Joint ICTP-EAIFR-IUGG Workshop on Computational Geodynamics: Towards Building a New Expertise Across Africa

GEODYNAMIC MODELS & SCALING

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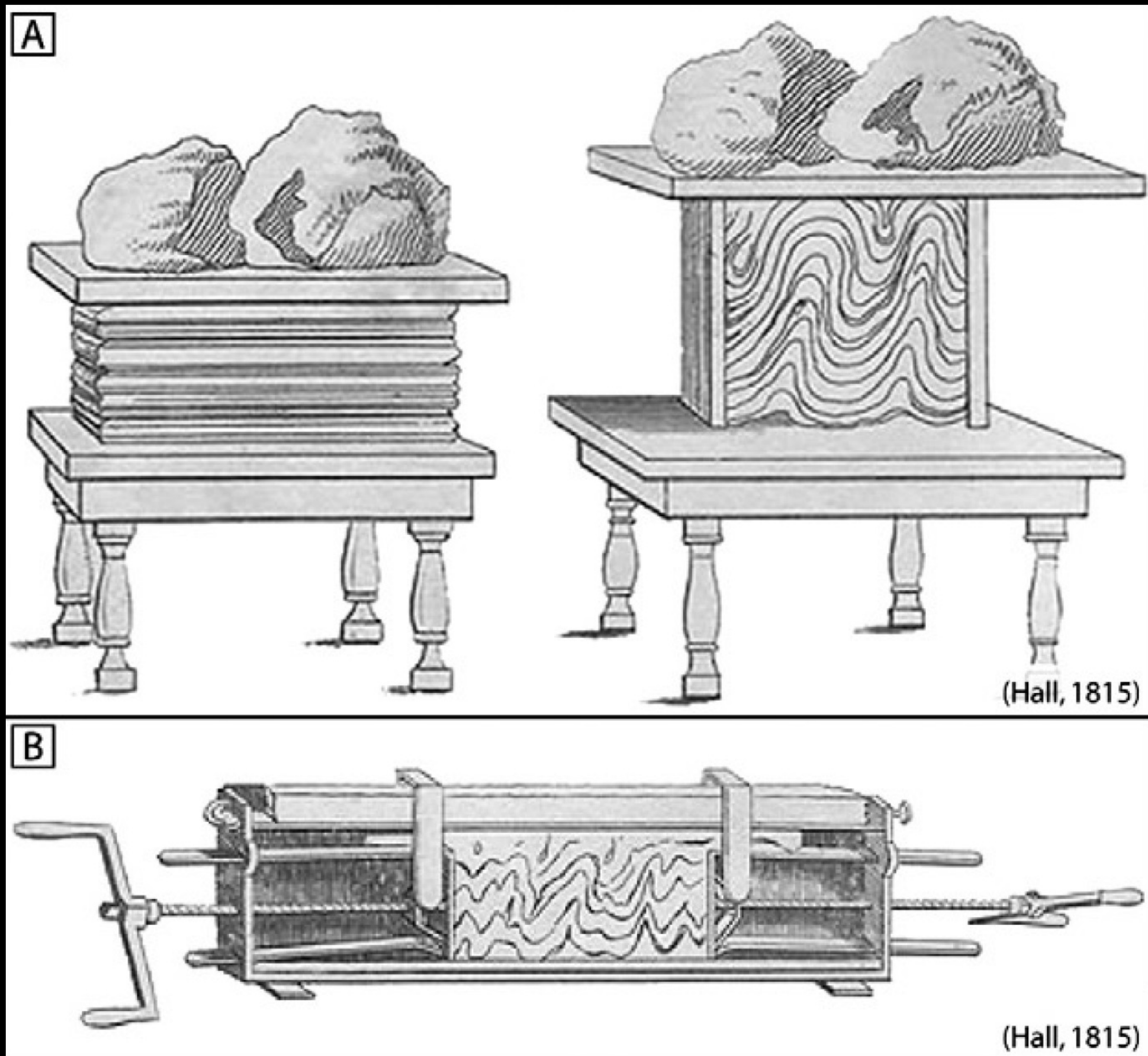
Outline of lecture

Scaling of models:

1. A brief history of geodynamic modelling.
2. Modelling approaches.
3. Geometric, kinematic & dynamic similarity.
4. Scaling of physical parameters (e.g. length, time, density (contrast), stress).
5. Reynolds number & dynamic flow regime.
6. Rheological similarity.
7. Scaling of topography.

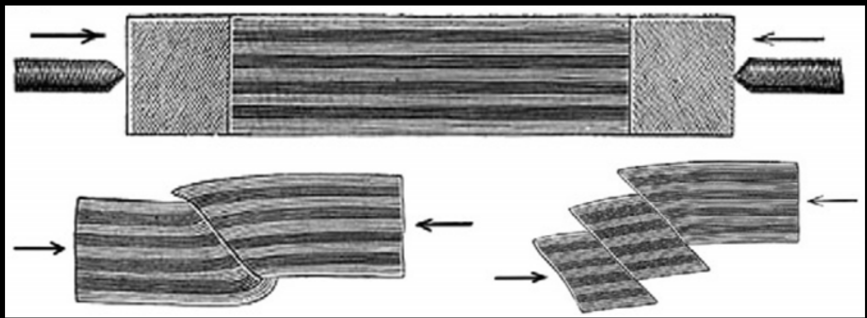
Brief history of geodynamic modelling

1815: First geodynamic models (analogue) to investigate folding of rocks [Hall, 1815]



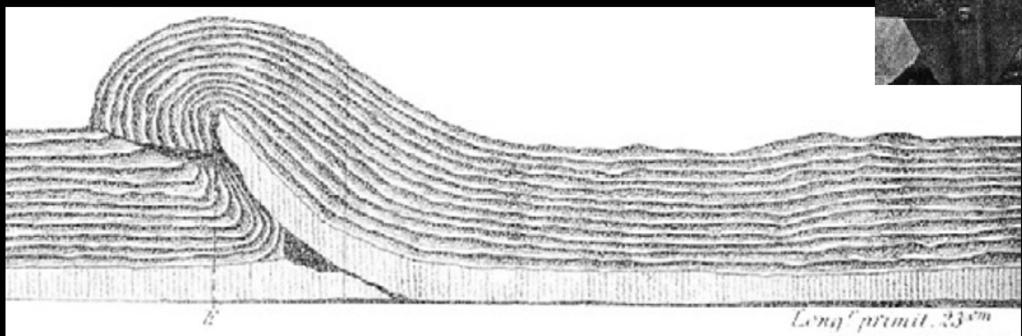
Brief history of geodynamic modelling

Other 19th century geodynamic models (analogue) also investigated shortening structures/processes



Reverse faulting [Daubree, 1879]

Thrust wedge formation [Cadell, 1888]

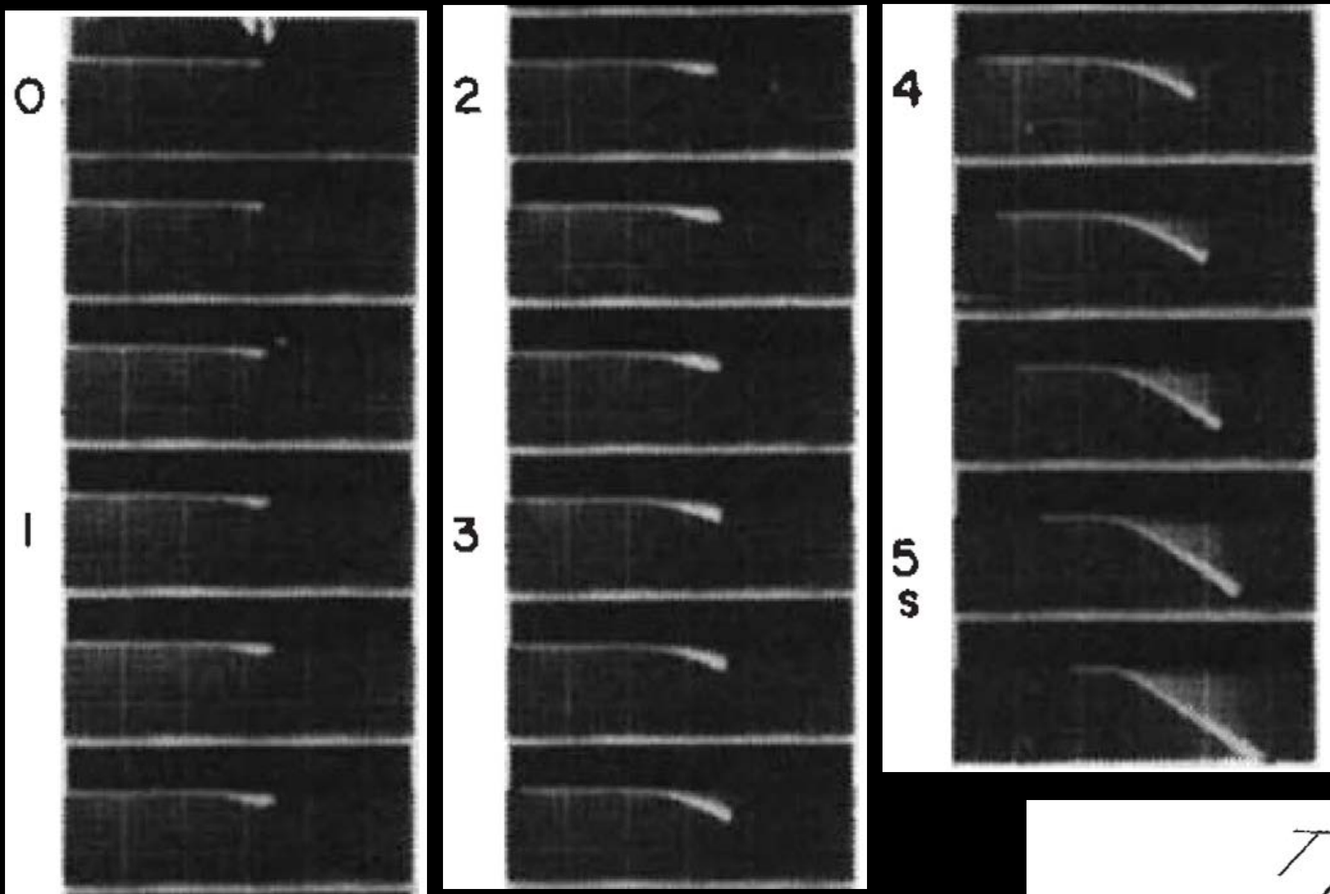


Fault-propagation folding [Schardt, 1884]

Brief history of geodynamic modelling

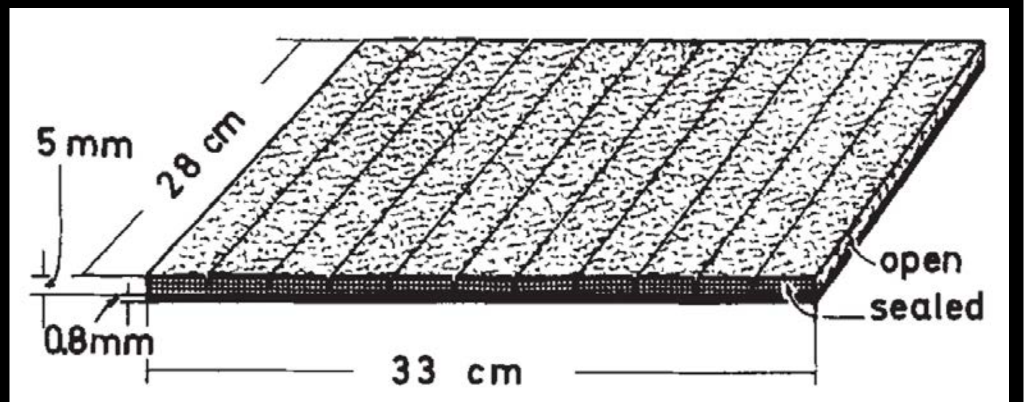
First subduction geodynamic model (analogue)

[Jacoby, Nat. Phys. Sci. 1973]



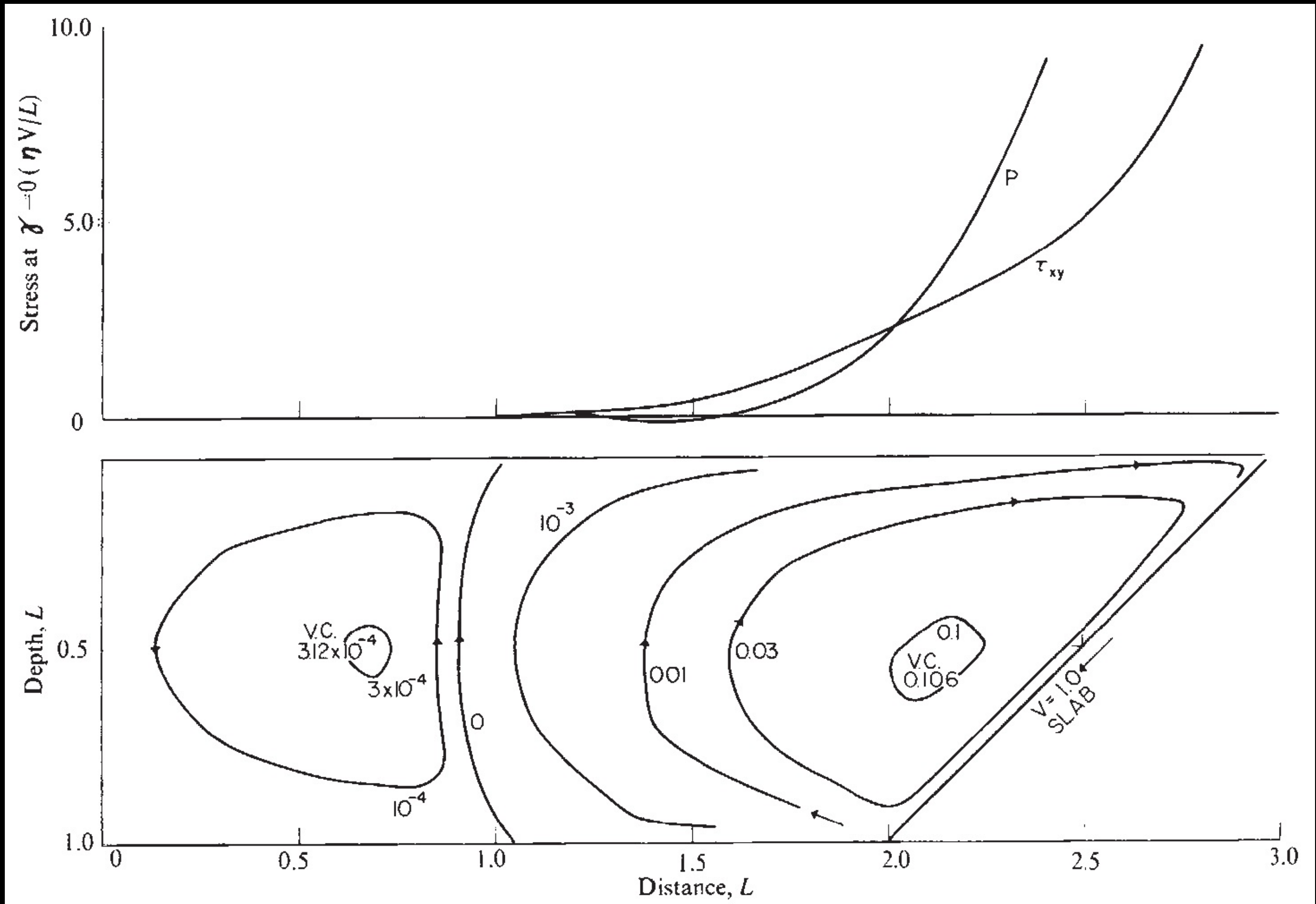
Experimental set up with thin rubber sheet covered with water-absorbing foam layer floating on water reservoir.

Side view of buoyancy-driven subduction experiment. Note experimental duration = ~5 s.



Brief history of geodynamic modelling

First subduction geodynamic models (numerical)



[*Sleep & Toksoz*, Nature 1971]

***Bottom*: cross-section of 2D subduction experiment with imposed slab geometry and velocity showing stream lines.**
***Top*: Shear stress and pressure at base of lithosphere.**

Why geodynamic modelling?

- **Viability**
  To test the physical/mechanical viability of a hypothesis or conceptual model.

- **Parametric**
  To investigate the influence of a particular parameter on a certain geological process.

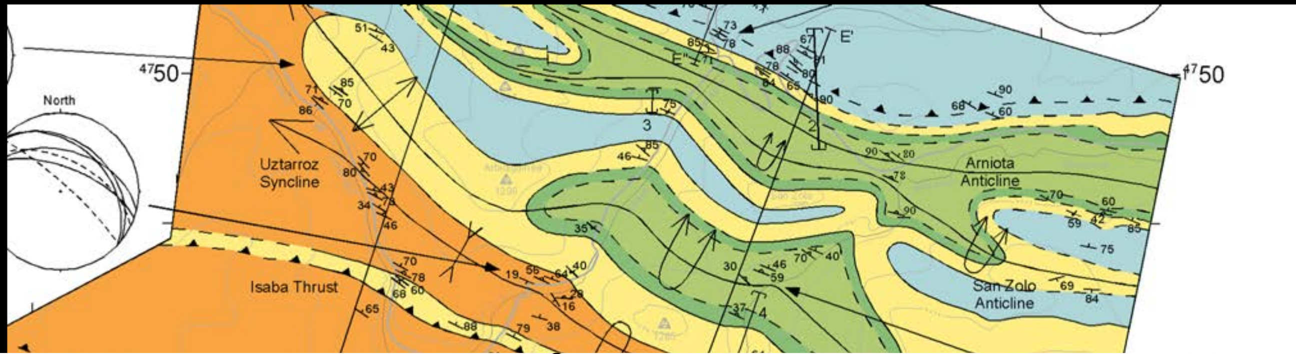
- **Time evolution and/or spatial distribution**
  To be able to visualize and quantify the geometry/ structure/flow field/velocity field/stress field of a particular geological process or phenomenon with progressive time and/or in 2D/3D space.

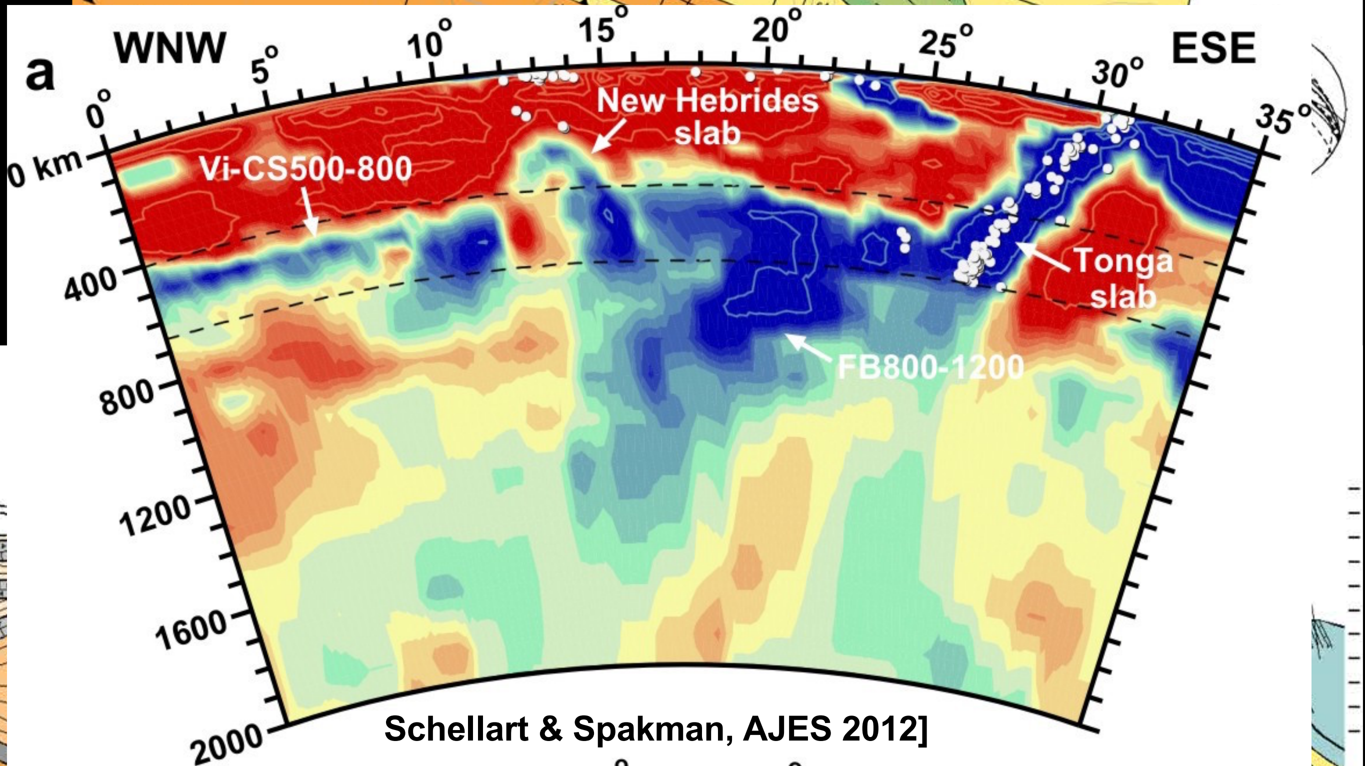
Geometric models (static models)

-Geological map, structural map, geomorphological map, hydrological map, gravity map, ……

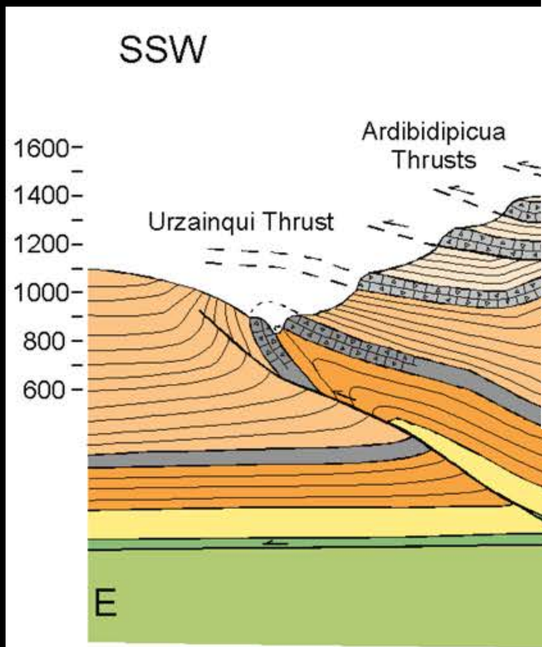
-Geological cross-section, …….

-Mantle tomography model, …….





Schellart & Spakman, AJES 2012]



[Schellart, JVE, 2002]

Kinematic models (displacements / velocities)

Tectonic reconstructions, restored balanced cross-sections.

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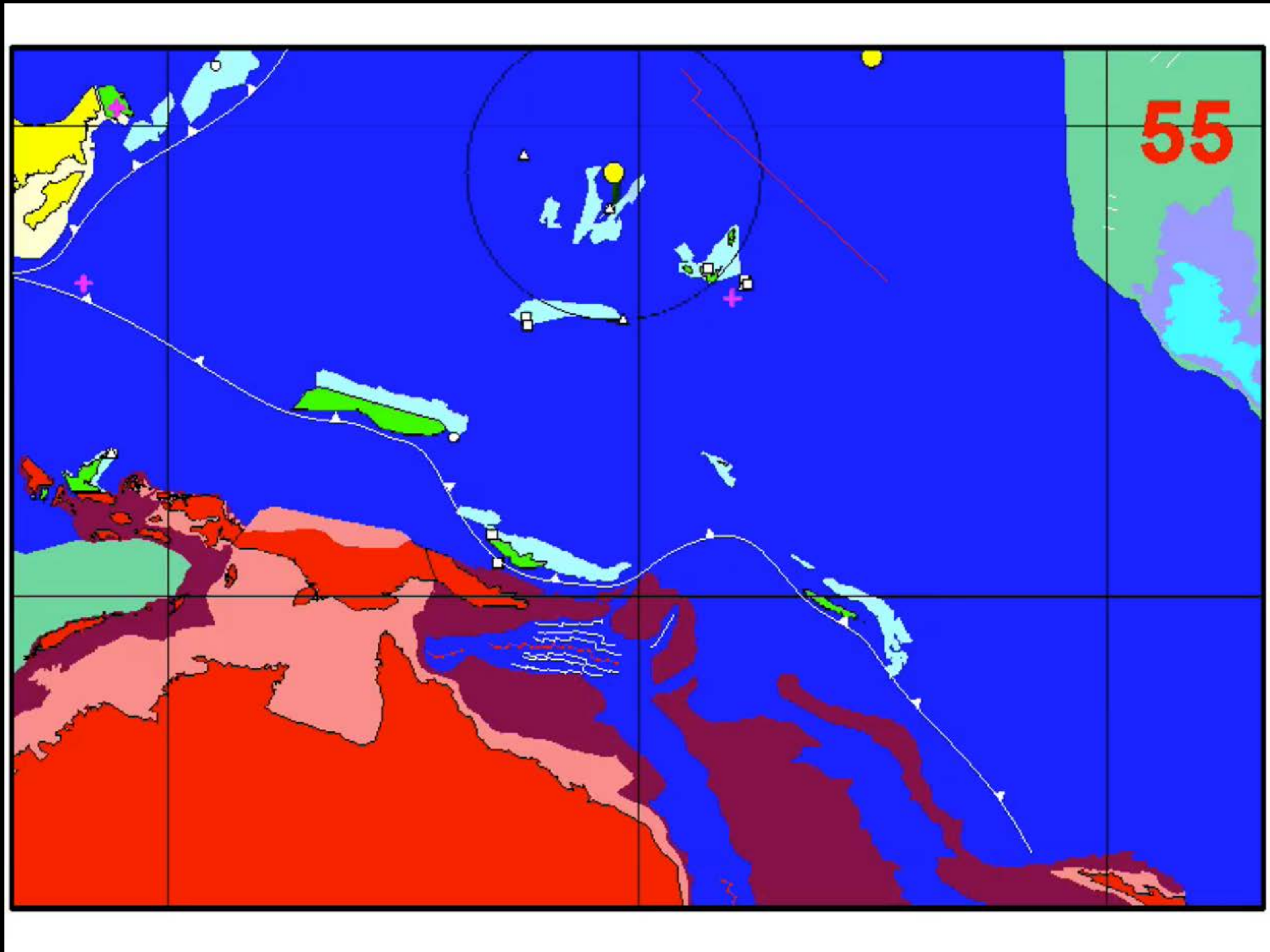
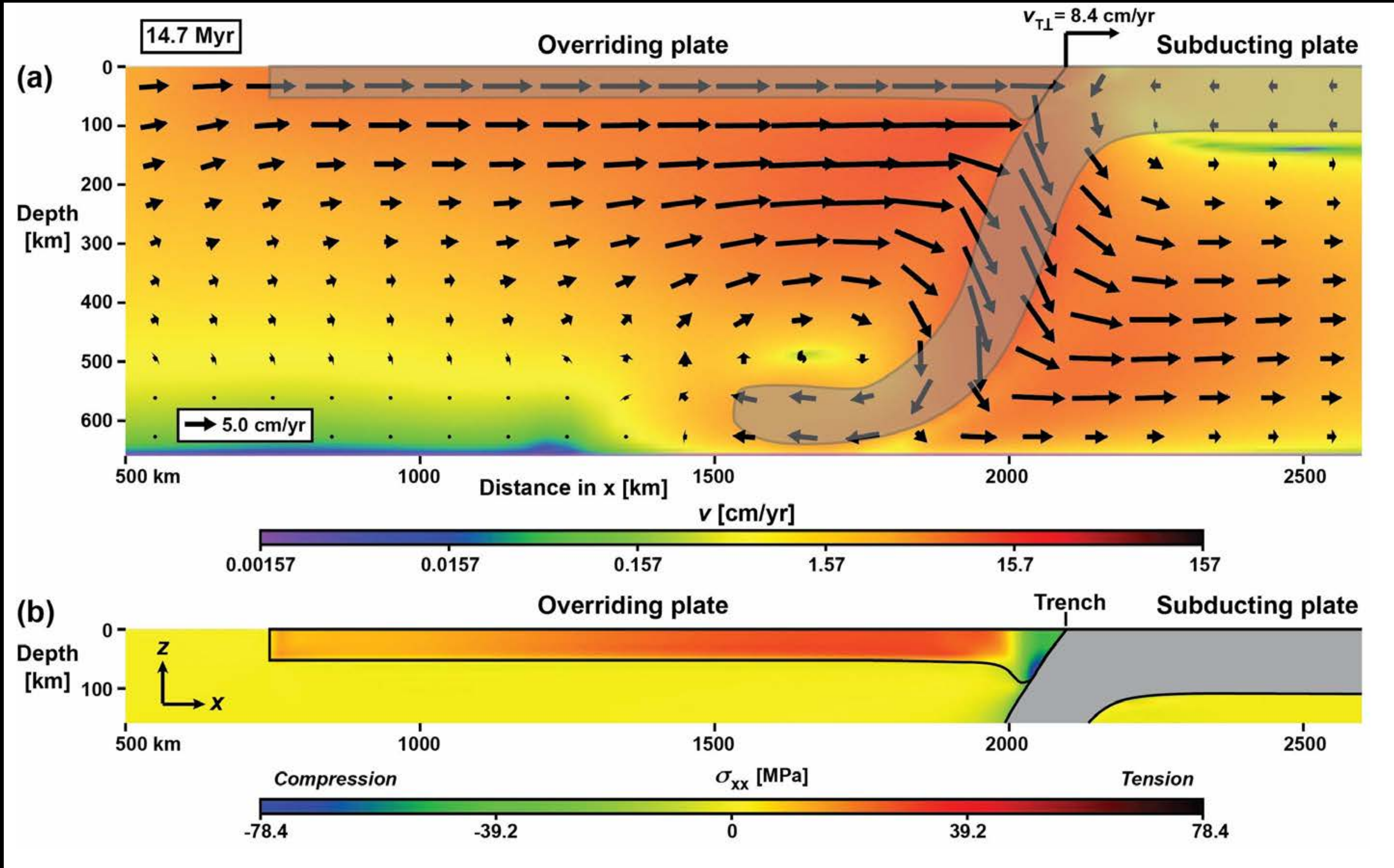


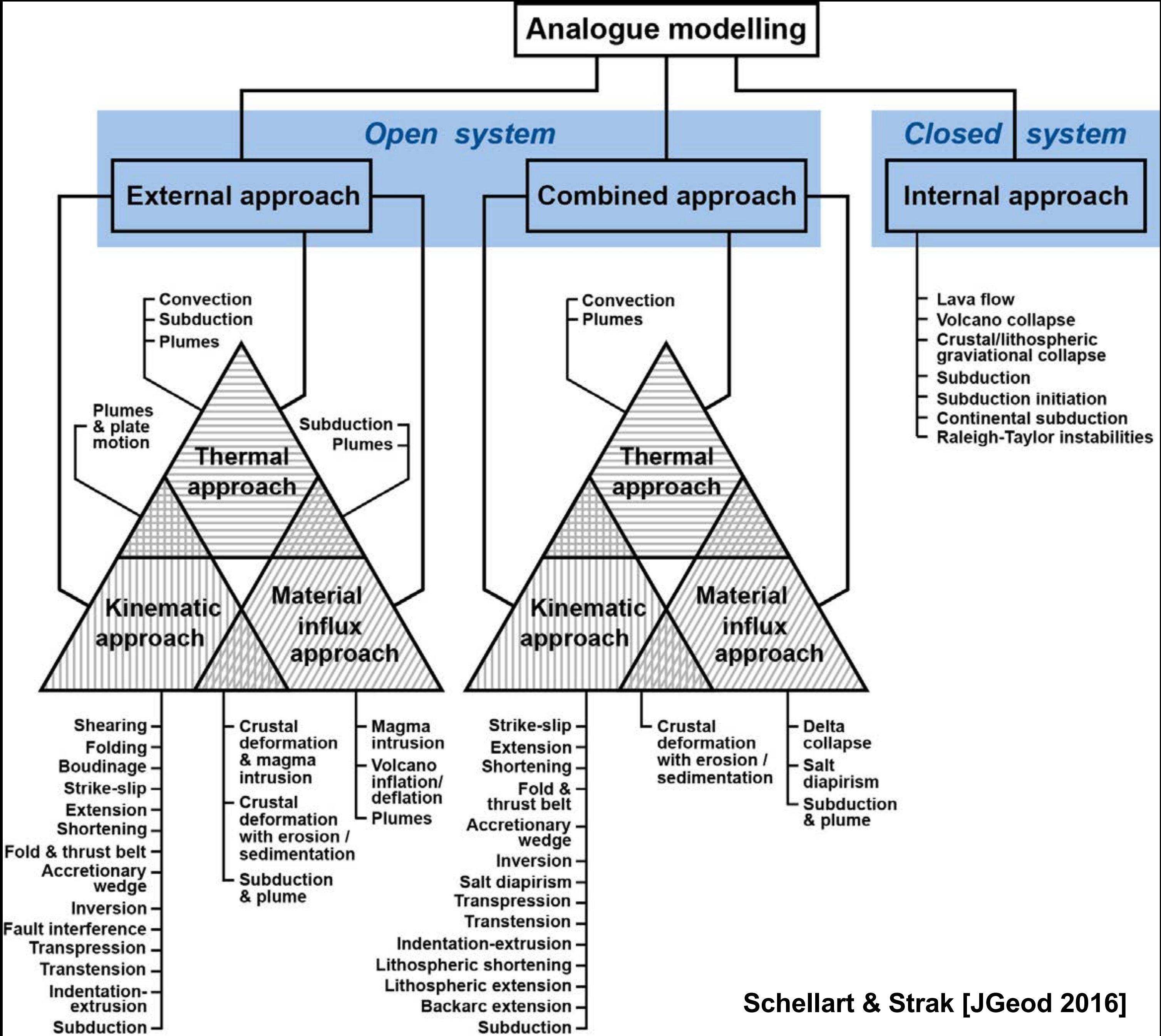
Tectonic reconstruction from Hall [AJES, 2002]

Dynamic models (linking forces, stresses, strain, velocities)



4D numerical subduction model from Schellart and Moresi [JGR 2013]

Approaches in analogue & numerical modelling



Schellart & Strak [JGeod 2016]

CLOSED SYSTEM

Internal:

All deformation driven by internal energy

OPEN SYSTEM

External:

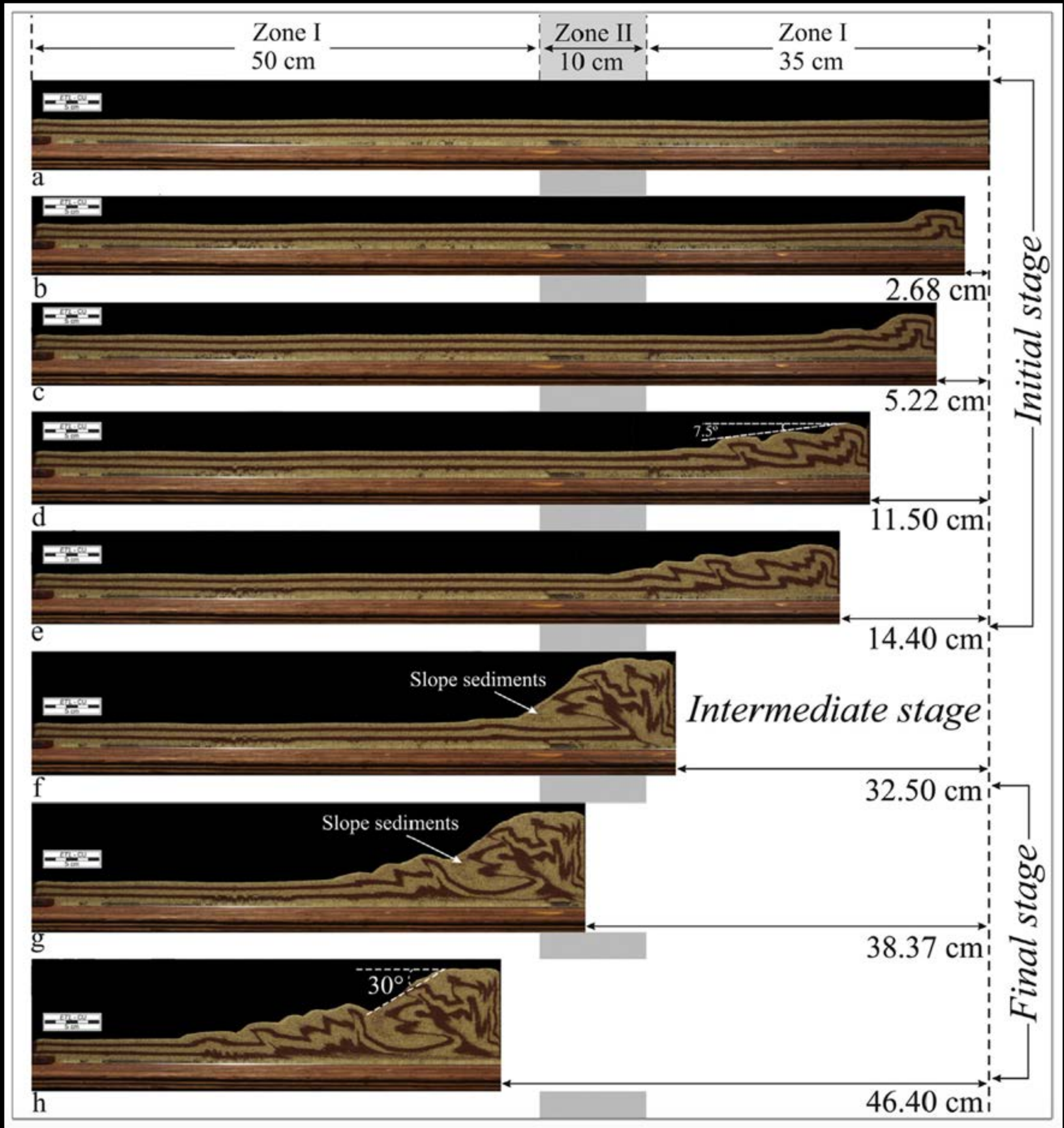
All deformation driven by externally added energy

Combined:

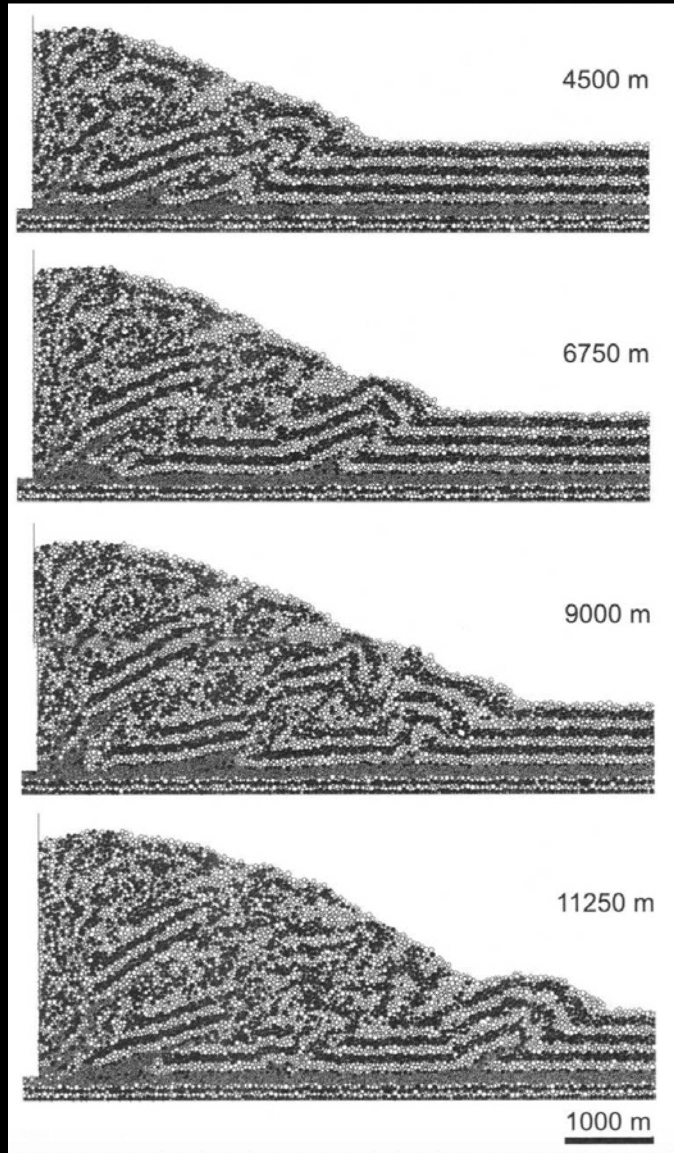
Deformation driven by internal & external energy

Geodynamic model: Open system external approach

Accretionary wedge simulations

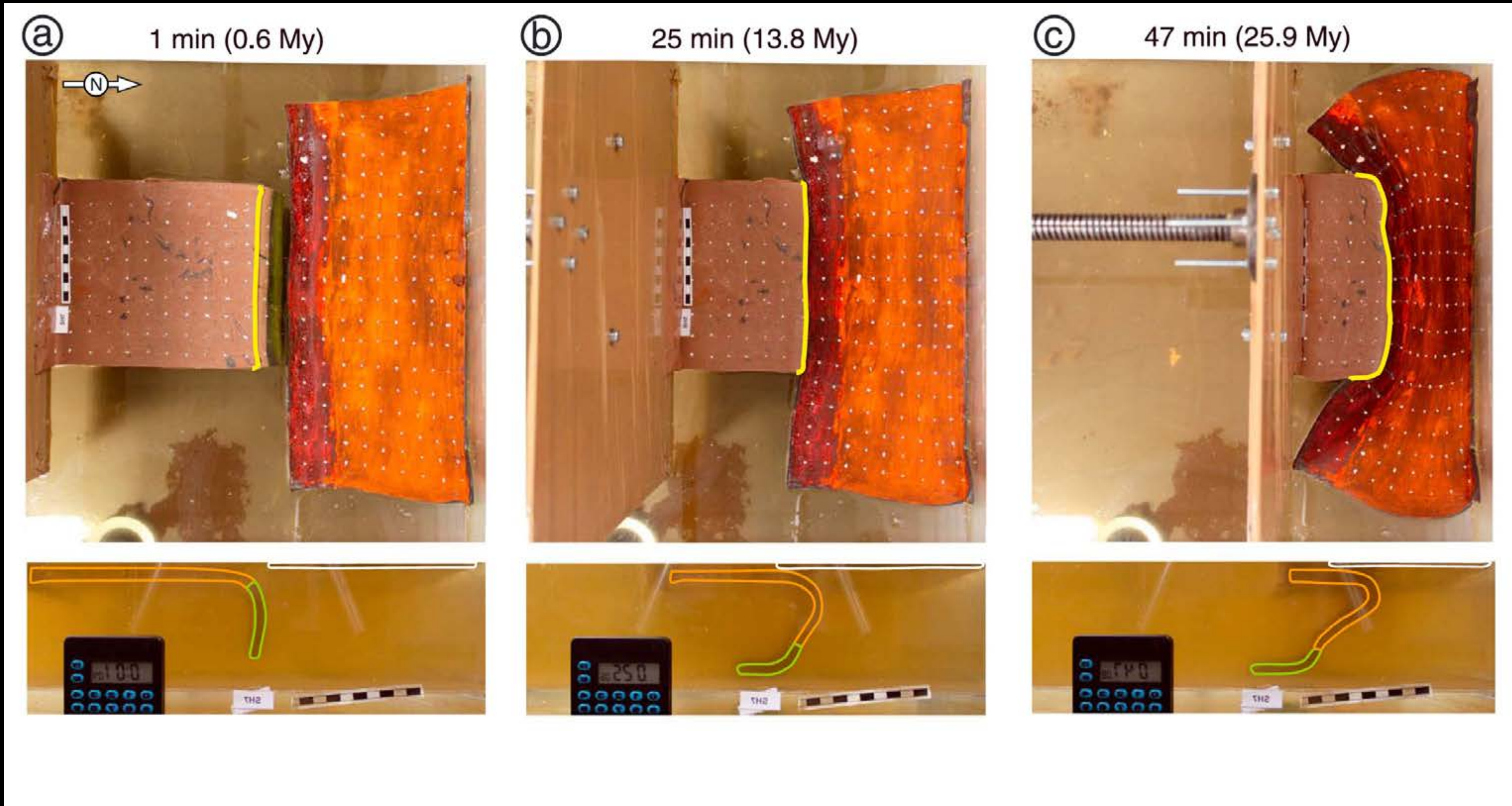


Analogue experiment **[Bose et al. J. Geod. 2015]**



Numerical model
[Yamato et al. GSSP 2006]

Geodynamic model: Open system combined approach



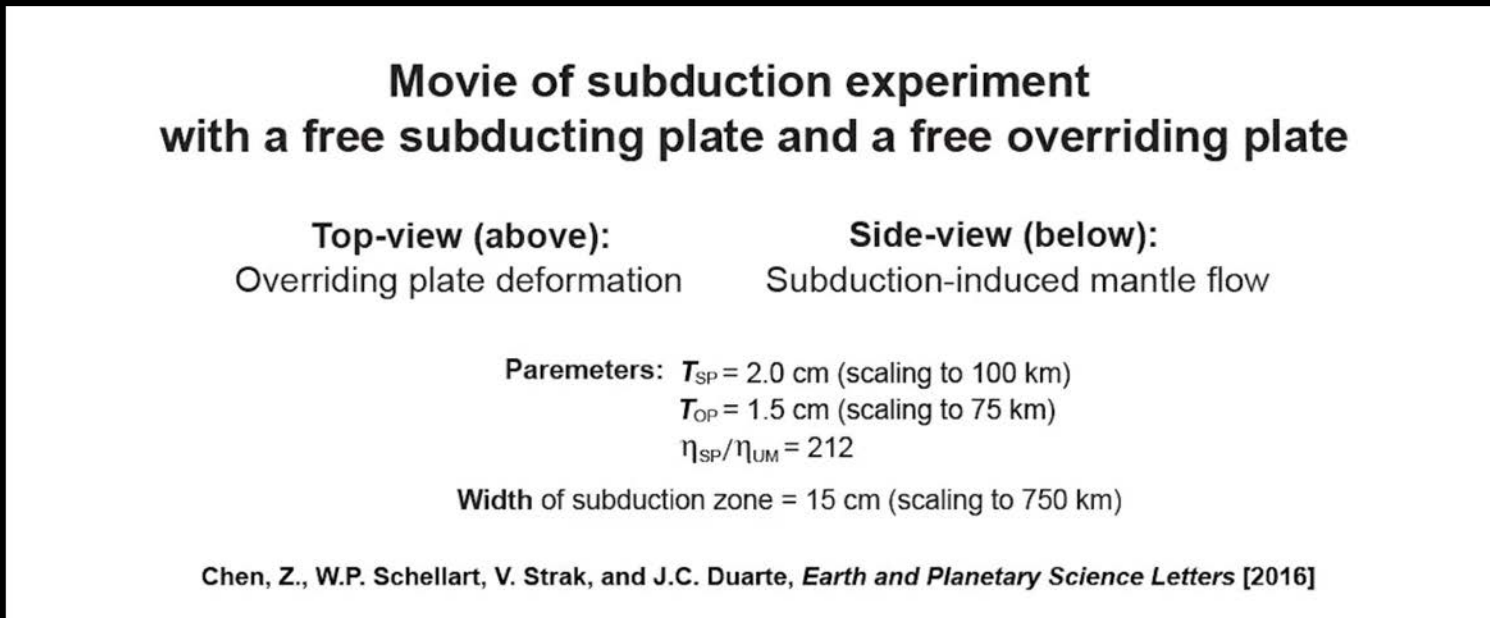
India-Eurasia subduction-collision experiment [Bajolet et al., Tect. 2013]

Geodynamic model: Closed system internal approach

Buoyancy-driven subduction models

Numerical model simulating flat slab subduction

[Schellart & Strak, GJI 2021]

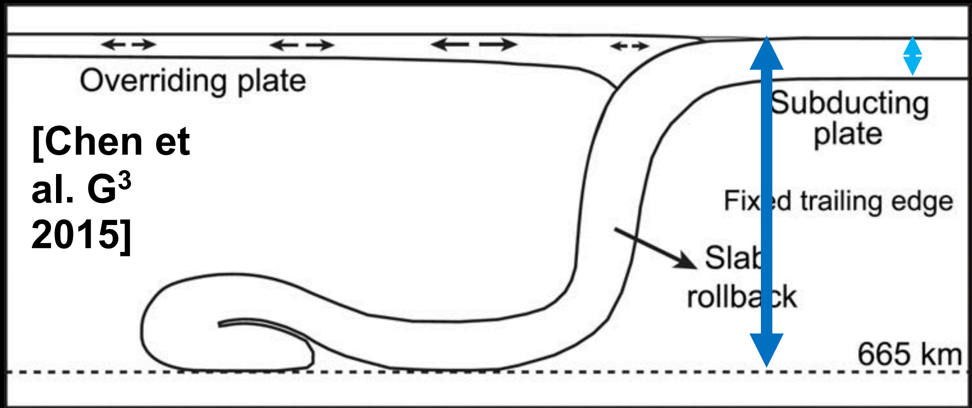


[Chen et al., EPSL 2016]

Analogue subducting plate-overriding plate-mantle flow experiment

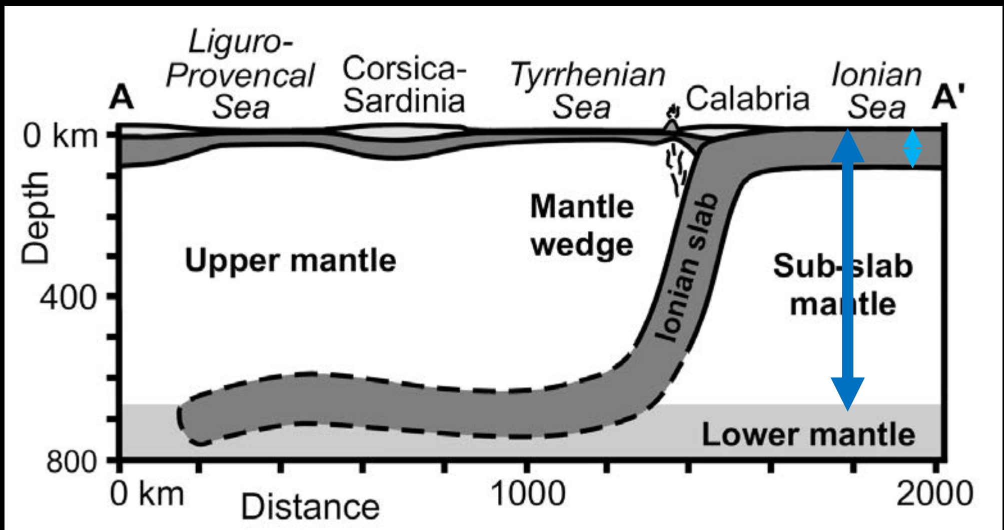
Geometric, kinematic & dynamic similarity

Geometric similarity: ***All corresponding lengths are proportional and all corresponding angles are equal in model and nature.***



Subduction experiment
[Chen et al. G-cubed 2015]

$$\frac{\text{Plate thickness}}{\text{Upper mantle thickness}}$$

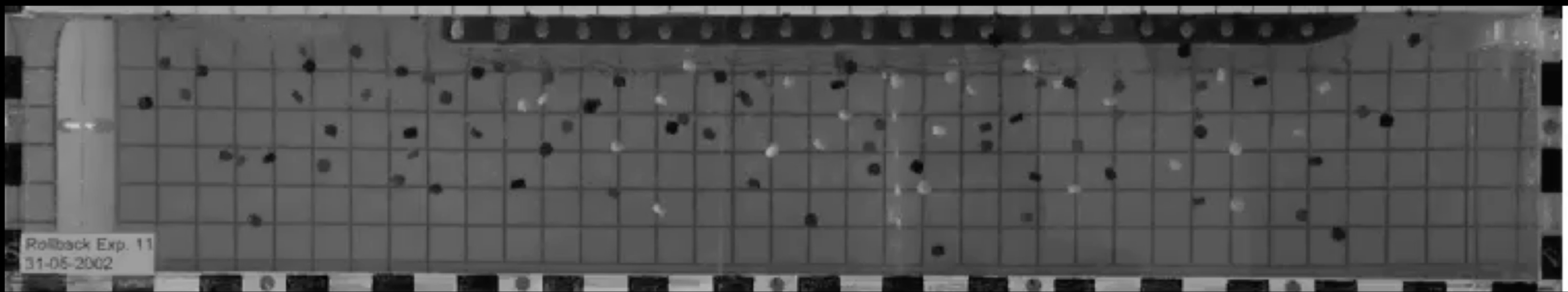


Calabria subduction zone

$$\frac{\text{Plate thickness}}{\text{Upper mantle thickness}}$$

Geometric, kinematic & dynamic similarity

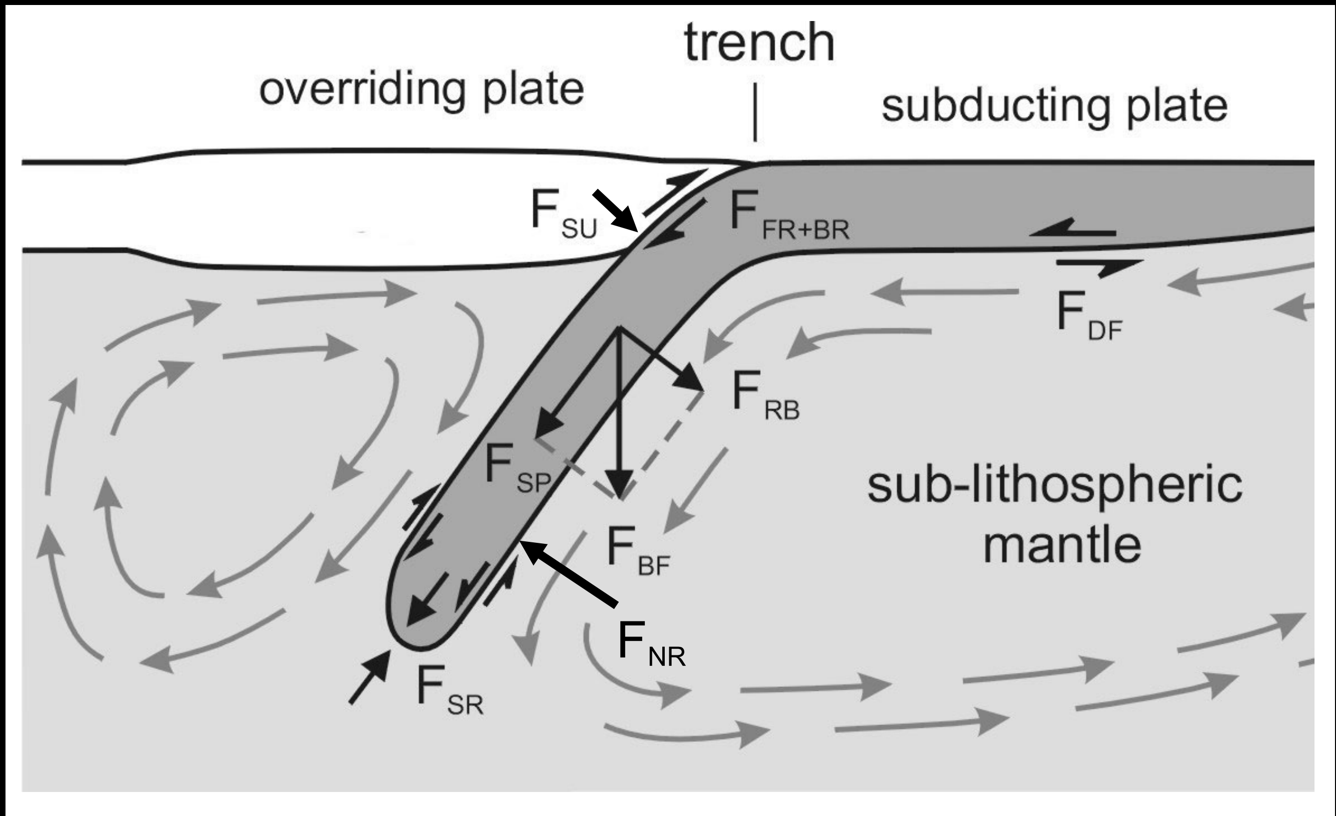
Kinematic similarity: ***Model & nature have to undergo similar changes of shape and/or position, where the time required for change in the model is proportional to that for the corresponding change in nature.***



Analogue subduction experiment with free trailing edge [Schellart JGR 2004]

Geometric, kinematic & dynamic similarity

Dynamic similarity: ***Similar distribution of driving forces and resistive forces in model and nature.***



Driving force (F_{BF}) and resistive forces (e.g. F_{SR}, F_{BR}, F_{NR}, F_{DF}, F_{FR}) in a subduction zone.

For example:

$$(F_{SR} / F_{BF})^{MODEL} = (F_{SR} / F_{BF})^{NATURE}$$

Scaling of length & time in mechanical models

Length (l) choose a convenient length scale):

$$\frac{l_1^m}{l_1^p} = \frac{l_2^m}{l_2^p} = \frac{l_3^m}{l_3^p} = \frac{l_n^m}{l_n^p}$$

Angles (α) no choice:

$$\frac{\alpha_1^m}{\alpha_1^p} = \frac{\alpha_2^m}{\alpha_2^p} = \frac{\alpha_3^m}{\alpha_3^p} = \frac{\alpha_n^m}{\alpha_n^p} = 1$$

Time (t) choose a convenient time scale:

$$\frac{t_1^m}{t_1^p} = \frac{t_2^m}{t_2^p} = \frac{t_3^m}{t_3^p} = \frac{t_n^m}{t_n^p}$$

Note for time scale: In practice depends on rheology

-For brittle only, free choice except that F_i = negligible.

-For viscous, time is scaled from viscosity.

(superscript *m* for model, superscript *p* for natural prototype)

Scaling of density or density contrast in mechanical models

Density (ρ), choose a convenient density scale:

$$\frac{\rho_{CC}^{m}}{\rho_{CC}^{p}} = \frac{\rho_{CLM}^{m}}{\rho_{CLM}^{p}} = \frac{\rho_{OL}^{m}}{\rho_{OL}^{p}} = \frac{\rho_{SLUM}^{m}}{\rho_{SLUM}^{p}}$$

Density contrast ($\Delta\rho$), choose a convenient density scale:

$$\frac{\left(\rho_{SLUM}^{m} - \rho_{CC}^{m}\right)}{\left(\rho_{SLUM}^{p} - \rho_{CC}^{p}\right)} = \frac{\left(\rho_{SLUM}^{m} - \rho_{CLM}^{m}\right)}{\left(\rho_{SLUM}^{p} - \rho_{CLM}^{p}\right)} = \frac{\left(\rho_{SLUM}^{m} - \rho_{OL}^{m}\right)}{\left(\rho_{SLUM}^{p} - \rho_{OL}^{p}\right)}$$

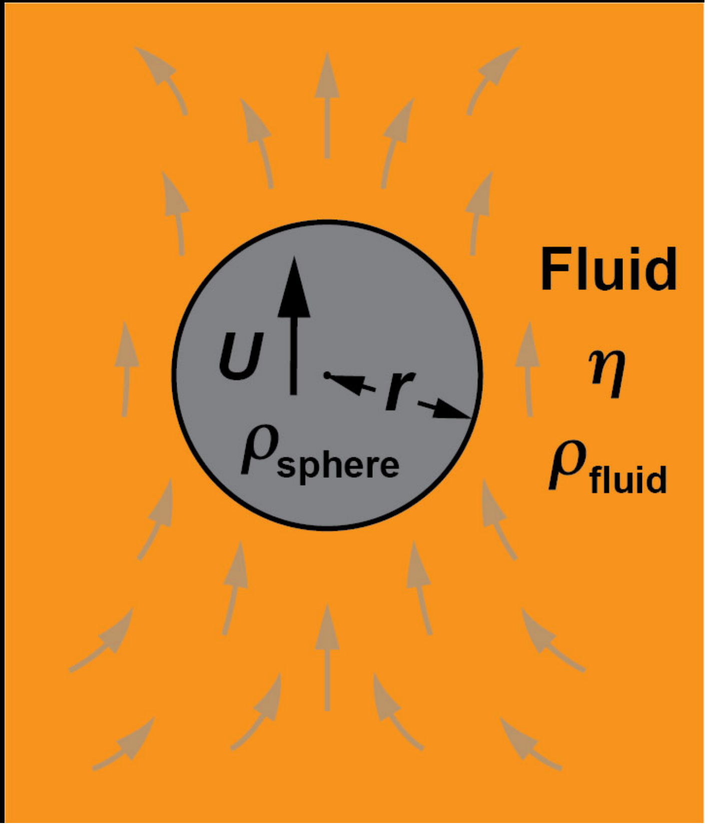
In analogue experiments, in practice, rheologically suitable materials are chosen, and densities might be altered through adding dense/light fillers or by diluting.

Scaling of velocity, stresses, viscosity, time in mechanical models using density contrasts

Use Stokes solution for a rising/sinking solid sphere

Exact solution to calculate the vertical velocity *U* of a solid sphere in an infinite volume, linear-viscous (Newtonian), fluid at low Reynolds number (Re <<1) during steady-state flow:

$$U = \frac{2r^2 \Delta\rho g}{9\eta}$$



U = vertical velocity (upward is positive) [m/s]

r = radius of the sphere [m]

$\Delta\rho$ = density contrast ($\rho_{fluid} - \rho_{sphere}$) [kg/m³]

g = gravitational acceleration (~9.8 m/s²)

η = dynamic shear viscosity of surrounding fluid [Pa s]

Approximate solution for solid ellipsoid

***Stokes-like rising/sinking of a solid ellipsoid in an infinite volume viscous fluid at low Reynolds number (Re << 1)* [*Kerr and Lister*, 1991]:**

$$u = \frac{S(D^*)^2 \Delta\rho g}{18\eta}$$

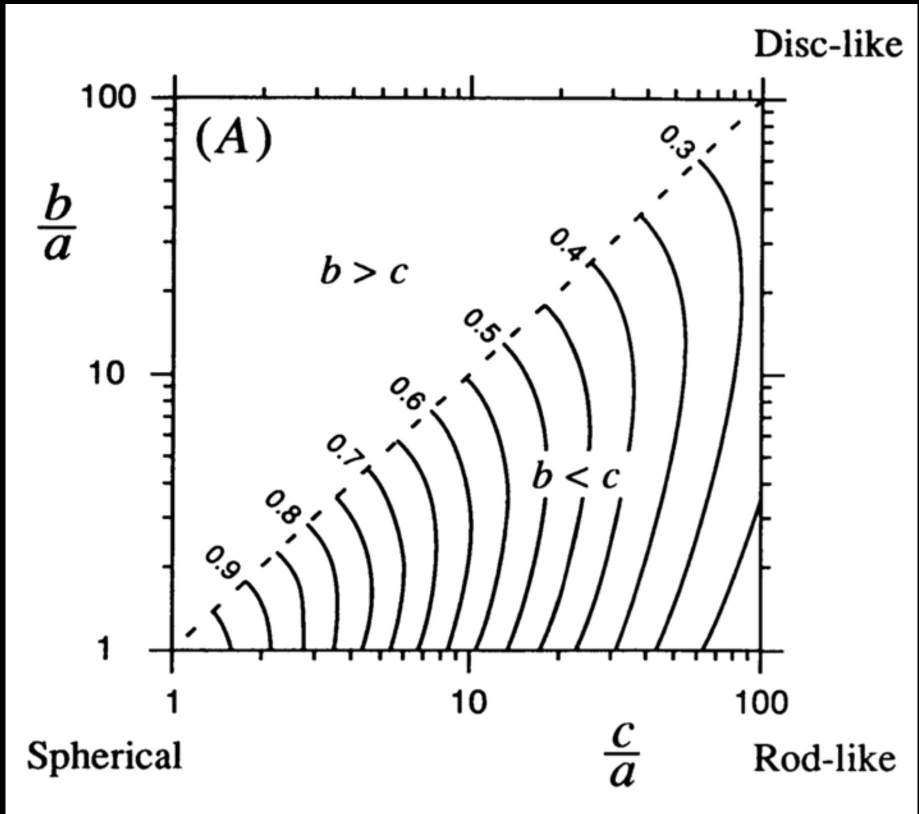
S	**= shape factor**
D*	**= effective diameter = (*abc*)$^{1/3}$**
a,b,c	**= axes of ellipsoid (normally the convention is: *a* ≤ *b* ≤ *c*)**

In the special case where *a* = *b* = *c* (sphere), then *S* = 1 and:

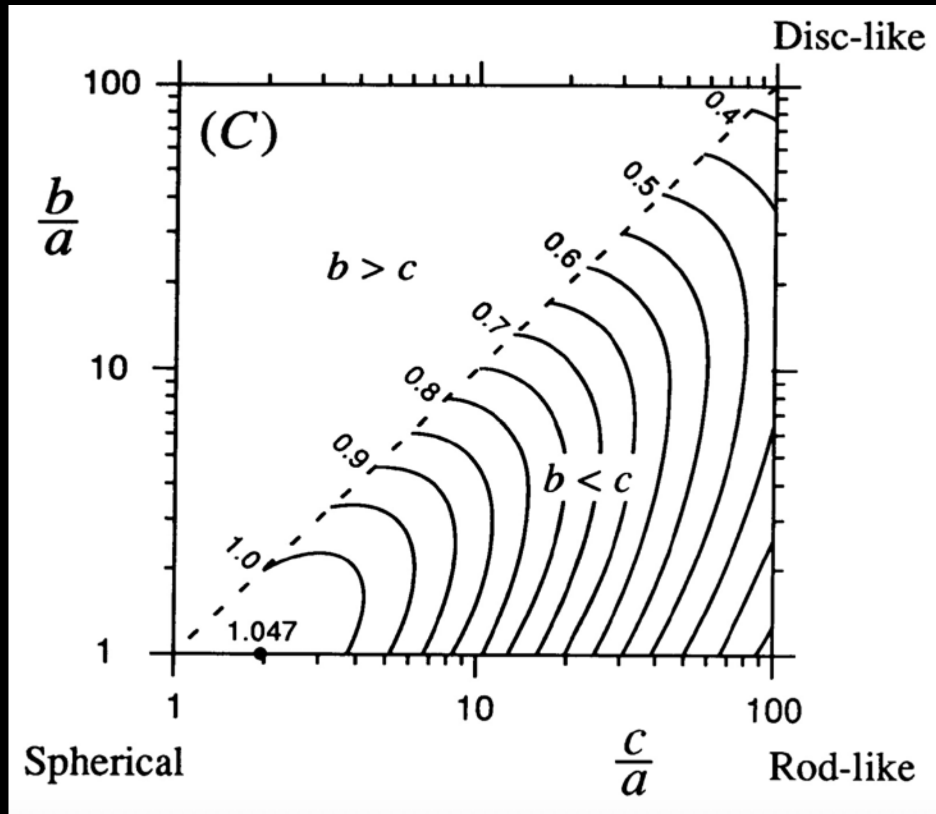
$$u = \frac{S\left(\left[8r^3\right]^{1/3}\right)^2 \Delta\rho g}{18\eta} = \frac{2r^2 \Delta\rho g}{9\eta}$$

Solid ellipsoid

a-axis is vertical



c-axis is vertical



Contours of the shape factor *S* for a sinking/rising ellipsoid with axes $a \leq b \leq c$ [Kerr and Lister, J Geol. 1991].

Scaling of velocity, stresses, viscosity, time in mechanical models using density contrasts

Approximate Stokes solution for sinking rigid object at Re << 1:

$$v \sim C\frac{\Delta\rho l^2 g}{\eta}$$

Writing for model (*m*) and natural prototype (*p*):

$$\frac{\Delta\rho^m\left(l^m\right)^2 g^m}{\eta^m v^m} = \frac{\Delta\rho^p\left(l^p\right)^2 g^p}{\eta^p v^p}$$

Writing for velocity (*v*):

$$\frac{v^m}{v^p} = \frac{\Delta\rho^m (l^m)^2 g^m \eta^p}{\Delta\rho^p (l^p)^2 g^p \eta^m}$$

For lab experiments, with $g^m = g^p$

C = constant
g = gravitational acceleration
η = dynamic shear viscosity

Scaling of velocity, stresses, viscosity, time in mechanical models using density contrasts

$$\frac{\Delta\rho^m (l^m)^2 g^m}{\eta^m v^m} = \frac{\Delta\rho^p (l^p)^2 g^p}{\eta^p v^p}$$

Writing for viscosity, with *v ~ l/t*:

$$\frac{\eta^m}{\eta^p} = \frac{\Delta\rho^m \cancel{g^m} l^m t^m}{\Delta\rho^p \cancel{g^p} l^p t^p}$$

Note: Time can be scaled when setting the viscosity ratio

Writing for stress (σ), with $\sigma \sim \eta/t$:

$$\frac{\sigma^m}{\sigma^p} = \frac{\Delta\rho^m \cancel{g^m} l^m}{\Delta\rho^p \cancel{g^p} l^p}$$

For lab experiments, with $g^m = g^p$

Scaling of stresses in mechanical models using densities

Cauchy's equation of motion:

$$\rho \frac{D^2 x_i}{Dt^2} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i \qquad (i, j = 1, 2, 3)$$

Negligible inertial forces, resulting in:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = -\rho g_i$$

Integrating w.r.t. x_j, with boundary conditions $\sigma_{ij} = 0$ at $x_j = 0$:

$$\sigma_{ij} = -\rho g_i x_j$$

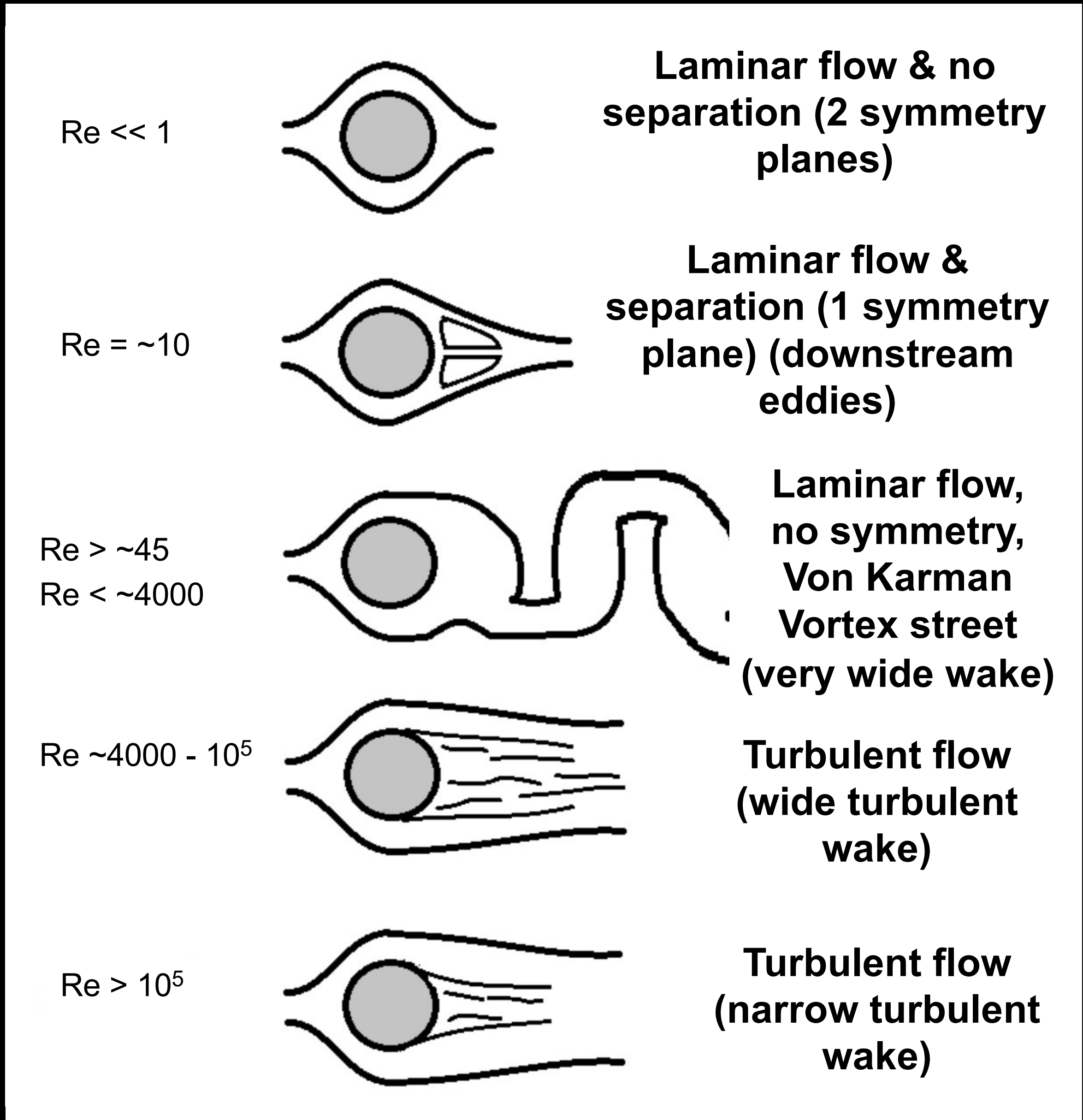
Writing for model (m) and natural prototype (p), with x_j being just a length scale l:

$$\frac{\sigma_{ij}^m}{\sigma_{ij}^p} = \frac{\rho^m \cancel{g^m} l^m}{\rho^p \cancel{g^p} l^p}$$

For lab experiments, with $g^m = g^p$

Reynolds number & dynamic flow regime

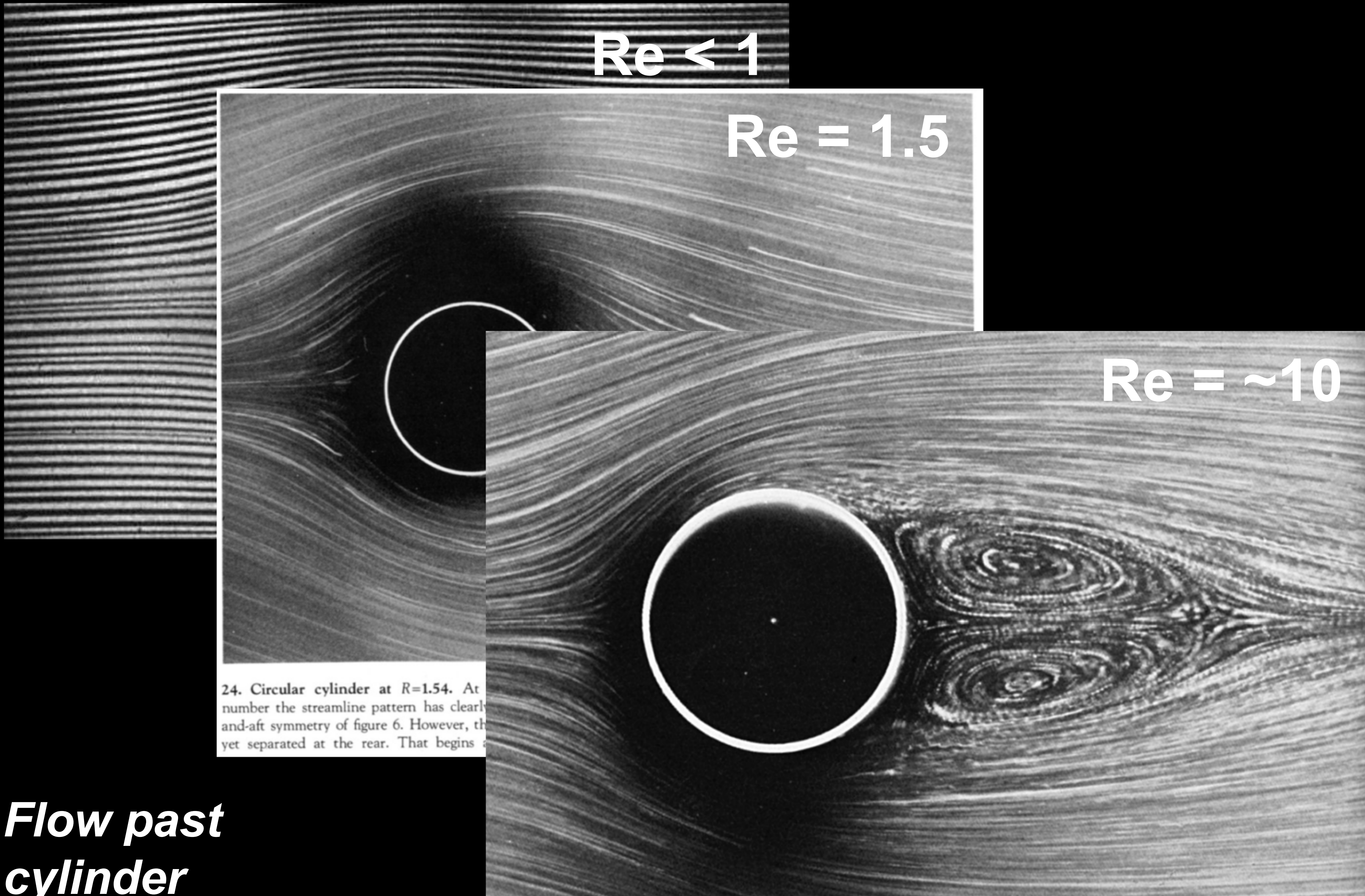
Dynamic similarity requires same flow regime



$$Re = d\rho v/\eta$$

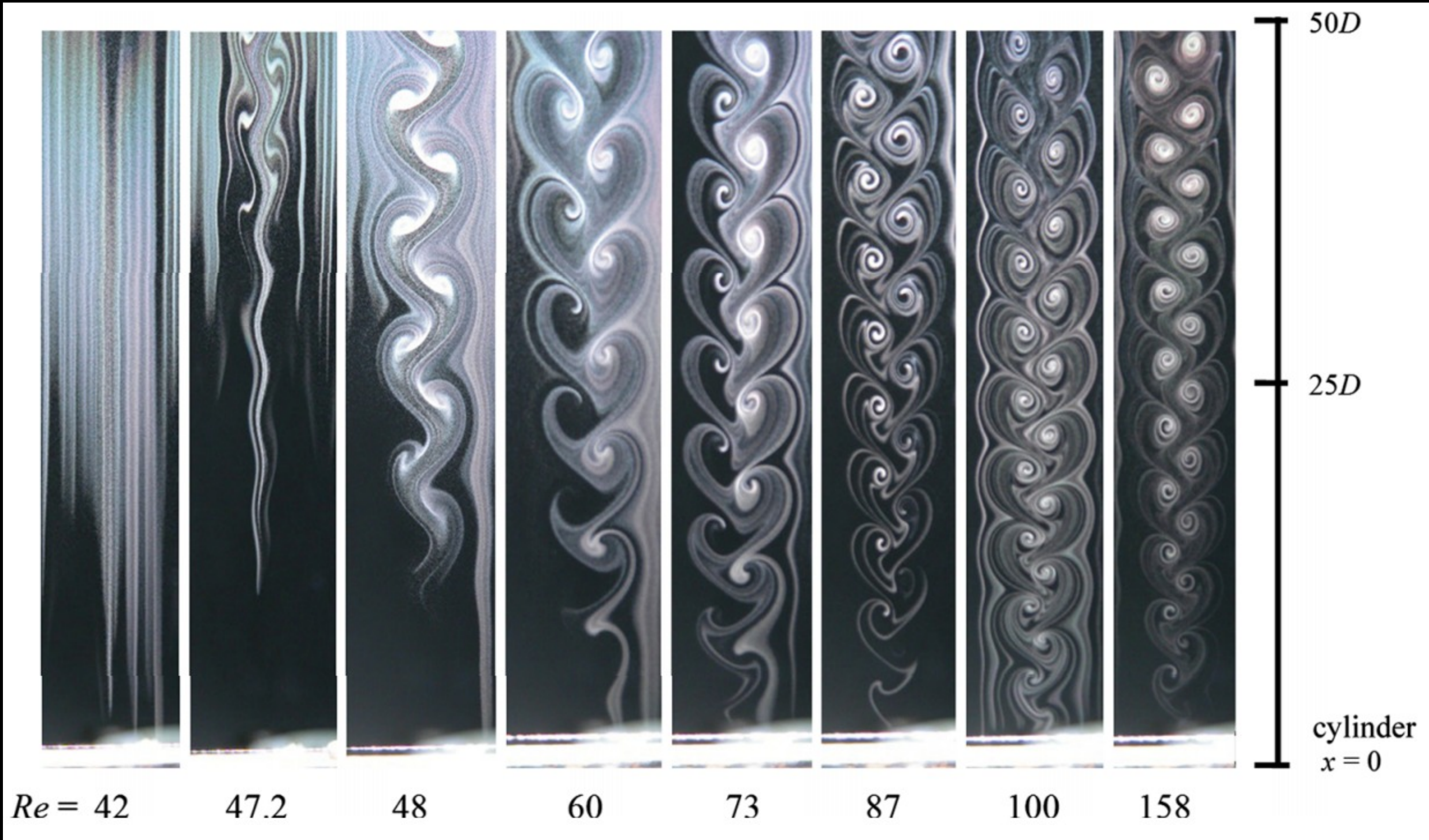
d **= characteristic length scale (e.g. diameter) of object;**
$\boldsymbol{\rho}$ **= density of fluid;**
v **is velocity of object w.r.t. fluid;**
$\boldsymbol{\eta}$ **= dynamic shear viscosity of fluid.**

Reynolds number & dynamic flow regime



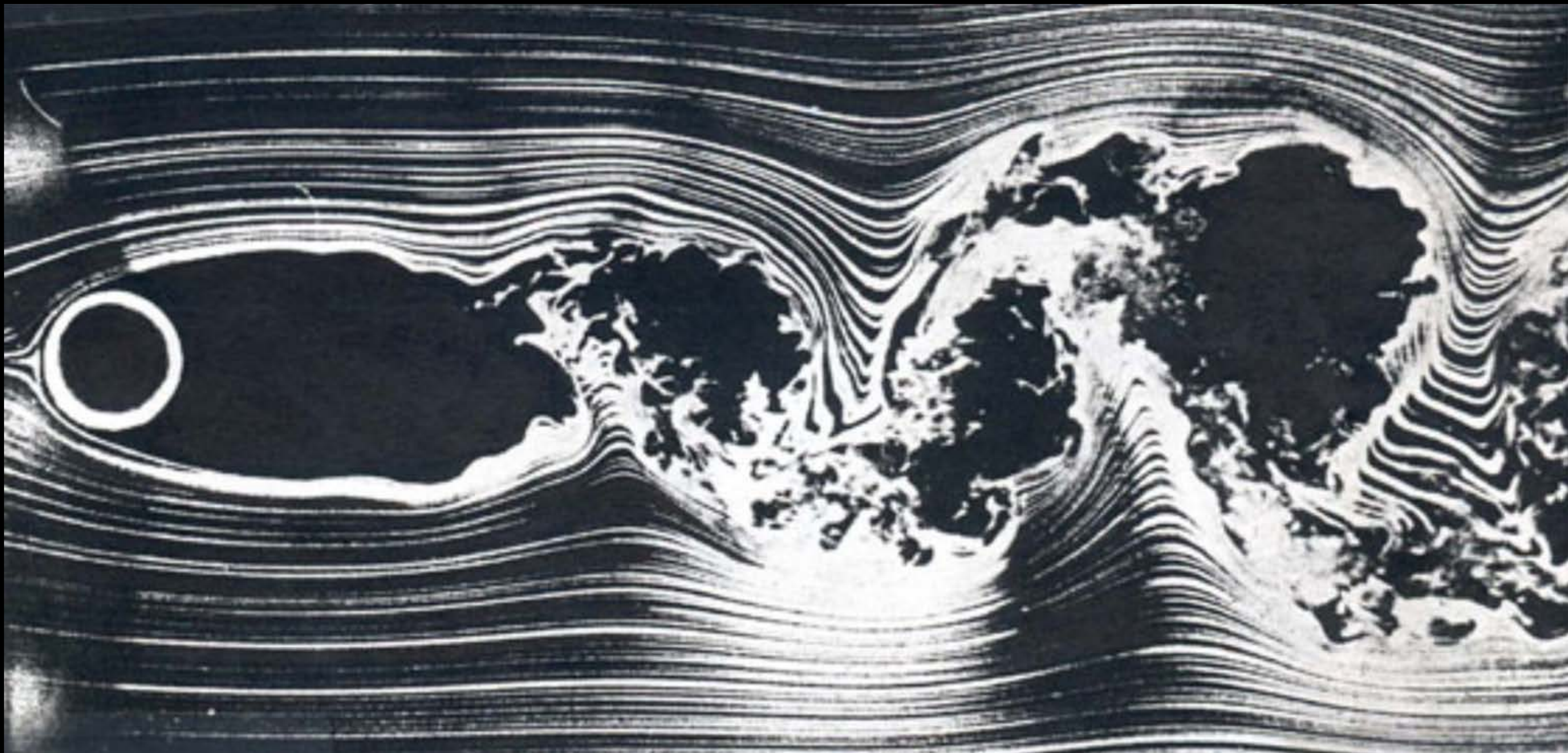
Flow past cylinder

Reynolds number & dynamic flow regime



Flow past cylinder   Von Karman vortex street

Reynolds number & dynamic flow regime



Turbulent flow (Re = ~10^4)

Flow past cylinder

Reynolds number & dynamic flow regime

$$\mathbf{Re} = d\rho v/\eta$$

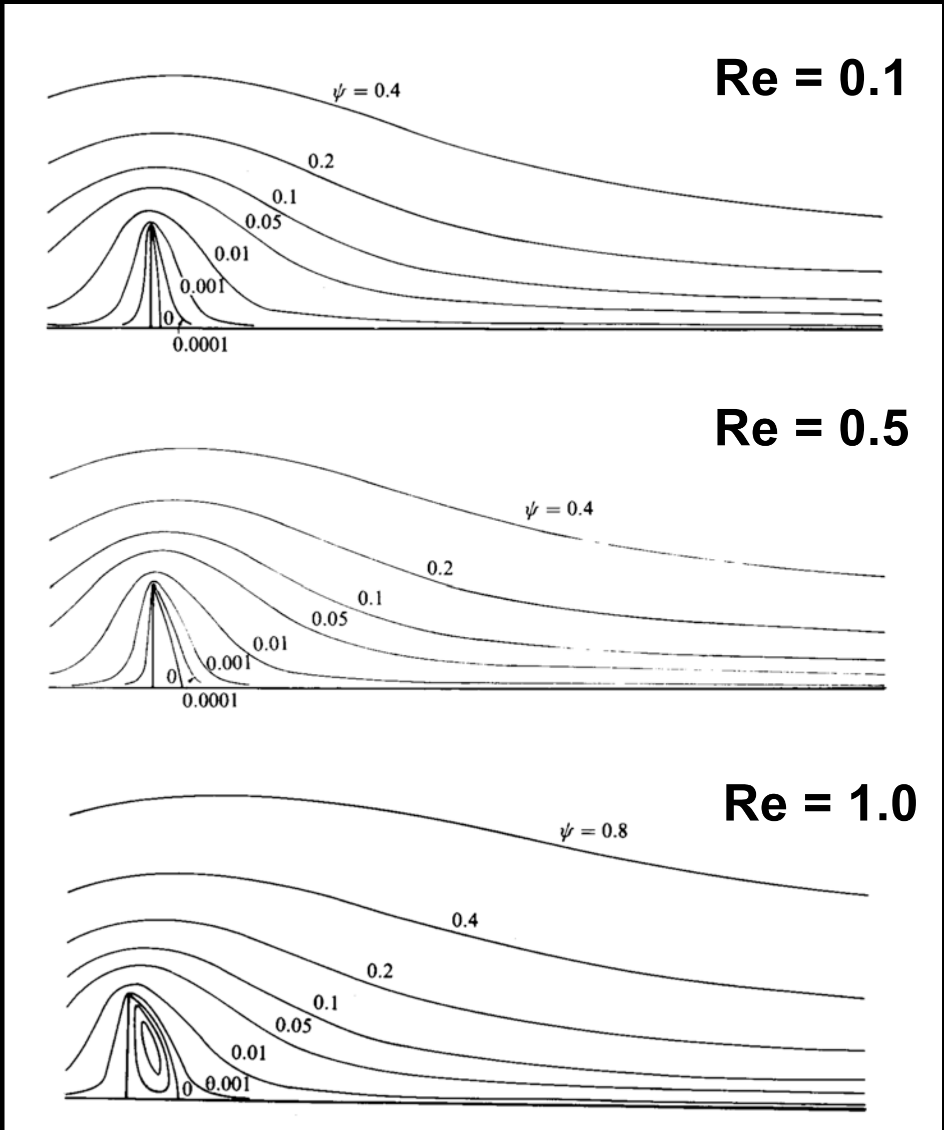
For subducted slab on Earth:

d = ~10^5 m
ρ = ~3300 kg/m^3
v = ~0.1/(3600*24*365) = ~3.2×10^{-9} m/s
η = ~10^{20} Pa s.

Re = ~ 1.0×10^{-20} <<<<<<<<< 1

Reynolds number & dynamic flow regime

Flow past vertical plate [Hudson and Dennis, *J. Fluid Mechanics*, 1985]



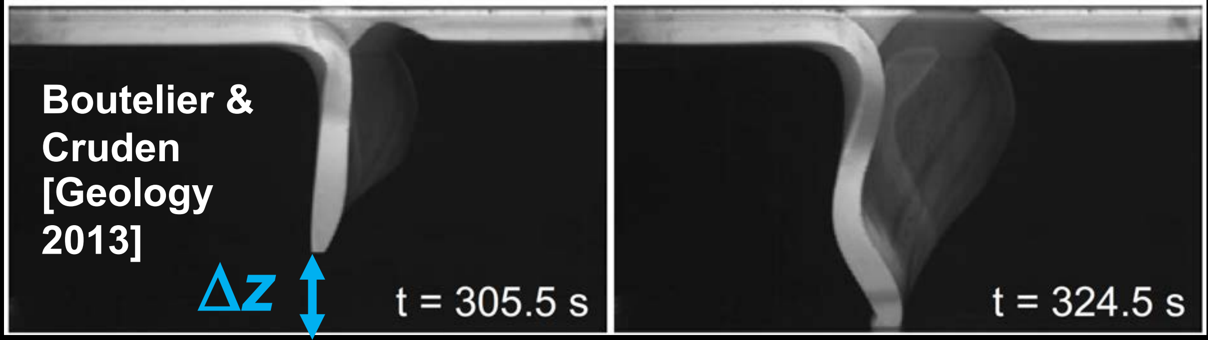
Small asymmetry develops

Asymmetry is well pronounced

Flow separation in wake of plate with formation of eddy

Exercise: *Re & flow regime*
in 3 subduction experiments

Re = $d\rho v/\eta$



***Boutelier & Cruden*:**

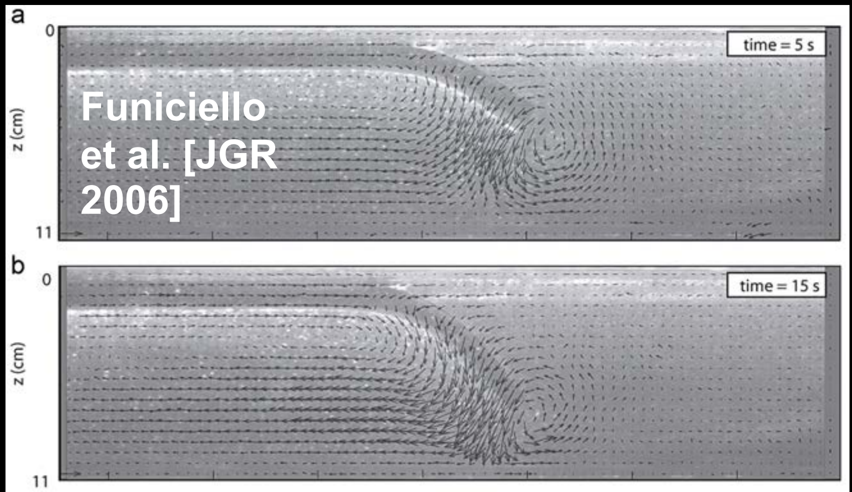
d = 17 cm (L) or 40 cm (W)

ρ = 1000 kg/m^3

Δz = 5.7 cm, Δt = 19 s, v = ….

η = 0.001 Pa s

Re_L = ~ ….. & Re_W = ~ …..



***Funiciello et al.*:**

d = 10 cm (length L) or 30 cm (width W)

ρ = 1382 kg/m^3

Δz = 2.5 cm, Δt = 10 s, so v = ….. m/s

η = 1 Pa s

Re_L = ~ ….. & Re_W = ~ …..

***Xue et al.*:**

d = 5 cm (L) or 10 cm (W)

ρ = 1408 kg/m^3

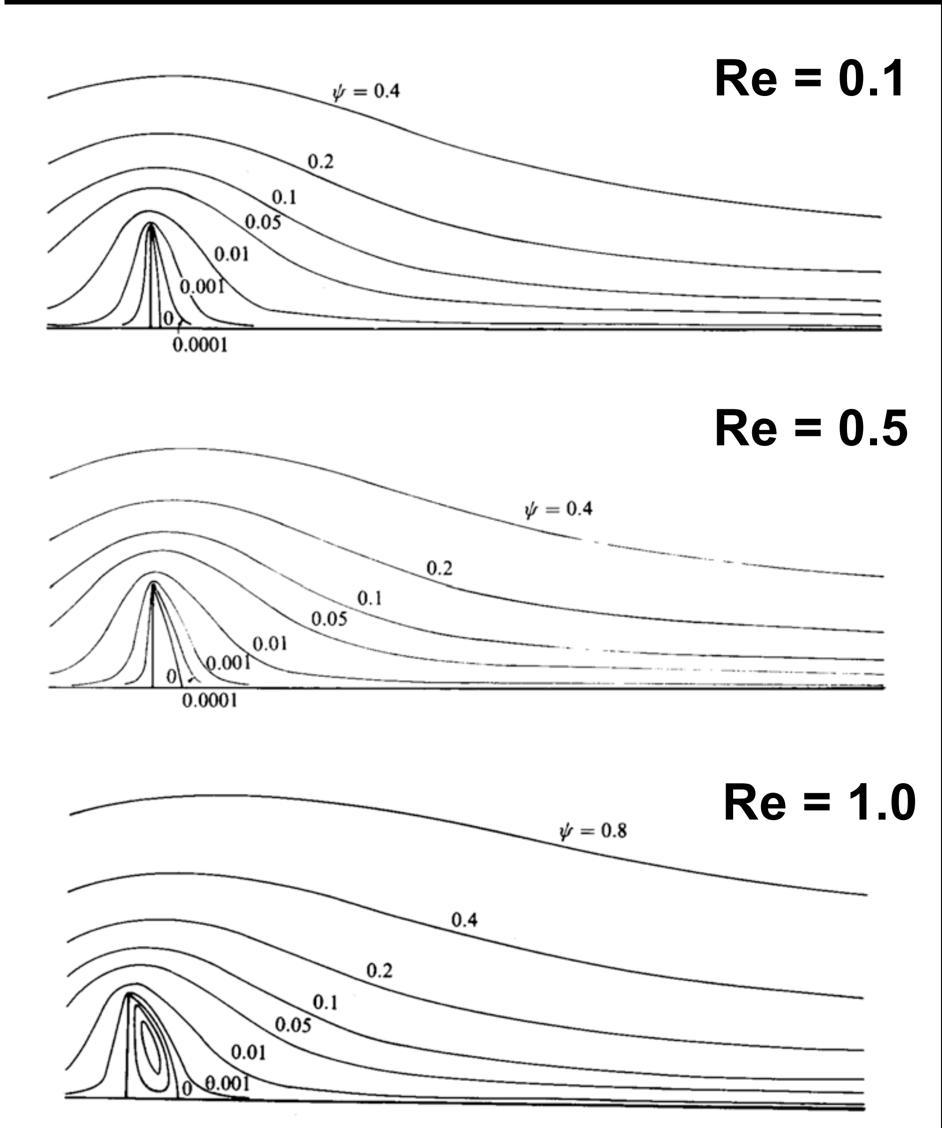
Δz = 4.0 cm, Δt = 280 s, so v = ….. m/s

η = 38 Pa s

Re_L = ~ ….. & Re_W = ~ …..

Exercise: *Re & flow regime*

$Re = d\rho v/\eta$



	Re_L	Re_W
Boutelier	510	1200
Funiciello	0.35	1.04
Xue	2.6×10^{-4}	5.3×10^{-4}

[Hudson and Dennis, *J. Fluid Mechanics*, 1985]

Rheological similarity

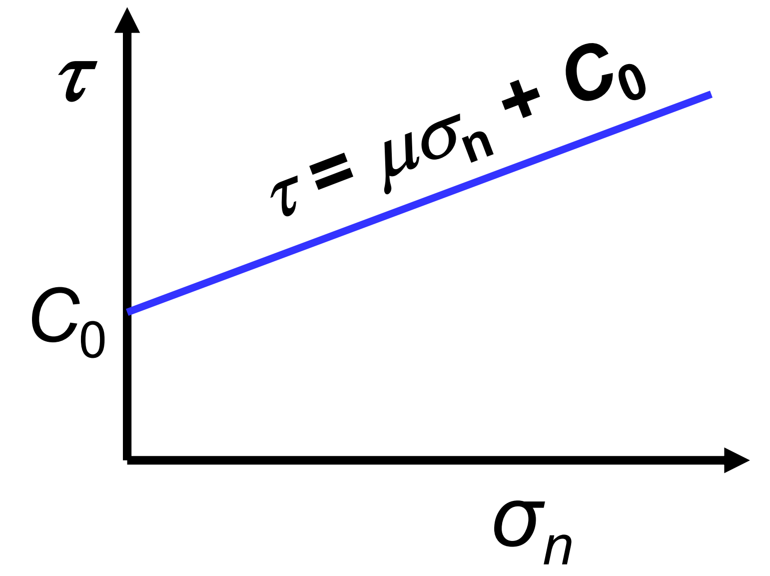
Dynamic similarity requires rheological similarity.

For brittle materials: $\tau = \mu\sigma_n + C_0$

(Coulomb failure envelope)

As μ is non-dimensional, then: $\mu^m = \mu^p$

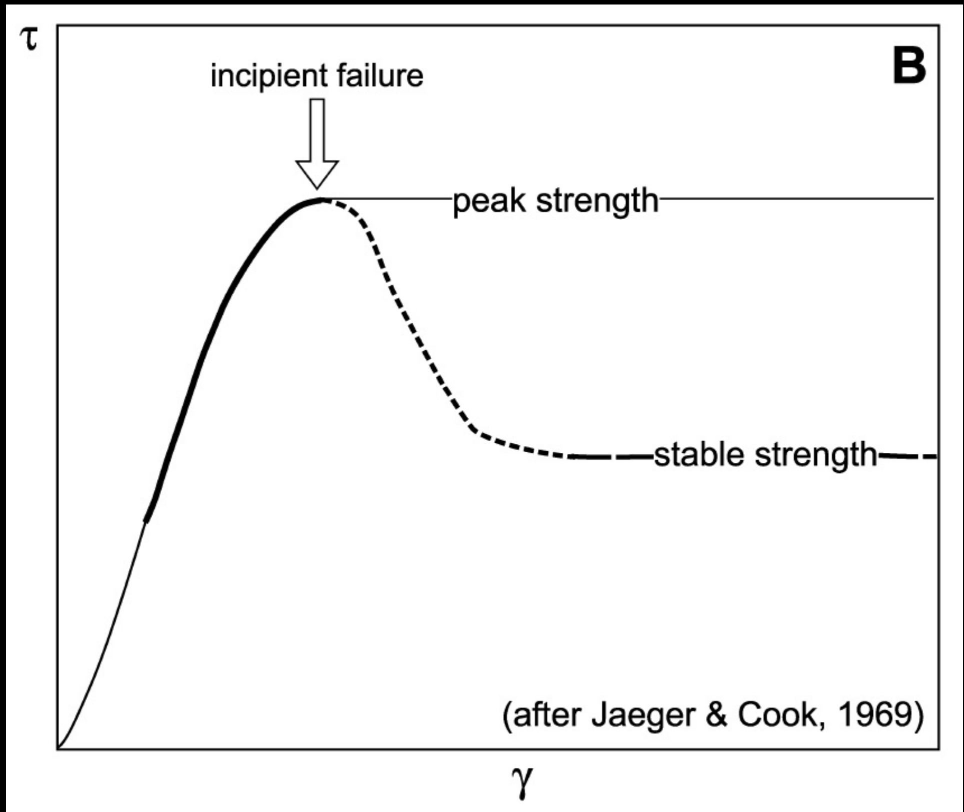
As C_0 has the dimensions [Pa], it scales as stresses.

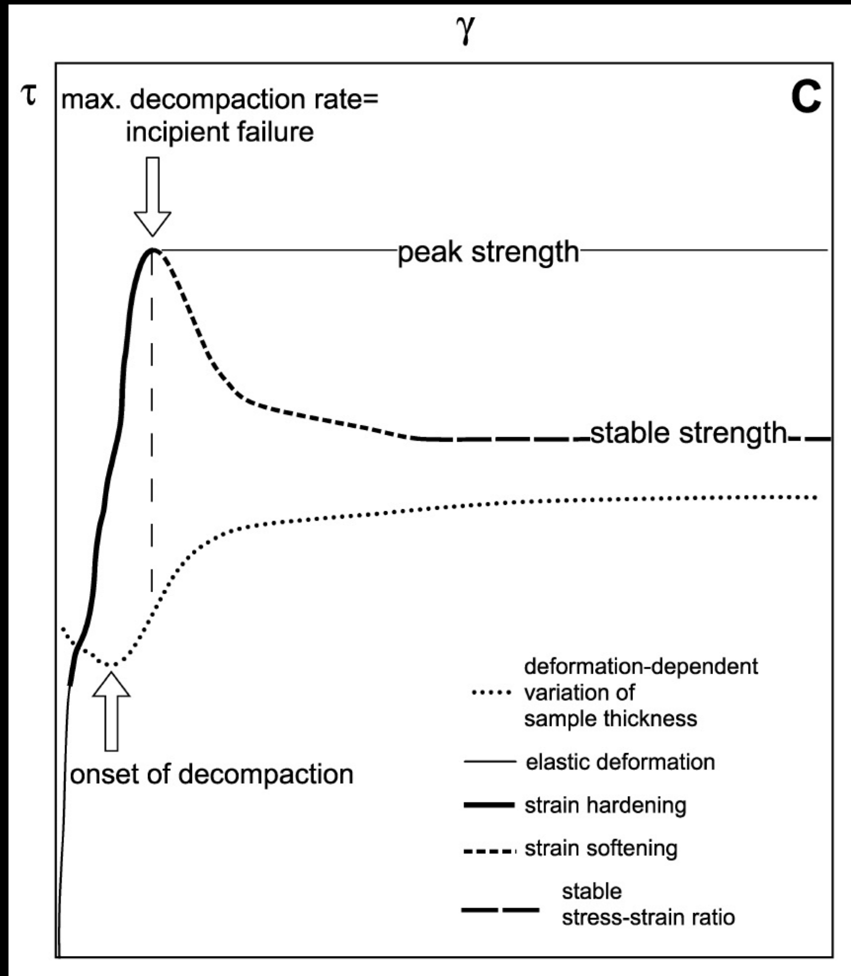


τ = shear stress
μ = friction coefficient
σ_n = normal stress
C_0 = cohesion

Rheological similarity

Brittle (frictional-plastic) materials in analogue models



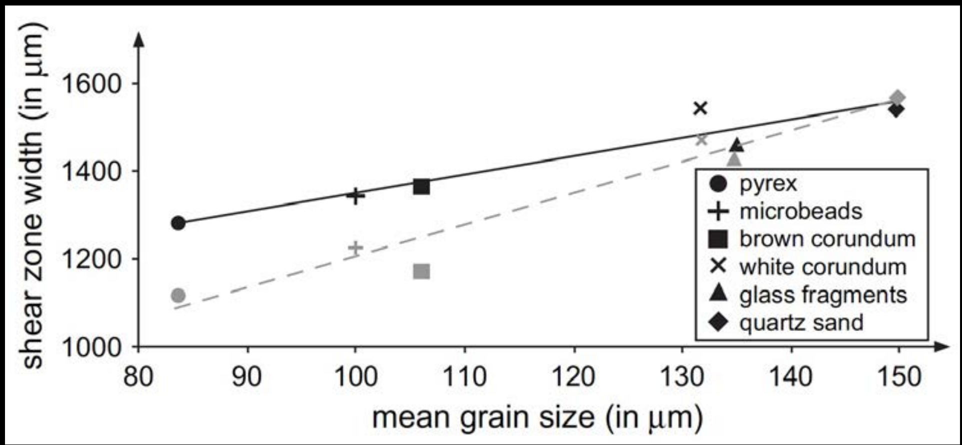


Shear stress in rocks (left) and granular material (right) with increasing angular shear [Lohrmann et al., JSG 2003].

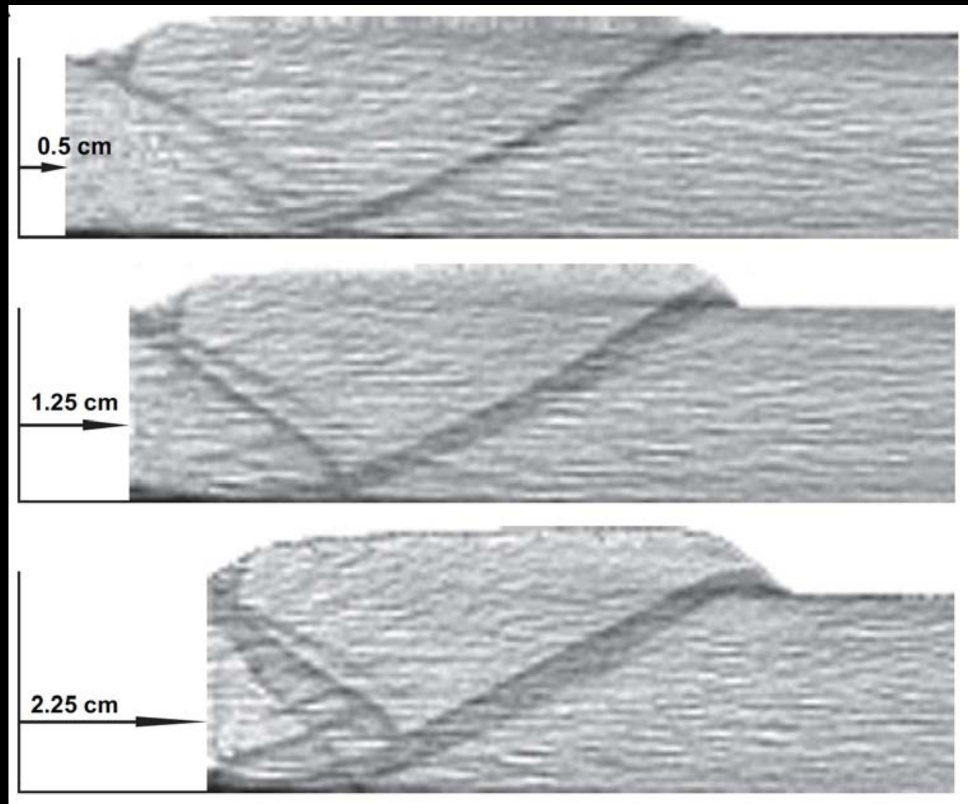
Rocks and granular materials show comparable rheological behaviour.

Rheological similarity

Faults/shear zones in brittle (frictional-plastic) materials in analogue experiments



Initial shear zone thickness as a function of mean grain size, with initial thickness ~ 11-16 x mean grain size [Panien et al., JSG 2006].



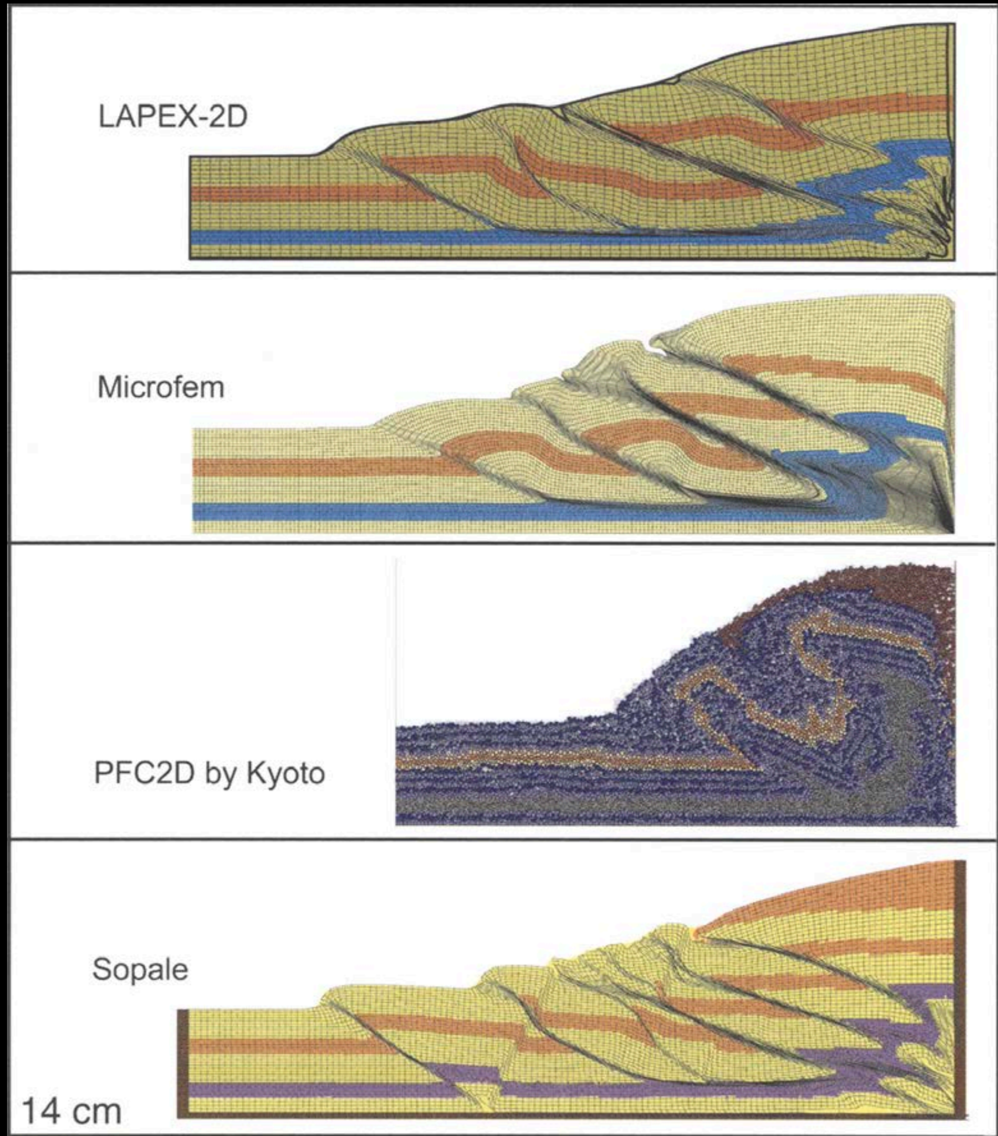
Cross-sections with CT-scanner of thrust experiment showing increasing thickness of shear zones with progressive deformation [Panien et al., JSG 2006].

Rheological similarity

Faults/shear zones in visco-plastic materials in numerical models

Glarus thrust, Alps: Triassic rocks on top of Eocene rocks

Shear strain $\gamma \geq 10\,000$ with thrust fault thickness = ~1 m & thrust displacement ≥ 10 km.



[Buiter et al., GSSP 2006]

Rheological similarity

Dynamic similarity requires rheological similarity.

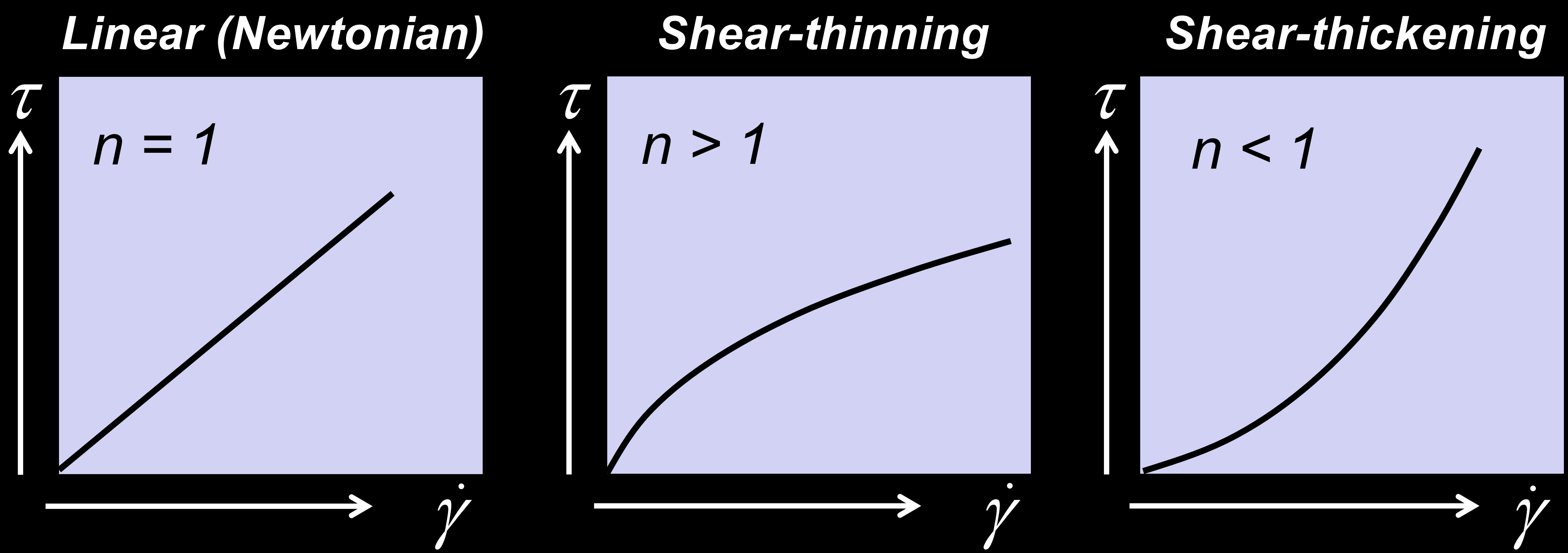
For viscous materials: $$\tau^n = \eta\,\dot{\gamma}$$

τ = stress;
n = non-dimensional stress exponent;
$\dot{\gamma}$ = strain rate;
η = the dynamic shear viscosity

Linear viscous for n = 1

n is non-dimensional, so: $n^m = n^p$

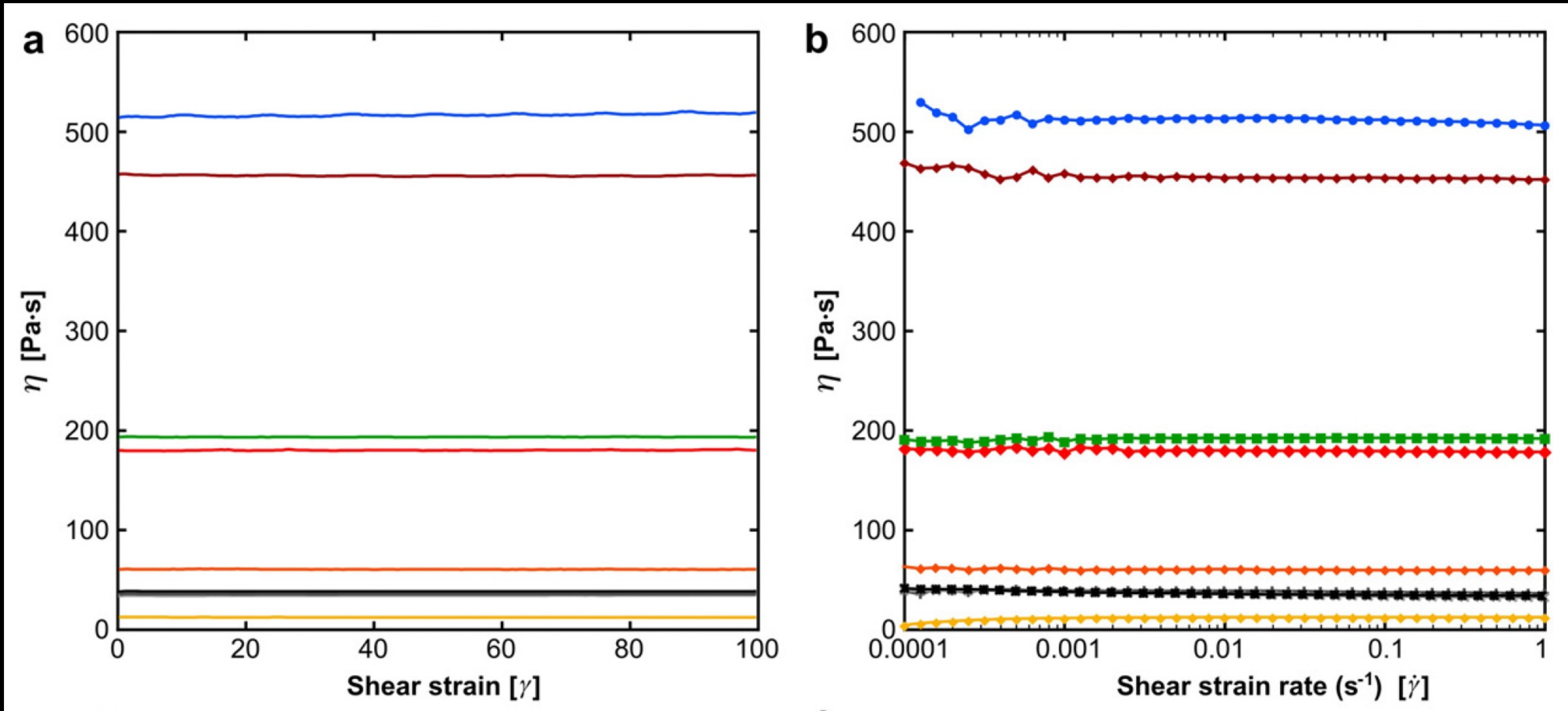
Strain rate [s^{-1}] scales with the inverse of time.



Rheological similarity

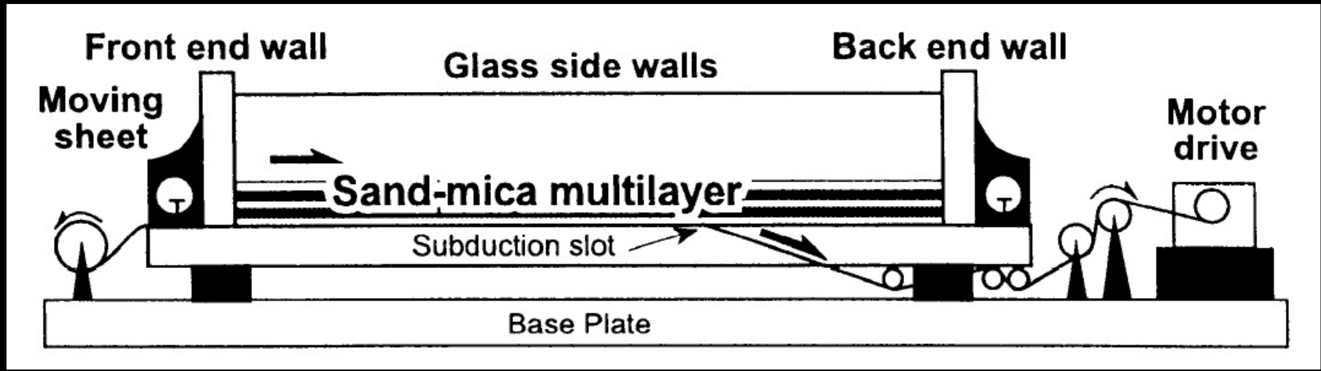
Linear viscous (Newtonian) materials in analogue models

Glucose syrups & honeys



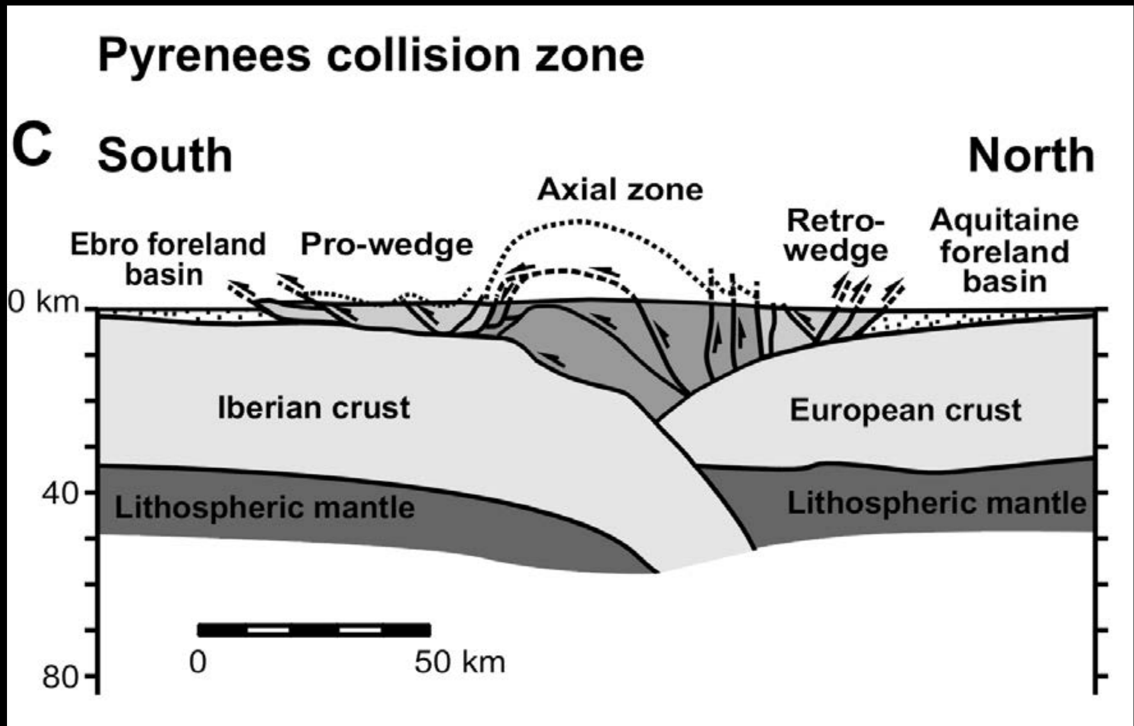
Viscosity of several glucose syrups and honeys, showing it is not dependent on shear strain and shear rate. [*Schellart*, JSG 2011]

Topography in geodynamic models

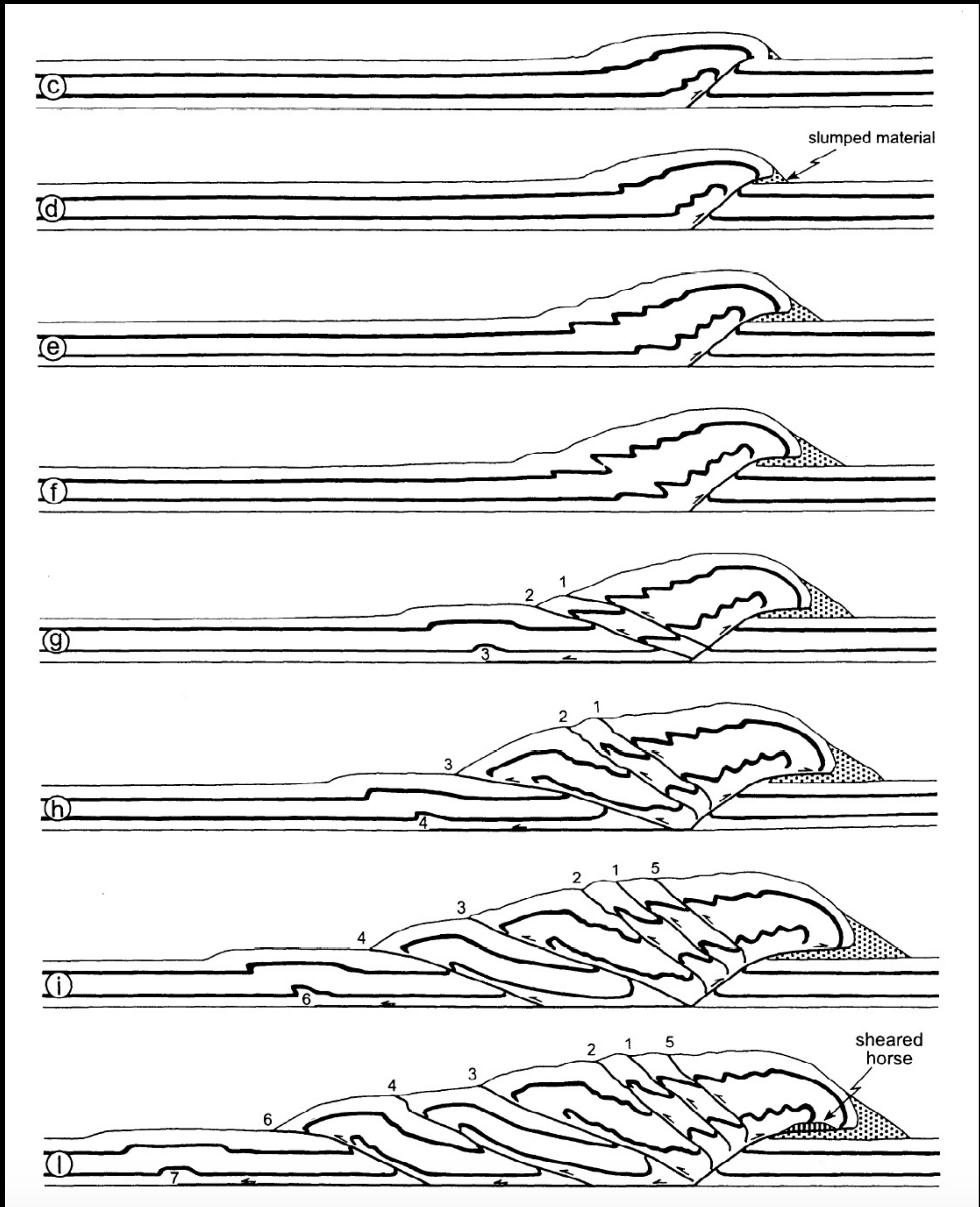


Model set-up to simulate Pyrenees orogeny

[Storti et al., Tectonics 2000]

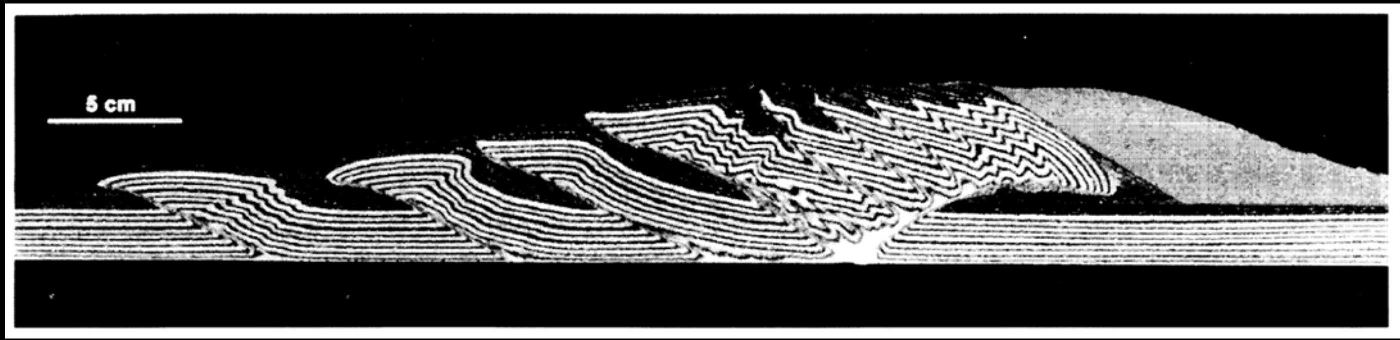


Topography in geodynamic models



Model results
[Storti et al., Tectonics 2000]

Topography in geodynamic models

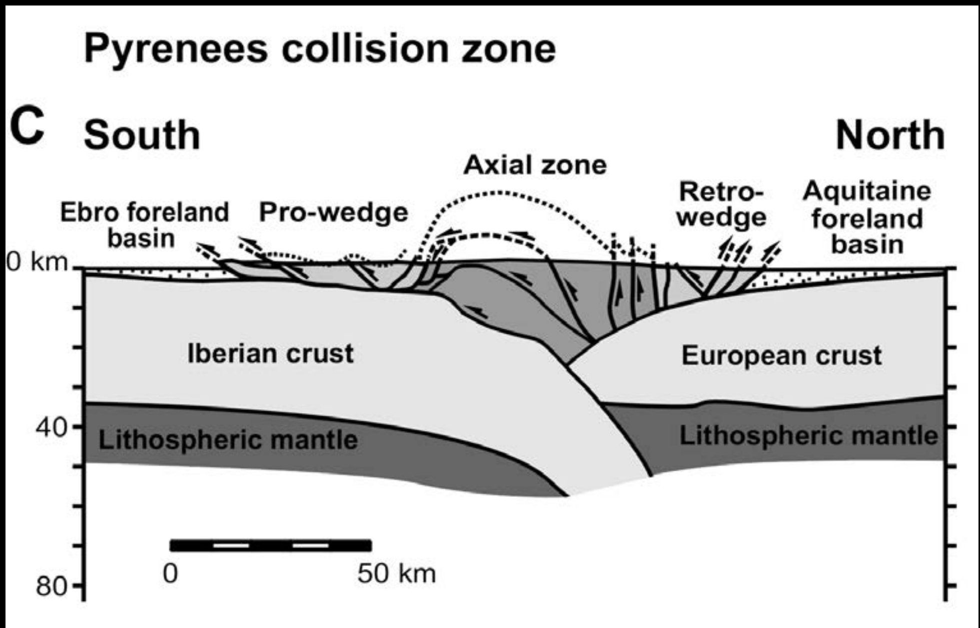


Final stage [Storti et al., Tectonics 2000]

-Width orogen 42 km (42 cm);
-Total shortening 52 km (52 cm);
-Height central zone 5 km (5 cm).

Difference in topography between model & nature?

-Isostatic compensation
-Erosion



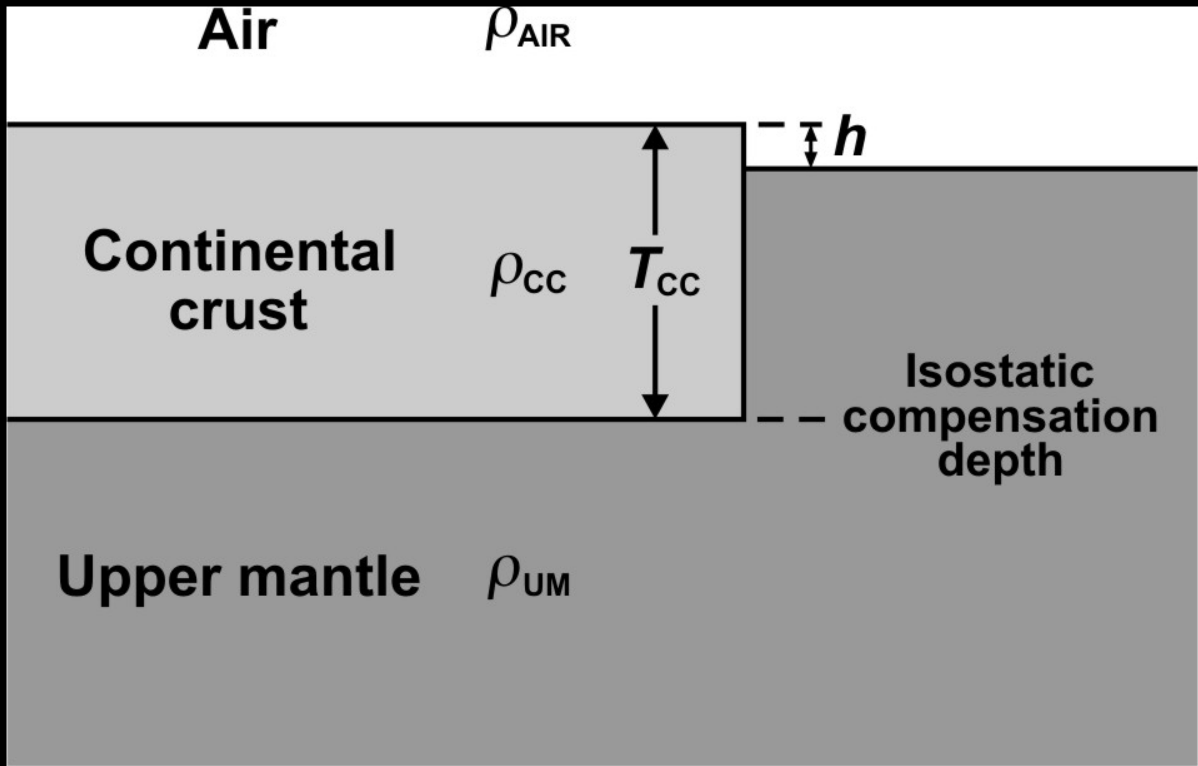
Pyrenees

-Width orogen 150 km;
-Total shortening ~150 km;
-Height central part ~2.5 km.

Scaling of topography when using ρ or $\Delta\rho$

Calculating elevation h of crustal layer assuming local isostacy:

$$\rho_{CC} g T_{CC} = \rho_{UM} g \left(T_{CC} - h\right) + \rho_{AIR} g h$$



Scaling of topography when using ρ

Rearranging to write for elevation *h*:

$$h = T_{CC}\frac{(\rho_{UM} - \rho_{CC})}{(\rho_{UM} - \rho_{AIR})}$$

Since ρ_{AIR} = ~10^{-3} x ρ_{UM} & ρ_{CC},
this simplifies to:

$$h = T_{CC}\frac{(\rho_{UM} - \rho_{CC})}{\rho_{UM}}$$

Rearranging again:

$$h = T_{CC}\left(1 - \frac{\rho_{CC}}{\rho_{UM}}\right)$$

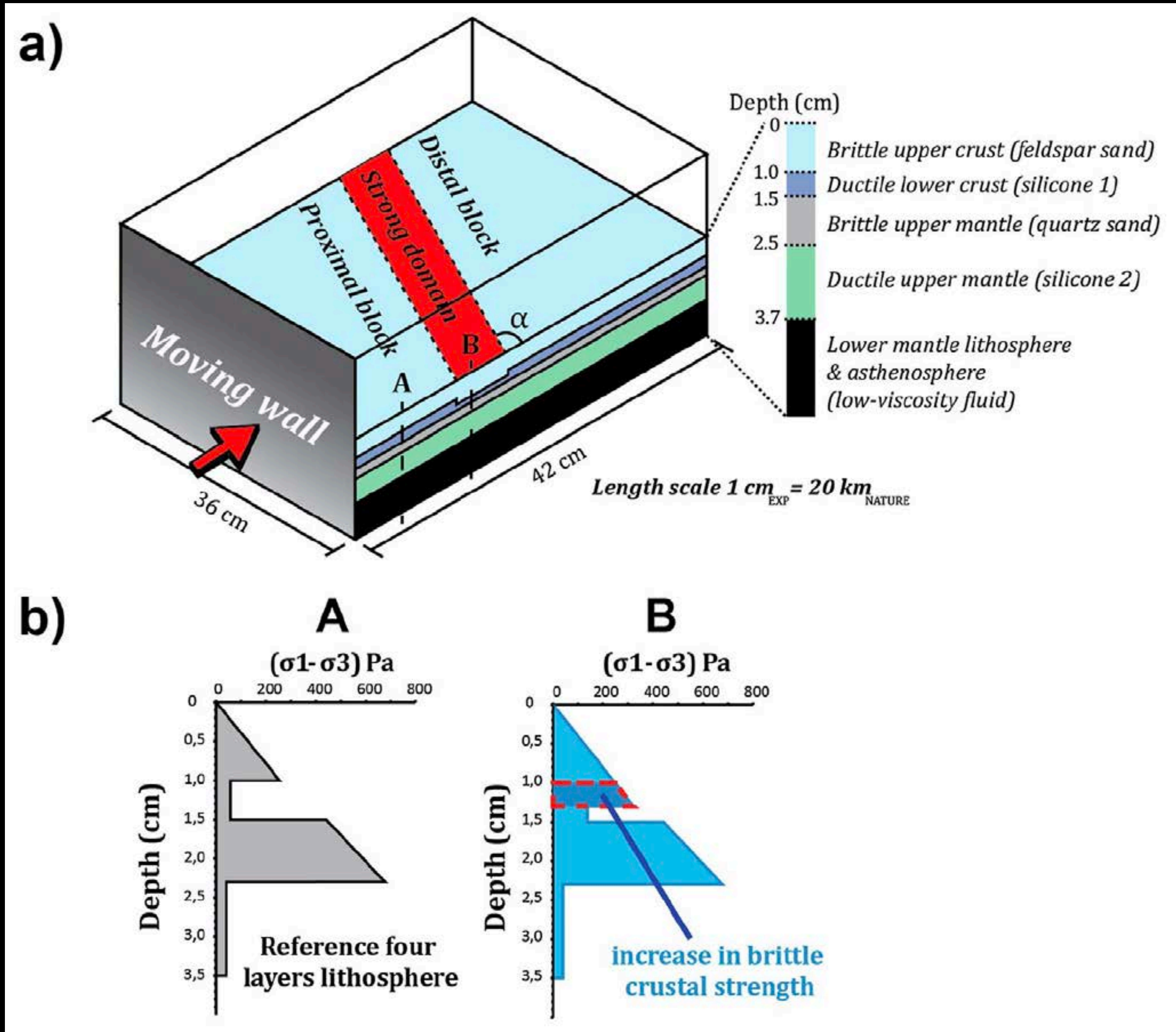
Writing for model & nature, with
$(1-\rho_{CC}/\rho_{UM})^m = (1-\rho_{CC}/\rho_{UM})^p$:

$$\frac{h^m}{h^p} = \frac{T_{CC}^m}{T_{CC}^p}$$

As T_{CC} is just a length scale we get:

$$h^p = \frac{l^p}{l^m}h^m$$

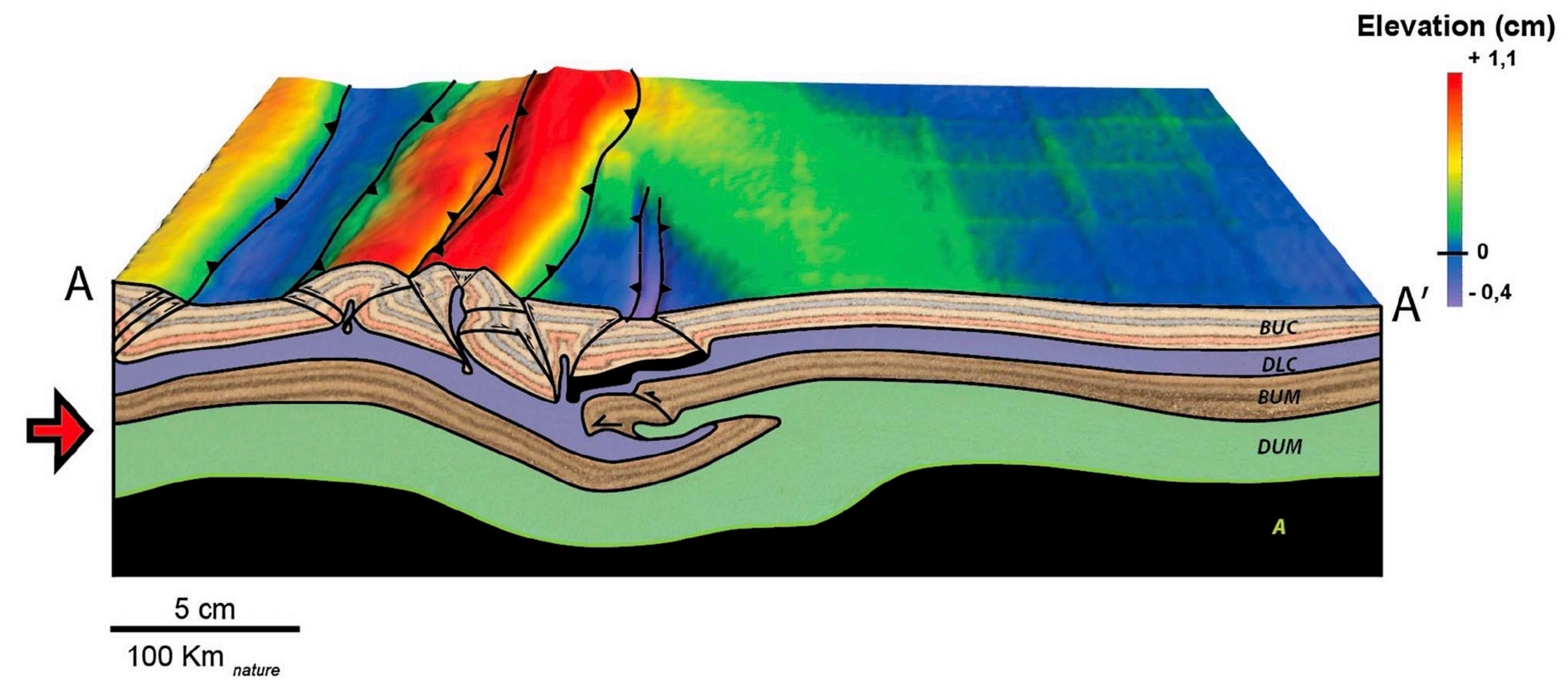
Scaling of topography **when using ρ**



Analogue 4D model of lithospheric shortening with external velocity boundary condition & isostatic support.

Calignano et al. [Tectonics 2017]

Scaling of topography when using ρ



Late stage showing surface topography & cross-sectional structure
[*Calignano et al.* Tectonics 2017]

Length scaling: 1 cm represents 20 km

Maximum scaled elevation (red colour):

$$h^p = (l^p/l^m)h^m = (20\,000/0.01) \times 0.011 = 22 \text{ km}$$

Scaling of topography when using $\Delta\rho$

As earlier, $\rho_{AIR} = \sim 10^{-3} \times \rho_{UM}$ & ρ_{CC}, so we have:

$$h = T_{CC} \frac{(\rho_{UM} - \rho_{CC})}{\rho_{UM}}$$

Writing for model and nature:

$$\frac{h^m}{h^p} = \frac{T_{CC}^m}{T_{CC}^p} \frac{\rho_{UM}^p (\rho_{UM}^m - \rho_{CC}^m)}{\rho_{UM}^m (\rho_{UM}^p - \rho_{CC}^p)}$$

As T_{CC} is just a length scale we get:

$$h^p = C_{Topo} \frac{l^p}{l^m} h^m$$

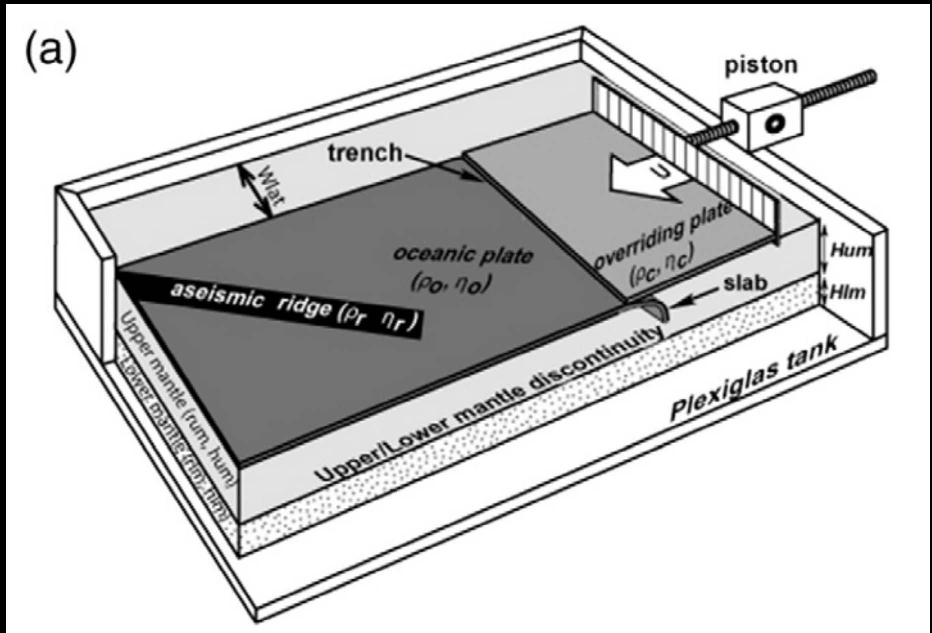
With the topographic correction factor:

$$C_{Topo} = \frac{\rho_{UM}^m (\rho_{UM}^P - \rho_{CC}^P)}{\rho_{UM}^p (\rho_{UM}^M - \rho_{CC}^M)}$$

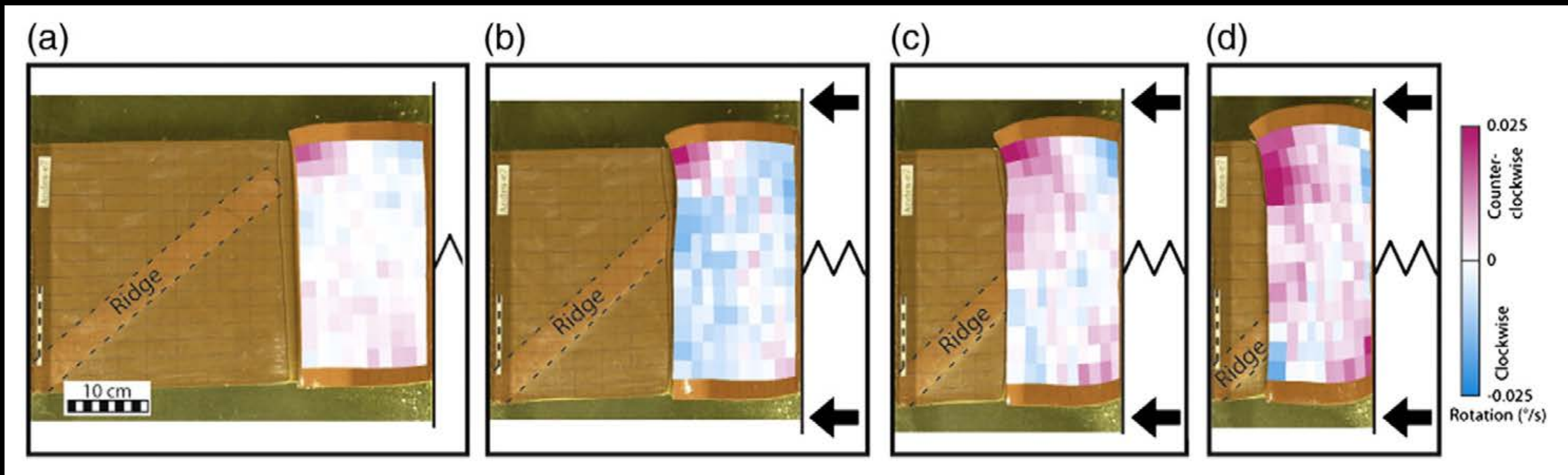
When $(\Delta\rho)^m = (\Delta\rho)^p$ then this simplifies to:

$$C_{Topo} = \frac{\rho_{UM}^m}{\rho_{UM}^p}$$

Scaling of topography when using $\Delta\rho$

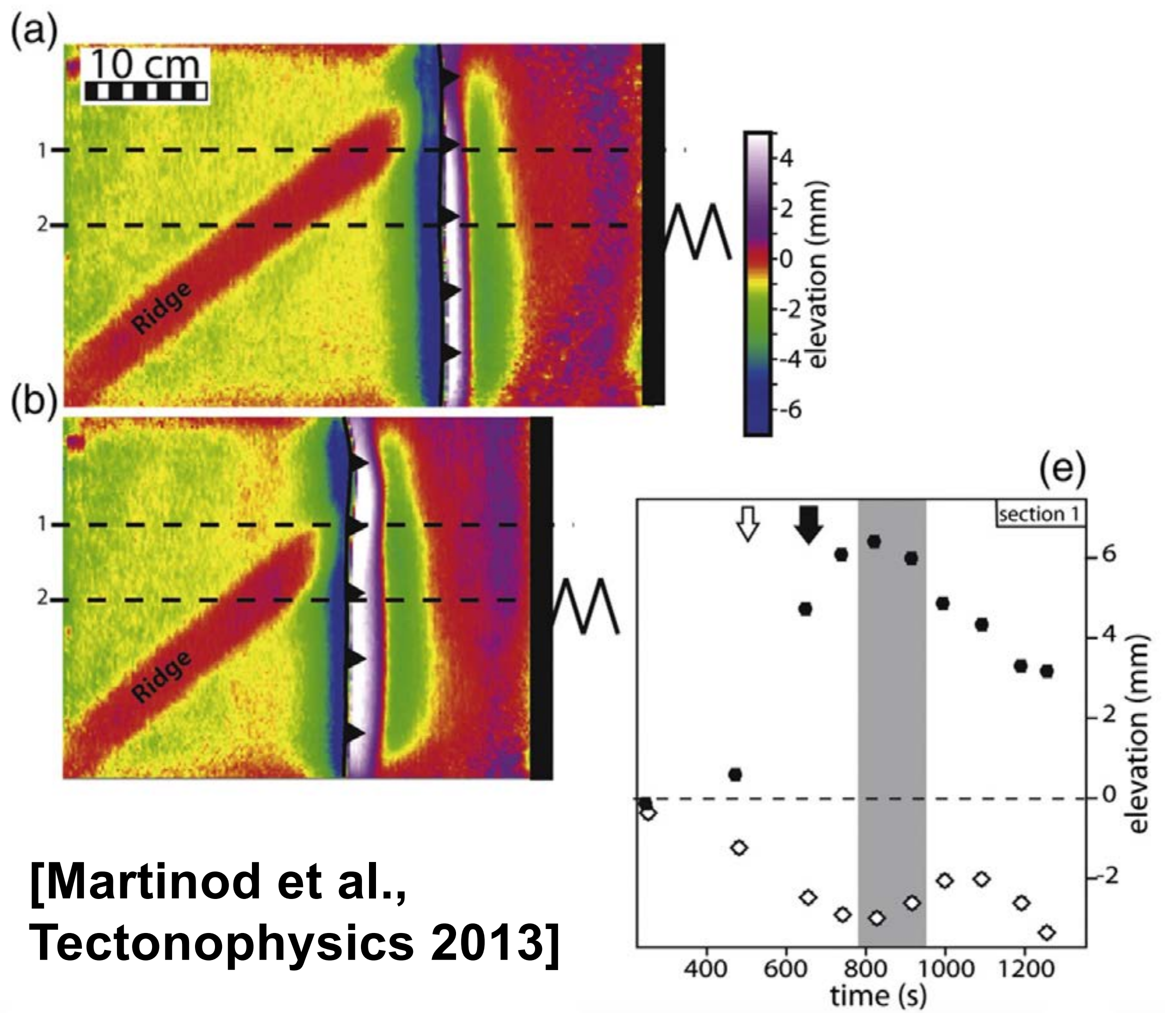


Laboratory model set-up of South American subduction experiment with aseismic ridge subduction
[Martinod et al., Tectonophysics 2013]



Model results **[Martinod et al., Tectonophysics 2013]**

Scaling of topography when using $\Delta\rho$



[Martinod et al., Tectonophysics 2013]

Topographic evolution during subduction & ridge subduction

Length scaling:
1 mm scales to 6.6 km

Topo forearc-arc:
6 mm scales to ~40 km
Topo backarc:
-3 mm scales to ~ -20 km

Andes:
Max. elevation plateau = ~4 km

With a topographic correction factor C_{Topo} = ~0.2, then:

Topo forearc-arc = ~8 km & topo backarc = ~ -4 km

Summary scaling of geodynamic models

- Different modelling approaches: internal, external and combined.
- Scaling requires: geometric, kinematic & dynamic similarity.
- Scaling of models: length, time, density (contrast), velocity, viscosity, stress, appropriate rheology.
- Scaling requires: Same flow regime as determined by Reynolds number.
- Dynamic similarity requires rheological similarity.
- Scaling of topography: Different scaling of topography for density and density contrasts (topographic correction).

Thank you!