The Abdus Salam International Centre for Theoretical Physics

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Title: Spectral Universality in High-Dimensional Statistics

Spectral universality refers to the empirical observation that asymptotic properties of a high-dimensional stochastic system driven by a structured random matrix are often determined only by the spectrum (or singular values) of the underlying matrix - the singular vectors are irrelevant provided they are sufficiently ``generic''. Consequently, the properties of the underlying system can be accurately predicted by analyzing the system under the mathematically convenient assumption that the singular vectors as uniformly random (or Haar-distributed) orthogonal matrices. This general phenomenon has been observed in numerous contexts, including statistical physics, communication systems, signal processing, statistics, and randomized numerical linear algebra. In this talk, I will describe recent progress toward a mathematical understanding of this universality phenomenon. In the context of penalized linear regression with strongly convex regularizers, I will describe nearly deterministic conditions on the design (or feature) matrix under which this universality phenomenon occurs. I will show that these conditions can be easily verified for highly structured design matrices constructed with limited randomnesses like randomly subsampled Hadamard transforms and signed incoherent tight frames. Due to this universality result, the performance of regularized least squares estimators on many structured sensing matrices with limited randomness can be characterized using the rotationally invariant sensing model with uniformly random (or Haar distributed) singular vectors as an equivalent yet mathematically tractable surrogate.