Large Scale Structure of the Universe

Rogerio Rosenfeld
IFT-UNESP/ICTP-SAIFR/LIneA

Dark Energy Survey & LSST

Legacy Survey of Space and Time

The Abdus Salam
International Centre
for Theoretical Physics

Giambiagi Winter School on Cosmology
Buenos Aires, July 2023

Juan José Giambiagi (1924 - 1996)

Juan José Giambiagi, better known as "Bocha" to his friends and colleagues, studied at the University of Buenos Aires where he graduated in physics in 1948 and completed his doctorate in 1950. He was a pioneer in the development of physics in Latin America, a charismatic leader and inspiring teacher.

Professor Giambiagi was one of the founders of the Escuela Latinoamericana di Física (ELAF) in the 1960s and Director of the Centro Latinoamericano de Física (CLAF) from 1986 to 1994.

He collaborated with Carlos Guido Bollini and made important contributions to "dimensional regularization".

Professor Giambiagi's ties with ICTP started prior to its foundation when he participated in the 1962 preparatory conference, which led to the creation of the Centre. He was an early ICTP Associate, Senior Associate and a close friend and strong supporter of ICTP. Professor Giambiagi was a member of the ICTP Scientific Council from 1987 to 1995.

Worked with Leon Rosenfeld in Manchester
CBPF: 1953-56
Renounced from UBA in 1966
CBPF: 1977-96

www.abc.org.br/membro/juan-jose-giambiagi

Ciências Matemáticas | MEMBRO ASSOCIADO

Juan José Giambiagi
(GIAMBIAGI, J.J.)

Nascimento: 18/06/1924

Nacionalidade: Brasileira

Five lectures on Cosmology and Large Scale Structure

Lecture I: The average Universe
Lecture II: Distances and thermal history
Lecture III: The perturbed Universe
Lecture IV: Theoretical challenges and surveys
Lecture V: Observational cosmology with LSS

Plan for Lecture I:

I.0 – Introduction & Motivation

I.1 – Brief review of GR

I.2 – Dynamics of the average Universe

“Our whole universe was in a hot dense state
Then nearly fourteen billion years ago expansion started”

I.0- Introduction & Motivation

A bit of historical context: cosmology's best moments (personal take)

1915: Einstein finishes GR
1917: First papers in modern cosmology
Einstein – static, closed, matter+Λ; de Sitter – Λ-dominated Universe
1919: GR confirmed in solar eclipse
1922: Friedmann – dynamical Universe (mathematical)
1927: Lemaître – dynamical Universe (physical)
1929+: Hubble & Humason – "expansion" of the Universe
1932: First standard cosmology – Einstein and de Sitter (EdS) – flat, Λ=0
1948+: Gamow et al. – Hot Big Bang (BBN & CMB)
1965: Penzias & Wilson – Discovery of CMB

1970’s: Peebles, Rubin+... – solid theoretical basis, dark matter
1980’s: Inflation
1990 - today: Detailed study of CMB (COBE, WMAP, Planck)
1998+: Accelerated expansion discovered
2000 – 2020’s: Large surveys of galaxies: SDSS, DES, KiDS,...
2000+: Precision cosmology – emergence of the current Standard Cosmological Model: ΛCDM
2023: Giambiagi School on Cosmology!

Nobel Prizes in Cosmology:

The Nobel Prize in Physics 1978

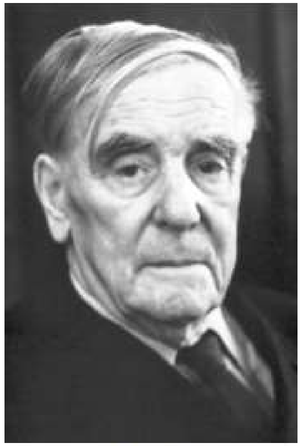

Photo from the Nobel Foundation archive.

Pyotr Leonidovich Kapitsa

Prize share: 1/2

Photo from the Nobel Foundation archive.

Arno Allan Penzias

Prize share: 1/4

Photo from the Nobel Foundation archive.

Robert Woodrow Wilson

Prize share: 1/4

The Nobel Prize in Physics 1978 was divided, one half awarded to Pyotr Leonidovich Kapitsa "for his basic inventions and discoveries in the area of low-temperature physics", the other half jointly to Arno Allan Penzias and Robert Woodrow Wilson "for their discovery of cosmic microwave background radiation"

The Nobel Prize in Physics 2006

Photo: P. Izzo

John C. Mather

Prize share: 1/2

Photo: J. Bauer

George F. Smoot

Prize share: 1/2

The Nobel Prize in Physics 2006 was awarded jointly to John C. Mather and George F. Smoot "for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation."

The Nobel Prize in Physics 2011

Saul Perlmutter, Brian P. Schmidt, Adam G. Riess

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The Nobel Prize in Physics 2011

Photo: U. Montan

Saul Perlmutter

Prize share: 1/2

Photo: U. Montan

Brian P. Schmidt

Prize share: 1/4

Photo: U. Montan

Adam G. Riess

Prize share: 1/4

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*.

The Nobel Prize in Physics 2019

© Nobel Media. Photo: A. Mahmoud

James Peebles

Prize share: 1/2

© Nobel Media. Photo: A. Mahmoud

Michel Mayor

Prize share: 1/4

© Nobel Media. Photo: A. Mahmoud

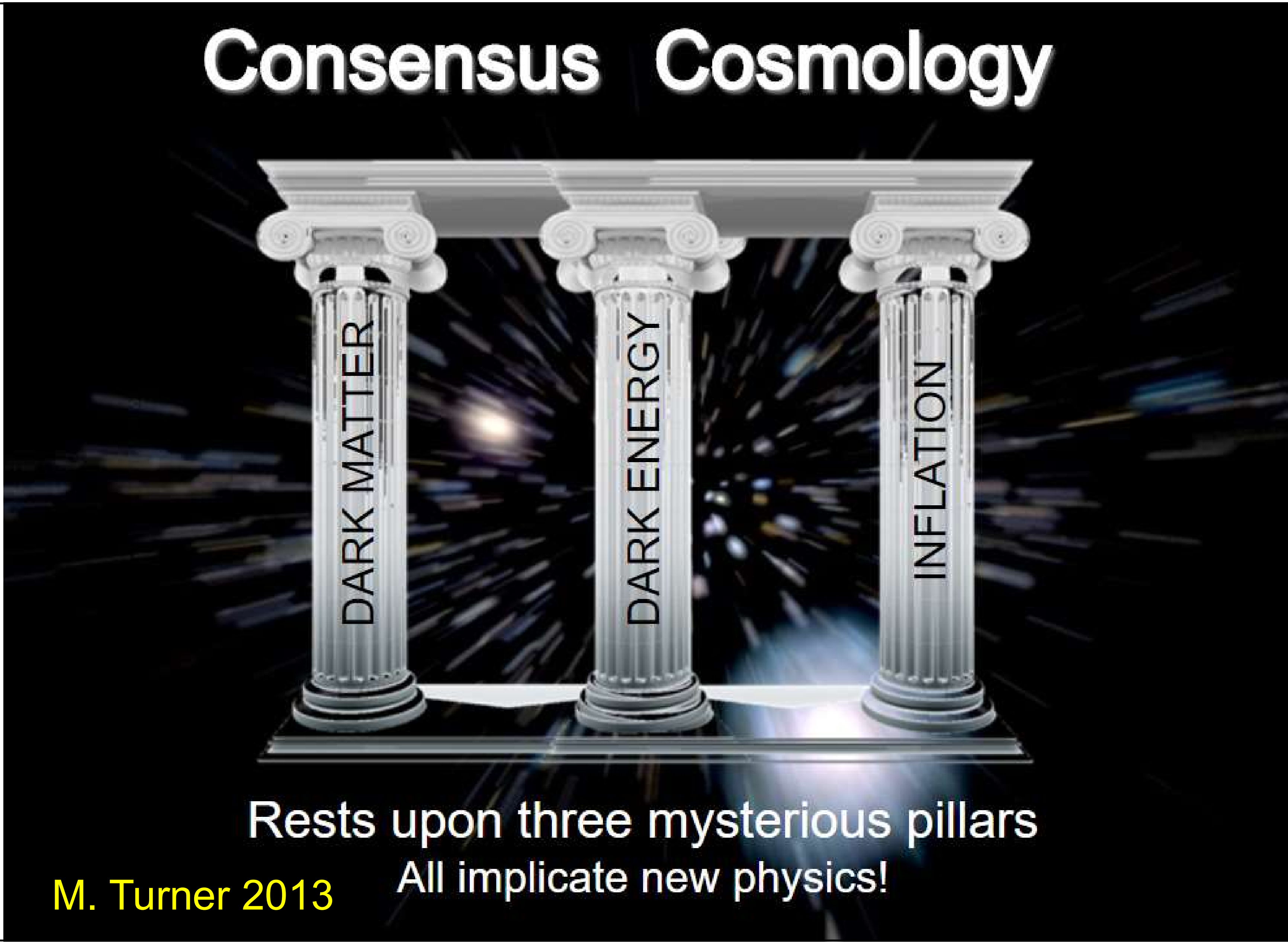
Didier Queloz

Prize share: 1/4

The Nobel Prize in Physics 2019 was awarded "for contributions to our understanding of the evolution of the universe and Earth's place in the cosmos" with one half to James Peebles "for theoretical discoveries in physical cosmology", the other half jointly to Michel Mayor and Didier Queloz "for the discovery of an exoplanet orbiting a solar-type star"

Modern cosmology is based on three unexpected discoveries that requires New Physics:

- There is more matter than expected $\rightarrow$ Dark Matter
- Universe was very homogeneous $\rightarrow$ Inflation
- Universe is accelerating $\rightarrow$ Dark Energy


Consensus Cosmology
DARK MATTER
DARK ENERGY
INFLATION
Rests upon three mysterious pillars
All implicate new physics!
M. Turner 2013

Theory: ΛCDM

General Relativity + known particle physics + cold dark matter + cosmological constant (= dark energy) + inflation (initial conditions for perturbations)

Observations:

Test different epochs of the Universe

Big Bang Nucleosynthesis - BBN (~few minutes)

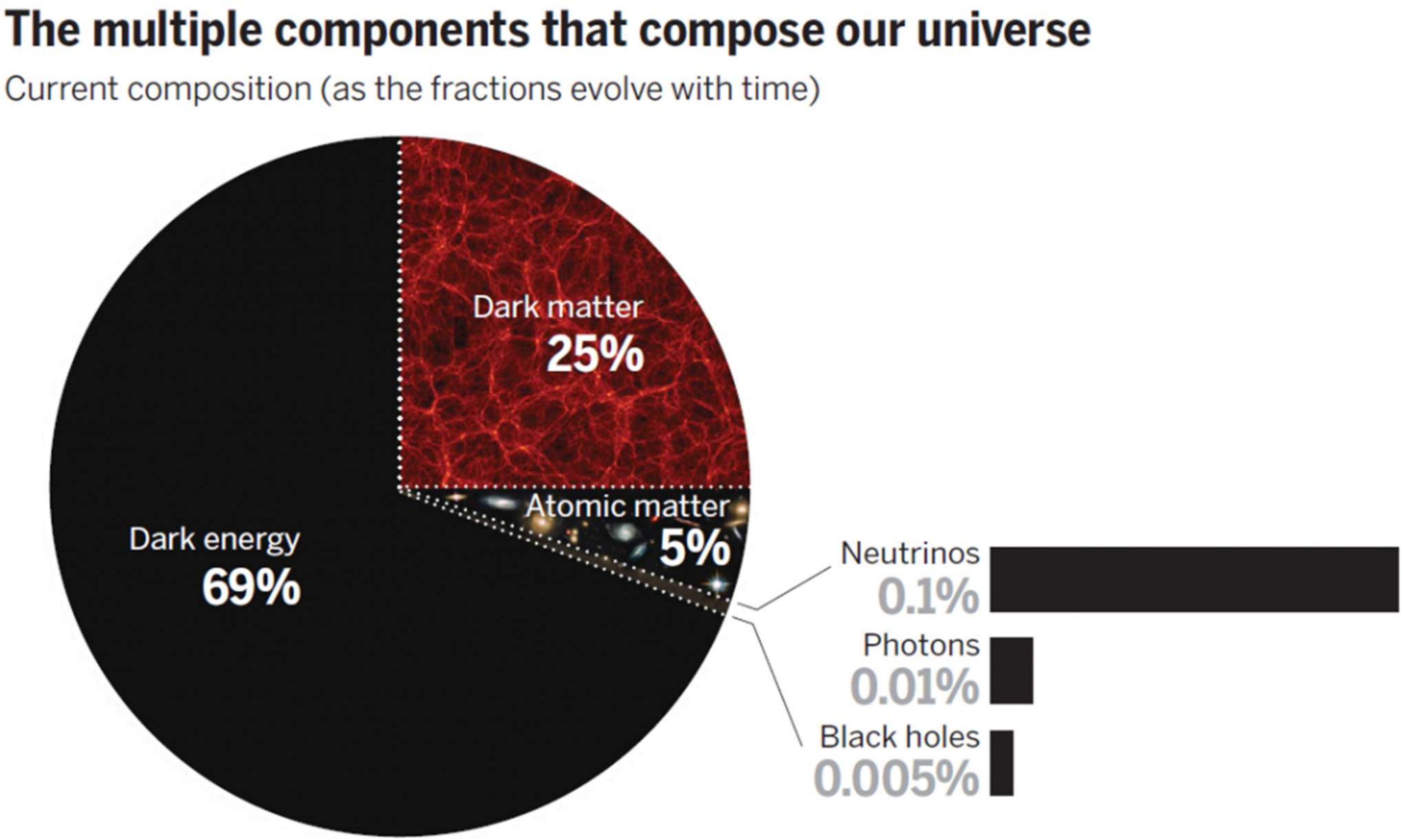
Cosmic Microwave Background - CMB (~380,000 years)

Supernovae Type Ia – SNIa (~ billion years)

Large Scale Structure – LSS (~ billion years)

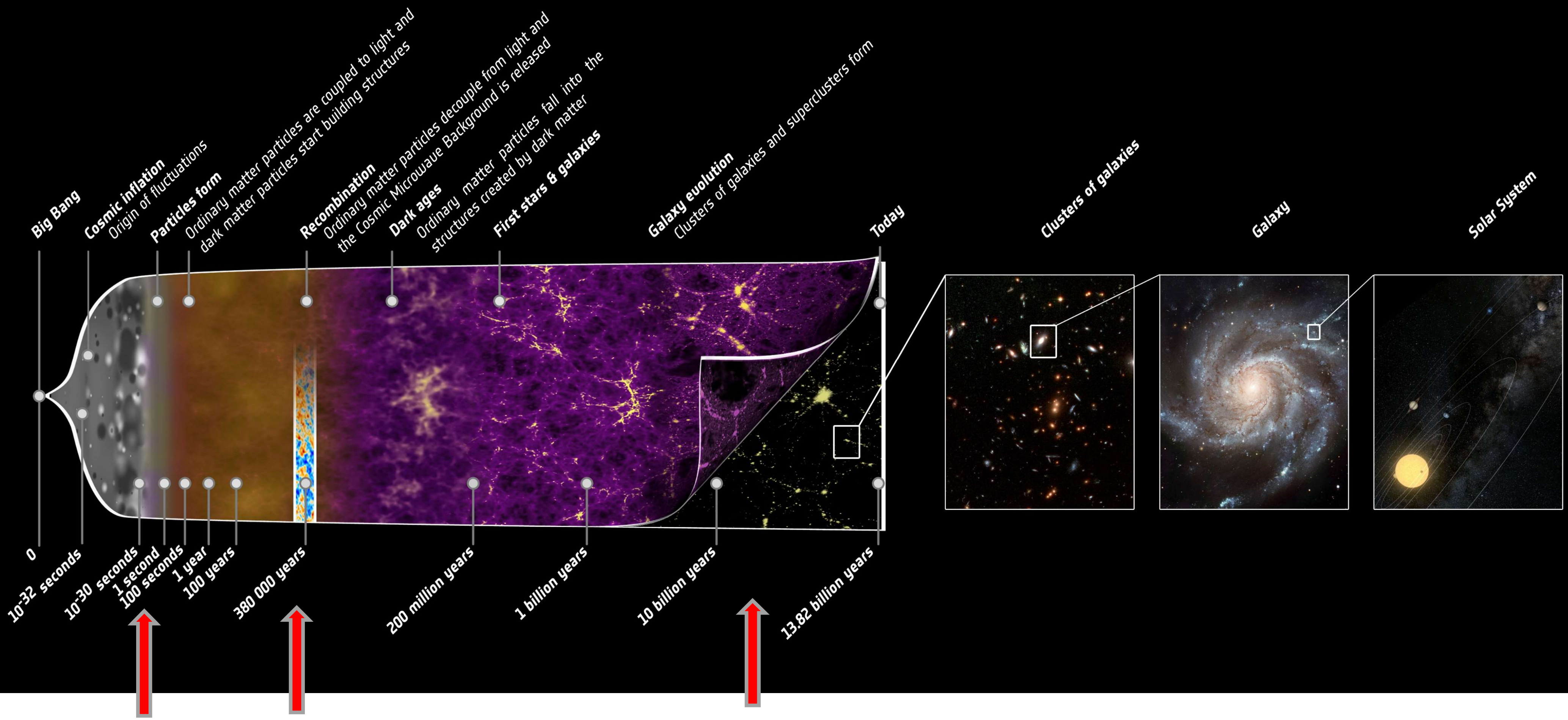
Comparing Theory with Observations:

Determination of the best values of cosmological parameters (and their uncertainties) that characterizes the model.



phys.org/news/2015-03-dark-side-cosmology.html

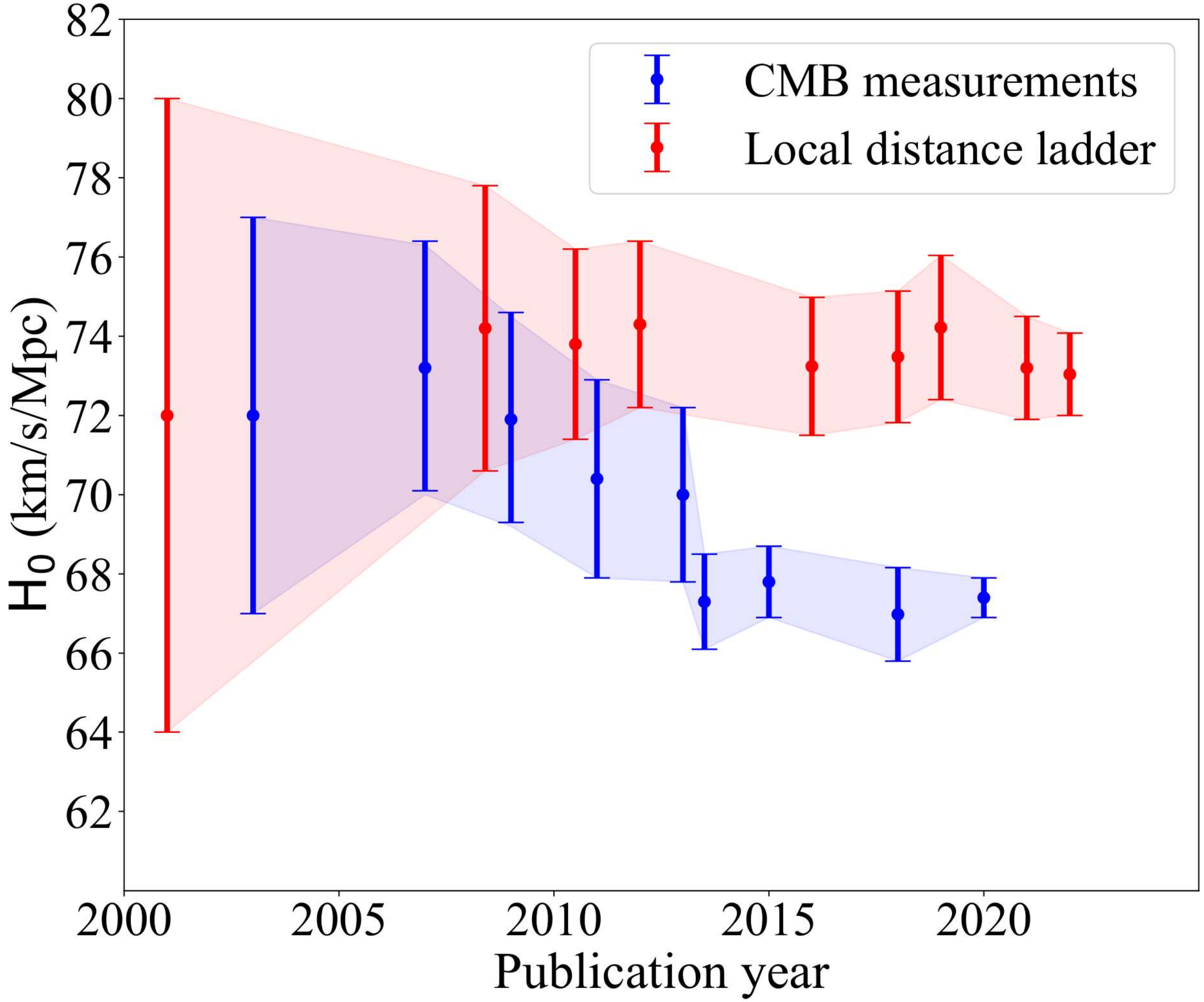
History of the Universe consistent with a single model: ΛCDM



BBN **CMB** **LSS, SNIa**

Precision can bring trouble: Hubble tension!

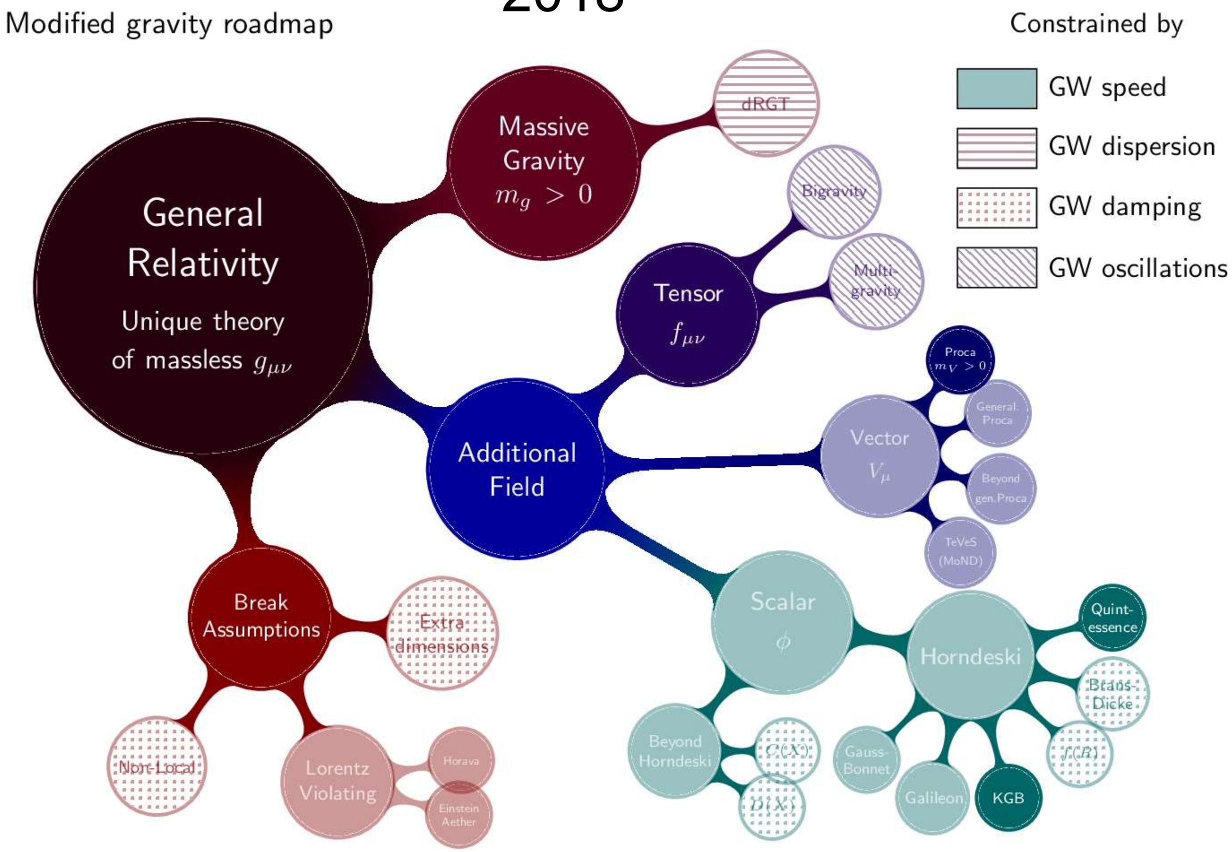
~5 σ discrepancy!
First hint of new physics?



If you are not happy with ΛCDM be my guest:

Bertone & Tait
2018

Ezquiaga & Zumalacárregui
2018



I.1- Brief Review of GR

I.1.0 – Classical field theory in a nutshell

I.1.1 – Fundamental degrees of freedom

I.1.2 – Einstein-Hilbert action

I.1.3 – The cosmological constant

I.1.4 – Adding matter/radiation to the Universe

I.1.5 – Energy-momentum tensor

I.1.6 – Einstein's equation

I.1- Brief Review of GR

General Relativity rules the Universe at large scales!
Classical description is sufficient in most cases.

I.1.0 – Classical field theory in a nutshell

$$\underbrace{\text{Fields}}_{\substack{\text{Fundamental degrees}\\ \text{of freedom}\\ \text{(not always physical)}}} \longrightarrow \underbrace{\text{lagrangian} \longrightarrow \text{action}}_{\substack{\text{Dynamics}\\ \text{(use of symmetries)}}} \longrightarrow \underbrace{\text{equations of motion}}_{\delta S = 0}$$

I.1.1 – Fundamental degrees of freedom

Fundamental field of gravity: the metric $g_{\mu\nu}$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Symmetric 4x4 matrix: 10 degrees of freedom
(not all physical!)

$$g_{\mu\alpha} g^{\alpha\nu} = \delta^\nu_\mu$$
$$g_{\mu\nu} g^{\mu\nu} = 4$$

Flat space-time – Minkwoski metric

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$p_\mu p^\mu = E^2 - (\vec{p})^2$$

I.1.2 – Einstein-Hilbert action

$$S_{\mathrm{E-H}}[g_{\mu\nu}] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\;R[g_{\mu\nu}]$$

For the Hilbert-Einstein dispute see:
L. Corry, J. Renn, and J. Stachel, Science 278, 1270 (1997)
F. Winterberg, Z. Naturforsch. **59a,** 715 – 719 (2004)

- Action is invariant under general coordinate transformations:

$$x^\mu \to x'^\mu(x^\mu)$$

- $R[g_{\mu\nu}]$ is the Ricci scalar: second order in derivatives of the metric

- $g = \det(g_{\mu\nu})$

- Christoffel symbols (aka metric conection, affine connection) – first derivative of the metric :

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left\{ \frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right\}$$

- Ricci tensor – second derivative of the metric:

$$R_{\mu\nu} = \frac{\partial}{\partial x^{\alpha}} \Gamma^{\alpha}_{\mu\nu} - \frac{\partial}{\partial x^{\nu}} \Gamma^{\alpha}_{\alpha\mu} + \Gamma^{\alpha}_{\mu\nu} \Gamma^{\beta}_{\alpha\beta} - \Gamma^{\beta}_{\alpha\mu} \Gamma^{\alpha}_{\beta\nu}$$

- Ricci scalar: $R = g^{\mu\nu} R_{\mu\nu}$

22

• G: Newton's constant

$$G = \frac{1}{M_{\mathrm{Pl}}^2} \quad (\hbar = c = 1)$$

$$M_{\mathrm{Pl}} = 1.2 \times 10^{19}\ \mathrm{GeV}$$

Obs.: sometimes the *reduced* Planck mass is used:

$$\tilde{M}_{\mathrm{Pl}} = \frac{M_{\mathrm{Pl}}}{\sqrt{8\pi}} = 2.4 \times 10^{18}\ \mathrm{GeV}$$

- Dimensional analysis: $(\hbar = c = 1)$

$$[g]:\ \text{dimensionless};\quad [R]:\ E^2$$

$$[d^4x]: E^{-4};\quad [S]:\ \text{dimensionless}$$

$$\Rightarrow \boxed{[G] : E^{-2}}$$

$$G = \frac{1}{M_{\mathrm{Pl}}^2}$$

- Einstein equation in vacuum (no matter) is obtained from:

$$\frac{\delta S_{\mathrm{E-H}}}{\delta g_{\mu\nu}} = 0$$

- Einstein equation in vacuum:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

I.1.3 – The cosmological constant

February 1917 (~100 years ago): Einstein's "Cosmological Considerations in the General Theory of Relativity" introduces the cosmological constant in the theory without violating symmetries: a new constant of Nature!

It has an "anti-gravity" effect (repulsive force) and it was introduced to stabilize the Universe.

$$S_{\mathrm{E-H}} + S_{\Lambda} = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \Lambda \right)$$

With the discovery of the expansion of the Universe (Hubble, 1929) it was no longer needed – "Einstein's biggest blunder".

George Gamow – My Worldline

correct, and changing it was a mistake. Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder he ever made in his life. But this "blunder," rejected by Einstein, is still sometimes used by cosmologists even today, and the cosmological constant denoted by the Greek letter Λ rears its ugly head again and again and again.

I.1.4 – Adding matter/radiation to the Universe

Matter/radiation is described by fields in a lagrangian:

$$S_{\text{matter}} = \int d^4x\sqrt{-g}\,\mathcal{L}_{\text{matter}}$$

Examples:

Electromagnetism: $$S_{\text{EM}} = -\frac{1}{4}\int d^4x\sqrt{-g}\, g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

Real scalar field: $$S_{\phi} = \int d^4x\sqrt{-g}\,\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\right]$$

I.1.5 – Energy-momentum tensor

Matter/radiation in GR is described by an energy-momentum tensor.

Definition:

$$\delta S_{\rm matter} = \frac{1}{2}\int d^4x\sqrt{-g}\,T^{\mu\nu}(x)\delta g_{\mu\nu}$$

which implies

$$T^{\mu\nu}(x) = \frac{2}{\sqrt{-g}}\frac{\delta S_{\rm matter}}{\delta g_{\mu\nu}}$$

I.1.6 – Einstein's equation

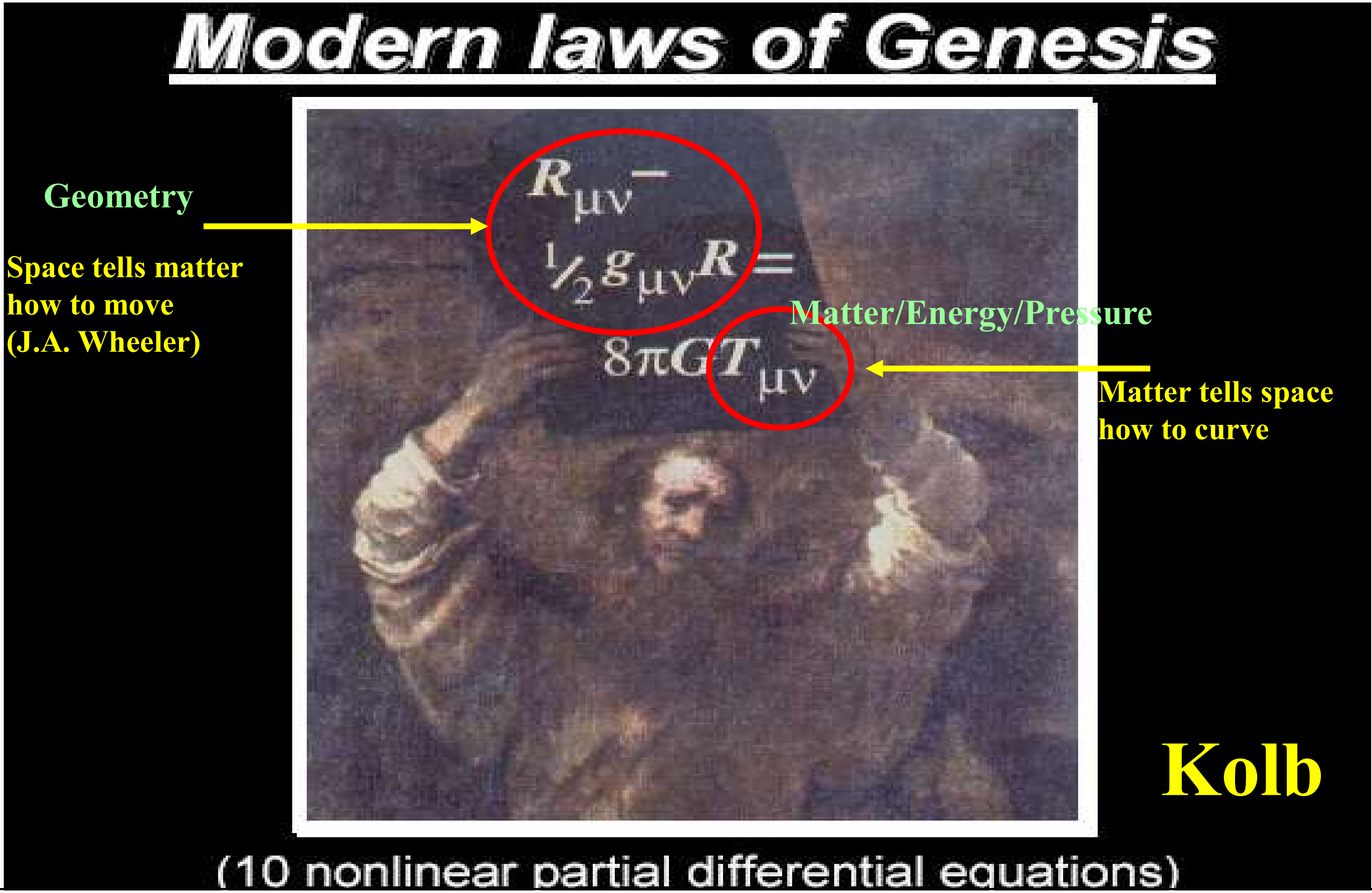
Einstein's equation for GR is obtained from the requirement:

$$\delta\left(S_{\mathrm{total}}\right) = \delta\left(S_{\mathrm{E-H}} + S_{\Lambda} + S_{\mathrm{matter}}\right) = 0$$

$$G_{\mu\nu} \equiv \overbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}^{\text{gravity}} = 8\pi G \overbrace{T_{\mu\nu}}^{\text{particle physics } + \Lambda}$$

10 nonlinear differential equations. In general it must be solved numerically, eg gravitational waves from coalescence of binary black holes.

Standard Cosmological Model



I.2- Dynamics of the average Universe

I.2.1 – Friedmann-Lemaître-Robertson-Walker metric

I.2.2 – Expansion of the Universe

I.2.3 – The right-hand side of Einstein equation:
the energy-momentum tensor simplified

I.2.4 – Friedmann's equations

I.2.5 – Evolution of different fluids

I.2.6 – Time evolution of the scale fator

I.2.7 – Inflation

I.2.8 – Recipe of the Universe

I.2- Dynamics of the average Universe

Here we will be interested on how the Universe evolves on average.

The averaged quantities only depend on time.

Averaged Einstein equation:

$$\bar{G}_{\mu\nu}(t) = 8\pi G \bar{T}_{\mu\nu}(t)$$

I.2.1 – Friedmann-Lemaître-Robertson-Walker metric

Universe is spatially homogeneous and isotropic on average.

It is described by the FLRW metric (for a spatially flat universe):

$$ds^2 = dt^2 - a(t)^2 \left[dx^2 + dy^2 + dz^2\right]$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 \\ 0 & 0 & -a^2 & 0 \\ 0 & 0 & 0 & -a^2 \end{pmatrix}$$

FLRW metric is determined by one time-dependent function: the so-called scale factor a(t).
Distances in the universe are set by the scale factor.

Scale factor is the key function to study how the average universe evolves with time.

convention: a=1 today

OBS: conformal time (light cone has the usual 45^0 angle).

$$d\eta = \frac{dt}{a(t)} \qquad ds^2 = a^2(t)[d\eta^2 - d\vec{r}^2]$$

dτ sometimes

For a spatially flat FLRW metric the Ricci tensor and the Ricci scalar are given by:

$$R_{00} = -3\frac{\ddot{a}}{a}; \;\; R_{ii} = a\ddot{a} + 2\dot{a}^2$$

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$$

Average evolution of the universe

- measurement of large scale distances, velocities and acceleration

$$a(t) \qquad \dot{a}(t) \qquad \ddot{a}(t)$$

- measured through standard candles and/or standard rulers

Redshift z: $\boxed{a(t) = \frac{1}{1+z}}$ z=0 today.

I.2.2 – Expansion of the Universe

Hubble parameter:

Expansion rate of the universe

Hubble constant: Hubble parameter today (H_0)

$$\boxed{H = \frac{\dot{a}(t)}{a(t)}}$$

Analogy of the expansion of the universe with a balloon:

Space itself expands and galaxies get a free "ride".

I.2.3 – The right-hand side of Einstein equation: the energy-momentum tensor simplified

It is usually assumed that one can describe the components of the Universe as "perfect fluids": at every point in the medium there is a locally inertial frame (rest frame) in which the fluid is homogeneous and isotropic (consistent with FLRW metric):

$$T^{00} = \rho(t); \; T^{ij} = \delta^{ij} P(t); \; T^{0i} = 0$$

density

isotropy

pressure

Rest-frame

Homegeneity: density and pressure depend only on time.

Energy-momentum in the rest frame (indices are important):

$$T^{\mu}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

In a frame with a given 4-velocity:

$$T^{\mu\nu} = -Pg^{\mu\nu} + (\rho + P)u^{\mu}u^{\nu}$$

$$u^{\mu} = \gamma\,(1, \vec{v})$$

I.2.4– Solving Einstein equation for the average Universe: Friedmann's equations

00 component:

$$R_{00} - \frac{1}{2} g_{00} R = 8\pi G T_{00} \implies$$

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho}$$

1st Friedmann equation
Expansion rate is determined by energy density.

ii component:

$$R_{ii} - \frac{1}{2} g_{ii} R = 8\pi G T_{ii} \implies$$

$$\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{a}}{a} = -8\pi G P \implies$$

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)}$$

2nd Friedmann equation
(De)acceleration is determined by energy density and pressure.

I.2.5 – Evolution of different fluids

Taking a time derivative of 1st Friedmann equation one arrives at the so-called continuity equation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

In order to study the evolution of a fluid we need to find a relation between density and pressure: the equation of state

Assume a simple equation of state: $$P = \omega\rho$$

ω is called the equation of state parameter.

Examples:

- Non-relativistic matter (dust): $P \ll \rho \longrightarrow \omega = 0$

- Relativistic matter (radiation): $\omega = 1/3$

- Cosmological constant: $\omega = -1$

$$T^{\mu}_{\nu,\Lambda} = \begin{pmatrix} \Lambda & 0 & 0 & 0 \\ 0 & \Lambda & 0 & 0 \\ 0 & 0 & \Lambda & 0 \\ 0 & 0 & 0 & \Lambda \end{pmatrix} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

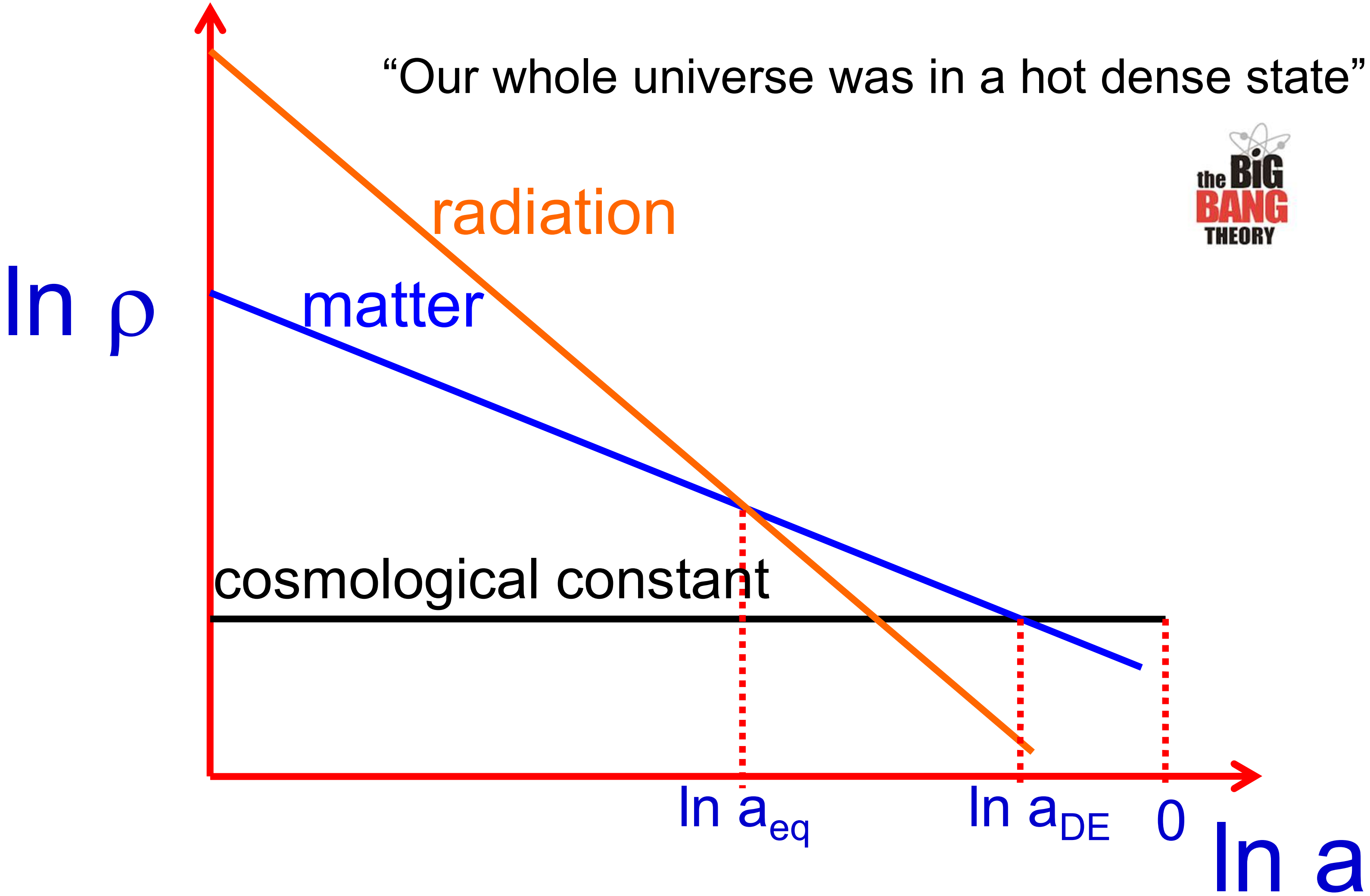
From the continuity equation it is easy to show that the evolution of the energy density for a constant equation of state is:

$$\rho(t) = \rho(t_i) \left(\frac{a(t)}{a(t_i)} \right)^{-3(1+\omega)}$$

OBS: It's easy to generalize to a time-dependente equation of state

- Non-relativistic matter (dust): $\rho \propto a^{-3}$
- Relativistic matter (radiation): $\rho \propto a^{-4}$
- Cosmological constant: $\rho \propto a^{0}$
- Kination (w=1): $\rho \propto a^{-6}$

“Our whole universe was in a hot dense state”



I.2.6 – Time evolution of the scale fator (expansion history)

Using 1st Friedmann equation and the result from last section:

$$\frac{\dot{a}}{a} \propto \sqrt{\rho},\ \rho \propto a^{-3(1+\omega)}$$

it is easy to show that:

$$a(t) \propto t^{\frac{2}{3(1+\omega)}} = \begin{cases} t^{2/3} \ (\text{matter}) \\ t^{1/2} \ (\text{radiation}) \end{cases}$$

but for the case of a cosmological constant one has an **exponential growth**:

$$\frac{\dot{a}}{a} = \text{const.} = H \rightarrow a(t) \propto e^{Ht}$$

Exponential growth: universe is accelerating!

2nd Friedmann equation is (for w=-1):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) = \frac{8\pi G}{3}\rho > 0$$

Exponential expansion: inflation

I.2.7 – Inflation (lectures by Mehrdad)

We think that the very early universe went through a phase of exponential expansion called inflation (Guth, Linde, Starobinsky, ... early 1980's).

We do not know what drove inflation. The simplest model involves a new scalar field: the inflaton. Equation of state was close to w=-1 during inflation.

During inflation the matter and radiation contents were rapidly diluted to ~nothing.

Any spatial curvature was erased – explains why universe is flat.

Put regions in our horizon in causal contact – explains homogeneity.

Quantum fluctuations of the inflaton field provided the initial small perturbations in homogeneity that evolved to form structures in the Universe. They also produced primordial gravitational waves.

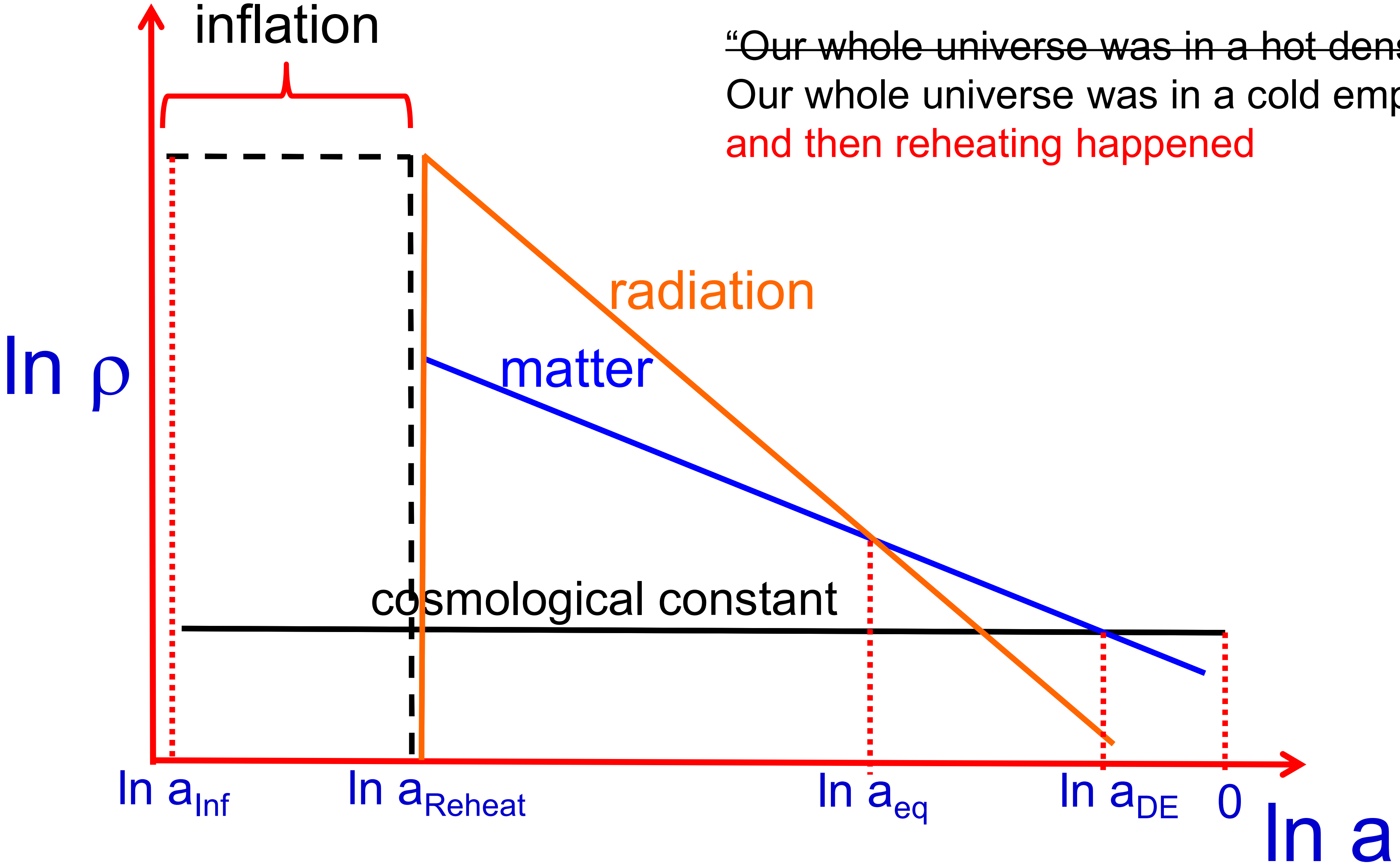
Inflation must end and the universe must be “reheated”. Energy density stored in the inflaton field is released to produce radiation.

We do not have a “smoking gun” signal for inflation yet.

~~“Our whole universe was in a hot dense state”~~

Our whole universe was in a cold empty state

and then reheating happened

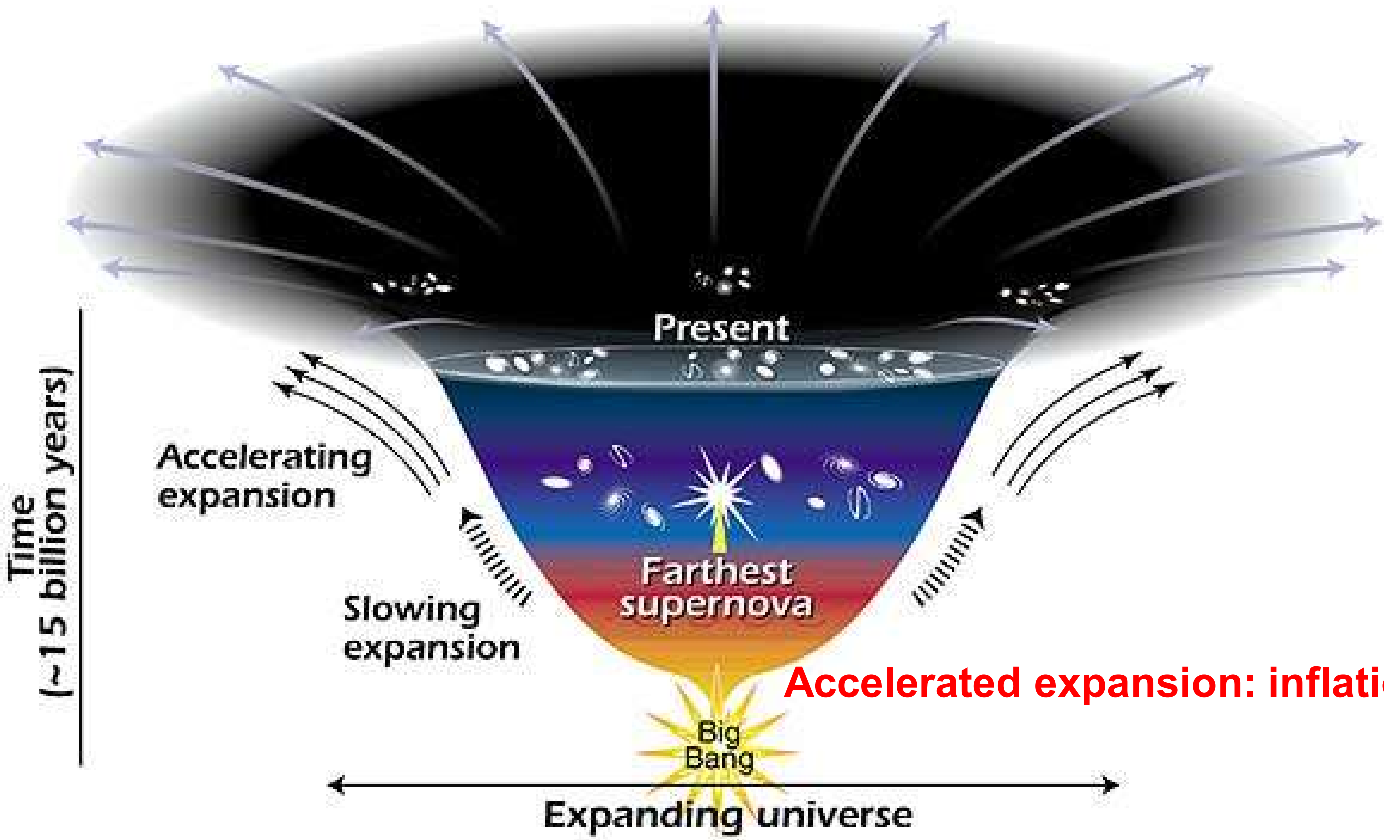


Encyclopædia Inflationaris

http://arxiv.org/1303.3787

Jérôme Martin,[a] Christophe Ringeval[b] and Vincent Vennin[a]

We are now in another inflationary phase. But if it is due to the cosmological constant it will not end!



Accelerated expansion: inflation

I.2.8 – Recipe of the Universe

Critical density: density at which the Universe is spatially flat today.

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Different contributions to the energy density budget of the Universe (i=baryons, photons, neutrinos, dark matter, dark energy,...)

$$\Omega_i = \frac{\rho_i}{\rho_c}$$

Spatially flat universe:

$$\sum_i \Omega_i = 1$$

1st Friedmann equation:

$$\frac{H(t)^2}{H_0^2} = \sum_i \Omega_i^{(0)} a^{-3(1+\omega_i)}$$

Exercise: estimate the critical density today in units of GeV/m^3 using the time-honored convention H_0 = 70 km/s/Mpc.

Exercise: estimate H^{-1}_0 in years for H_0 = 70 km/s/Mpc.

Exercise: assuming that inflation happened at an energy scale of 10^{12} GeV and approximating the whole expansion history as radiation-dominated, estimate at what time inflation took place in the universe and the scale factor at that time.

Exercise: given a Universe with composition

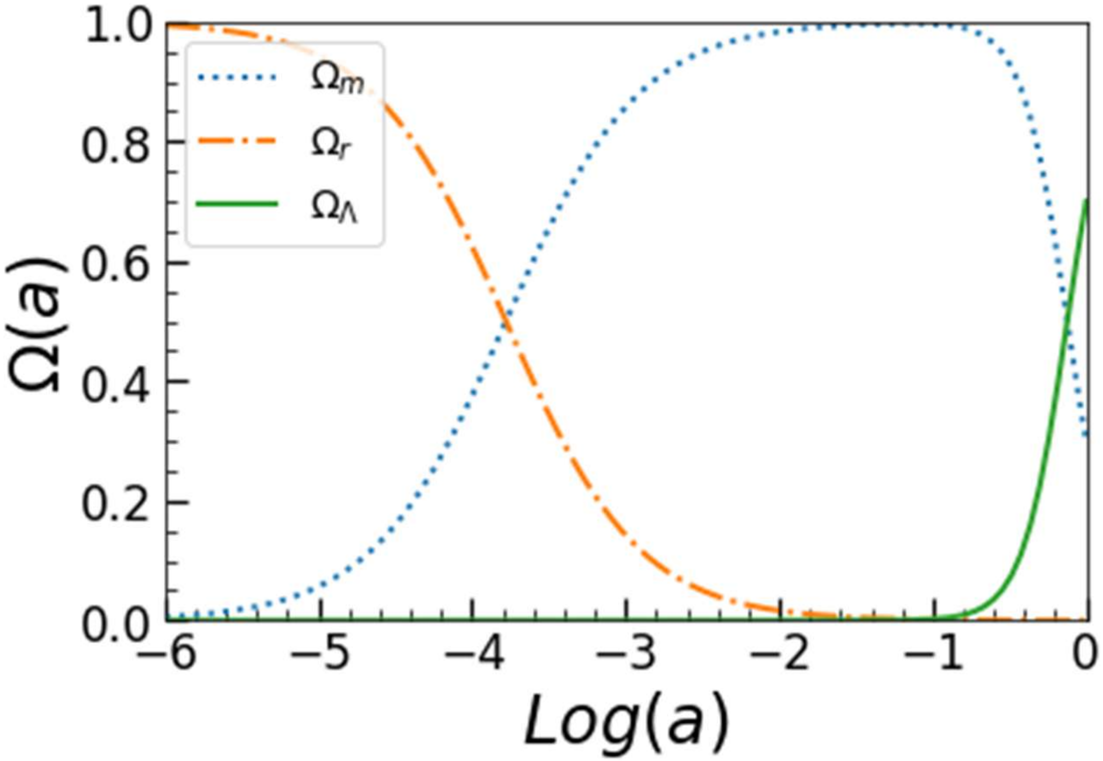
$$\Omega_{\Lambda}^{(0)} = 0.7,\ \Omega_{\rm matter}^{(0)} = 0.3,\ \Omega_{\rm rad}^{(0)} = 5 \times 10^{-5}$$

a. estimate the redshift z_{eq}

b. estimate the redshift z_{Λ}

c. plot these different Ω's as a function of log(a)

(see python colab notebook)



End of first lecture

Extra slides (time permitting)

I.2.9 – Vacuum energy: the elephant in the room

Quantum mechanics – zero point energy of a harmonic oscillator:

$$E = \hbar\omega(n + 1/2)$$

In Quantum Field Theory, the energy density of the vacuum is (free scalar field of mass m):

$$\rho_{vac} = \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2}$$

and is infinite! Integral must be cut-off at some physical energy scale - goes as $(\text{cut-off})^4$.

If integral is cutoff at the Planck scale, disagreement of ~ 10^{120} with data. This is know as the cosmological constant problem.

I.2.10 – Beyond Λ: dynamical dark energy

For a real homogeneous scalar field the energy-momentum tensor gives:

$$T_\phi^{00} = \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi);$$

$$T_\phi^{ii} = -g^{ii}P = -\left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right)g^{ii}$$

and therefore the time-dependent equation of state in this case is:

$$\omega(t) = \frac{P}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \Rightarrow -1 \leq \omega \leq 1$$

If potential energy dominates w~-1 and scalar field resembles a cosmological constant: quintessence field. Can be ultralight (~H_0)!

Klein-Gordon equation for a scalar field in an arbitrary metric

$$\mathcal{L}_\phi = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

$$S_\phi = \int d^4x \sqrt{-g} \mathcal{L}_\phi, \ \delta S_\phi = 0 \Rightarrow$$

$$\frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g}\ \partial^\mu \phi] + \frac{dV}{d\phi} = 0$$

Klein-Gordon equation for a homogeneous scalar field in FRWL metric

$$\sqrt{-g} = a^3(t)$$

$$\ddot{\phi} + 3H(t)\dot{\phi} + \frac{dV}{d\phi} = 0$$