w^2) - \gamma^5 \right]$



$= i \frac{g}{4c_w} \gamma^\mu (1 - \gamma^5)$

w/ similar rules for the quarks.

Fermion Masses and Mixings

There is one more set of terms we can write in the Lagrangian with $D \leq 4$, The "Yukawa couplings": $\mathcal{L} \supset y_u H Q u + y_d H Q d + y_e H L e$

Let's understand their structure and implications in detail.

Note: $SU(3)_c$ forbids any Yukawa coupling between quarks and leptons.

Lepton Yukawas

Must be $SU(2)_L \times U(1)_Y$ invariant

Recall $L_L \in 2_{-1/2}$, $e_R \in 1_{-1}$, $H \in 2_{1/2}$

Lorentz invariance for spinors requires $L\bar{e}$

Note $Y[L\bar{e}] = 1/2 \Rightarrow$ Must contract w/ H:

$$(H^*)^\alpha L_{L,\alpha} = H^* \cdot L$$

$$\Rightarrow \mathcal{L} \supset -y_e \bar{e}_R H^* \cdot L_L - y_e^* \bar{L}_L \cdot H e_R$$

Plugging in the Higgs vev in unitary gauge, we have

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \qquad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\Rightarrow \mathcal{L} \supset -\frac{y_e}{\sqrt{2}}(v+h)\bar{\ell}_R \ell_L - \frac{y_e^*}{\sqrt{2}}(v+h)\bar{\ell}_L \ell_R$$

$$\underset{\uparrow}{=} \frac{y_e}{\sqrt{2}} v \bar{\ell}\ell - \frac{y_e}{\sqrt{2}} h \bar{\ell}\ell$$

rephase ℓ_L or ℓ_R to make y_e real

$\Rightarrow$ Charged lepton has a Dirac mass $m_e = \frac{y_e v}{\sqrt{2}}$

and there is a new Feynman vertex:

$$= -i\frac{y_e}{\sqrt{2}} = -i\frac{m_e}{v}.$$

*The neutrino stays massless.

From the masses of the leptons, we can determine

$$y_e = \frac{\sqrt{2} m_e}{v} = \left(\underset{e}{3 \times 10^{-6}},\ \underset{\mu}{6 \times 10^{-4}},\ \underset{\tau}{10^{-2}} \right)$$

$\Rightarrow$ Couplings between Higgs boson and leptons is tiny and typically can be ignored.

Quark Yukawas

To write the up-type Yukawas, we will need

$$H^c_\alpha \equiv \varepsilon_{\alpha\beta} (H^*)^\beta = \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} h_u^* \\ h_d^* \end{pmatrix}$$

This is a doublet of $SU(2)$ w/ hypercharge $-1/2$

It transforms like a doublet because $\varepsilon_{\alpha\beta}$ "lowers" $SU(2)$ indices. We can check explicitly.

Using $\sigma_2 \tau^{a*} \sigma_2 = -\tau^a \Rightarrow \sigma_2 e^{-i\alpha^a \tau^{*a}} \sigma_2 = e^{i\alpha^a \tau^a}$

$$\Rightarrow H^c \equiv i\sigma_2 H^* \xrightarrow{SU(2)} i\sigma_2 e^{-i\alpha^a \tau^{a*}} H^* = e^{i\alpha^a \tau^a} \underbrace{i\sigma_2 H^*}_{= H^c}$$

Then the quark Yukawas are

$$\mathcal{L} \supset -y_d \bar{Q}_L \cdot H d_R + h.c. \quad \left(Y = -\tfrac{1}{6} + \tfrac{1}{2} - \tfrac{1}{3} = 0 \right)$$

$$- y_u \bar{Q}_L \cdot H^c u_R + h.c. \quad \left(Y = -\tfrac{1}{6} - \tfrac{1}{2} + \tfrac{2}{3} = 0 \right)$$

In Unitary gauge, $H^c = \begin{pmatrix} \frac{v+h}{\sqrt{2}} \\ 0 \end{pmatrix} \Rightarrow$ gives mass to up-type quarks

$$\Rightarrow \mathcal{L} \supset -\frac{y_d}{\sqrt{2}} v \bar{d} d - \frac{y_d}{\sqrt{2}} h \bar{d} d - \frac{y_u}{\sqrt{2}} v \bar{u} u - \frac{y_u}{\sqrt{2}} h \bar{u} u$$

$$m_{u/d} = \frac{y_{u/d}}{\sqrt{2}} v$$

$$= -i \frac{y_d}{\sqrt{2}} = -i \frac{m_d}{v}$$

Yukawas are small except for the top quark:

$$m_t \simeq 173 \text{ GeV} \Rightarrow y_t \simeq 1$$

This is the largest coupling in the SM (above the QCD confinment scale)

Impact of family structure

This was correct for a 1 family model.

But the SM has 3 families $\Rightarrow$ can have mixing between the different families.

The point is that we are allowed to do field redefinitions without changing any of the physical observables.

In this case, the field redefinitons are rotations among the families and rephasing of the fields. We will see that we can remove many of these mixing parameters using such field redefinitions.

3 families of leptons

First we will show that the mixing parameters have no physical impact in the lepton sector (as long as the neutrinos are massless).

Take the most general lepton Yukawa matrix:

$$\mathcal{L} \supset -(Y_e)_i{}^j \, \bar{L}_L^i H e_{R,j} - (Y_e^\dagger)^i{}_j \, \bar{e}^j H^\dagger \cdot L_{L,i}$$

w/ $i = 1, 2, 3$

It seems we now have 9 complex parameters to specify. We will now show that we can absorb all but 3 real parameters into field redefs.

Note that the kinetic term and gauge interactions are diagonal:

$$\mathcal{L} \supset \sum_{i=1}^{3} \bar{L}_L^{\,i}\, i\not{D} L_{L,i} + \sum_{j=1}^{3} \bar{e}_R^{\,j}\, i\not{D}\, e_{R,j}$$

$\Rightarrow$ There is a $U(3)_L \times U(3)_e$ global symmetry group

$$L_{L,i} \to (V_L)_i^{\ j} L_{L,j} \qquad L_L \in (3,1)$$

$$\bar{L}_L^{\,i} \to (V_L^\dagger)^i{}_j\, \bar{L}_L^{\,j} \qquad \bar{L}_L \in (\bar{3},1)$$

$$e_{R,i} \to (V_e)_i^{\ j}\, e_{R,j} \qquad e_R \in (1,3)$$

$$\bar{e}_R^{\,i} \to (V_e^\dagger)^i{}_j\, \bar{e}_R^{\,j} \qquad \bar{e}_R \in (1,\bar{3})$$

where $V^\dagger V = 1$ (the V's are unitary)

Performing this transformation does not change the kinetic and gauge terms. It will change the Yukawa terms. We can always write

$$Y_e = U_L\, Y_e^D\, U_R^\dagger \quad \text{w/} \quad U_{L,R}^\dagger U_{L,R} = 1$$

by doing a Singular Value Decomposition.

w/

$$Y_e^D = \begin{pmatrix} Y_e & & 0 \\ & Y_\mu & \\ 0 & & Y_\tau \end{pmatrix}$$

is diagonal and has positive eigenvalues

$\Rightarrow$ We can write (supressing all the indices)

$$\mathcal{L} \supset -\bar{\vec{L}}_L H\, U_L\, Y_e^D\, U_e\, \vec{e}_R + h.c.$$

Then we can perform a $U(3)_L \times U(3)_e$ transformation $V_{L,e} = U_{L,R}$

$$\Rightarrow \mathcal{L} \supset -\bar{\vec{L}}\, H\, Y_e^D\, \vec{e}_R + h.c.$$

This has profound implications. It tells us that the most general $\mathcal{L}$ with operators $d \leq 4$ does not allow lepton flavor transitions.

There is an exact $U(1)$ symmetry left over after rotating to the diagonal basis:

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \to e^{i\alpha_\ell} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\bar{e}_R \to e^{-i\alpha_\ell}\, \bar{e}_R \qquad \Rightarrow U(1)_\ell \quad \text{"lepton number"}$$

There is one for each family $\Rightarrow$ 3 conserved charges "electron number", "muon number" and "tau number".

These are "accidental symmetries" of the SM.

They can be violated by $d > 4$ operators.

So processes like $\mu \to e\gamma$, $\mu \to eee$, ...
are forbidden in the SM.

Quark masses and mixings

Now we have

$$\mathcal{L} \supset \sum_{i=1}^{3} \overline{Q}_L^{\,i}\, i\not{D}\, Q_{L,i} + \sum_{j=1}^{3} \overline{u}_R^{\,j}\, i\not{D}\, u_{R,j} + \sum_{k=1}^{3} \overline{d}_R^{\,k}\, i\not{D}\, d_{R,k}$$

w/ flavor group $U(3)_Q \times U(3)_u \times U(3)_d$

$\Rightarrow$ Can remove 3 unitary matrices worth of parameters in the Yukawa couplings.

The new feature is we would need 4 independent unitary matrices to diagonalize both Yukawa couplings in general, i.e., u_L and d_L are linked.

Let us try to do the same trick:

$$Y_u = U_{L,u} Y_u^D U_{R,u}^\dagger \;;\; Y_d = U_{L,d} Y_d^D U_{R,d}^\dagger$$

Set $V_Q = U_{L,u}$, $V_u = U_{R,u}$, $V_d = U_{R,d}$

$$\Rightarrow \mathcal{L} \supset -\overline{\vec{Q}}_L H^c Y_u^D \vec{u}_R - \overline{\vec{Q}}_L V_Q^\dagger U_{L,d} Y_d^D \vec{d}_R + h.c.$$

So we have the 6 real parameters

$$Y_u = \mathrm{diag}(Y_u, Y_c, Y_t)$$

$$Y_d = \mathrm{diag}(Y_d, Y_s, Y_b)$$

and a unitary matrix worth of parameters

$$V_{CKM} \equiv V_Q^\dagger \cdot U_{L,d} = U_{L,u}^\dagger \cdot U_{L,d}$$

"Cabibbo - Kobayashi - Maskawa" matrix

$\Rightarrow$ There is no exact notion of quark flavor in the SM. So the 3 would be $U(1)^3$ global symmetry is broken down to a single $U(1)_B$ "Baryon number".

The CKM matrix has another important impact: it can break CP symmetry. To see this, we need to understand the CP structure of the Yukawa couplings. We know:

$$\psi \xrightarrow{P} \gamma^0 \psi \quad , \quad H \xrightarrow{P} H \quad , \quad H^c \xrightarrow{P} H^c$$

$$\Rightarrow \bar{\psi}_1 H \left(\frac{1-\gamma^5}{2}\right) \psi_2 \xrightarrow{P} \bar{\psi}_1 H \left(\frac{1+\gamma^5}{2}\right) \psi_2 \qquad \left(\begin{matrix}\text{used} \\ \{\gamma^\mu, \gamma^5\} = 0\end{matrix}\right)$$

Under C: $\psi \to i\gamma^2\gamma^0 \bar{\psi}^t$; $H \to H^*$

$$\Rightarrow \bar{\psi}_1 H \left(\frac{1+\gamma^5}{2}\right)\psi_2 \xrightarrow{CP} \psi_1{}^t H^* (i\gamma^2\gamma^0)^2 \left(\frac{1-\gamma^5}{2}\right)\bar{\psi}_2{}^t$$

$$\left[(i\gamma^2\gamma^0)^2 = \mathbb{1}\right] \qquad = -\psi_1{}^t H^* \left(\frac{1-\gamma^5}{2}\right)\bar{\psi}_2{}^t$$

$$\text{anti-commute} \quad = +\bar{\psi}_2 H^* \left(\frac{1-\gamma^5}{2}\right)\psi_1$$

$$\left[(\bar{\psi}_1\gamma^5\psi_2)^\dagger = -\bar{\psi}_2\gamma^5\psi_1\right] \quad = \left(\bar{\psi}_1 H \left(\frac{1+\gamma^5}{2}\right)\psi_2\right)^\dagger$$

$\Rightarrow$ Yukawa operator $\mathcal{O}_Y$ transforms to its conjugate under CP.

$$\Rightarrow \mathcal{L}_Y = Y\mathcal{O}_Y + Y^*\mathcal{O}_Y^\dagger \xrightarrow{CP} Y\mathcal{O}_Y^\dagger + Y^*\mathcal{O}_Y$$

$$= \mathcal{L}_Y \quad \text{iff} \quad Y = Y^*$$

So complex Yukawa couplings break CP.

In our basis above, we diagonalized the up-type Yukawas using SVD. So the up-type sector preserves CP.

The down-type sector depends on V_{CKM}.

Since Y_d^D is real, the down-type sector preserves CP if $V_{CKM}^* = V_{CKM}$.

Since V_{CKM} is unitary, it in principle has many complex entries. But we did not exhaust our field redefinition freedom yet, i.e., some of the V_{CKM} parameters are unphysical.

Our Lagrangian is invariant under 3 $U(1)$ phase rotations in the up-sector (just like for the leptons):

$$Q_L^j \to e^{i\alpha_j} Q_L^j \; ; \; u_R^j \to e^{i\alpha_j} u_R^j \quad j=1,2,3$$

We can also rephase all the d_R's:

$$d_R^j \to e^{i\beta_j} d_R^j \quad w/ \; j=1,2,3$$

The down-type Yukawas break all 6 of these $U(1)$ global symmetries down to $U(1)$ Baryon number: $\alpha_j = \beta_j \equiv \alpha_B \Rightarrow$ There are 5 phase rotations we can do to remove phases in V_{CKM}:

$$\mathcal{L} \supset - \bar{\vec{Q}}_L H \cdot (V_{CKM} \cdot Y_d^D) \vec{d}_R + h.c.$$

both diagonal so they commute

$$\to - \bar{\vec{Q}}_L H \cdot \left(e^{-i\,\mathrm{diag}(\vec{\alpha})} V_{CKM} \, e^{i\,\mathrm{diag}(\vec{\beta})} Y_d^D\right) \vec{d}_R + h.c.$$

So we can redefine

$$V_{CKM} \to e^{-i\,\mathrm{diag}(\vec{\alpha})} V_{CKM}\, e^{i\,\mathrm{diag}(\vec{\beta})}$$

Taking $\alpha_i = \beta_i = \alpha_B$ has no impact, so this is how we remove 5 phases.

Let us count parameters:

We know that an $N \times N$ unitary matrix has N^2 real parameters. Of these,

$N(N-1)/2$ are angles and

$N(N+1)/2$ are phases.

- Assume $N=1$ generations $\Rightarrow$ 1 phase

Can absorb this phase w/ chiral phase rotation $\alpha = -\beta$ (confirms what we already knew for $N=1$ model). $\alpha = \beta$ rotation does nothing.

- Assume $N=2$ generations $\Rightarrow$ 1 angle + 3 phases.

Now we have 2 α's and 2 β's that can absorb phases, but $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 =$ baryon # has no impact $\Rightarrow$ remove 3 phases $\Rightarrow$ no $\cancel{CP}$ and one angle.

This is the "Cabibbo Angle"

$$\Rightarrow V_{CKM}^{(2)} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

w/ $\sin\theta_c \simeq 0.23$

- Assume $N=3$ generations $\Rightarrow$ 3 angles and 6 phases. Can use 5 of 6 phases $\vec{\alpha} + \vec{\beta}$ to remove 3 phases. $\Rightarrow V_{CKM}$ specified by 3 angles and 1 CP phase.

Now that we know the physical content of the CKM matrix, we can study how it enters the Feynman rules. There are two basis choices that are typically discussed. The first is the "gauge eigenstate" basis, where the couplings to the W boson are diagonal in flavor space, the up-type quark masses are diagonal, but the down-type Yukawas take the form (in unitary gauge):

$$\mathcal{L} \supset -\vec{\bar{Q}}_L H \cdot V_{CKM} \cdot Y_d^D \vec{d}_R + h.c.$$

$$= -\frac{v+h}{\sqrt{2}} \vec{\bar{d}}_L \cdot V_{CKM} \cdot Y_d^D \vec{d}_R + h.c.$$

$\Rightarrow$ non-diagonal mass matrix, so mixing effects must be included in propagators.

The other basis is the "mass eigenstate" basis, where we diagonalize the down-type Yukawa couplings:

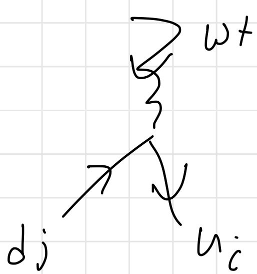
$$\begin{pmatrix} d_{L,1}^{(g)} \\ d_{L,2}^{(g)} \\ d_{L,3}^{(g)} \end{pmatrix} = V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

$(g) \leftarrow$ gauge basis

$$\Rightarrow \mathcal{L} \supset \frac{g}{\sqrt{2}} (V_{CKM})_i{}^j W_\mu^+ \bar{u}_L^i \gamma^\mu d_{L,j}$$

$$+ \frac{g}{\sqrt{2}} (V_{CKM}^*)^i{}_j W_\mu^- \bar{d}_L^j \gamma^\mu u_{L,i}$$

All other interactions are flavor diagonal.

$\Rightarrow$ 

$$= i\frac{g}{\sqrt{2}} (V_{CKM})_{ij} \gamma^\mu (1-\gamma^5)$$

+ conjugate diagram

The CKM matrix is often written as

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

So that  $\sim V_{us}$

There is a common phenomenological parametrization due to Wolfenstein:

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^2)$$

w/ $\lambda \simeq \sin\theta_c \simeq 0.23$

$A \simeq 0.8$, $\rho \simeq 0.14$, $\eta \simeq 0.35$

$\Rightarrow$ $1 \leftrightarrow 2$ transitions most likely, $2 \leftrightarrow 3$ somewhat likely, $1 \leftrightarrow 3$ transitions least likely

Since $\cancel{CP}$ requires 3-families, we can put the phase in 1-3 entry $\Rightarrow$ tiny effect.

It is often a good approx to take

$$V_{CKM} \simeq \begin{pmatrix} V_{Cabibbo} & 0 \\ 0 & 1 \end{pmatrix}$$

Intro to Effective Field Theory (EFT)

TASI
1903.03622

We are familiar with the idea that one does not need to know the detailed microscopic properties of a system to accurately model it.

e.g. Friction, Thermodynamics, ...

QM e.g. Hydrogen: treat proton as pointlike is good approx since $r_B = 10^{-7}$ cm while $r_p \sim 10^{-13}$ cm

ratio of scales $r_p / r_B \sim 10^{-6}$

EFT makes these types of approximations systemati

We will use toy scalar theory to illustrate how to generate sys expansion E/M

E = low energy of experiments + M = heavy mass scale

Termanology

relevant couplings: positive mass dim $(m^2 \phi^2)$

marginal couplings: zero mass dim $(\lambda \phi^4)$

irrelevant couplings: negative mass dim $\left(\frac{1}{M^2} \phi^6\right)$

fundamental theory: UV

Effective theory: IR

Two real scalar fields φ + η w/ $m_\varphi \ll m_\eta$

Want effective description for $E \ll m_\eta$

Impose a Z_2 symmetry $\varphi \to -\varphi$ + $\eta \to +\eta$

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m_\varphi^2\varphi^2 + \frac{1}{2}(\partial\eta)^2 - \frac{1}{2}m_\eta^2\eta^2 - V$$

w/ $V = \frac{\lambda}{4!}\varphi^4 + \frac{g}{2}\varphi^2\eta + \frac{g'}{3!}\eta^3 + \frac{\lambda'}{4}\varphi^2\eta^2 + \frac{\lambda''}{4!}\eta^4$

Path integral to compute φ correlation functions:

$$Z[J] = \int \mathcal{D}\varphi\, \mathcal{D}\eta\, \exp\left[i S[\varphi,\eta] + \int J\varphi\right]$$

Note only include source for φ, since we do not have energy to produce η

Path integral for EFT

$$Z_{eff}[J] = \int \mathcal{D}\varphi\, \exp\left[i S_{eff}[\varphi] + \int J\varphi\right]$$

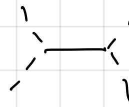
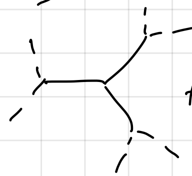
with $\exp\left[i S_{eff}[\varphi]\right] = \int \mathcal{D}\eta\, e^{iS[\varphi,\eta]}$

This could be useful <u>if</u> S_{eff} is "local"

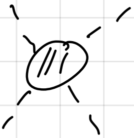
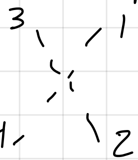
i.e., $\mathcal{L}_{eff}$ is polynomial in fields and derivaties of fields

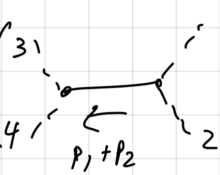
Call procedure for deriving S_{eff} "integrating out" the heavy field η.

Let's compute a few terms

$\Rightarrow i S_{eff}[\varphi] = i S'_{\varphi} +$  $+$  $+ \ldots$

Focus on 4-point:

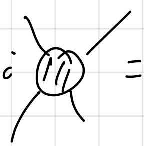
 $=$ 

$+ \Big($ 

$+ \; t + u \Big)$

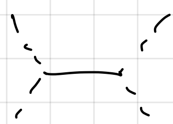
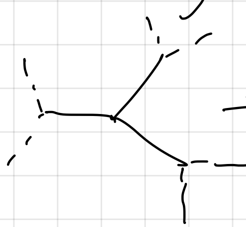
$$= -i\lambda - i g^2 \left[\frac{1}{s - m_\eta^2} + \frac{1}{t - m_\eta^2} + \frac{1}{u + m_\eta^2} \right]$$

Assume $E \ll m_\eta$: $\dfrac{1}{p^2 - m_\eta^2 + i\varepsilon} = -\dfrac{1}{m_\eta^2} - \dfrac{p^2}{m_\eta^4} + \ldots$

$\Rightarrow -i$  $= \lambda + \dfrac{g^2}{m_\eta^2} + \dfrac{g^2}{m_\eta^4}(s+t+u) + \ldots$

$$\Rightarrow \mathcal{L}_{eff}^{(4)} = -\frac{1}{4!}\left(\lambda - \frac{3g^2}{m_\eta^2}\right)\varphi^4 - \frac{g^2}{8 m_\eta^4}\varphi^2 \Box \varphi^2 + \ldots$$

What are we doing? Shrinking heavy line to point:

 $\rightarrow$   $\rightarrow$ 

$\Rightarrow$ local!

etc.

Power Counting

Integrating out η generates an ∞ # of terms

Can we organize them?

Assume: Fundamental params

$$g, g' \sim m_\eta \equiv M \quad + \quad \lambda, \lambda', \lambda'' \sim \mathcal{O}(1)$$

$$\Rightarrow \mathcal{L}_{eff} \sim \sum_{n,m} \frac{1}{M^{n+m-4}} \partial^n \phi^m$$

w/ only even powers due to Symmetries

eg at $\mathcal{O}(1/M^2)$: $\phi^6, \partial^2\phi^4, \partial^4\phi^2$

Truncating to $\mathcal{O}(1/M^2) \Rightarrow$ computing amplitudes to accuracy E^2/M^2.

$\Rightarrow$ Power counting determines accuracy of calculation

Integrate out field using equations of motion

"Semiclassical expansion": evaluate action on a solution to EOM

$$S_{eff}[\phi] = S[\phi, \eta_{cl}] + \mathcal{O}(\hbar)$$

$$\text{w/} \quad \left. \frac{\delta S[\phi, \eta]}{\delta \eta} \right|_{\eta = \eta_{cl}} = 0$$

EOM

$$\Rightarrow \Box \eta + m_\eta^2 \eta + \frac{g}{2}\phi^2 + \frac{g'}{2}\eta^2 + \frac{\lambda'}{2}\eta\phi^2 + \frac{\lambda''}{6}\eta^3 = 0$$

Solve itteratively: $\eta_{c1}^{(1)} = \frac{-g}{2m_\eta^2}\varphi^2 \sim \mathcal{O}\left(\frac{1}{M}\right)$

$$\Rightarrow \eta_{c1} = \underbrace{\frac{-g}{2m_\eta^2}\varphi^2}_{\mathcal{O}\left(\frac{1}{M}\right)} - \underbrace{\frac{1}{m_\eta^2}\Box\eta_{c1}}_{\mathcal{O}(1/M^3)} - \underbrace{\frac{\lambda'}{2m_\eta^2}\eta_{c1}\varphi^2}_{\mathcal{O}(1/M^3)} - \underbrace{\frac{g'}{2m_\eta^2}\eta_{c1}^2}_{\mathcal{O}(1/M^3)} - \underbrace{\frac{\lambda''}{6m_\eta^2}\eta_{c1}^3}_{\mathcal{O}(1/M^5)}$$

$\Rightarrow$ To go to $\mathcal{O}(1/M^3)$ sub $\eta_{c1}^{(1)}$ into EOM keeping terms up to $\mathcal{O}(1/M^3)$

$$\Rightarrow \eta_{c1}^{(3)} = \frac{-g}{2m_\eta^2}\varphi^2 + \frac{g}{2m_\eta^4}\Box\varphi^2 + \left(\frac{g\lambda'}{4m_\eta^4} + \frac{g^2g'}{4m_\eta^6}\right)\varphi^4 + \mathcal{O}\left(\frac{1}{M^5}\right)$$

Sub into $\mathcal{L}_{UV}$

$$\Rightarrow \mathcal{L}_{eff} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m_\varphi^2\varphi^2 - \frac{1}{4!}\left(\lambda - \frac{3g^2}{m_\eta^2}\right)\varphi^4$$
$$- \frac{1}{6!}\left(\frac{45\lambda' g^2}{m_\eta^4} - \frac{15g'g^3}{m_\eta^6}\right)\varphi^6$$
$$+ \frac{g^2}{8m_\eta^4}(\partial_\mu\varphi^2)(\partial^\mu\varphi^2) + \mathcal{O}(1/M^4)$$

which agrees w/ previous diagramatic approach upon integration by parts of $(\partial\varphi^2)^2$ term.

Simplifying $\mathcal{L}_{eff}$

Two stradigies: (1) integration by parts
(2) field redefinitions

Ex: Classify all possible terms of the form $\partial^2 \varphi^n$

Using $\partial_\mu \varphi^r = r\varphi^{r-1}\partial_\mu \varphi$ rewrite operator so each derivative acts on single field.

$\Rightarrow$ Most general operator is linear combo of

$$\varphi^{n-1}\Box\varphi \quad \text{and} \quad \varphi^{n-2}\partial^\mu\varphi\,\partial_\mu\varphi$$

Then
$$\varphi^{n-2}\partial^\mu\varphi\,\partial_\mu\varphi = \frac{1}{n-1}\partial^\mu\varphi^{n-1}\,\partial_\mu\varphi$$
$$= -\frac{1}{n-1}\varphi^{n-1}\Box\varphi + \text{total der}$$

$\Rightarrow$ Only single independent operator for each n.

Ex: Field redefinitions (aka "using the equations of motion")

Let $\varphi \to \varphi + f(\varphi)$ and expand in powers of $f(\varphi)$:

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - V \to \frac{1}{2}(\partial\varphi)^2 - V - f(\varphi)\underbrace{\left[\Box\varphi + V'\right]}_{\text{EOM}} + \mathcal{O}(f^2)$$

Ex: Let's simplify our previous example

$$\varphi \to \varphi + c\frac{g^2}{m_\eta^4}\varphi^3$$

$$\Rightarrow \mathcal{L}_{eff} \to \mathcal{L}_{eff} + \frac{cg^2}{m_\eta^4}\varphi^3\left[\Box\varphi + m_\varphi^2\varphi + \frac{\lambda}{3!}\varphi^3\right] + \mathcal{O}(\Lambda^{-4})$$

Taking $c = \frac{1}{2} \Rightarrow$

$$\mathcal{L}_{eff} \to \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m_\varphi^2\varphi^2 - \frac{1}{4!}\left(\lambda - \underbrace{\frac{3g^2}{m_\eta^2}}_{\mathcal{O}(1)} - \underbrace{\frac{6g^2m_\varphi^2}{m_\eta^4}}_{\mathcal{O}(1/\Lambda^2)}\right)\varphi^4$$

$$+ \frac{1}{6!}\left[\frac{g^2(45\lambda' - 60\lambda)}{m_\eta^4} - \frac{15g'g^3}{m_\eta^6}\right]\varphi^6 + \mathcal{O}(1/\Lambda^4)$$

We have eliminated the $\partial^2\varphi^4$ term!

$\Rightarrow$ All indirect effects from η can be modeled by modified φ^4 and φ^6 terms up to $\mathcal{O}(E^2/\Lambda^2)$

This justifies using the classical EOMs to rewrite the $\mathcal{L}$ into a more convenient form.

Universality

Different UV Theories can yield same IR Theory.

Call this "universality".

Ex: Let's add N heavy fields η_i

w/ Same Z_2 Sym $\varphi \to -\varphi$ and $\eta_i \to \eta_i$

$$\mathcal{L} = \mathcal{L}_{kin} - V$$

$$V = \frac{\lambda}{4!}\varphi^4 + \frac{g_i}{2}\varphi^2\eta_i + \frac{g'_{ijk}}{3!}\eta_i\eta_j\eta_k + \frac{\lambda'_{ij}}{4}\varphi^2\eta_i\eta_j + \frac{\lambda''_{ijkl}}{4!}\eta_i\eta_j\eta_k\eta_l$$

Power counting: $M \sim m_{\eta_i} \sim g_i \sim g'_i \gg m_\varphi$

$$\lambda \sim \lambda'_{ij} \sim \lambda''_{ijkl} \sim \mathcal{O}(1)$$

Claim: $\mathcal{L}_{eft} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m_\varphi^2\varphi^2 - \frac{\lambda_{4\,eff}}{4!}\varphi^4 - \frac{\lambda_{6\,eff}}{6!M^2}\varphi^6 + \mathcal{O}(1/M^4)$

w/ $\lambda_{4\,eff}$ + $\lambda_{6\,eff} \sim 1$

and we used int by parts + EOM to elliminate $\partial^2\varphi^4$ term

Using EOMs: $\eta_{cl\,i} = \frac{-g_i}{2m_{\eta_i}^2}\varphi^2 + \mathcal{O}(1/M^3)$

$$\Rightarrow \lambda_{4\,eff} = \lambda - \sum_i \frac{3g_i^2}{m_{\eta_i}^2} + \mathcal{O}(1/M^2)$$

Note "decoupling" when $m_{\eta_i} \gg m_{\eta_j}$

Bottom Up EFT

Leading terms in $\mathcal{L}_{eff}$ consist of all relevant and marginal interactions that are compatible w/ symmetries that are inherited from fundamental theory.

Impact of higher dimension operators are supressed by powers of E/M

Ex: Standard Model

$$\Delta \mathcal{L}_{eff} \sim \frac{1}{M} (LH)^2 \xrightarrow[\langle H \rangle = v]{} m_\nu = \frac{v^2}{M}$$

$\Rightarrow$ "explain" small neutrino masses.

Ex: Baryon + Lepton number are accidental symmetries $\Rightarrow$ proton decay via higher dimension operators

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Interpreting irrelevant interactions

$$\mathcal{L}_{eff} = \frac{\lambda_6}{6!} \phi^6 \quad \text{w/} \quad [\lambda_6] = -2$$

Assume more fundamental theory w/ scale M and $\mathcal{O}(1)$ couplings

Then $\lambda_6^{-1/2} \overset{?}{\sim} M$

From our example, we had

$$\lambda_6 = \frac{g^2(45\lambda' - 60\lambda)}{m_\eta^4} - \frac{15 g' g^2}{m_\eta^6}$$

So rule of thumb holds for $g \sim g' \sim m_\eta \sim M$
and $\lambda \sim \lambda' \sim 1$

But could have taken couplings small
$\Rightarrow \quad \lambda_6^{-1/2} \gg m_\eta$

Could we have taken couplings large?
$\Rightarrow$ breakdown of pert theory

Can formalize as violating partial wave unitarity
to find $\quad \underline{M} \lesssim 100\, \lambda_6^{-1/2}$

$\Rightarrow$ If we observe higher dim op
$\Rightarrow$ upper bound on new physics scale

Bottom up marginal + relevant couplings

- marginal: dimensionless ⇒ no info about heavy new physics scale (loops will induce logarithmic sensitivity)
- relevant couplings:

$$\mathcal{L}_{eff} = -\frac{\lambda_3}{3!}\varphi^3$$

Naively if $\lambda_3 \sim M \Rightarrow$ dim analysis $\Rightarrow \mathcal{M} \sim \frac{\lambda_3}{E} \sim \frac{M}{E} \gg 1$

⇒ EFT does not make sense

Must have $\lambda_3 \ll M$. Can be enforced by a symmetry (e.g. $\varphi \to -\varphi$) that is broken by small coupling (spurion).

We will revisit this.

- Mass: $\mathcal{L}_{eff} = -\frac{1}{2} m_\varphi^2 \varphi^2$

Loops will shift this by $\sim \frac{1}{16\pi^2} M^2$

⇒ Hierarchy problem.

Can use symmetry to solve this problem: SUSY or "shift symmetry" $\varphi \to \varphi + c$

Expanded perturbation theory:

Now we are doing dual expansion

1) loop ($\hbar$) expansion

2) E/M expansion

Clearly distinguished at tree, but this becomes much more subtle at loop level.

EFTs and the SM:

- $E \ll m_e$: Euler-Heisenberg for photons
- $E \ll \Lambda_{QCD}$: Chiral $\mathcal{L}$ for light mesons
- $E \ll m_W$: Fermi theory for quarks and leptons
- $E \ll \Lambda_{\text{new physics}}$: SMEFT (w/ $\Lambda_{np} \gg v$)

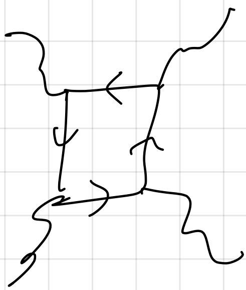
  HEFT (w/ $\Lambda_{NP} \sim v$)

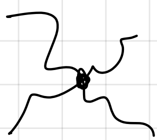
EFTs w/ kinematic restrictions

- HQET: $m_Q \gg \Lambda_{QCD}$ $p \ll m_Q$
- NRQCD: $m_Q \gg \Lambda_{QCD}$ $p \ll m_Q$ $p \sim mv$, $E \sim \frac{1}{2}mv^2$, $p \sim E$
- SCET: $p \ll \sqrt{\hat{S}}$ w/ p "collinear" or "soft"

Euler - Heisenberg

Integrate out the electron to generate light-by-light scattering

$i\mathcal{M} =$  Full Theory

$i\mathcal{M} =$  EFT

$$\mathcal{L}_{EFT} = \frac{c}{m_e^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{c'}{m_e^4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2$$

$$\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$\alpha = \frac{e^2}{4\pi}$

Matching calculation $\Rightarrow$ $c = \frac{\alpha^2}{90}$ & $c' = \frac{7\alpha^2}{360}$

Can be used to compute

$$\sigma_{\gamma\gamma\to\gamma\gamma} = \frac{973}{10125\pi}\alpha^4\frac{\omega^6}{m^8}$$

(see Schwartz 33.4.2)

Fermi Theory

Now let's apply this to the SM at energies $E \ll m_W$. (Let us continue to ignore the fact that QCD becomes non-perturbative at $E \sim 1\,\text{GeV}$)

This will allow us to derive "Fermi Theory" from the top down.

Start with the Lagrangian

$$\mathcal{L} = m_W^2 |W_\mu|^2 + \frac{m_Z^2}{2} Z_\mu^2 + \frac{g}{2\sqrt{2}} W_\mu^+ J^{+,\mu}$$

$$+ \frac{g}{2\sqrt{2}} W_\mu^- J^{-,\mu} + \frac{g}{c_W} Z_\mu J^{\mu,0}$$

where "charged current" is

$$J_\mu^+ \equiv \sum_{\text{doublets}}' \bar{\psi}_d \gamma_\mu (1-\gamma^5) \psi_u$$

$$= \bar{l} \gamma_\mu (1-\gamma^5) \nu + \bar{d} \gamma_\mu (1-\gamma^5) u + \text{other families}$$

and "neutral current" is

$$J_\mu^0 \equiv \sum_{\substack{\text{all} \\ \text{fermions}}}' \bar{\psi}_i \gamma^\mu (g_V(i) - g_A(i)\gamma^5) \psi_i$$

This neglects the kinetic terms, since we are only keeping the leading term in the derivative expansion (∂^0).

Neglects W/Z self interactions since this would lead to more powers of $1/m_{W/Z}$. Neglects Higgs interactions, since would either give more powers of $1/m_{W/Z/H}$ or would be proportional to tiny Yukawa couplings.

In this approximation, the EOMs for the W and Z are

$$m_W^2 W_\mu^+ + \frac{g}{2\sqrt{2}} J_\mu^+ = 0 \qquad\qquad W_\mu^+ = -\frac{g}{2\sqrt{2}\, m_W^2} J_\mu^+$$

$$\Rightarrow$$

$$m_Z^2 Z_\mu + \frac{g}{c_W} J_\mu^0 = 0 \qquad\qquad Z_\mu^+ = \frac{-g}{c_W m_Z^2} J_\mu^0$$

Plugging these back into $\mathcal{L}$

$$\Rightarrow \mathcal{L} \supset \frac{g^2}{8m_W^2} J^+ \cdot J^- - 2\frac{g^2}{8m_W^2} J^+ \cdot J^- + \frac{g^2}{2c_W^2 m_Z^2} J_0 \cdot J_0 - \frac{g^2}{c_W^2 m_Z^2} J_0 \cdot J_0$$

$$= -\frac{g^2}{8m_W^2} J^+ \cdot J^- - \frac{g^2}{2W_W^2} J_0 \cdot J_0 \quad \left(\text{used } c_W^2 m_Z^2 = m_W^2\right)$$

We define the "Fermi Constant"

$$\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8m_W^2} = \frac{1}{2v^2} \qquad [G_F] = -2 \qquad G_F = 1.2\times 10^{-5}/\mathrm{GeV}^2$$

QED; charged current interaction; neutral current interaction

$$\Rightarrow \mathcal{L} \supset \sum_{\substack{\text{light}\\\text{fermions}}} \bar{\psi} i \not{D} \psi - \frac{G_F}{\sqrt{2}} J^{+}\cdot J^{-} - \frac{4G_F}{\sqrt{2}} J^{0}\cdot J^{0}$$

w/

$$J^{\mu,-} = (J^{\mu,+})^* = \sum_{\substack{\text{light}\\\text{families}}} \left(\bar{\nu}\gamma^\mu(1-\gamma^5)e + \bar{u}\gamma^\mu(1-\gamma^5)d\right)$$

$$J^{\mu,0} = \sum_{\substack{\text{light}\\\text{families}}} \bar{\psi}\gamma^\mu(g_V - g_A\gamma^5)\psi$$

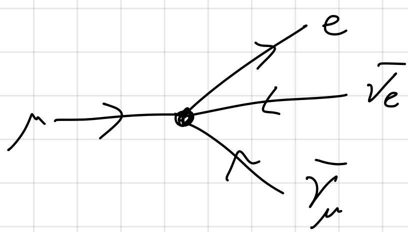
w/ $g_V = \frac{1}{4}\left[2T^3 - 4s_W^2 Q\right]$, $g_A = \frac{1}{4}\left[2T^3\right]$

We see that the weak interactions provide a correction to QED by introducing irrelevant operators.

We can capture a lot of the consequences of electroweak physics using this approximation.

For example, we can compute the decay rates of fermions. For example, the muon decay is determined by

$$\mathcal{L} \supset -\frac{G_F}{\sqrt{2}}\left[\bar{e}\gamma^\mu(1-\gamma^5)\nu_e\right]\left[\bar{\nu}_\mu\gamma_\mu(1-\gamma^5)\mu\right]$$

$\Rightarrow$ 

$$\Rightarrow \Gamma(\mu \to e\bar{\nu}_e\bar{\nu}_\mu) = \frac{G_F^2}{192\pi^3} m_\mu^5$$

Note that this is why the weak interactions are "weak". It is not due to small coupling since $g \sim g' \sim e$ in the SM. It is instead due to suppression by mass of heavy particles!

We could have guessed this answer using dimensional analysis:

$\mathcal{M} \sim G_F \Rightarrow \Gamma \sim G_F^2$

$[\Gamma] = 1 \Rightarrow$ assuming e, ν_e, ν_μ massless, only other dimensionful quantity is m_μ

$\Rightarrow \Gamma \sim G_F^2 m_\mu^5$

We also know 2-body $\Gamma \sim \frac{1}{4\pi}$

3-body $\Gamma \sim \frac{1}{4\pi(16\pi^2)} = \frac{1}{64\pi^3}$

$\Rightarrow \Gamma \sim \frac{1}{64\pi^3} G_F^2 m_\mu^5$ is pretty close :)

This explains why the muon lives for a long time. We have $\Gamma_\mu / m_\mu \sim 3 \times 10^{-18}$

Naively expect $\Gamma/m \sim 10^{-2}$, so this is a big supression.

SMEFT

Approach to parametrize indirect BSM effects

Write down all $SU(3) \times SU(2) \times U(1)$ invariant operators supressed by heavy scale Λ.

A dim 5, there is one unique choice:

$$\mathcal{L}_{\dim 5} = \frac{1}{\Lambda}(H\bar{L}^c)(HL) \rightarrow \mathcal{L} \supset \frac{v^2}{\Lambda} \bar{\nu}^c \nu$$

$\Rightarrow$ Majorana neutrino mass.

At dim 6 there are 3045 independent operators. (Must be careful about redundancies)
1512.03433