Giambiagi Summer School, Buenos Aires, 17-21.07.2023

CMB physics

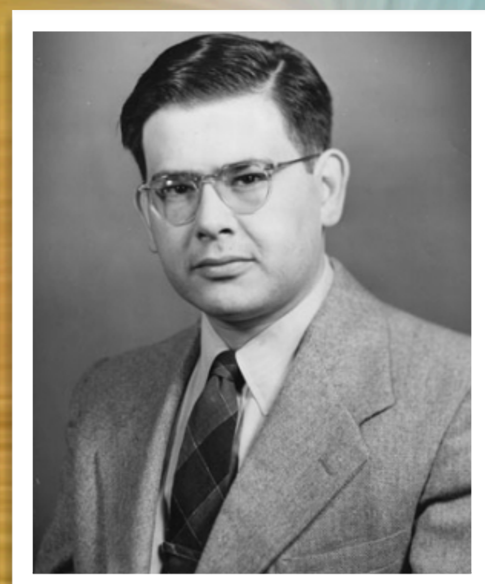
J. Lesgourgues
Institut für Theoretische Teilchenphysik und Kosmologie (TTK),
RWTH Aachen University

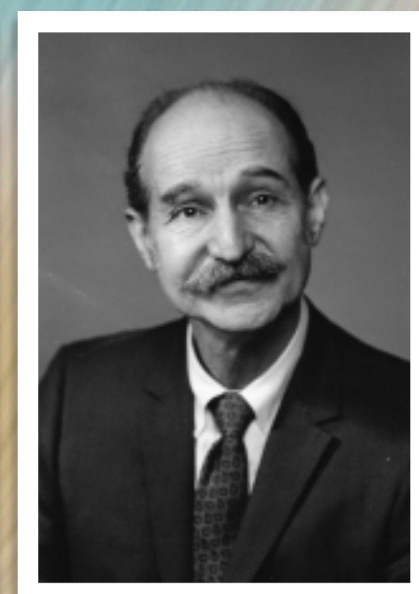
Version 17_11:51

Cosmic Microwave Background: an early prediction!

Cosmic photons with blackbody spectrum and $T \sim \mathcal{O}(10)K$ predicted on basis of models of Nucleosynthesis by

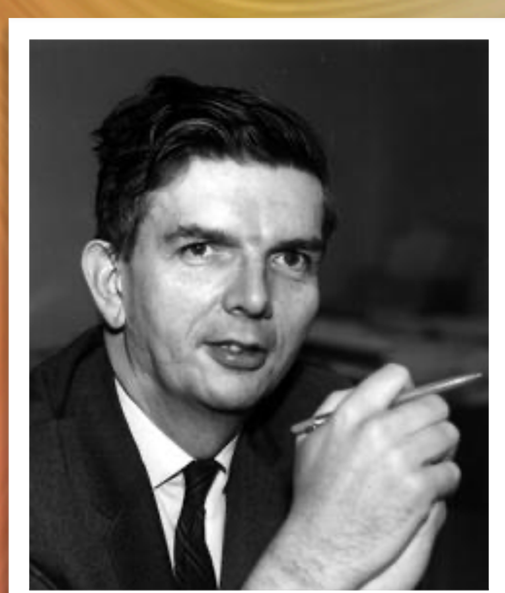
Gamow, Alpher, Hermann (40-50's),

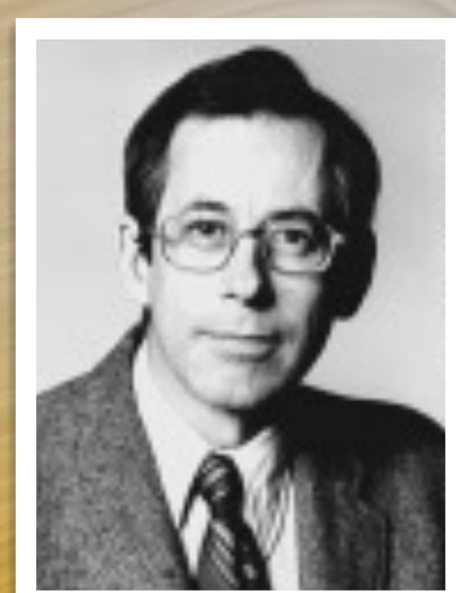




Zel'dovitch (60's),

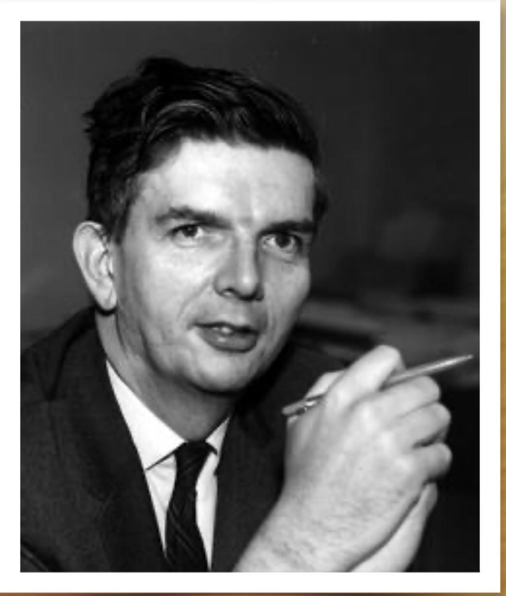
Dicke, Peebles (60's)…

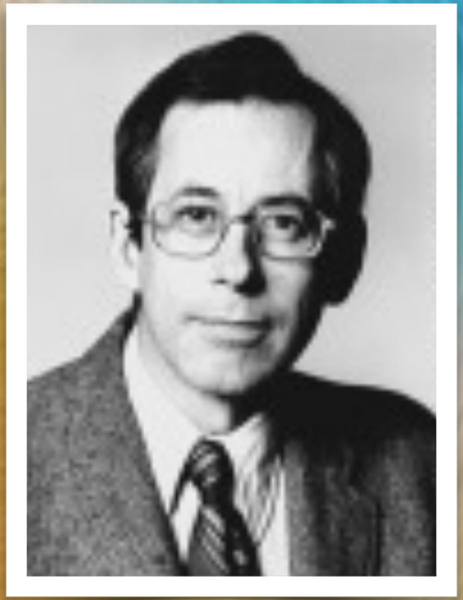




The CMB is no pigeon shit!

Dicke, Peebles (60's)…





While discussing about dedicated experiment…

… contacted thanks to luck and coffee room discussions by…

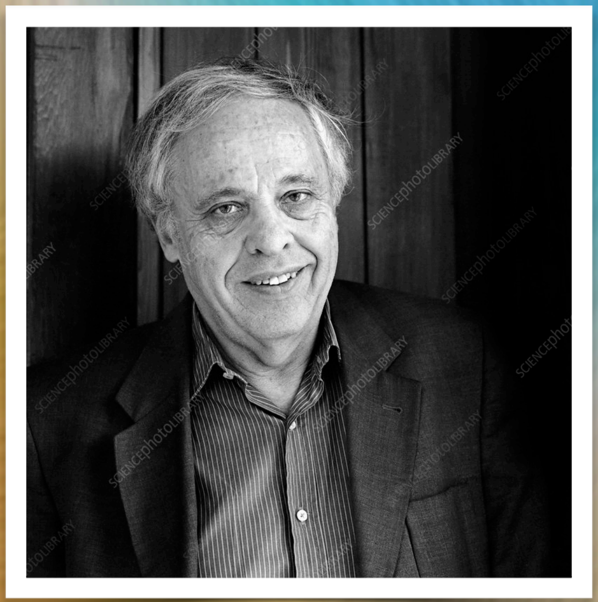
Penzias & Wilson (1964)

↓

Nobel prize Penzias & Wilson (1978)

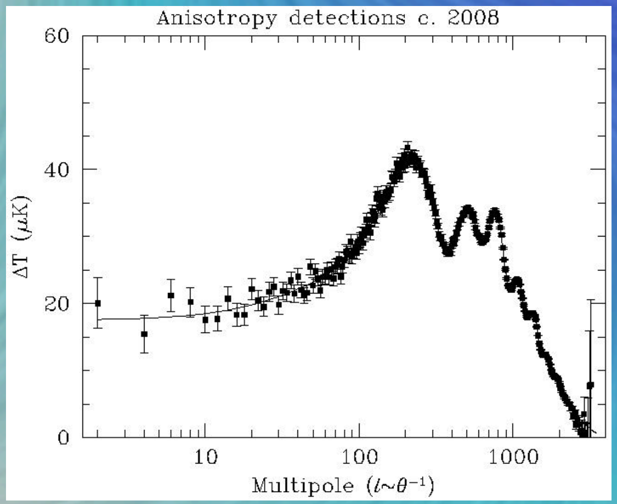
Prediction of non-trivial correlations in CMB anisotropies!

70-80's : Peebles, Silk, Sunyaev and respective collaborators (70-80's)…



…discuss information contain in CMB temperature spectrum and acoustic oscillations !

90's: precise prediction of CMB spectra:
Bertschinger, Hu, Kamionkowski, Ma, Seljak, Sujiyama, White, Zaldarriga + many others…



Confirmation by COBE, Boomerang, WMAP, Planck …

↓

Nobel prize Mather & Smoot 2006
Nobel prize Peebles 2019

The CMB is the Rosetta Stone of cosmology

Plan

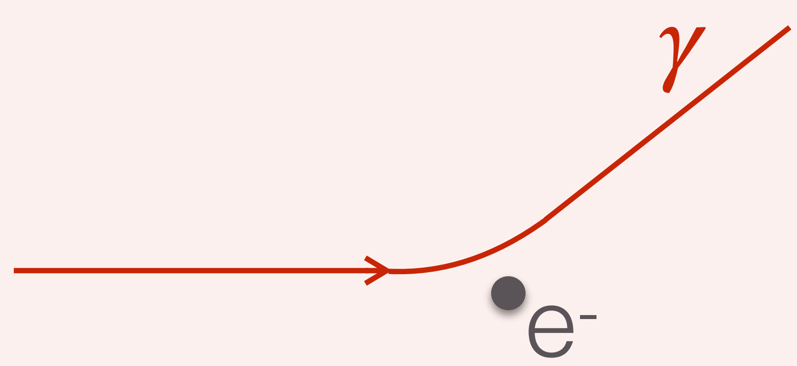
- Thomson scattering
- Linear theory of stochastic cosmological perturbations
- Spectrum of temperature anisotropies
- Polarisation and tensors
- Observation summary; Planck results for ΛCDM model
- Why can we measure independently the ΛCDM parameters ?
- Beyond ΛCDM: CMB & neutrino masses, N_{eff}, inflation, DM …
- Future CMB surveys

…following:

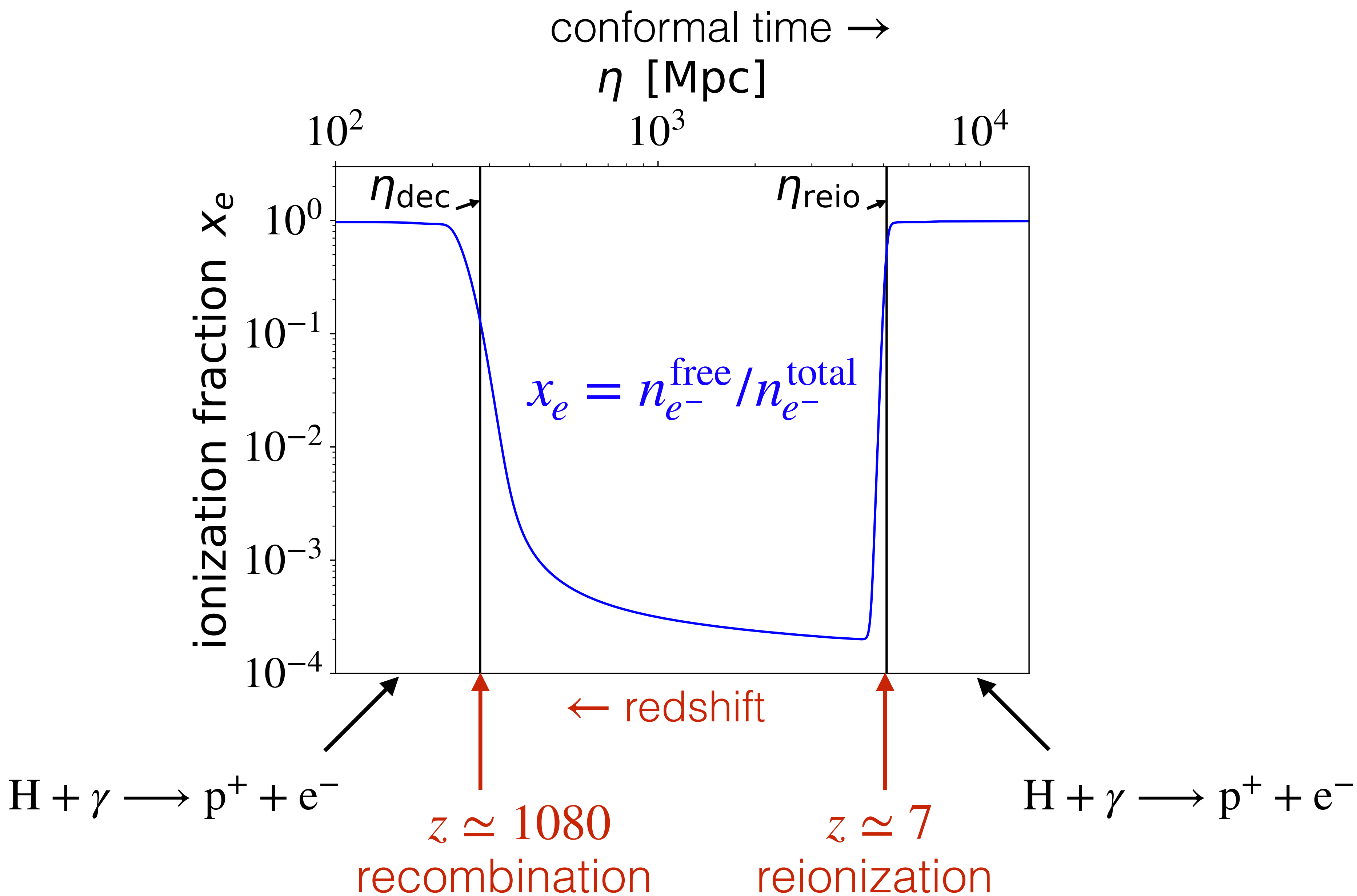
The Young Universe: Primordial Cosmology,
edited by R. Taillet (John Wiley & Sons, 2022) ISBN : 1789450322
→ Chapter 2: Cosmological Microwave Background, by JL

(also: Chapter 5 of: Neutrino Cosmology, JL et al., CUP 2013)

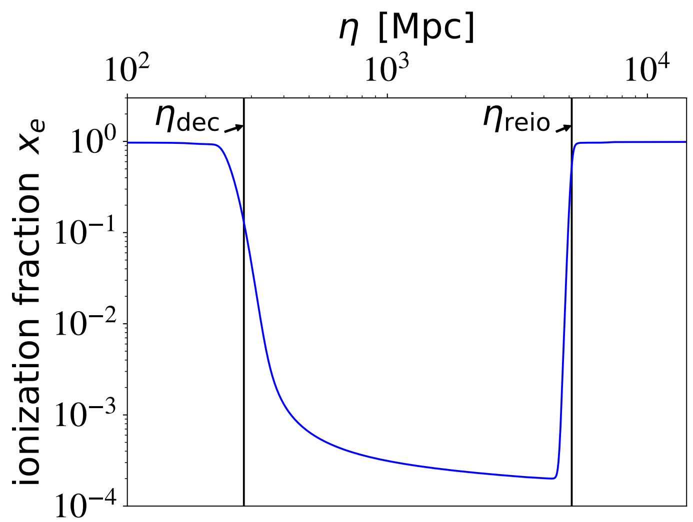
Thomson scattering

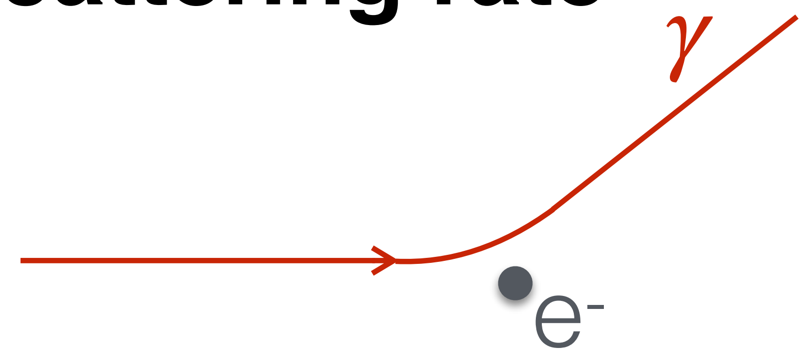


Ionisation fraction in the Universe

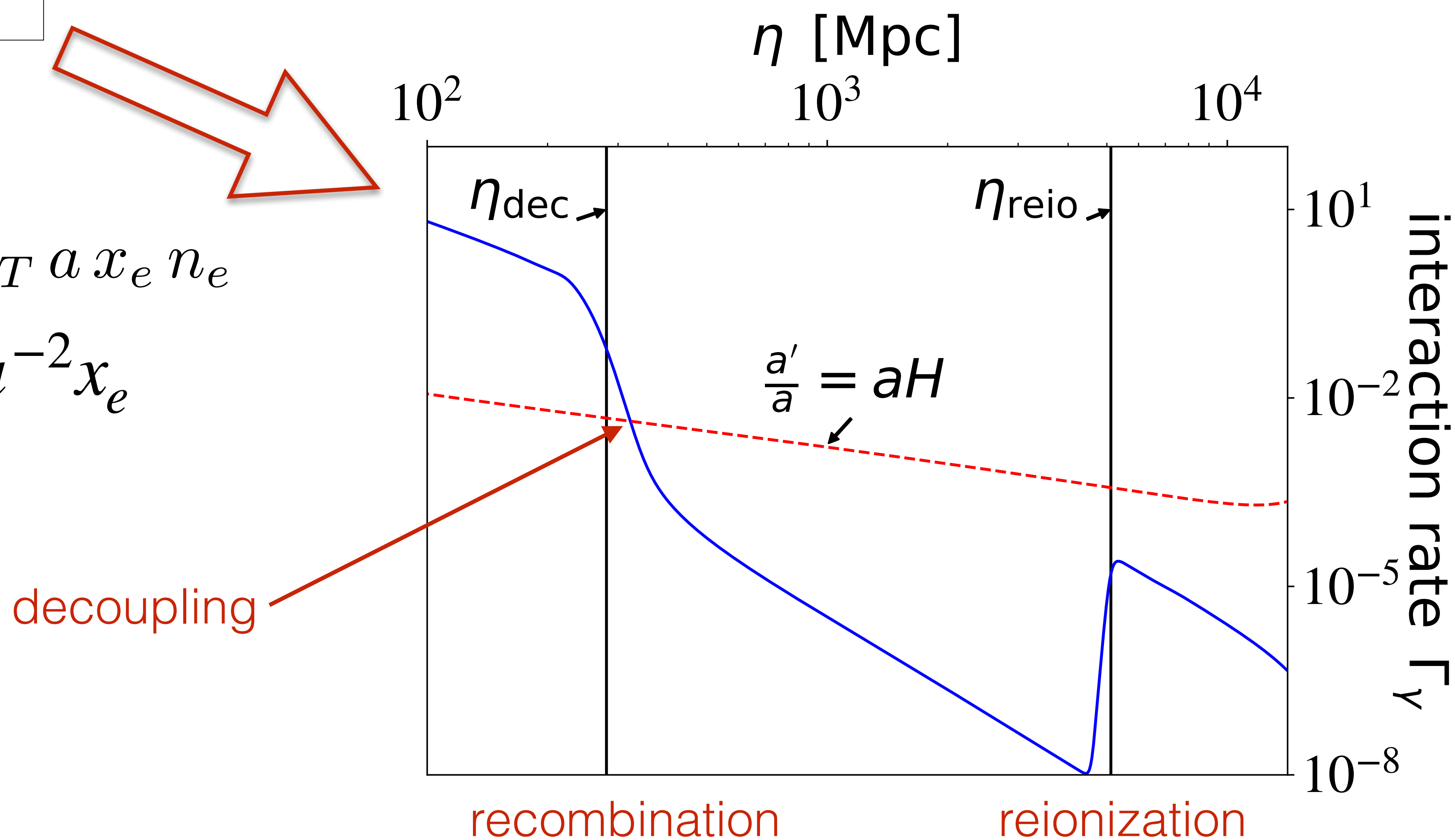


Thomson scattering rate

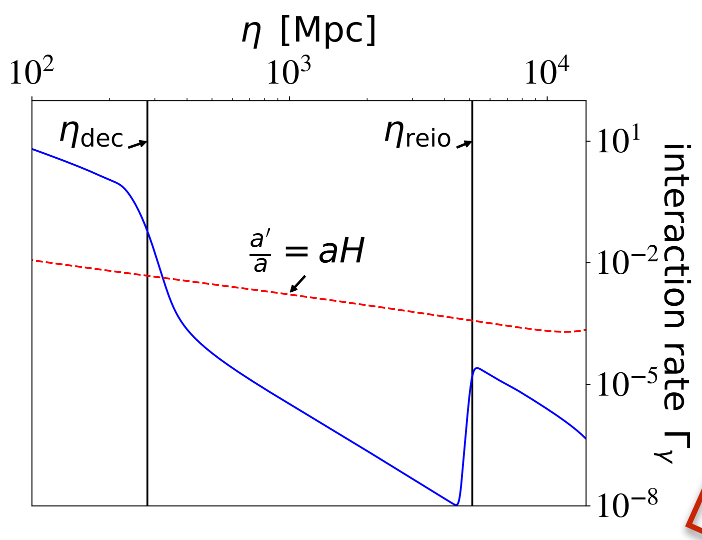




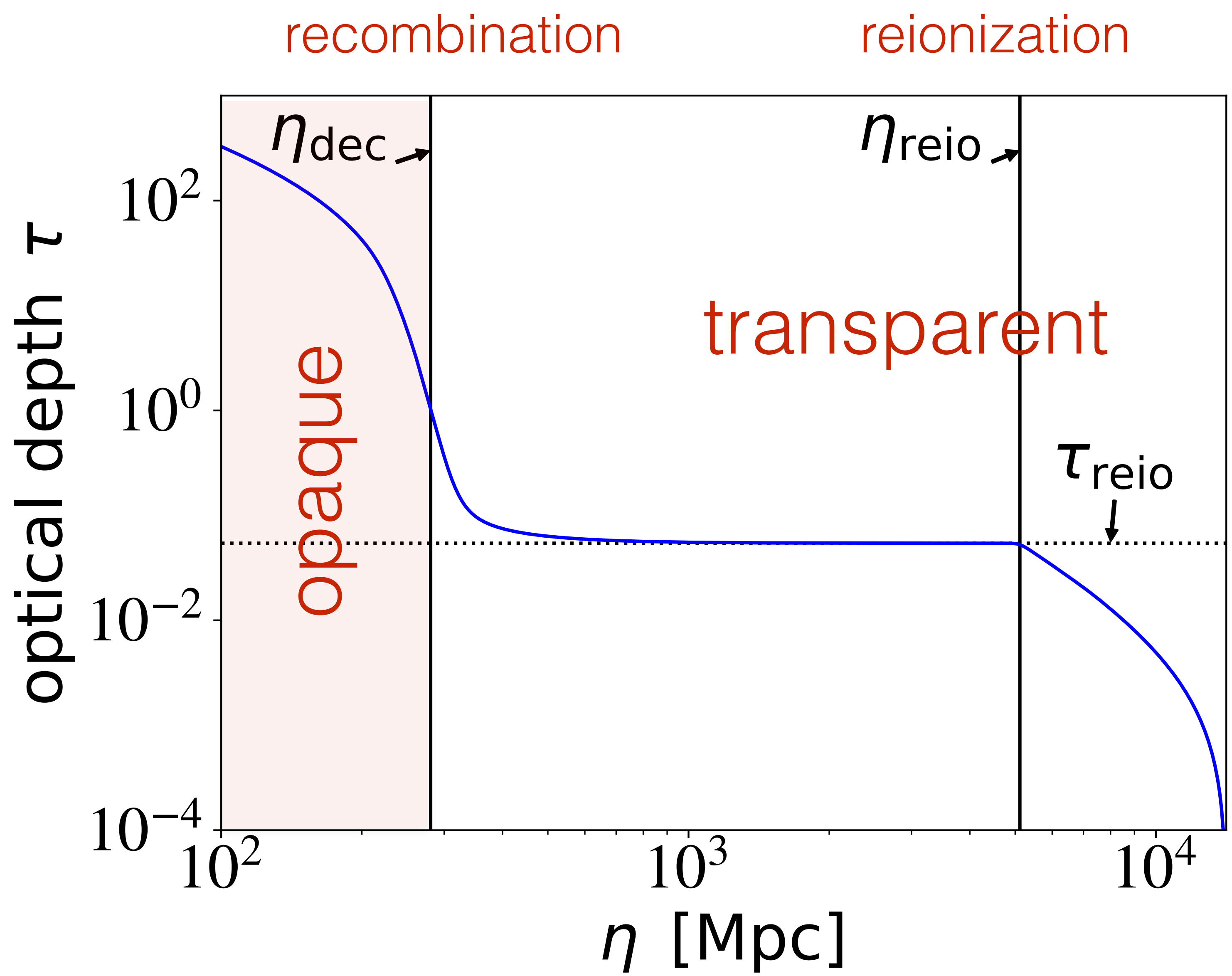
$$\Gamma_\gamma \equiv \frac{dN}{d\eta} = \sigma_T \, a \, x_e \, n_e$$
$$\propto a^{-2} x_e$$



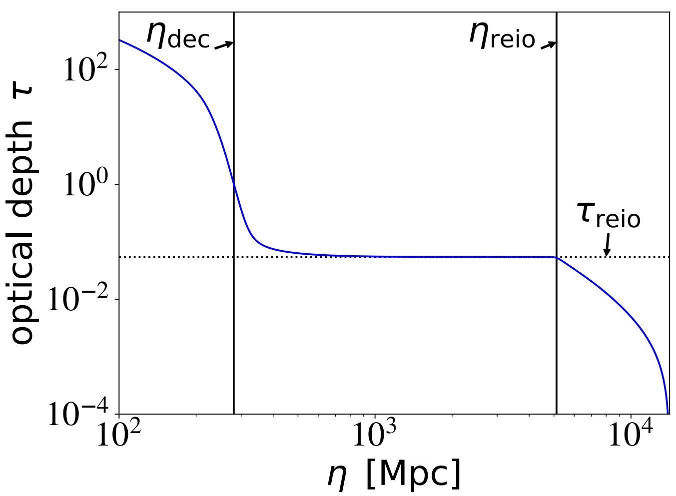
Optical depth of cosmic fog



$$\tau(\eta) \equiv \int_\eta^{\eta_0} d\eta\, \Gamma_\gamma(\eta)$$

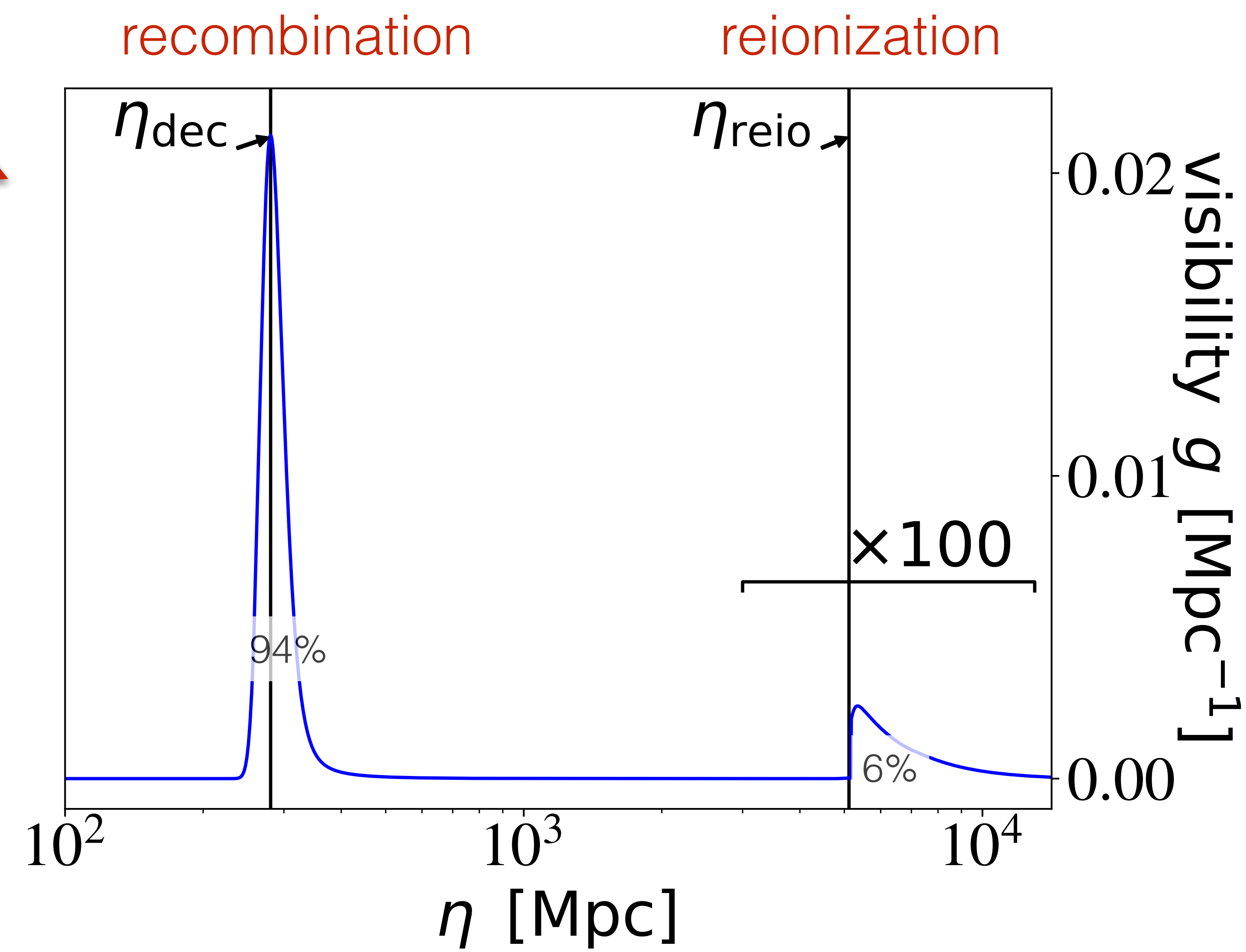


Visibility function

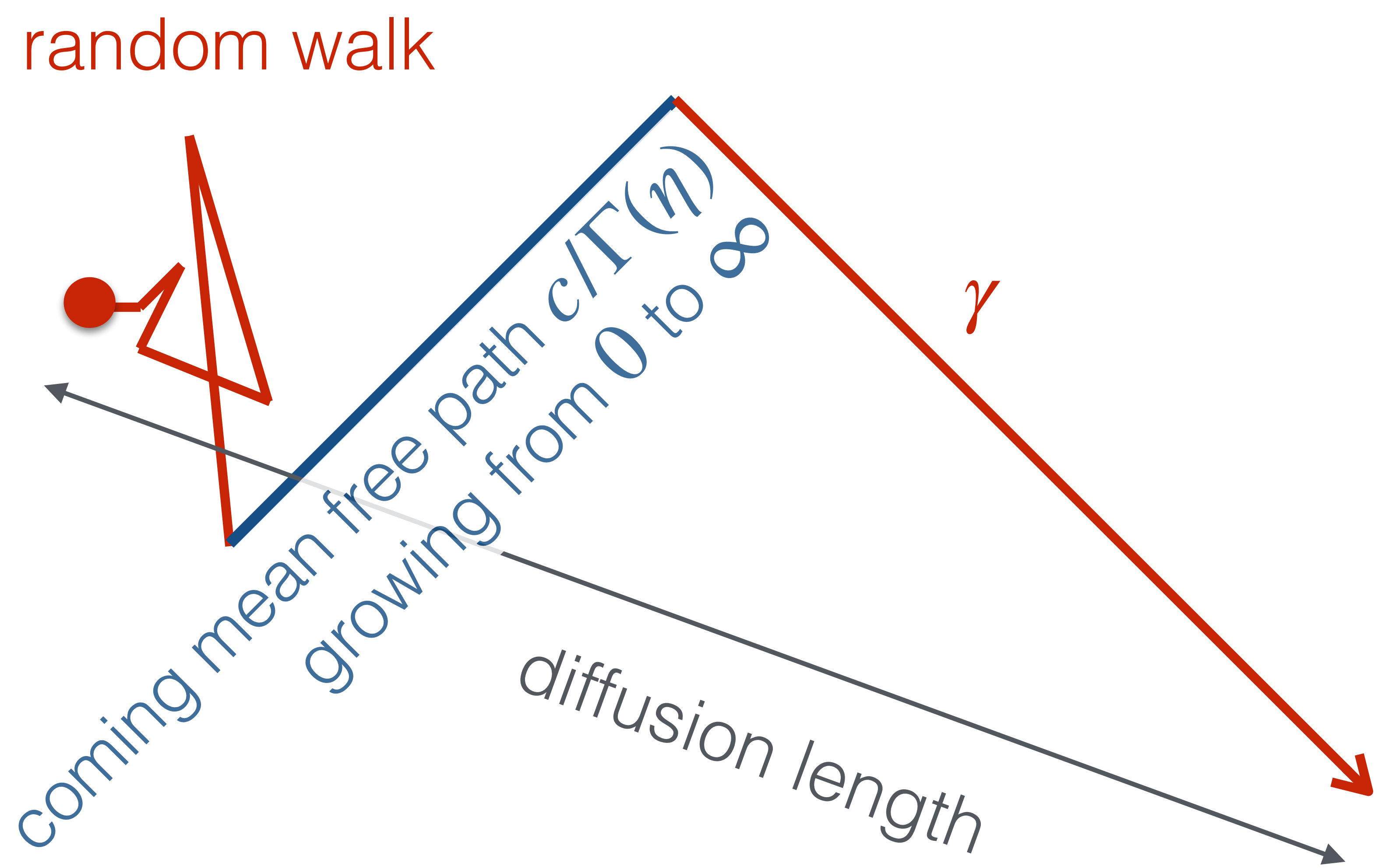


$$g(\eta) \equiv -\tau' e^{-\tau}$$

probability of last interaction at η
= probability of interaction at η
x (1-probability of interaction after η)



Mean free path and diffusion length



$$\lambda_\mathrm{d}(\eta) = a(\eta)\, r_\mathrm{d}(\eta) \simeq a(\eta) \left[\int_{\eta_\mathrm{ini}}^{\eta} d\tilde{\eta}\, c\; \Gamma_\gamma^{-1}(\tilde{\eta}) \right]^{1/2}$$

Linear cosmological perturbations

Bardeen decomposition

$$g_{\mu\nu}(\eta,\vec{x}) = \bar{g}_{\mu\nu}(\eta) + \delta g_{\mu\nu}(\eta,\vec{x}) \quad \text{and} \quad T^{\mu\nu}(\eta,\vec{x}) = \bar{T}^{\mu\nu}(\eta) + \delta T^{\mu\nu}(\eta,\vec{x})$$

background

perturbations
10 d.o.f

(for each species!)

James Bardeen 1986

FLRW background invariant under spatial rotations

$\Rightarrow$ irreducible representations of SO(3) $\rightarrow$ decoupled sectors

$\Rightarrow$ Bardeen | scalars (gravity forces) : 4 d.o.f.
vectors (graviton magnetism) : 2 d.o.f.
tensors (gravitational waves) : 2 d.o.f.

Newtonian gauge

Gauge freedom: perturbations depends on choice of coordinates

$$\Downarrow$$

Newtonian gauge: eliminate 2 scalar d.o.f. to stick to diagonal $\delta g_{\mu\nu}$

$$\begin{aligned} ds^2 &= -(1+2\psi)dt^2 + (1-2\phi)a^2(t)d\vec{x}^{\,2} \\ &= a^2(\eta)\left[-(1+2\psi)d\eta^2 + (1-2\phi)d\vec{x}^{\,2}\right] \end{aligned}$$

Local distorsion of time
= generalised
gravitational potential

Local distorsion
of expansion rate

Matter perturbations

$\delta T_{\mu\nu,X}$: still 4 d.o.f. per species X

$\Downarrow$

δ_X : relative fluctuation of energy density

θ_X : divergence of bulk velocity

δp_X : fluctuation of (isotropic) pressure

σ_X : anisotropic stress = quadrupole of (anisotropic) pressure

Perfect fluids (strong interactions)

$\Downarrow$

Pressure is isotropic ($\sigma_X = 0$)

Local pressure relates to local density (e.g. $\delta p_X \leftrightarrow \delta_X$)

$\Downarrow$

only 2 d.o.f.

Comoving Fourier space

Thereafter: only flat cosmologies for simplicity

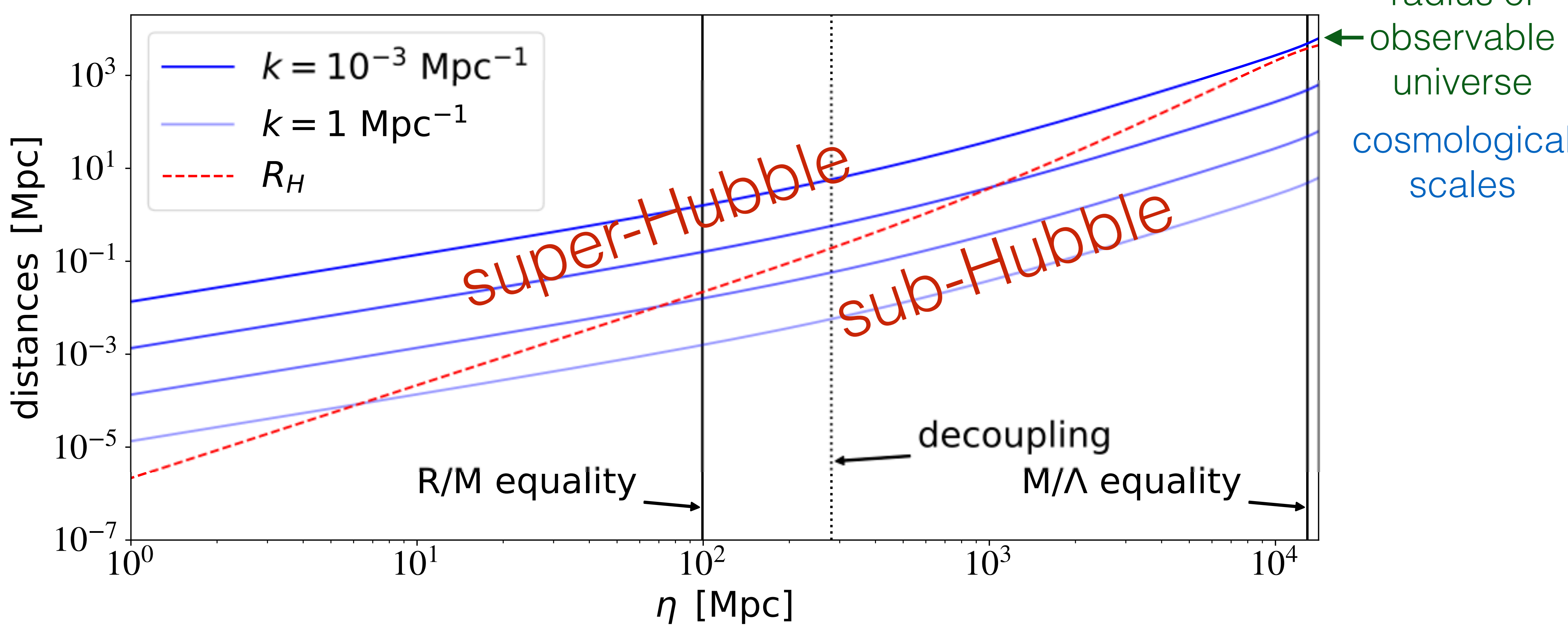
coming wavevector

comoving coordinate

$$\delta_X(\eta,\vec{k}) = \int \frac{d^3\vec{x}}{(2\pi)^{3/2}} \delta_X(\eta,\vec{x}) e^{-i\vec{k}\cdot\vec{x}}$$

Wavelengths $\lambda(\eta) = a(\eta)\ 2\pi/k$

Decelerated expansion ⇒ grow slower than Hubble radius $R_H(\eta) = c\, a(\eta)/\dot{a}(\eta)$



Condition for Hubble crossing : $k \sim aH \sim 1/\eta$

Linearised Einstein Equations

- from $G^0_0 = 8\pi G\, T^0_0$:

$$\frac{2}{a^2}\left[k^2\phi + 3\frac{a'}{a}\left(\phi' + \frac{a'}{a}\psi\right)\right] = -8\pi G \sum_X \bar{\rho}_X \delta_X$$

dominates on sub-Hubble

$\Downarrow$

Poisson equation :

$$-\frac{k^2}{a^2}\phi = 4\pi G\, \bar{\rho}_{\rm total}\, \delta_{\rm total}$$

dominates on super-Hubble

$\Downarrow$

Using Friedmann :

$$2\psi = -\,\delta_{\rm total}$$

- from $G^i_j = 8\pi G\, T^i_j$:

$$\frac{2}{3}\frac{k^2}{a^2}(\phi - \psi) = 8\pi G \sum_X (\bar{\rho}_X + \bar{p}_X)\sigma_X$$

perfect fluids $\Rightarrow \sigma_X = 0 \Rightarrow \phi = \psi$

- other scalar equations redundant with upcoming equations of motion (Bianchi identity)

Equations of motion

Without details: each decoupled species fulfils energy/momentum conservation:

✦ continuity equation: $\delta'_X = -(1+w_X)(\theta_X - 3\phi') - 3\frac{a'}{a}(c^2_{s\,X} - w_X)\delta_X$

✦ Euler equation: $\theta'_X = -\frac{a'}{a}(1-3c^2_{a\,X})\theta_X + \frac{c^2_{s\,X}}{1+w_X}k^2\delta_X - k^2\sigma_X + k^2\psi$

(featuring sound speed c_s and adiabatic sound speed c_a)

In ΛCDM:

- CDM : negligible pressure/stress: closed system
- e-/baryons: negligible pressure/stress but Thomson scattering: $+\frac{4}{3}\frac{\bar{\rho}_\gamma}{\bar{\rho}_\mathrm{b}}\tau'(\theta_\mathrm{b} - \theta_\gamma)$
- photons, neutrinos: when not strongly coupled, anisotropic stress

→need Boltzmann equation

$$\frac{d}{d\eta}f_\gamma = C\left[f_\gamma, f_e\right] \qquad\qquad \frac{d}{d\eta}f_\nu = 0$$

Photon phase-space distribution

Blackbody shape: $f_\gamma(\eta,\vec{x},p,\hat{n}) = \dfrac{1}{e^{\frac{p}{T(\eta,\vec{x},\hat{n})}} - 1}$

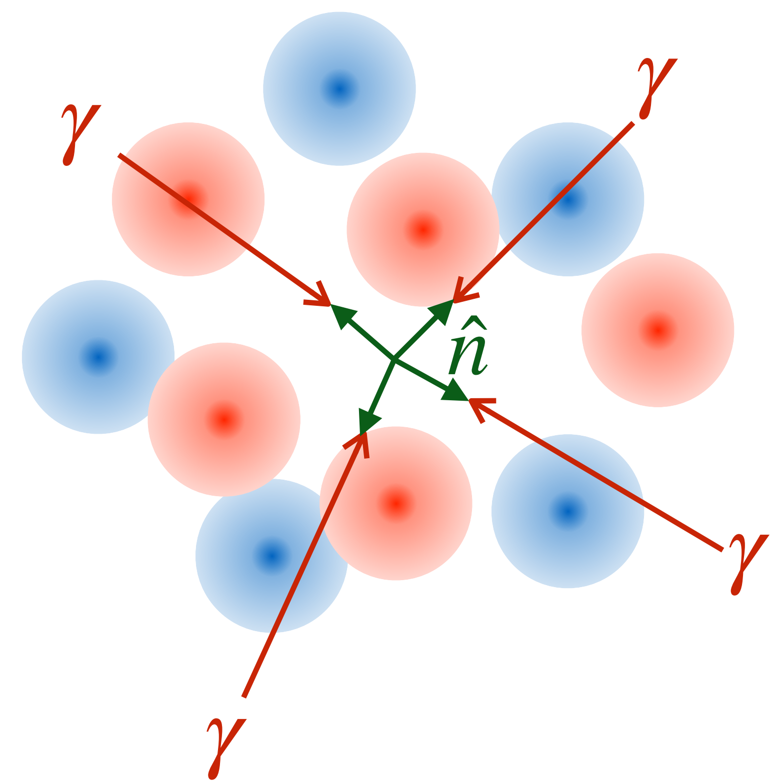
Up to very good approximation: preserved even when leaving thermal equilibrium, but becomes direction-dependent due to gravitational interactions:

redshifting along geodesics:

$$\frac{d\ln(a\,p)}{d\eta} = \phi' - \hat{n}\cdot\vec{\nabla}\psi$$

dilation

gravitational Doppler



Then: $T(\eta,\vec{x},\hat{n}) = \bar{T}(\eta)\,\big(1+\Theta(\eta,\vec{x},\hat{n})\big)$

Linearised Boltzmann equation

Monopole and dipole of Θ account for local density & bulk velocity:

$$\Theta(\eta, \vec{x}, \hat{n}) = \boxed{\frac{1}{4}\delta_\gamma(\eta, \vec{x})} + \boxed{\hat{n} \cdot \vec{v}_\gamma(\eta, \vec{x})} + \text{higher multipoles}$$

Thomson scattering wants to align velocity of photons vs. electron/baryons, and to wash out higher multipoles:

$$\Theta' + \hat{n} \cdot \vec{\nabla}\Theta - \underbrace{\boxed{\phi'}}_{\text{dilation}} + \underbrace{\boxed{\hat{n} \cdot \vec{\nabla}\psi}}_{\text{gravitational Doppler}} = -\underbrace{\boxed{\Gamma_\gamma}}_{\text{Thomson scattering}}\left(\hat{n} \cdot (\vec{v}_\gamma - \vec{v}_\mathrm{b}) + \text{higher multipoles}\right)$$

Boltzmann hierarchy

After:

- Fourier transformation
- Legendre expansion $\Theta(\eta,\vec{k},\alpha) = \sum_l (-i)^l (2l+1)\Theta_l(\eta,\vec{k})P_l(\cos\alpha)$

one gets:

$$\delta'_\gamma + \frac{4}{3}\theta_\gamma - 4\phi' = 0$$

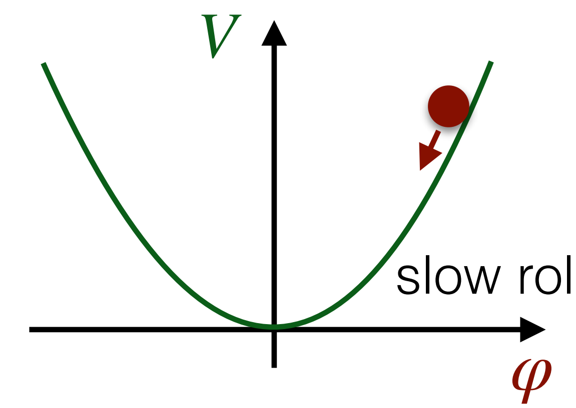
$$\theta'_\gamma + k^2\left(-\frac{1}{4}\delta_\gamma + \sigma_\gamma\right) - k^2\psi = \tau'(\theta_\gamma - \theta_{\rm b})$$

$$\Theta'_l - \frac{kl}{2l+1}\Theta_{l-1} + \frac{k(l+1)}{2l+1}\Theta_{l+1} = \tau'\Theta_l \quad \forall l \geq 2$$

⇒ Solved together with previous equations by Einstein-Boltzmann solvers (CMBFAST, CAMB, CLASS…)

Stochastic theory of cosmological perturbations

Initial conditions



Canonical single-field inflation guarantees:

A. stochastic perturbations with independent Fourier modes

B. gaussian statistics for each Fourier mode / each d.o.f.

$\Rightarrow$ described by variance(wavenumber) = power spectrum

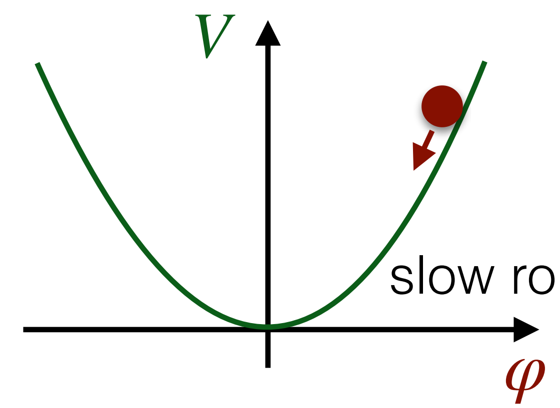
C. for each Fourier mode, all d.o.f. related to each other (fully correlated) on super-Hubble scales: "adiabatic initial conditions"

e.g. during RD: $$-2\psi = -2\phi \underset{\substack{\uparrow \\ \text{Einstein eq.}}}{=} \delta_\gamma = \delta_\nu = \frac{4}{3}\delta_{\rm b} = \frac{4}{3}\delta_{\rm c} \underset{\substack{\uparrow \\ \text{Einstein eq.}}}{=} \text{constant}$$

(Comes from $A(\eta,\vec{x}) = \bar{A}(\eta+\delta\eta(\vec{x})) = \bar{A}(\eta) + \bar{A}'(\eta)\,\delta\eta(\vec{x})$)

perturbation $\delta A(\eta,\vec{x})$
in adiabatic case

Primordial power spectrum



Canonical single-field inflation guarantees:

A. stochastic perturbations with independent Fourier modes

B. gaussian statistics for each Fourier mode / each d.o.f.

$\Rightarrow$ described by variance(wavenumber) = power spectrum

C. for each Fourier mode, all d.o.f. related to each other (fully correlated) on super-Hubble scales: “adiabatic initial conditions”

$\Rightarrow$ need power spectrum for single degree of freedom, e.g. curvature perturbation $\mathcal{R} \equiv \phi - \frac{a'}{a}\frac{v_{\mathrm{tot.}}}{a^2}$ in Newt. Gauge

$\Rightarrow$ Primordial spectrum: $\langle \mathcal{R}(\eta_{\mathrm{ini}},\vec{k})\mathcal{R}^*(\eta_{\mathrm{ini}},\vec{k}')\rangle = \delta_D(\vec{k}'-\vec{k})\ \boxed{\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(k)}$

D. Power law, nearly scale-invariant spectrum: $\mathcal{P}_{\mathcal{R}}(k) = A_{\mathrm{s}}\left(\frac{k}{k_*}\right)^{n_{\mathrm{s}}-1}$

Transfer functions

For each Fourier mode $\vec{k}$:

- all perturbations → system of linear coupled differential equations
- adiabatic ICs → single constant of integration $\mathcal{R}(\eta_{\rm ini},\vec{k})$
- e.g. for densities $\forall X, \quad \delta_X(\eta,\vec{k}) = \tilde{\alpha}_X(\eta,k)\,\mathcal{R}(\eta_{\rm ini},\vec{k})$

stochastic Fourier mode

stochastic IC

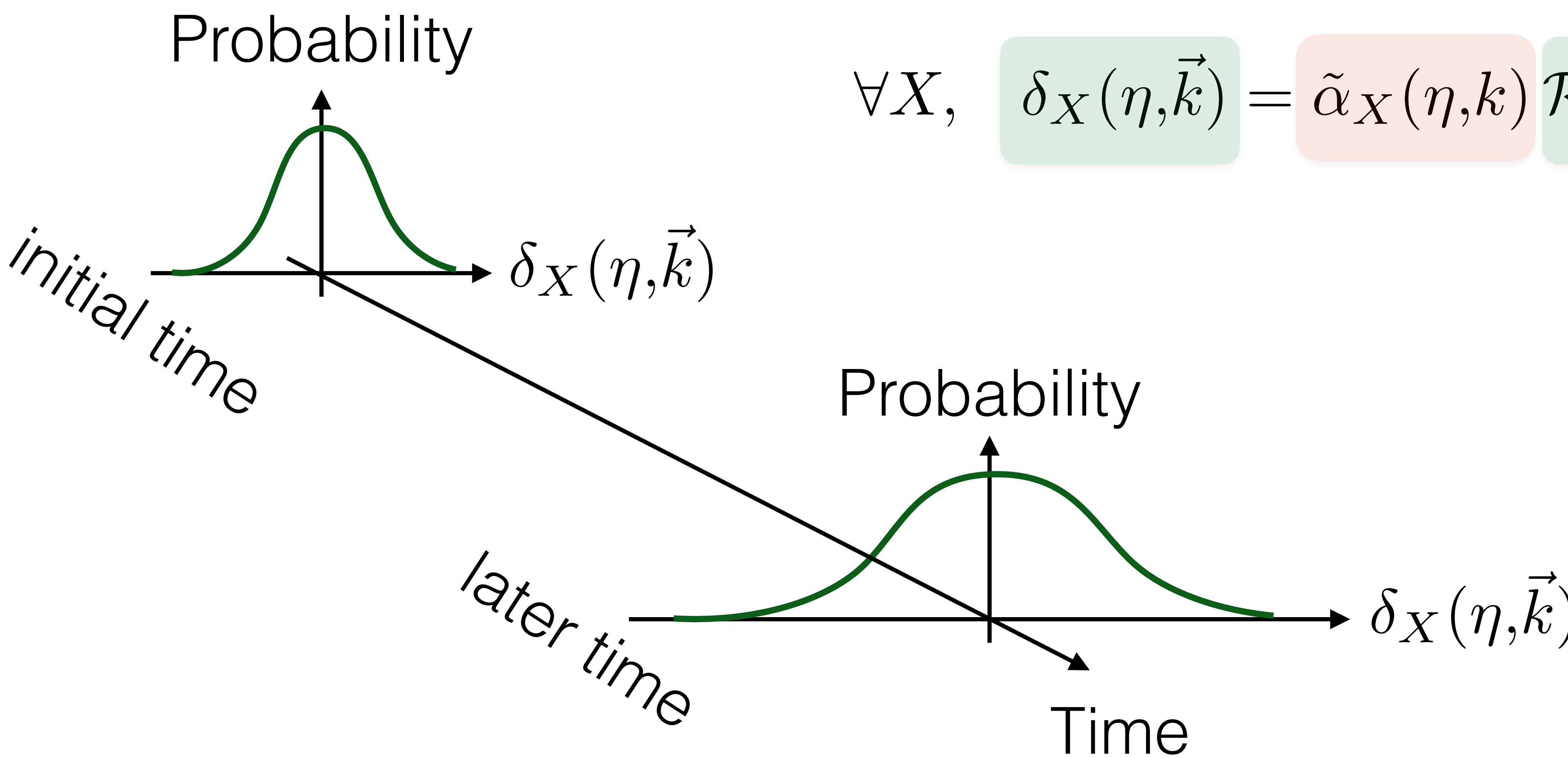
Deterministic solution of e.o.m. normalised to $\mathcal{R}=1$

= transfer function of δ_X

Isotropic background ⇒ depends only on k

⇒ denoted later as $\delta_X(t,k)$

Linear transport of probability



$$\forall X, \quad \delta_X(\eta,\vec{k}) = \tilde{\alpha}_X(\eta,k)\,\mathcal{R}(\eta_{\rm ini},\vec{k})$$

Linearity of solutions ⇒ probability shape always preserved (standard model: Gaussian)

⇒ variance evolves like squared transfer function

Power spectrum

Adiabatic initial conditions

$\Rightarrow$ for <u>any</u> perturbation at <u>any</u> time:

$$\langle A(\eta,\vec{k})\,A^*(\eta,\vec{k}')\rangle = A(\eta,k)\,A^*(\eta,k')\,\langle \mathscr{R}(\eta_\mathrm{i},\vec{k})\,\mathscr{R}^*(\eta_\mathrm{i},\vec{k}')\rangle$$

$$= \boxed{|\underbrace{A(\eta,k)}_{\text{transfer function of } A}|^2\,\frac{2\pi^2}{k^3}\,\underbrace{\mathscr{P}_{\mathscr{R}}(k)}_{\text{dimensionless primordial spectrum}}}\;\delta_D(\vec{k}-\vec{k}')$$

power spectrum of A at η

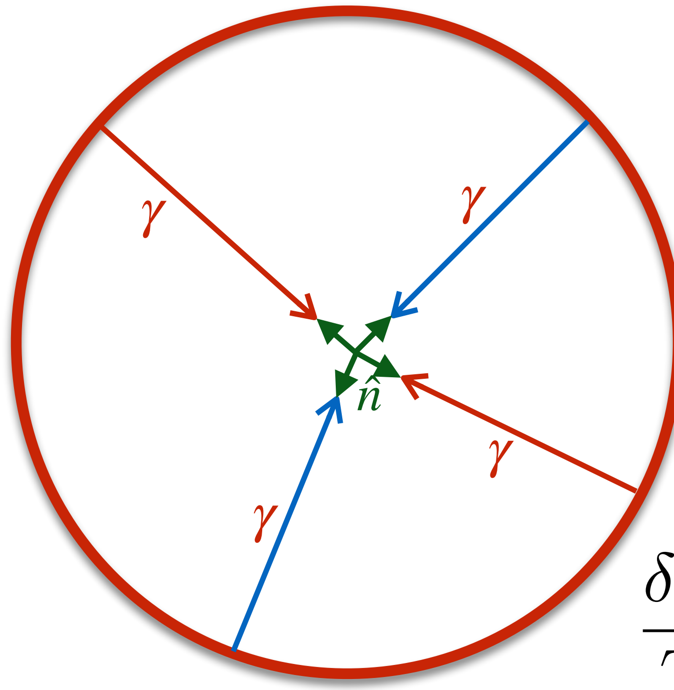
Spectrum of temperature anisotropies

Temperature multipoles

$g(\eta)$ very peaked at $\eta_{\rm dec}$

$\Downarrow$

last scattering sphere



$$\frac{\delta T}{\bar{T}}(\hat{n}) = \Theta(\eta_0, \vec{o}, -\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

inversion + Fourier + Legendre $\Rightarrow$ $$a_{lm} = (-i)^l \int \frac{d^3\vec{k}}{2\pi^2} Y_{lm}(\hat{k}) \Theta_l(\eta_0, \vec{k})$$

stochastic, Gaussian ← stochastic, Gaussian

photon transfer function

primordial spectrum

correlation/variance $\Rightarrow$ $$\langle a_{lm} a^*_{l'm'} \rangle = \delta^K_{ll'} \delta^K_{mm'} \left[\frac{1}{2\pi^2} \int \frac{dk}{k} \Theta_l^2(\eta_0, k) \mathcal{P}_\mathcal{R}(k) \right]$$

Temperature power spectrum

Multipole correlation/variance:

transfer function — primordial spectrum

$$\langle a_{lm} a^*_{l'm'} \rangle = \delta^K_{ll'} \delta^K_{mm'} \boxed{\left[\frac{1}{2\pi^2} \int \frac{dk}{k} \Theta_l^2(\eta_0, k) \mathcal{P}_\mathcal{R}(k) \right]}$$

$$= C_\ell$$

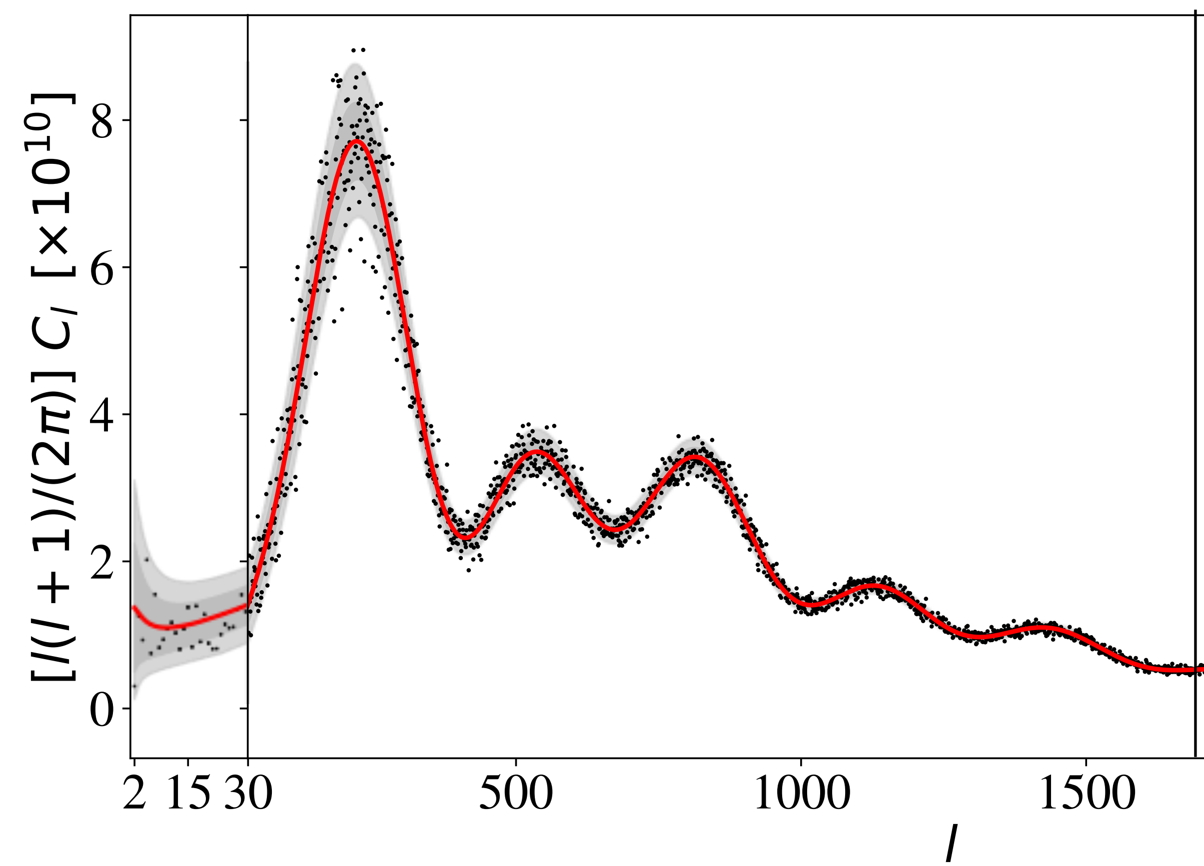
temperature power spectrum

theory $\longleftrightarrow$ observations

Observations: estimator:

$$\hat{C}_l(a_{lm}) \equiv \frac{1}{2l+1} \sum_{-l \leq m \leq l} |a_{lm}|^2$$

Cosmic variance: $\langle (\hat{C}_l - C_l)^2 \rangle = \frac{2}{2l+1} C_l^2$



Physics of temperature anisotropies

“Line-of-sight” integral in Fourier space

Boltzmann hierarchy ⇒ formal solution Zaldarriaga & Harari astro-ph/9504085:

$$\Theta_l(\eta_0,\vec{k}) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \, \{ g\,(\Theta_0+\psi)\; j_l(k(\eta_0-\eta))$$

$$+\, g\, k^{-1}\theta_{\rm b}\; j_l'(k(\eta_0-\eta))$$

$$+\, e^{-\tau}(\phi'+\psi')\; j_l(k(\eta_0-\eta)) \}$$

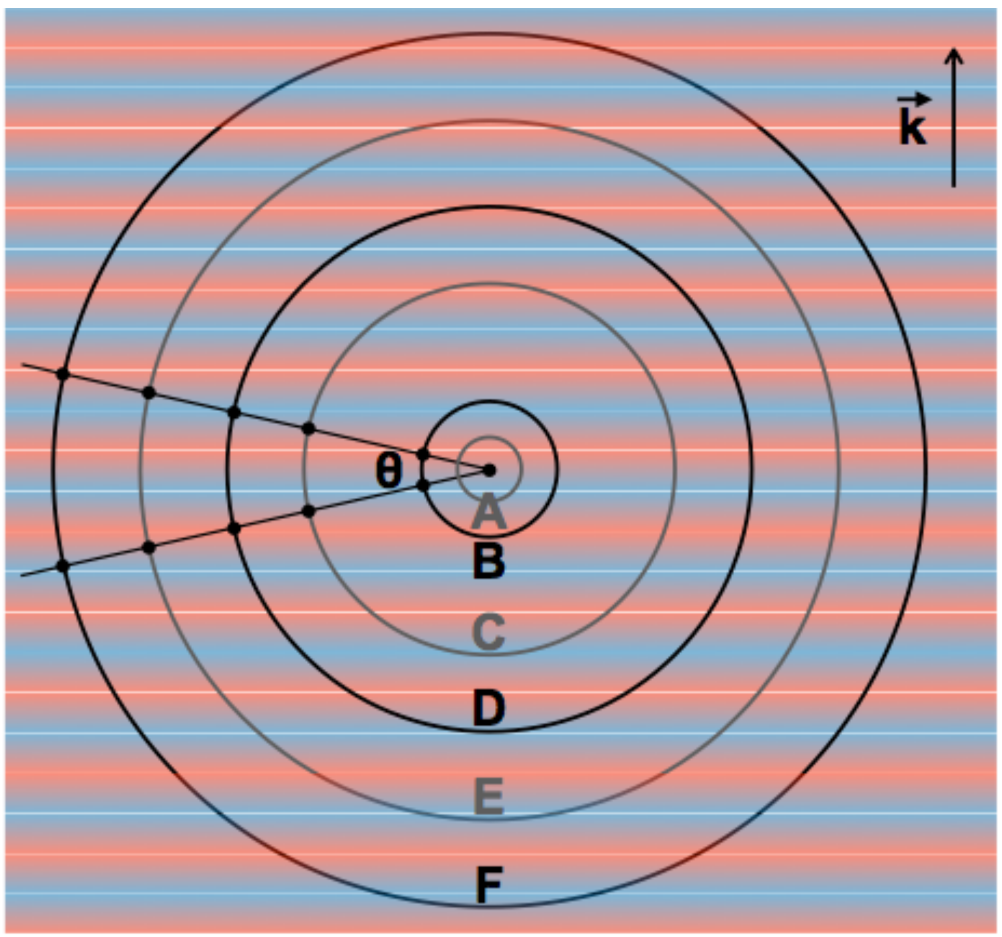
valid both for single mode $\vec{k}$ or transfer function with k

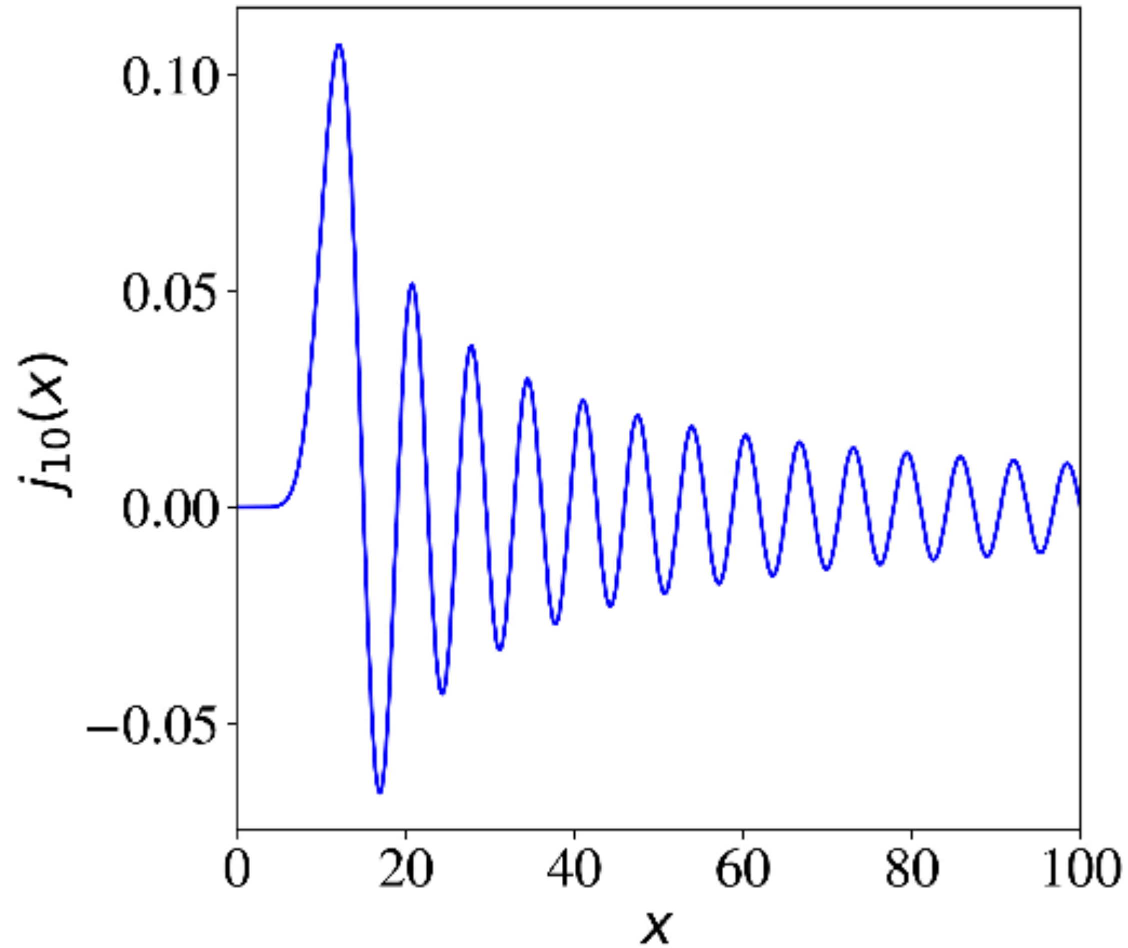
structure: $\int d\eta \left[f(\eta)\; A(\eta,\vec{k})\; j_\ell(k(\eta_0-\eta)) \right]$

“Physical effects relevant at times described by $f(\eta)$ imprint CMB photon anisotropies described in Fourier space by $A(\eta,\vec{k})$, that project to multipole space according to $j_\ell(k(\eta_0-\eta))$”

Angular projection of Fourier modes

Role of $j_\ell(k(\eta_0 - \eta))$?





Main contribution: $\theta = \frac{\pi}{l} = \frac{\lambda/2}{d_\mathrm{a}} = \frac{a(\eta)\,\pi/k}{a(\eta)\,(\eta_0-\eta)} \quad \Leftrightarrow \quad l = k(\eta_0 - \eta)$

Other contributions: harmonics

Sachs-Wolfe term

$$\Theta_l(\eta_0,\vec{k}) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \, \{ g \, (\Theta_0 + \psi) \; j_l(k(\eta_0 - \eta))$$

$$+ \, g \, k^{-1} \theta_{\rm b} \; j_l'(k(\eta_0 - \eta))$$

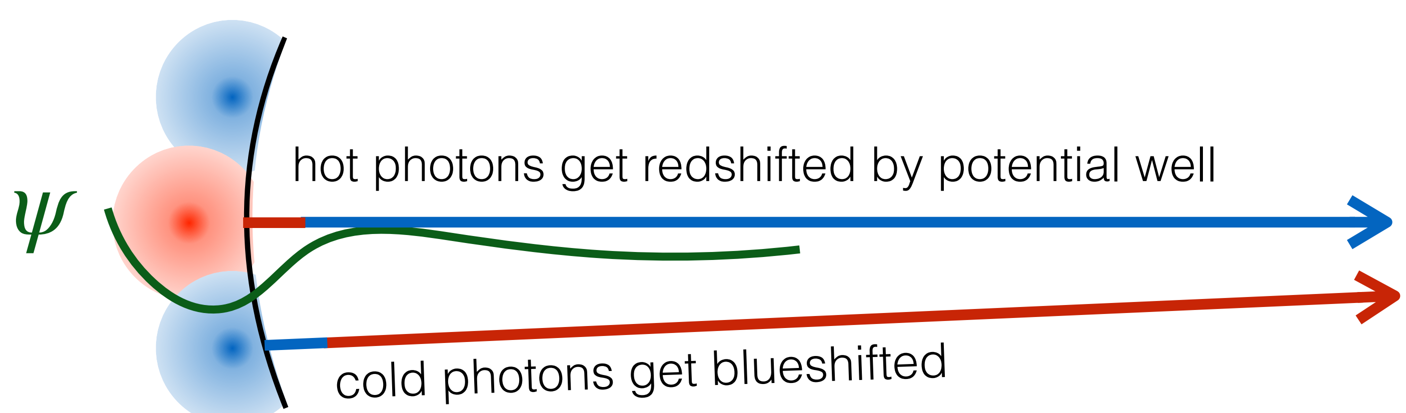
$$+ \, e^{-\tau} (\phi' + \psi') \, j_l(k(\eta_0 - \eta)) \}$$

Neglecting reionization: $g(\eta)$ very peaked at $\eta_{\rm dec}$

$\Rightarrow$ effect takes place only on last scattering sphere

$\Rightarrow$ mode k project to $\ell = k(\eta_0 - \eta_{\rm dec})$

$\Theta_0(\eta_{\rm dec}, \vec{k}) + \psi(\eta_{\rm dec}, \vec{k})$ = intrinsic fluctuation + gravitational Doppler shift



(super-Hubble modes with adiabatic IC: $\psi = -2\Theta_0$, Sachs-Wolfe effect wins, negative picture of last scattering sphere !)

Doppler term

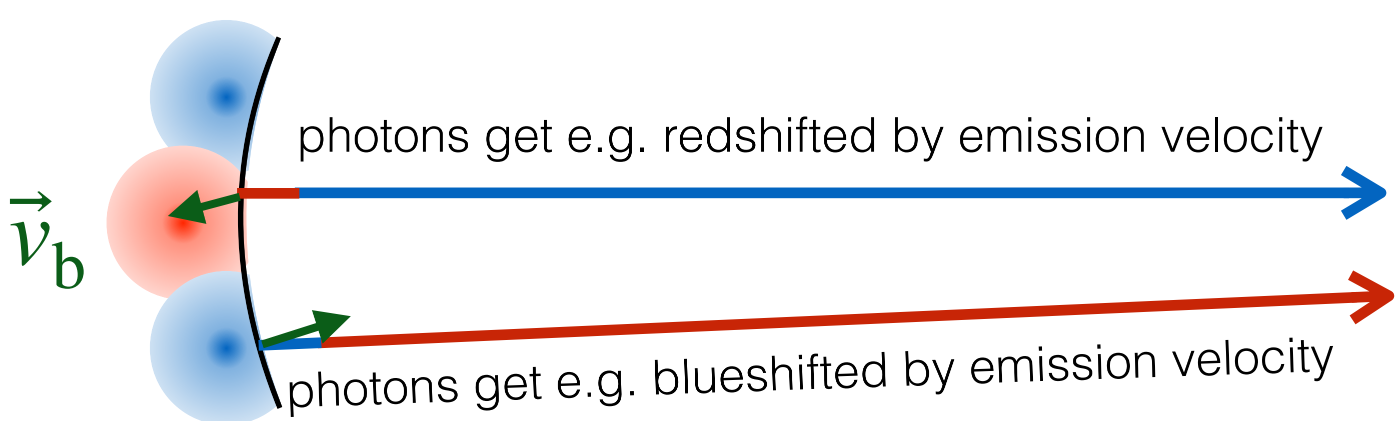
$$\Theta_l(\eta_0,\vec{k}) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \, \{g \, (\Theta_0 + \psi) \; j_l(k(\eta_0 - \eta))$$
$$+ \, g \, k^{-1}\theta_{\rm b} \; j_l'(k(\eta_0 - \eta))$$
$$+ \; e^{-\tau}(\phi' + \psi') \, j_l(k(\eta_0 - \eta))\}$$

Neglecting reionization: $g(\eta)$ very peaked at $\eta_{\rm dec}$

$\Rightarrow$ effect takes place only on last scattering sphere

$\Rightarrow$ mode k project to $\ell = k(\eta_0 - \eta_{\rm dec})$

$\vec{v}_{\rm b}^{\rm scalar} \rightarrow k^{-1}\theta_{\rm b}$ = velocity Doppler shift (j_ℓ' from a gradient)



Integrated Sachs-Wolfe (ISW) term

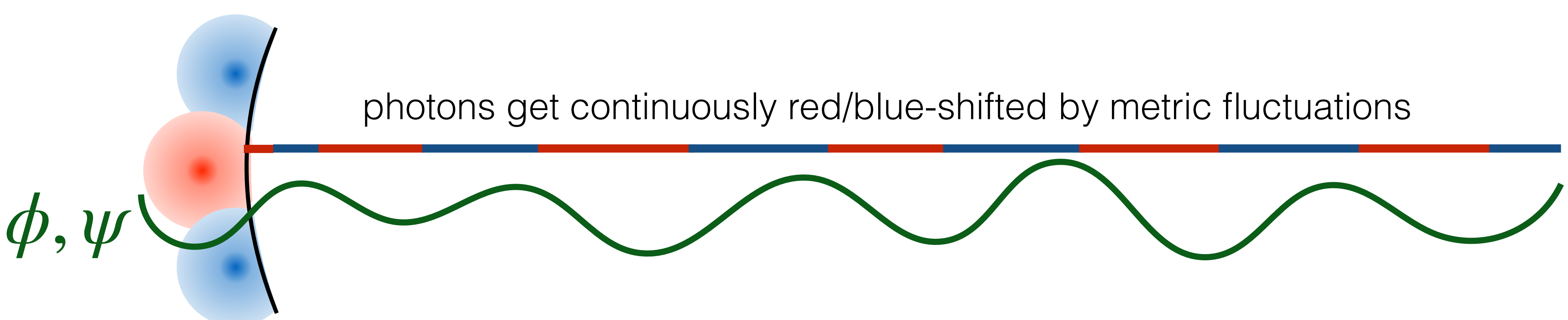
$$\Theta_l(\eta_0,\vec{k}) = \; \dots \; + \; e^{-\tau}(\phi' + \psi')\, j_l(k(\eta_0 - \eta))$$

Neglecting reionization: $e^{-\tau}$ negligible before $\eta_{\rm dec}$, $\simeq 1$after

$\Rightarrow$ effect takes place at all times $\eta > \eta_{\rm dec}$ along each line of sight

$\Rightarrow$ mode k projects from each sphere to $\ell = k(\eta_0 - \eta)$

$\partial_\eta\{\phi(\eta,\vec{k}) + \psi(\eta,\vec{k})\}$ comes from dilation + gravitational Doppler effects



- ϕ, ψ static: no dilation, gravitational Doppler effect is conservative: only $(\psi_{\rm dec} - \psi_{\rm obs})$
- ϕ, ψ time-dependent: net effect (e.g. net redshift when crosses deepening potential wells)

Summary

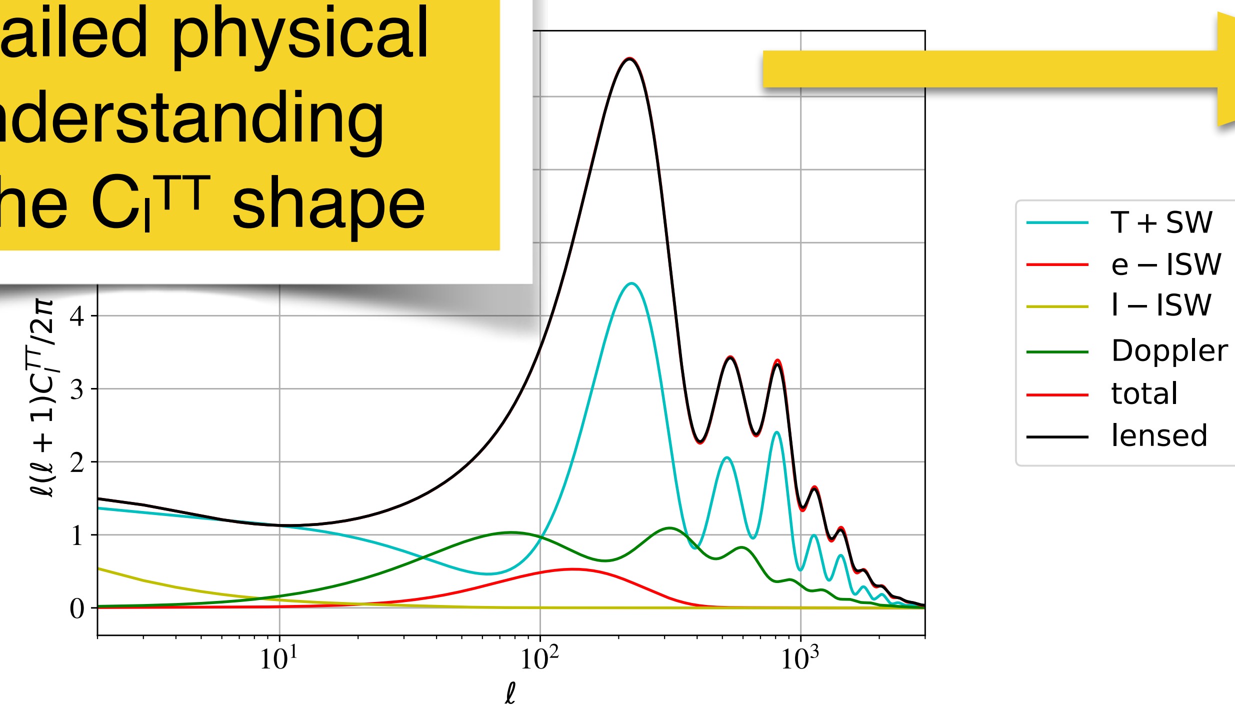
Final goal: compute

$$C_l \equiv \langle |a_{lm}|^2 \rangle = \frac{1}{2\pi^2} \int \frac{dk}{k} \Theta_l^2(\eta_0,k) \mathcal{P}_{\mathcal{R}}(k)$$

with transfer functions

$$\Theta_l(\eta_0,k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \, \{ g \left(\Theta_0 + \psi\right) j_l(k(\eta_0-\eta)) + g \, k^{-1}\theta_{\rm b} \, j_l'(k(\eta_0-\eta)) + e^{-\tau}(\phi' + \psi') \, j_l(k(\eta_0-\eta)) \}$$

Detailed physical understanding of the C_l^{TT} shape



behaviour of

$\Theta_0(\eta_{\rm dec}, k)$

$\theta_{\rm b}(\eta_{\rm dec})$

$\psi(\eta \geq \eta_{\rm dec}, k) \simeq \phi$

Tight-Coupling Approximation (TCA)

Tightly coupled baryon-photon fluid: $\begin{cases} \Theta_0 = \frac{1}{4}\delta_\gamma = \frac{1}{3}\delta_b \\ 3k\Theta_1 = \theta_\gamma = \theta_b \\ \Theta_{l\geq 2} = 0 \end{cases}$

→ from thermal equilibrium (first line)

→ from efficient Thomson scattering (second and third lines)

Photon Boltzmann hierarchy + baryon e.o.m. —> TCA equation:

$$\boxed{\underbrace{\Theta_0'' + \frac{R'}{1+R}\Theta_0'}_{\text{baryon damping}} + \underbrace{k^2 c_\mathrm{s}^2 \Theta_0}_{\text{pressure force}} = \underbrace{-\frac{k^2}{3}\psi}_{\text{gravity force}} + \underbrace{\frac{R'}{1+R}\phi'}_{\text{local baryon damping}} + \underbrace{\phi''}_{\text{dilation}}}$$

Sound speed / baryon-to-photon ratio: $c_\mathrm{s}^2 = \dfrac{1}{3(1+R)}\ , \qquad R \equiv \dfrac{3\bar{\rho}_\mathrm{b}}{4\bar{\rho}_\gamma} \propto a$

Newtonian gauge with $\quad ds^2 = -(1+2\psi)a^2 d\tau^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j$

Tight-coupling equation

$$\Theta_0'' + \frac{R'}{1+R}\Theta_0' + k^2 c_s^2 \Theta_0 = -\frac{k^2}{3}\psi + \frac{R'}{1+R}\phi' + \phi''$$

baryon damping — pressure force — gravity force — local baryon damping — dilation

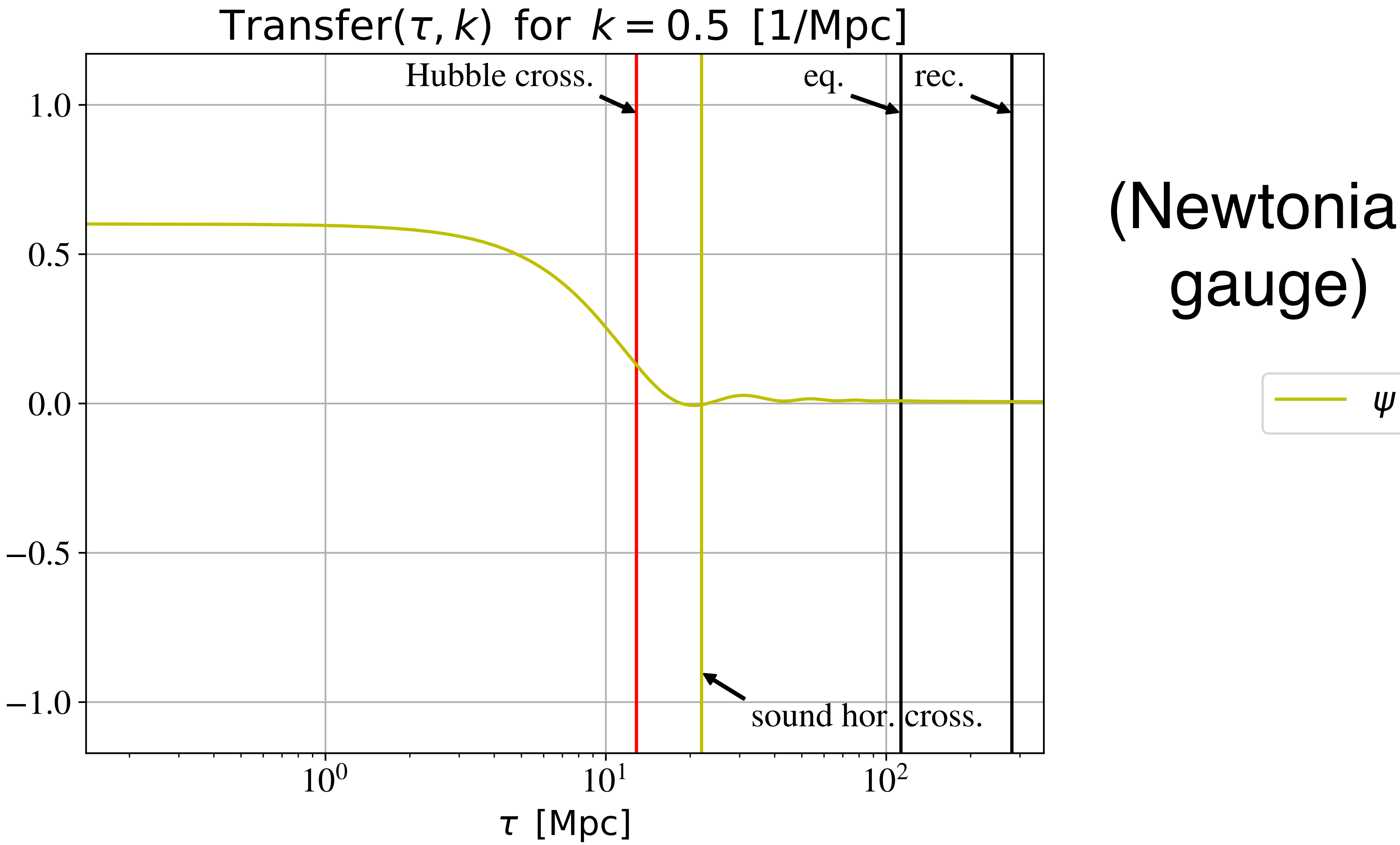
Sound speed / baryon-to-photon ratio: $c_s^2 = \frac{1}{3(1+R)}\,, \quad R \equiv \frac{4\bar{\rho}_b}{3\bar{\rho}_\gamma} \propto a$

Equilibrium point neglecting metric time derivatives: $\Theta_0^{\mathrm{equi.}} = -\frac{1}{3c_s^2}\psi = -(1+R)\psi$

WKB TCA solution " " " : $\Theta_0 = A(1+R)^{-1/4}\cos\left(k \int c_s(\tau)d\tau\right) - (1+R)\psi$

Very good approximation up to gravity boost + (Silk) damping/diffusion effects

Evolution for one mode

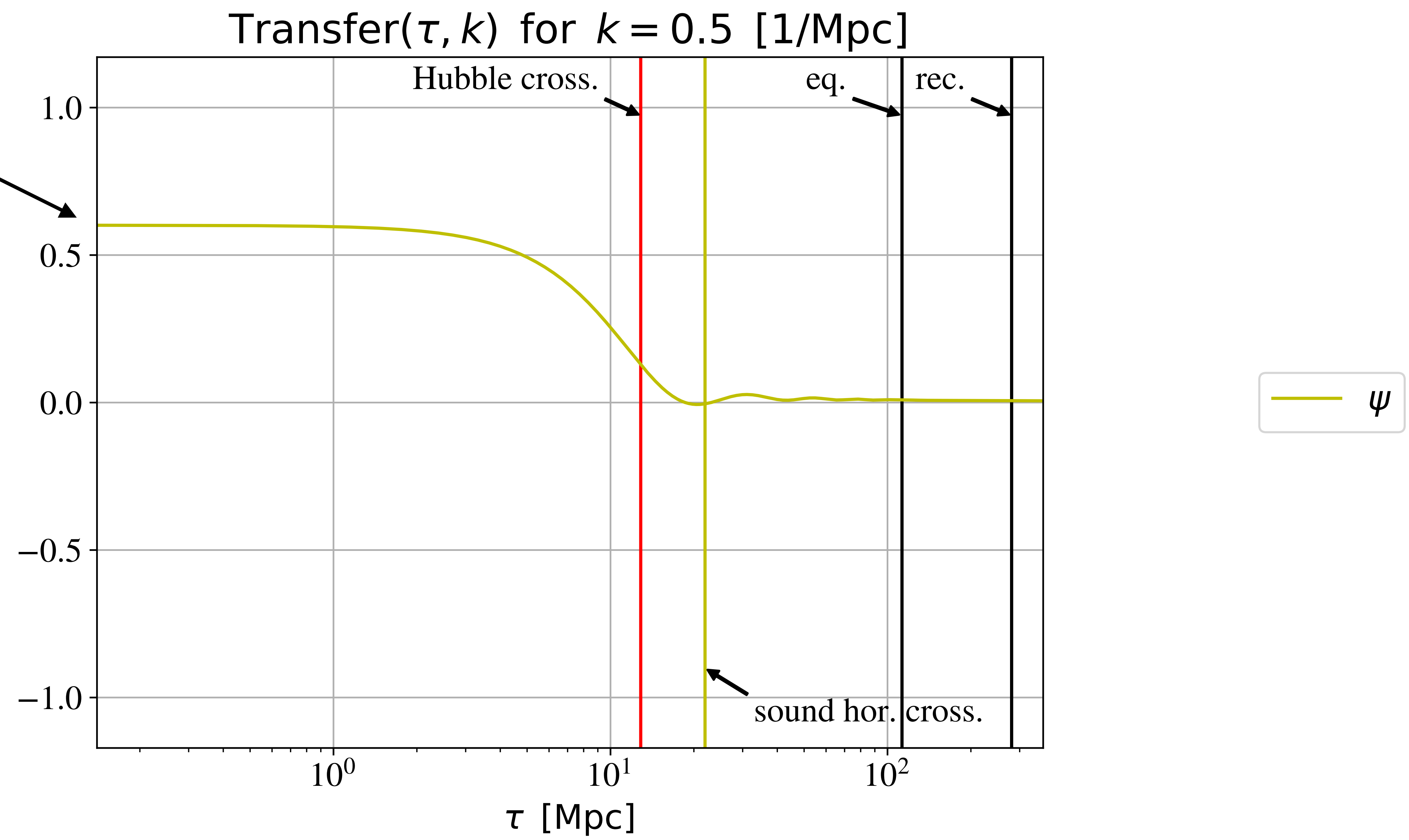


(Newtonian gauge)

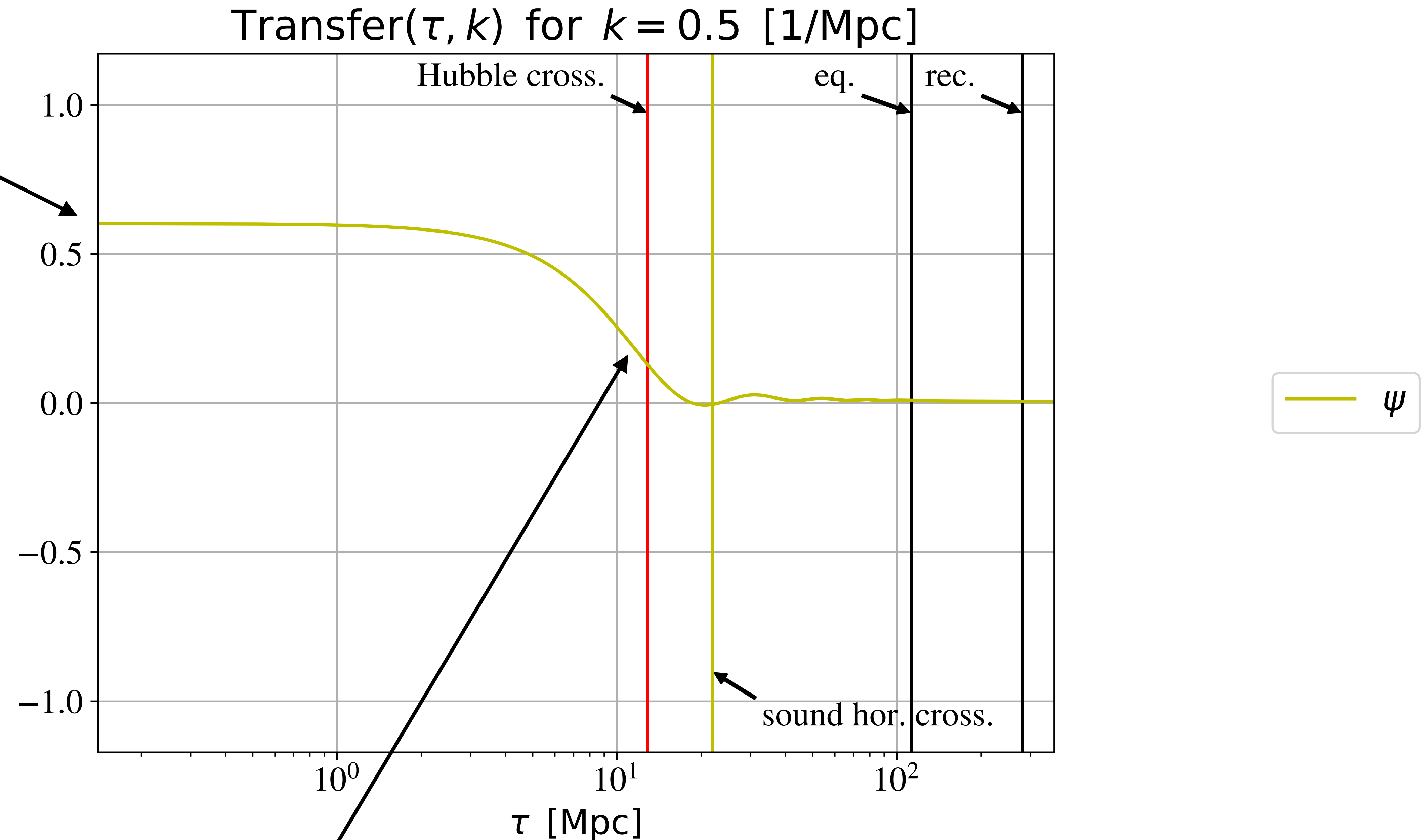
Evolution for one mode

Adiabatic IC
($\mathcal{R}$=1, ψ= 2/3)

positive
grav. pot.
Fourier mode
=>
negative
density
Fourier mode



Evolution for one mode



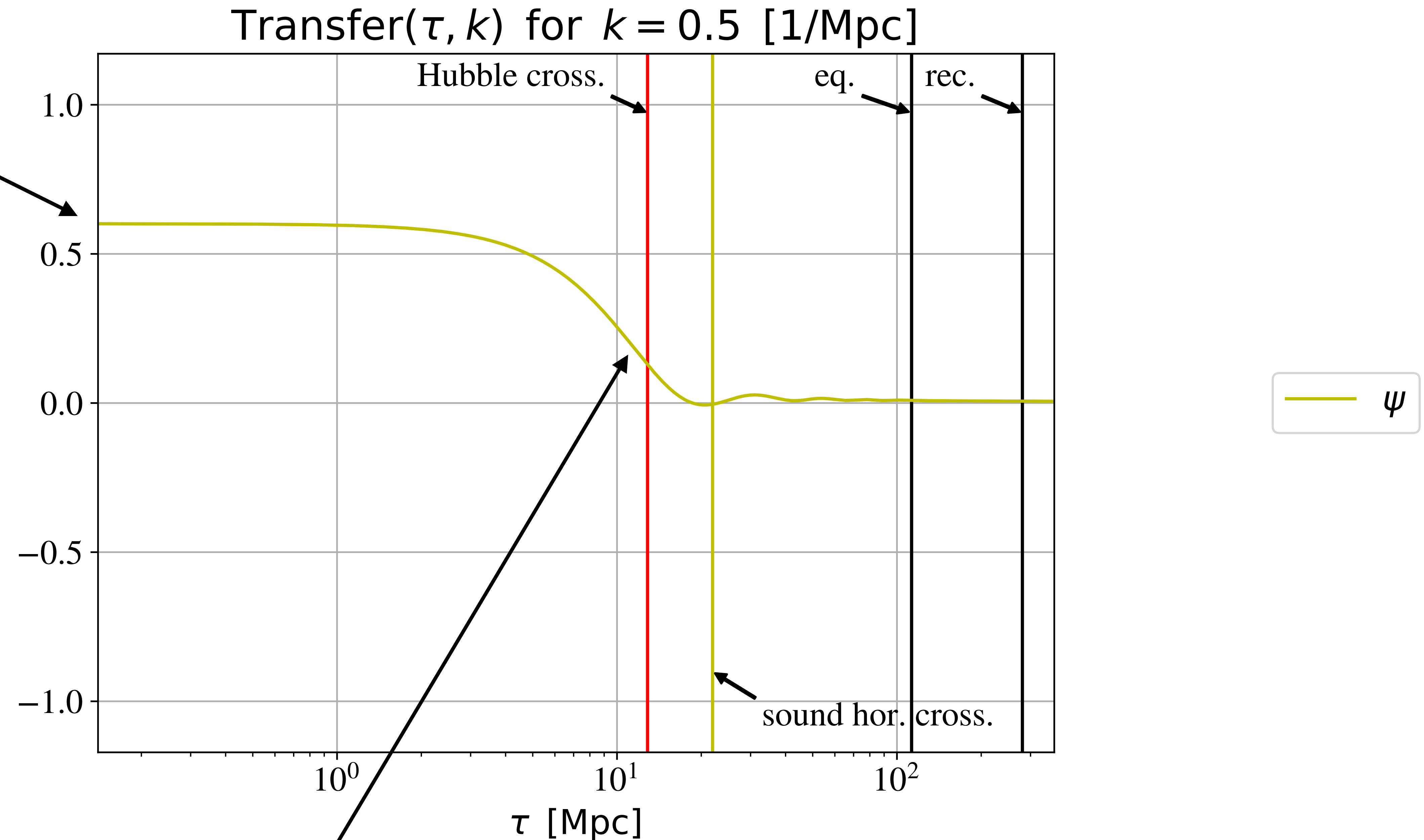
Adiabatic IC
($\mathcal{R}$=1, ψ= 2/3)

positive
grav. pot.
Fourier mode
=>
negative
density
Fourier mode

Metric damped near Hubble crossing during RD

—> photon pressure, Poisson: $-k^2\phi = 4\pi G\, a^2\, \delta\rho_r \propto a^2 \rho_r\, \delta_r \sim a^{2-4+0} \sim a^{-2}$

Evolution for one mode



Adiabatic IC
($\mathcal{R}$=1, ψ= 2/3)

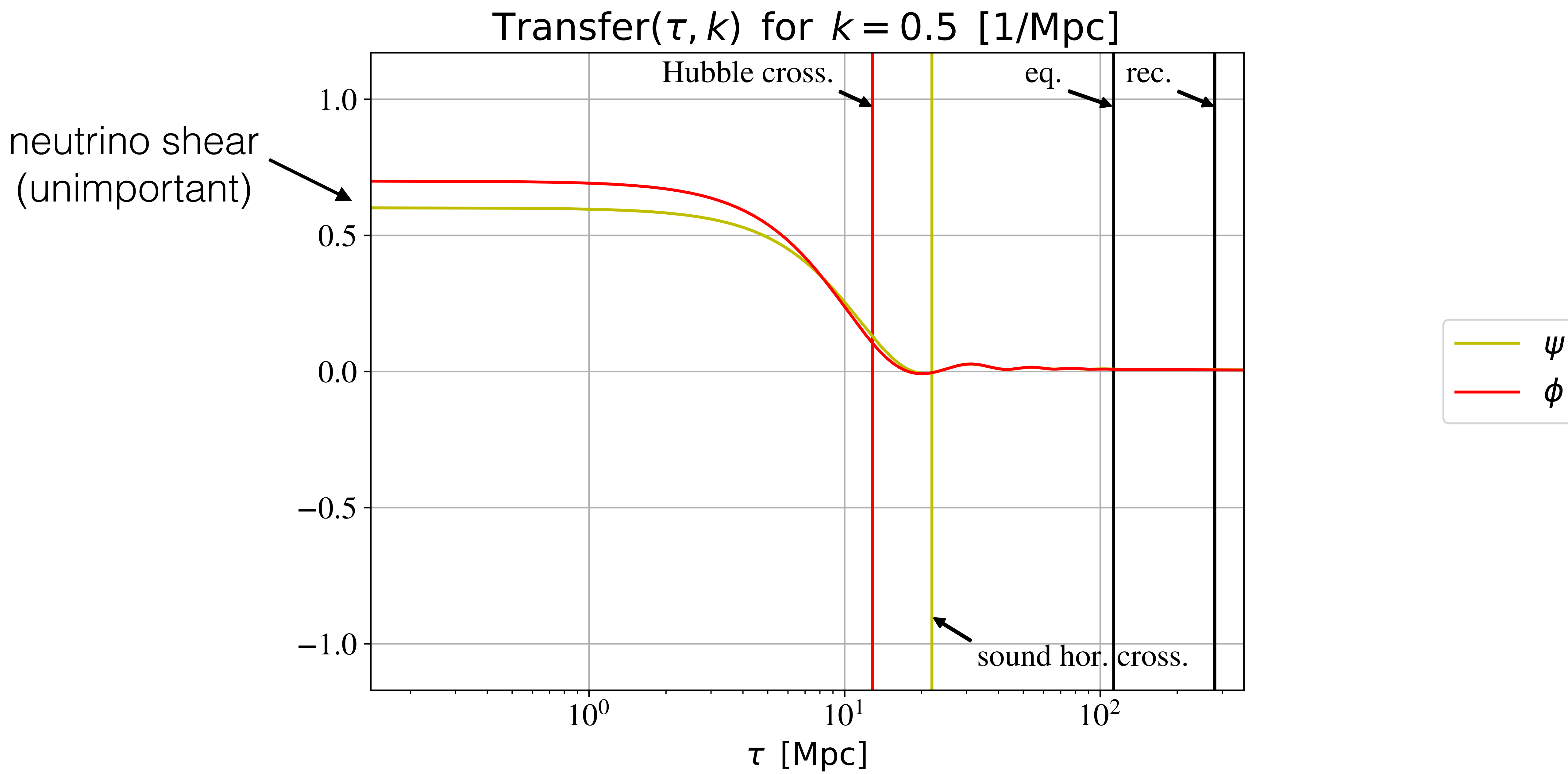
positive
grav. pot.
Fourier mode
=>
negative
density
Fourier mode

Metric damped near Hubble crossing during RD

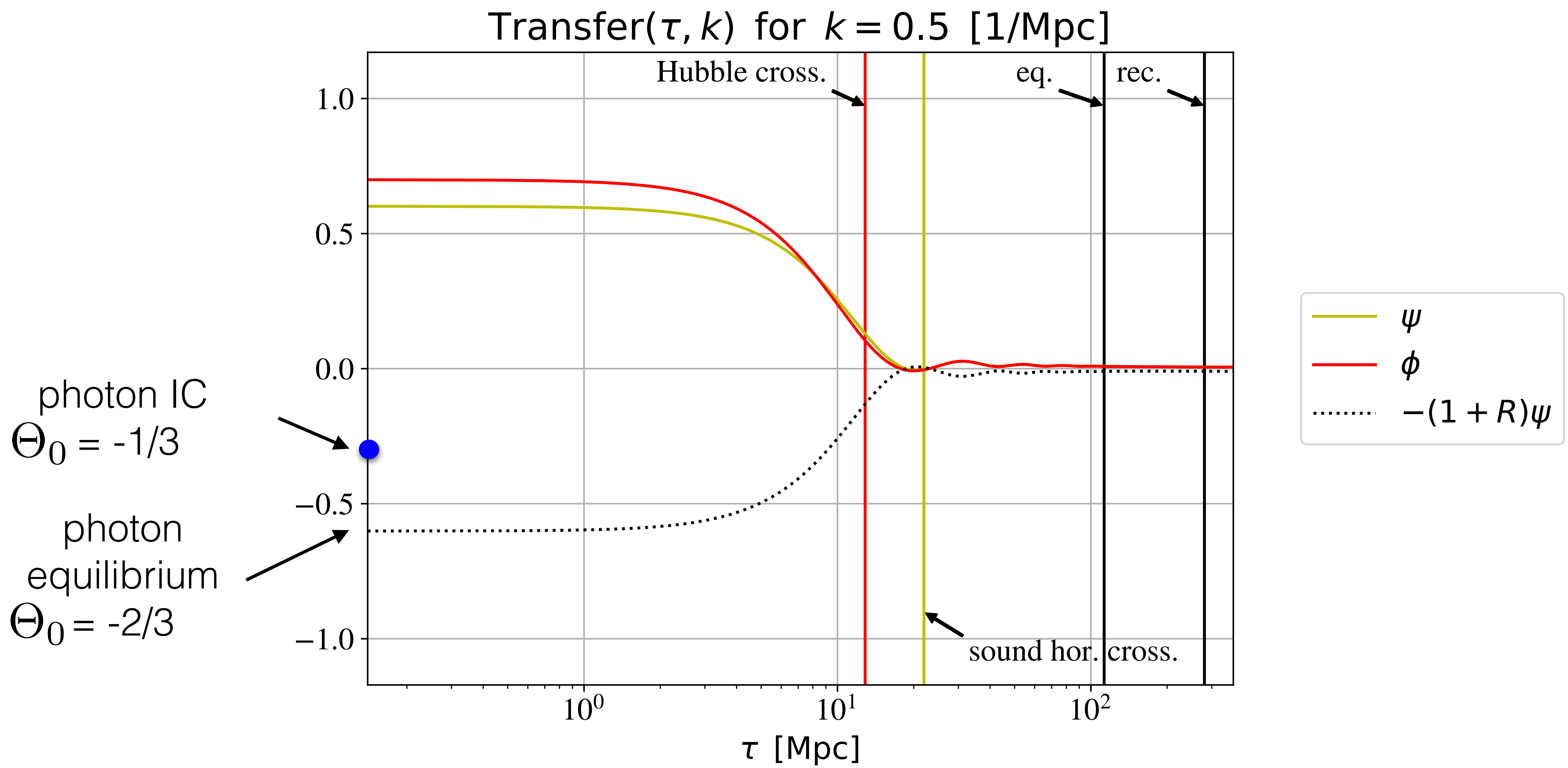
—> photon pressure, Poisson: $-k^2\phi = 4\pi G\, a^2\, \delta\rho_r \propto a^2 \rho_r\, \delta_r \sim a^{2-4+0} \sim a^{-2}$

—> very different from MD: $-k^2\phi = 4\pi G\, a^2\, \delta\rho_m \propto a^2 \rho_m\, \delta_m \sim a^{2-3+1} \sim \text{constant}$

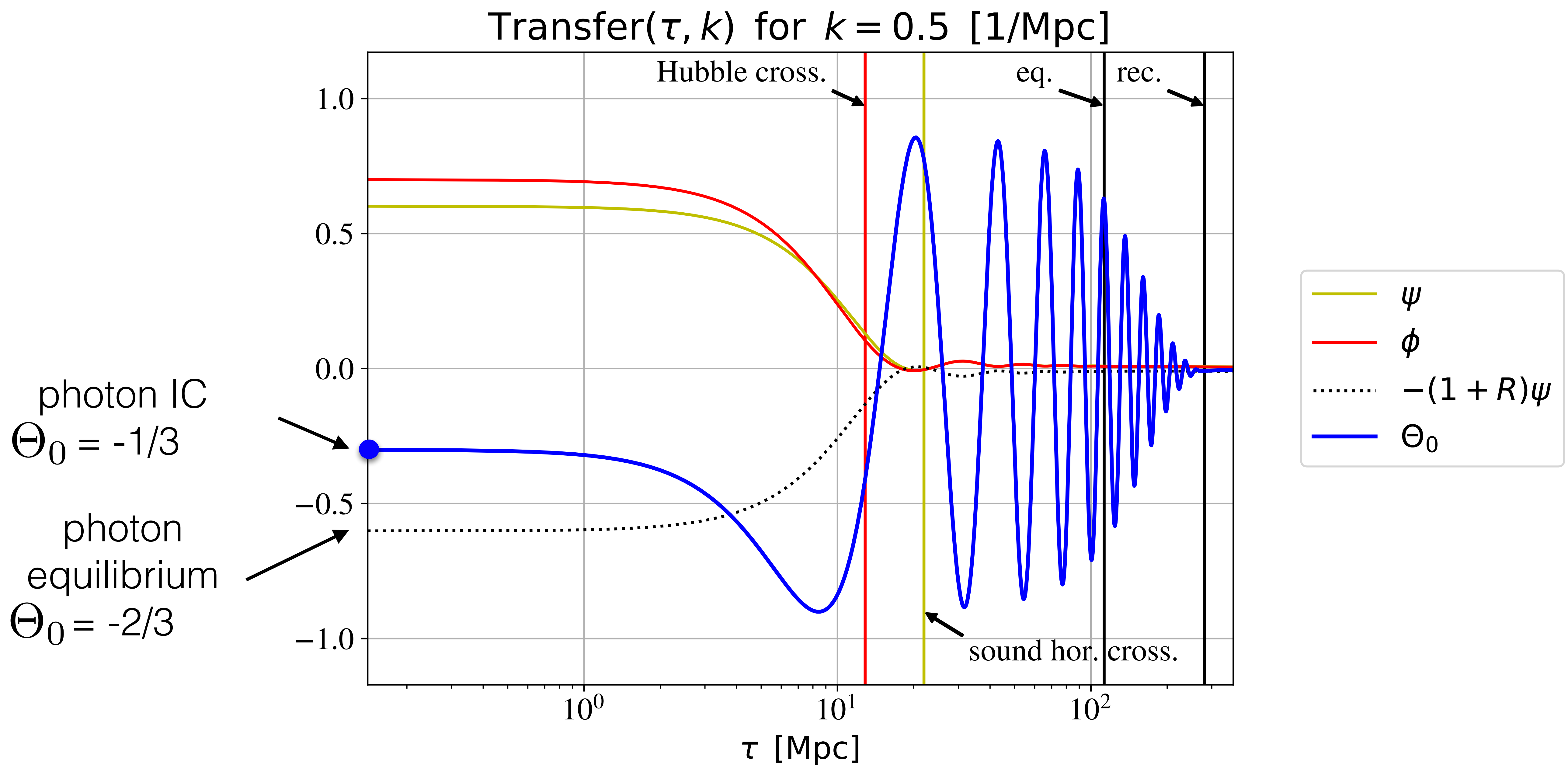
Evolution for one mode



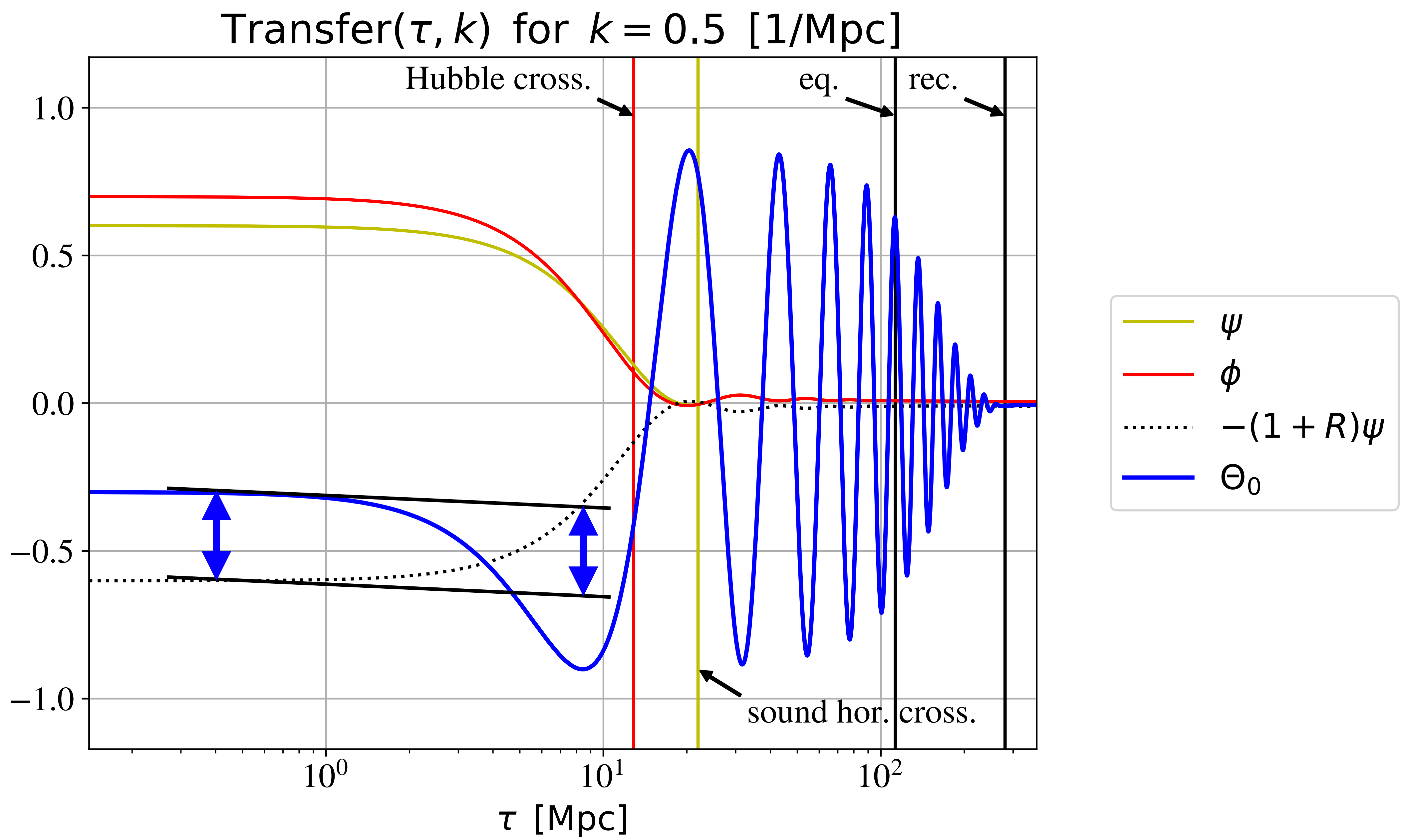
Evolution for one mode



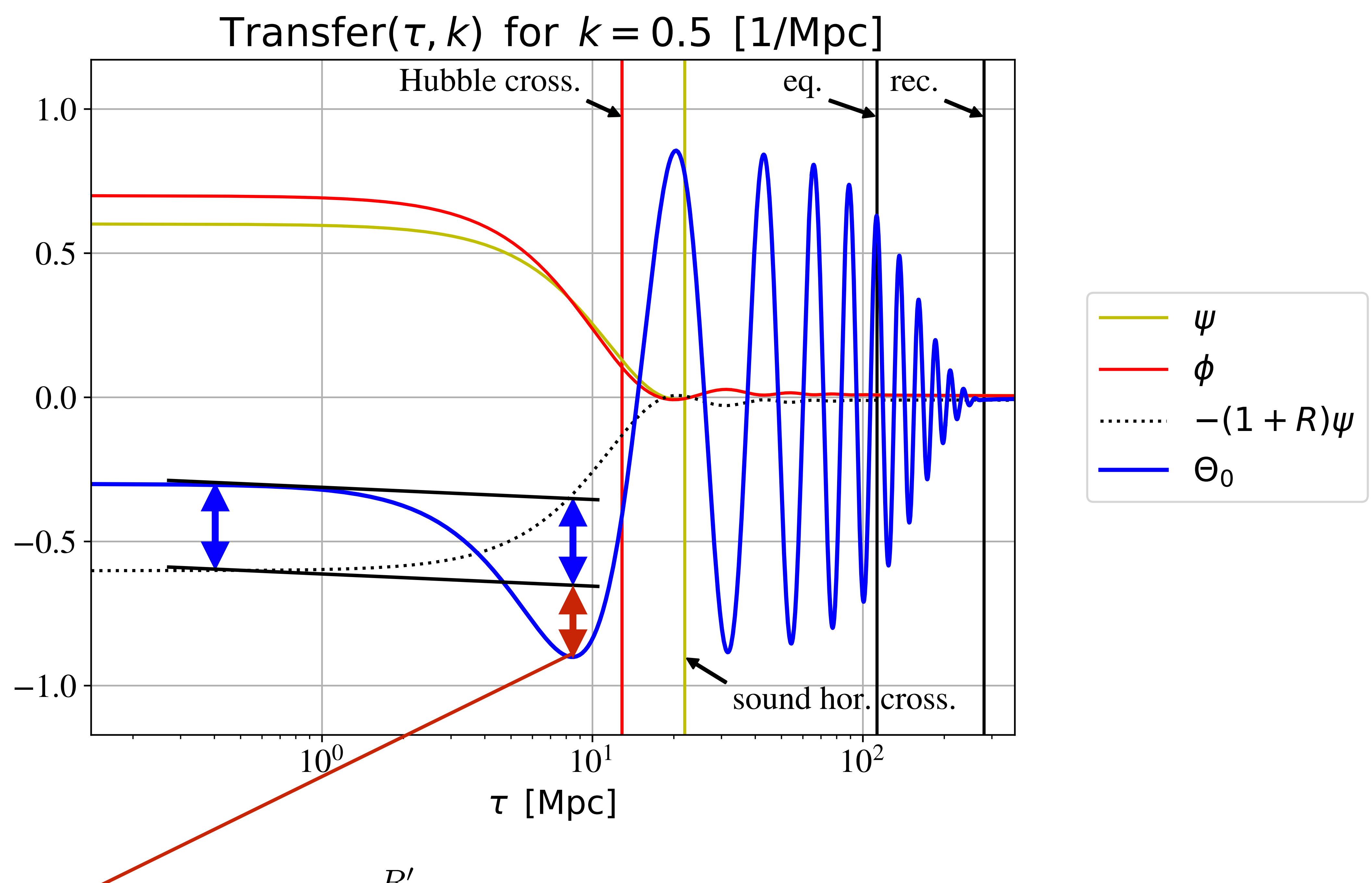
Evolution for one mode



Evolution for one mode



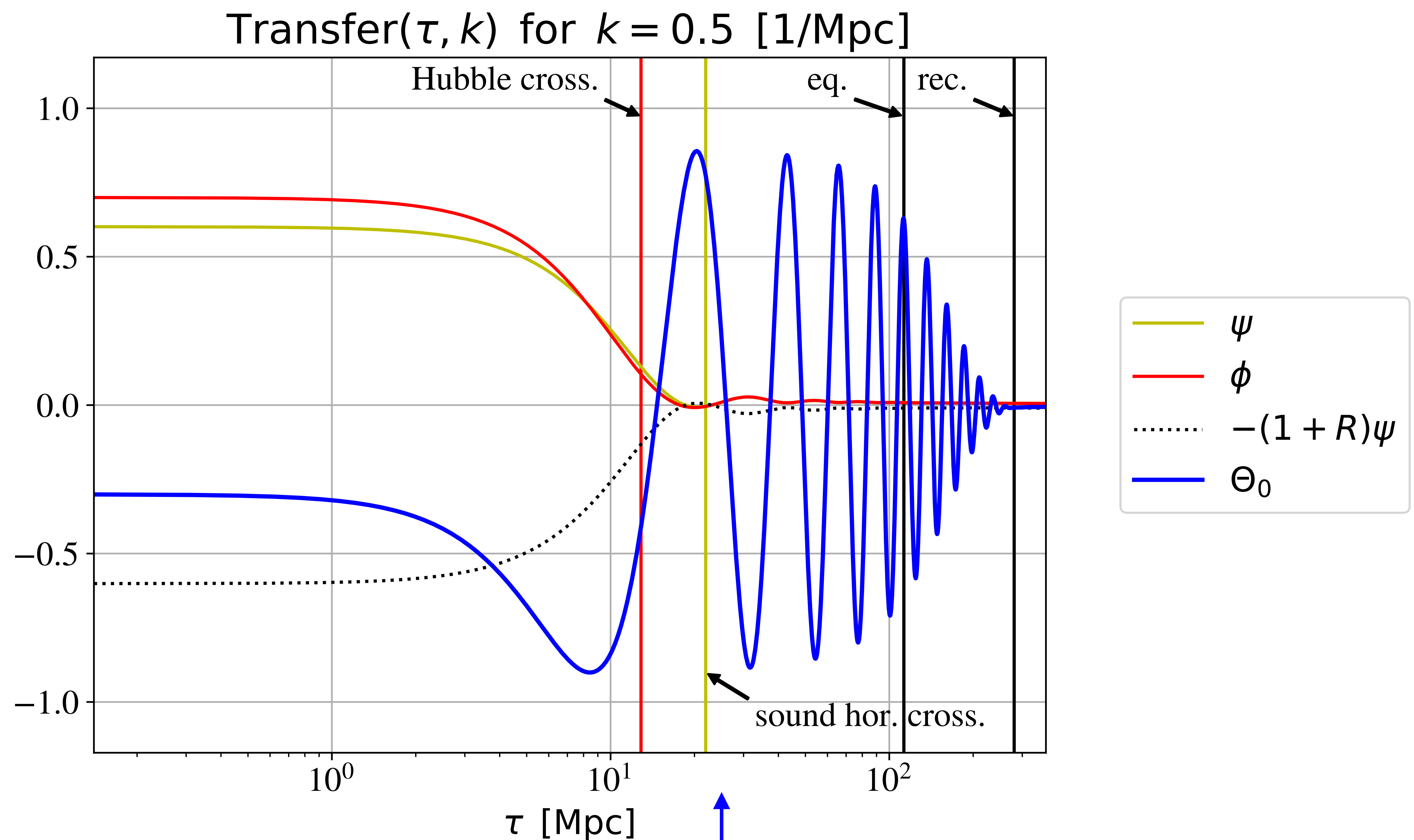
Evolution for one mode



Gravity boost effect from $\frac{R'}{1+R}\phi' + \phi''$

Will be important for effect of neutrinos, DR…

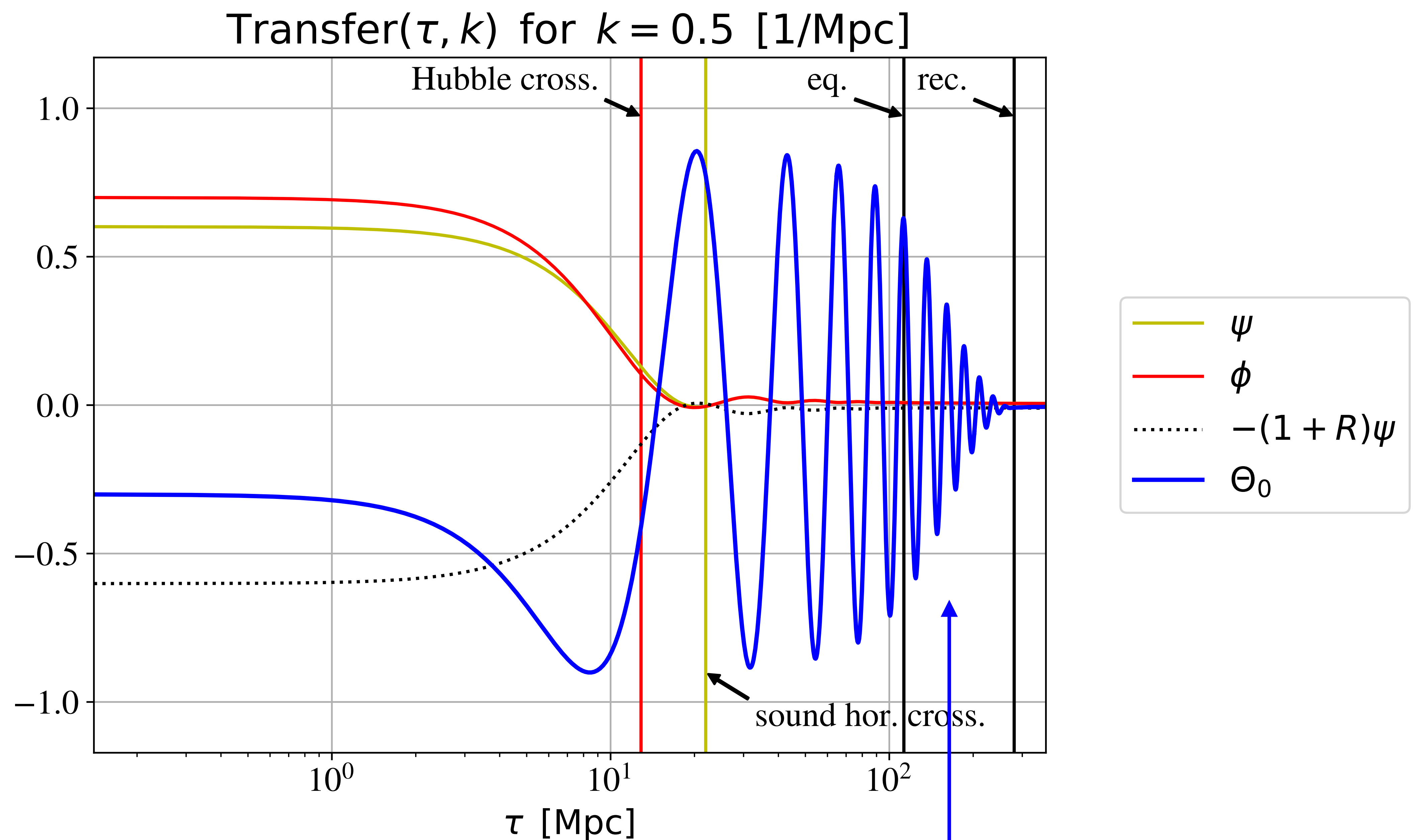
Evolution for one mode



symmetric and stationary oscillation

(deep sub-Hubble, deep DR)

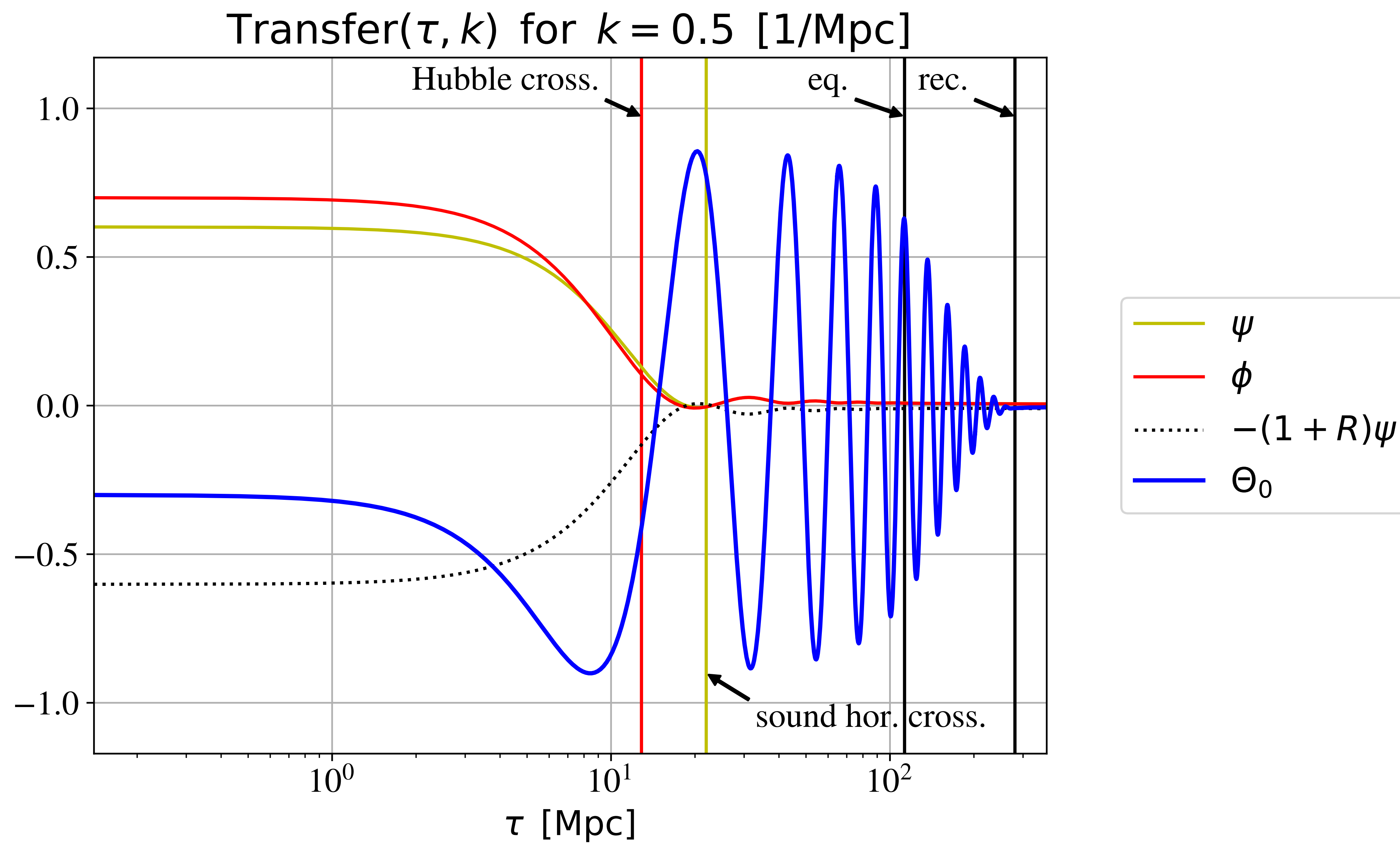
Evolution for one mode



exponentially damped oscillations

(approaching recombination)

Evolution for one mode

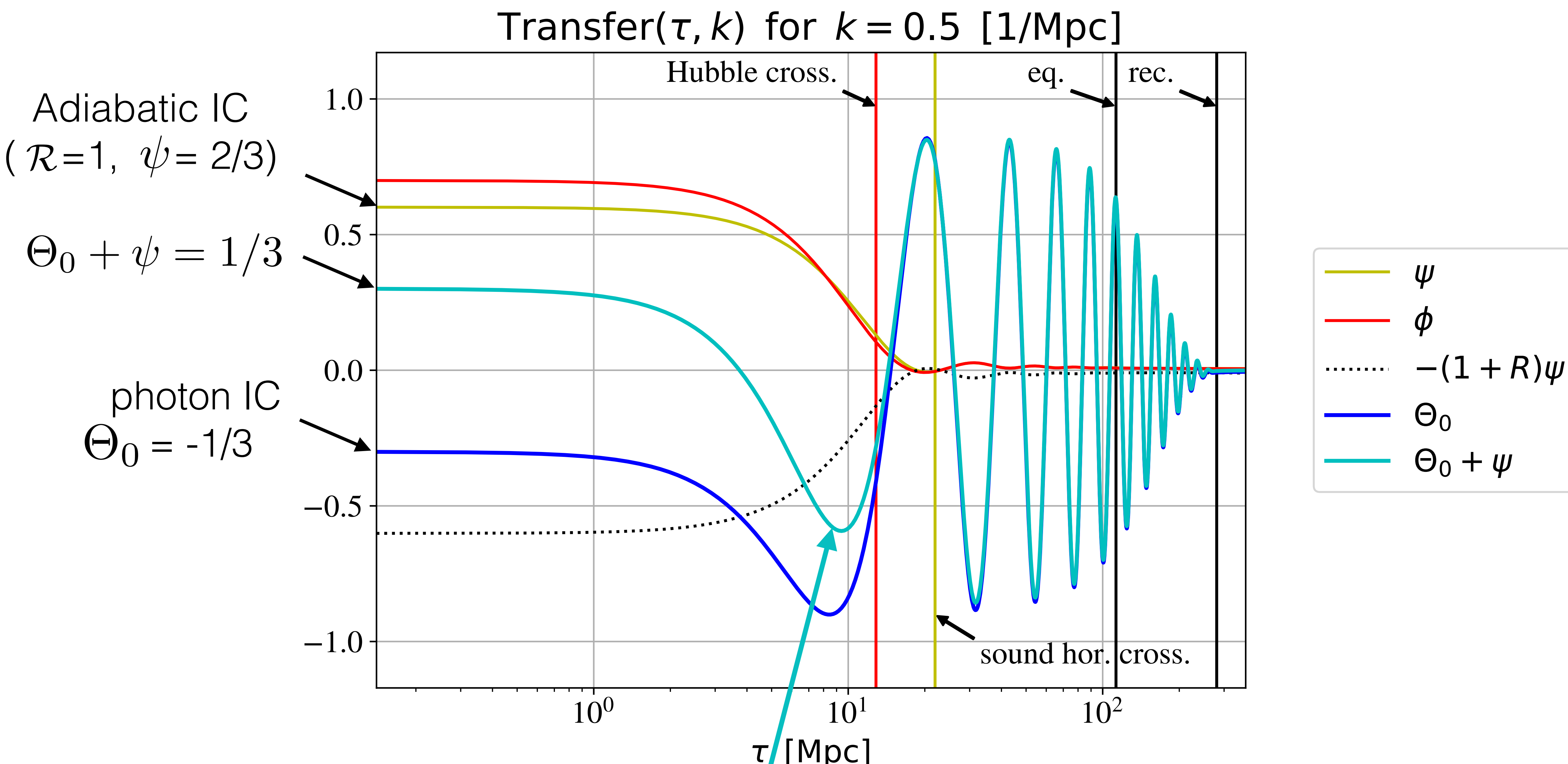


Final goal:

(MZ's line-of-sight integral)

$$\Theta_l(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; \{\underbrace{g\,(\Theta_0 + \psi)}_{\rm TSW} + \underbrace{\left(g\,k^{-2}\theta_{\rm b}\right)'}_{\rm Doppler} + \underbrace{e^{-\kappa}(\phi' + \psi')}_{\rm ISW}\}\; j_l(k(\tau_0 - \tau))$$

Evolution for one mode

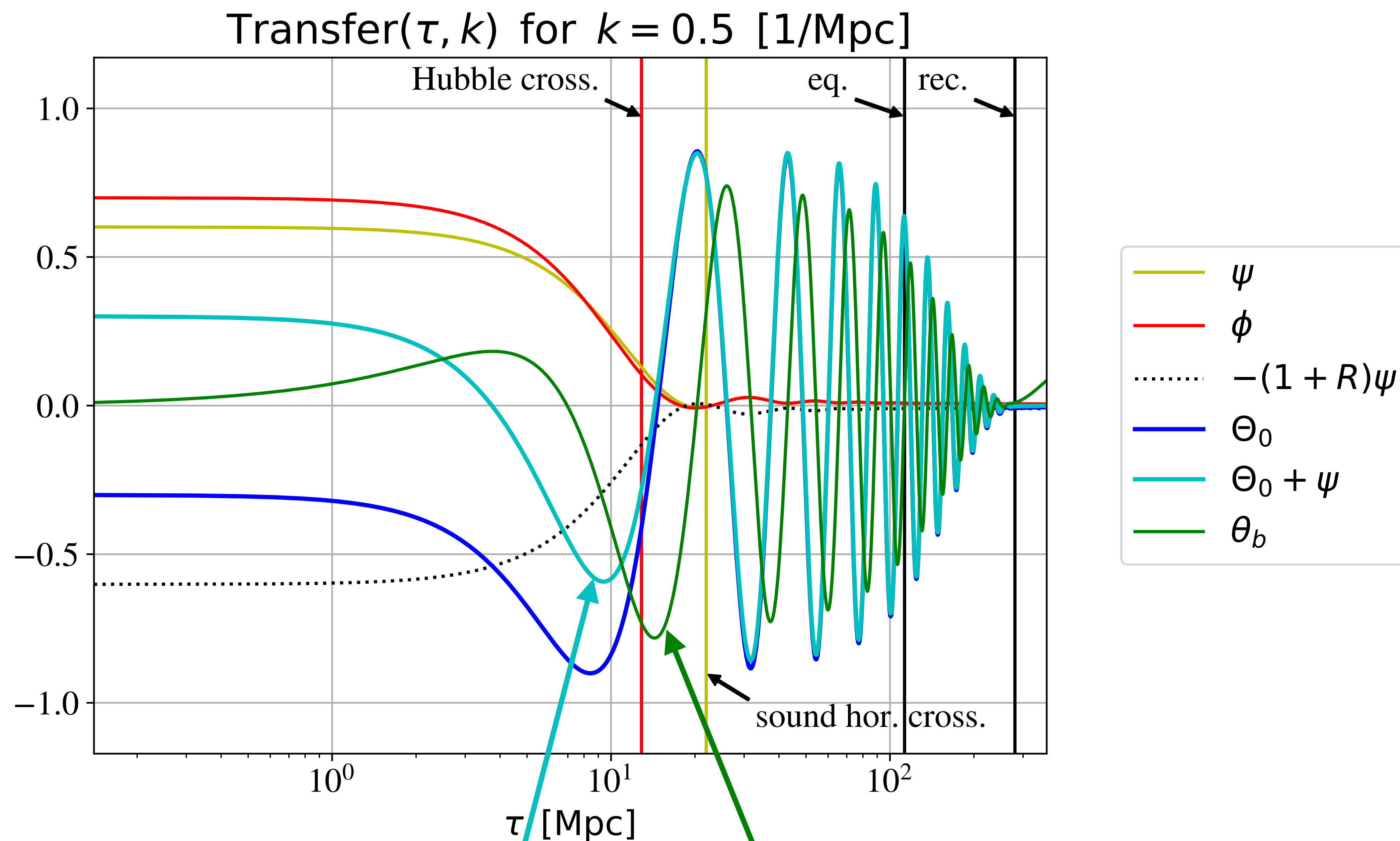


Final goal:

(MZ's line-of-sight integral)

$$\Theta_l(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; \{\underbrace{g\,(\Theta_0 + \psi)}_{\rm TSW} + \underbrace{\left(g\,k^{-2}\theta_{\rm b}\right)'}_{\rm Doppler} + \underbrace{e^{-\kappa}(\phi' + \psi')}_{\rm ISW}\}\; j_l(k(\tau_0 - \tau))$$

Evolution for one mode



Final goal:

(MZ's line-of-sight integral)

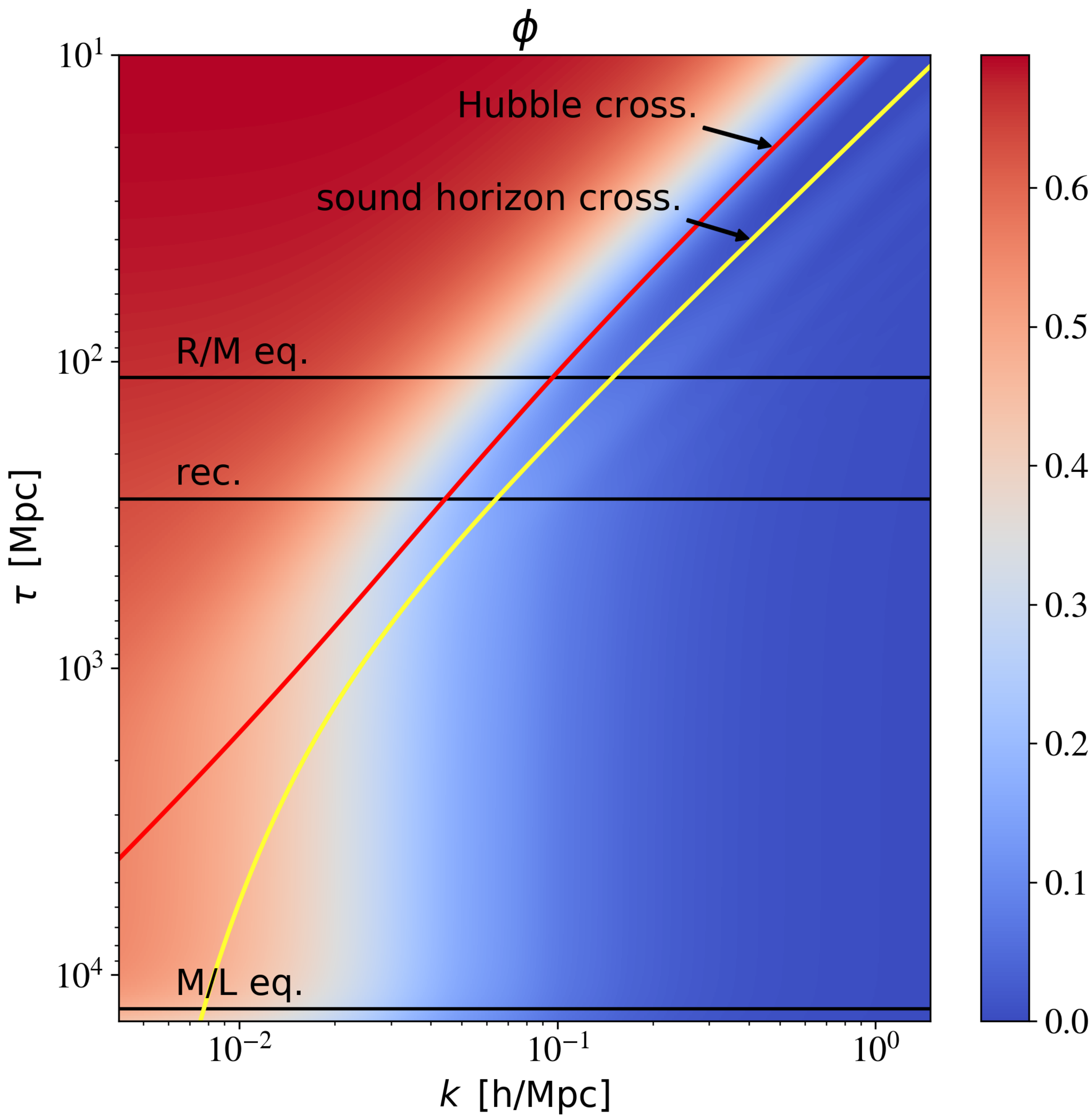
$$\Theta_l(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \,\{\underbrace{g\,(\Theta_0 + \psi)}_{\rm TSW} + \underbrace{\left(g\,k^{-2}\theta_{\rm b}\right)'}_{\rm Doppler} + \underbrace{e^{-\kappa}(\phi' + \psi')}_{\rm ISW}\}\; j_l(k(\tau_0 - \tau))$$

Evolution for one mode

Rest for the eyes

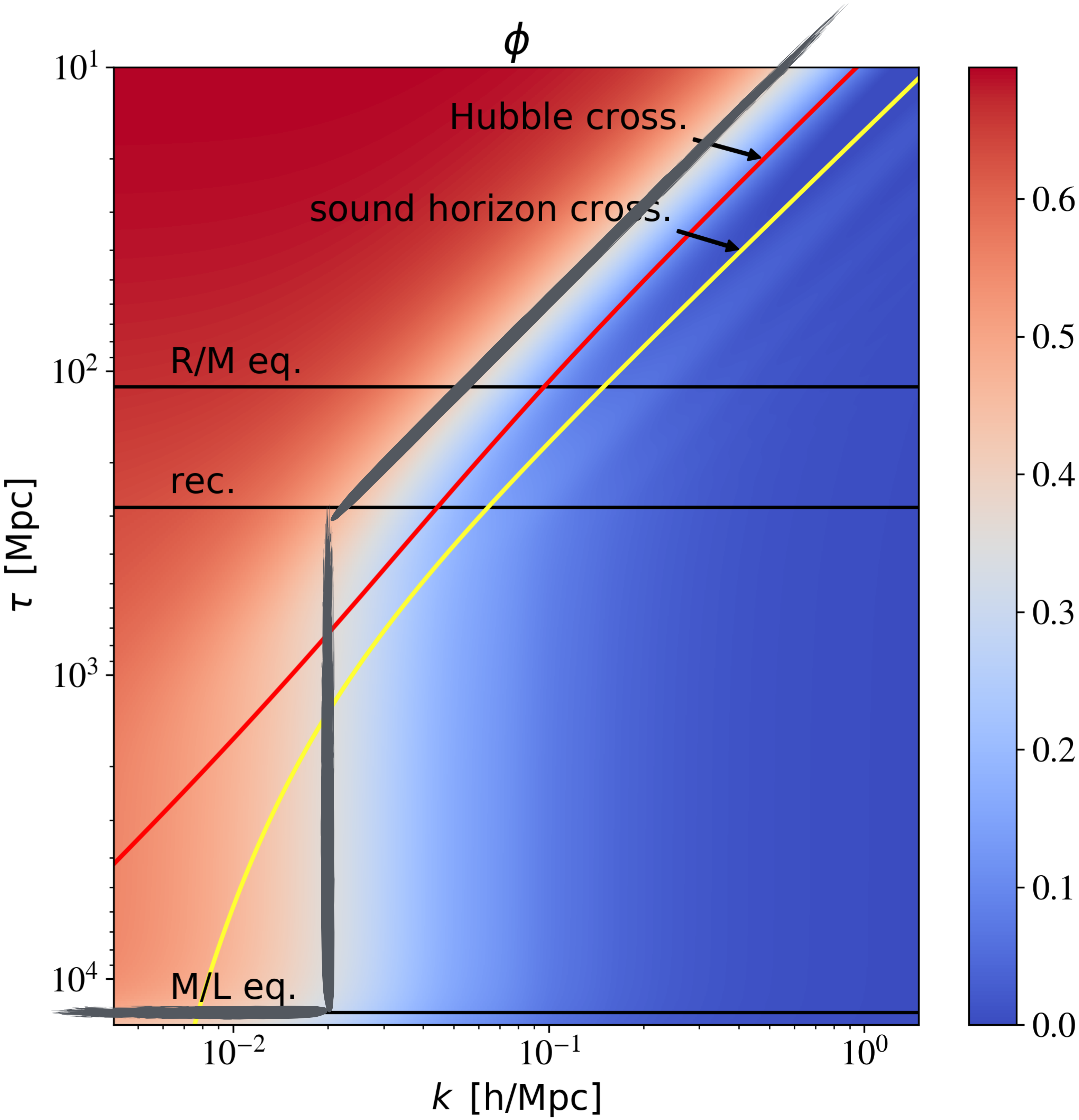
Evolution for all wavenumbers

Metric $\phi(\tau, k)$:



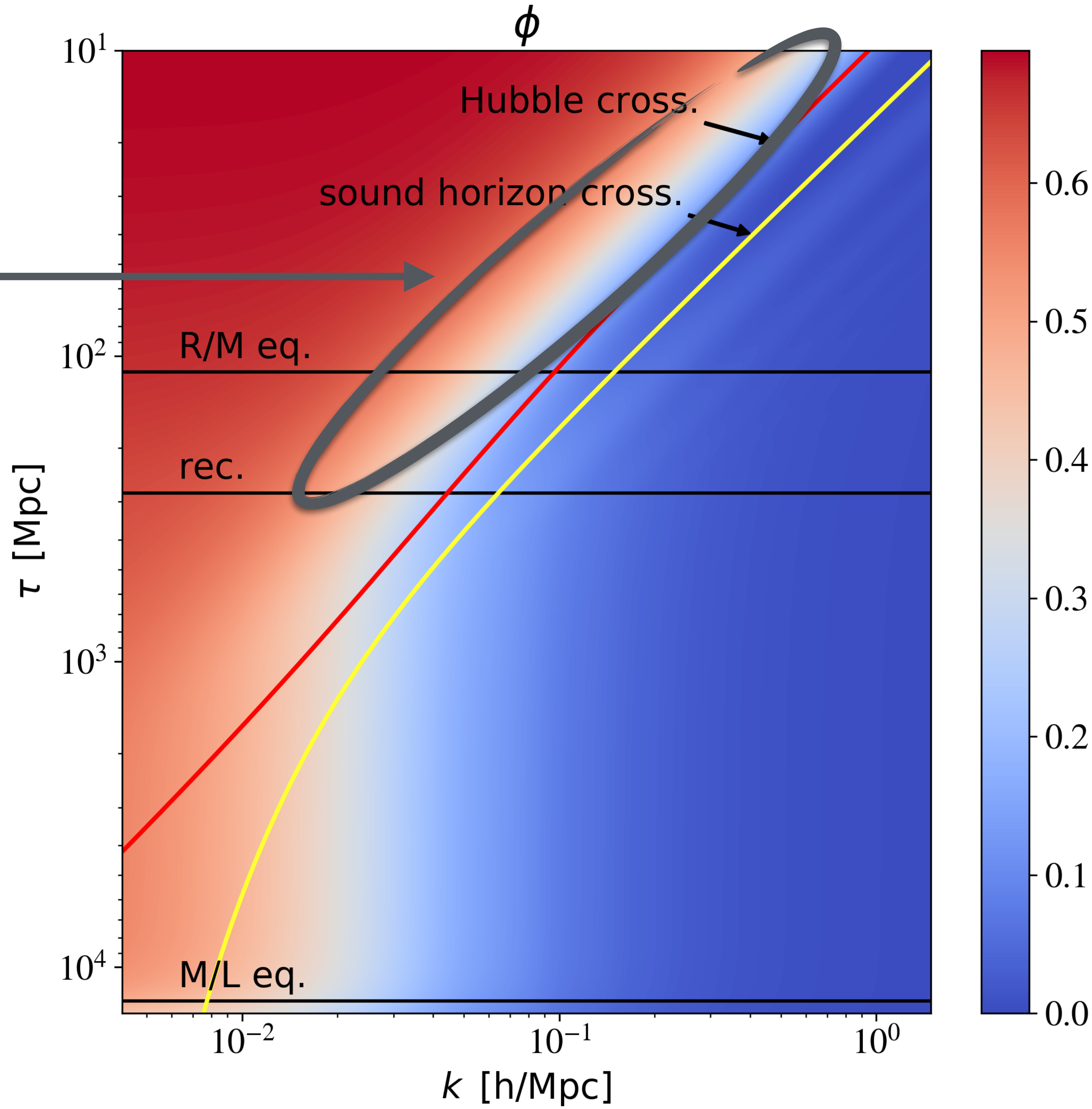
Evolution for all wavenumbers

Metric $\phi(\tau, k)$:



Evolution for all wavenumbers

$$\frac{R'}{1+R}\phi' + \phi''$$ gravity boost



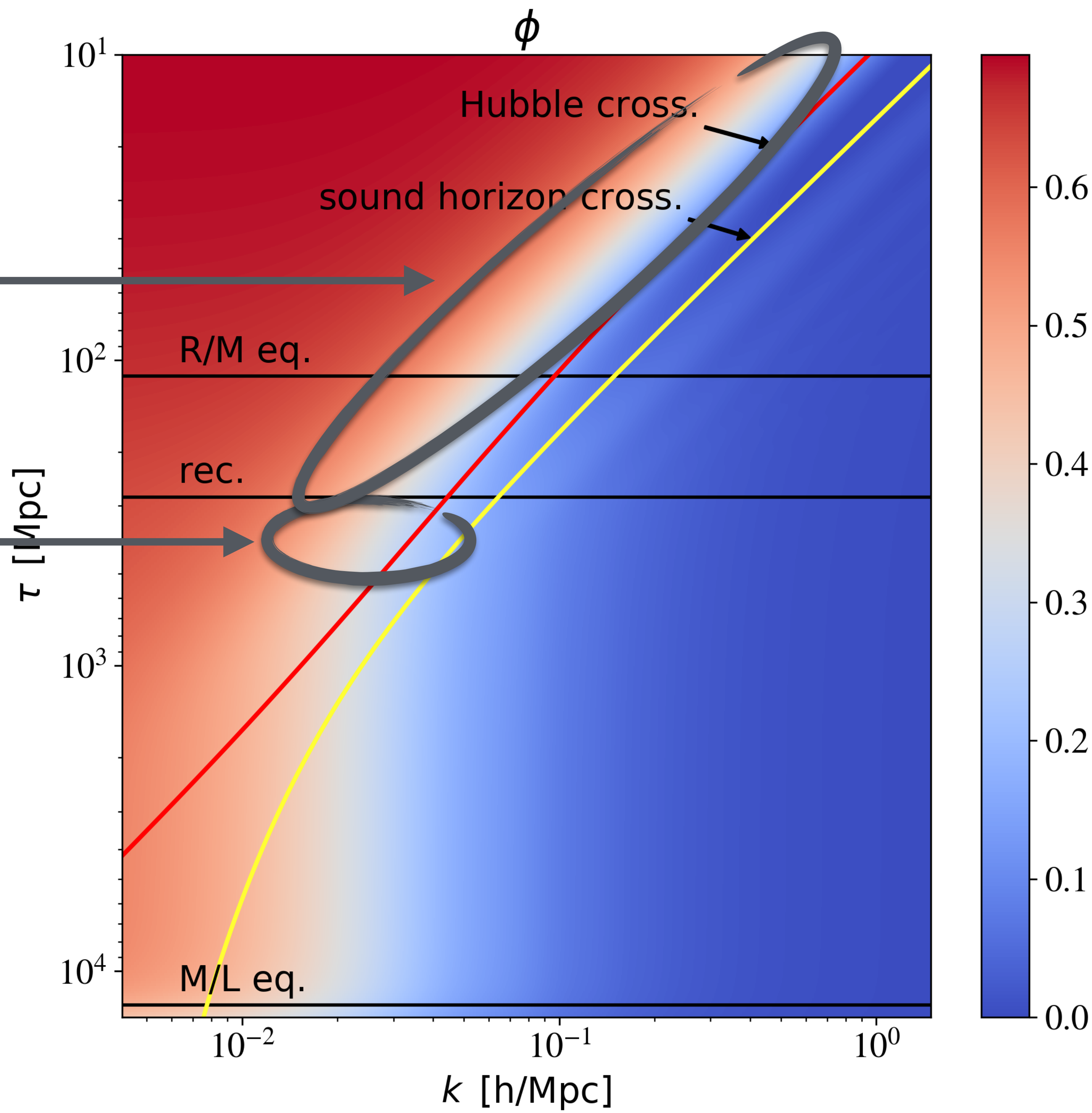
Evolution for all wavenumbers

$$\frac{R'}{1+R}\phi' + \phi''$$

gravity boost

$$\underbrace{e^{-\kappa}(\phi' + \psi')\}}_{\text{ISW}}$$

early ISW



Evolution for all wavenumbers

$$\frac{R'}{1+R}\phi' + \phi''$$

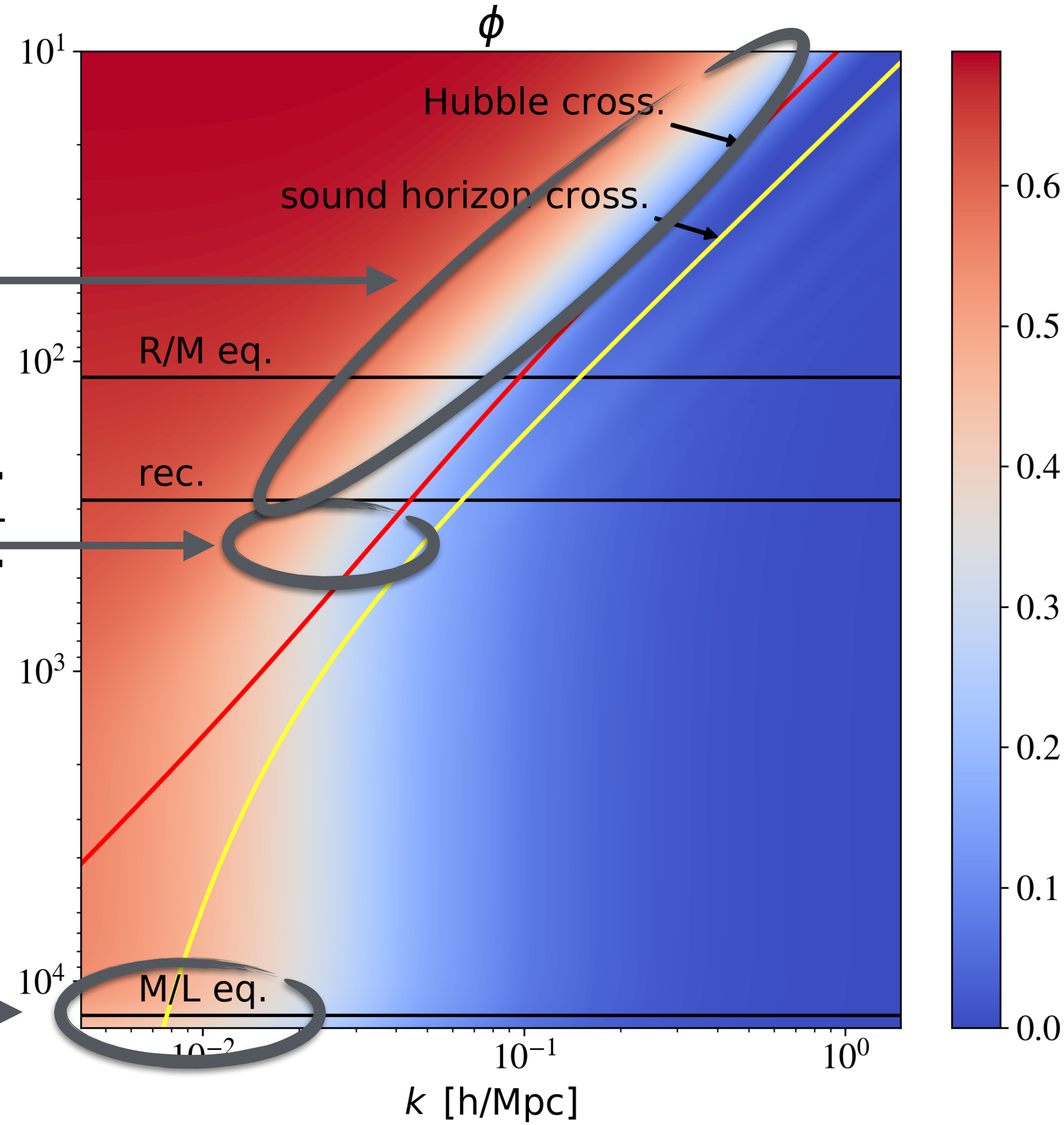
gravity boost

$$\underbrace{e^{-\kappa}(\phi' + \psi')\}}_{\text{ISW}}$$

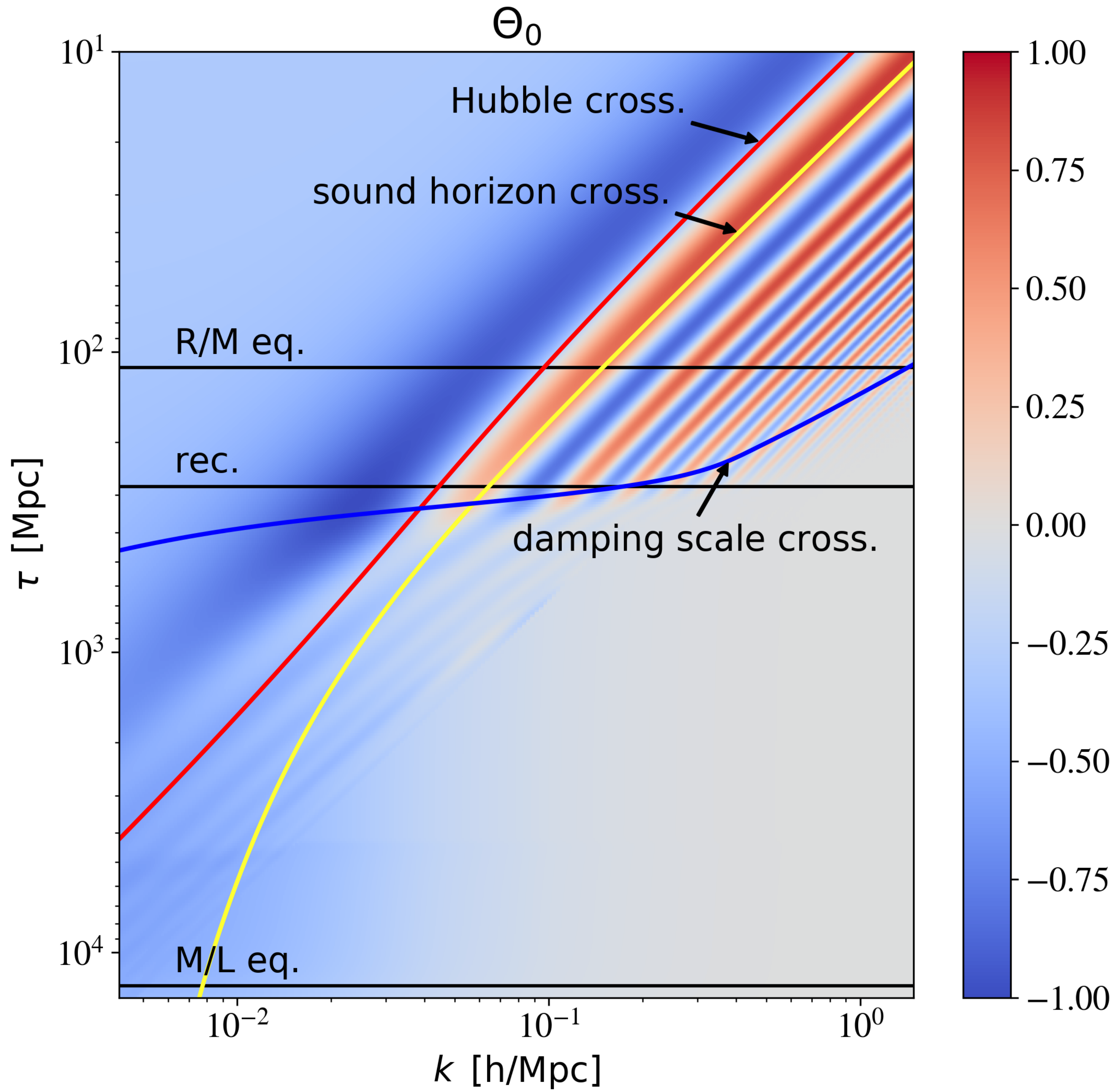
early ISW

$$\underbrace{e^{-\kappa}(\phi' + \psi')\}}_{\text{ISW}}$$

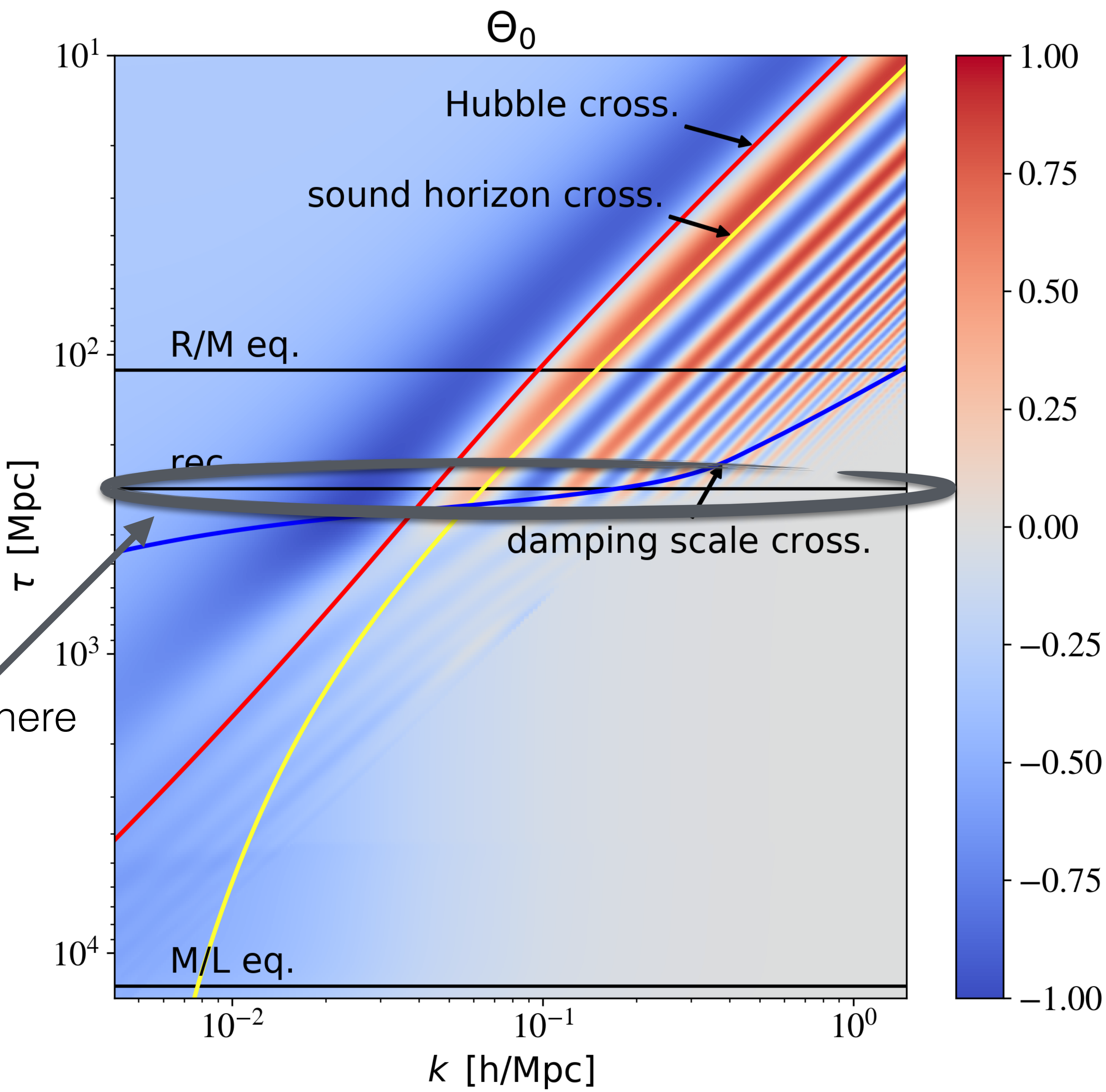
late ISW



Evolution for all wavenumbers



Evolution for all wavenumbers

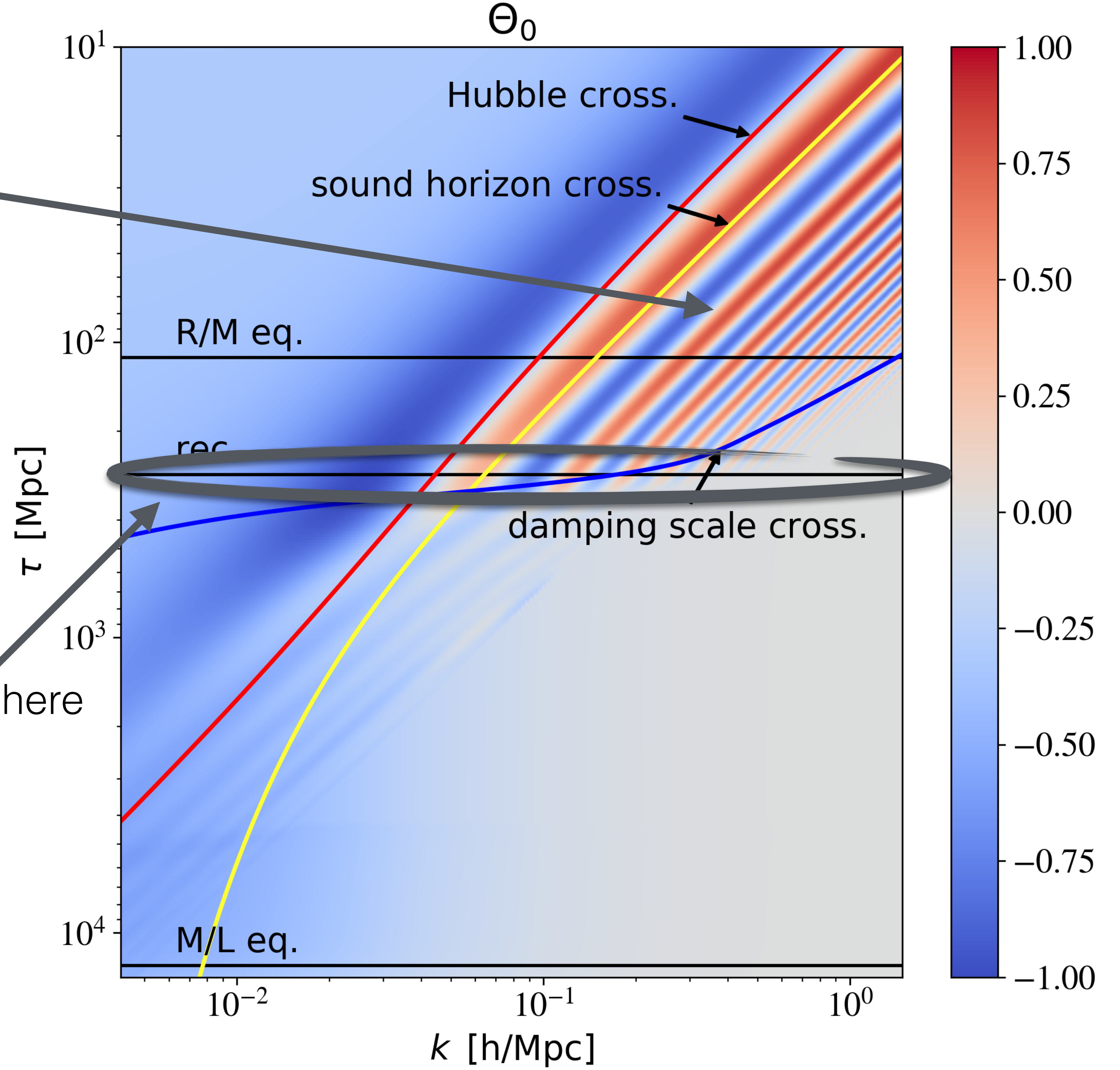


CMB patterns mainly come from here

Evolution for all wavenumbers

45° symmetry comes from

$$\cos(kc_s\tau)$$

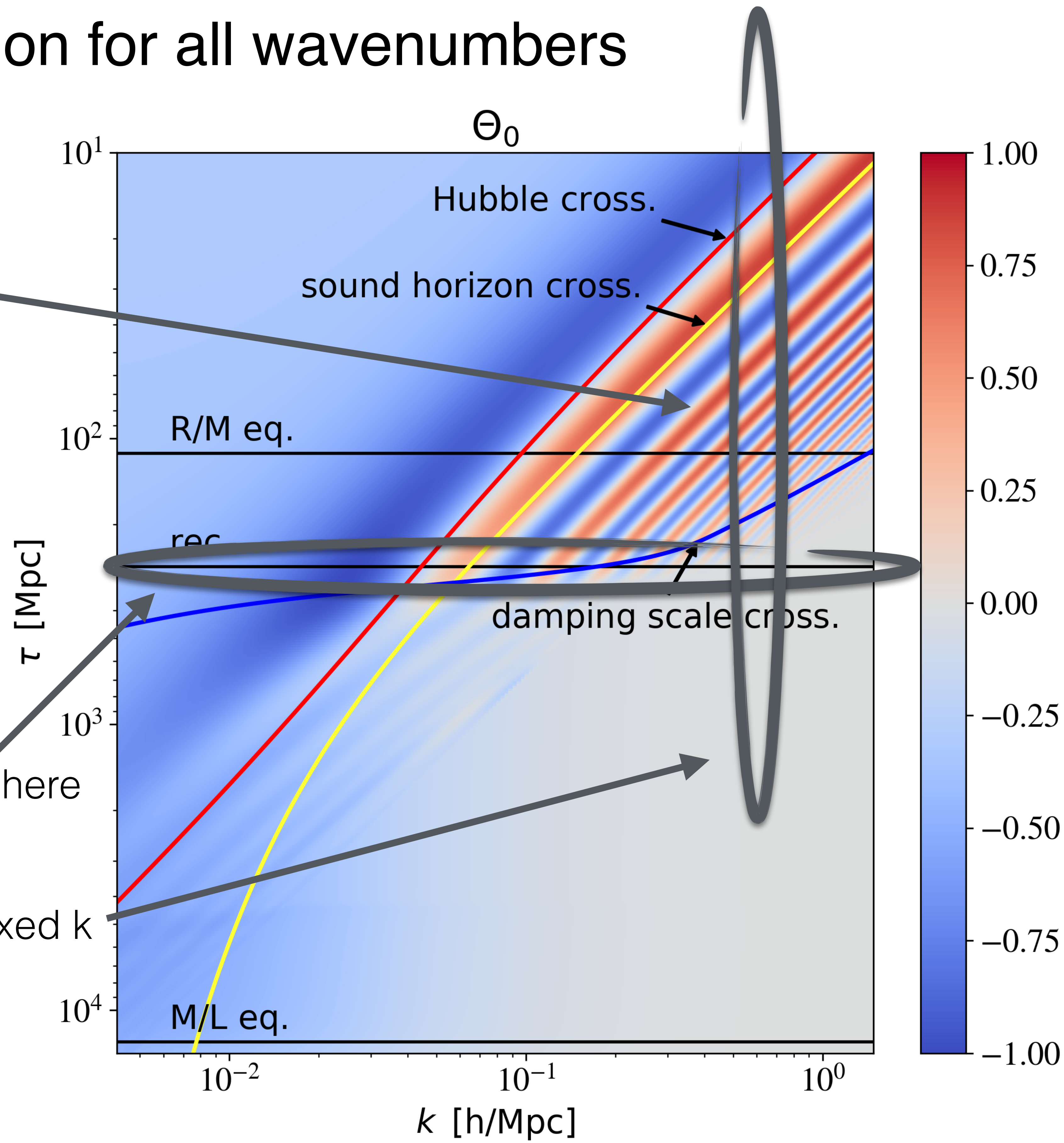


CMB patterns mainly come from here

Evolution for all wavenumbers

45o symmetry comes from

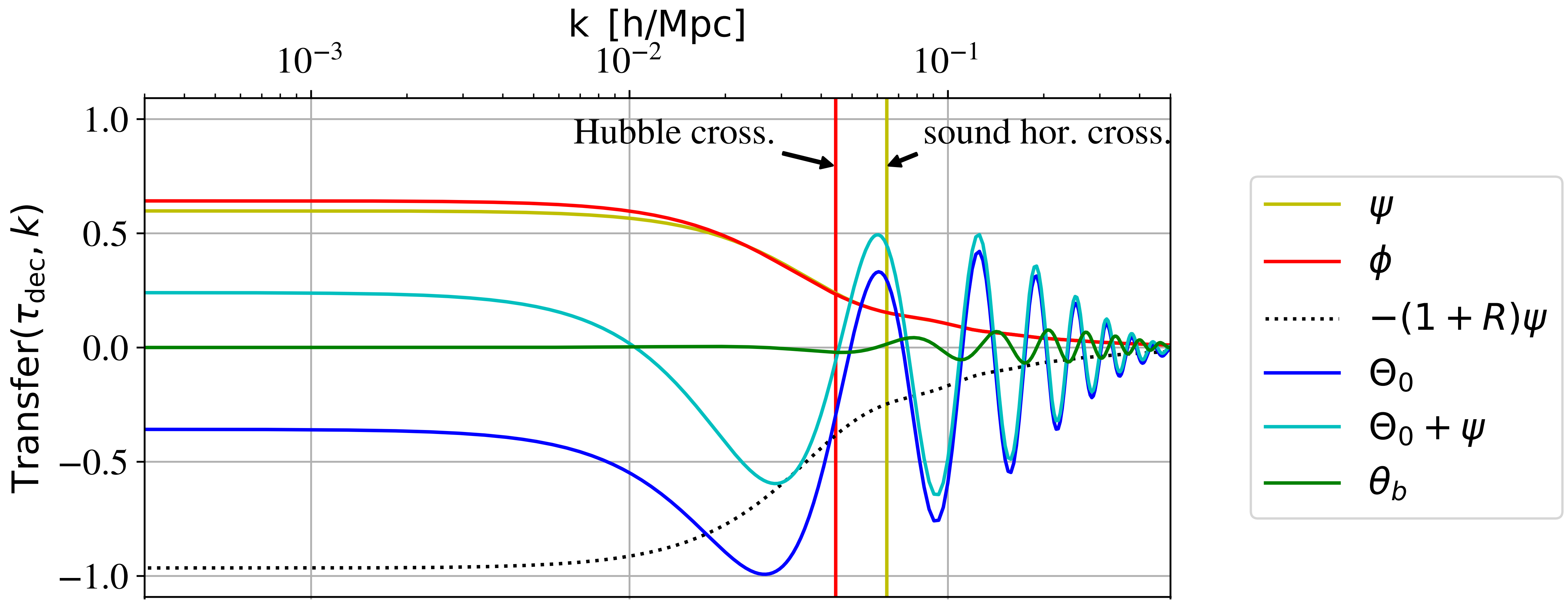
$$\cos(kc_s\tau)$$



CMB patterns mainly come from here

fixed time will look like previous fixed k

Transfer functions at recombination/decoupling



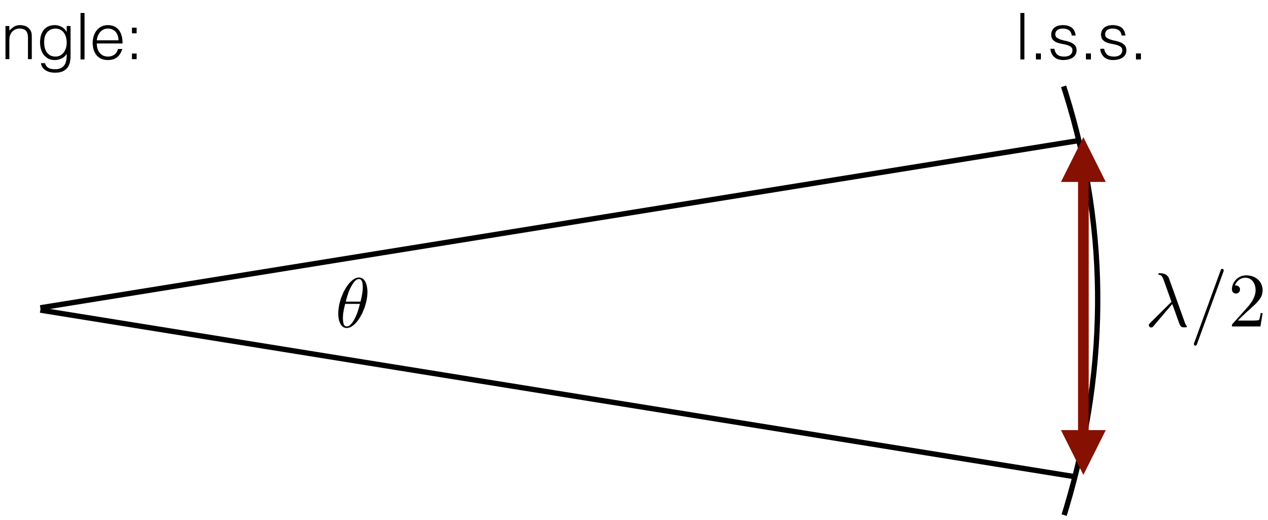
Rest for the brain

Projection effects

Instantaneous decoupling, small angle:

$$\theta = \frac{\pi}{l} = \frac{\lambda/2}{d_a(z_{\rm dec})} = \frac{\pi/k}{\tau_0 - \tau_{\rm rec}}$$

$$\Longrightarrow l = k(\tau_0 - \tau_{\rm rec})$$



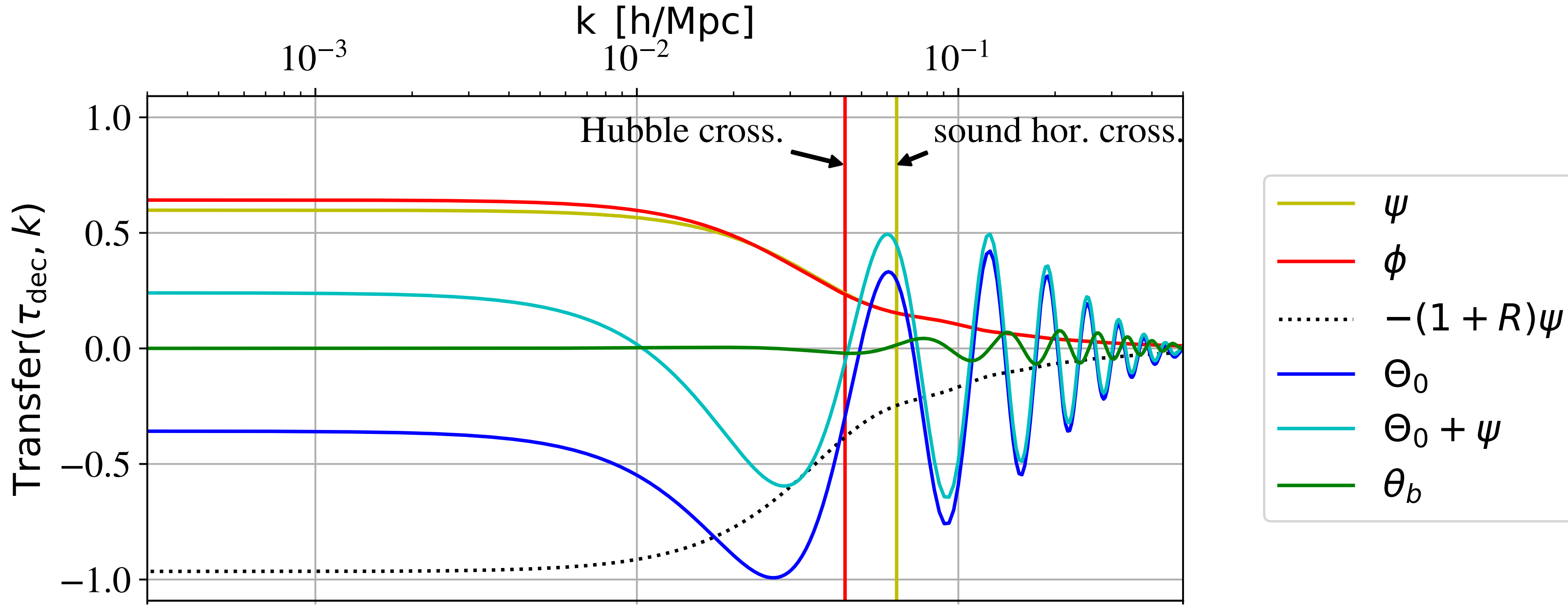
Recover it from the CMB equations with $g(\tau) \simeq \delta(\tau - \tau_{\rm rec})$, $j_l(x) \sim \delta(l - x)$:

$$\Theta_l(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; \{g\,(\Theta_0 + \psi) + \left(g\,k^{-2}\theta_{\rm b}\right)' + {\rm ISW}\} \; j_l(k(\tau_0 - \tau))$$

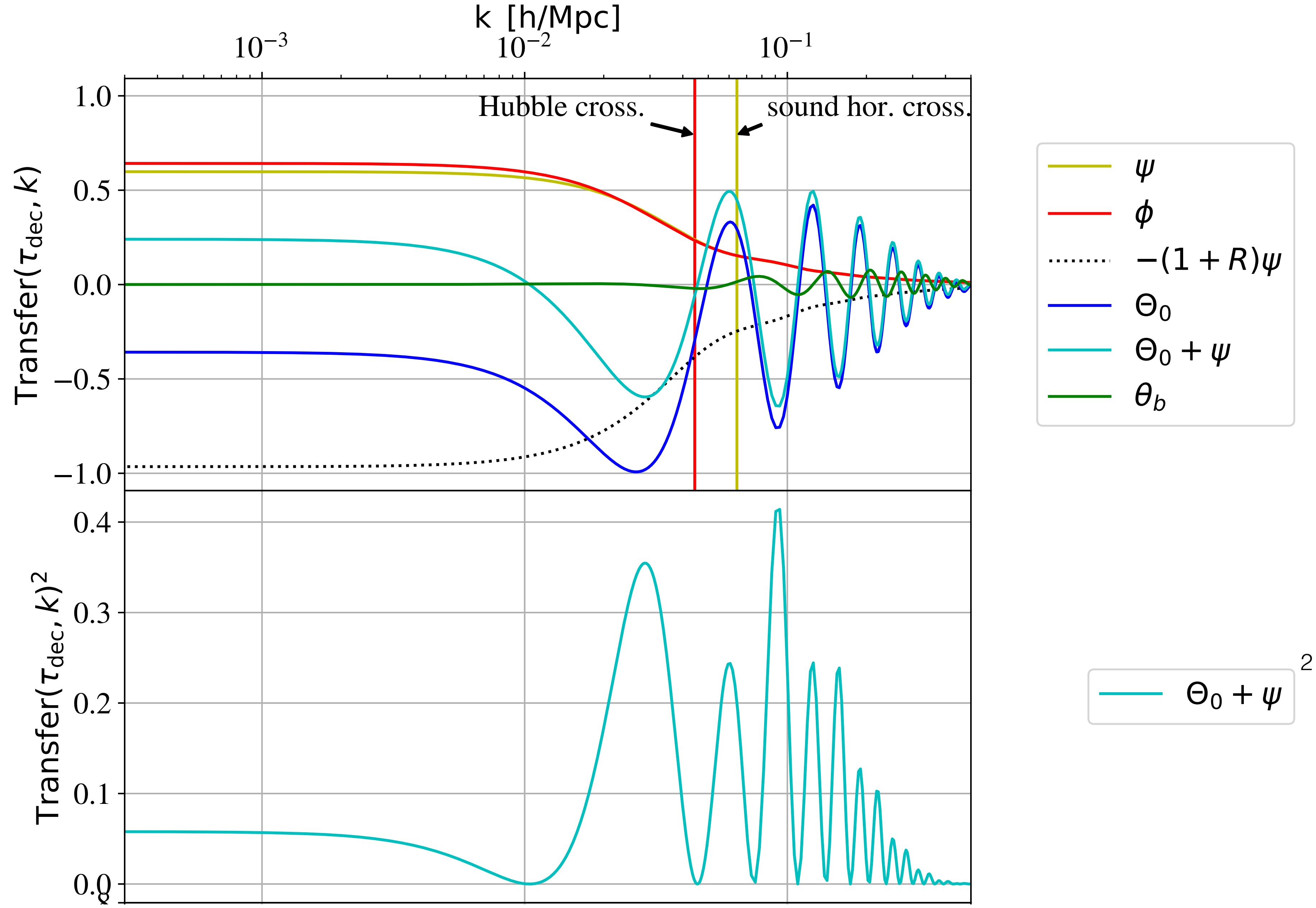
$$\Theta_l(\tau_0, k) = \{(\Theta_0 + \psi)_{\rm rec} + (...\theta_{\rm b}')_{\rm rec} + (...\theta_{\rm b} + {\rm ISW})_{\rm rec}\} \; j_l(k(\tau_0 - \tau_{\rm rec}))$$

$$C_l \sim (...) \left\{(\Theta_0 + \psi)^2_{(\tau_{\rm rec}, k = l/(\tau_0 - \tau_{\rm rec}))} + {\rm Doppler} + {\rm ISW}\right\}^2 \mathcal{P}_{\mathcal{R}}(k)$$

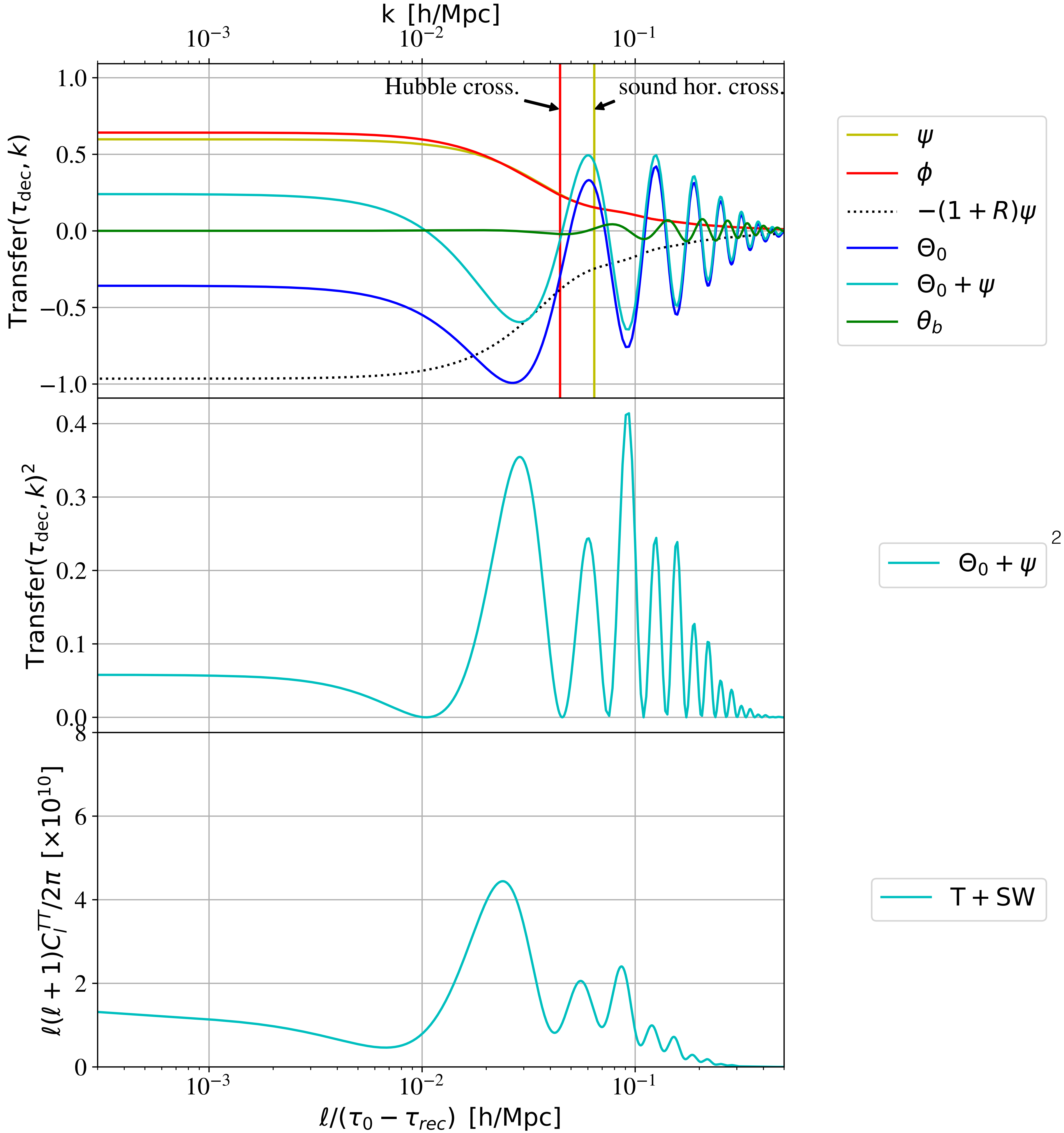
Projection effects:



Projection effects:



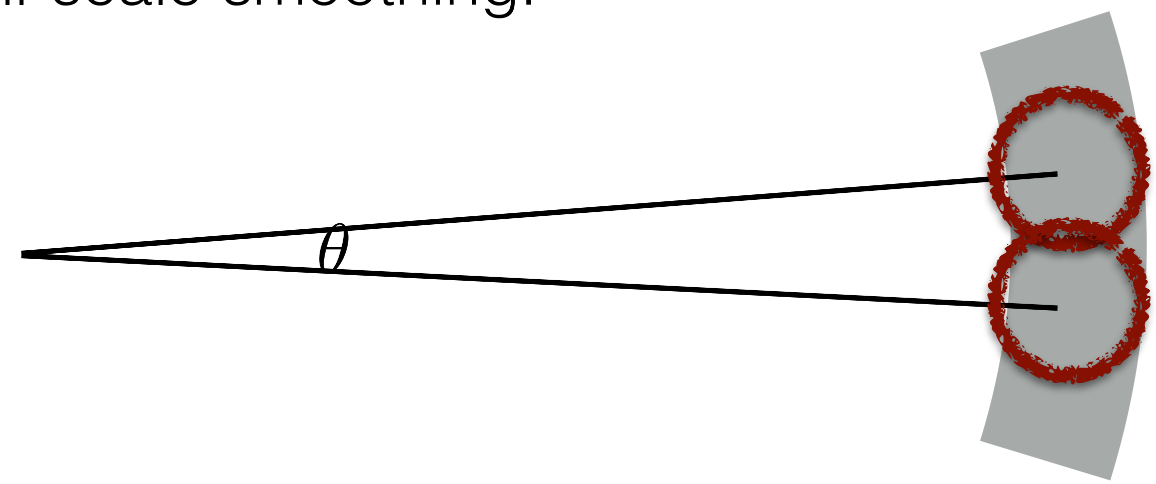
Projection effects:



Projection effects

- Thickness of l.s.s produces small-scale smoothing:

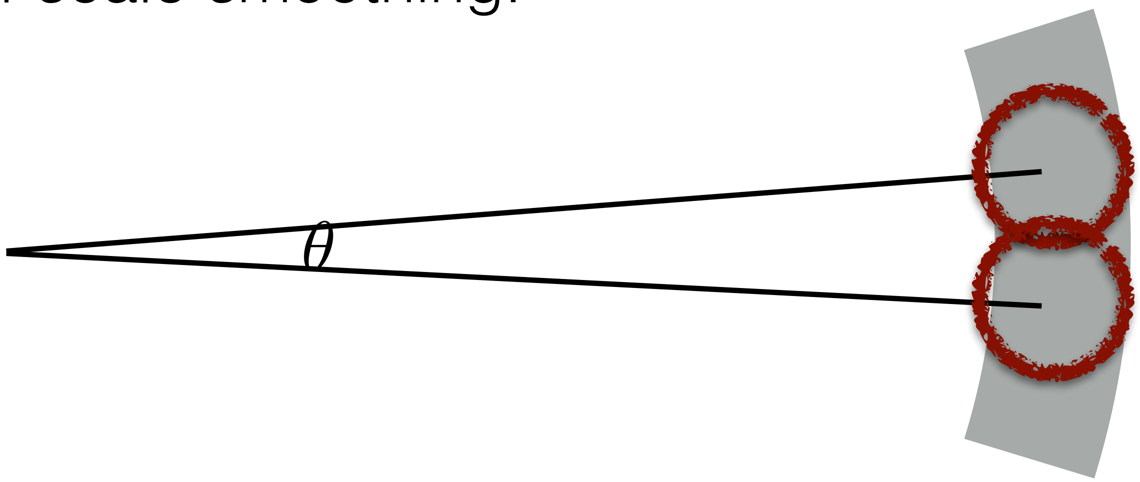
observed photons could carry temperature from wherever inside circles



Projection effects

- Thickness of l.s.s produces small-scale smoothing:

observed photons could carry temperature from wherever inside circles



- Mathematically, two types of smoothing:

$$\Theta_l(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; \{g\,(\Theta_0 + \psi) + \left(g\,k^{-2}\theta_{\rm b}\right)' + {\rm ISW}\} \; j_l(k(\tau_0 - \tau))$$

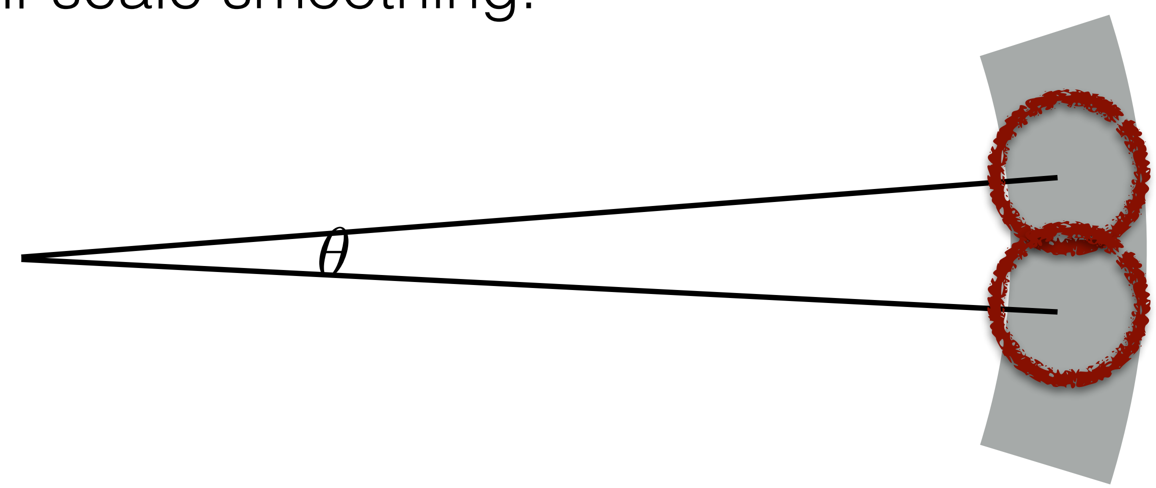
$$C_l = 4\pi \int dk \; k^2 \left(\Theta_l(\tau_0, k)\right)^2 \mathcal{P}_{\mathcal{R}}(k)$$

—> contribution of wide range of *times* and *wavenumber* to single C_l

Projection effects

- Thickness of l.s.s produces small-scale smoothing:

observed photons could carry temperature from wherever inside circles



- Mathematically, two types of smoothing:

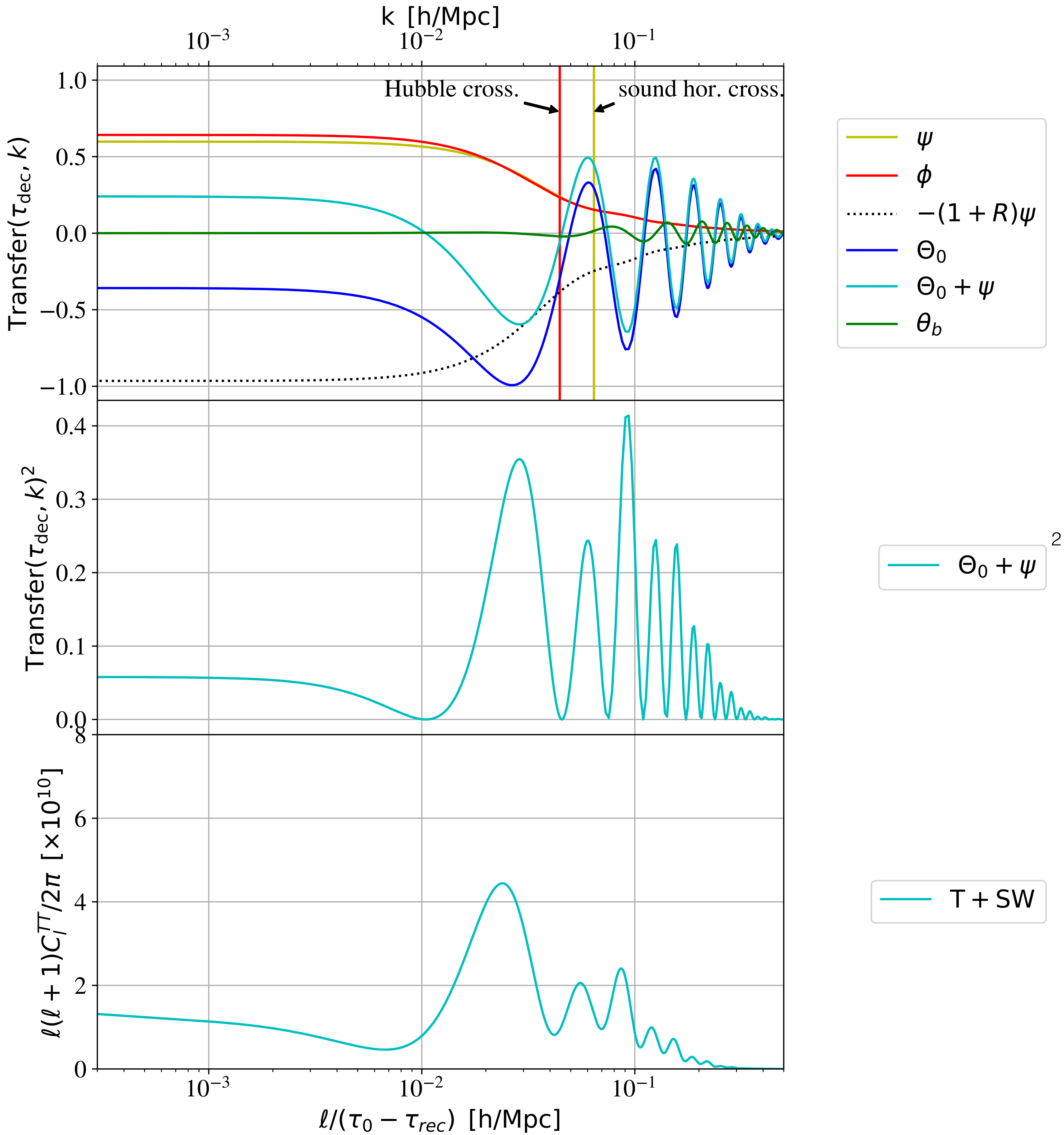
$$\Theta_l(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; \{g\,(\Theta_0 + \psi) + \left(g\,k^{-2}\theta_{\rm b}\right)' + {\rm ISW}\}\; j_l(k(\tau_0 - \tau))$$

$$C_l = 4\pi \int dk\; k^2\, (\Theta_l(\tau_0, k))^2\, \mathcal{P}_{\mathcal{R}}(k)$$

—> contribution of wide range of *times* and *wavenumber* to single C_l

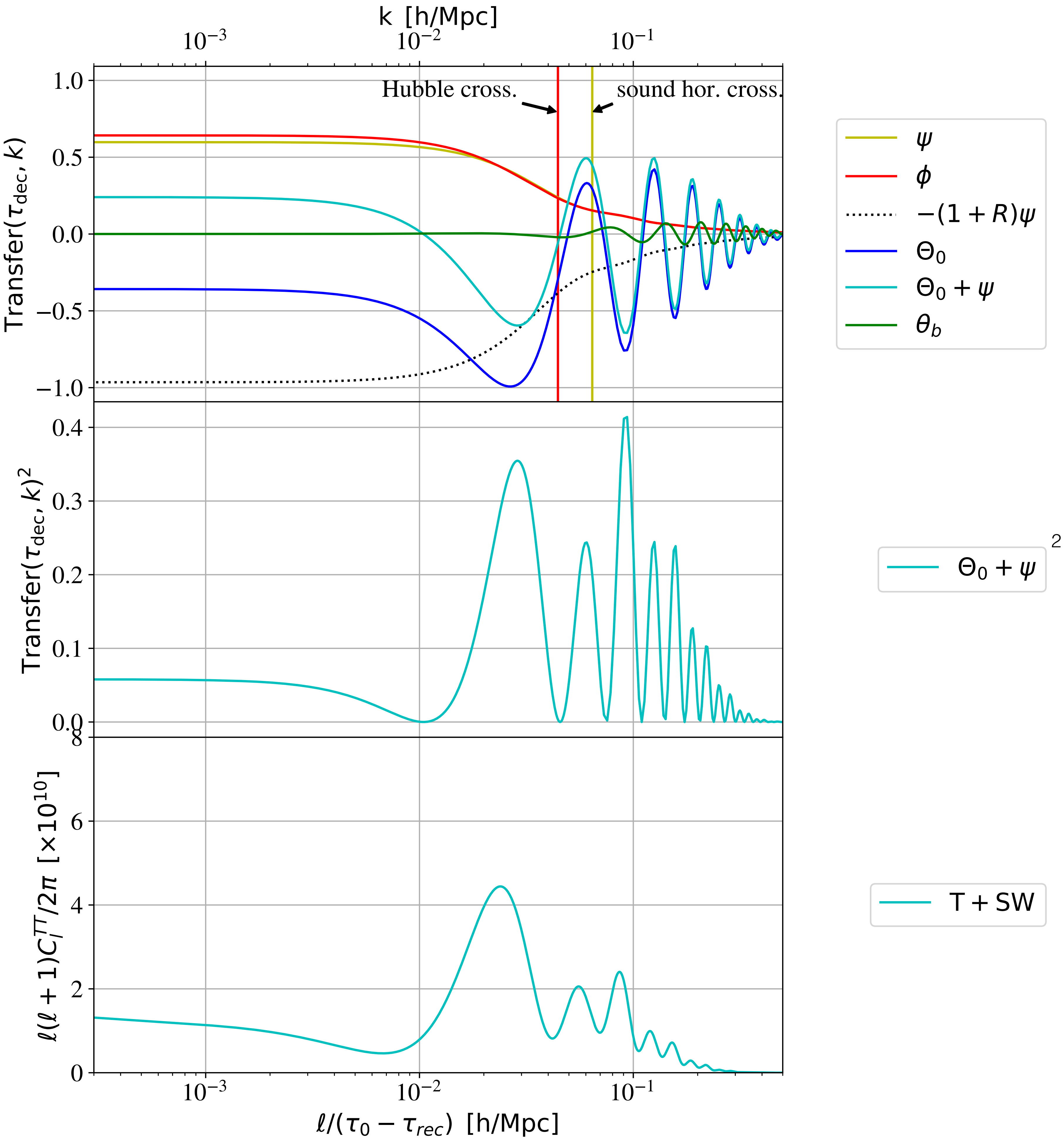
- Even in instantaneous limit: Spherical Bessel functions —> effects beyond small angle, effect of smaller non-transverse k, …: extra smoothing on all scales

Projection effects

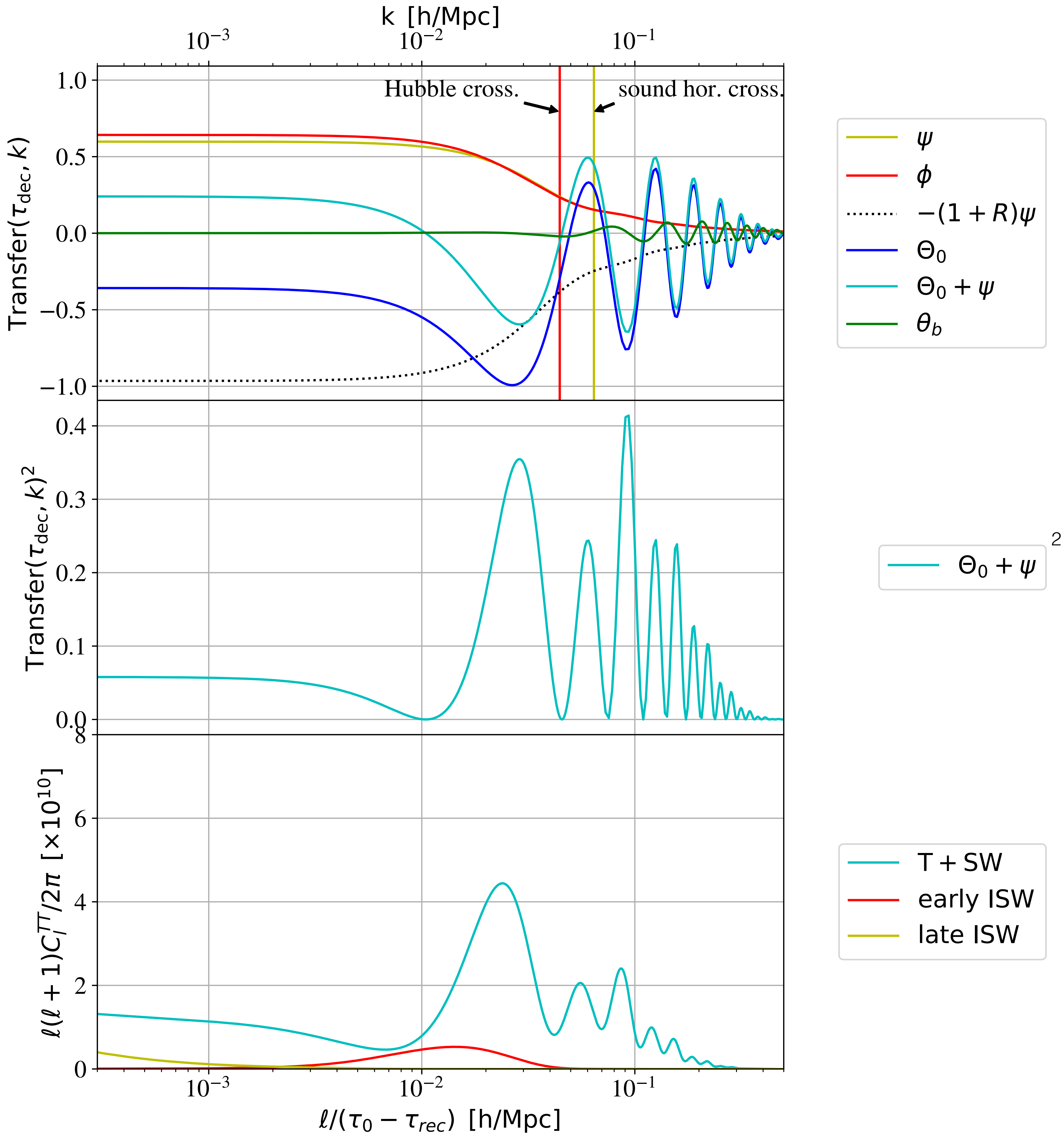


Rest for the ears

Towards final C_l^{TT}'s



Towards final C_l^{TT}'s

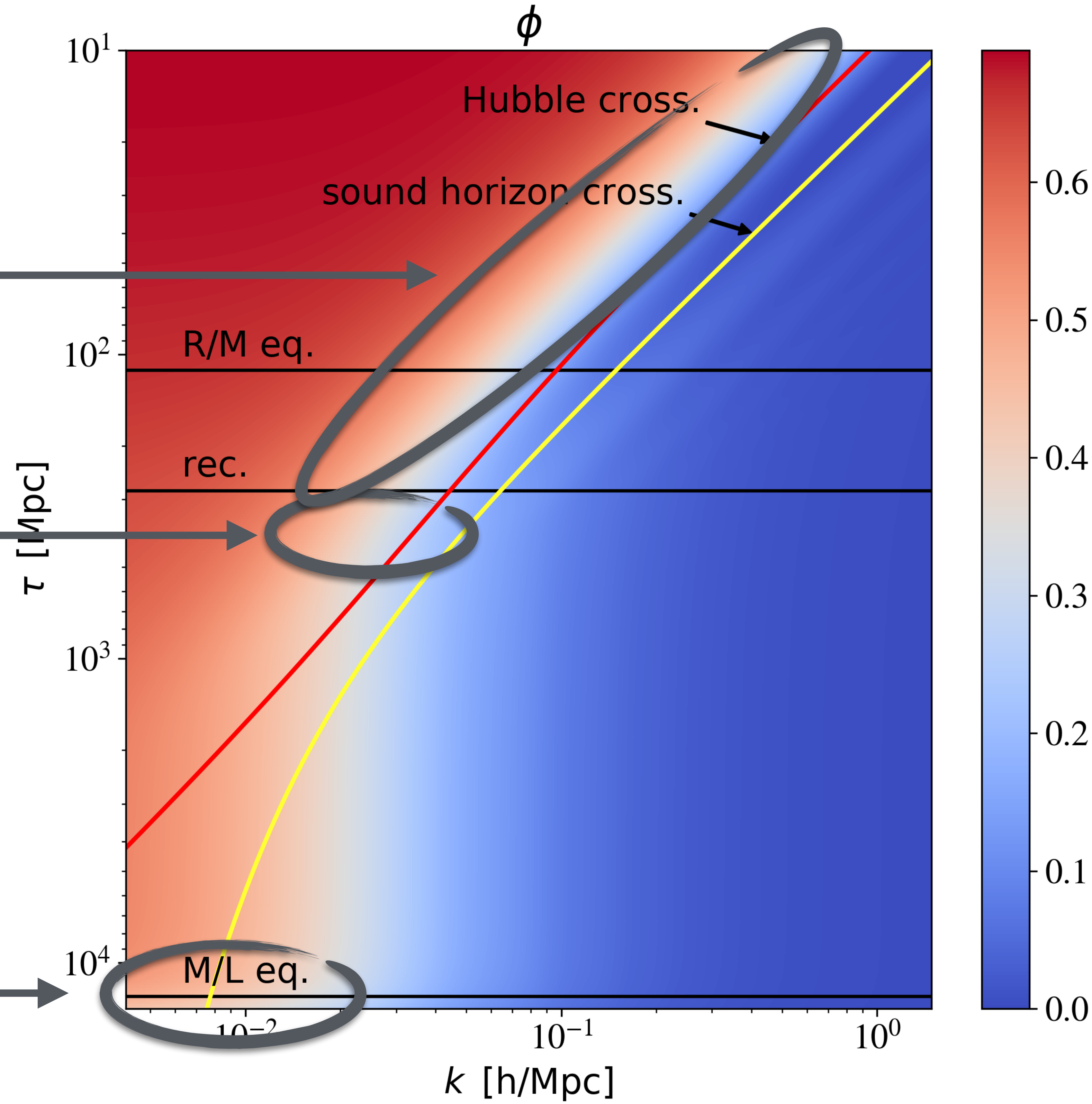


Evolution for all wavenumbers

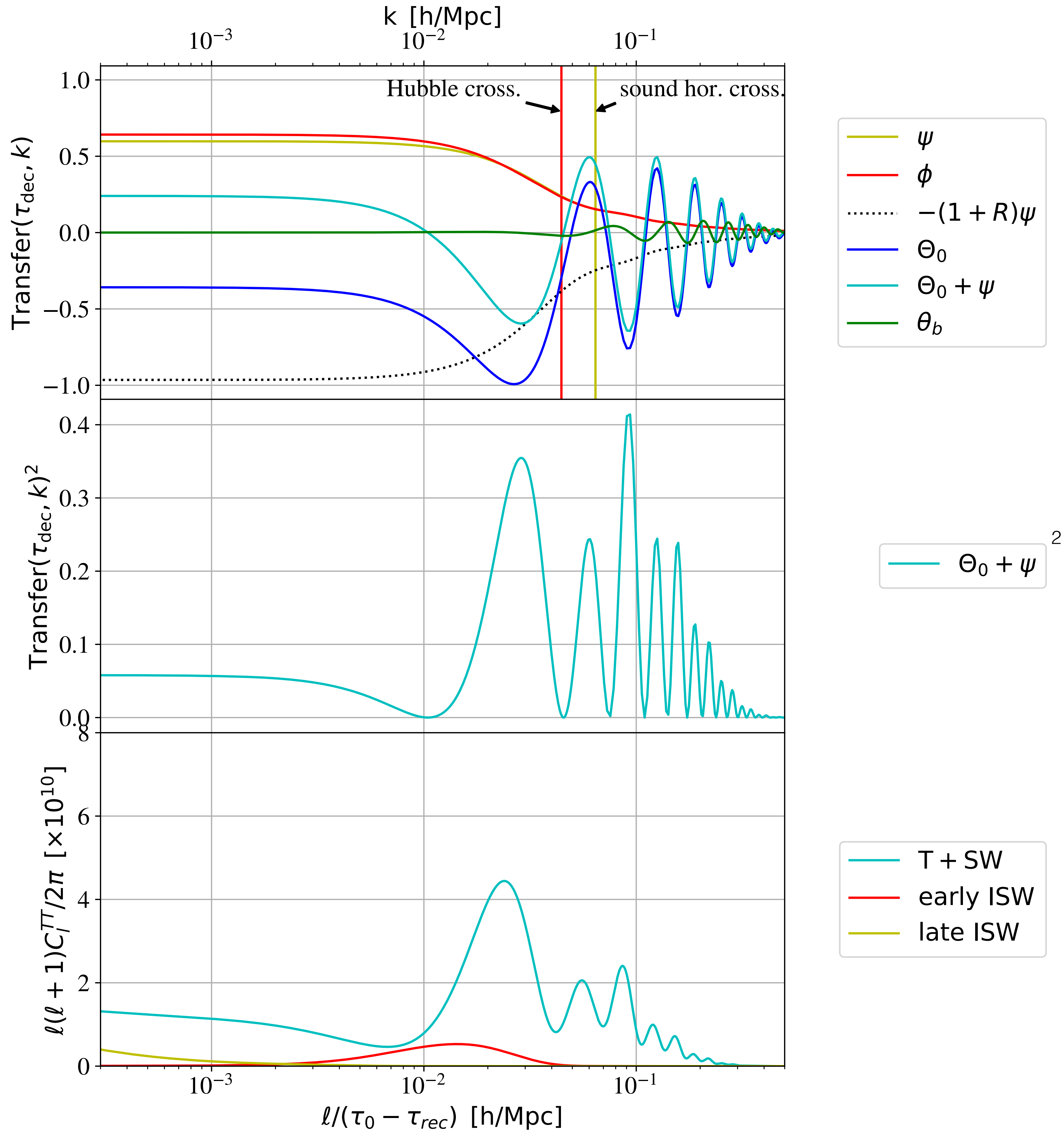
$$\frac{R'}{1+R}\phi' + \phi''$$ gravity boost

$$\underbrace{e^{-\kappa}(\phi' + \psi')\}}_{\text{ISW}}$$ early ISW

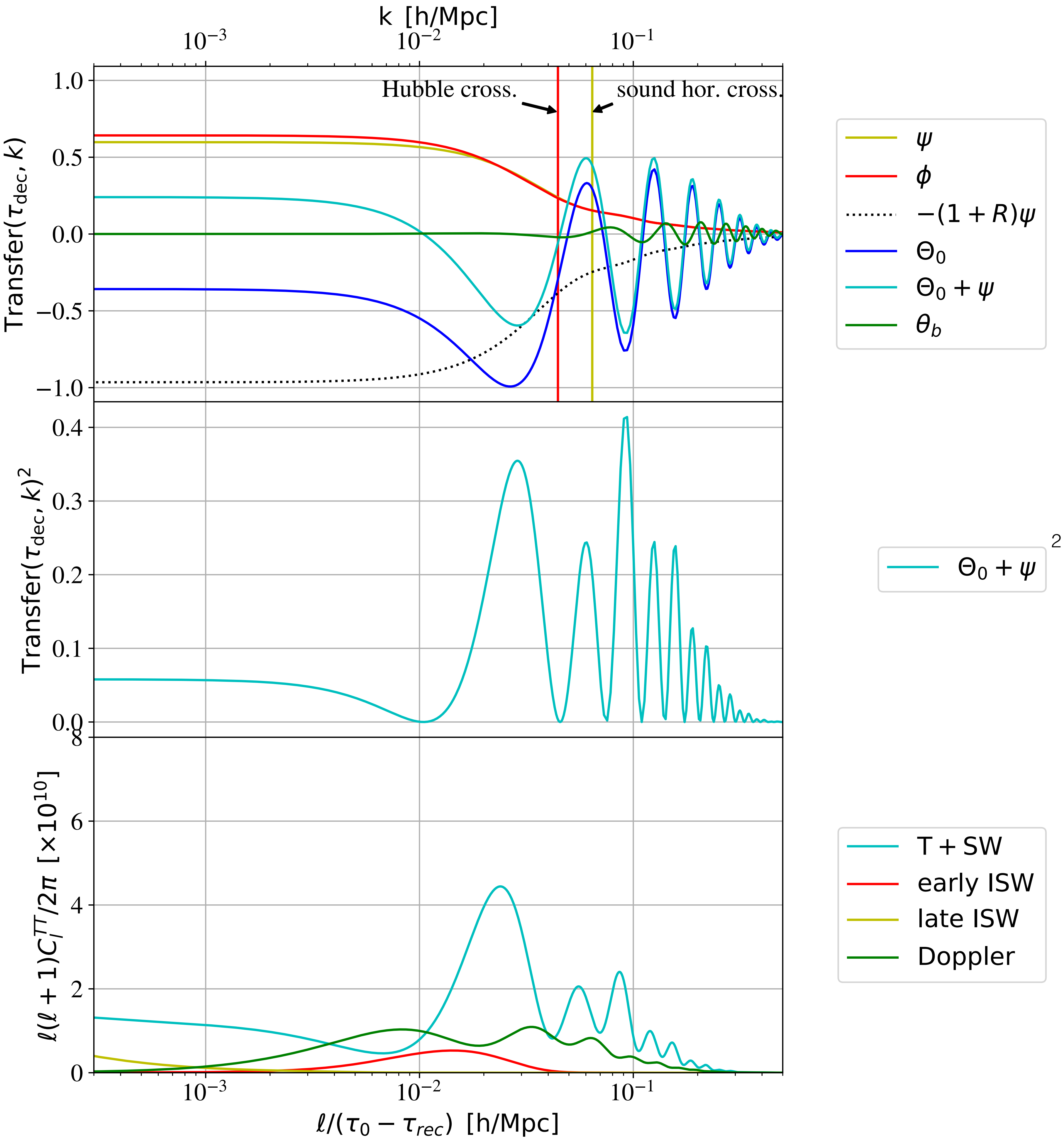
$$\underbrace{e^{-\kappa}(\phi' + \psi')\}}_{\text{ISW}}$$ late ISW



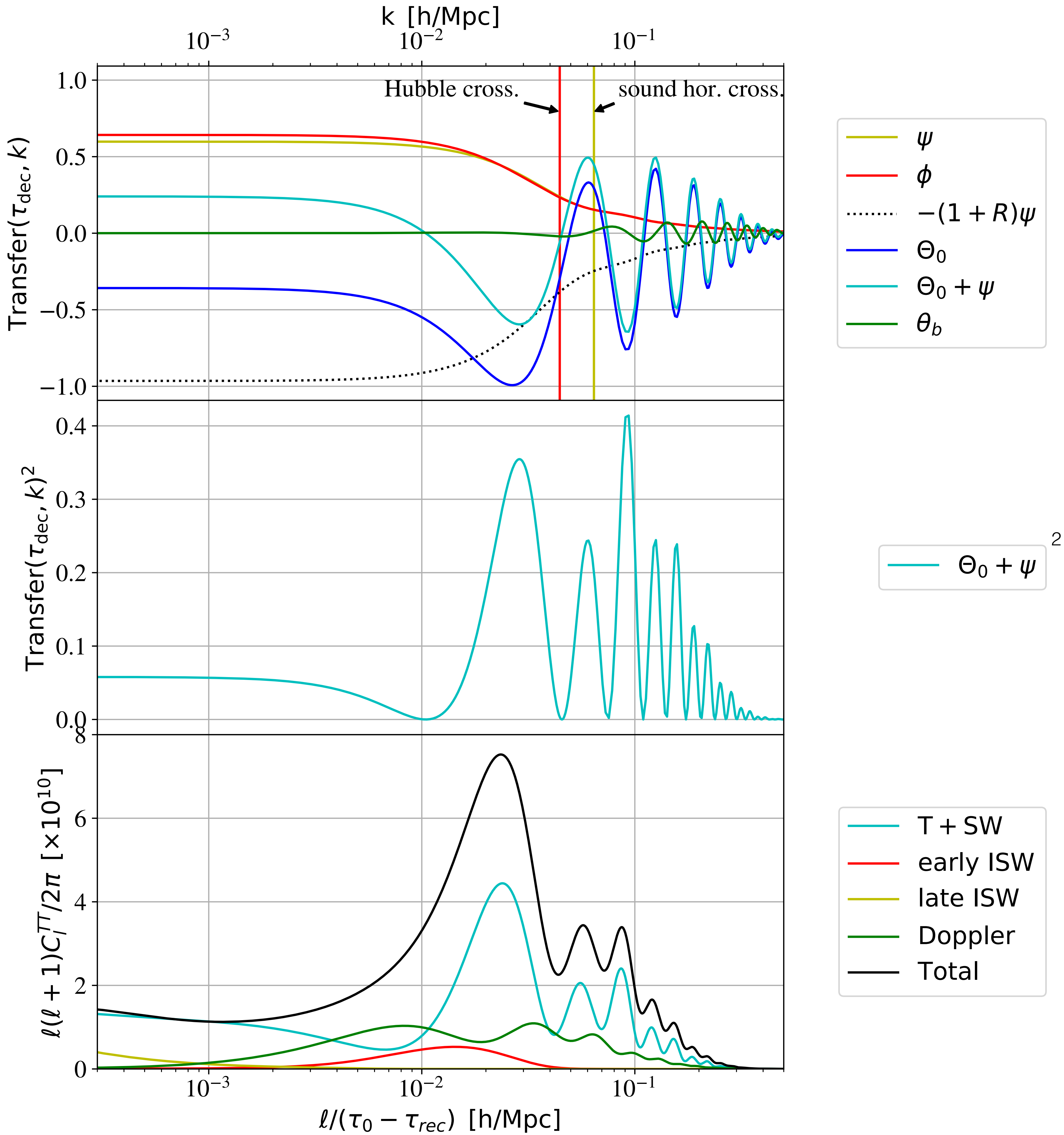
Towards final C_l^{TT}'s



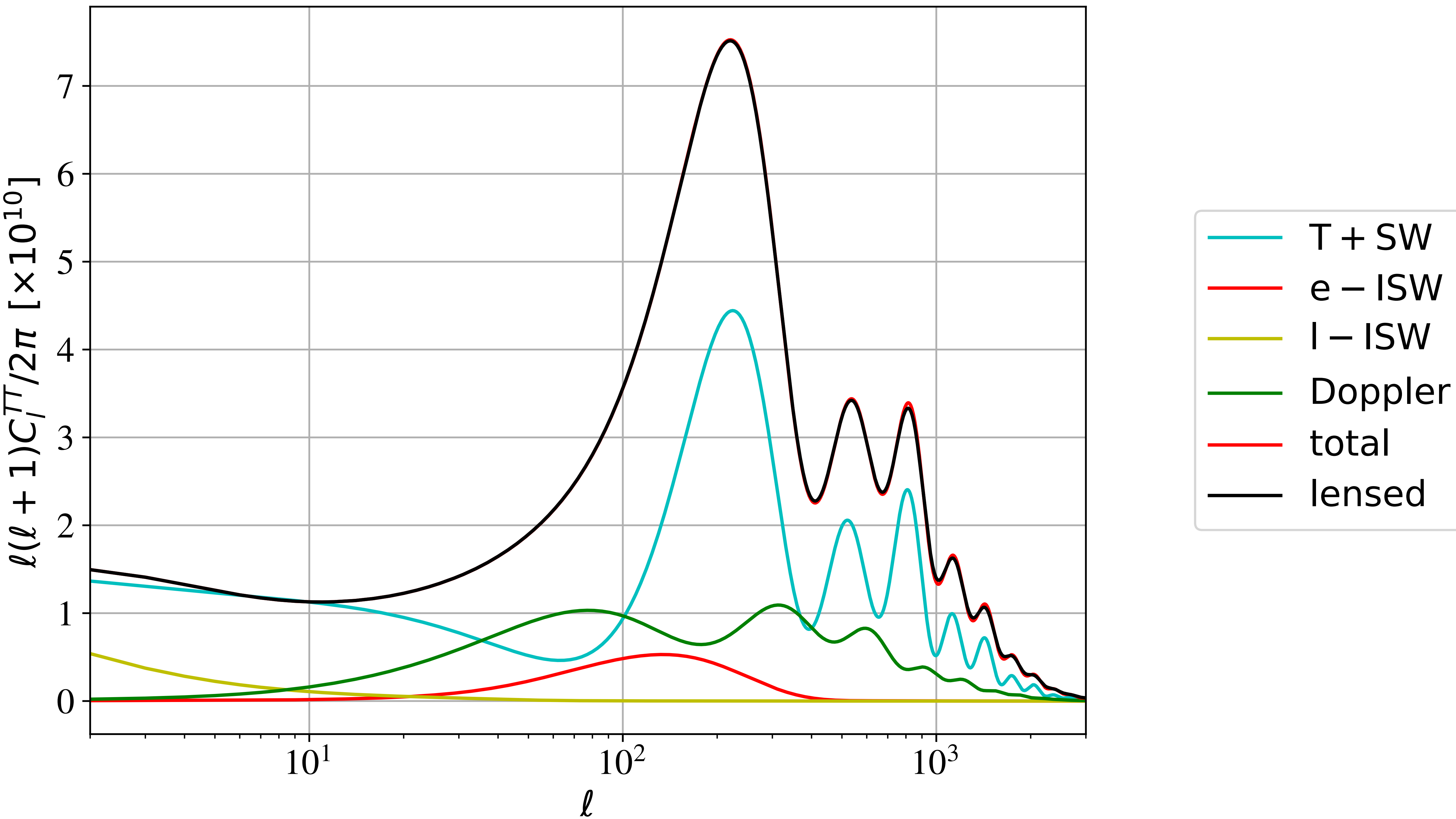
Towards final C_l^{TT}'s



Towards final C_l^{TT}'s

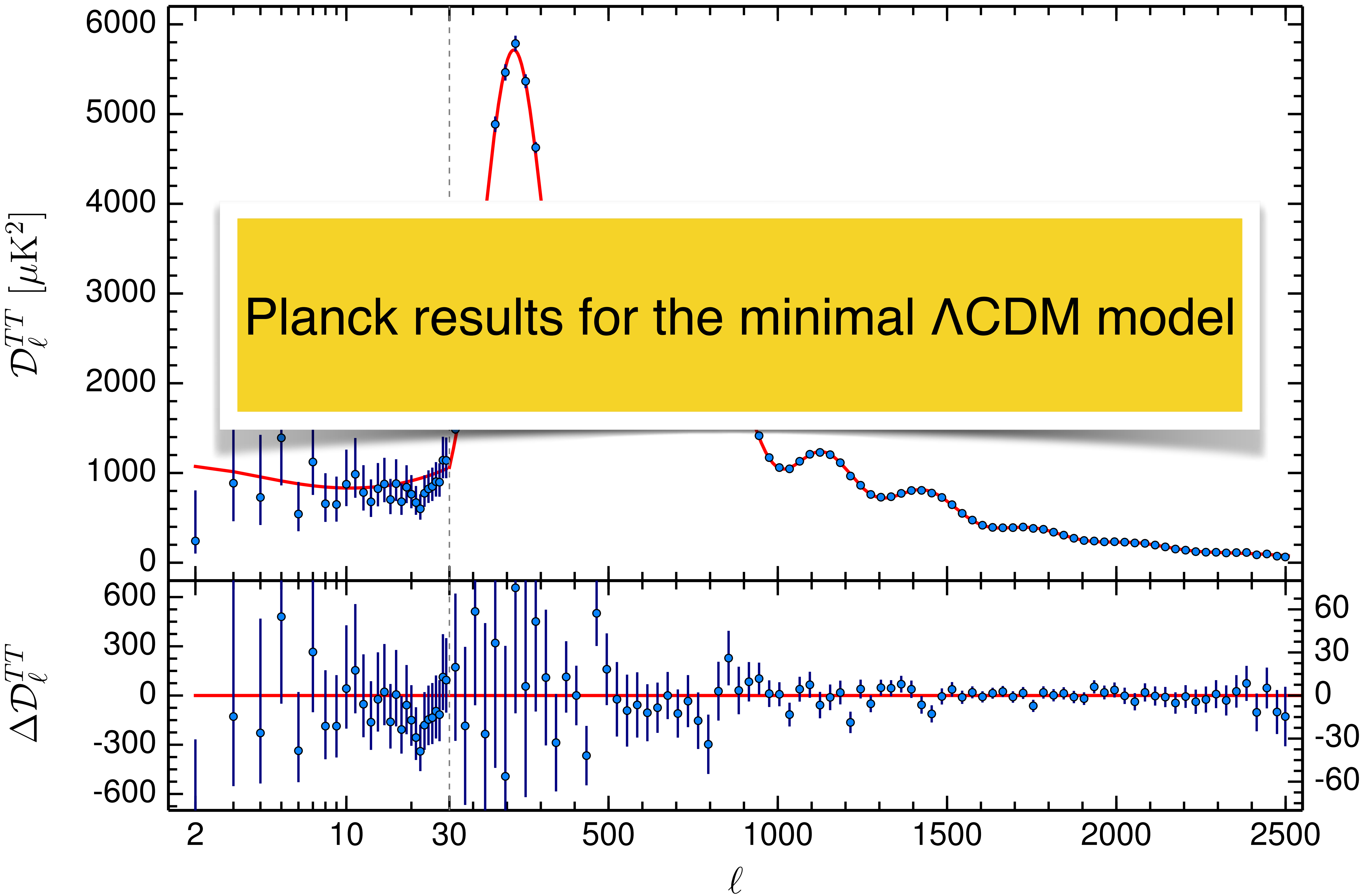


Detailed physical understanding of the C_l^{TT} shape

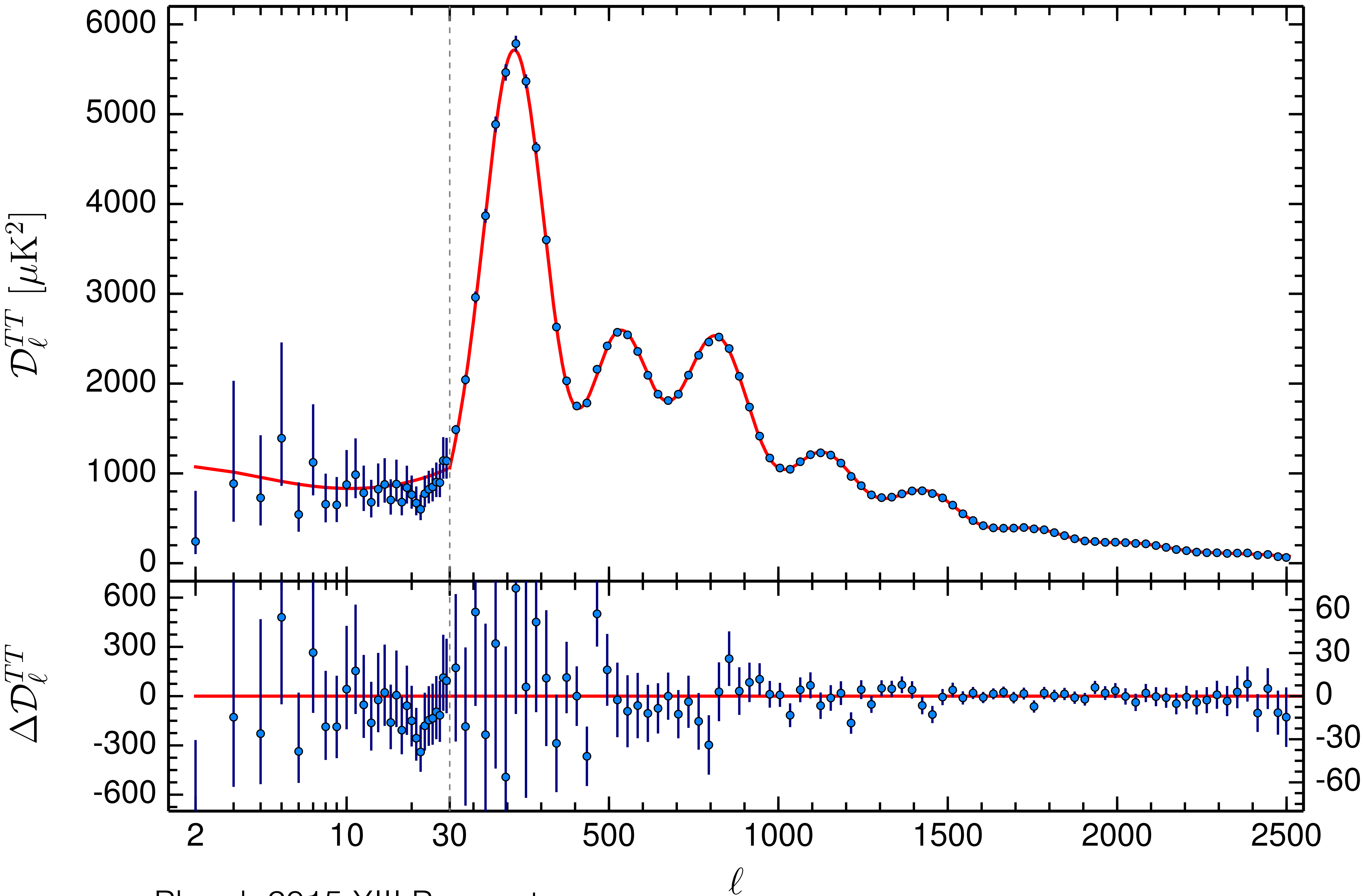


Institute for Theoretical Particle Physics and Cosmology

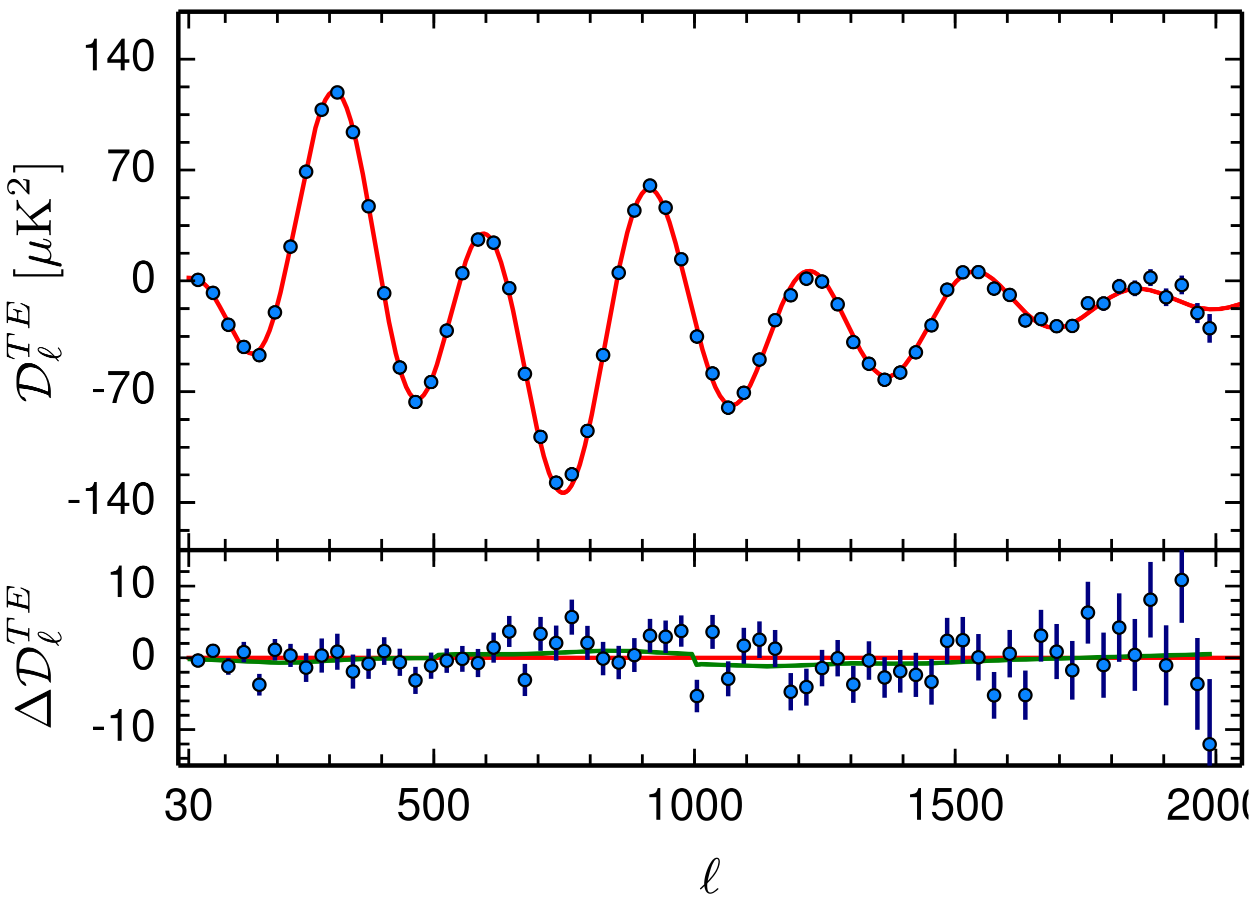
RWTH AACHEN UNIVERSITY

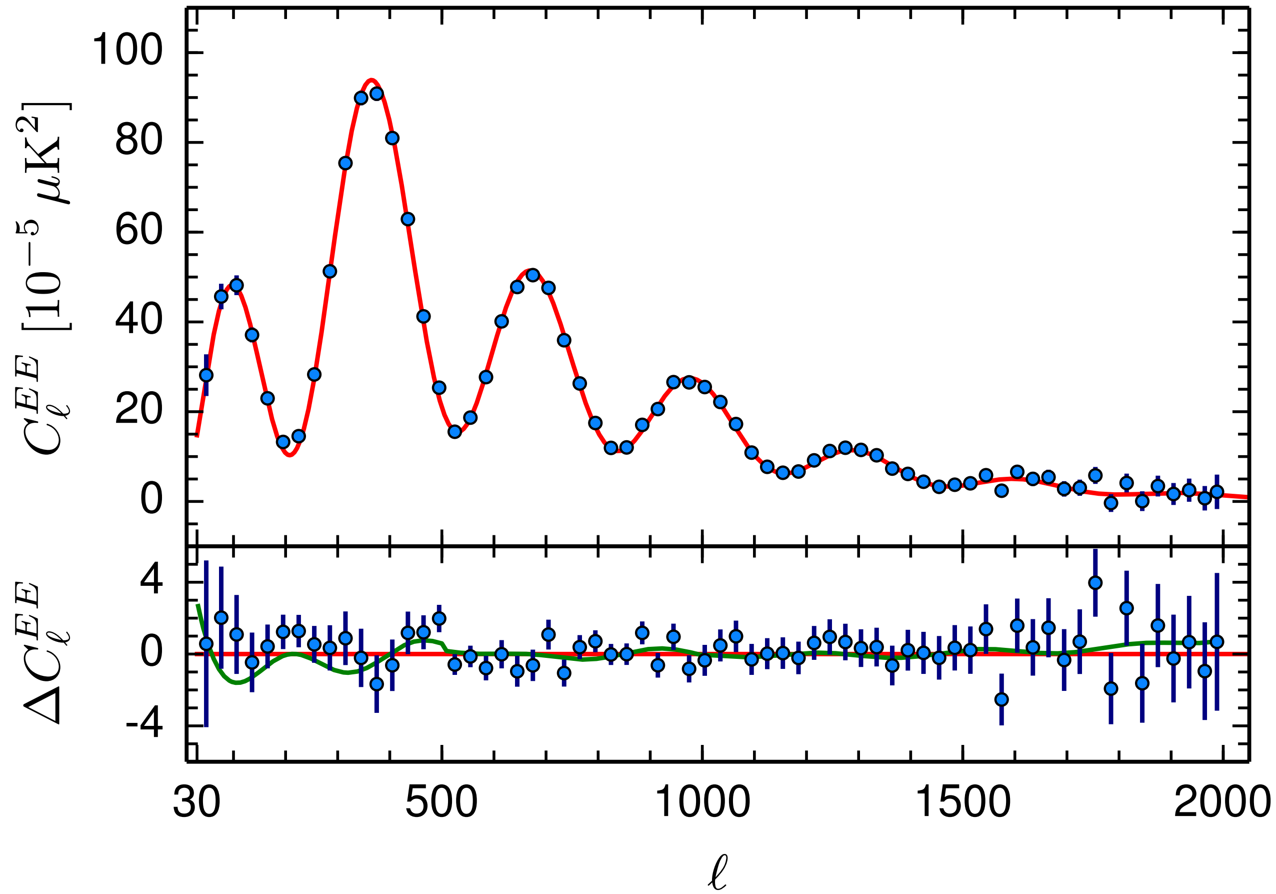


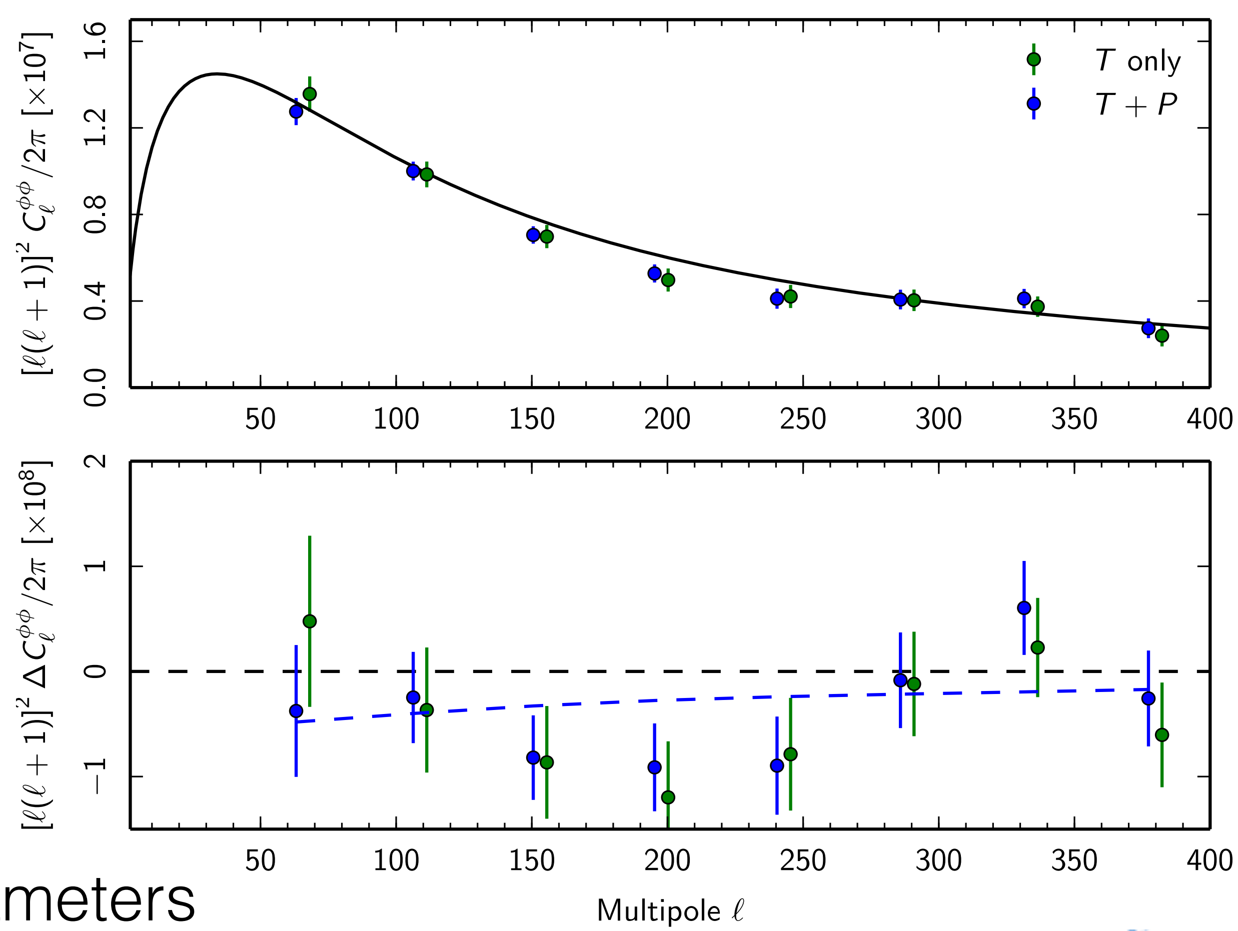
Planck results for the minimal ΛCDM model



Planck 2015 XIII Parameters





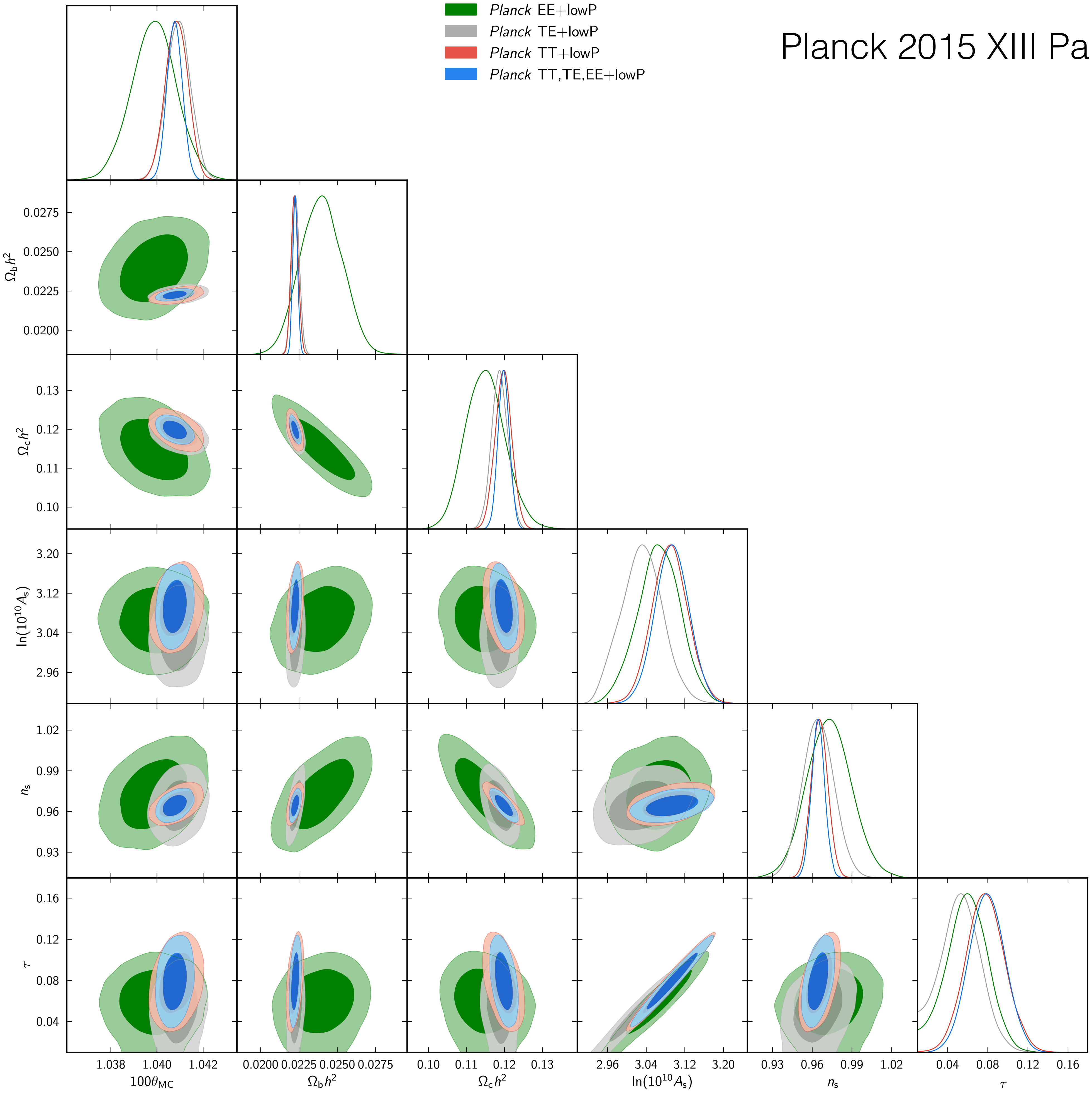


Planck 2015 XIII Parameters

TTK Institute for Theoretical Particle Physics and Cosmology

RWTH AACHEN UNIVERSITY

Planck 2015 XIII Parameters

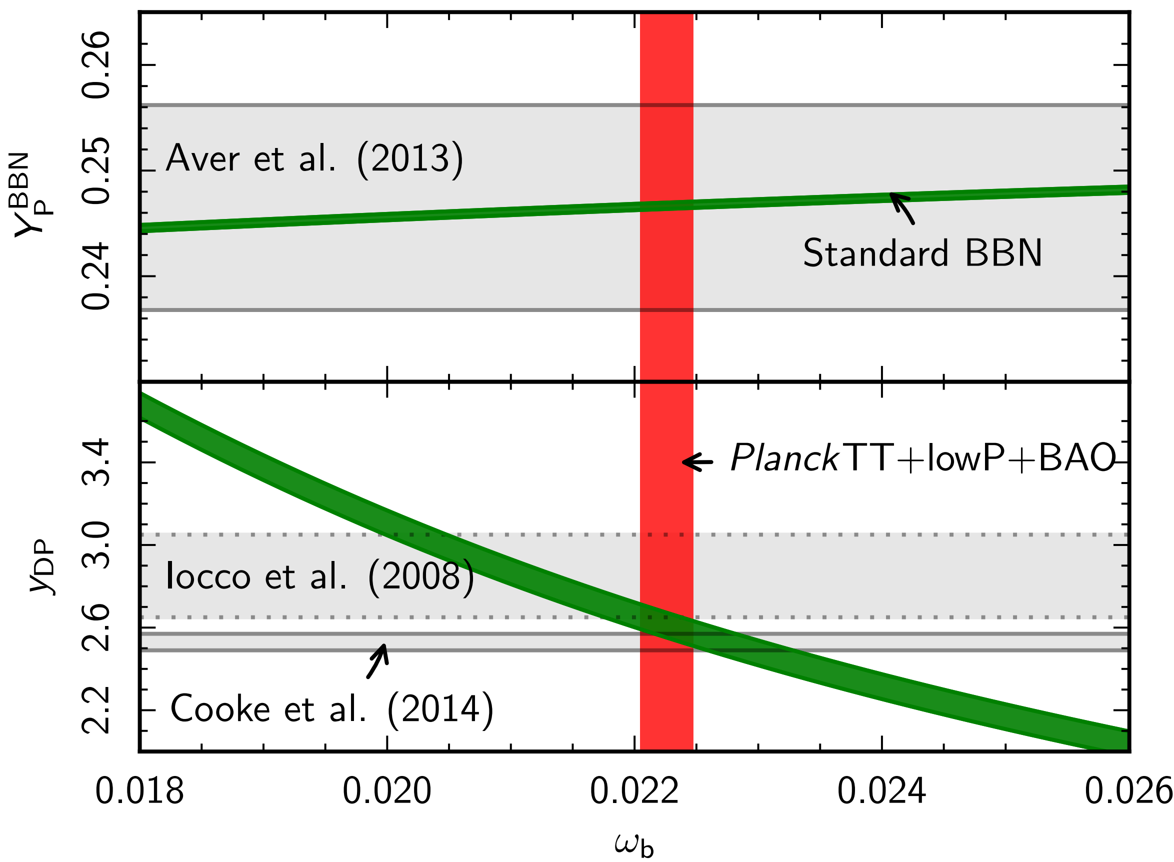


	Parameter	PlanckTTTEEE+SIMlow 68 % limits
Flat prior	$\Omega_b h^2$	0.02218 ± 0.00015
	$\Omega_c h^2$	0.1205 ± 0.0014
	$100\theta_{MC}$	1.04069 ± 0.00031
	τ	0.0596 ± 0.0089
	$\ln(10^{10} A_s)$	3.056 ± 0.018
	n_s	0.9619 ± 0.0045
Derived	H_0	66.93 ± 0.62
	Ω_m	0.3202 ± 0.0087
	σ_8	0.8174 ± 0.0081
	$\sigma_8 \Omega_m^{0.5}$	0.4625 ± 0.0091
	$\sigma_8 \Omega_m^{0.25}$	0.6148 ± 0.0086
	z_{re}	8.24 ± 0.88
	$10^9 A_s e^{-2\tau}$	1.886 ± 0.012
	Age/Gyr	13.826 ± 0.025

Planck 2016 intermediate results 1605.02985

	Parameter	PlanckTTTEEE+SIMlow 68 % limits
Flat prior	$\Omega_b h^2$	0.02218 ± 0.00015
	$\Omega_c h^2$	0.1205 ± 0.0014
	$100\theta_{MC}$	1.04069 ± 0.00031
	τ	0.0596 ± 0.0089
	$\ln(10^{10} A_s)$	3.056 ± 0.018
	n_s	0.9619 ± 0.0045
Derived	H_0	66.93 ± 0.62
	Ω_m	0.3202 ± 0.0087
	σ_8	0.8174 ± 0.0081
	$\sigma_8 \Omega_m^{0.5}$	0.4625 ± 0.0091
	$\sigma_8 \Omega_m^{0.25}$	0.6148 ± 0.0086
	z_{re}	8.24 ± 0.88
	$10^9 A_s e^{-2\tau}$	1.886 ± 0.012
	Age/Gyr	13.826 ± 0.025

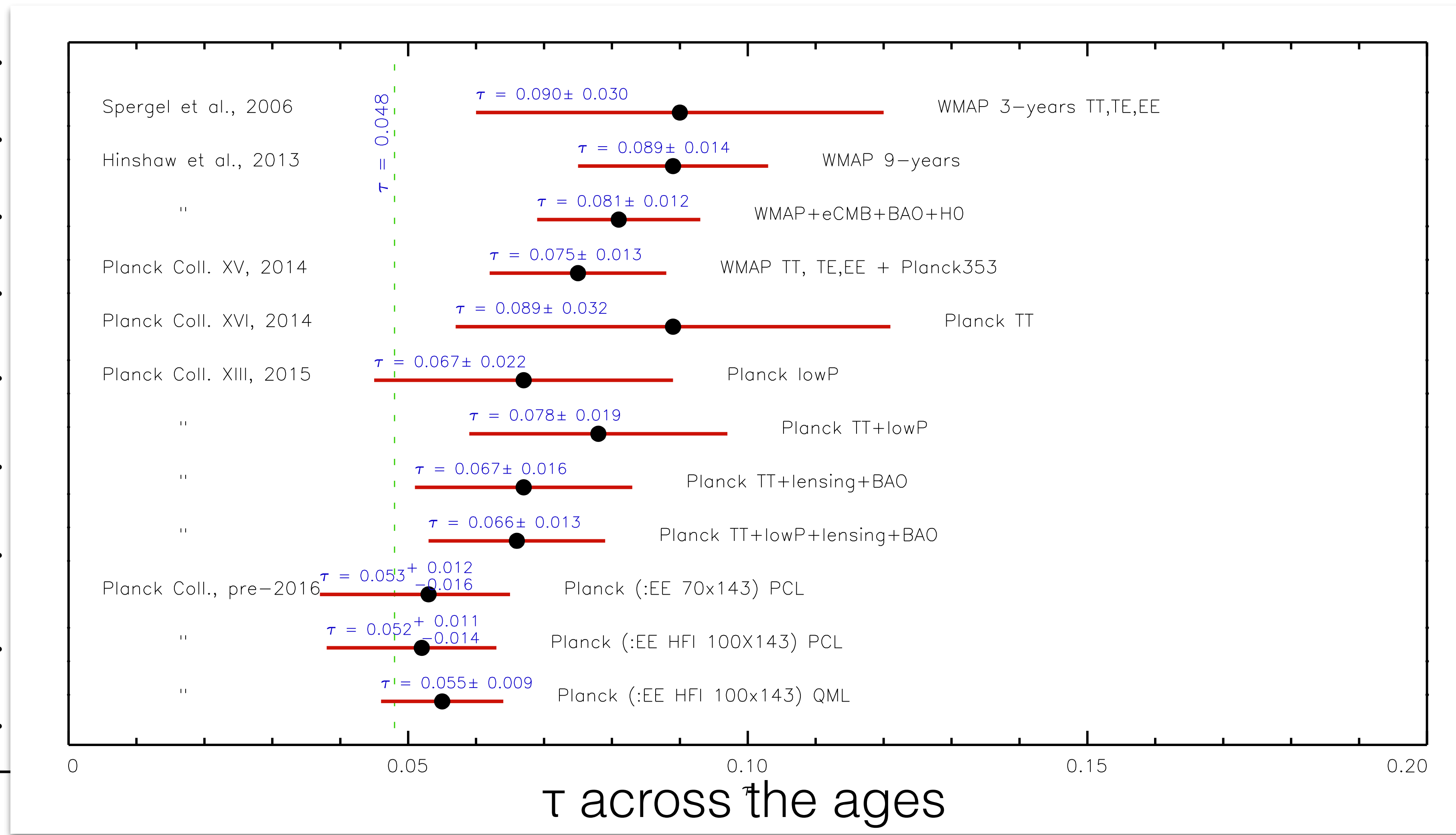
interesting to compare to BBN



Planck 2016 intermediate results 1605.02985

	Parameter	PlanckTTTEEE+SIMlow 68 % limits
Flat prior	$\Omega_b h^2$	0.02218 ± 0.00015
	$\Omega_c h^2$	0.1205 ± 0.0014
	$100\theta_{MC}$	1.04069 ± 0.00031
	τ	0.0596 ± 0.0089
	$\ln(10^{10} A_s)$	3.056 ± 0.018
	n_s	
Derived	H_0	
	Ω_m	
	σ_8	
	$\sigma_8 \Omega_m^{0.5}$	
	$\sigma_8 \Omega_m^{0.25}$	
	z_{re}	
	$10^9 A_s e^{-2\tau}$	
	Age/Gyr	

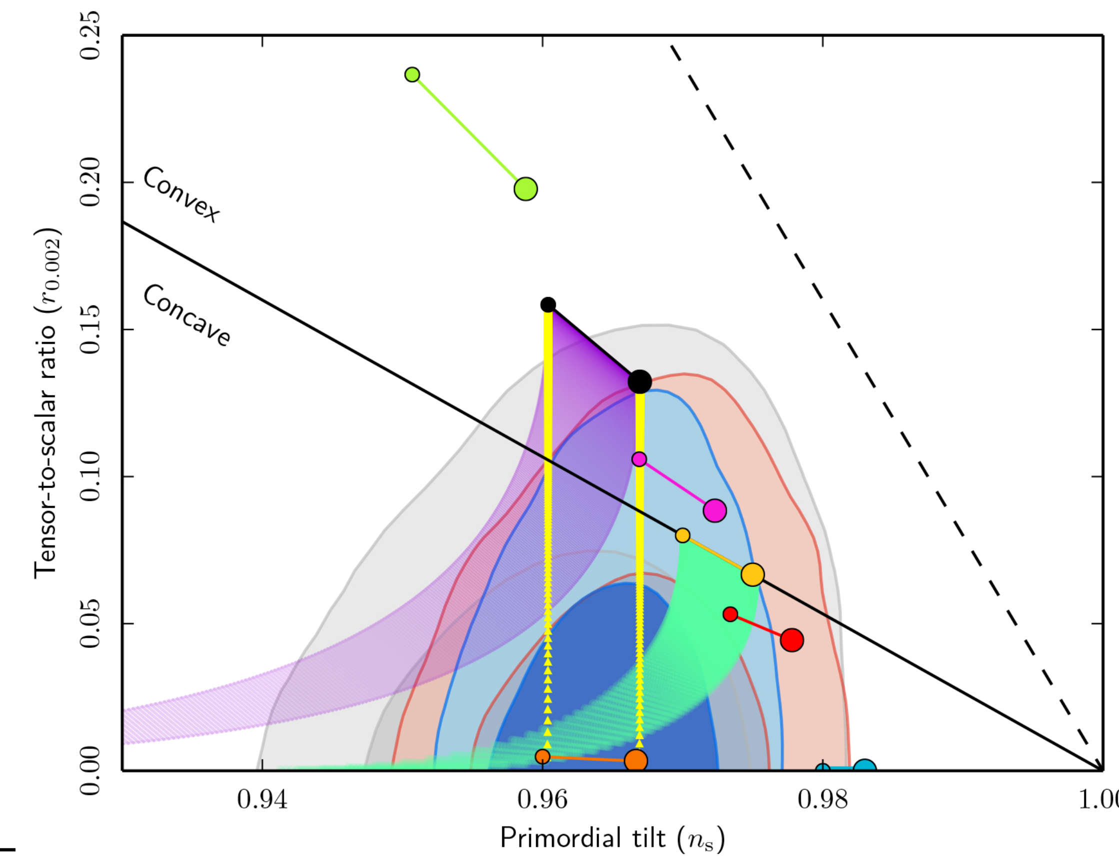
interesting to compare to quasars, Gunn-Peterson…



τ across the ages

Planck 2016 intermediate results 1605.02985

	Parameter	PlanckTTTEEE+SIMlow 68 % limits
Flat prior	$\Omega_b h^2$	0.02218 ± 0.00015
	$\Omega_c h^2$	0.1205 ± 0.0014
	$100\theta_{MC}$	1.04069 ± 0.00031
	τ	0.0596 ± 0.0089
	$\ln(10^{10} A_s)$	3.056 ± 0.018
	n_s	0.9619 ± 0.0045 ← interesting for inflation
Derived	H_0	66.93 ± 0.62
	Ω_m	0.3202 ± 0.0087
	σ_8	0.8174 ± 0.0081
	$\sigma_8 \Omega_m^{0.5}$	0.4625 ± 0.0091
	$\sigma_8 \Omega_m^{0.25}$	0.6148 ± 0.0086
	z_{re}	8.24 ± 0.88
	$10^9 A_s e^{-2\tau}$	1.886 ± 0.012
	Age/Gyr	13.826 ± 0.025

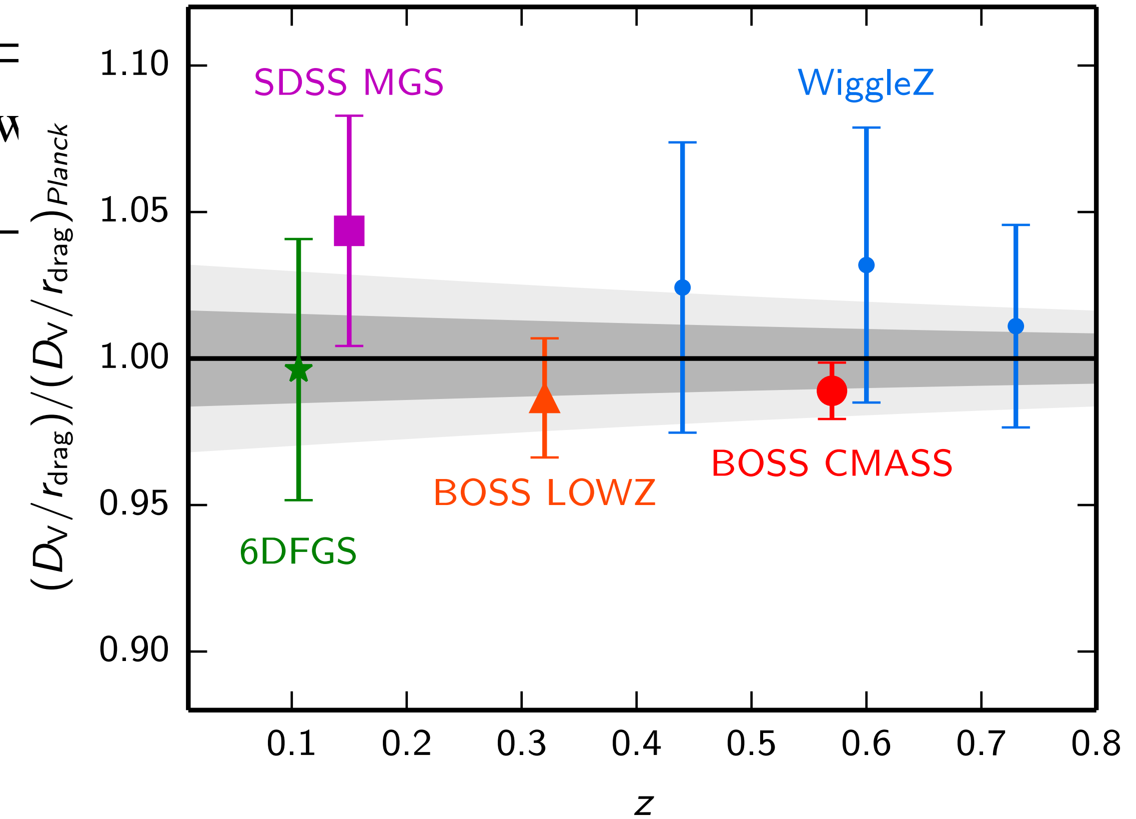


Planck 2016 intermediate results 1605.02985

Parameter	PlanckTTTEEE+SIMlow 68 % limits
$\Omega_b h^2$	0.02218 ± 0.00015
$\Omega_c h^2$	0.1205 ± 0.0014
$100\theta_{MC}$	1.04069 ± 0.00031
τ	0.0596 ± 0.0089
$\ln(10^{10}A_s)$	3.056 ± 0.018
n_s	0.9619 ± 0.0045
H_0	66.93 ± 0.62
Ω_m	0.3202 ± 0.0087
σ_8	0.8174 ± 0.0081
$\sigma_8\Omega_m^{0.5}$	0.4625 ± 0.0091
$\sigma_8\Omega_m^{0.25}$	0.6148 ± 0.0086
z_{re}	8.24 ± 0.88
$10^9 A_s e^{-2\tau}$	1.886 ± 0.012
Age/Gyr	13.826 ± 0.025

Flat prior

Derived



interesting to compare to expansion probes: BAO, SNIa, H_0

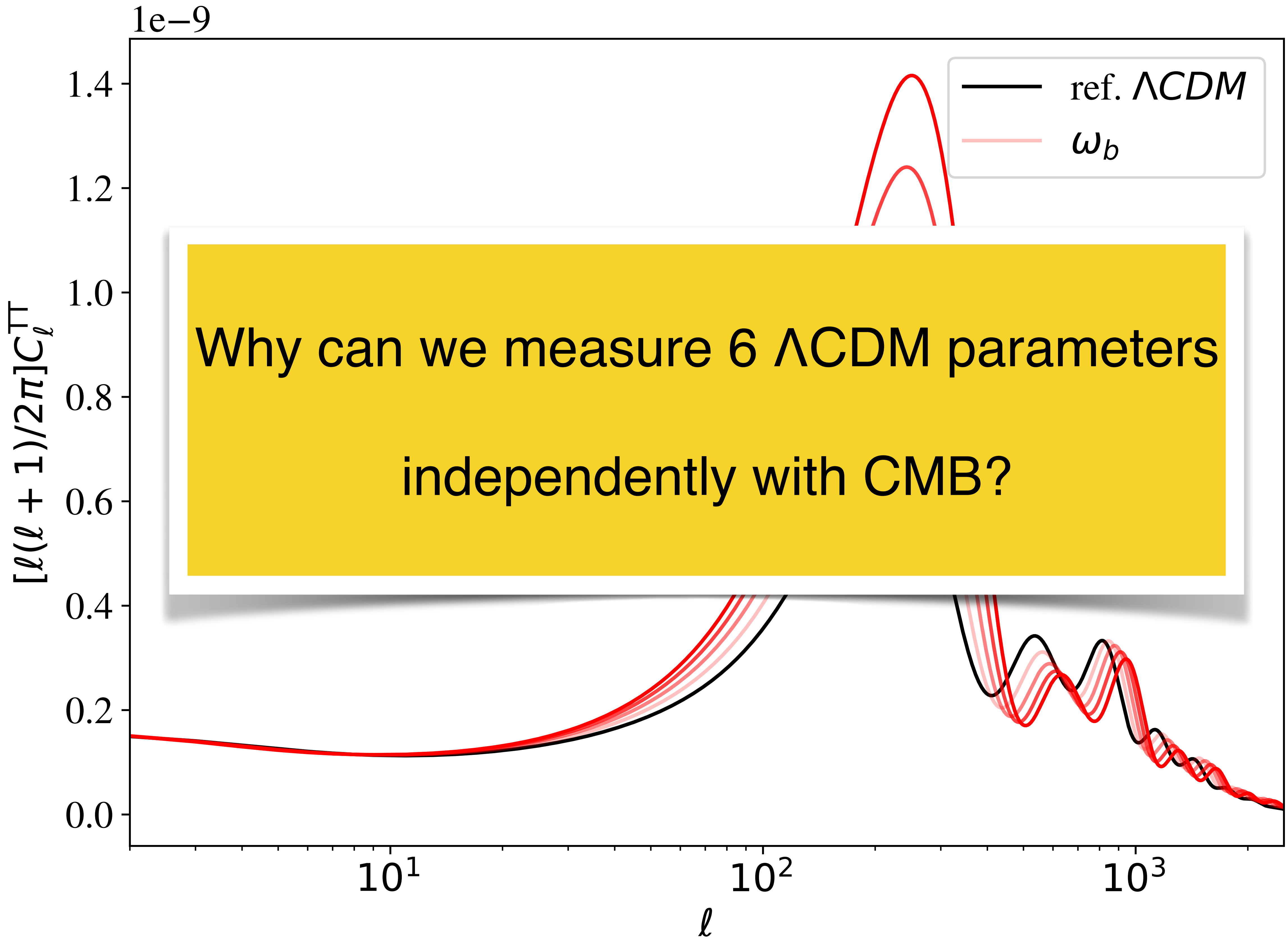
3.2sigma tension btw Riess et al. and Planck 2016 TT + SIMlow

Planck 2015 XIII Parameters

	Parameter	PlanckTTTEEE+SIMlow 68 % limits
Flat prior	$\Omega_b h^2$	0.02218 ± 0.00015
	$\Omega_c h^2$	0.1205 ± 0.0014
	$100\theta_{MC}$	1.04069 ± 0.00031
	τ	0.0596 ± 0.0089
	$\ln(10^{10} A_s)$	3.056 ± 0.018
	n_s	0.9619 ± 0.0045
Derived	H_0	66.93 ± 0.62
	Ω_m	0.3202 ± 0.0087
	σ_8	0.8174 ± 0.0081
	$\sigma_8 \Omega_m^{0.5}$	0.4625 ± 0.0091
	$\sigma_8 \Omega_m^{0.25}$	0.6148 ± 0.0086
	z_{re}	8.24 ± 0.88
	$10^9 A_s e^{-2\tau}$	1.886 ± 0.012
	Age/Gyr	13.826 ± 0.025

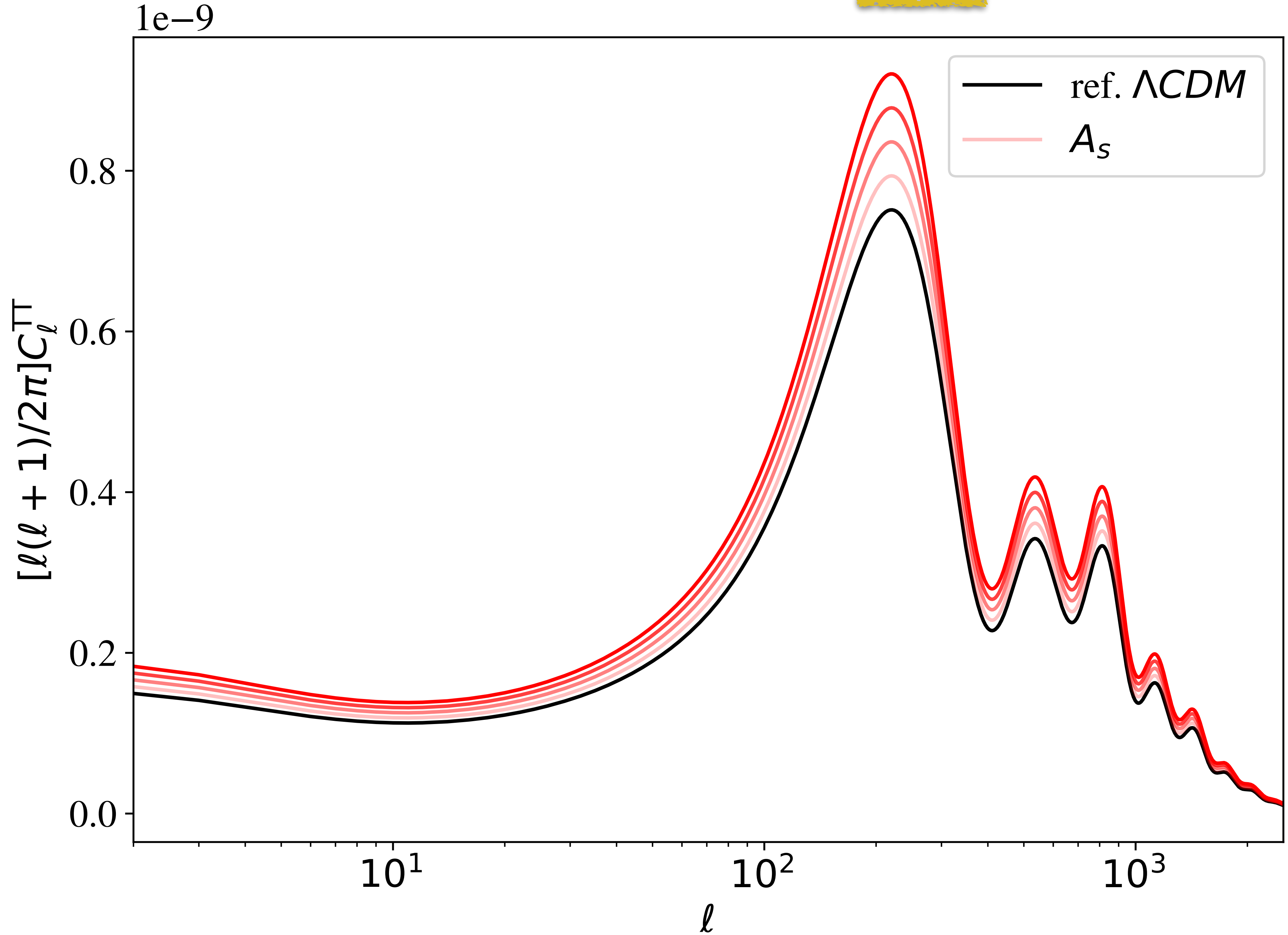
interesting to compare to
LSS data
(RSD, P(k), cosmic shear,
cluster mass function,
Lyman-alpha, …)

Planck 2015 XIII Parameters



Why can we measure 6 ΛCDM parameters independently with CMB?

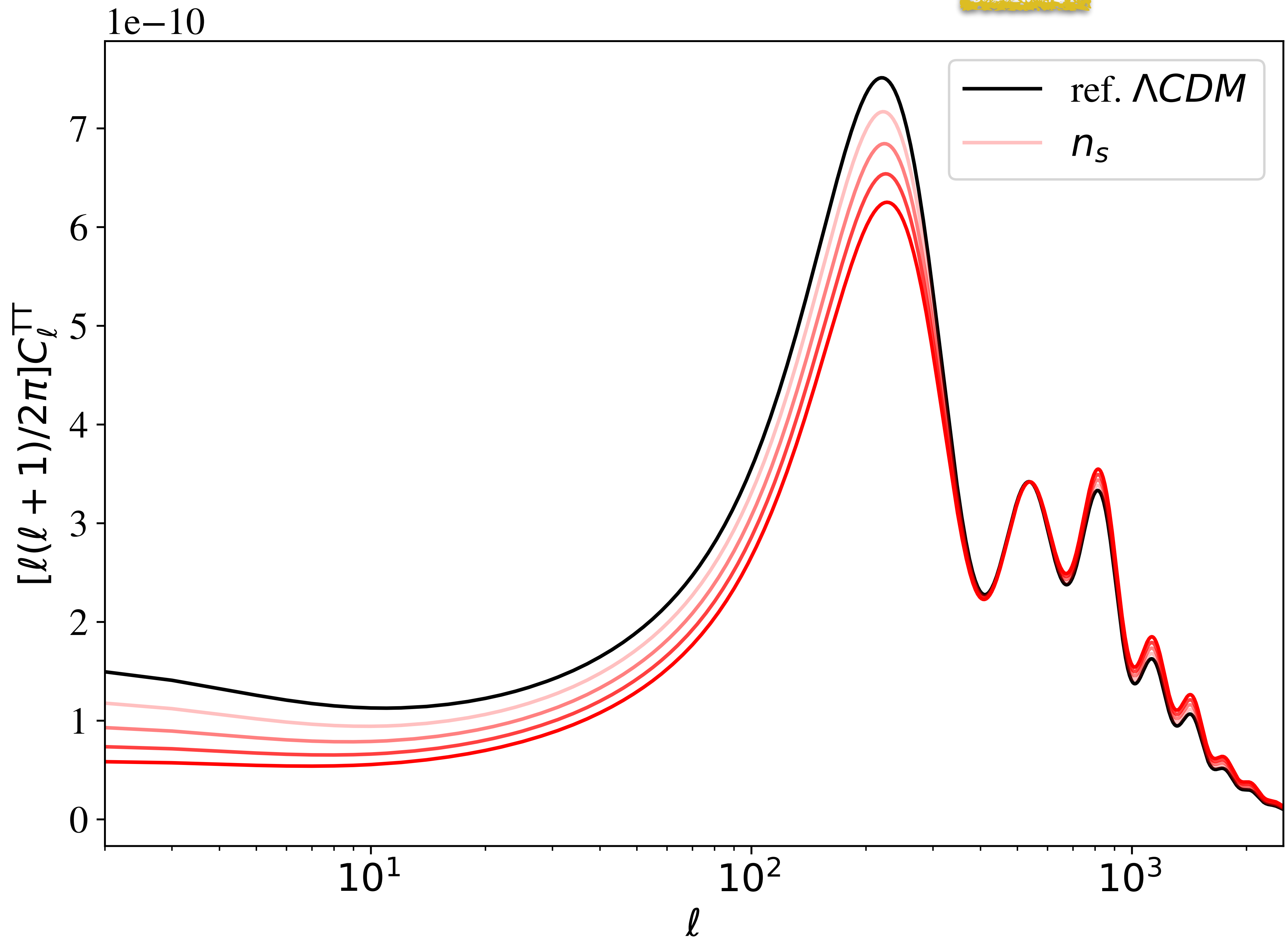
$$\{\omega_b, \omega_m, \Omega_\Lambda, \tau_{\rm reio}, \boxed{A_s}, n_s\}$$



$$C_l^{XY} = 4\pi \int dk\ k^2 \Delta_l^X(k) \Delta_l^Y(k) \mathcal{P}_\mathcal{R}(k) \qquad \mathcal{P}_\mathcal{R}(k) = A_s(k_*) \left(\frac{k}{k_*}\right)^{n_s - 1}$$

Institute for Theoretical Particle Physics and Cosmology
RWTH AACHEN UNIVERSITY

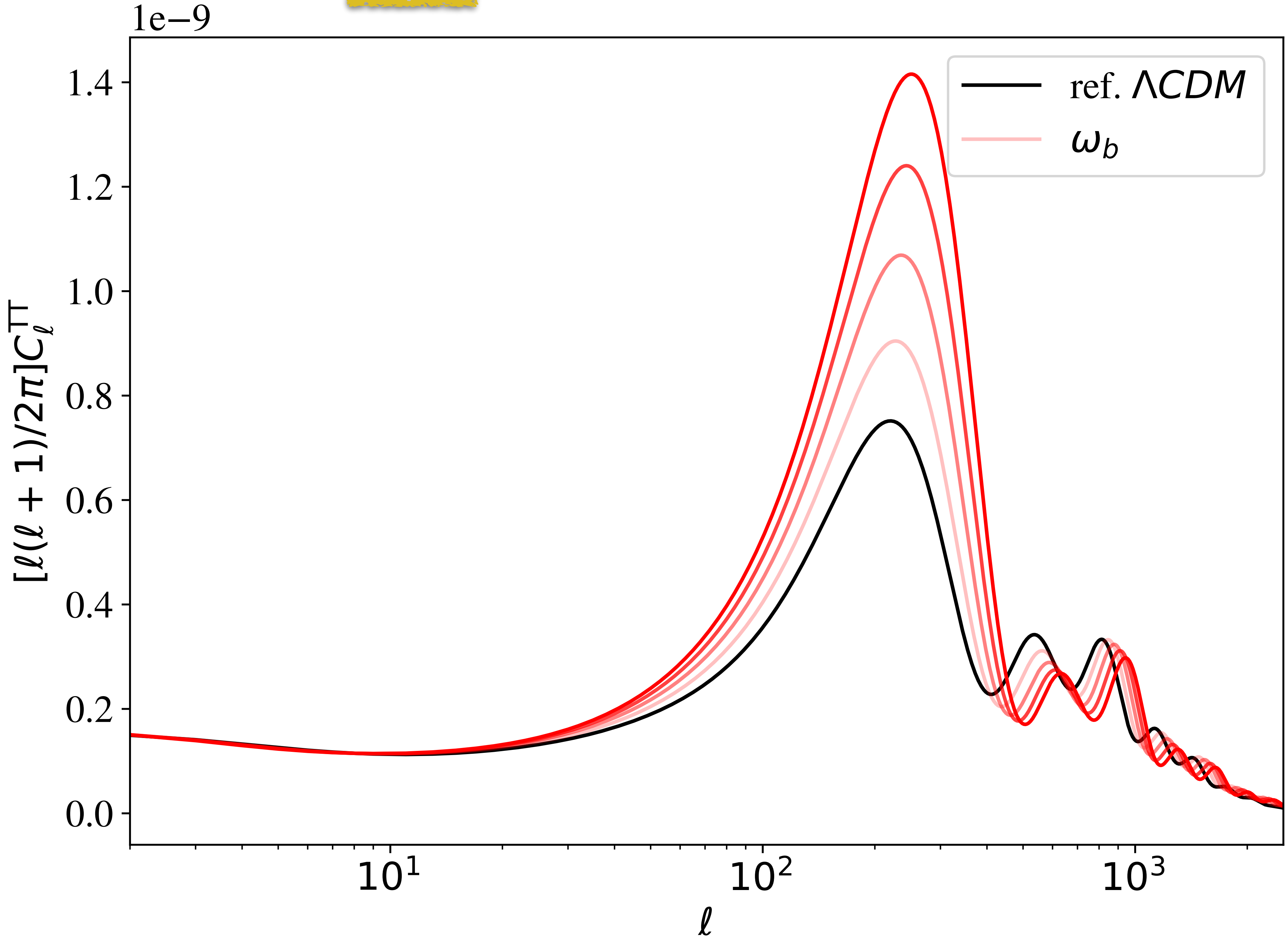
$$\{\omega_b, \omega_m, \Omega_\Lambda, \tau_{\text{reio}}, A_s, \boxed{n_s}\}$$



$$C_l^{XY} = 4\pi \int dk\ k^2 \Delta_l^X(k) \Delta_l^Y(k) \mathcal{P}_\mathcal{R}(k) \qquad \mathcal{P}_\mathcal{R}(k) = A_s(k_*) \left(\frac{k}{k_*}\right)^{n_s - 1}$$

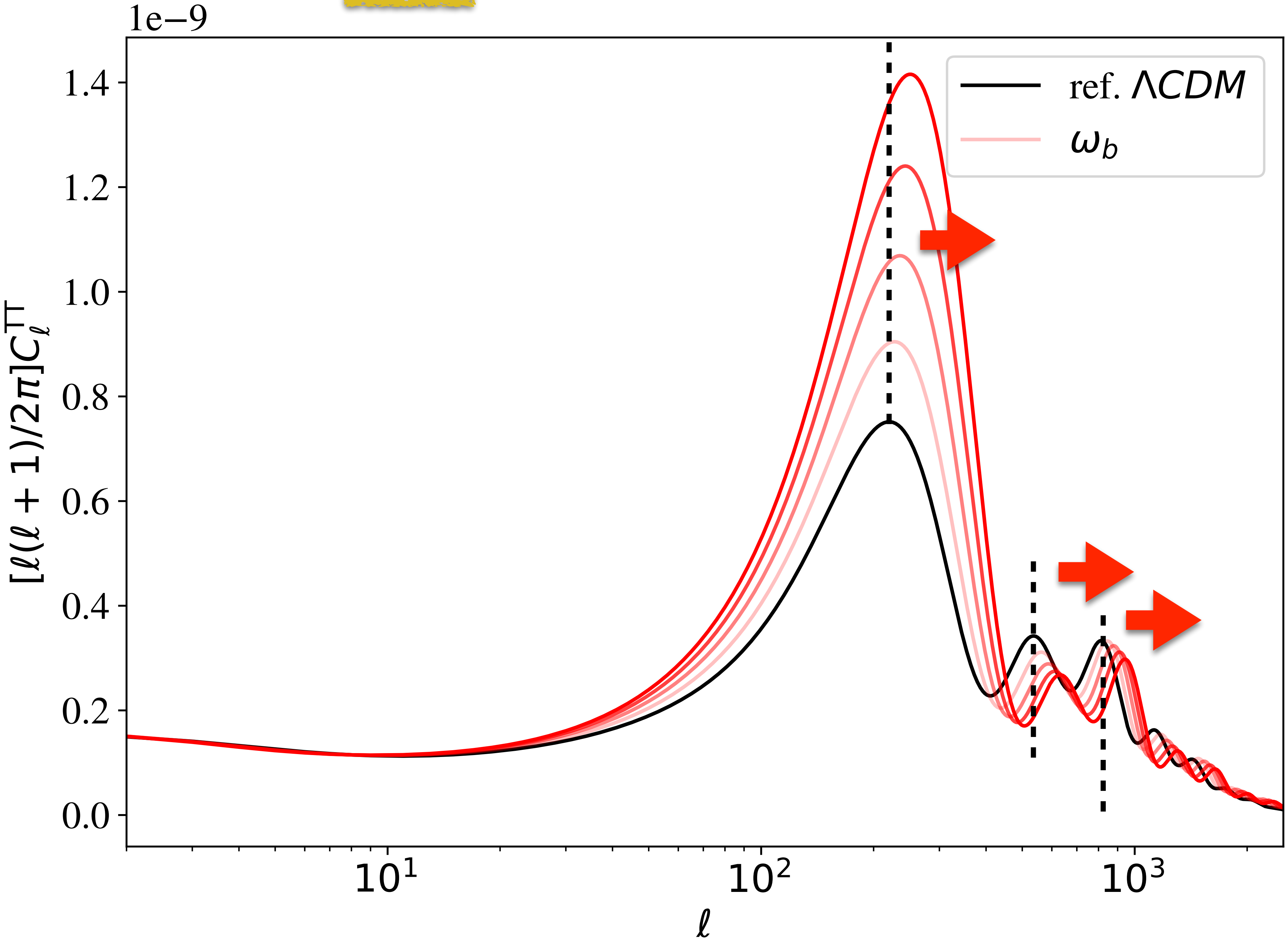
TTK Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

$$\{\boxed{\omega_b,}\,\omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$

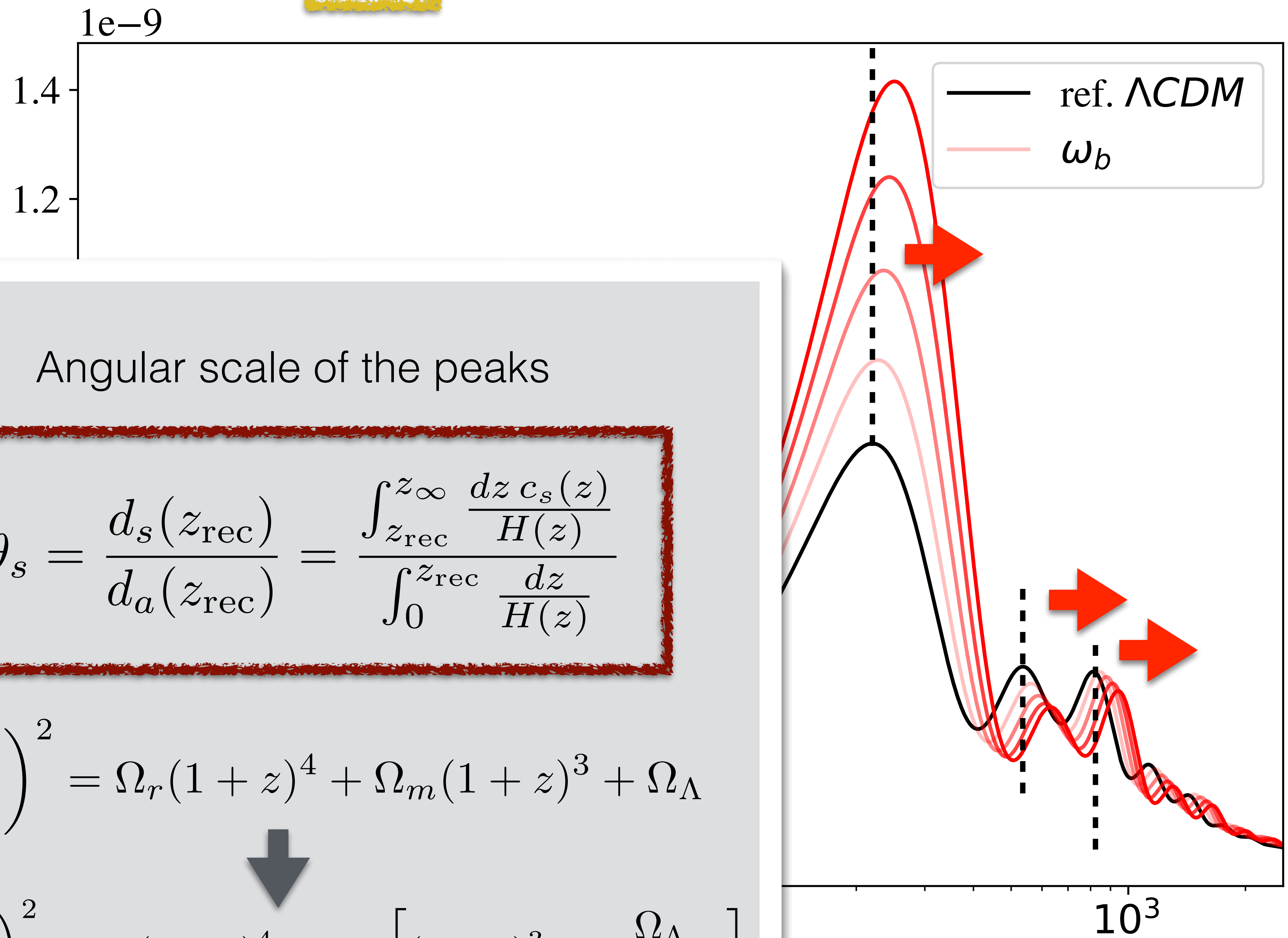


Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

$$\{\boxed{\omega_b,}\,\omega_m, \Omega_\Lambda, \tau_{\mathrm{reio}}, A_s, n_s\}$$



$$\{\omega_b, \omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



Angular scale of the peaks

$$\boxed{\theta_s = \frac{d_s(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{\int_{z_{\rm rec}}^{z_\infty} \frac{dz\, c_s(z)}{H(z)}}{\int_0^{z_{\rm rec}} \frac{dz}{H(z)}}}$$

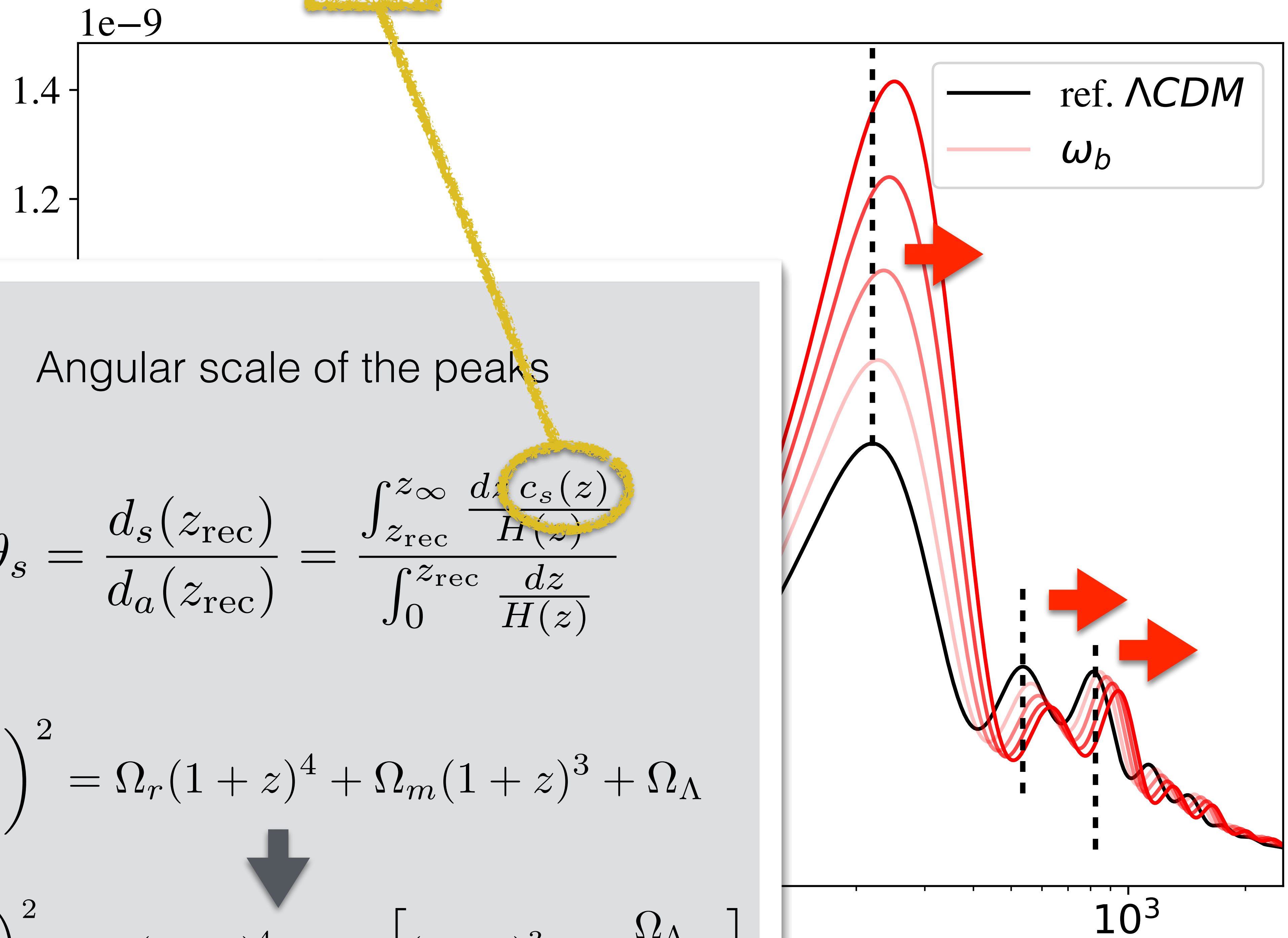
$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda$$

$$\Downarrow$$

$$\left(\frac{H(z)h}{H_0}\right)^2 = \omega_r(1+z)^4 + \omega_m\left[(1+z)^3 + \frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right]$$

$z_{\rm rec}$, $T_{\rm rec}$ fixed by recombination eqs.
(up to negligible $\log(\omega_b)$ dependence)

$$\{\boxed{\omega_b}, \omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



Angular scale of the peaks

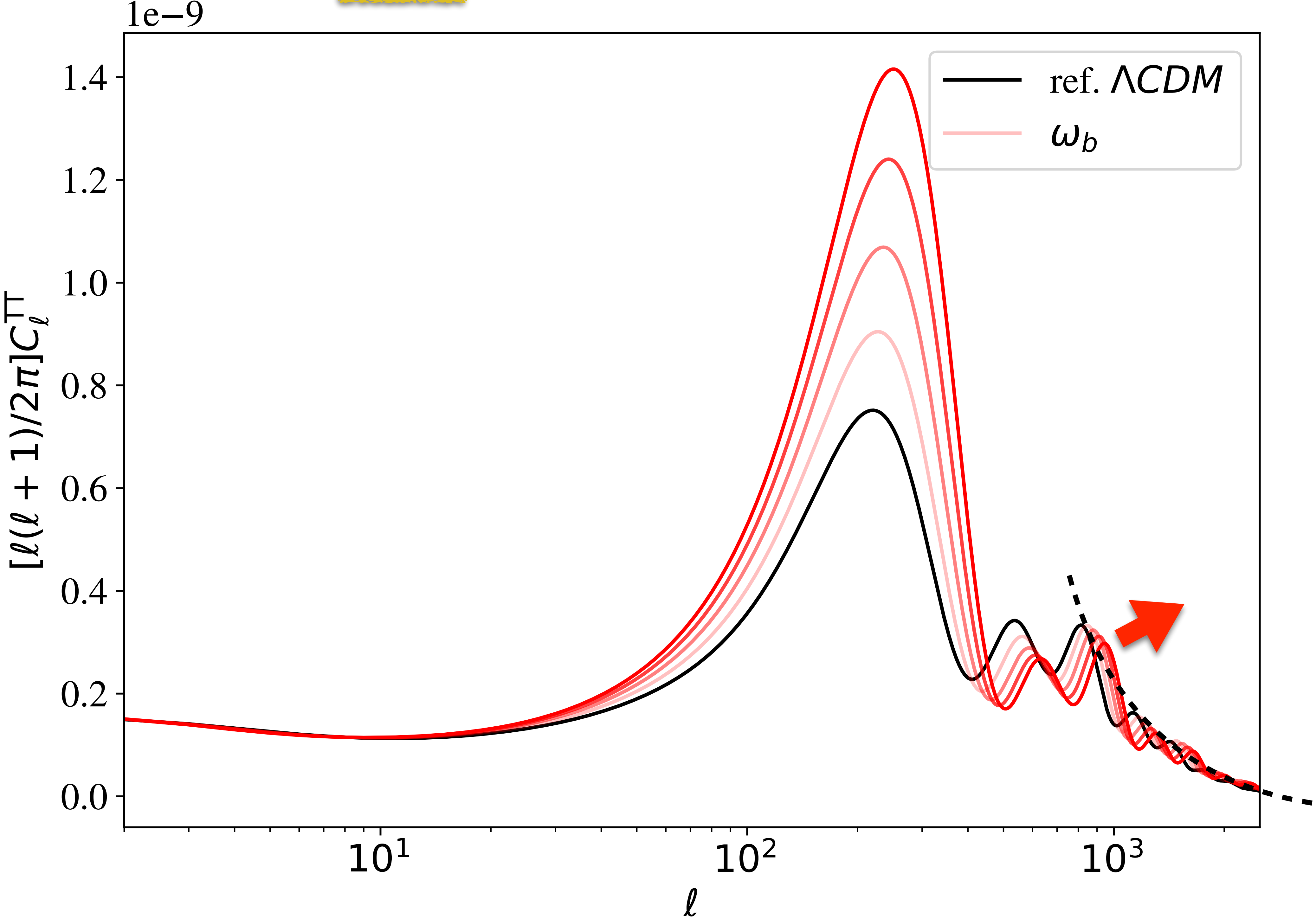
$$\theta_s = \frac{d_s(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{\int_{z_{\rm rec}}^{z_\infty} \frac{dz\, c_s(z)}{H(z)}}{\int_0^{z_{\rm rec}} \frac{dz}{H(z)}}$$

$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda$$

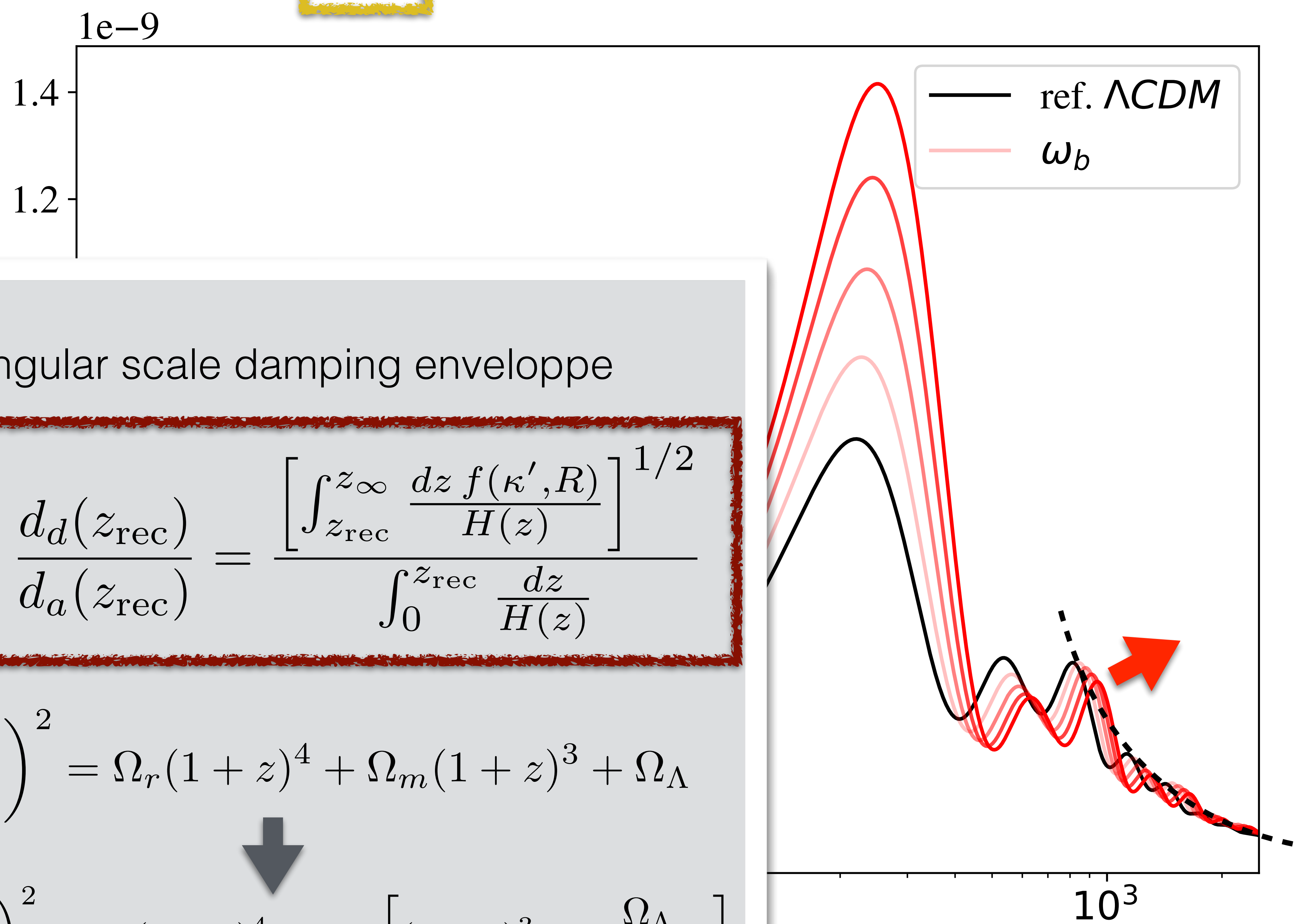
$$\Downarrow$$

$$\left(\frac{H(z)h}{H_0}\right)^2 = \omega_r(1+z)^4 + \omega_m\left[(1+z)^3 + \frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right]$$

$$\{\boxed{\omega_b,}\,\omega_m,\Omega_\Lambda,\tau_{\rm reio},A_s,n_s\}$$



$$\{\boxed{\omega_b,}\,\omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



Angular scale damping enveloppe

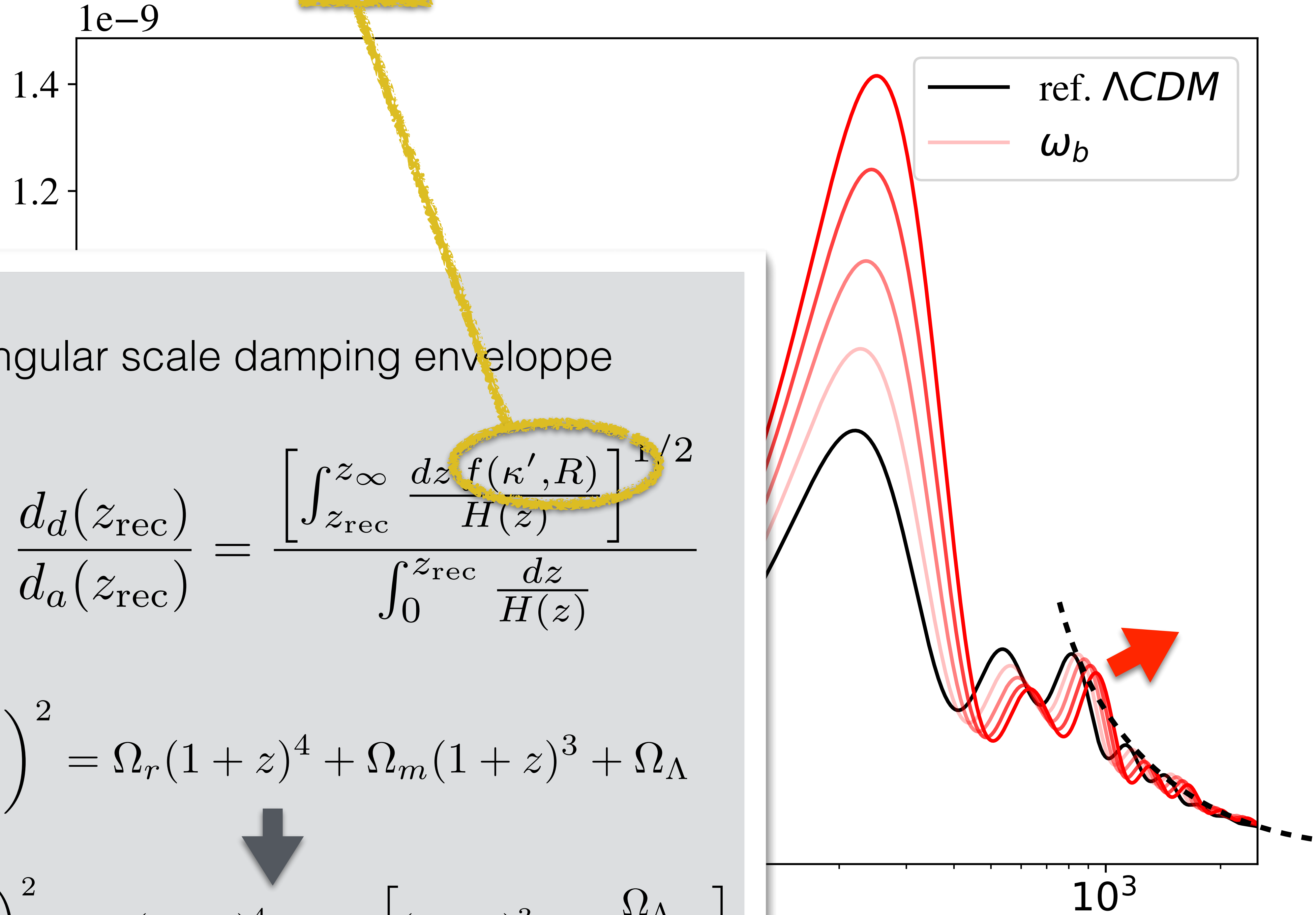
$$\boxed{\theta_d = \frac{d_d(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{\left[\int_{z_{\rm rec}}^{z_\infty} \frac{dz\, f(\kappa', R)}{H(z)}\right]^{1/2}}{\int_0^{z_{\rm rec}} \frac{dz}{H(z)}}}$$

$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda$$

$$\Downarrow$$

$$\left(\frac{H(z)h}{H_0}\right)^2 = \omega_r(1+z)^4 + \omega_m\left[(1+z)^3 + \frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right]$$

$$\{\omega_b, \omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



Angular scale damping enveloppe

$$\theta_d = \frac{d_d(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{\left[\int_{z_{\rm rec}}^{z_\infty} \frac{dz\, f(\kappa', R)}{H(z)}\right]^{1/2}}{\int_0^{z_{\rm rec}} \frac{dz}{H(z)}}$$

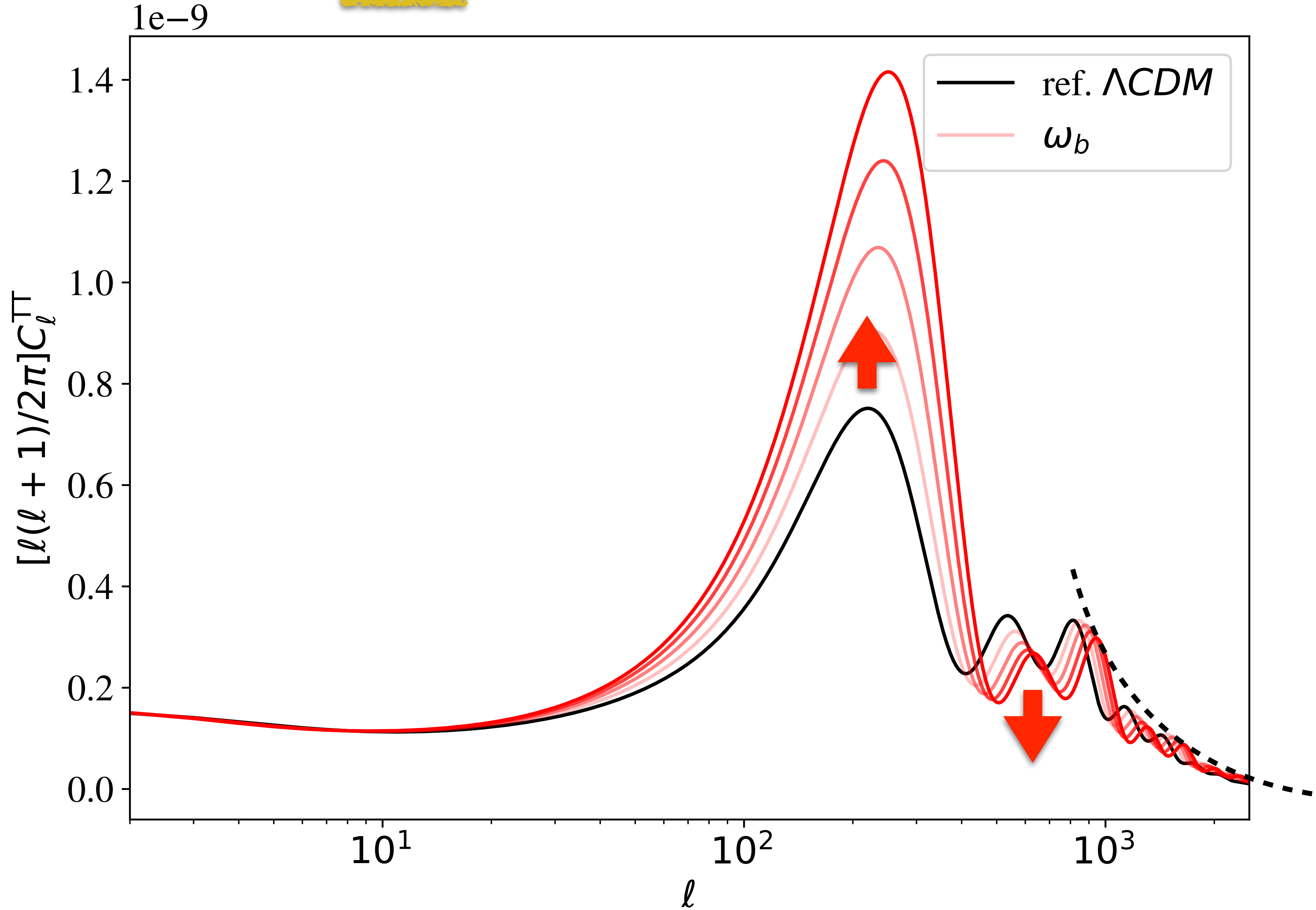
$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda$$

$$\Downarrow$$

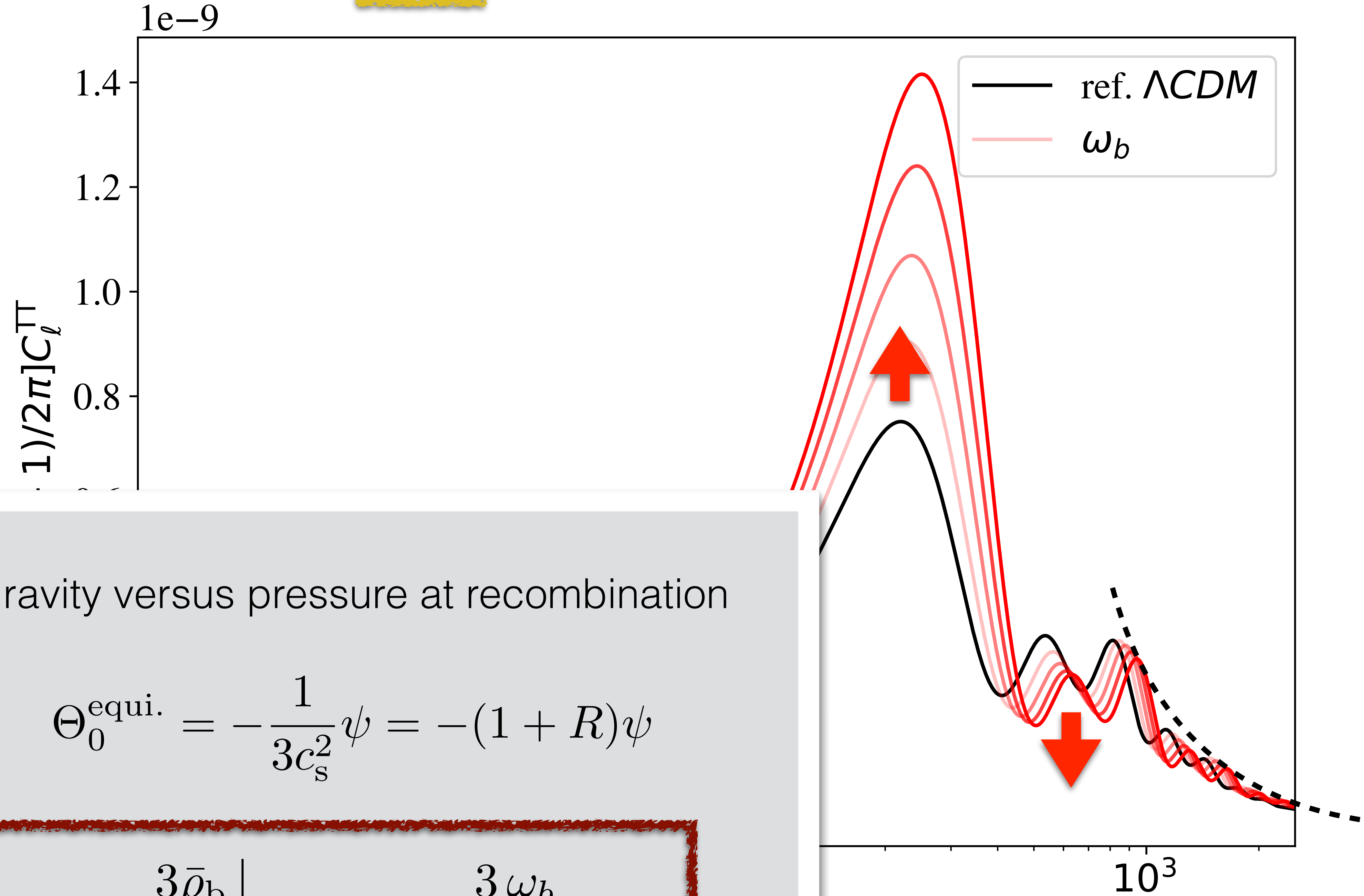
$$\left(\frac{H(z)h}{H_0}\right)^2 = \omega_r(1+z)^4 + \omega_m\left[(1+z)^3 + \frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right]$$

TTK Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

$$\{\boxed{\omega_b,}\, \omega_m, \Omega_\Lambda, \tau_{\mathrm{reio}}, A_s, n_s\}$$



$$\{\omega_b, \omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$

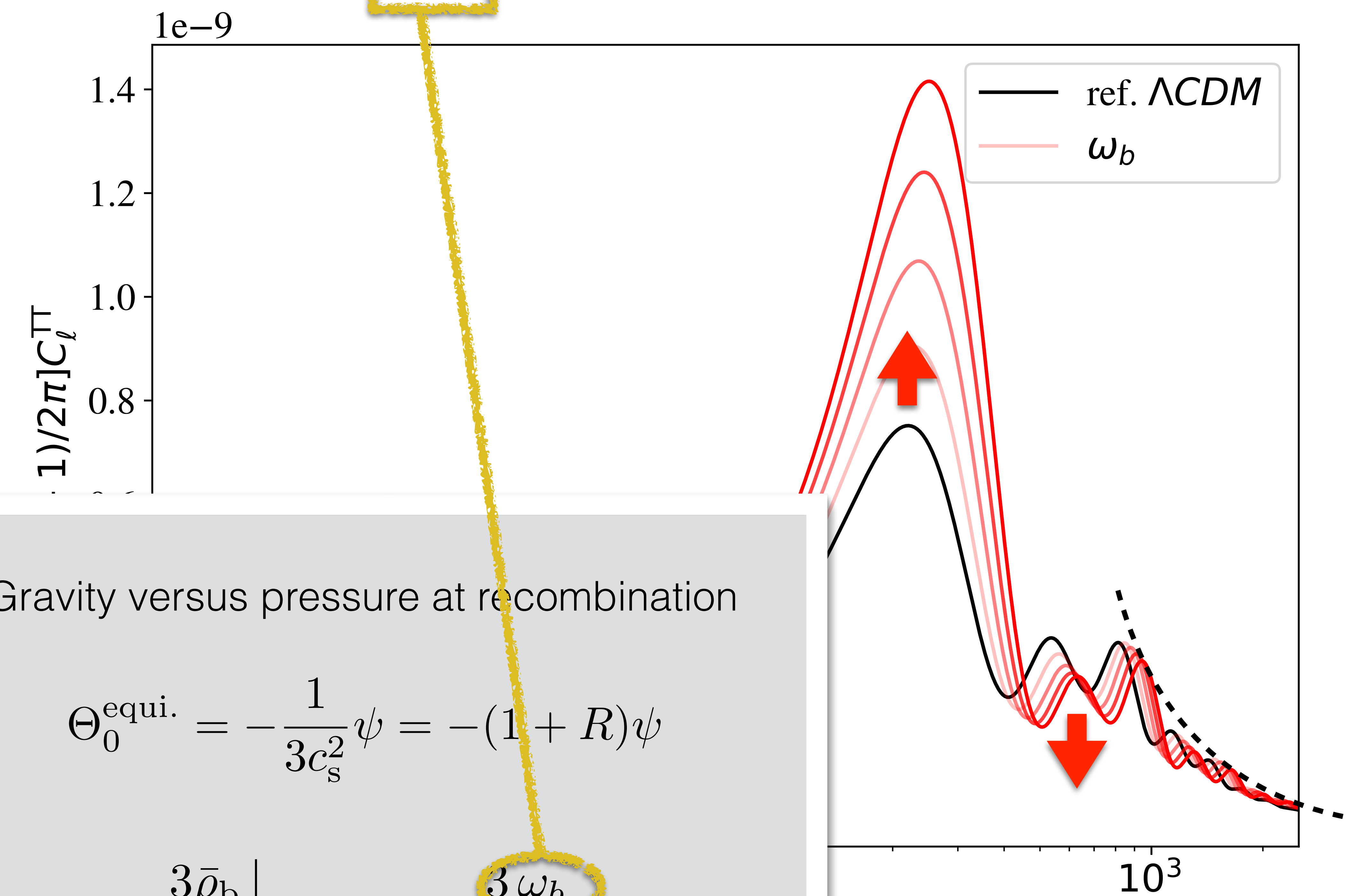


Gravity versus pressure at recombination

$$\Theta_0^{\rm equi.} = -\frac{1}{3c_{\rm s}^2}\psi = -(1+R)\psi$$

$$\boxed{R_{\rm rec} \equiv \left.\frac{3\bar{\rho}_{\rm b}}{4\bar{\rho}_\gamma}\right|_{\rm rec} = \frac{3\,\omega_b}{4\,\omega_\gamma(1+z_{\rm rec})}}$$

$$\{\boxed{\omega_b}, \omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



Gravity versus pressure at recombination

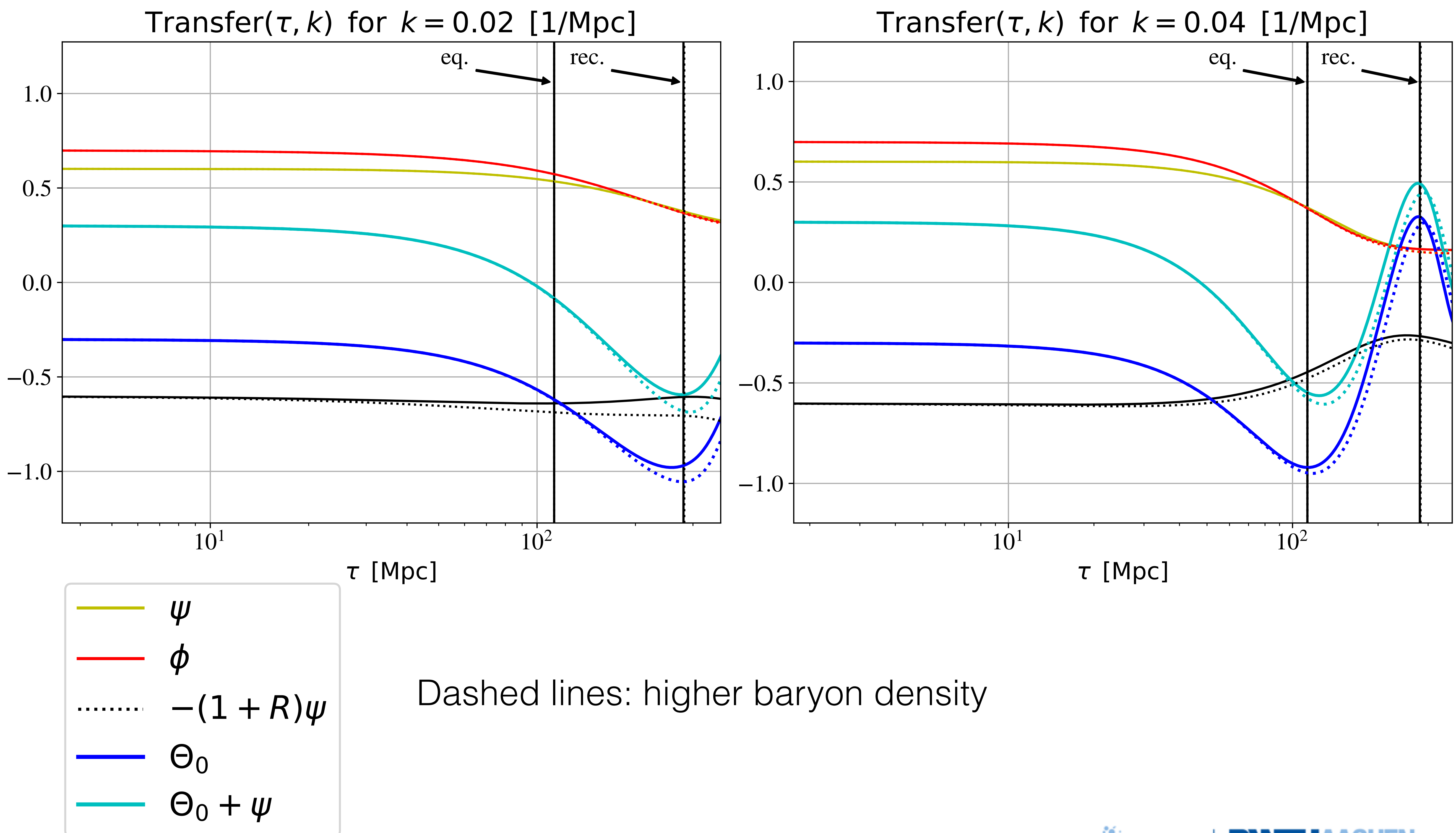
$$\Theta_0^{\rm equi.} = -\frac{1}{3c_{\rm s}^2}\psi = -(1+R)\psi$$

$$R_{\rm rec} \equiv \left.\frac{3\bar{\rho}_{\rm b}}{4\bar{\rho}_\gamma}\right|_{\rm rec} = \frac{3\,\omega_b}{4\,\omega_\gamma(1+z_{\rm rec})}$$

$$\{\boxed{\omega_b,}\,\omega_m,\Omega_\Lambda,\tau_{\rm reio},A_s,n_s\}$$

1st peak scale

2nd peak scale

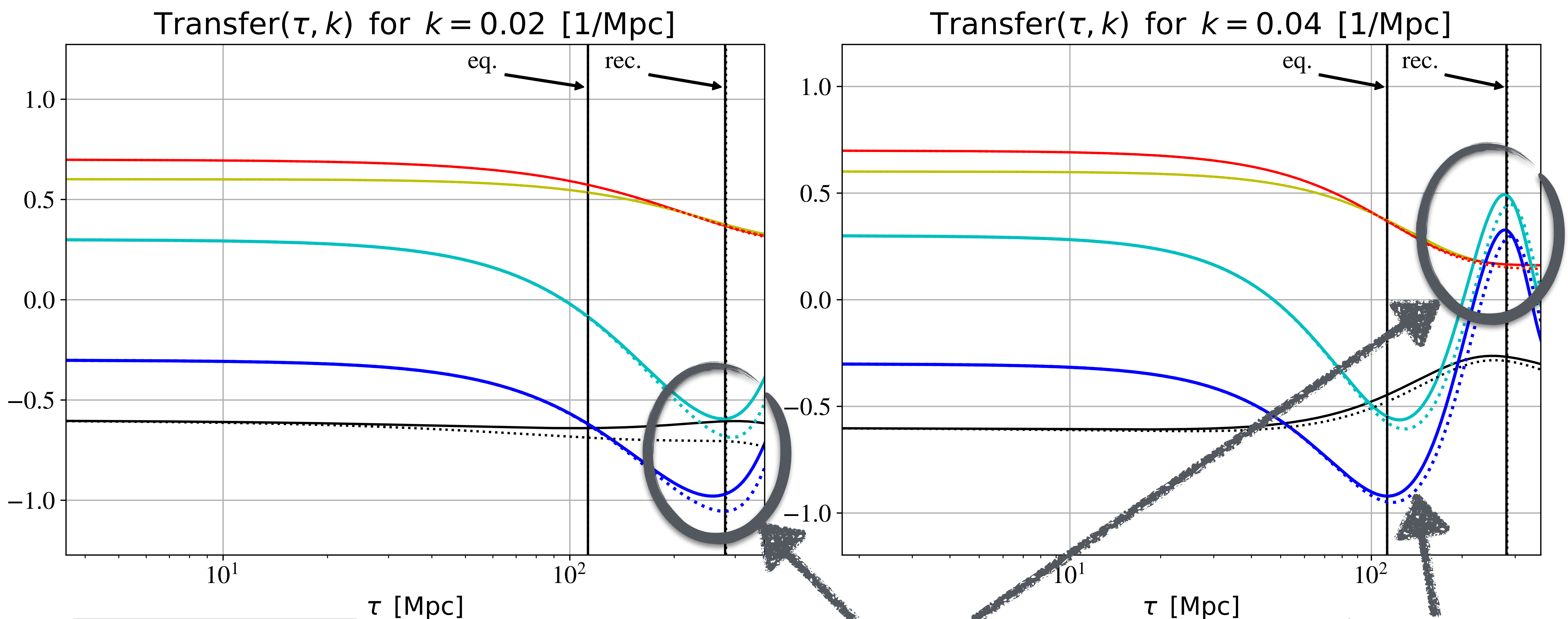


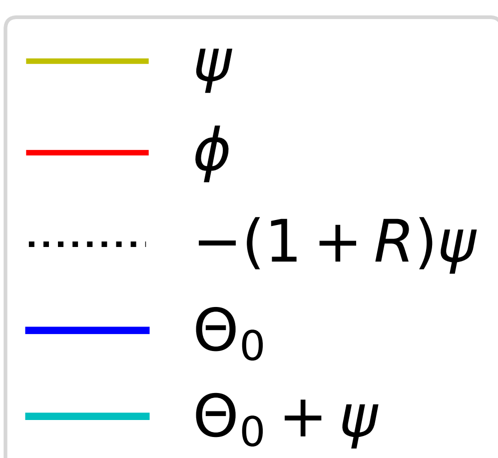
Dashed lines: higher baryon density

$$\{\boxed{\omega_b,}\,\omega_m,\Omega_\Lambda,\tau_{\rm reio},A_s,n_s\}$$

1st peak scale

2nd peak scale



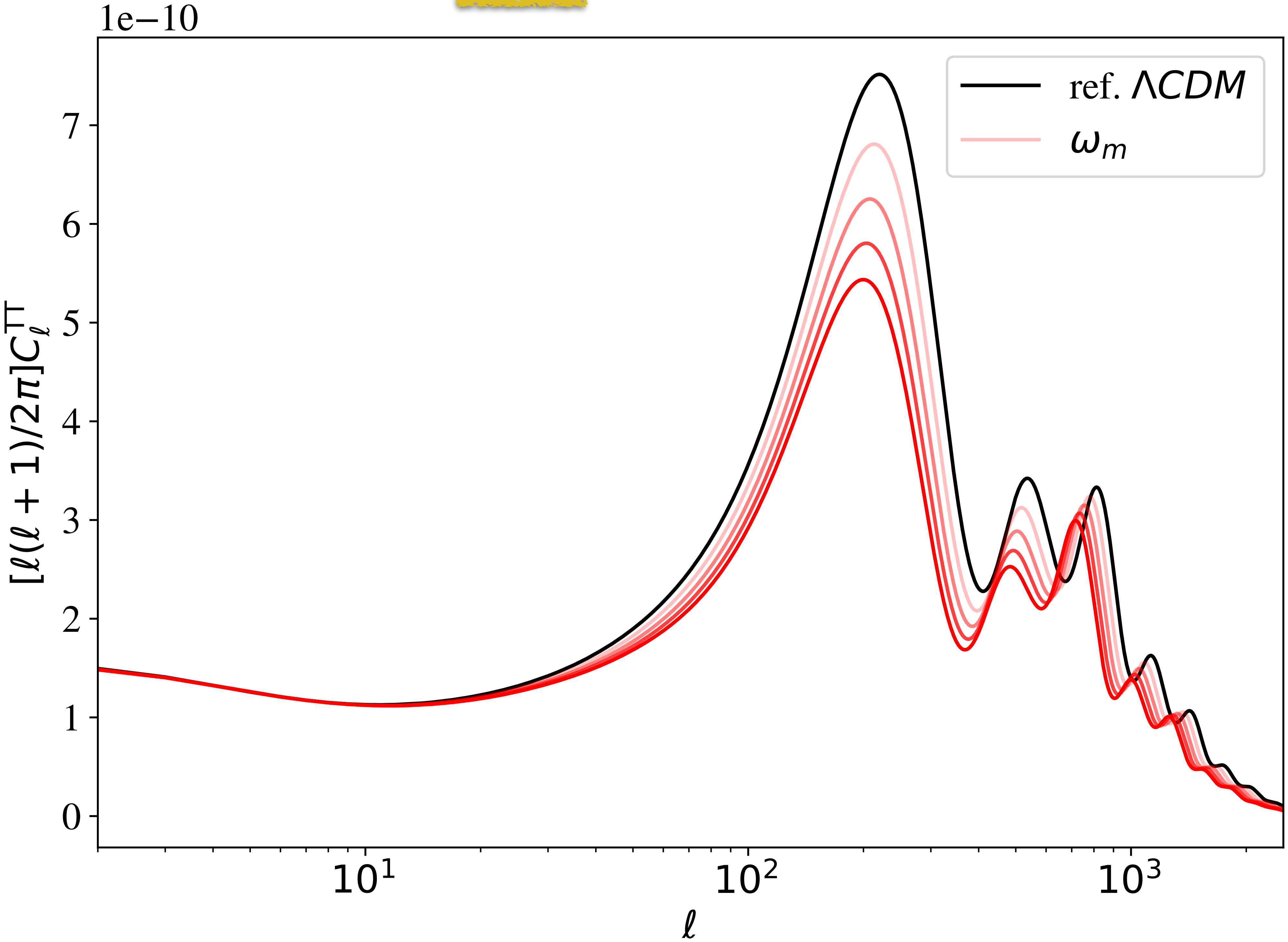


(Phase shift due to sound speed)

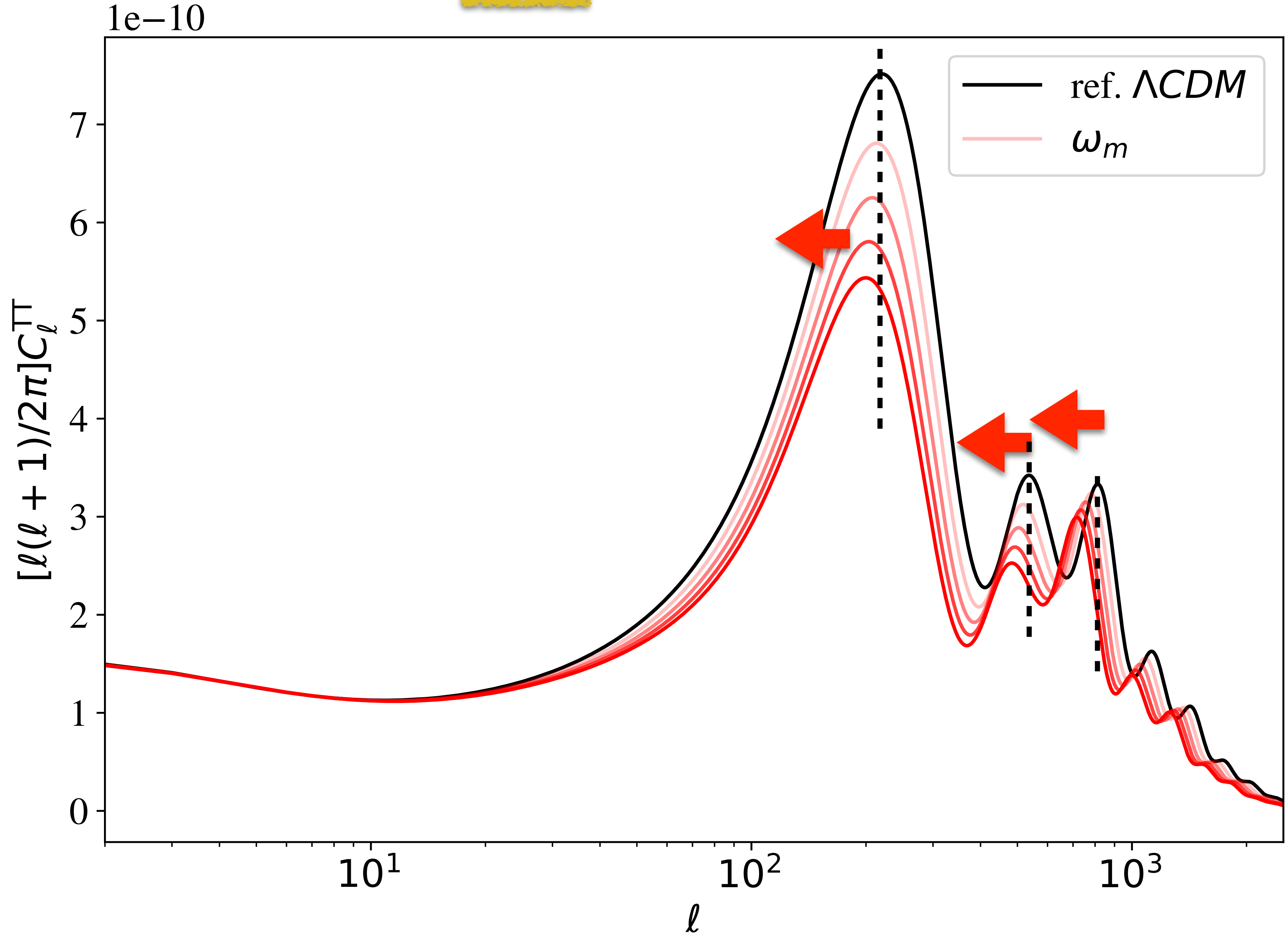
Less pressure for the same gravity, odd peaks enhanced, even peaks reduced

$$\Theta_0^{\rm equi.} = -\frac{1}{3c_s^2}\psi = -(1+R)\psi$$

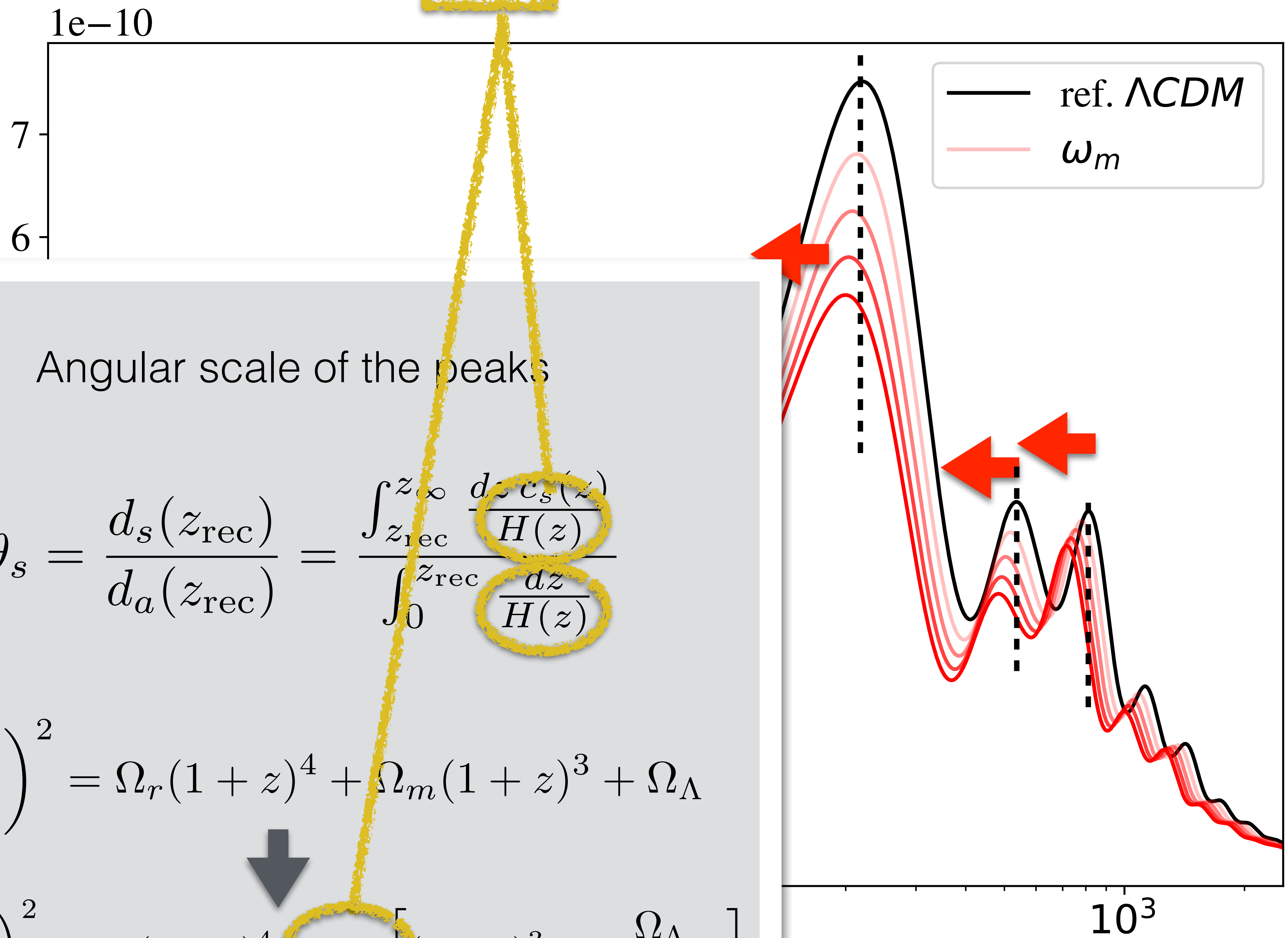
$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



$$\{\omega_b, \omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



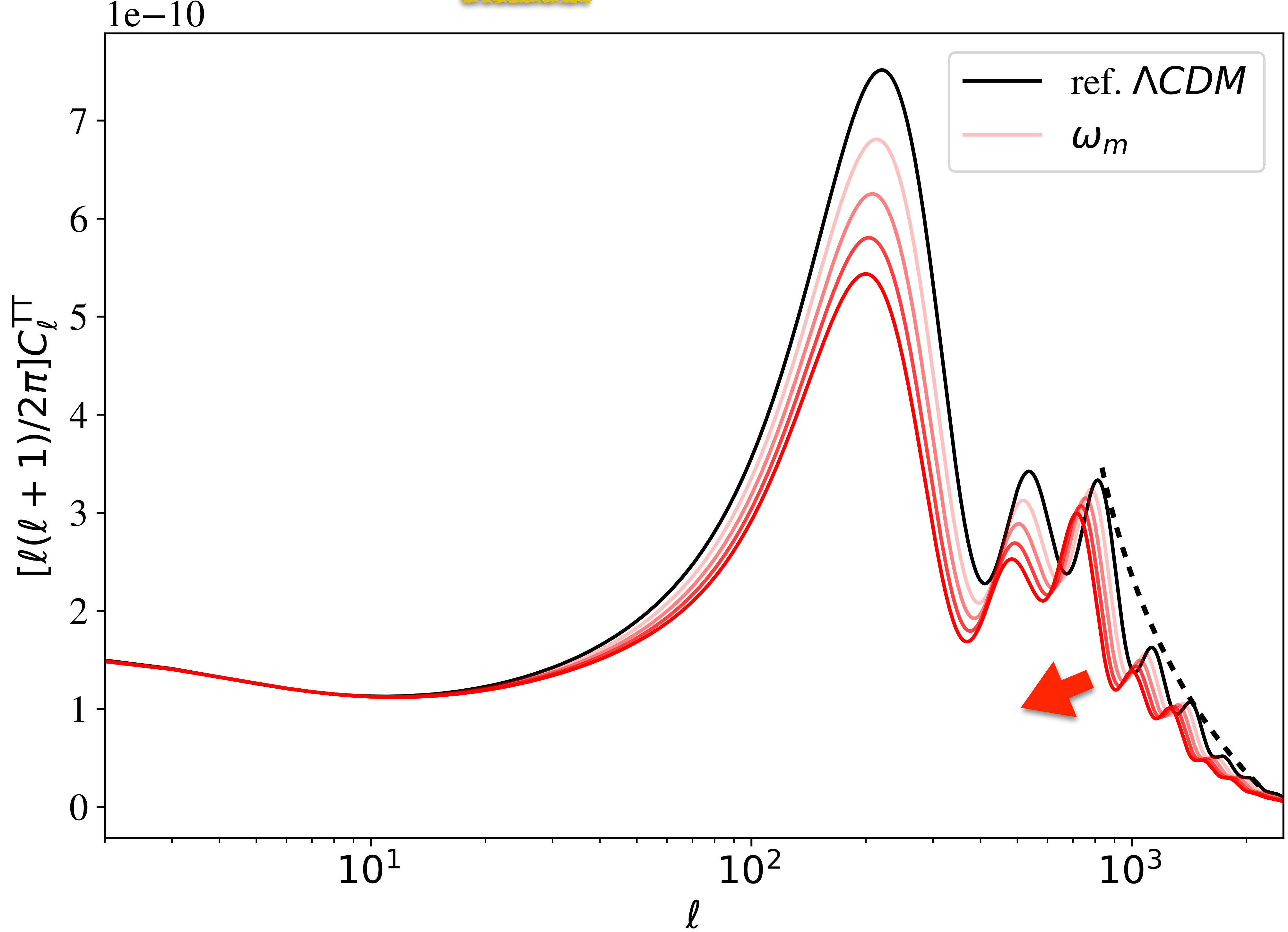
Angular scale of the peaks

$$\theta_s = \frac{d_s(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{\int_{z_{\rm rec}}^{z_\infty} dz \frac{c_s(z)}{H(z)}}{\int_0^{z_{\rm rec}} \frac{dz}{H(z)}}$$

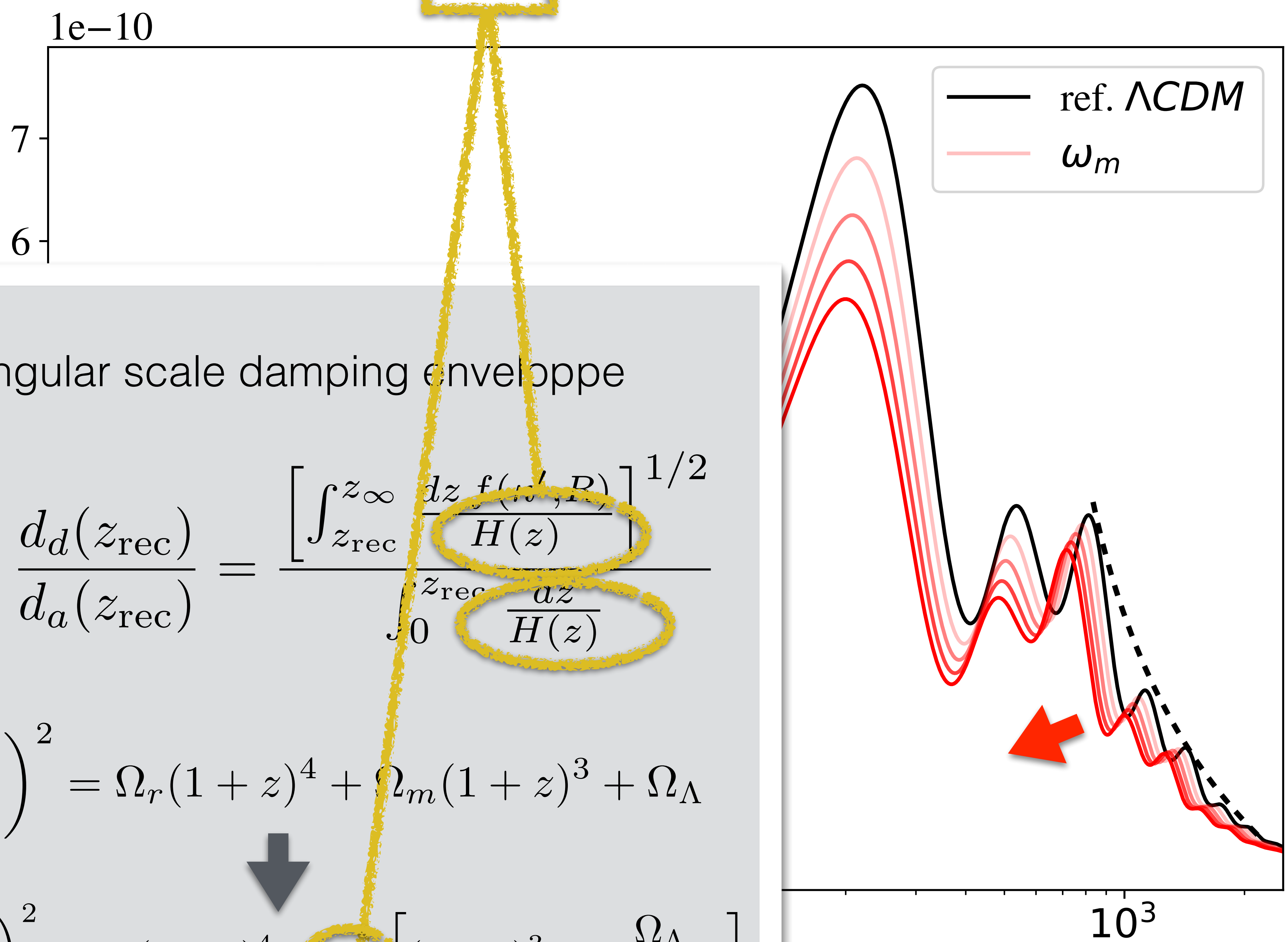
$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda$$

$$\left(\frac{H(z)h}{H_0}\right)^2 = \omega_r(1+z)^4 + \omega_m\left[(1+z)^3 + \frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right]$$

$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



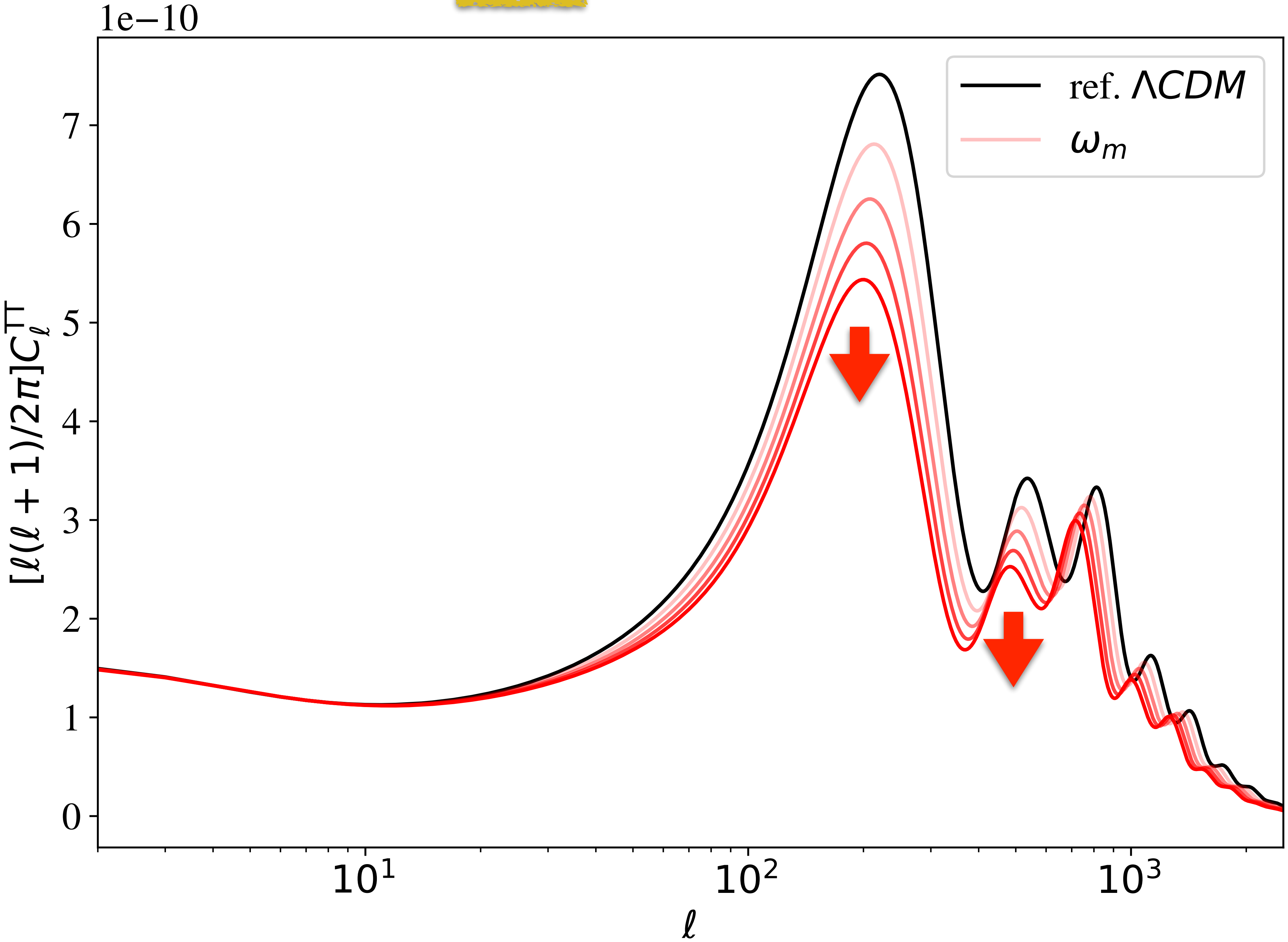
Angular scale damping enveloppe

$$\theta_d = \frac{d_d(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{\left[\int_{z_{\rm rec}}^{z_\infty} \frac{dz\, f(\omega', R)}{H(z)}\right]^{1/2}}{\int_0^{z_{\rm rec}} \frac{dz}{H(z)}}$$

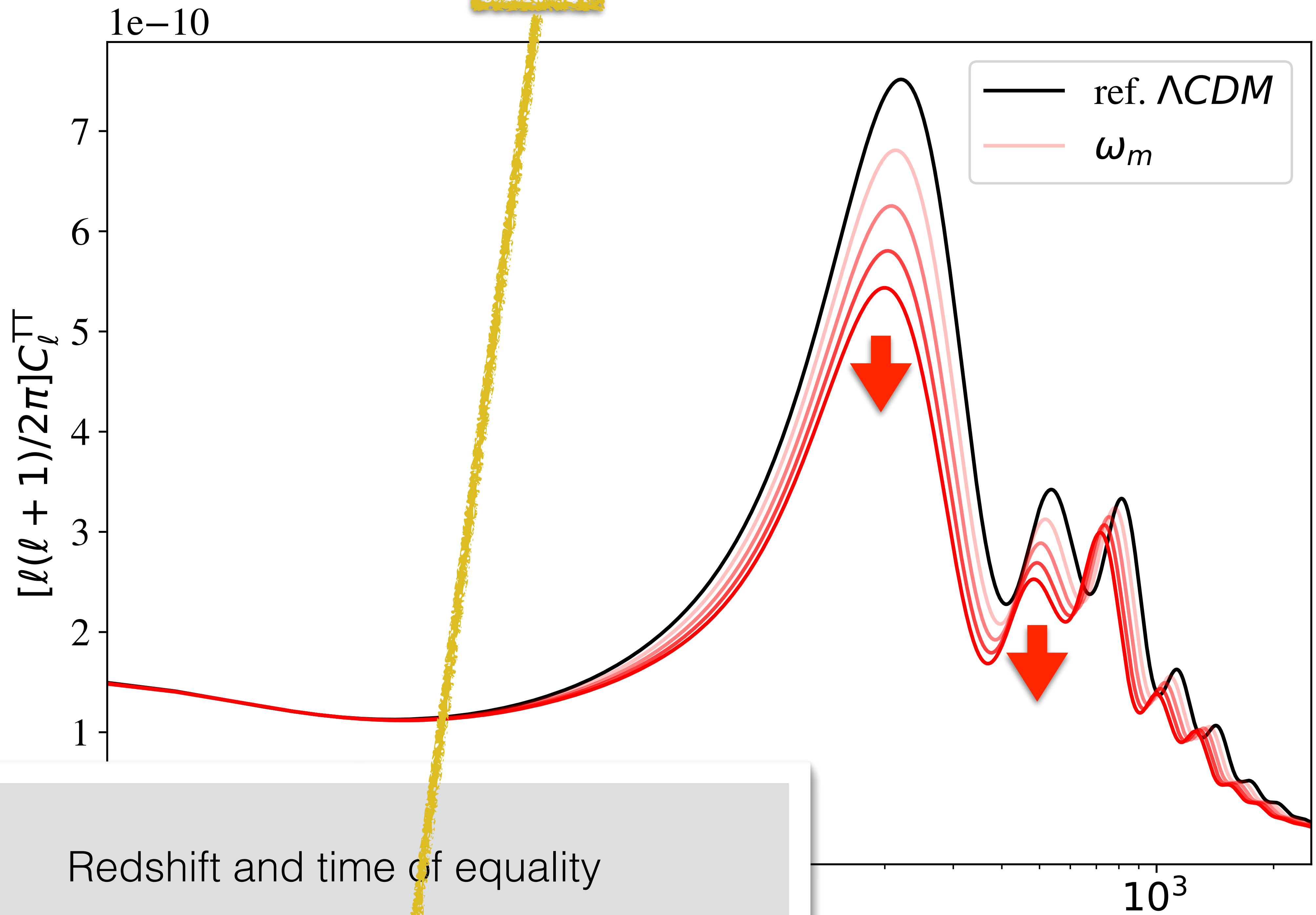
$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda$$

$$\left(\frac{H(z)h}{H_0}\right)^2 = \omega_r(1+z)^4 + \omega_m\left[(1+z)^3 + \frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right]$$

$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



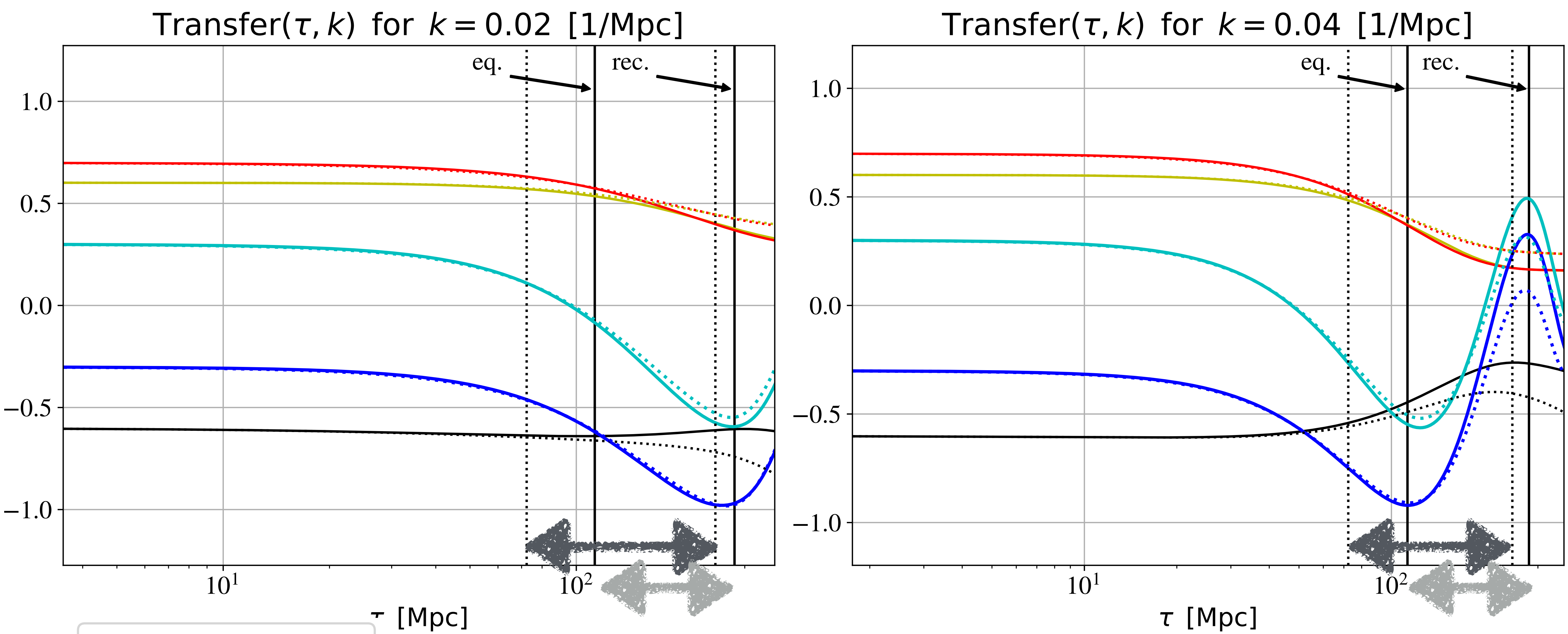
Redshift and time of equality

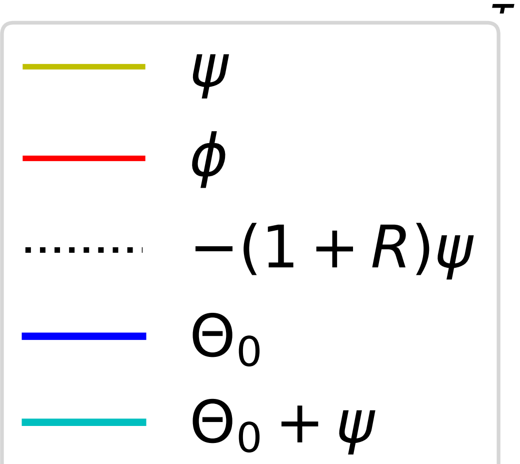
$$1 + z_{\rm rec} = \omega_m / \omega_r$$

$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$

1st peak scale

2nd peak scale





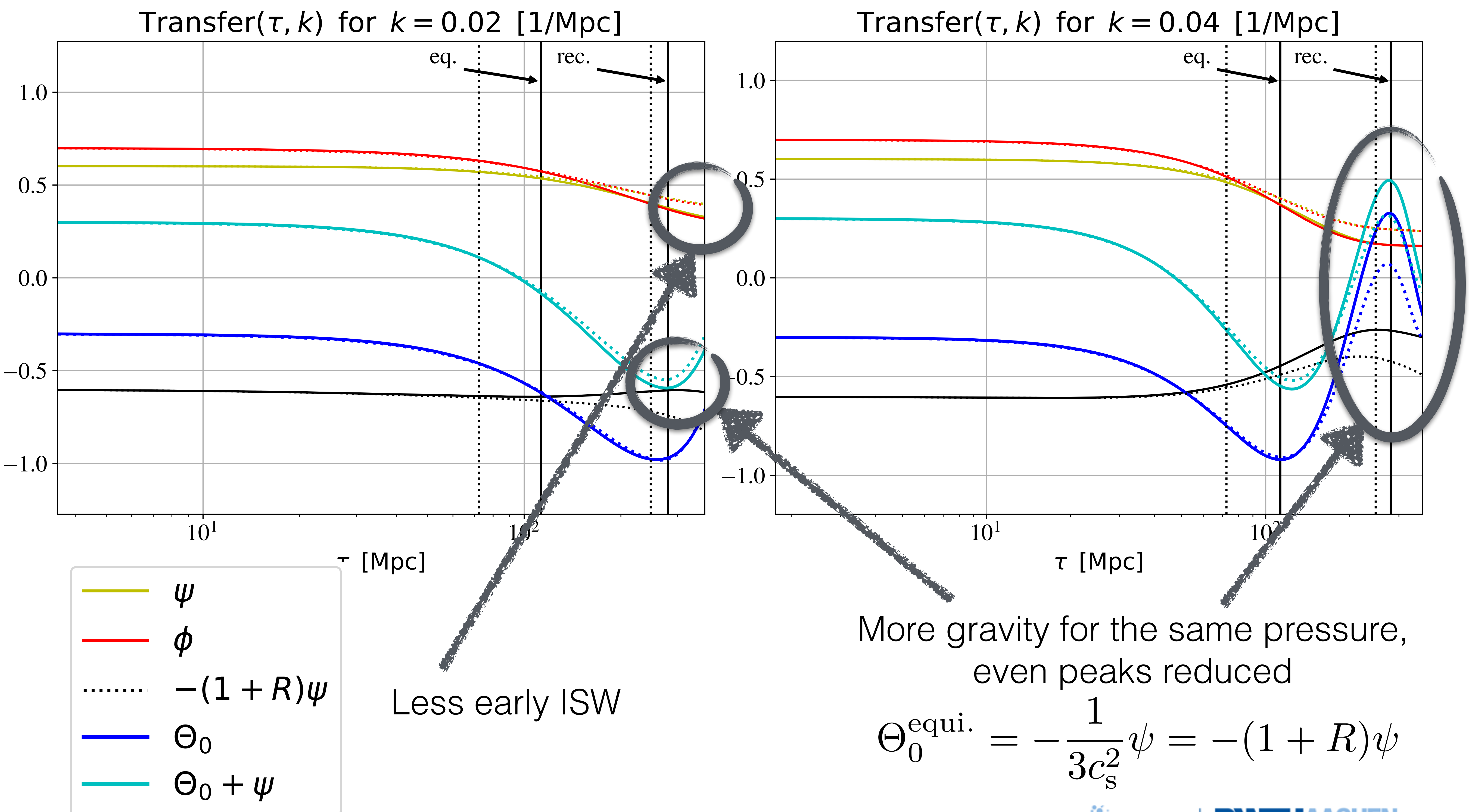
—> z_{rec} nearly fixed by T_{rec} but τ_{rec} shifts slightly due to expansion rate prior to recombination. R_{rec} is fixed. Most important change: (τ_{eq} - τ_{rec})

TTK Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$

1st peak scale

2nd peak scale

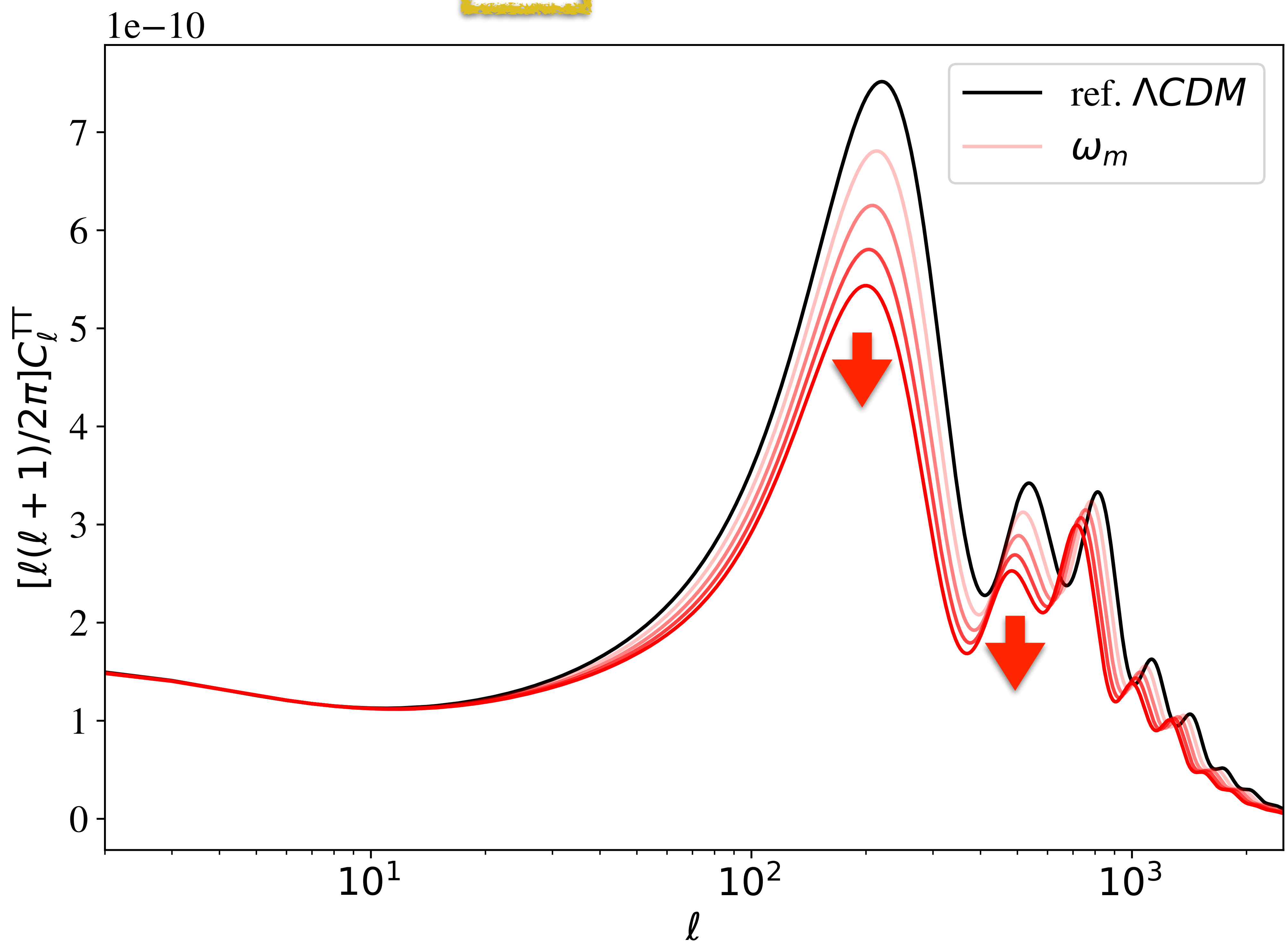


Less early ISW

More gravity for the same pressure, even peaks reduced

$$\Theta_0^{\rm equi.} = -\frac{1}{3c_{\rm s}^2}\psi = -(1+R)\psi$$

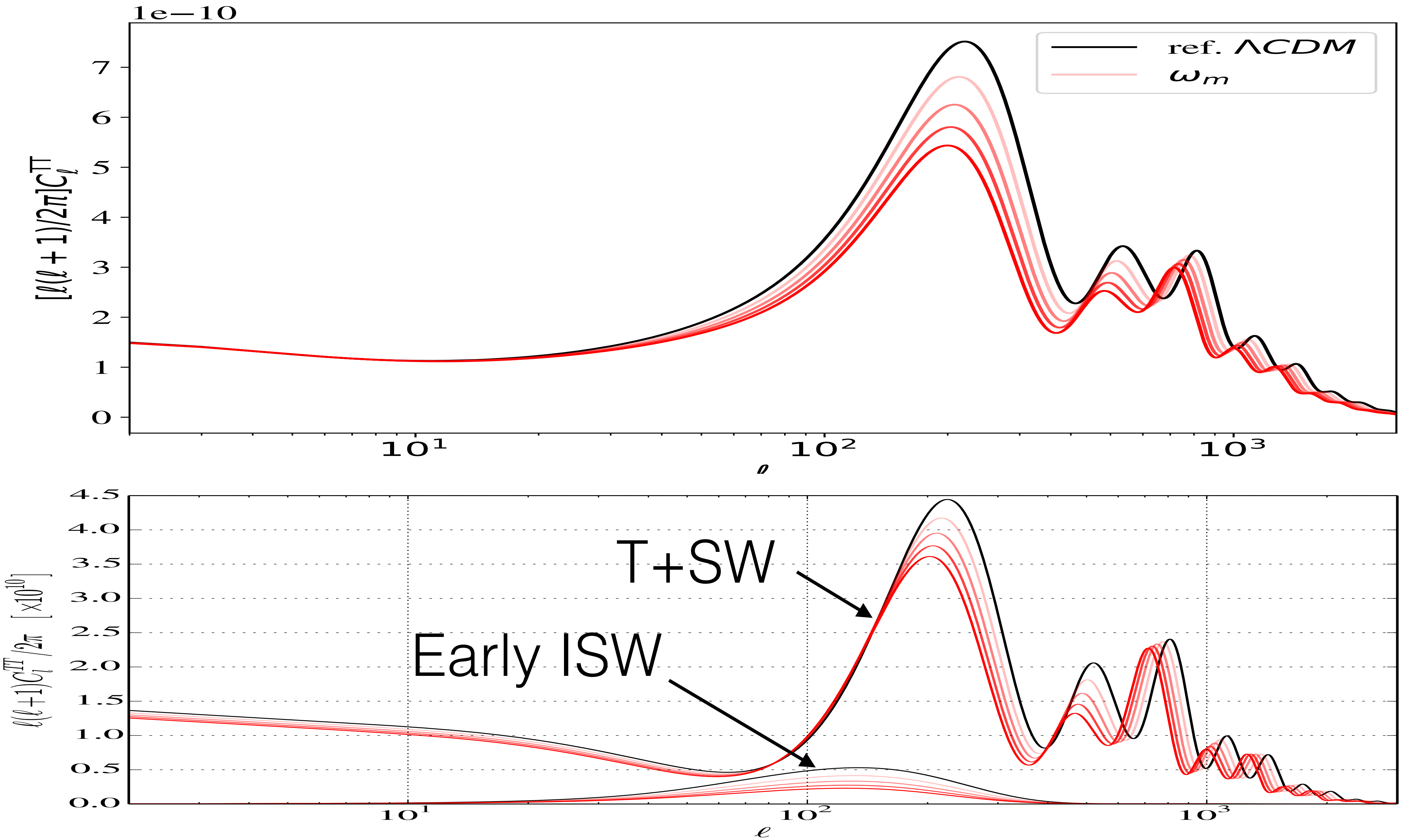
$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



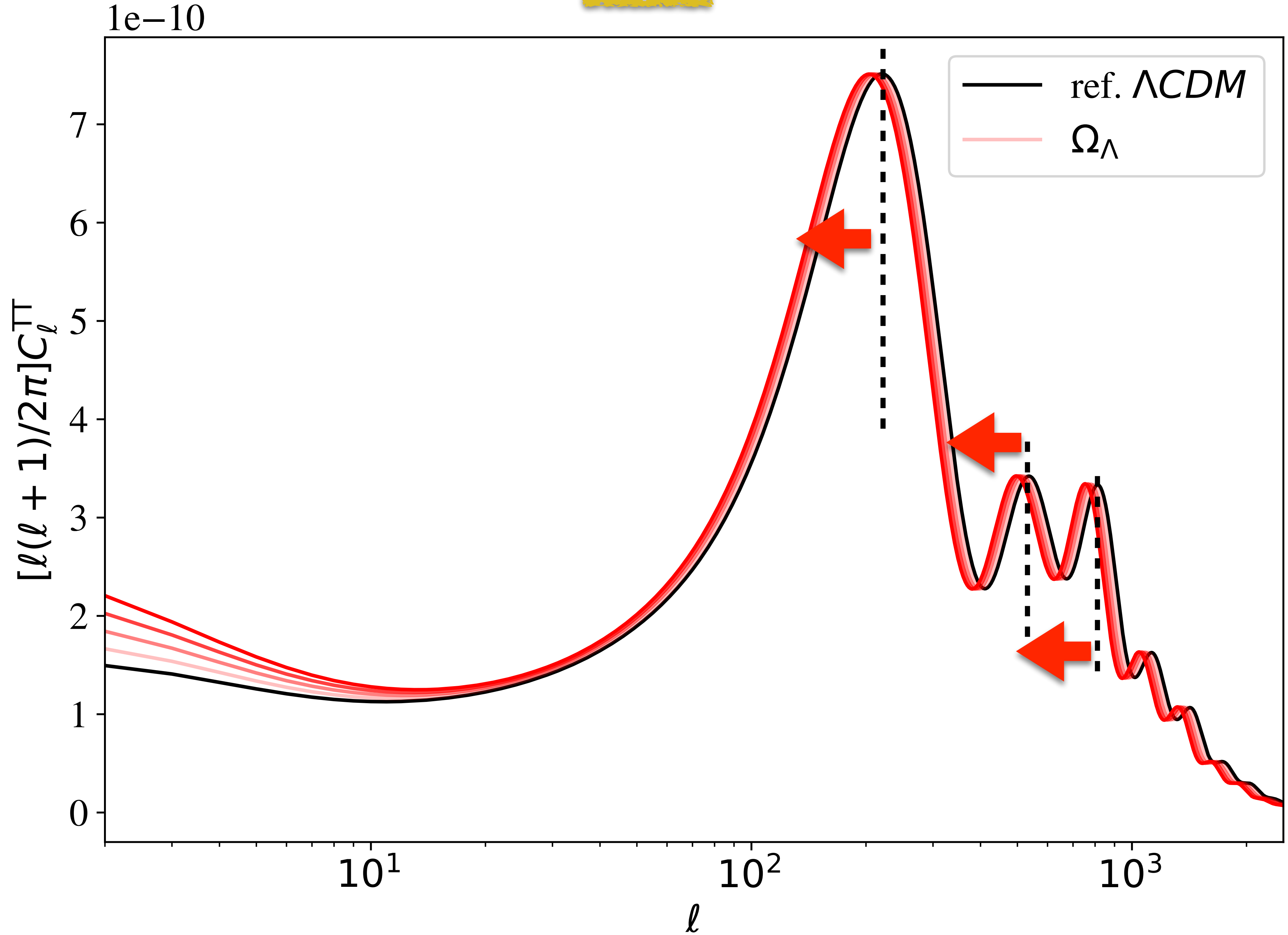
Exercise: add to this plots the individual effect of the matter density increase on the eISW and TSW components, to see which one dominates and in which range.

$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$

Exercise result (thank you Renan, Micol, Oscar…)

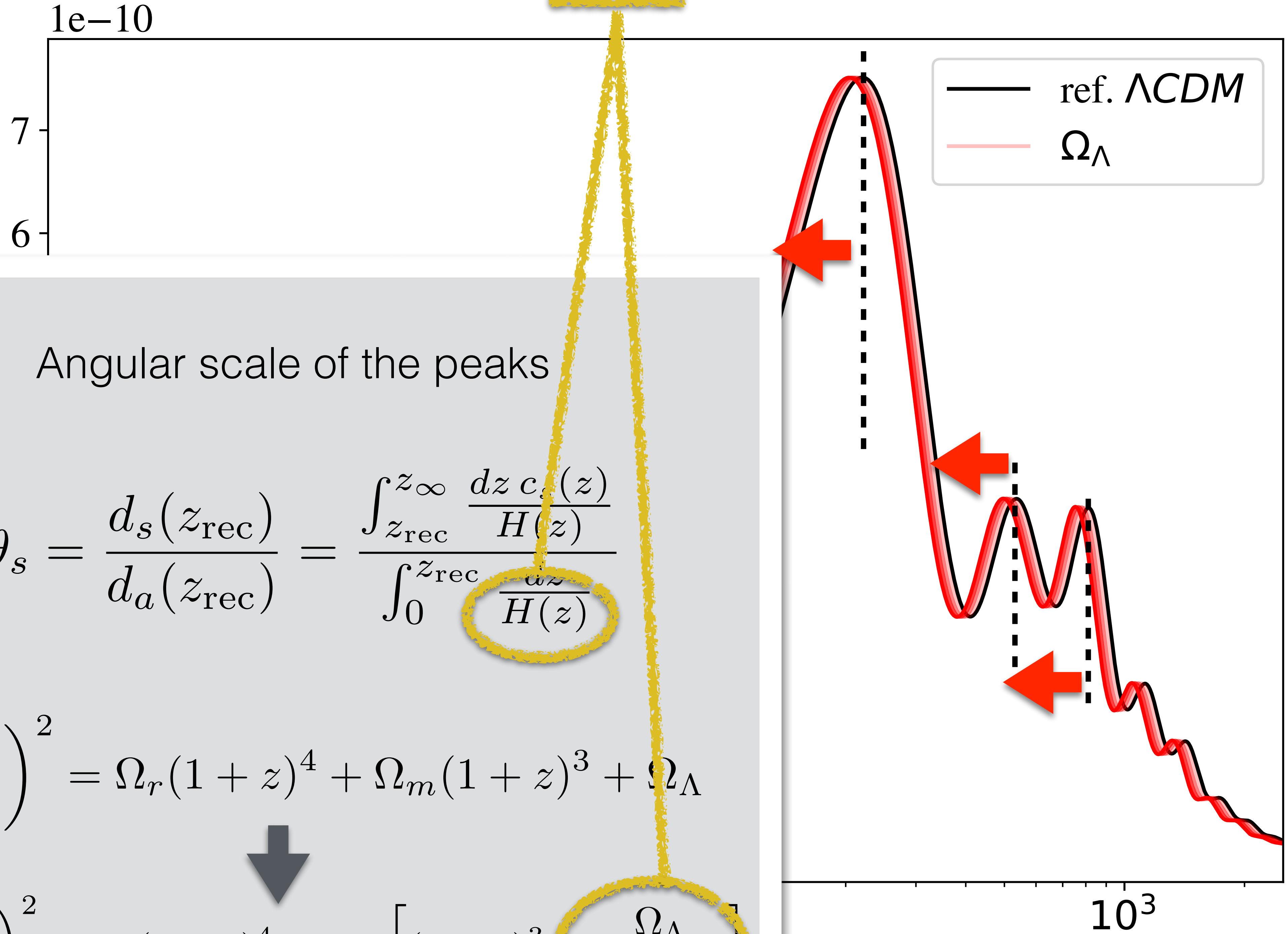


$$\{\omega_b, \omega_m, \boxed{\Omega_\Lambda}, \tau_{\text{reio}}, A_s, n_s\}$$



TTK Institute for Theoretical Particle Physics and Cosmology
RWTH AACHEN UNIVERSITY

$$\{\omega_b, \omega_m, \boxed{\Omega_\Lambda}, \tau_\mathrm{reio}, A_s, n_s\}$$



Angular scale of the peaks

$$\theta_s = \frac{d_s(z_\mathrm{rec})}{d_a(z_\mathrm{rec})} = \frac{\int_{z_\mathrm{rec}}^{z_\infty} \frac{dz\, c_s(z)}{H(z)}}{\int_0^{z_\mathrm{rec}} \frac{dz}{H(z)}}$$

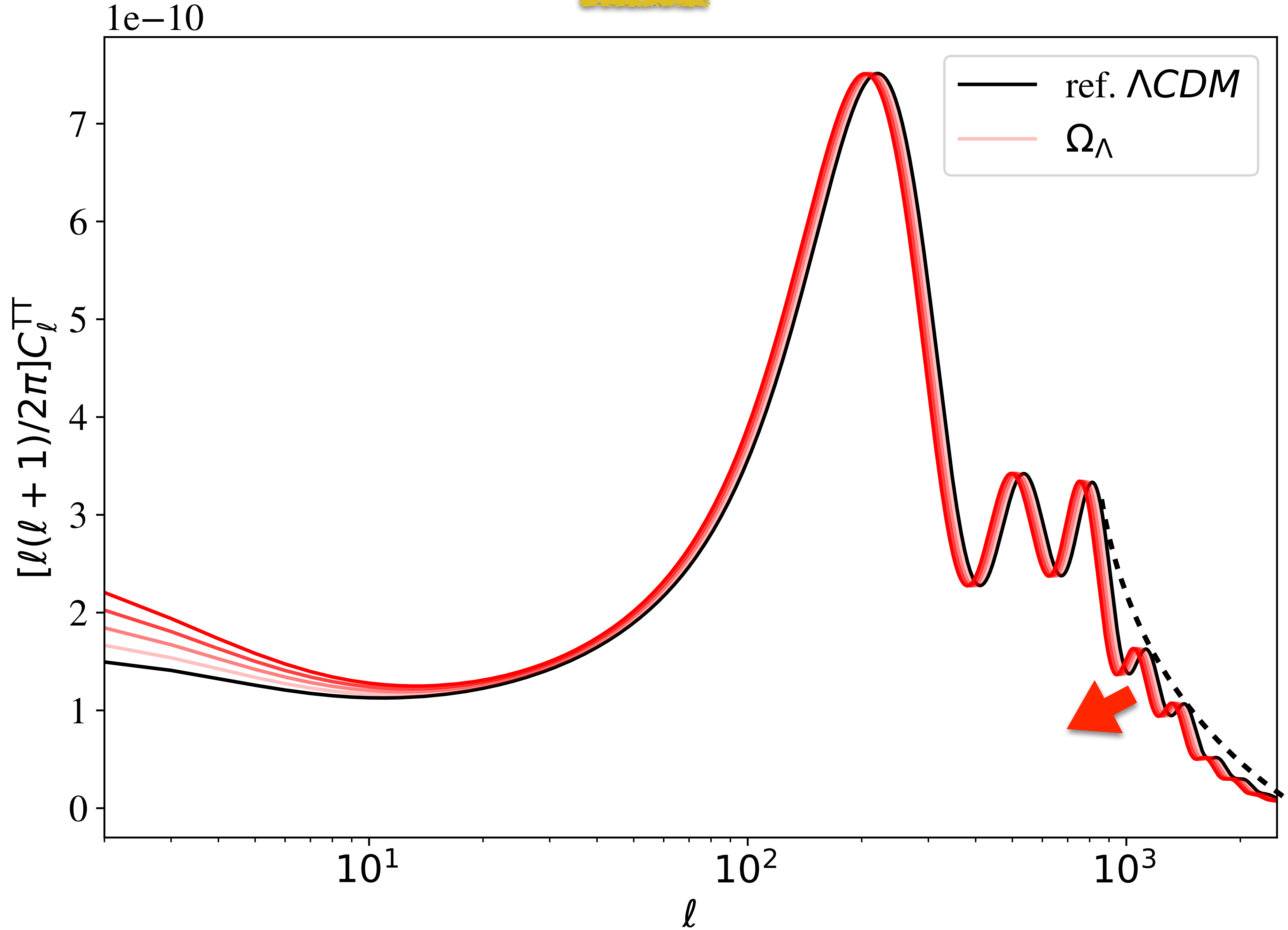
$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda$$

$$\Downarrow$$

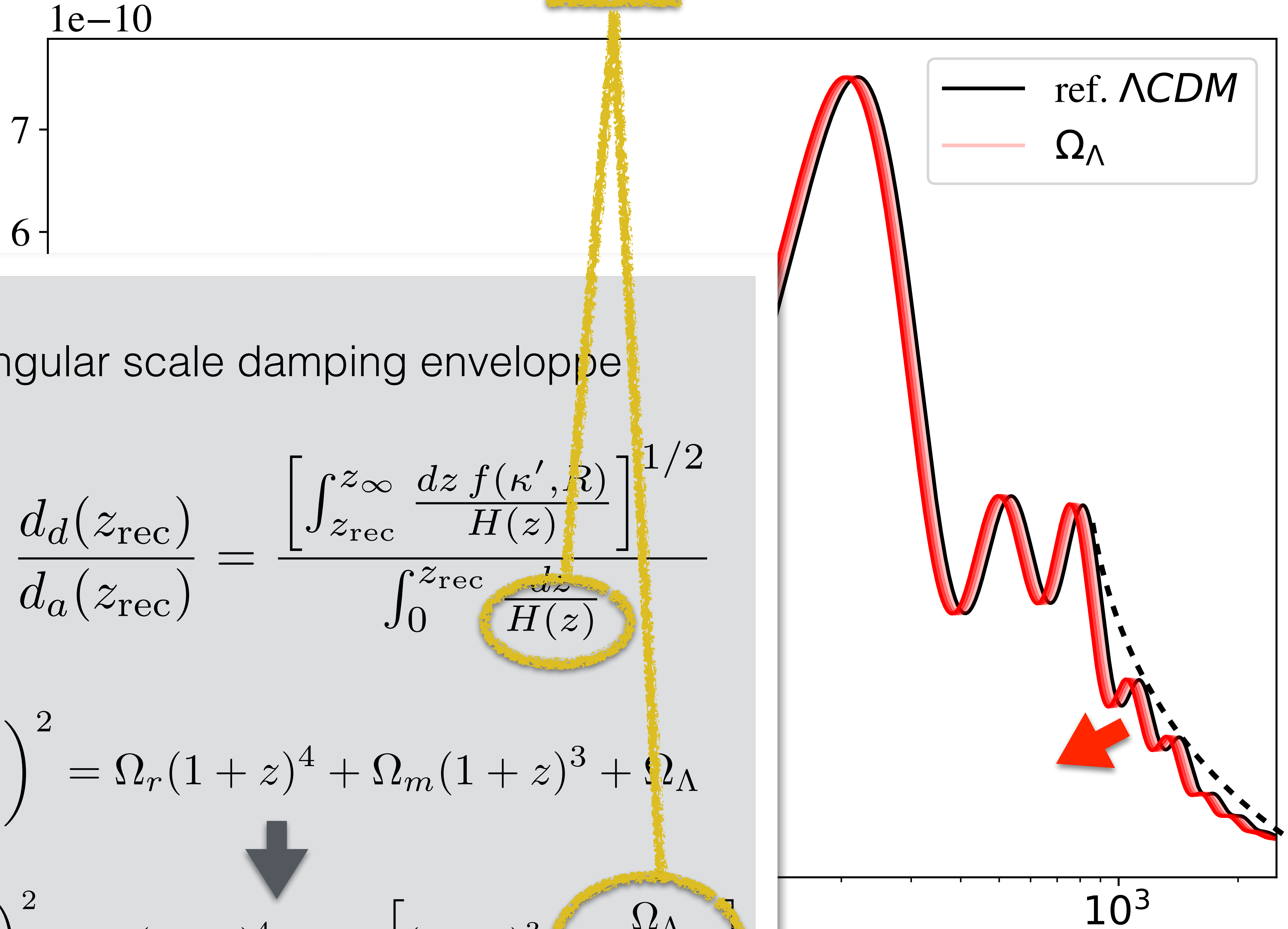
$$\left(\frac{H(z)h}{H_0}\right)^2 = \omega_r(1+z)^4 + \omega_m\left[(1+z)^3 + \frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right]$$

only impacts H(z) at small z

$$\{\omega_b, \omega_m, \boxed{\Omega_\Lambda}, \tau_{\rm reio}, A_s, n_s\}$$



$$\{\omega_b, \omega_m, \boxed{\Omega_\Lambda}, \tau_{\rm reio}, A_s, n_s\}$$



Angular scale damping envelope

$$\theta_d = \frac{d_d(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{\left[\int_{z_{\rm rec}}^{z_\infty} \frac{dz\, f(\kappa', R)}{H(z)}\right]^{1/2}}{\int_0^{z_{\rm rec}} \frac{dz}{H(z)}}$$

$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda$$

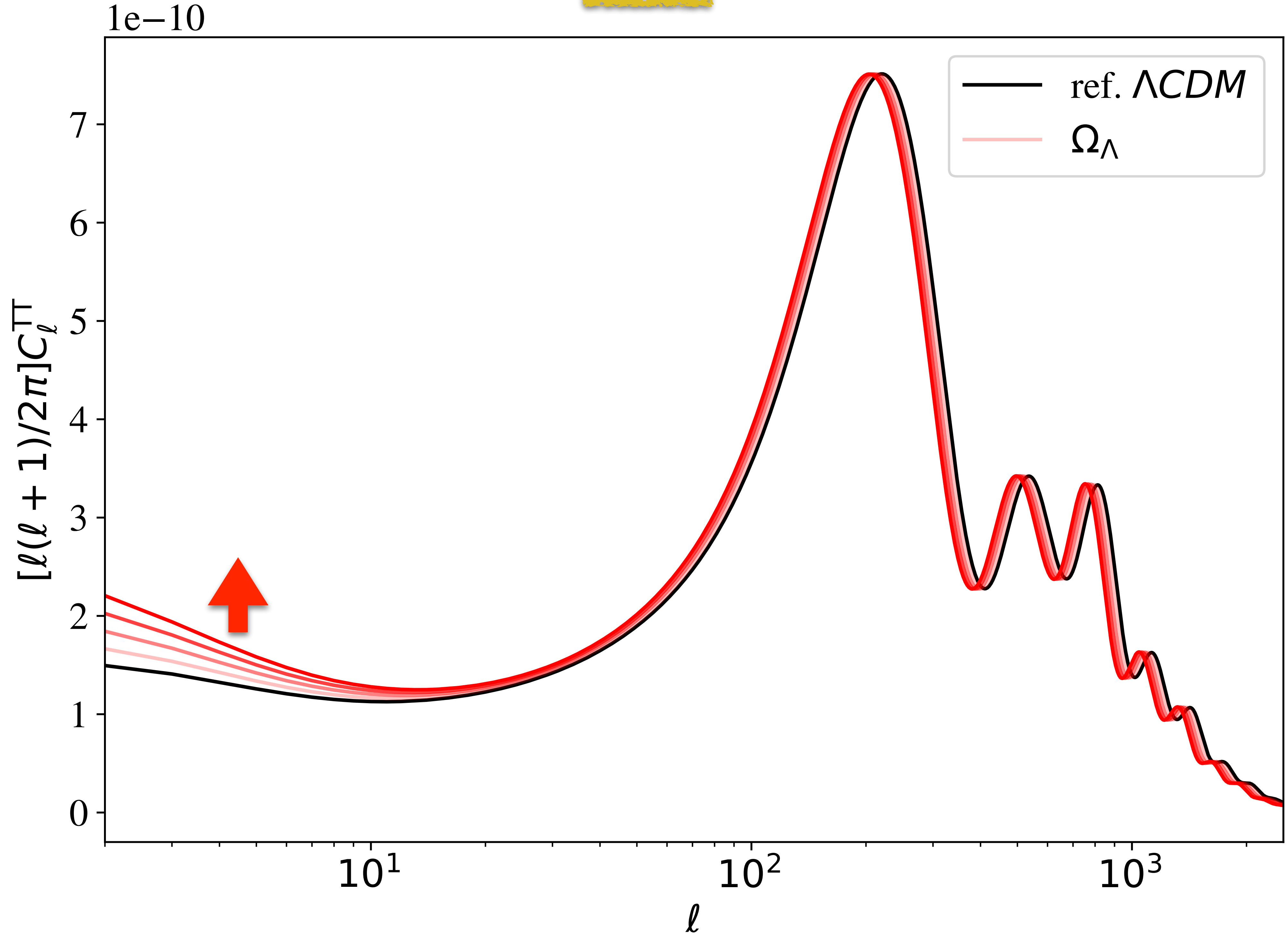
$$\Downarrow$$

$$\left(\frac{H(z)h}{H_0}\right)^2 = \omega_r(1+z)^4 + \omega_m\left[(1+z)^3 + \frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right]$$

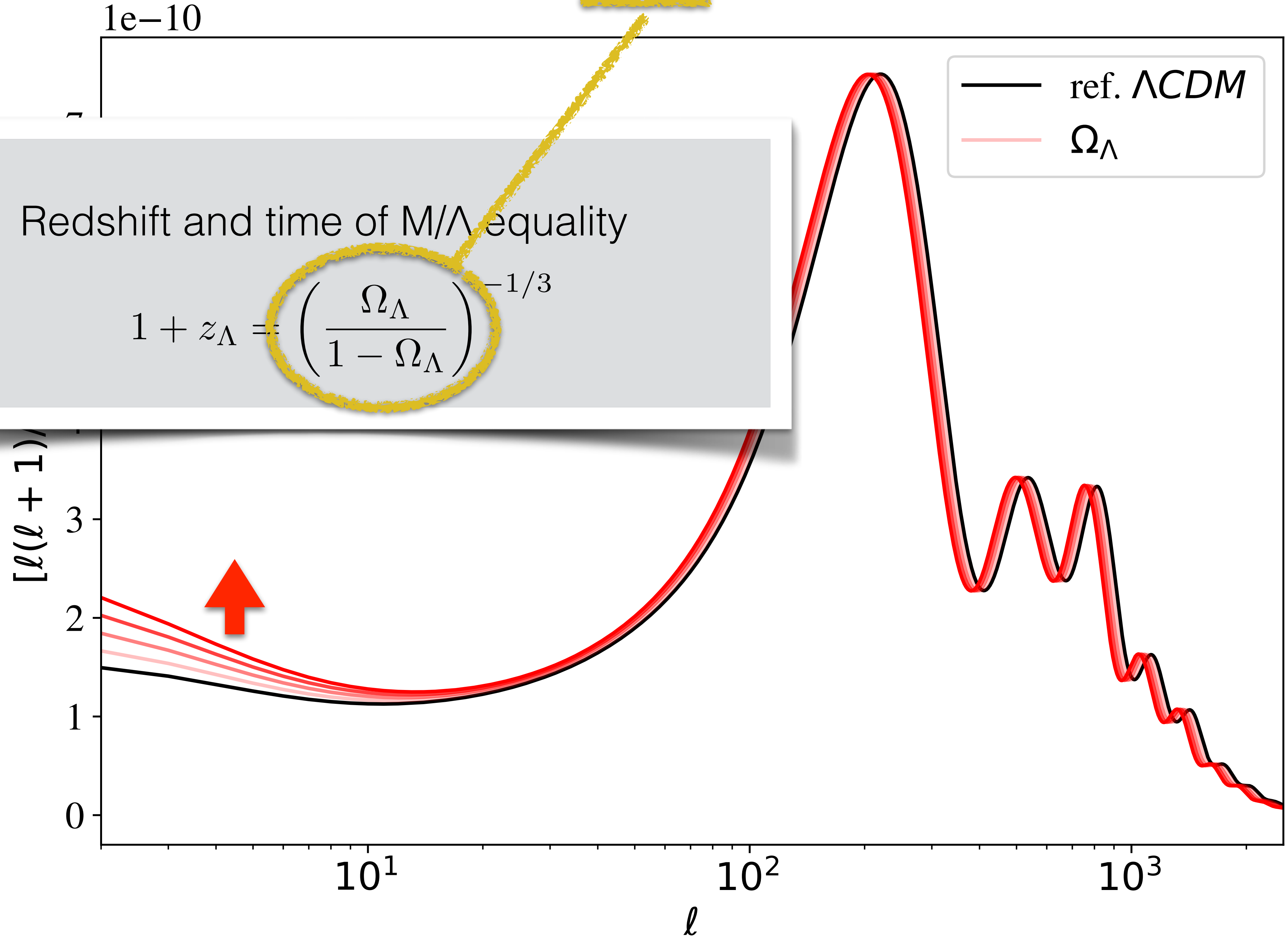
only impacts H(z) at small z

TTK Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

$$\{\omega_b, \omega_m, \boxed{\Omega_\Lambda}, \tau_{\mathrm{reio}}, A_s, n_s\}$$



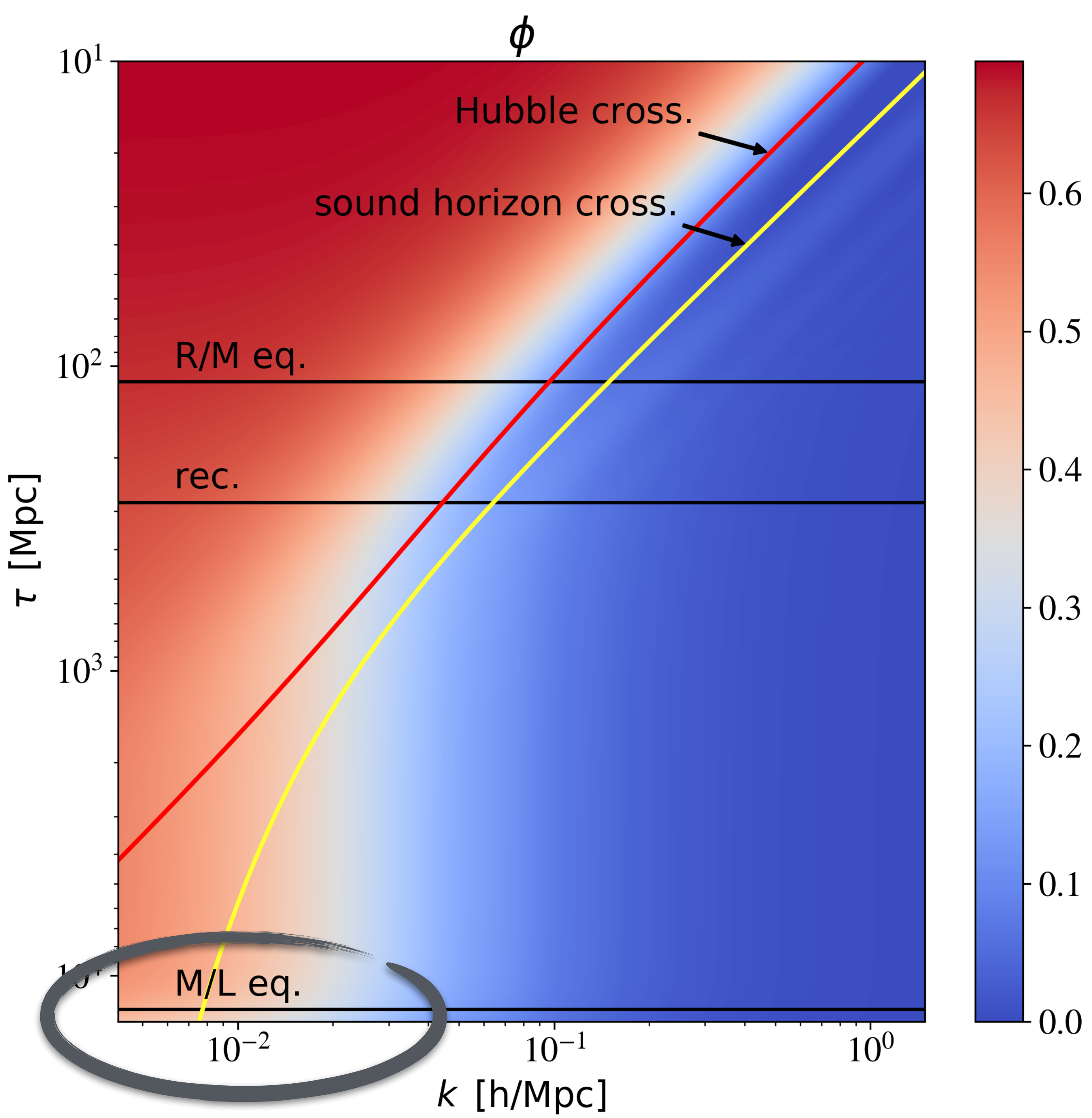
$$\{\omega_b, \omega_m, \boxed{\Omega_\Lambda}, \tau_{\rm reio}, A_s, n_s\}$$

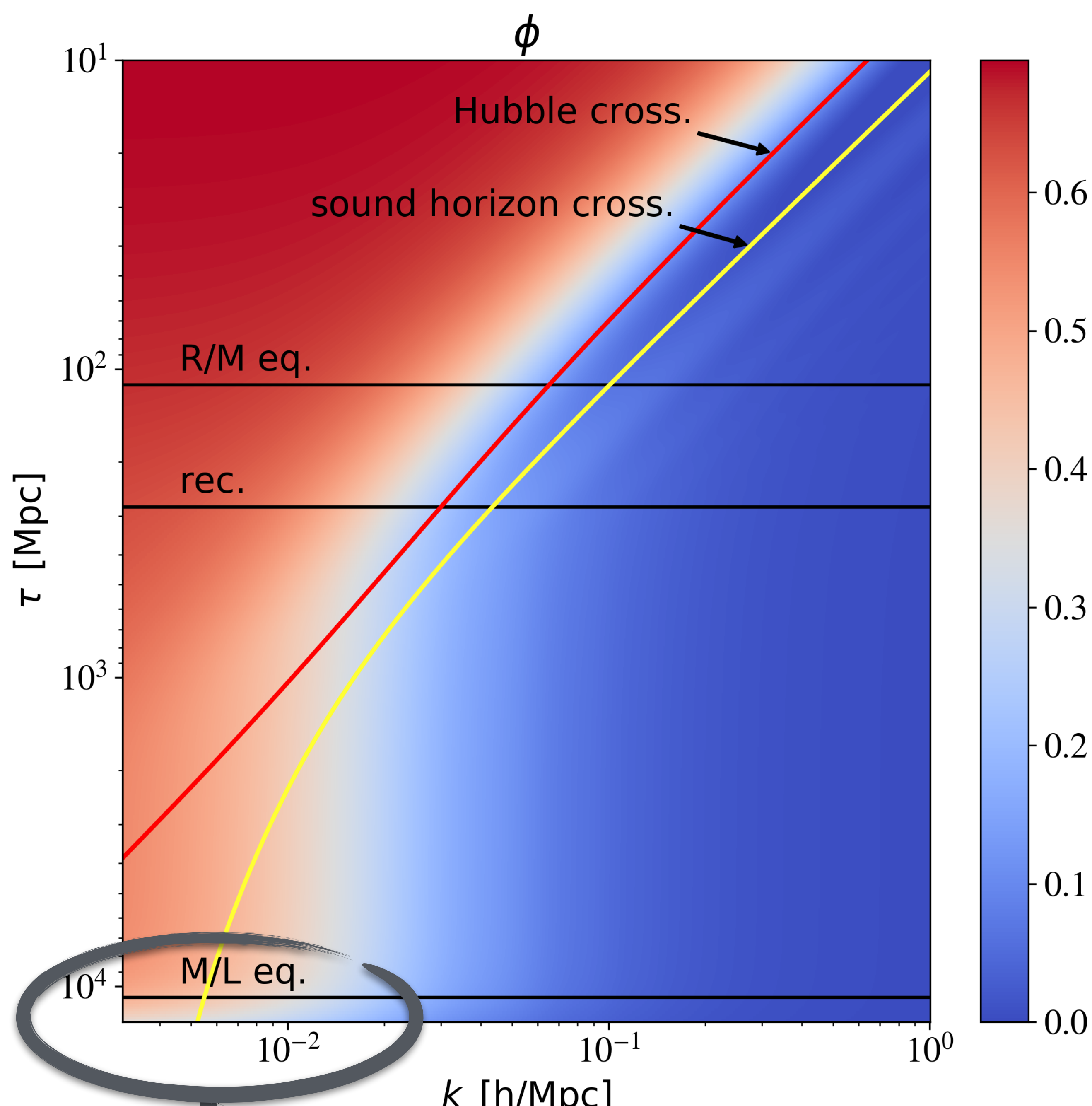


Redshift and time of M/Λ equality

$$1 + z_\Lambda = \left(\frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right)^{-1/3}$$

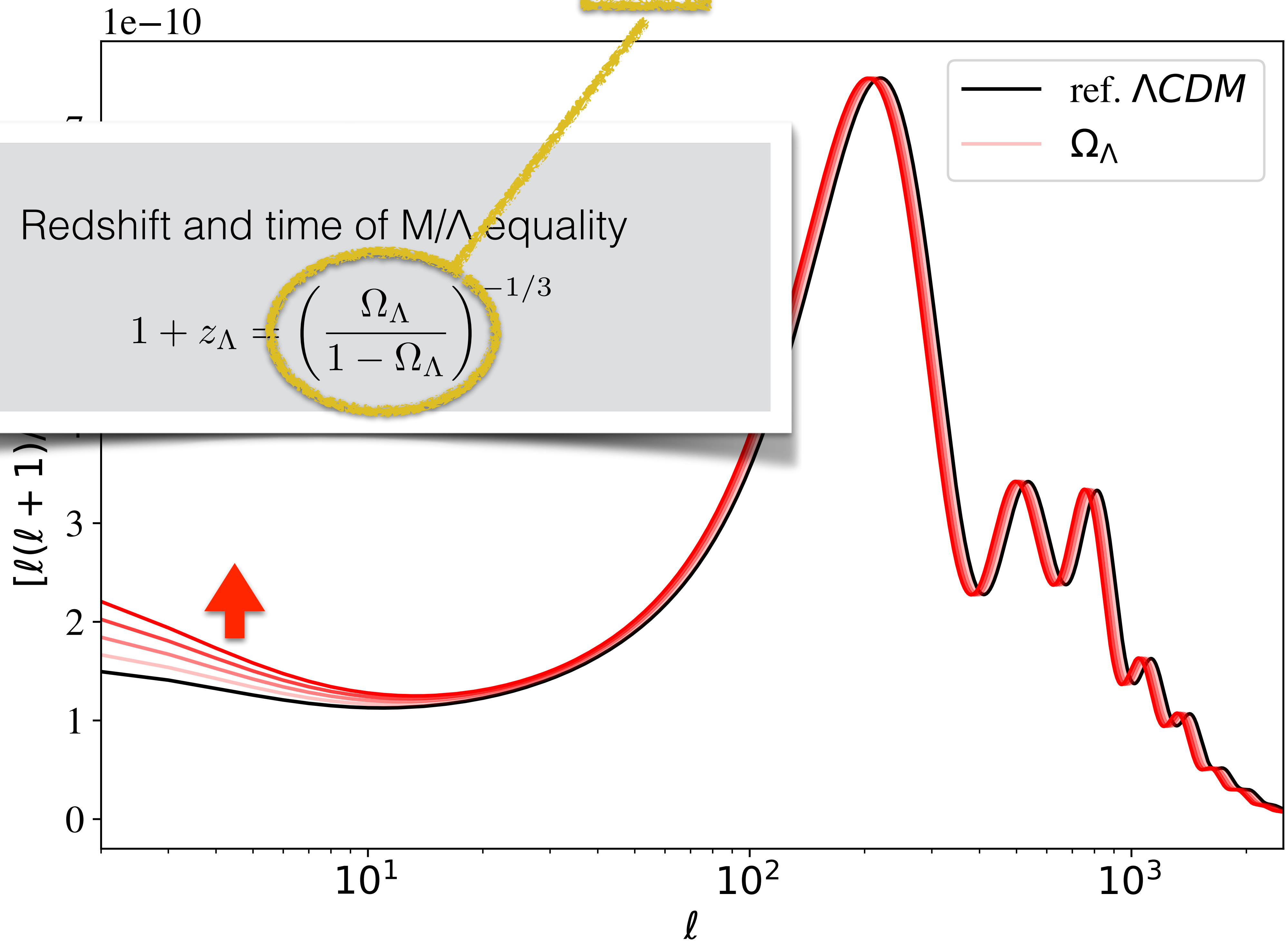
$$\{\omega_b, \omega_m, \boxed{\Omega_\Lambda}, \tau_{\rm reio}, A_s, n_s\}$$





increasing Ω_Λ

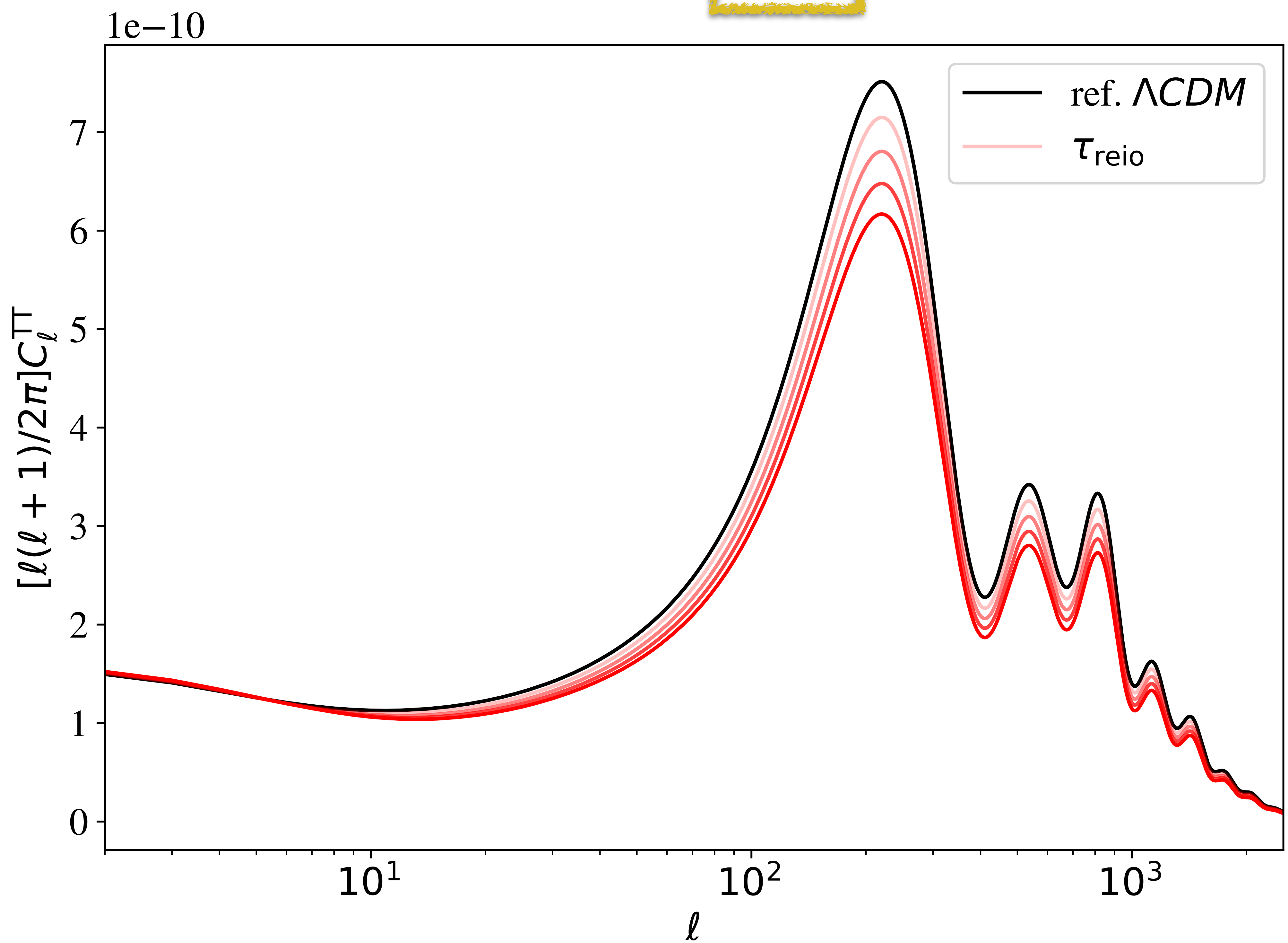
$$\{\omega_b, \omega_m, \boxed{\Omega_\Lambda}, \tau_{\rm reio}, A_s, n_s\}$$



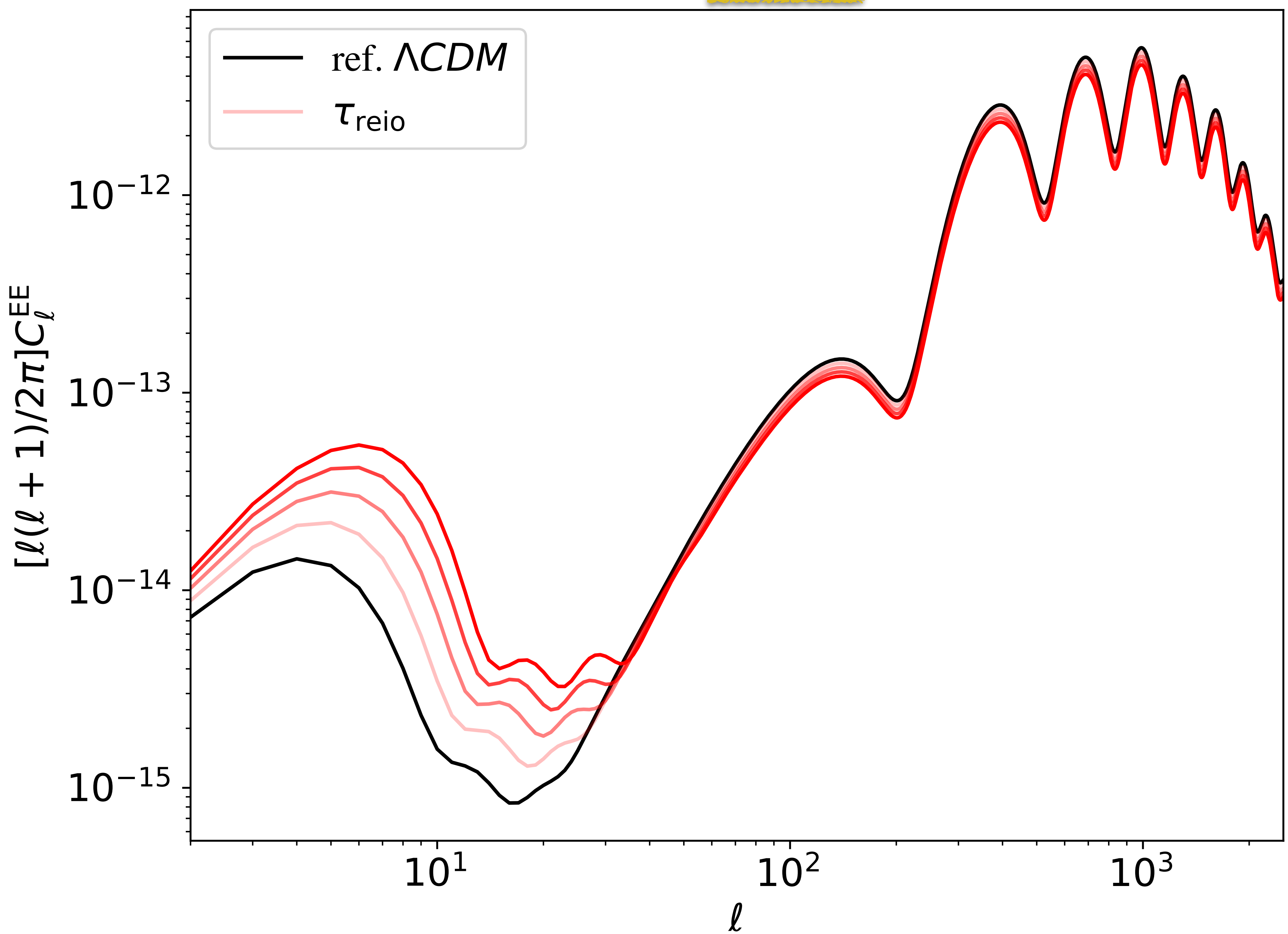
Redshift and time of M/Λ equality

$$1 + z_\Lambda = \left(\frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right)^{-1/3}$$

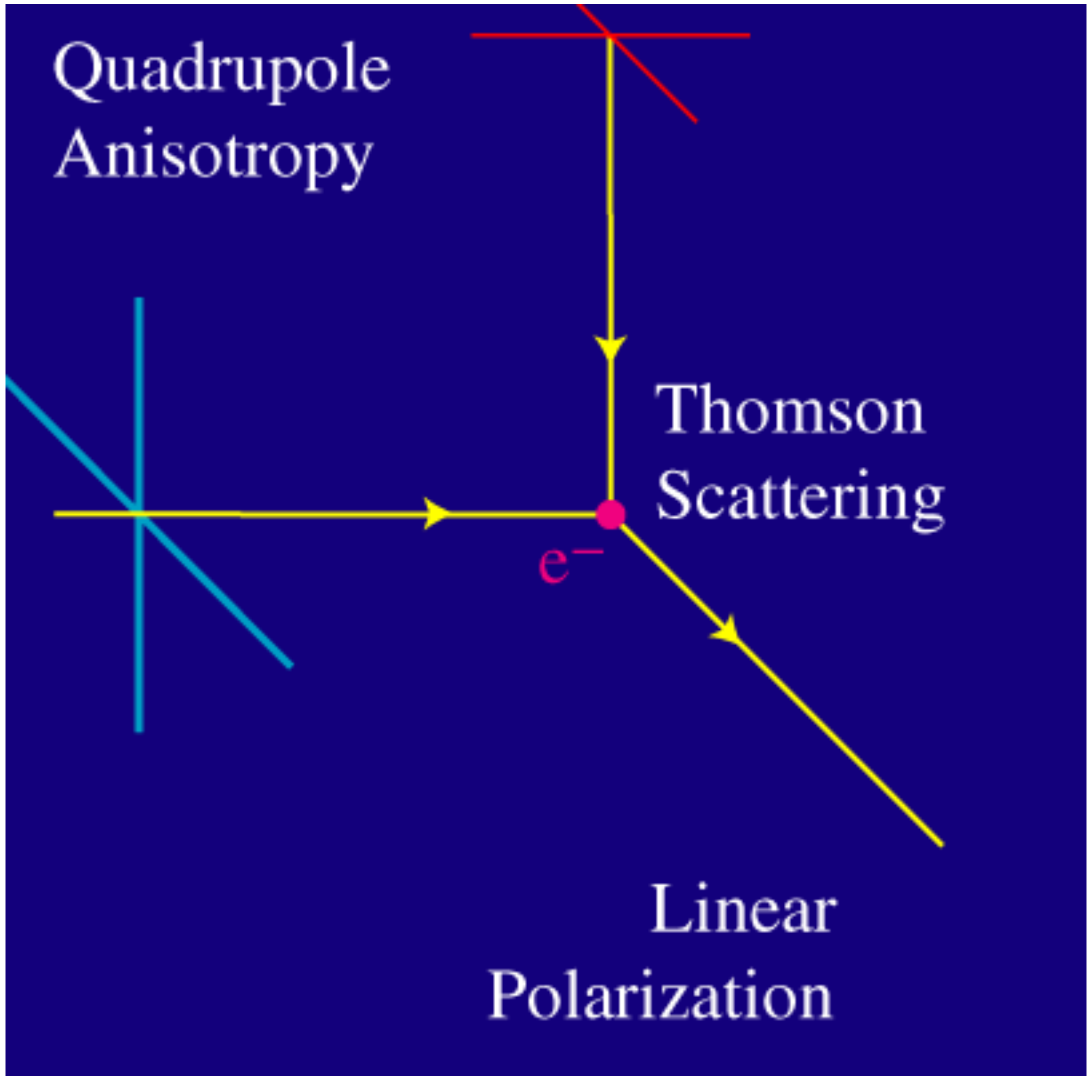
$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$



$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$



$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$



cosθ in Thomson —> coupling with 2nd Boltzmann hierarchy sourced by Θ_2

—> observable polarisation spectrum depends on ($g\ \Theta_2$) instead of $g(\Theta_0+\psi)$

Ma & Berstchinger 1995 (2 hierarchies, flat, scalar);

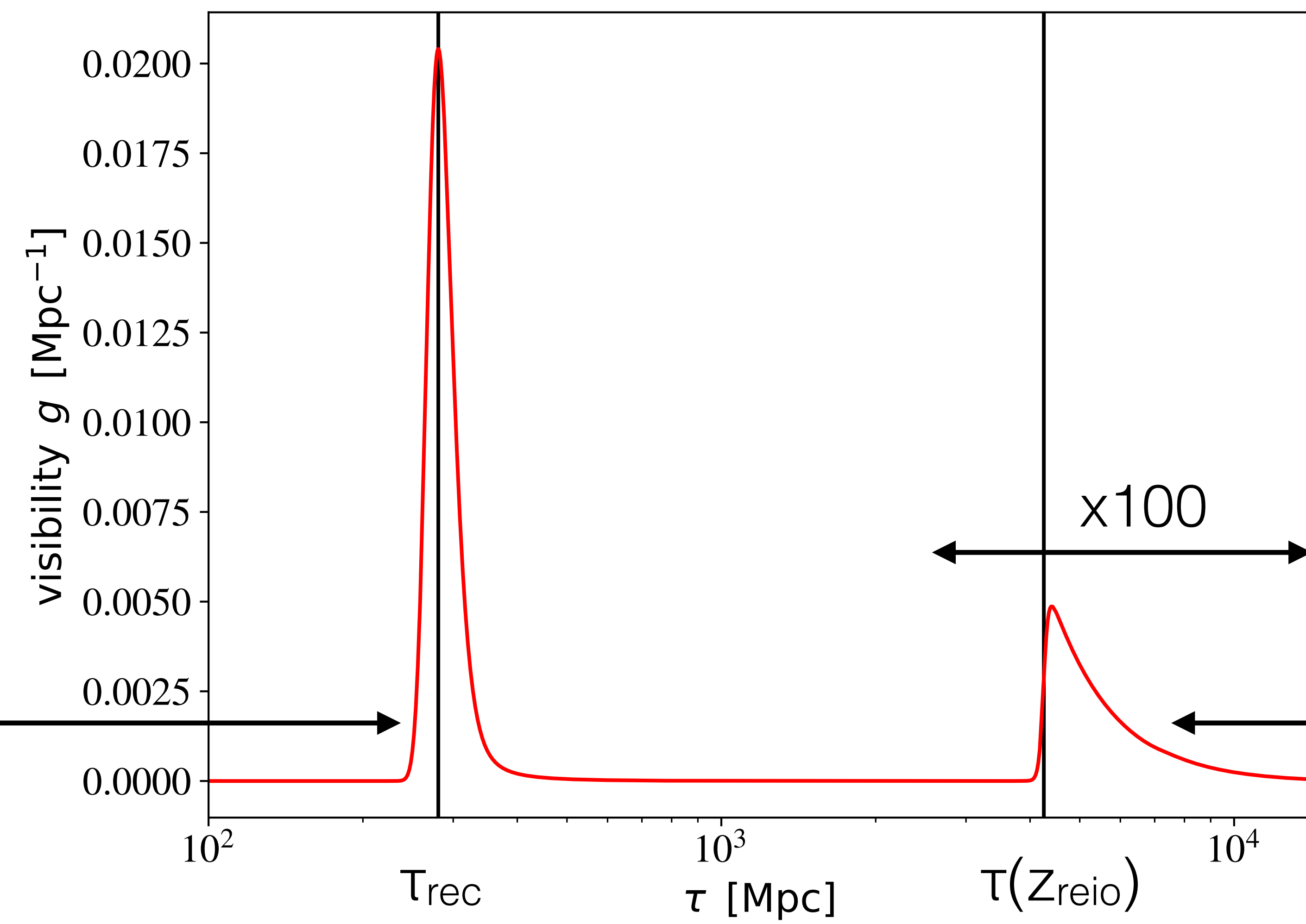
Tram & JL 2013 (2 hierarchies in flat/open/closed, scalar/vector/tensor)

$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$

Optical depth: $\kappa = \int_\tau^{\tau_0} d\tau \; \Gamma = \int_{\tau_0}^{\tau} d\tau \; \kappa'$

Visibility function: $g(\tau) = -\kappa' e^{-\kappa}$ by construction normalised to $\int_0^{\tau_0} g(\tau) d\tau = 1$

$e^{-\kappa_{\rm reio}}$ ~94%



$(1 - e^{-\kappa_{\rm reio}})$ ~ 6%

$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$

CMB temp. Transfer function: $\Delta_l^T(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; \{\underbrace{g\,(\Theta_0 + \psi)}_{\rm TSW} + ...\} \; j_l(k(\tau_0 - \tau))$

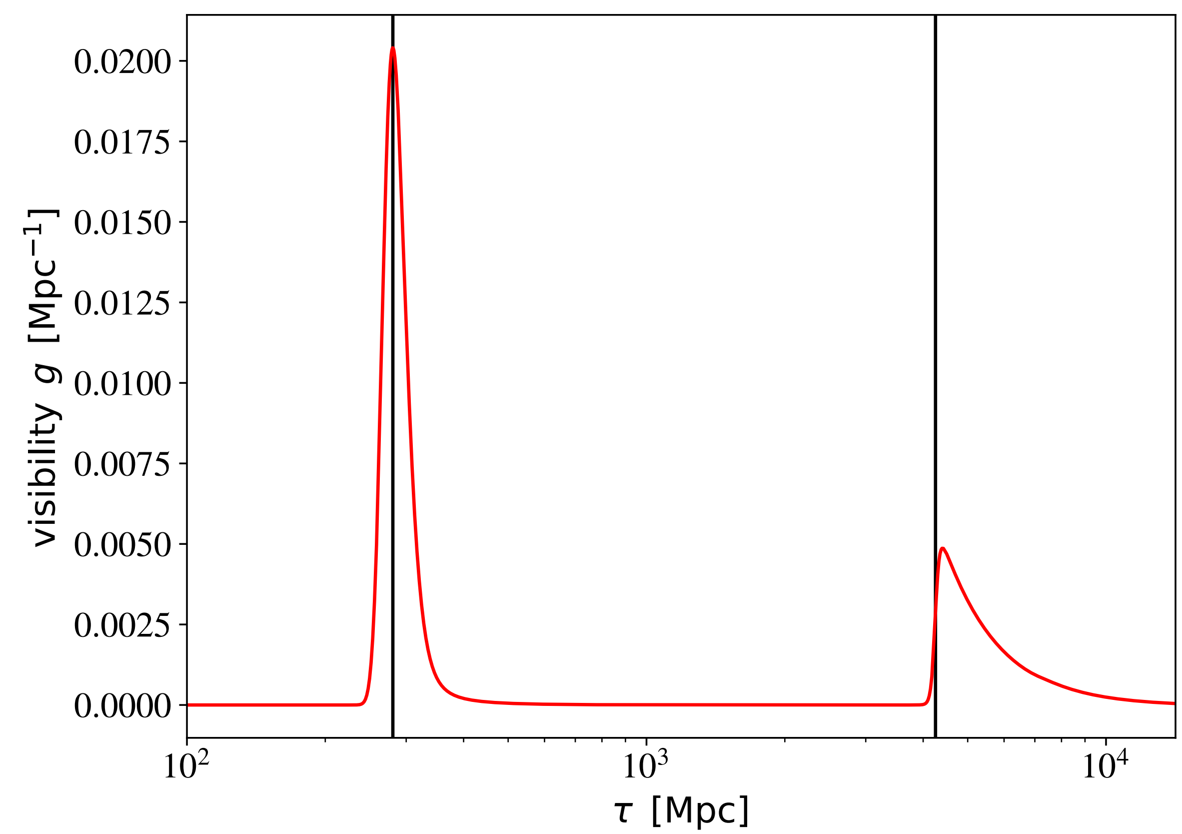
CMB pol transfer function: $\Delta_l^E(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; \{g\,\Theta_2 + ...\} \; j_l(k(\tau_0 - \tau))$

Finally: $C_l^{XY} = 4\pi \int dk \; k^2 \Delta_l^X(k) \Delta_l^Y(k) \mathcal{P}_\mathcal{R}(k)$

Usual contribution multiplied by $e^{-2\kappa_{\rm reio}}$

plus new contribution projected

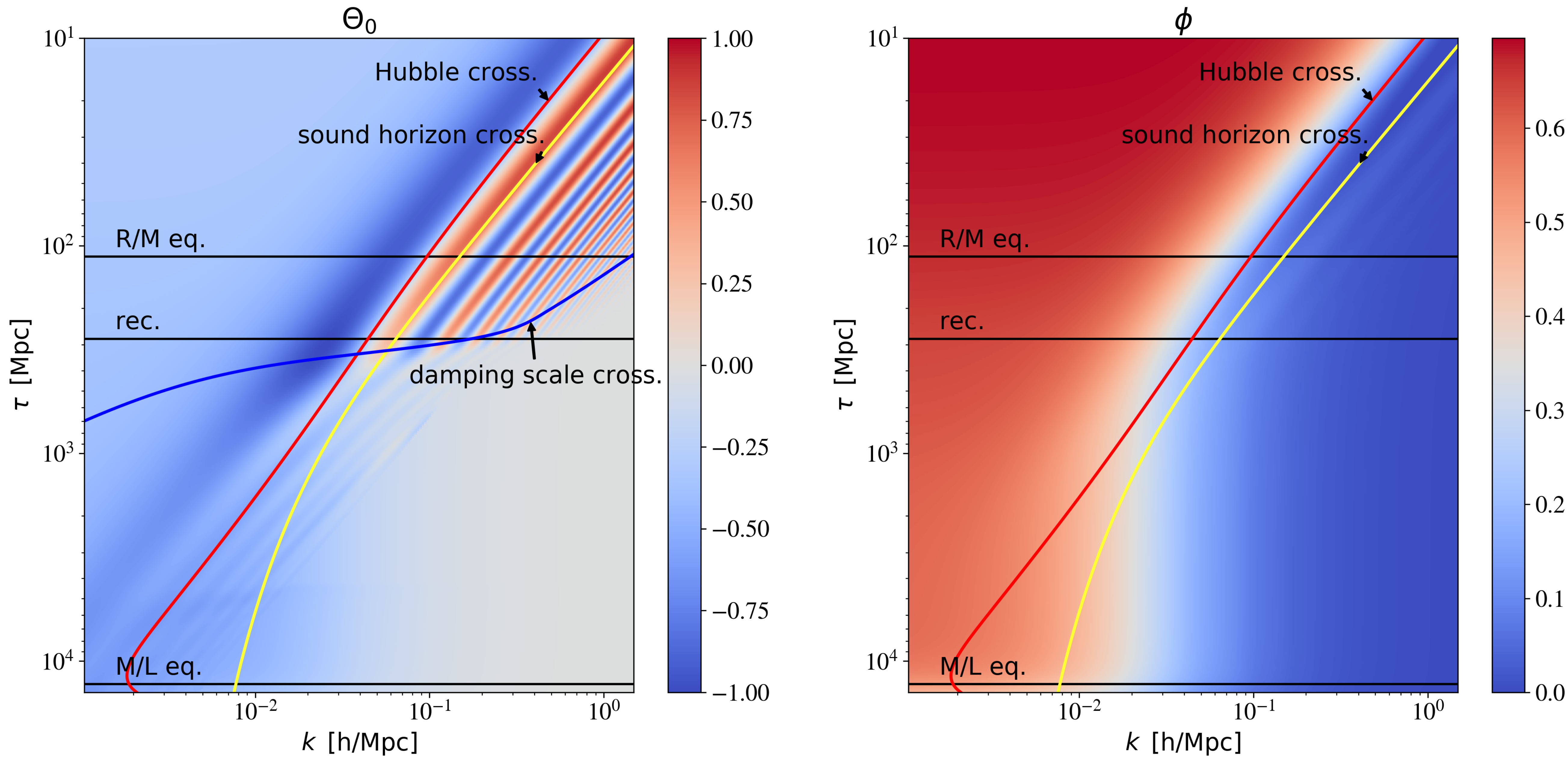
under $l \sim k(\tau_0 - \tau_{\rm reio})$



Institute for Theoretical Particle Physics and Cosmology

RWTH AACHEN UNIVERSITY

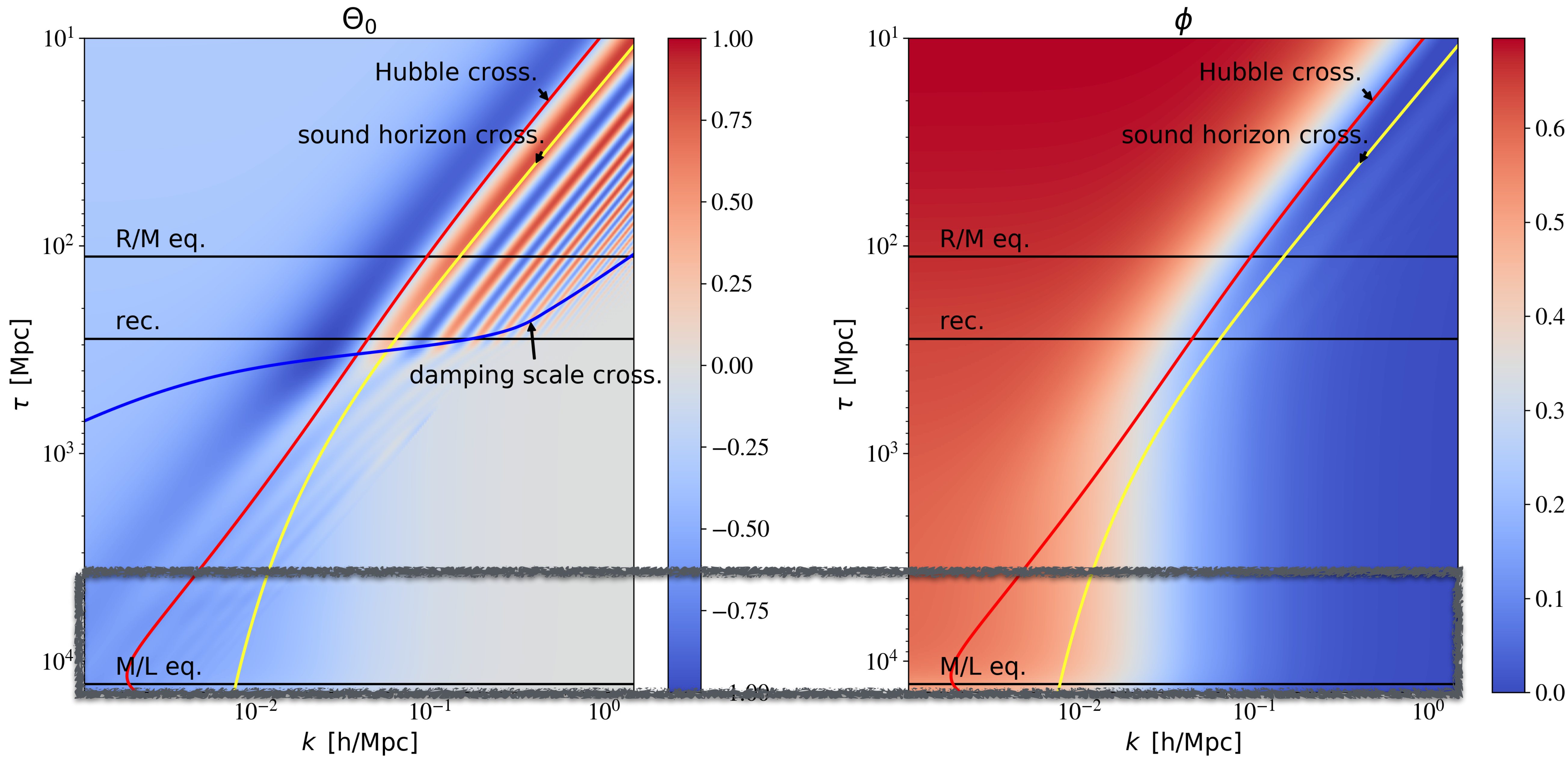
$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$



$$\Delta_l^T(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; \{\underbrace{g\,(\Theta_0 + \psi)}_{\rm TSW} + \ldots\} \; j_l(k(\tau_0 - \tau))$$

$$C_l^{XY} = 4\pi \int dk \; k^2 \Delta_l^X(k) \Delta_l^Y(k) \mathcal{P}_\mathcal{R}(k)$$

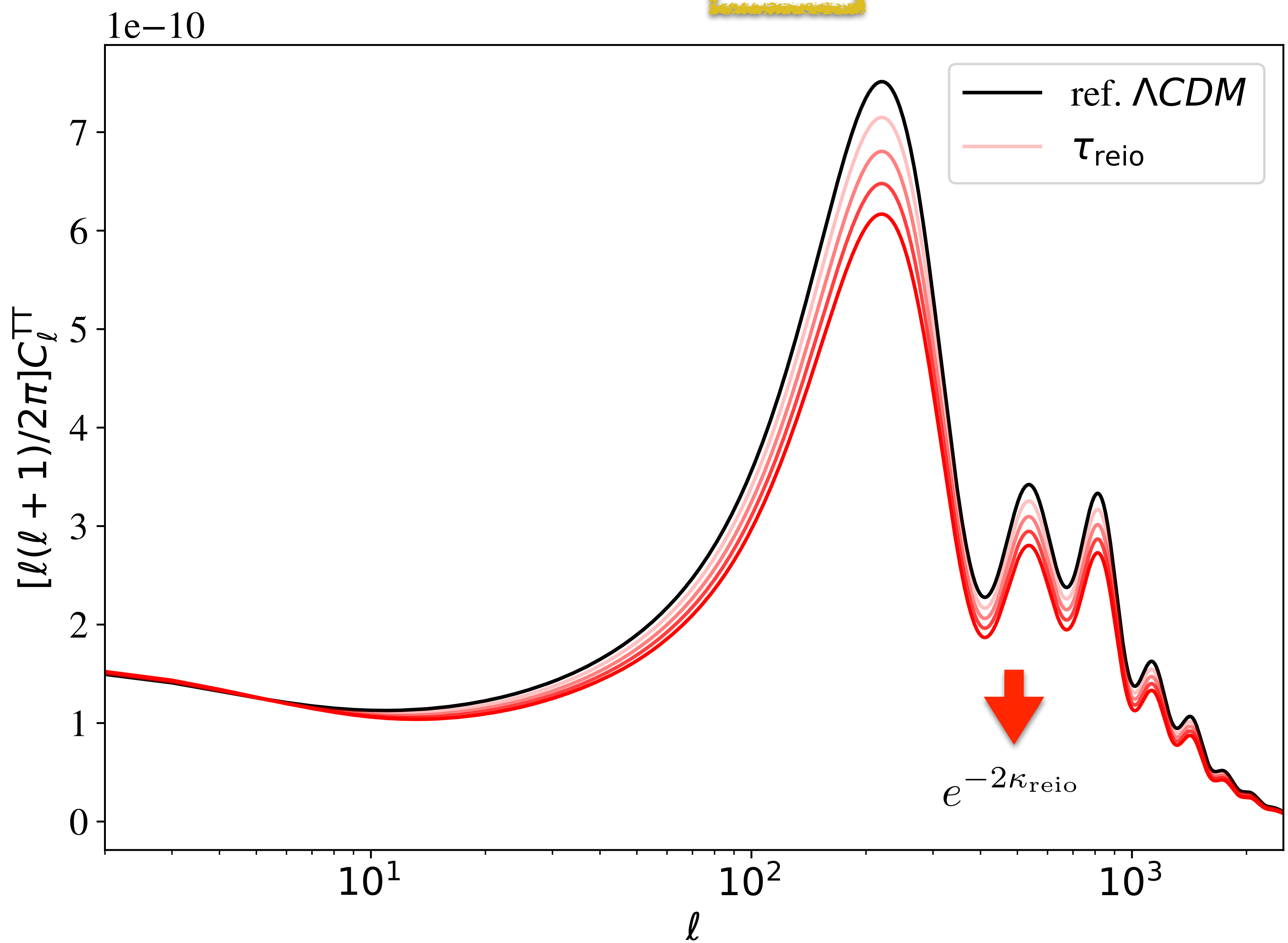
$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_\mathrm{reio}}, A_s, n_s\}$$



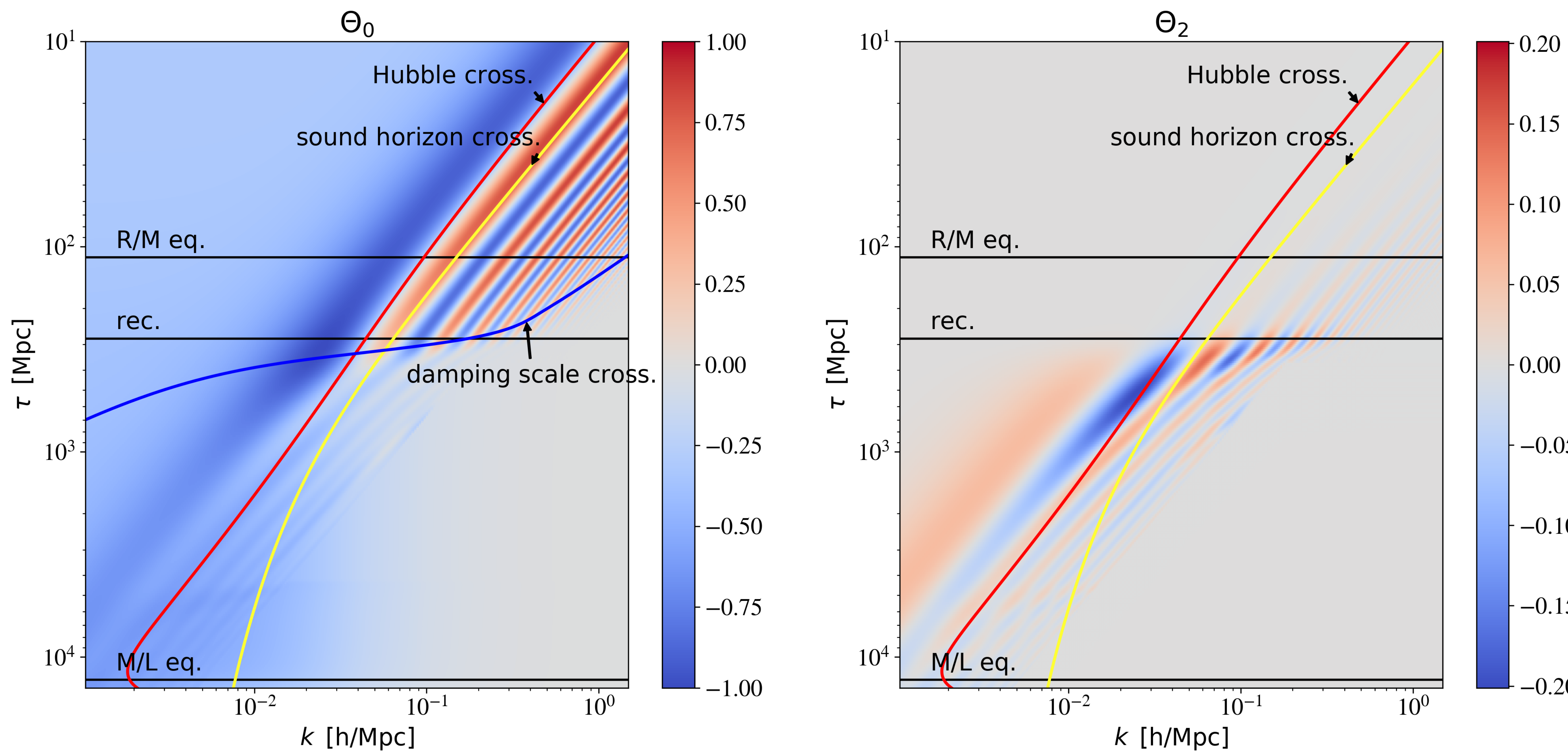
$$\Delta_l^T(\tau_0, k) = \int_{\tau_\mathrm{ini}}^{\tau_0} d\tau \; \{\underbrace{g\,(\Theta_0 + \psi)}_{\mathrm{TSW}} + \ldots\} \; j_l(k(\tau_0 - \tau))$$

$$C_l^{XY} = 4\pi \int dk \; k^2 \Delta_l^X(k) \Delta_l^Y(k) \mathcal{P}_\mathcal{R}(k)$$

$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$



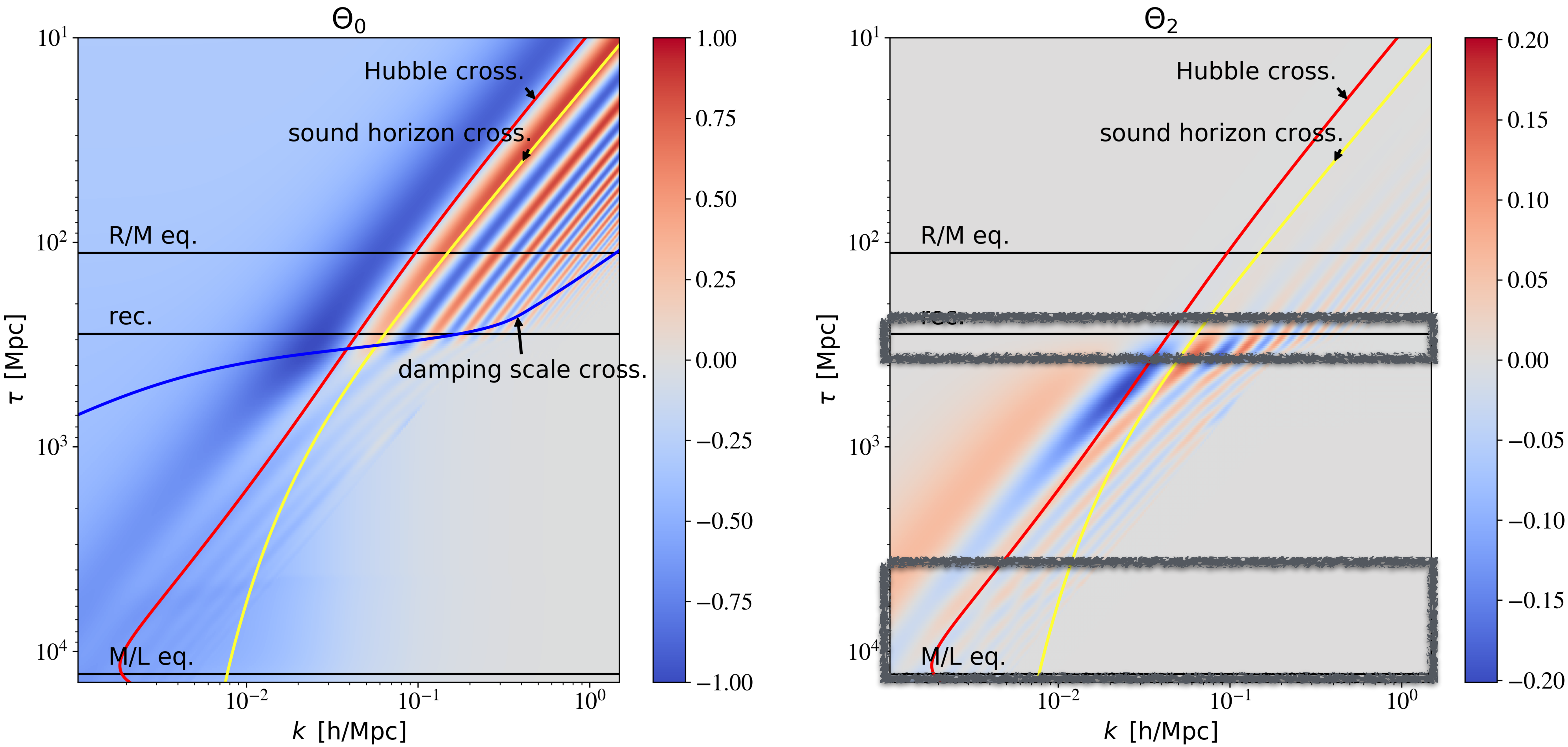
$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$



$$\Delta_l^E(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; \{g\,\Theta_2 + ...\}\; j_l(k(\tau_0 - \tau))$$

$$C_l^{XY} = 4\pi \int dk\; k^2 \Delta_l^X(k) \Delta_l^Y(k) \mathcal{P}_{\mathcal{R}}(k)$$

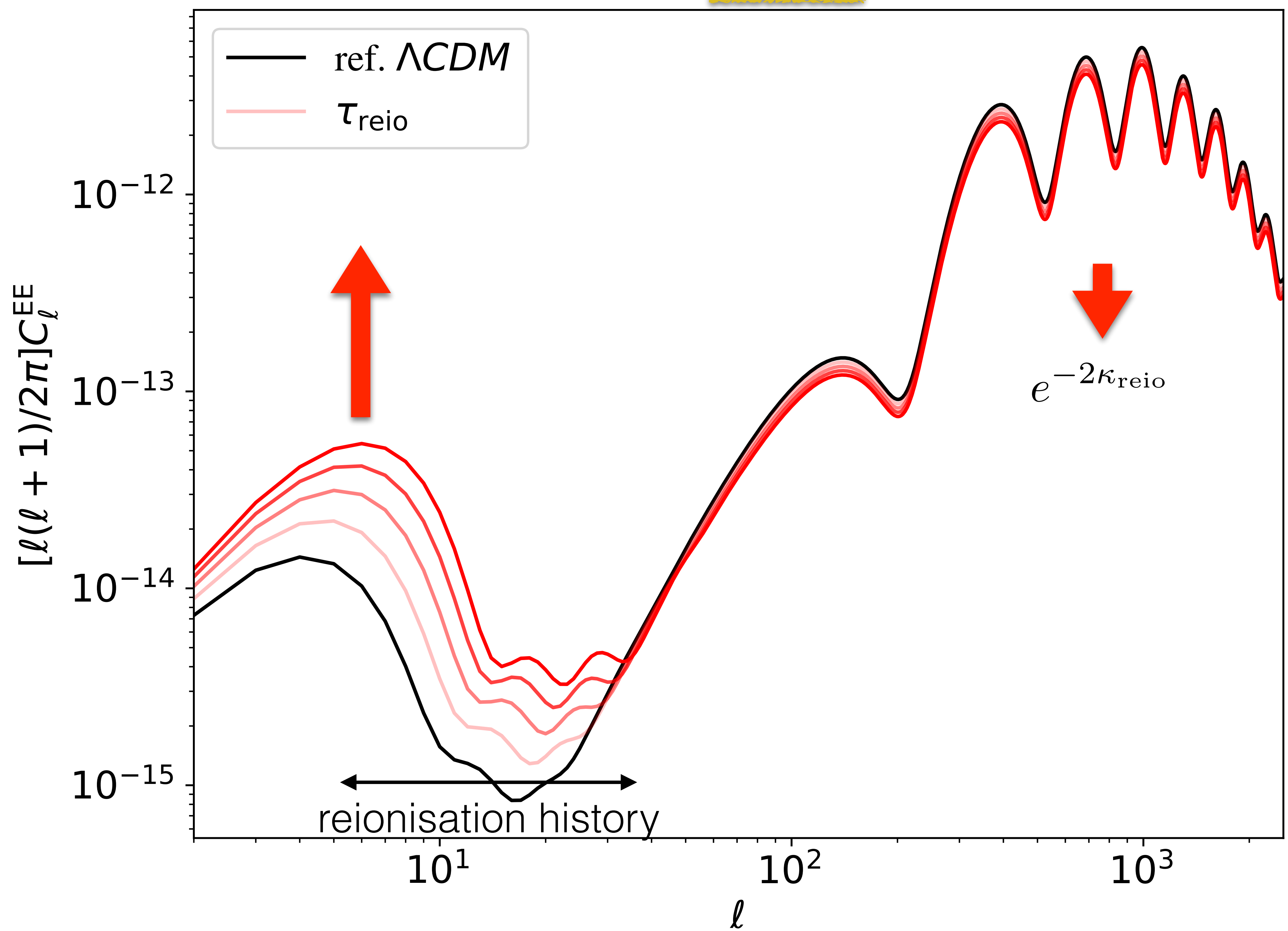
$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$



$$\Delta_l^E(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; \{g\,\Theta_2 + ...\} \; j_l(k(\tau_0 - \tau))$$

$$C_l^{XY} = 4\pi \int dk \; k^2 \Delta_l^X(k) \Delta_l^Y(k) \mathcal{P}_\mathcal{R}(k)$$

$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$



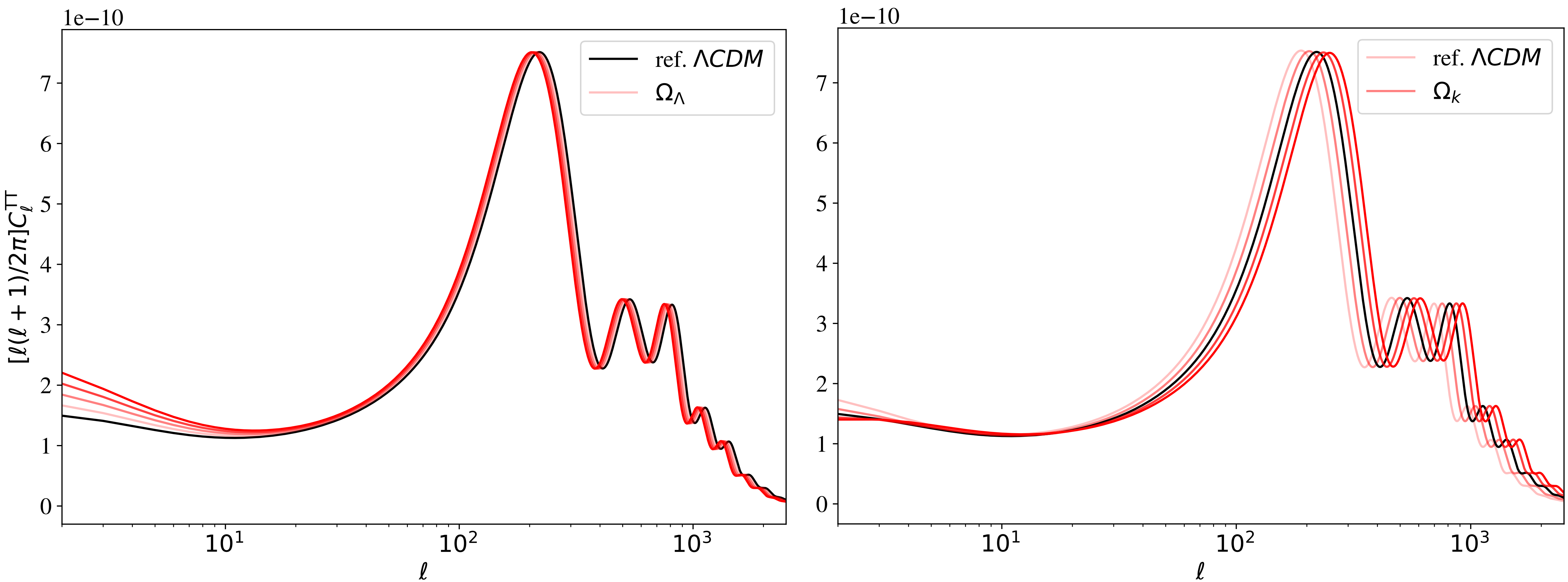
Why can we measure 6 ΛCDM parameters independently with CMB?

8 physical governing C_l's shape

- C1: angular scale of the peaks, θ_s
- C2: pressure at recombination, R_{rec}
- C3: metric (value and derivative) at recombination, z_{eq}
- C4: angular scale of damping enveloppe, θ_d
- C5: global amplitude
- C6: global tilt
- C7: plateau tilting by late ISW
- C8: reionisation effects

but all tight to 6 parameters in ΛCDM

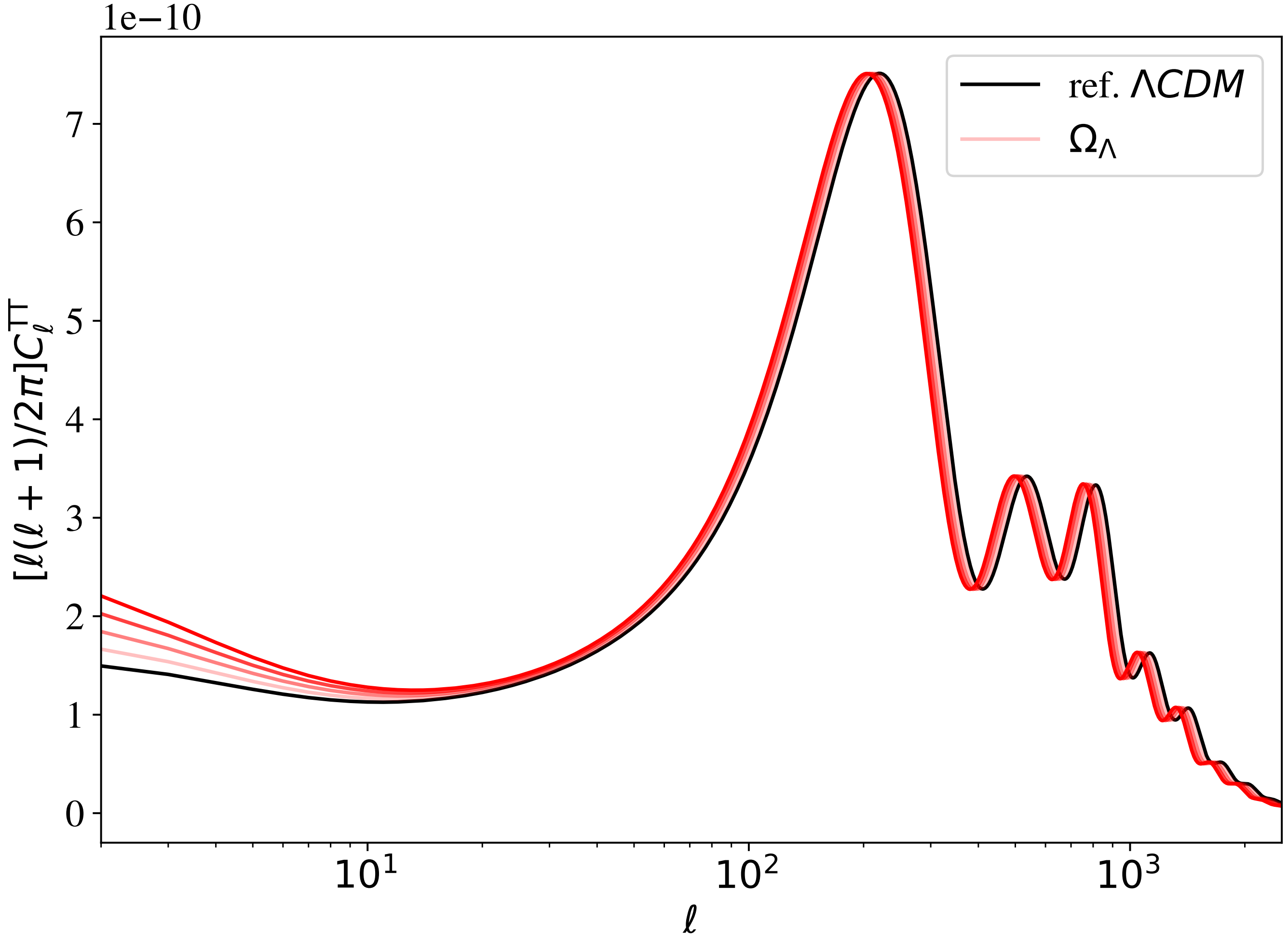
And even more parameters ….

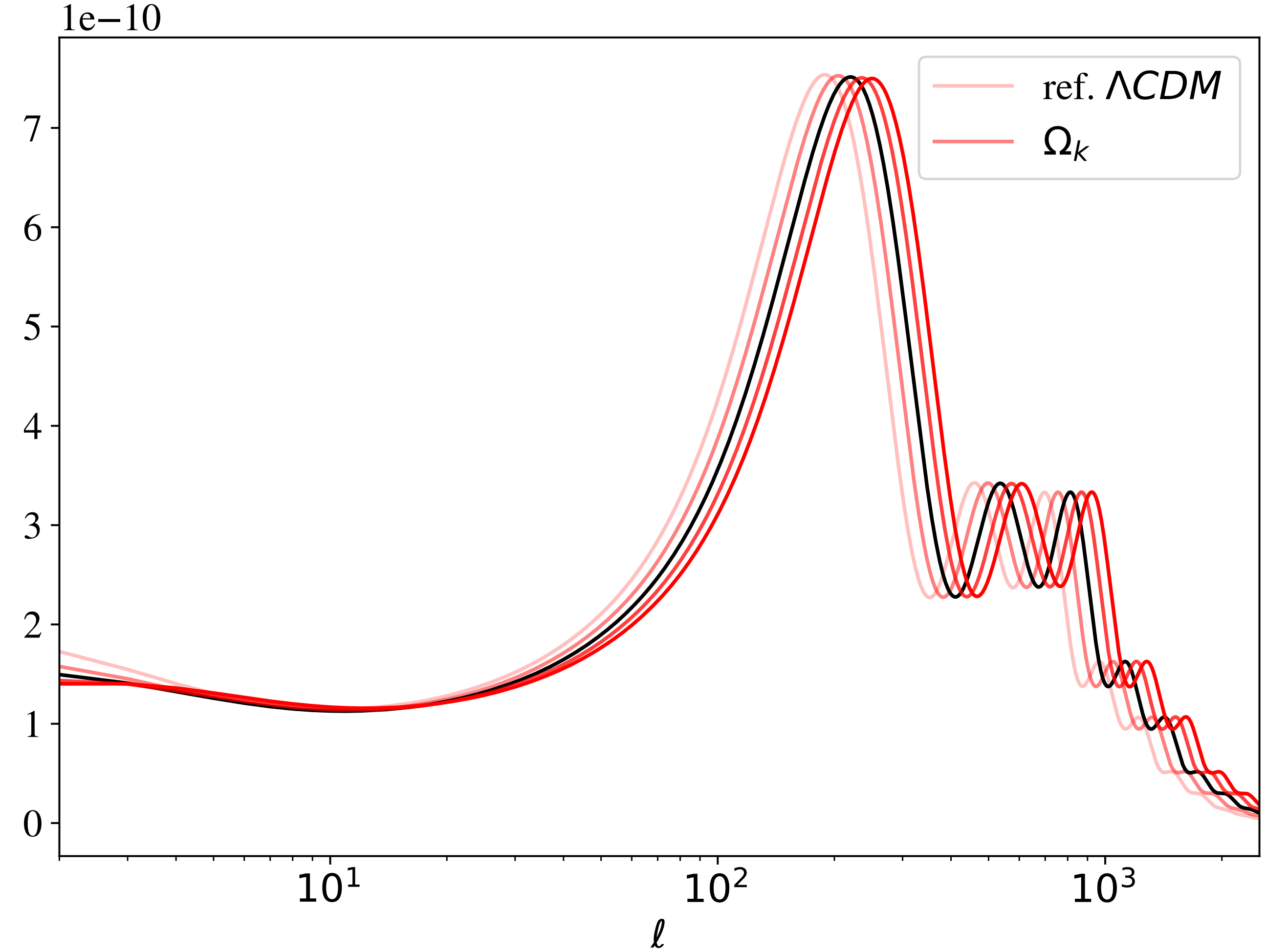


In ΛCDM + Ω_k, same 8 effects only, but tight to 7 parameters:

CMB only can also bound Ω_k (enters C1, C4 through $d_a(z_{rec})$)

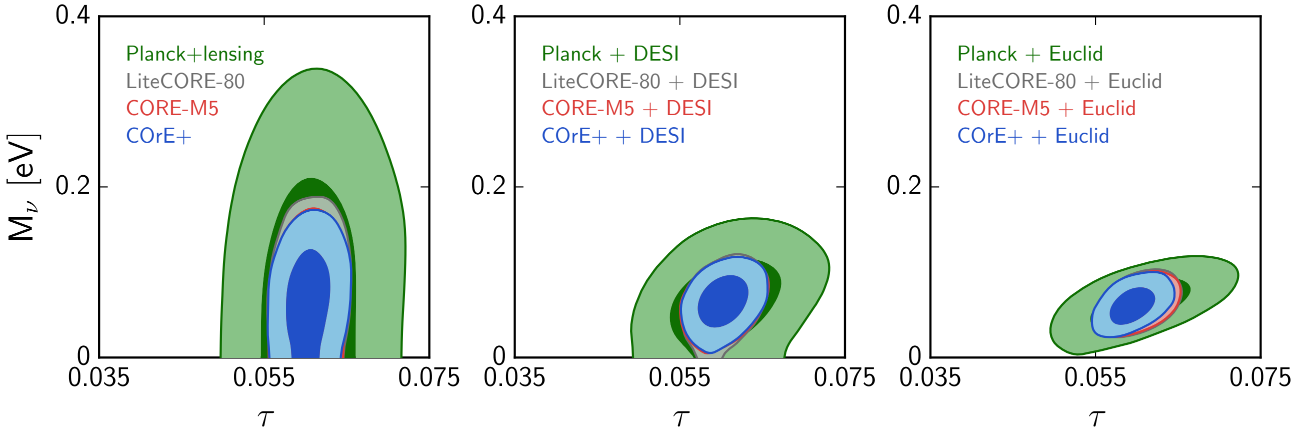
And even more parameters ….

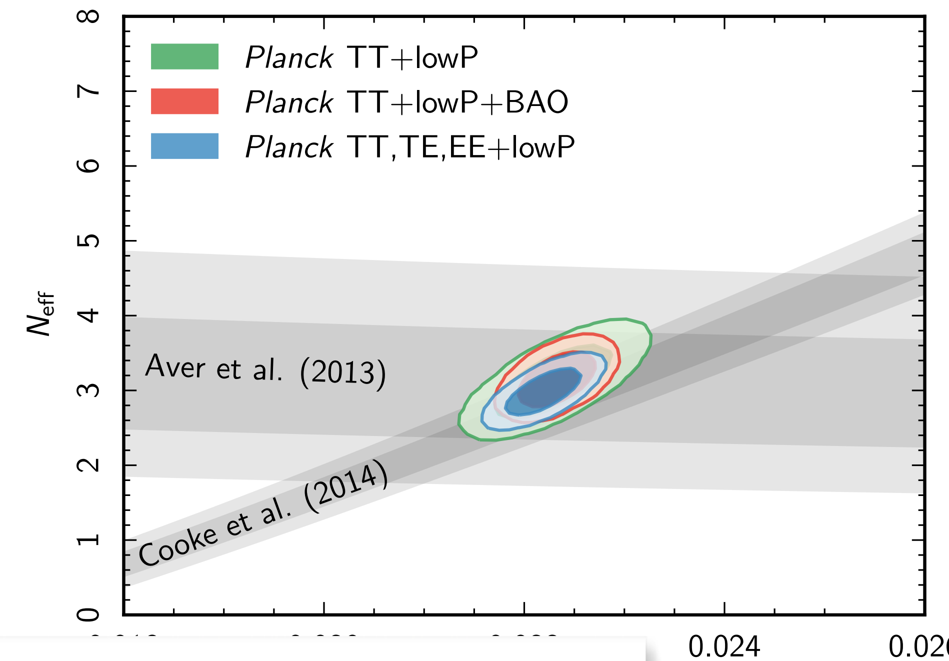




$$\Omega_K = -0.040^{+0.038}_{-0.041} \quad (95\%, \textit{Planck}\ \text{TT,TE,EE+lowP})$$

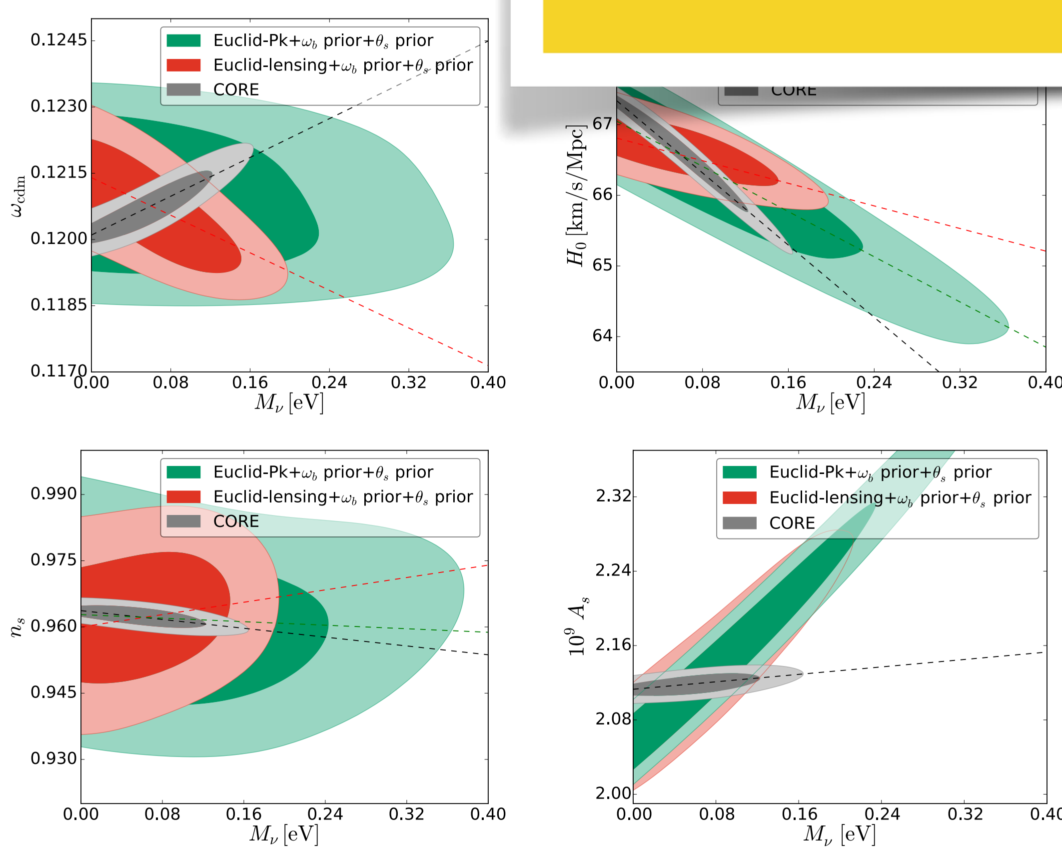
$$\Omega_K = 0.000 \pm 0.005 \quad (95\%,\ \textit{Planck}\ \text{TT+lowP+lensing+BAO})$$

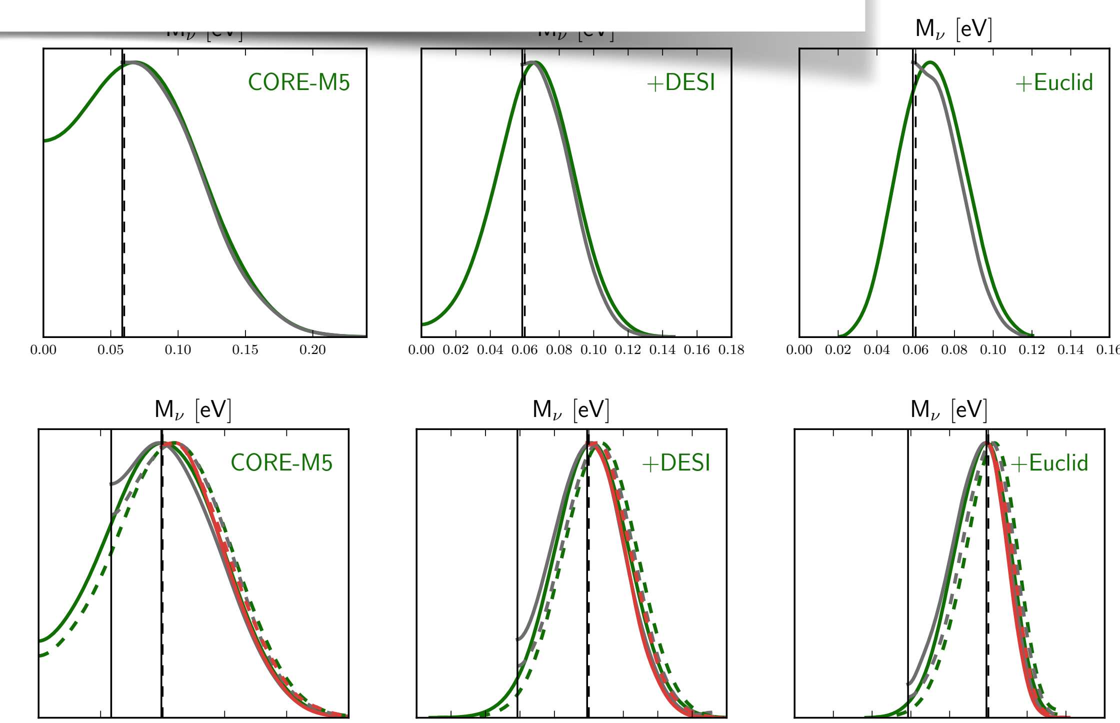




Parameter	LiteCORE-80, TEP	LiteCORE-120, TEP	CORE-M5, TEP	COrE+, TEP
$N_{\rm eff}$	3.06 ± 0.12	3.05 ± 0.10	3.05 ± 0.10	3.048 ± 0.092
$Y_{\rm P}^{\rm BBN}$	0.2463 ± 0.0069	0.2463 ± 0.0057	0.2464 ± 0.0056	0.2465 ± 0.0052
$\Omega_b h^2$	$0.022185^{+0.000085}_{-0.000084}$	0.022182 ± 0.000063	0.022181 ± 0.000058	$0.022183^{+0.000048}_{-0.000050}$
$\Omega_c h^2$	0.1206 ± 0.0018			
$100\theta_{MC}$	1.04067 ± 0.00044			
τ	0.0597 ± 0.0020			
n_s	$0.9620^{+0.0034}_{-0.0033}$			
$\ln(10^{10} A_s)$	$3.0565^{+0.0051}_{-0.0047}$			
H_0 [km/s/Mpc]	$67.00^{+0.74}_{-0.75}$			
σ_8	0.8175 ± 0.0037			

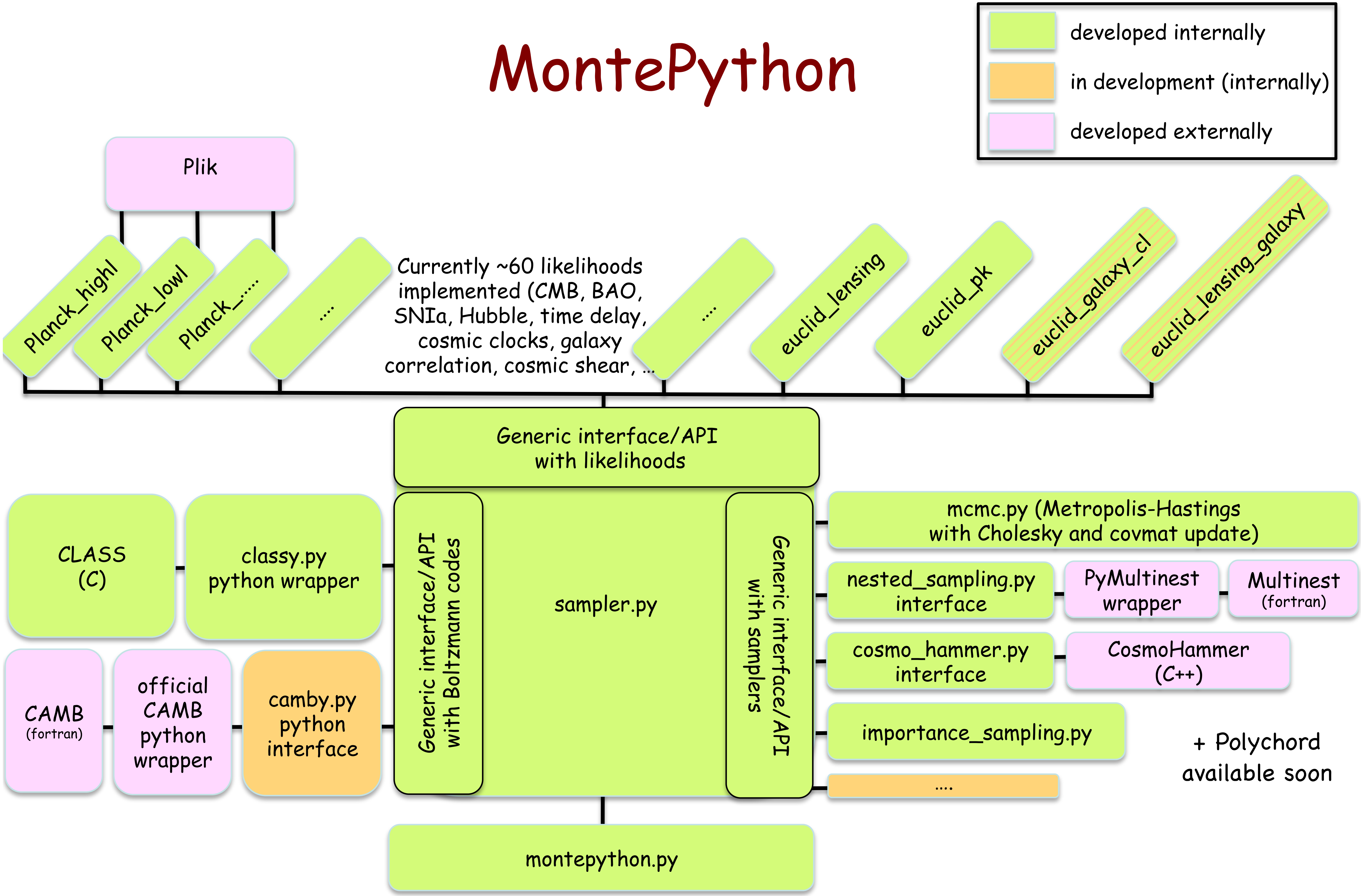
Bayesian parameter extraction codes





- Bayes theorem and inversion
- Metropolis-Hastings
- Proposal density, acceptance rate
- Covmat, jumping factor, covmat update, jumping update
- Number of chains: convergence, independence, communication
- Cycles, eigenvectors, fast/slow, Cholesky

MontePython



```
#------Experiments to test (separated with commas)-----
data.experiments=['Planck_highl','Planck_lowl','Planck_lensing']

#------ Settings for the over-sampling.
data.over_sampling=[1, 4]

#------ Parameter list -------
# Cosmological parameters list

data.parameters['omega_b']      = [ 2.2253,   None, None,  0.028, 0.01, 'cosmo']
data.parameters['omega_cdm']    = [0.11919,   None, None, 0.0027,    1, 'cosmo']
data.parameters['100*theta_s']  = [ 1.0418,   None, None,   3e-4,    1, 'cosmo']
data.parameters['ln10^{10}A_s'] = [ 3.0753,   None, None, 0.0029,    1, 'cosmo']
data.parameters['n_s']          = [0.96229,   None, None, 0.0074,    1, 'cosmo']
data.parameters['tau_reio']     = [0.09463,   0.04, None,  0.013,    1, 'cosmo']

# Nuisance parameter list
data.parameters['A_cib_217']         = [     61,    0,  200,     7,     1,'nuisance']
.....
data.parameters['A_planck']          = [100.028,   90,  110,  0.25,  0.01,'nuisance']

# Derived parameters

data.parameters['z_reio']          = [1, None, None, 0,     1,   'derived']
...
data.parameters['sigma8']          = [0, None, None, 0,     1,   'derived']

# Other cosmo parameters (fixed parameters, precision parameters, etc.)
data.cosmo_arguments['k_pivot'] = 0.05
data.cosmo_arguments['N_ur'] = 2.0328
data.cosmo_arguments['N_ncdm'] = 1
data.cosmo_arguments['m_ncdm'] = 0.06

# These two are required to get sigma8 as a derived parameter
# (class must compute the P(k) until sufficient k)
data.cosmo_arguments['output'] = 'mPk'
data.cosmo_arguments['P_k_max_h/Mpc'] = 1.
```

Run and analyse

```
mpr -p base2015.param -o saopaolo_test/ -N 1000 -b bestfit/base2015.bestfit -c covmat/base2015.covmat
mpr -p base2015.param -o saopaolo_test/ -N 1000 -b bestfit/base2015.bestfit -c covmat/base2015.covmat
mpr -p base2015.param -o saopaolo_test/ -N 1000 -b bestfit/base2015.bestfit -c covmat/base2015.covmat

...

mpi info saopaolo_test/
```

Parameter best fit, mean, convergence R-1

Bestfit and covmat files

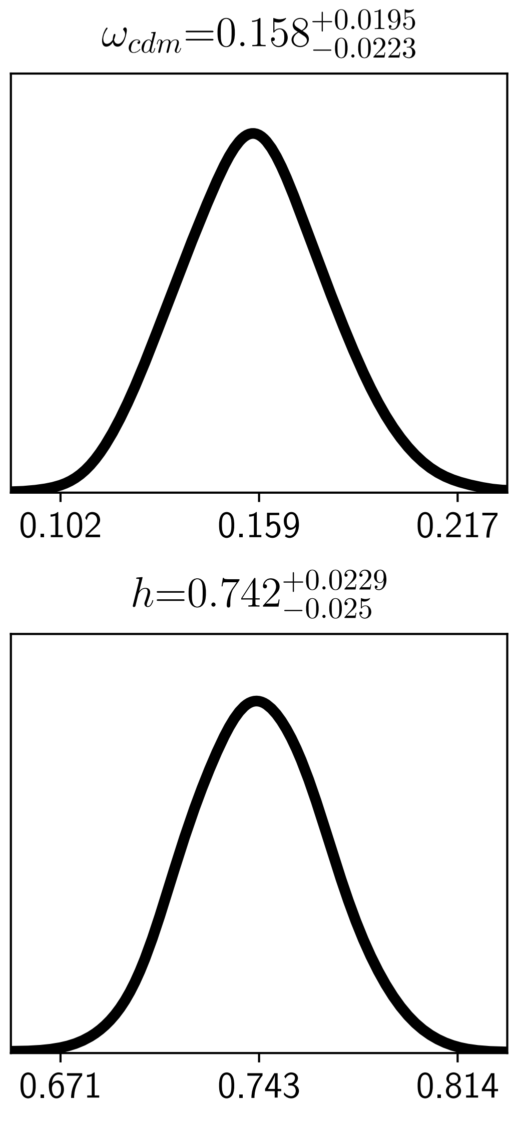
Table in latex

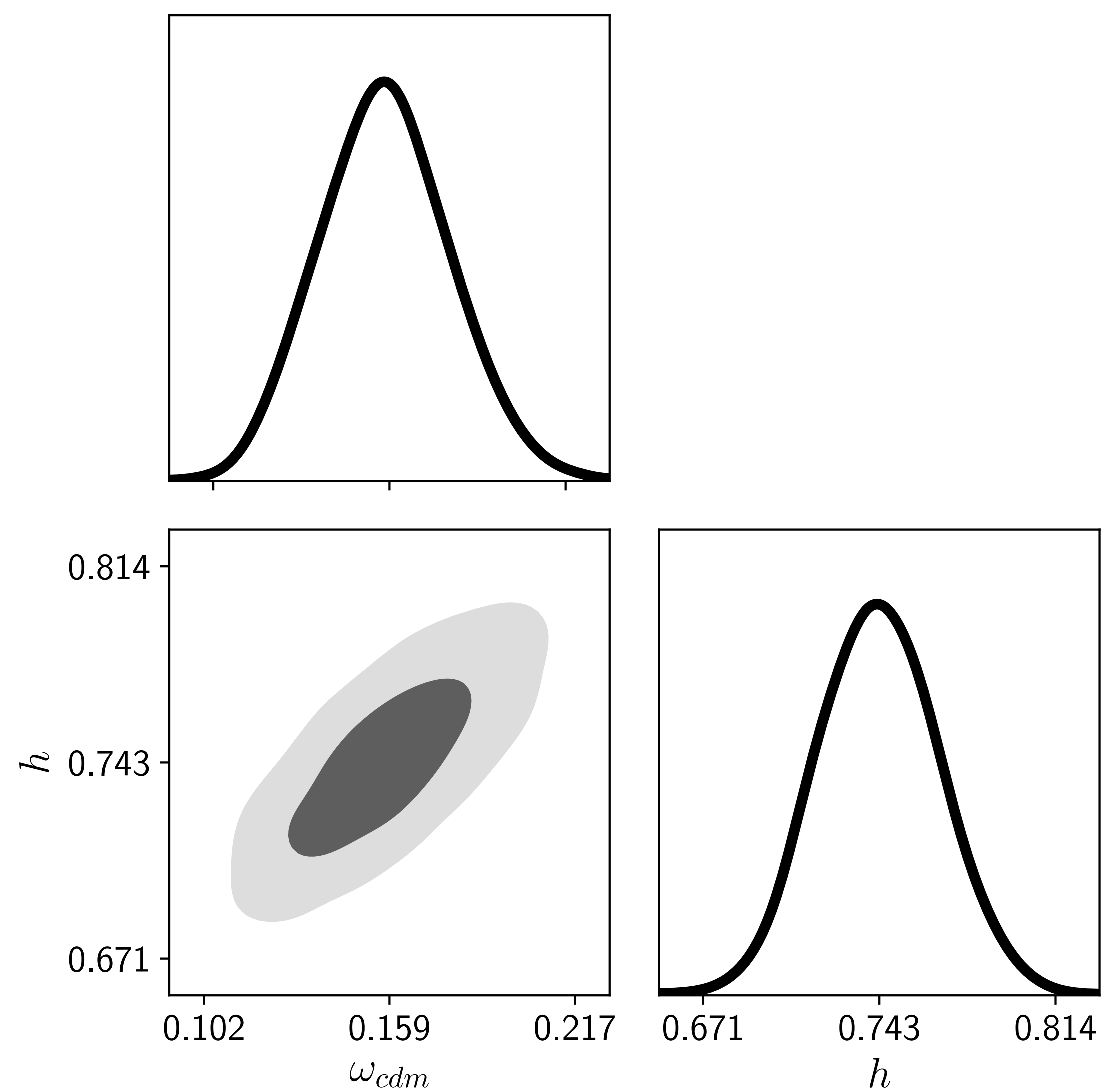
Triangle plot

1D probability plot

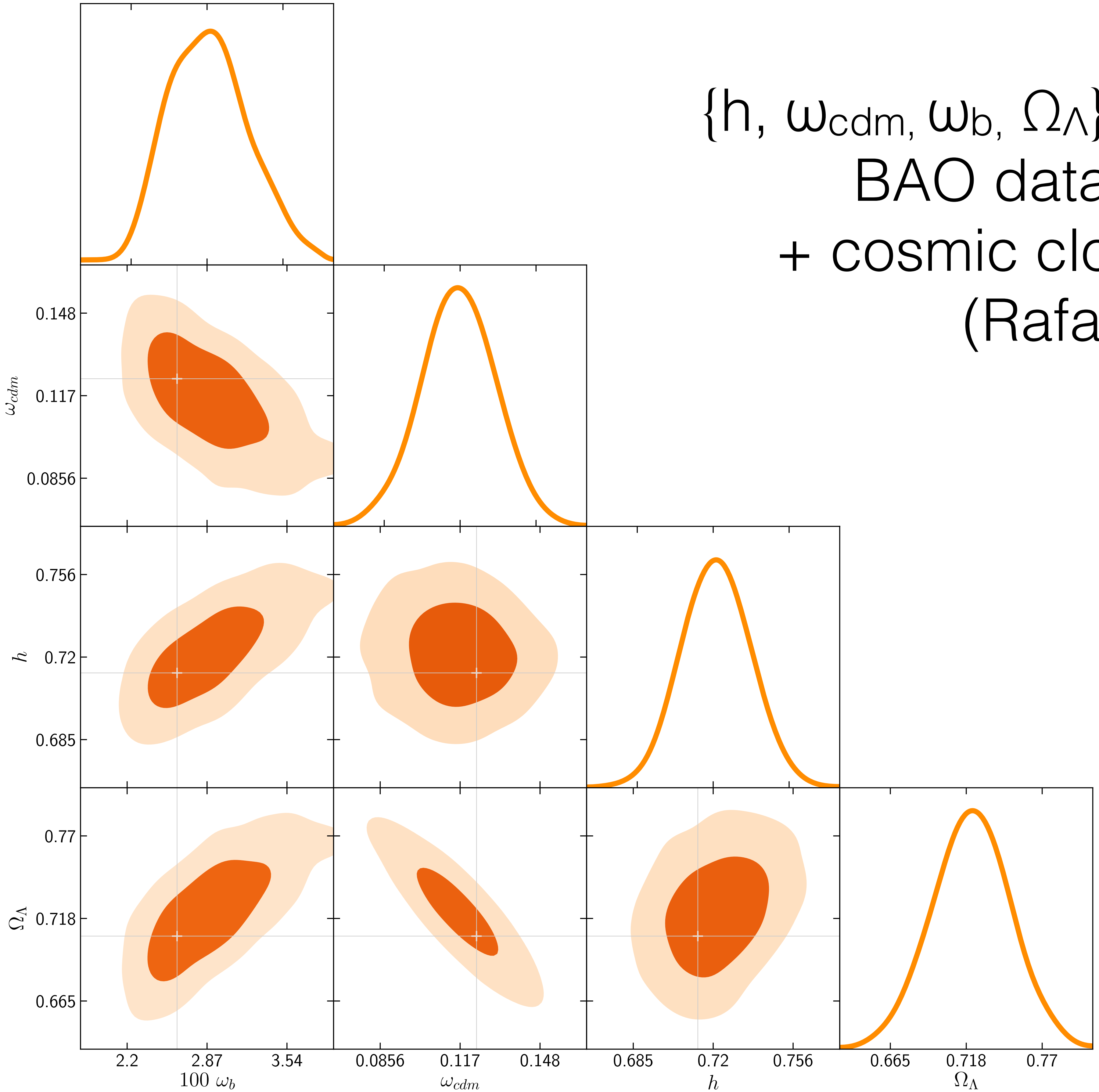
log.param file for next runs and records

{h, ω_{cdm}} from various BAO data + HST (Jorge & Javier):

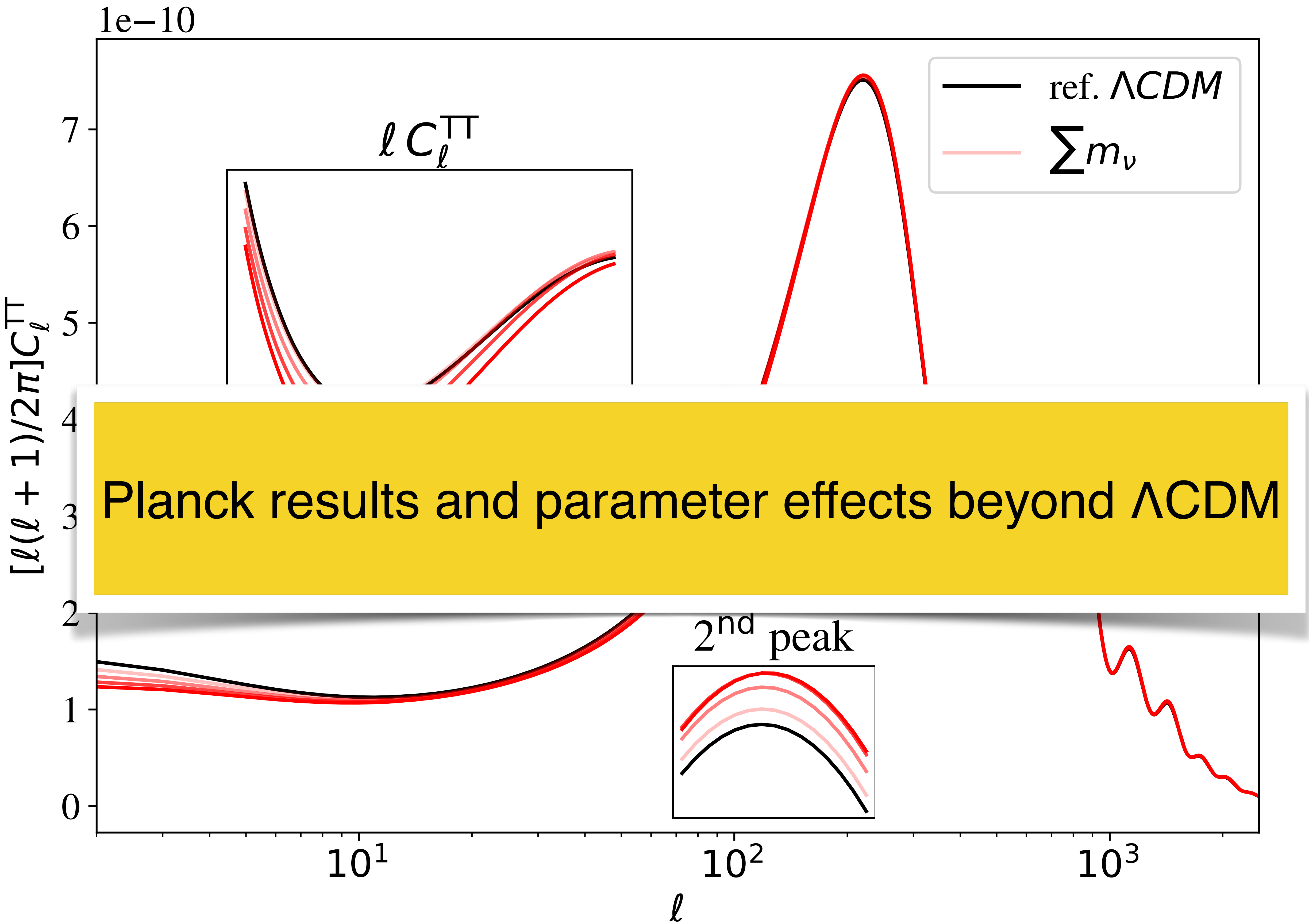




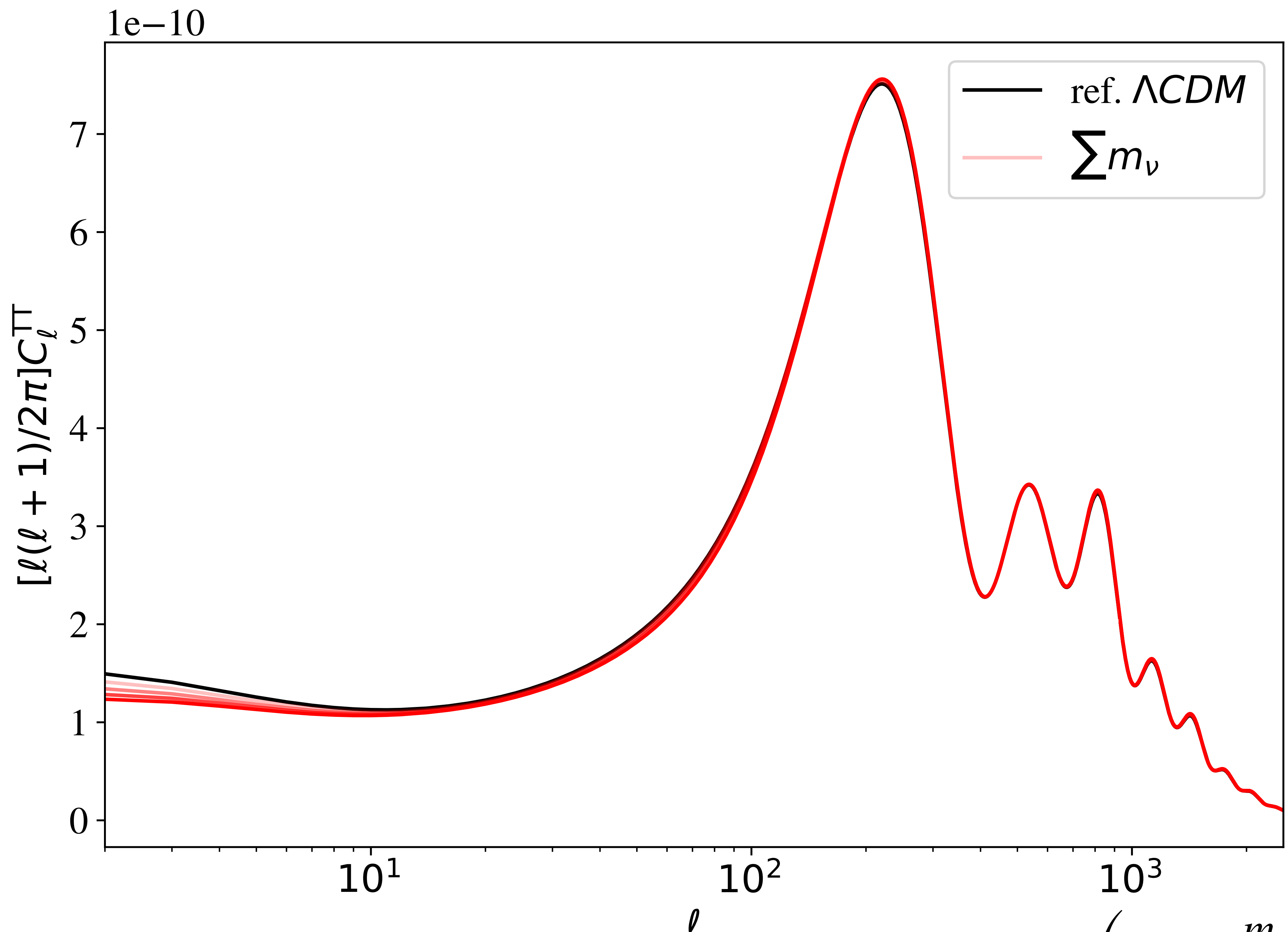
{h, ω_{cdm}, ω_b, Ω_Λ} from various BAO data + HST + cosmic clocks (H(z)) (Rafael):



Planck results and parameter effects beyond ΛCDM



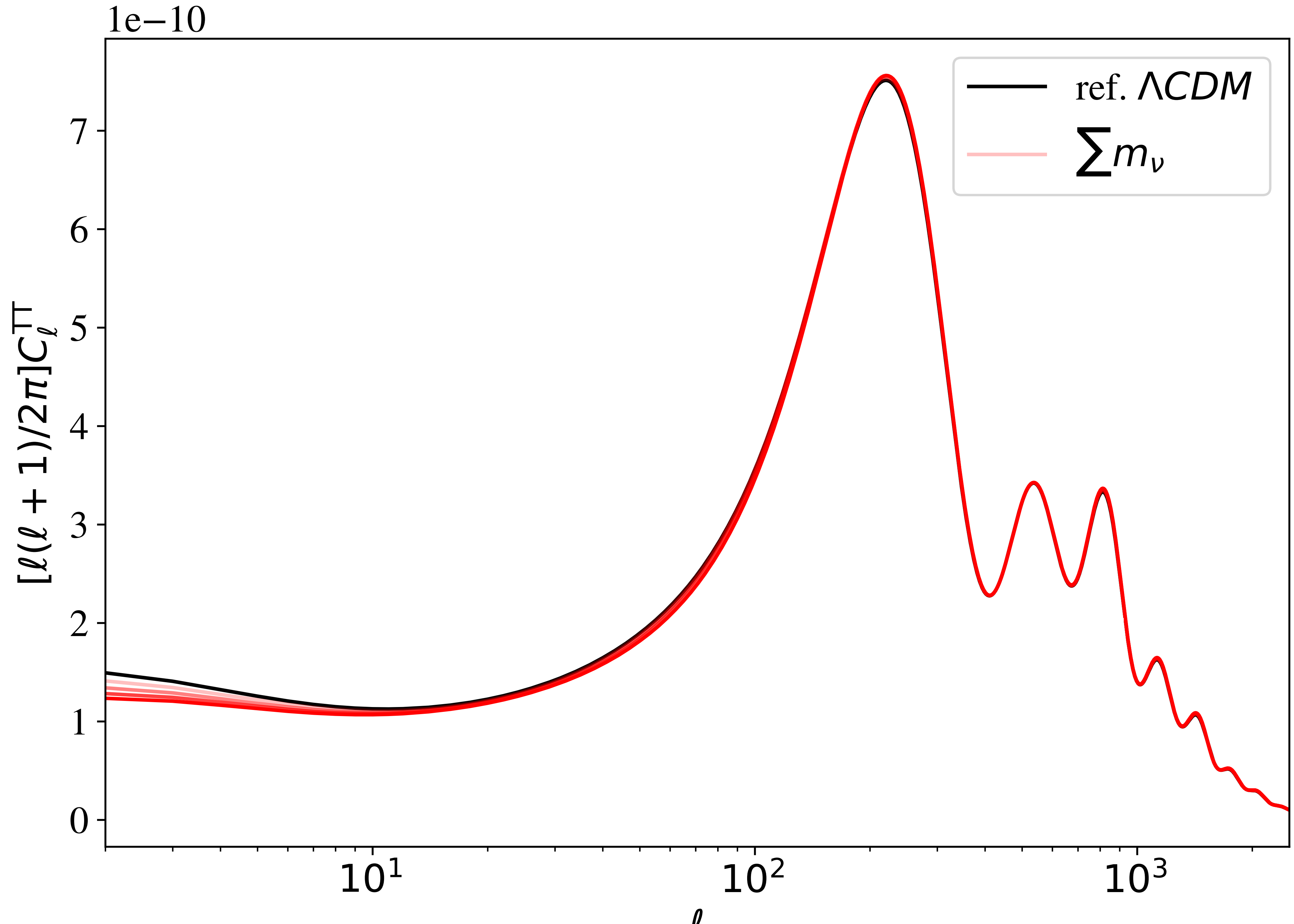
ΛCDM + M_ν



Cannot exaggerate the effects with large masses…

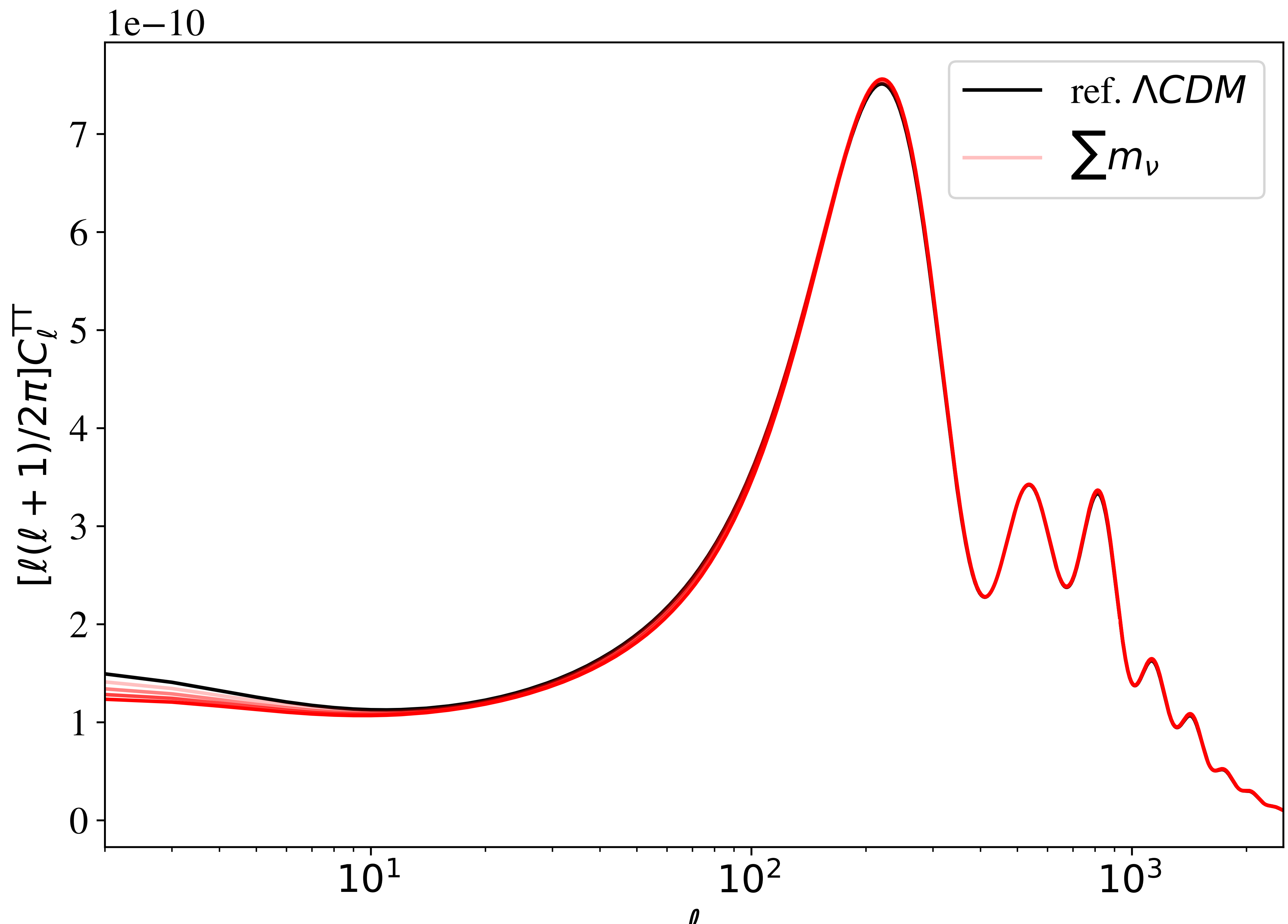
$$z_{\rm nr} = \left(\frac{m_\nu}{5.28 \times 10^{-4}\ {\rm eV}}\right) - 1$$

ΛCDM + M_ν



Fixed parameters: $\{\omega_b, \cancel{\omega_m, \Omega_\Lambda}, \tau_{\rm reio}, A_s, n_s\}$

ΛCDM + Mν

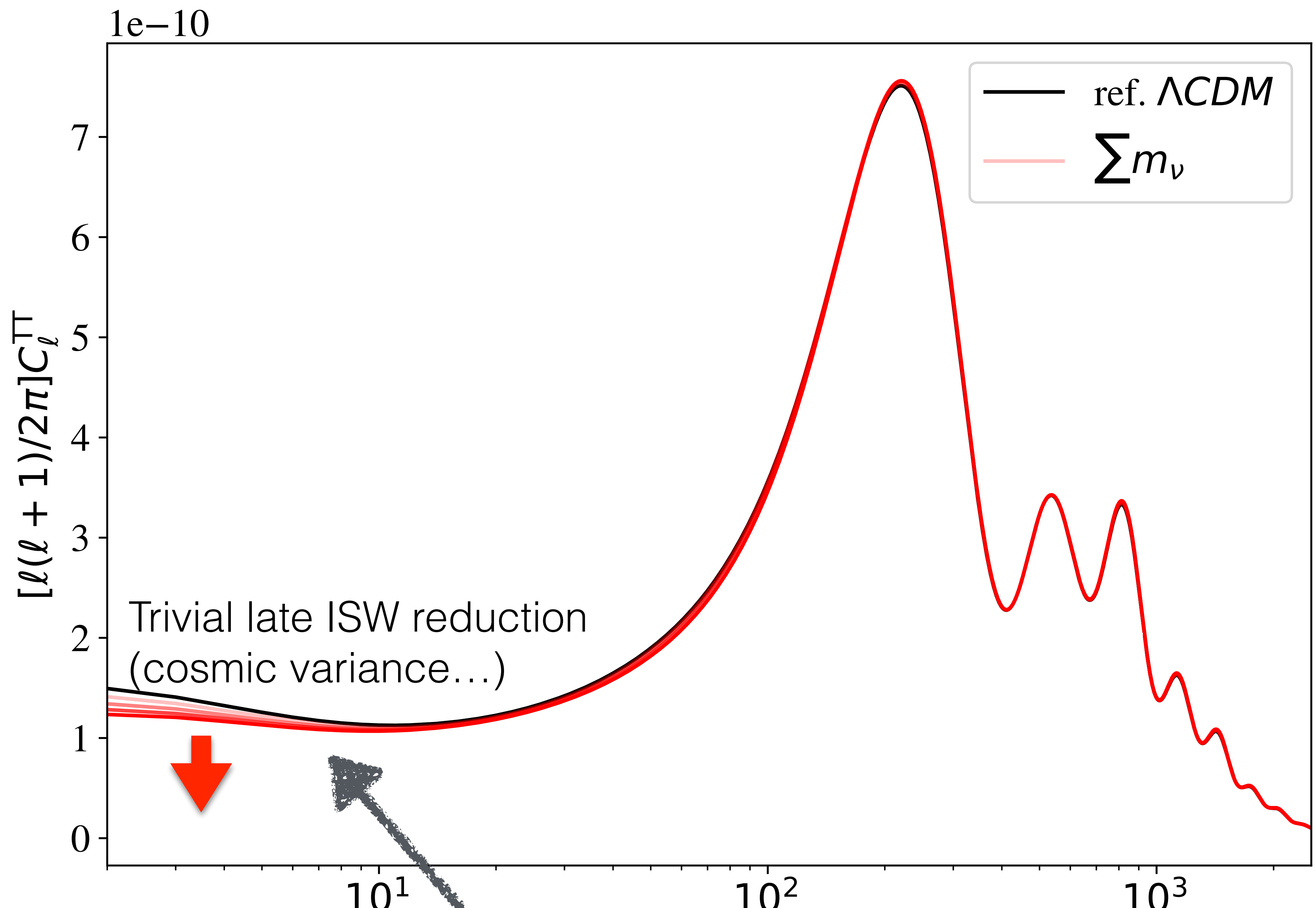


Fixed parameters: $\{\omega_b, \cancel{\omega_m, \Omega_\Lambda}, \tau_{\rm reio}, A_s, n_s\}$

ω_{dm}, θ_s

fixed $z_{\rm eq}$, not z_Λ

ΛCDM + M_ν



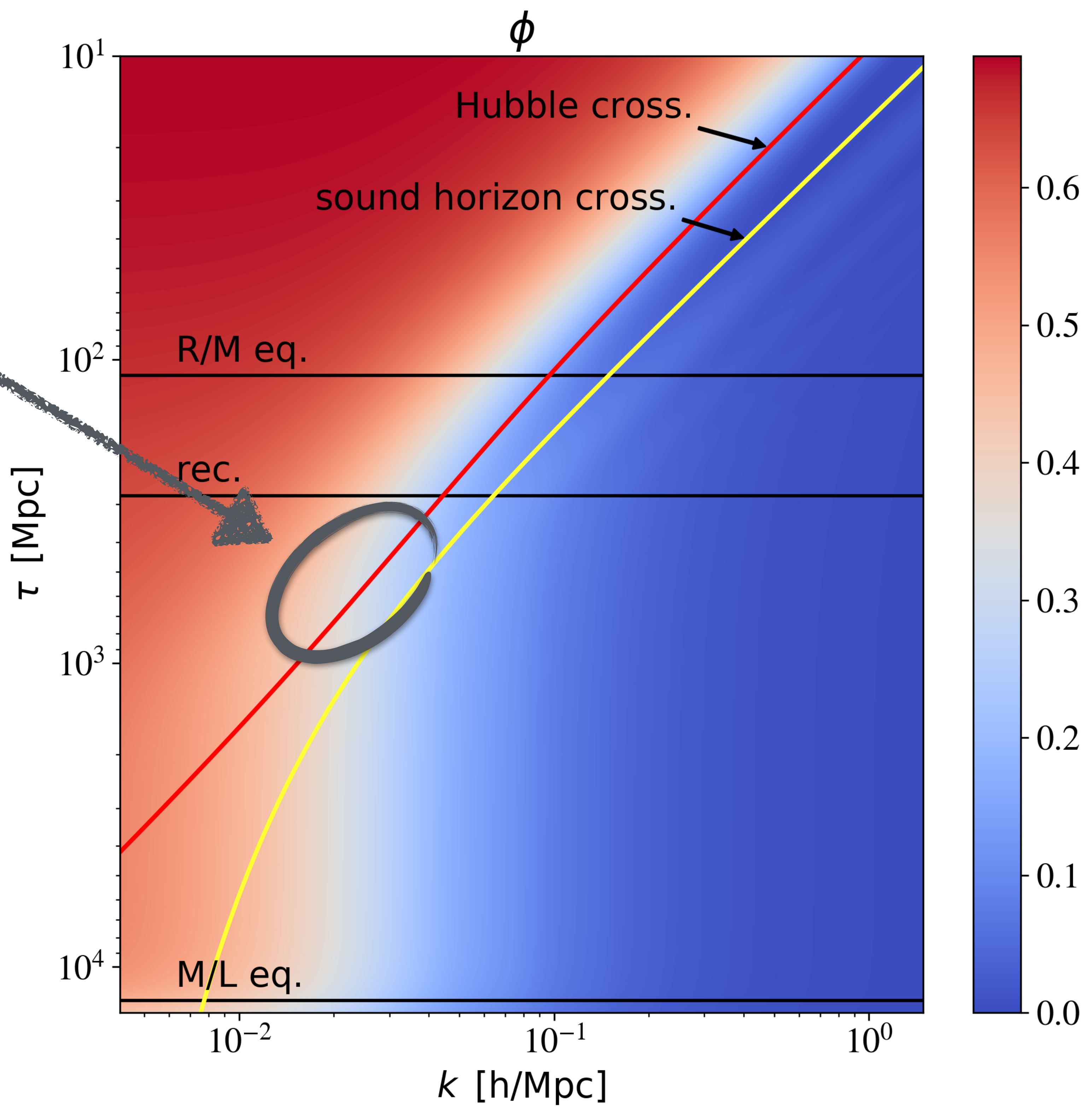
Fixed parameters: $\{\omega_b, \cancel{\omega_m, \Omega_\Lambda}, \tau_{\rm reio}, A_s, n_s\}$

ω_{dm}, θ_s

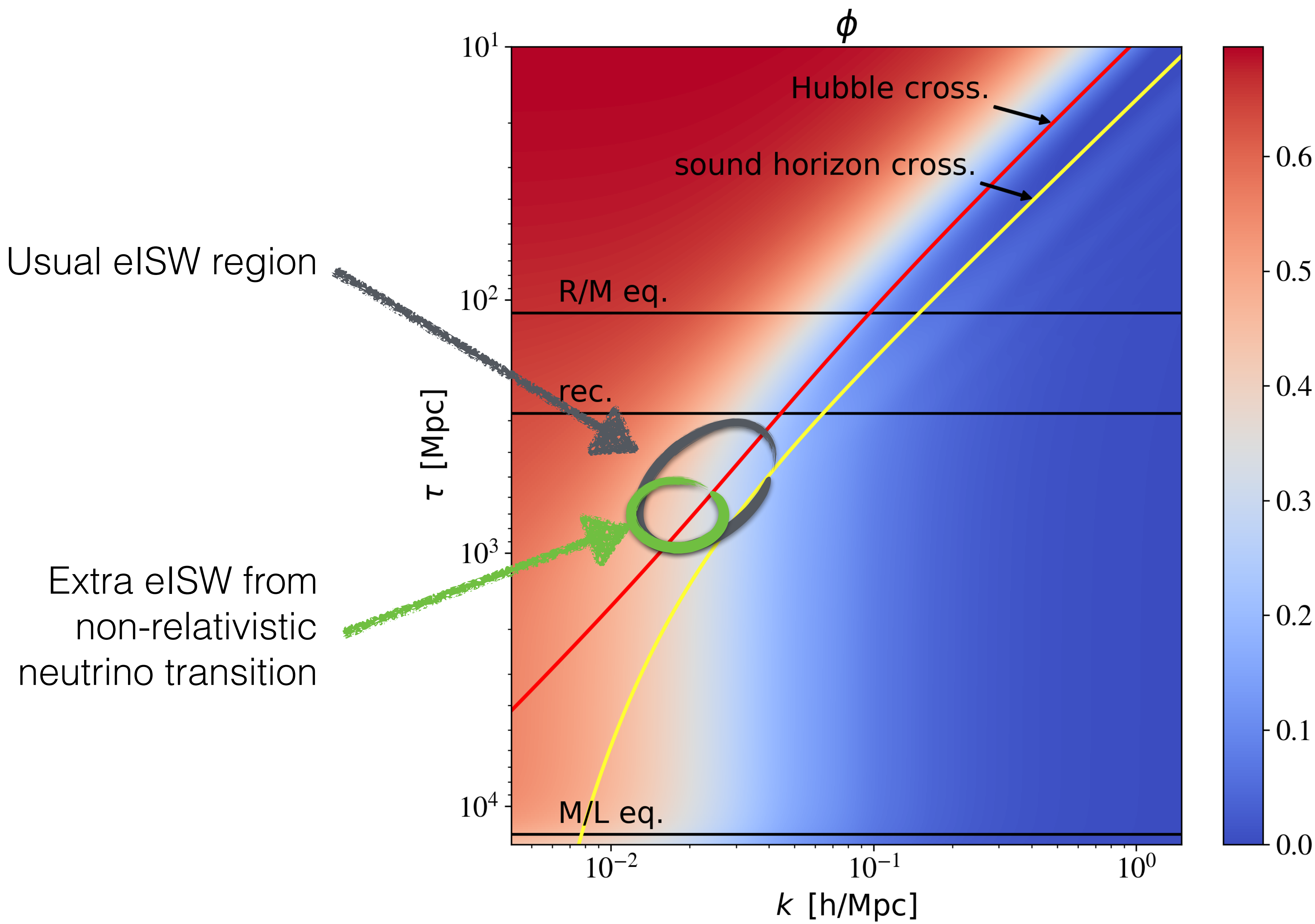
fixed $z_{\rm eq}$, not z_Λ

ΛCDM + M$_\nu$

Usual eISW region

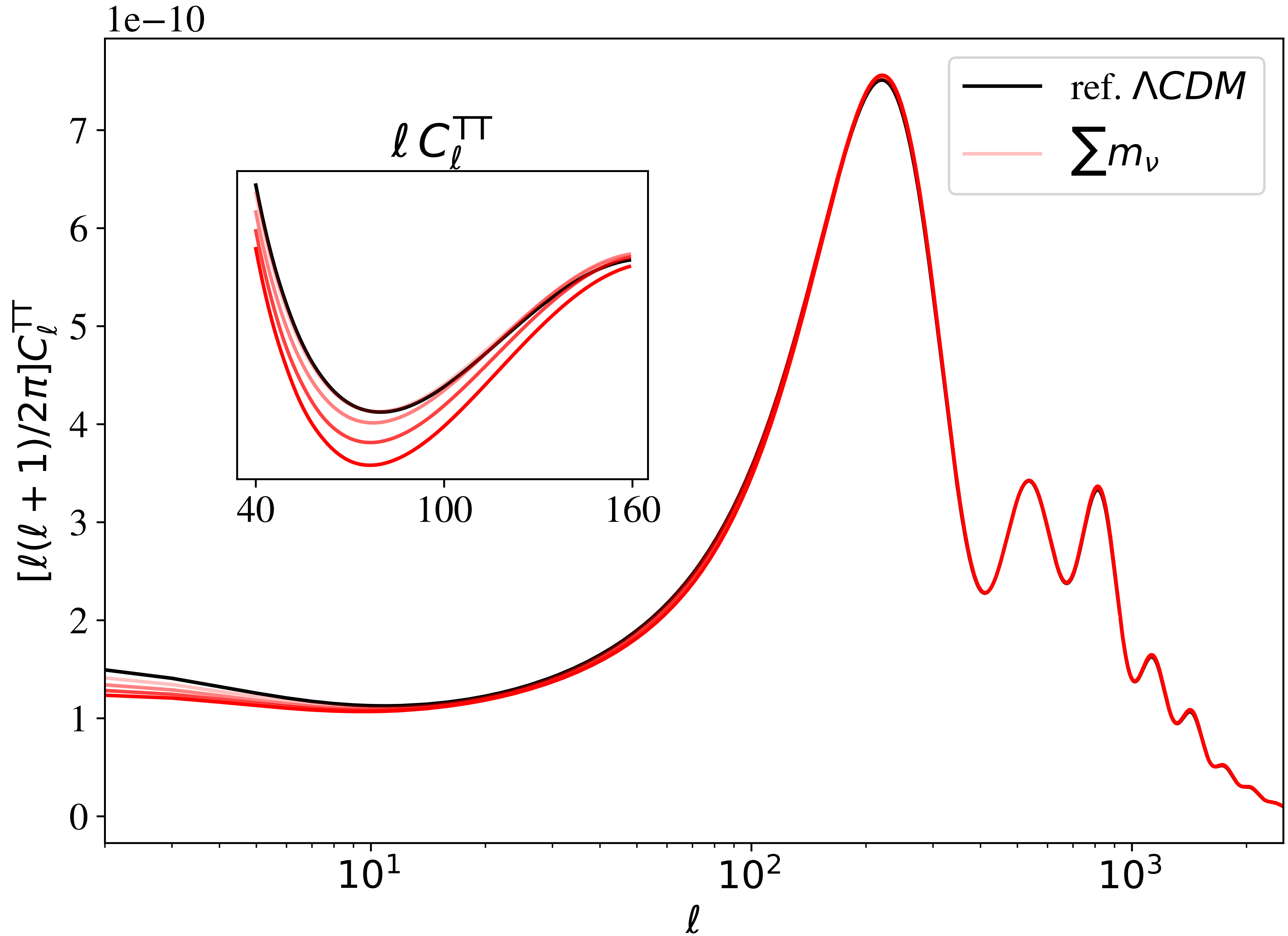


ΛCDM + Mν



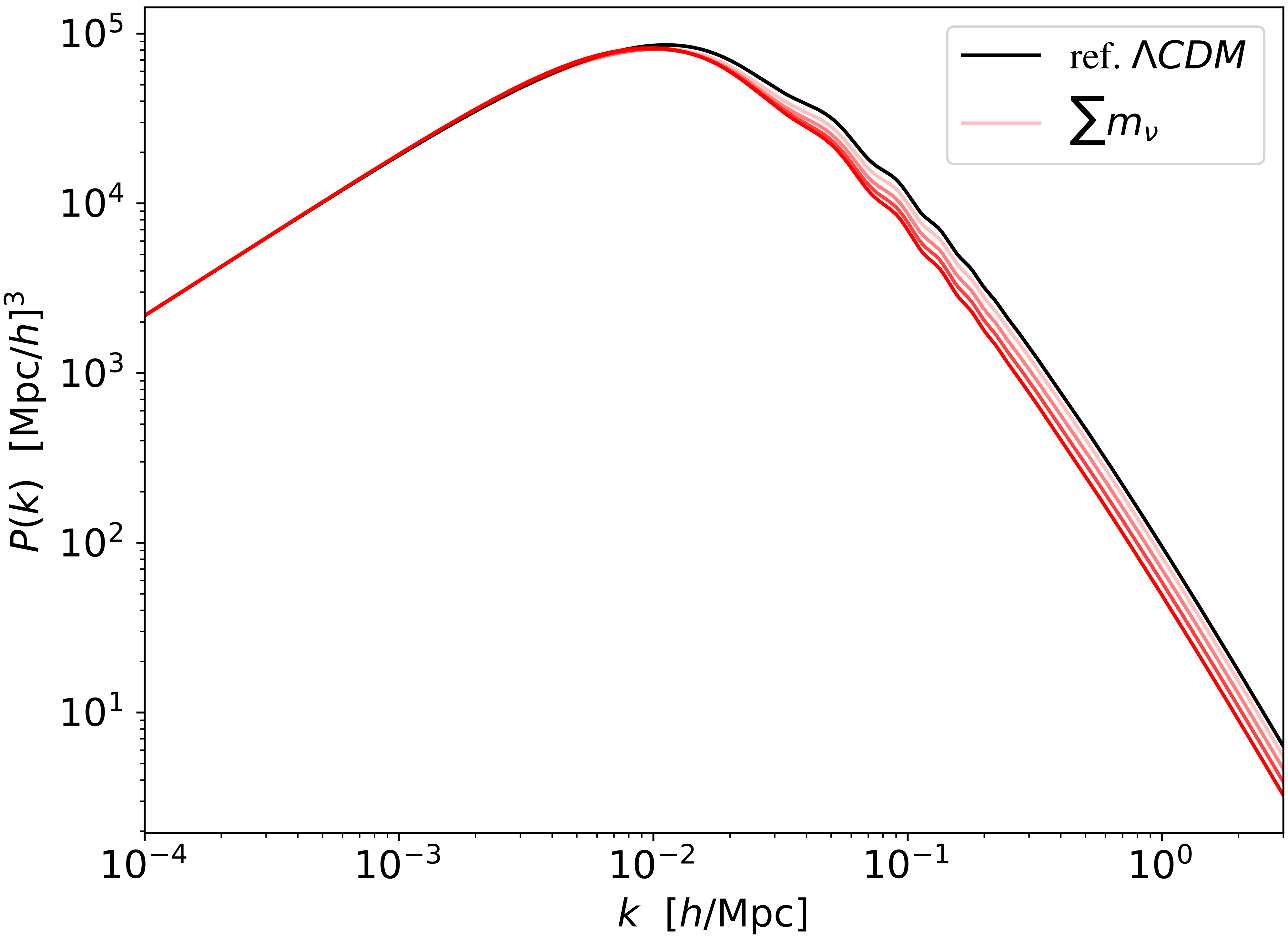
Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

ΛCDM + M_ν



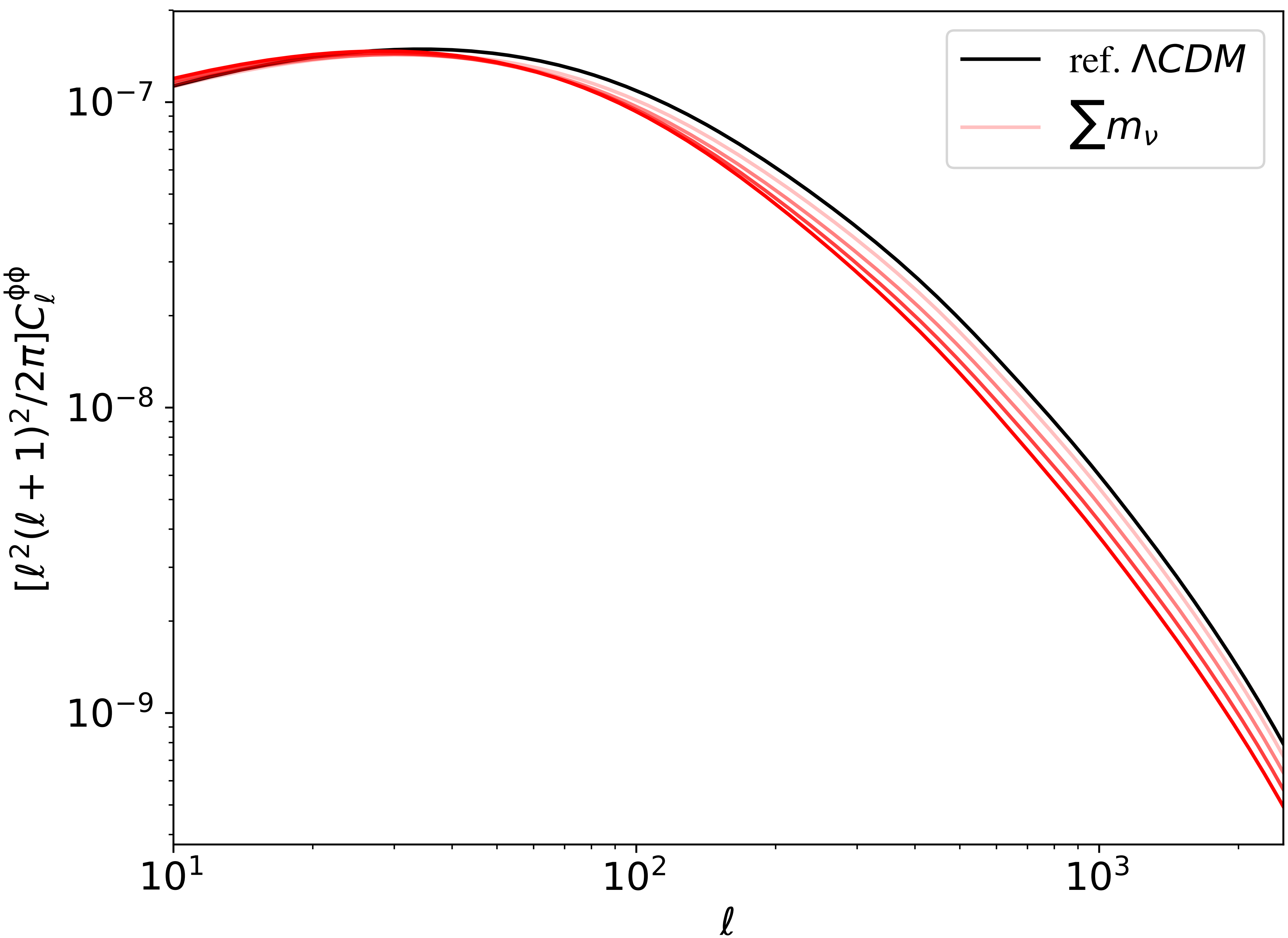
Neutrino Cosmology, CUP 2013, JL, Mangano, Miele, Pastor

ΛCDM + M_ν



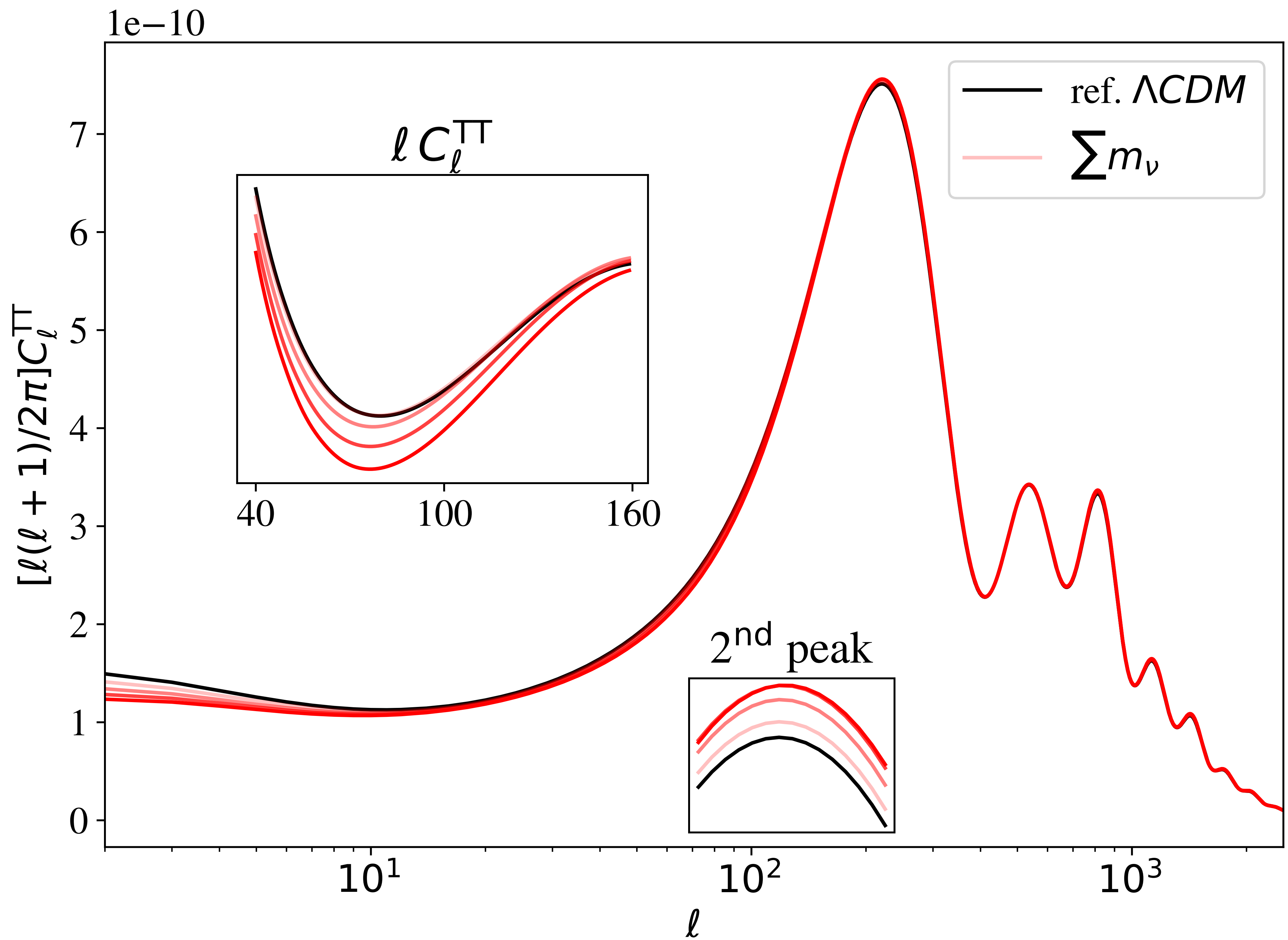
$1 - 8\omega_\nu/\omega_m$ Hu, Eisenstein, Tegmark 1997; Lesgourgues & Pastor 2006

ΛCDM + M_ν



ref. ΛCDM
$\sum m_\nu$
$[\ell^2(\ell+1)^2/2\pi]C_\ell^{\phi\phi}$
10^{-7}
10^{-8}
10^{-9}
10^1
10^2
10^3
ℓ

ΛCDM + M_ν



1e−10
ref. ΛCDM
$\sum m_\nu$
$\ell\, C_\ell^{TT}$
40
100
160
2^{nd} peak
$[\ell(\ell+1)/2\pi]C_\ell^{TT}$
0
1
2
3
4
5
6
7
10^1
10^2
10^3
ℓ

ΛCDM + M_ν

$$\sum m_\nu < 0.72\,\mathrm{eV} \quad \text{(PlanckTT+lowP)}$$

$$\sum m_\nu < 0.59\,\mathrm{eV} \quad \text{(PlanckTT+SimLow)}$$

$$\sum m_\nu < 0.34\,\mathrm{eV} \quad \text{(PlanckTTTEEE+SimLow)}$$

$$\sum m_\nu < 0.17\,\mathrm{eV} \quad \text{(PlanckTT+SimLow+lensing+BAO)}$$

$$\sum m_\nu < 0.14\,\mathrm{eV} \quad \text{(PlanckTTTEEE+SimLow+lensing)}$$

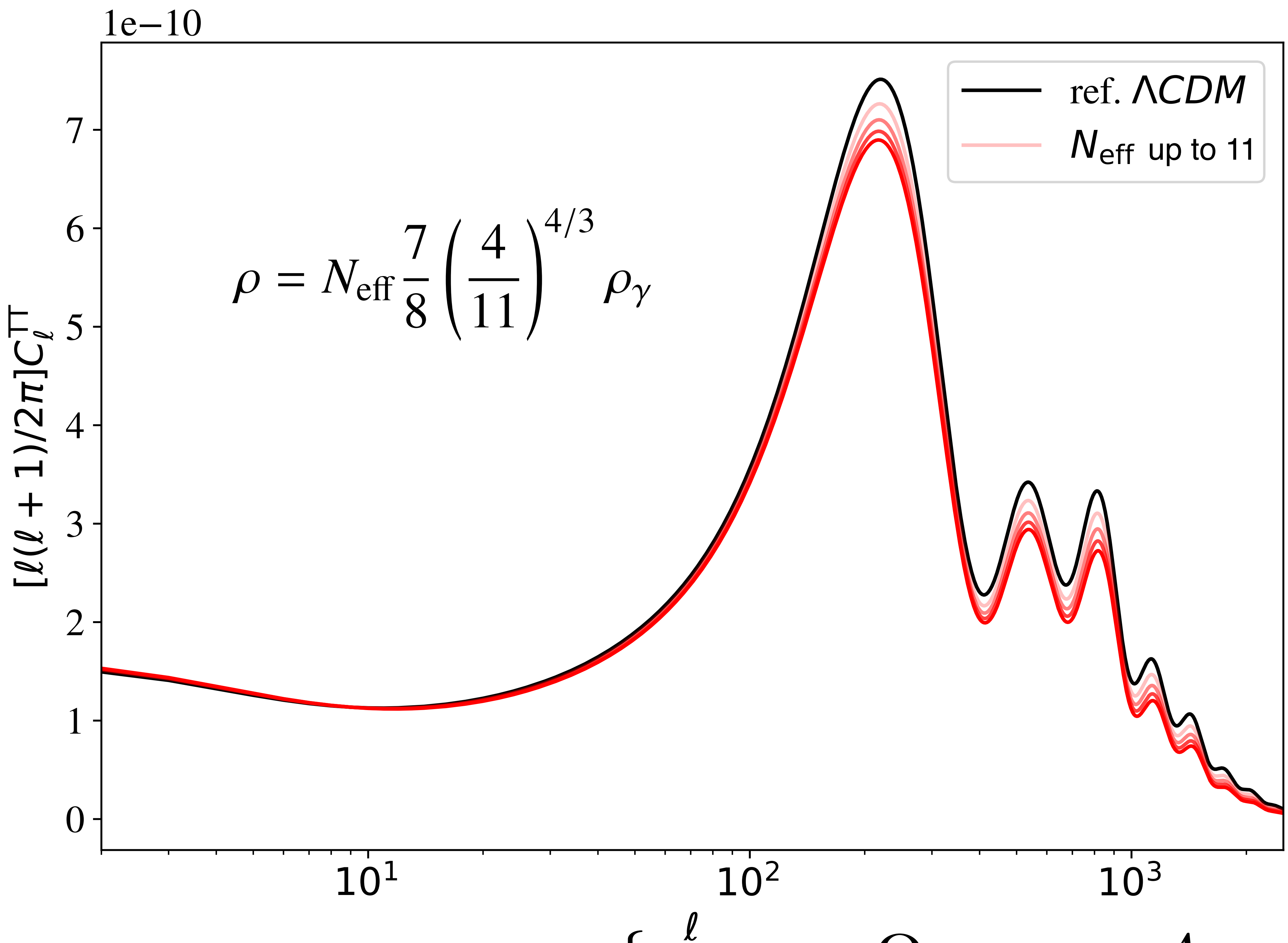
Planck 2016 intermediate results 1605.02985

all at 95%CL

Institute for Theoretical Particle Physics and Cosmology

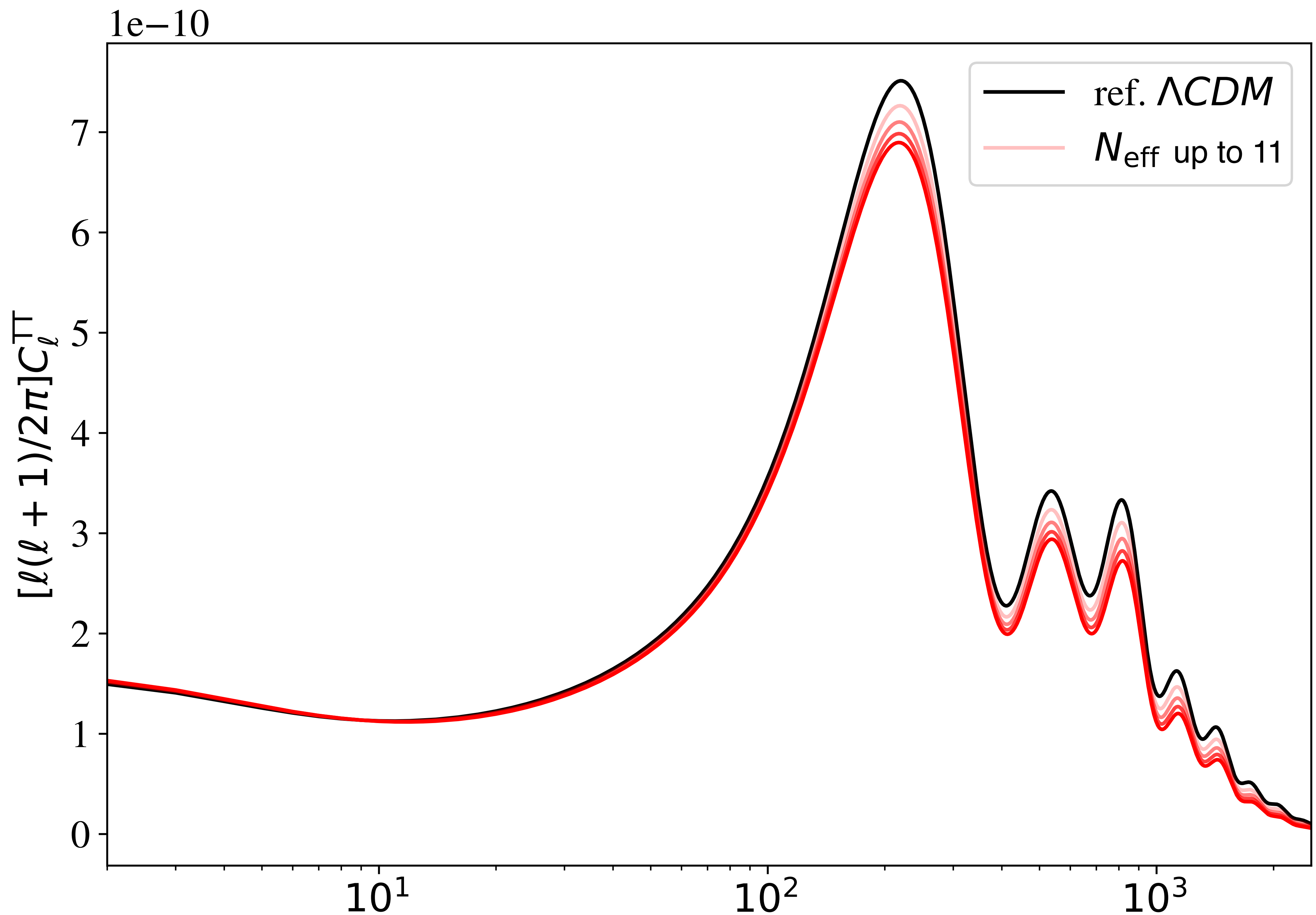
RWTH AACHEN UNIVERSITY

ΛCDM + $N_{\rm eff}$



Fixed parameters: $\{\omega_b, \cancel{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$

ΛCDM + $N_{\rm eff}$

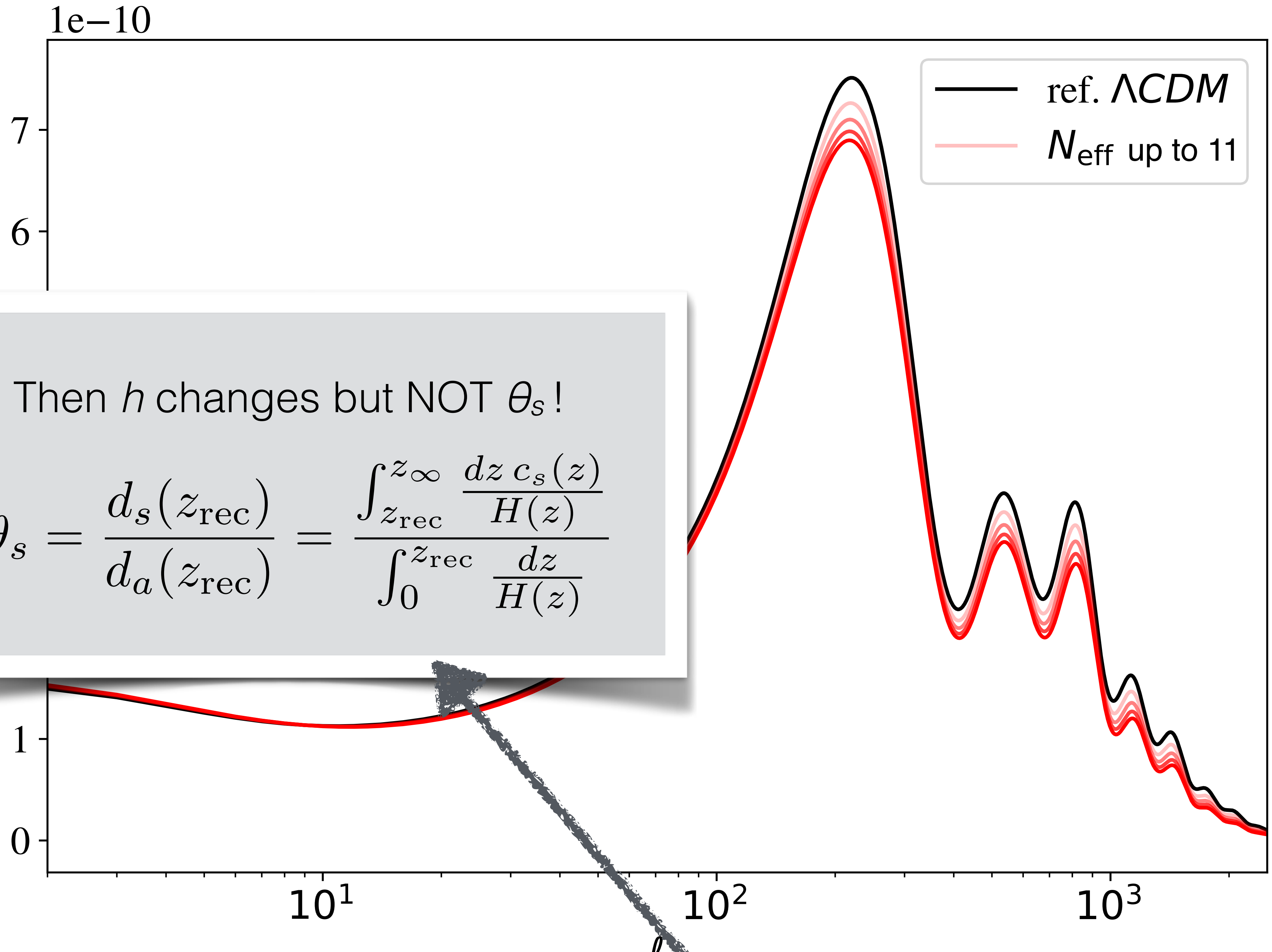


Fixed parameters: $\{\omega_b, \cancel{\omega_m}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$

$z_{\rm eq}$

(ρ_r, ρ_m, ρ_Λ) rescaled equally

ΛCDM + N_{eff}



Then h changes but NOT θ_s !

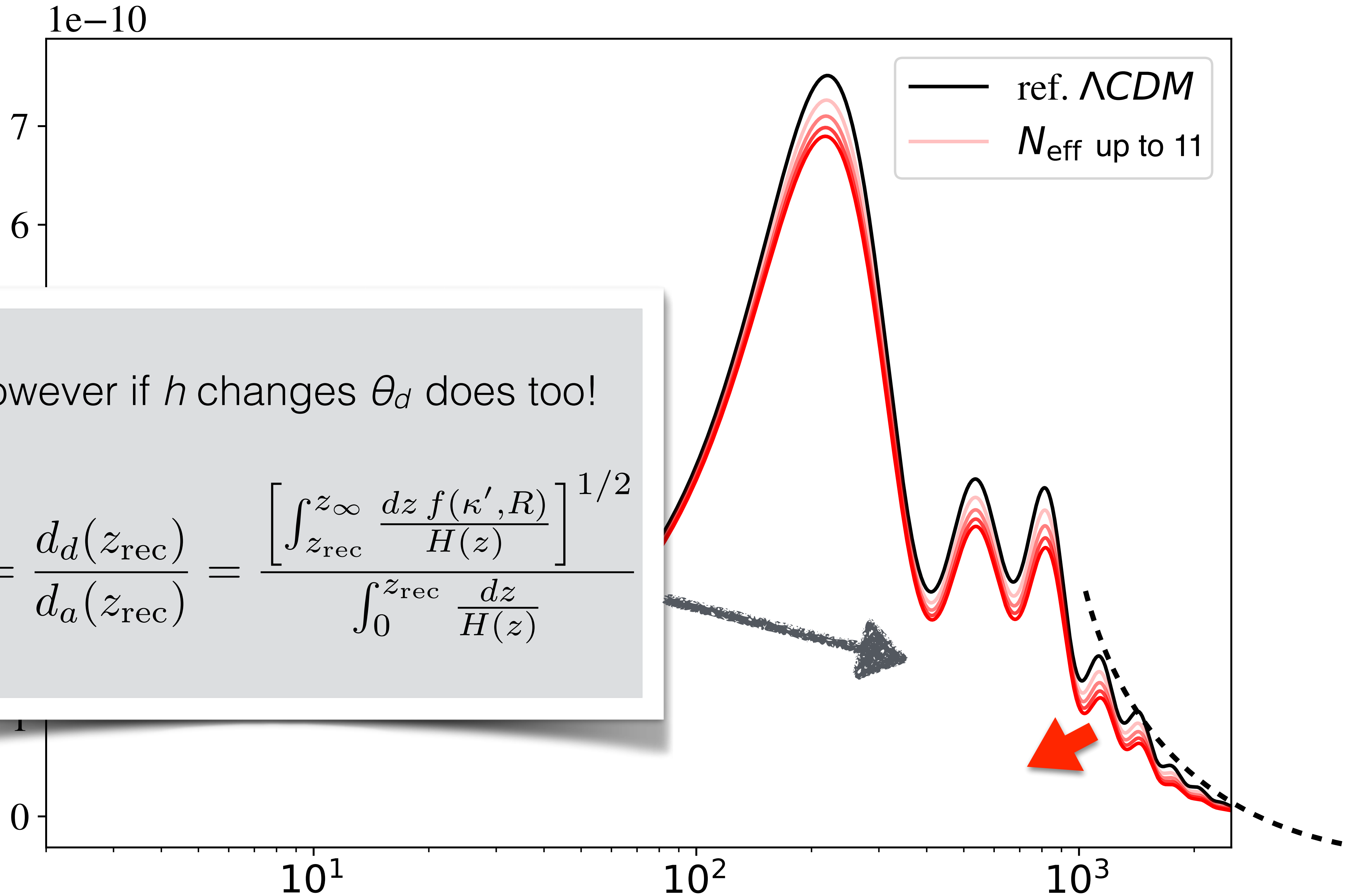
$$\theta_s = \frac{d_s(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{\int_{z_{\rm rec}}^{z_\infty} \frac{dz\, c_s(z)}{H(z)}}{\int_0^{z_{\rm rec}} \frac{dz}{H(z)}}$$

Fixed parameters: $\{\omega_b, \underset{z_{\rm eq}}{\cancel{\omega_m}}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$

(ρ_r, ρ_m, ρ_Λ) rescaled equally

Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

ΛCDM + $N_{\rm eff}$



However if h changes θ_d does too!

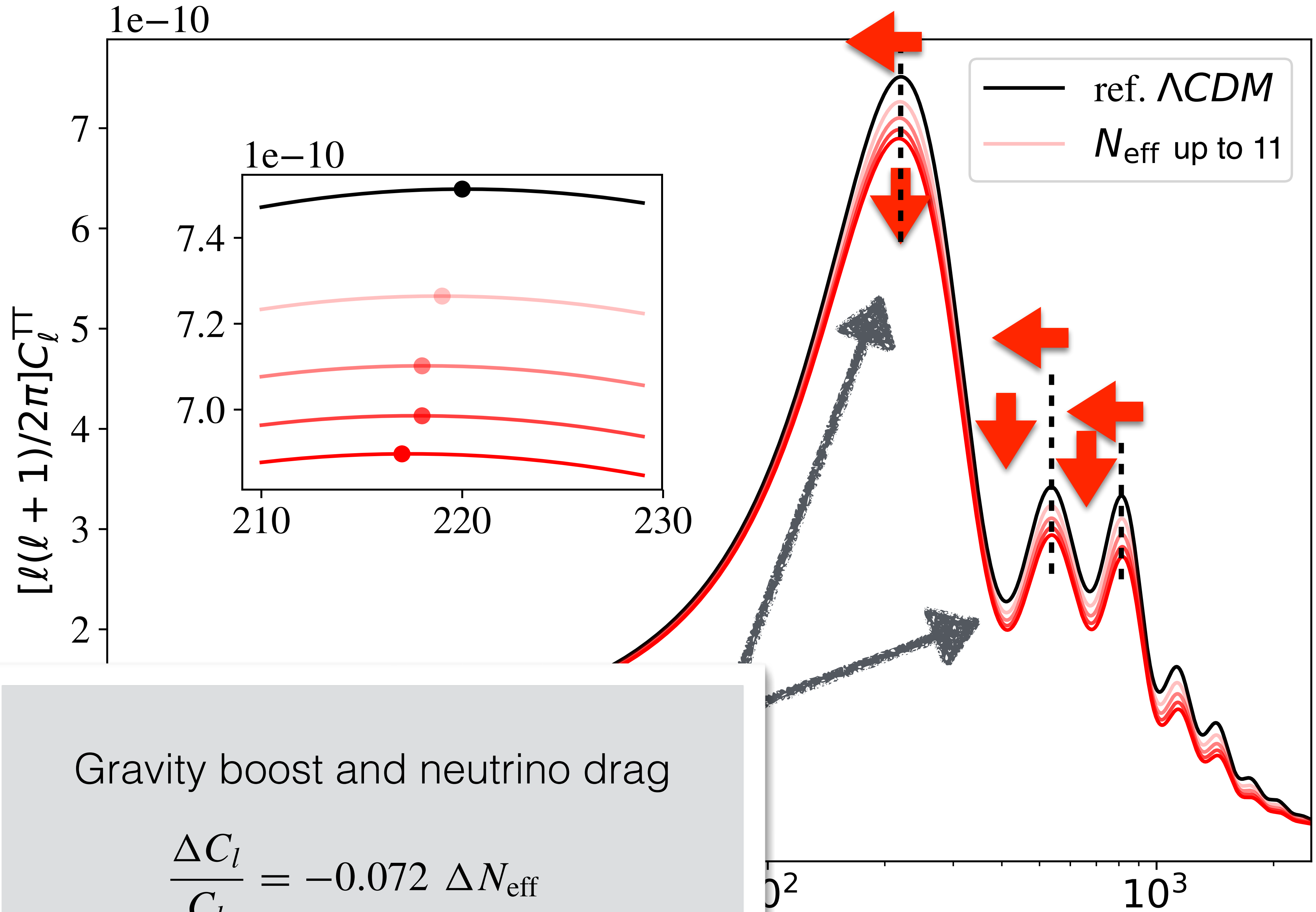
$$\theta_d = \frac{d_d(z_{\rm rec})}{d_a(z_{\rm rec})} = \frac{\left[\int_{z_{\rm rec}}^{z_\infty} \frac{dz\, f(\kappa', R)}{H(z)}\right]^{1/2}}{\int_0^{z_{\rm rec}} \frac{dz}{H(z)}}$$

Fixed parameters: $\{\omega_b^{\ell}, \underset{z_{\rm eq}}{\cancel{\omega_m}}, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$

(ρ_r, ρ_m, ρ_Λ) rescaled equally

Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

ΛCDM + $N_{\rm eff}$



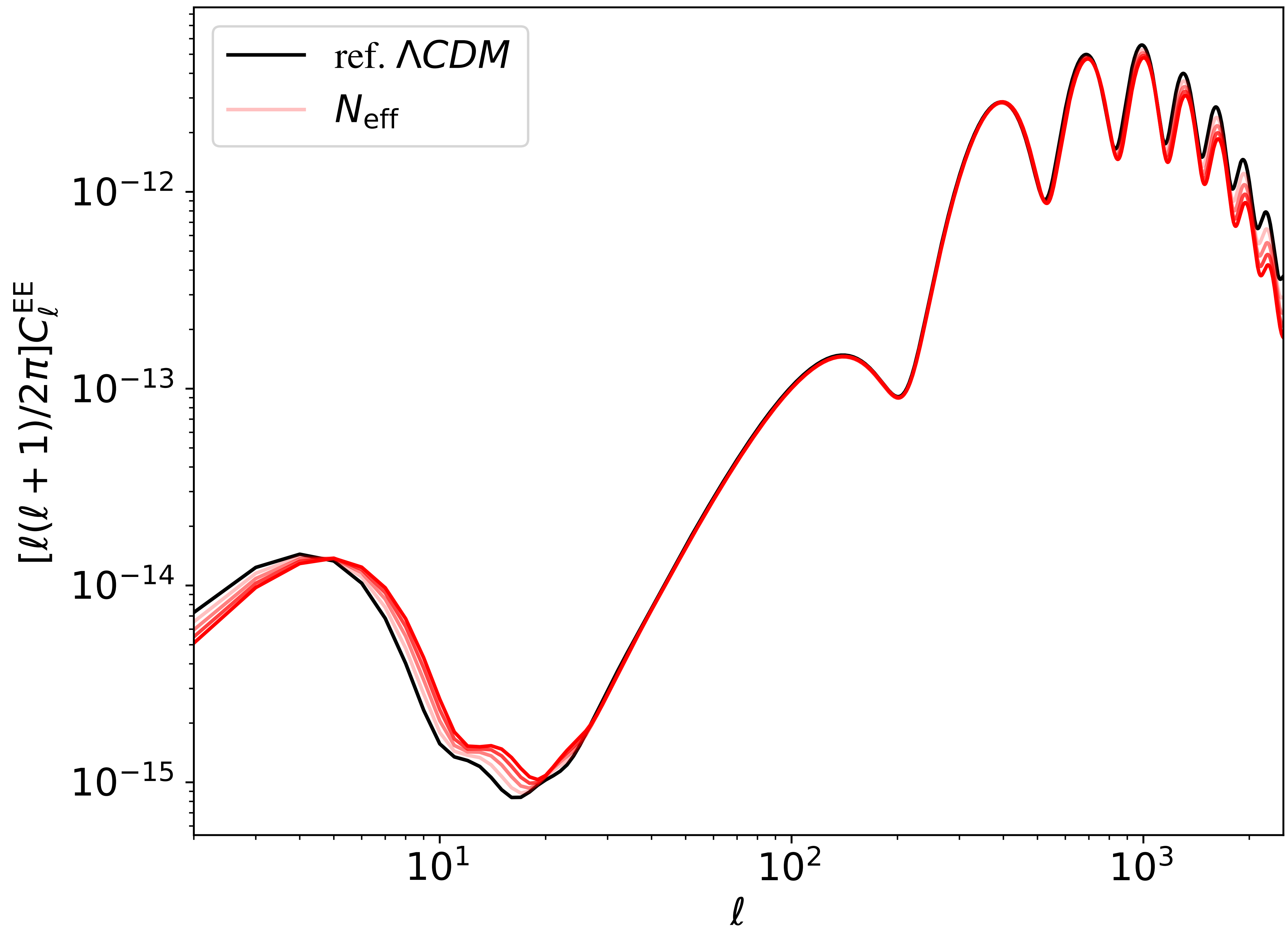
Gravity boost and neutrino drag

$$\frac{\Delta C_l}{C_l} = -0.072 \ \Delta N_{\rm eff}$$

$$\Delta l_{\rm peak} = -\frac{r_A(\eta_{LS})}{r_s(\eta_{LS})} \left(0.1912\pi\, R_\nu + O(R_\nu^2)\right)$$

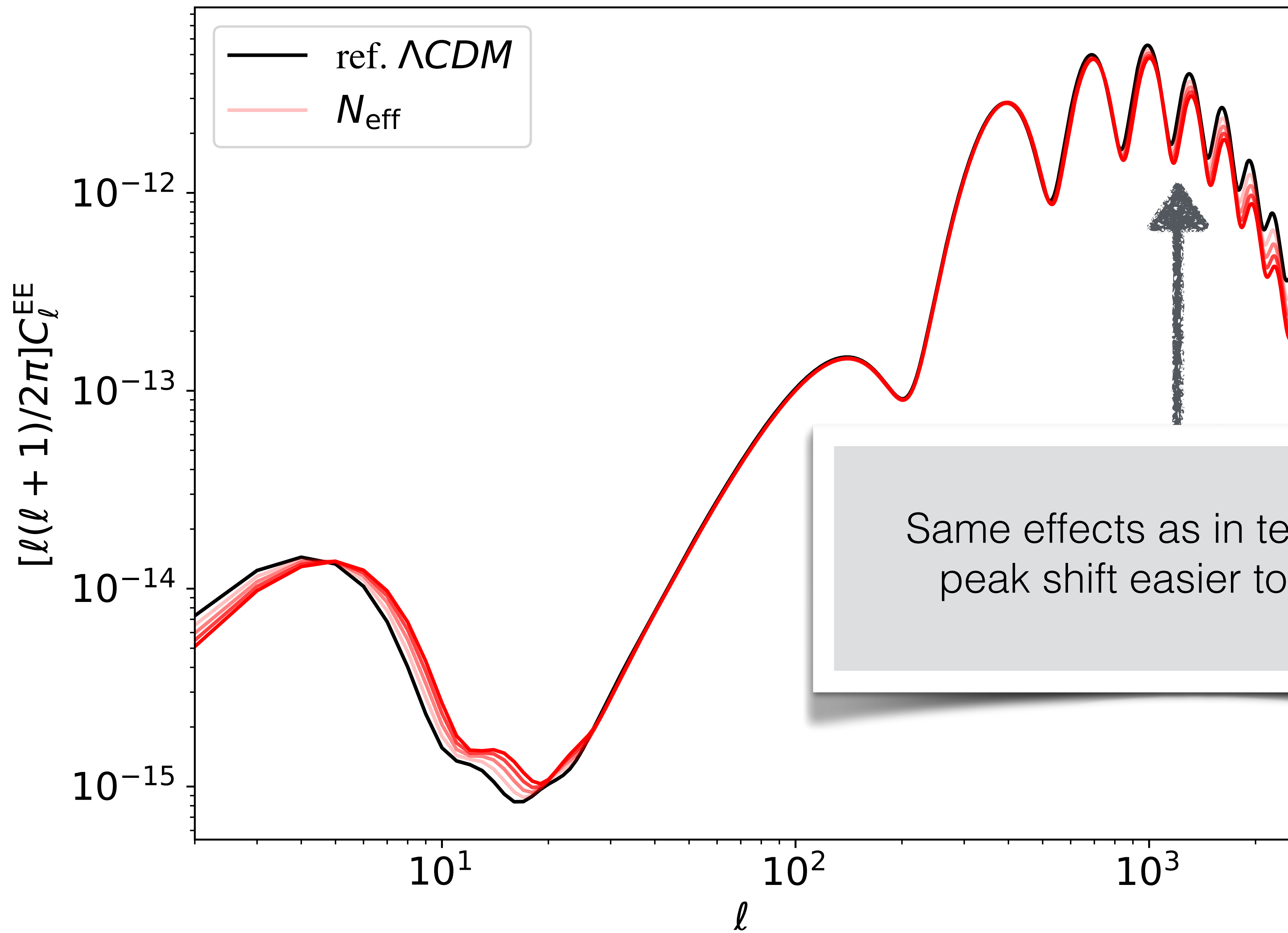
Bashinsky & Seljak 2004;
Neutrino Cosmology, CUP 2013

ΛCDM + N$_{\rm eff}$



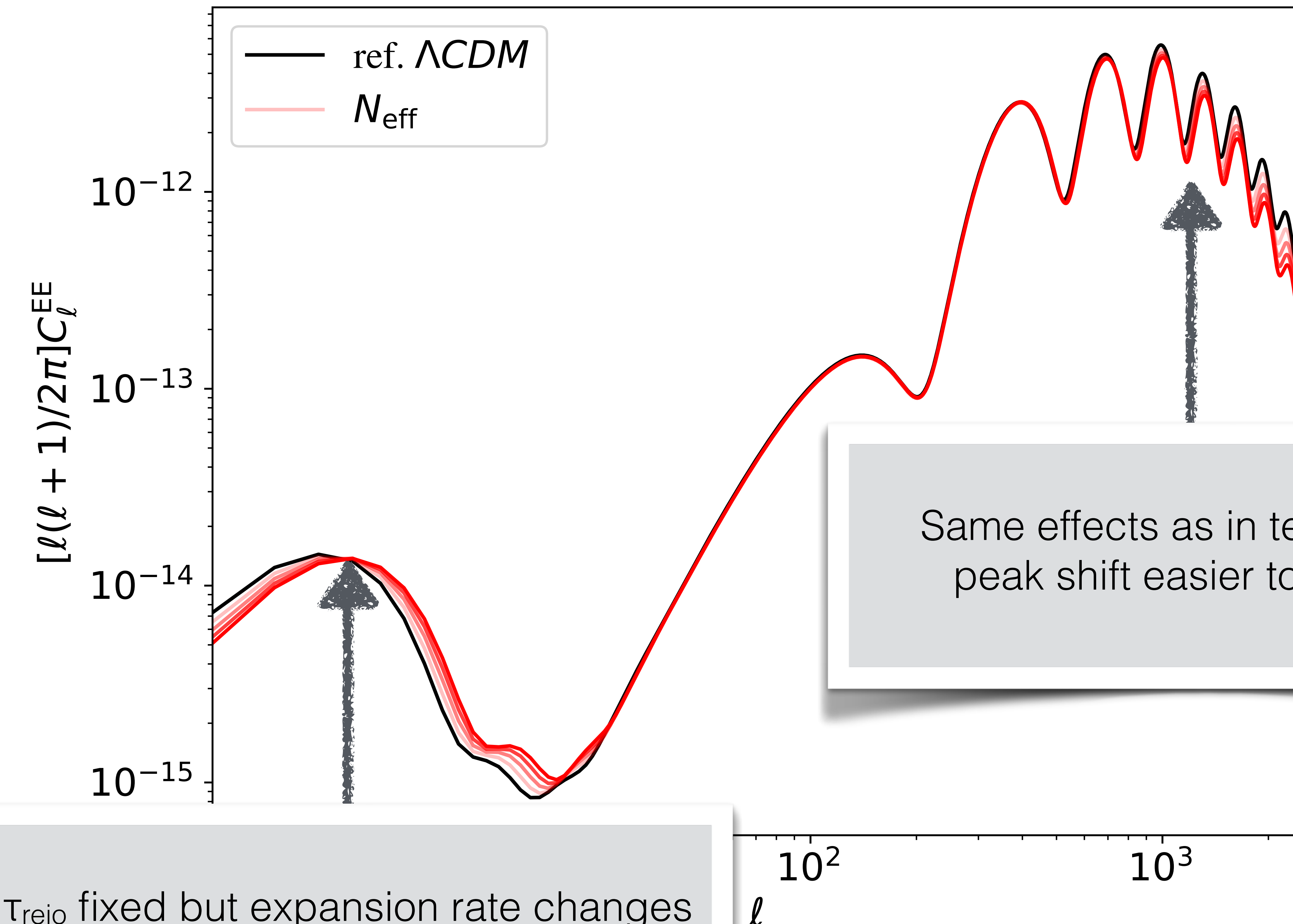
TTK Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

ΛCDM + N_{eff}



Same effects as in temperature, peak shift easier to measure

ΛCDM + N_{eff}



Same effects as in temperature, peak shift easier to measure

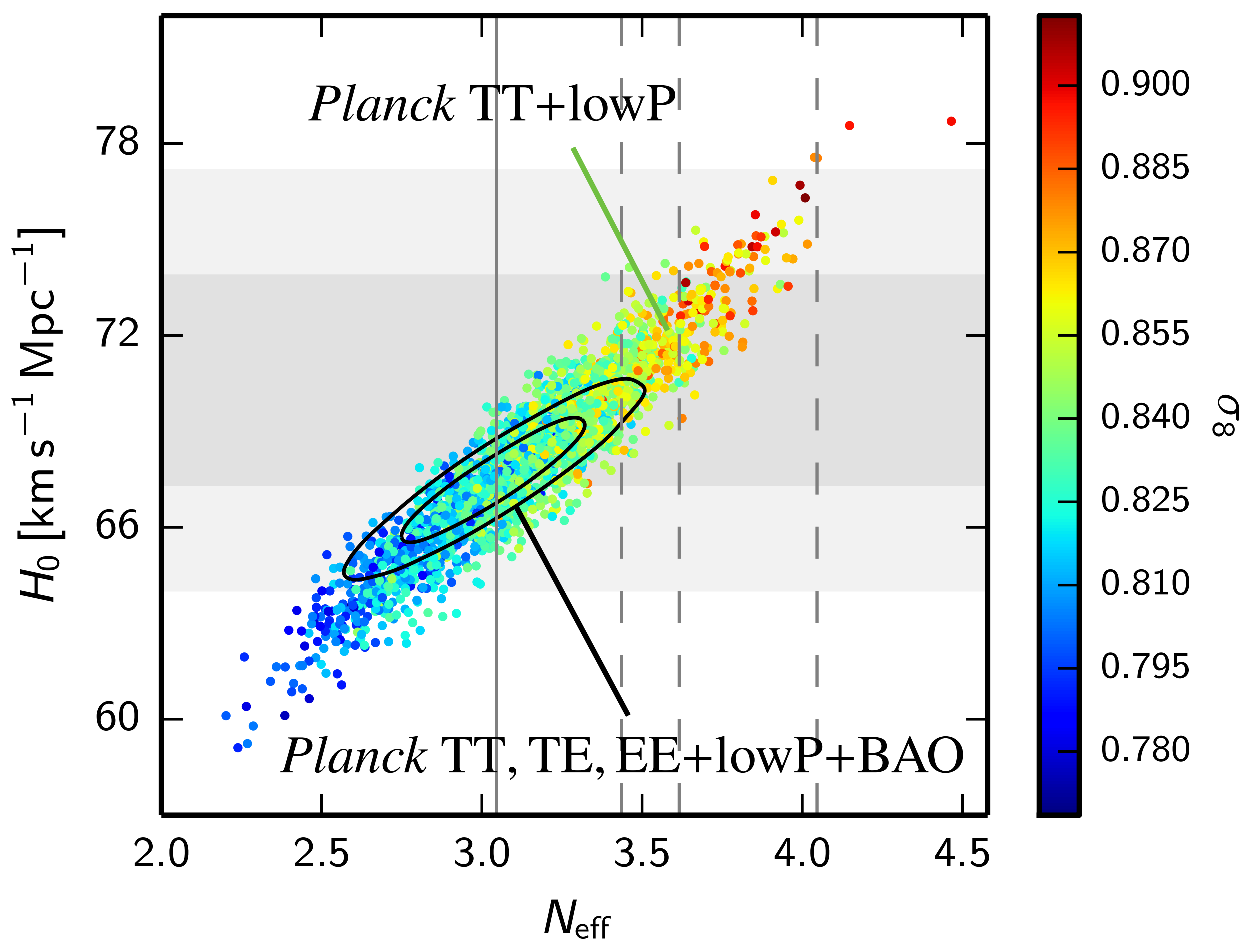
τ_{reio} fixed but expansion rate changes with *h*: then z_{reio} must also vary

Institute for Theoretical Particle Physics and Cosmology

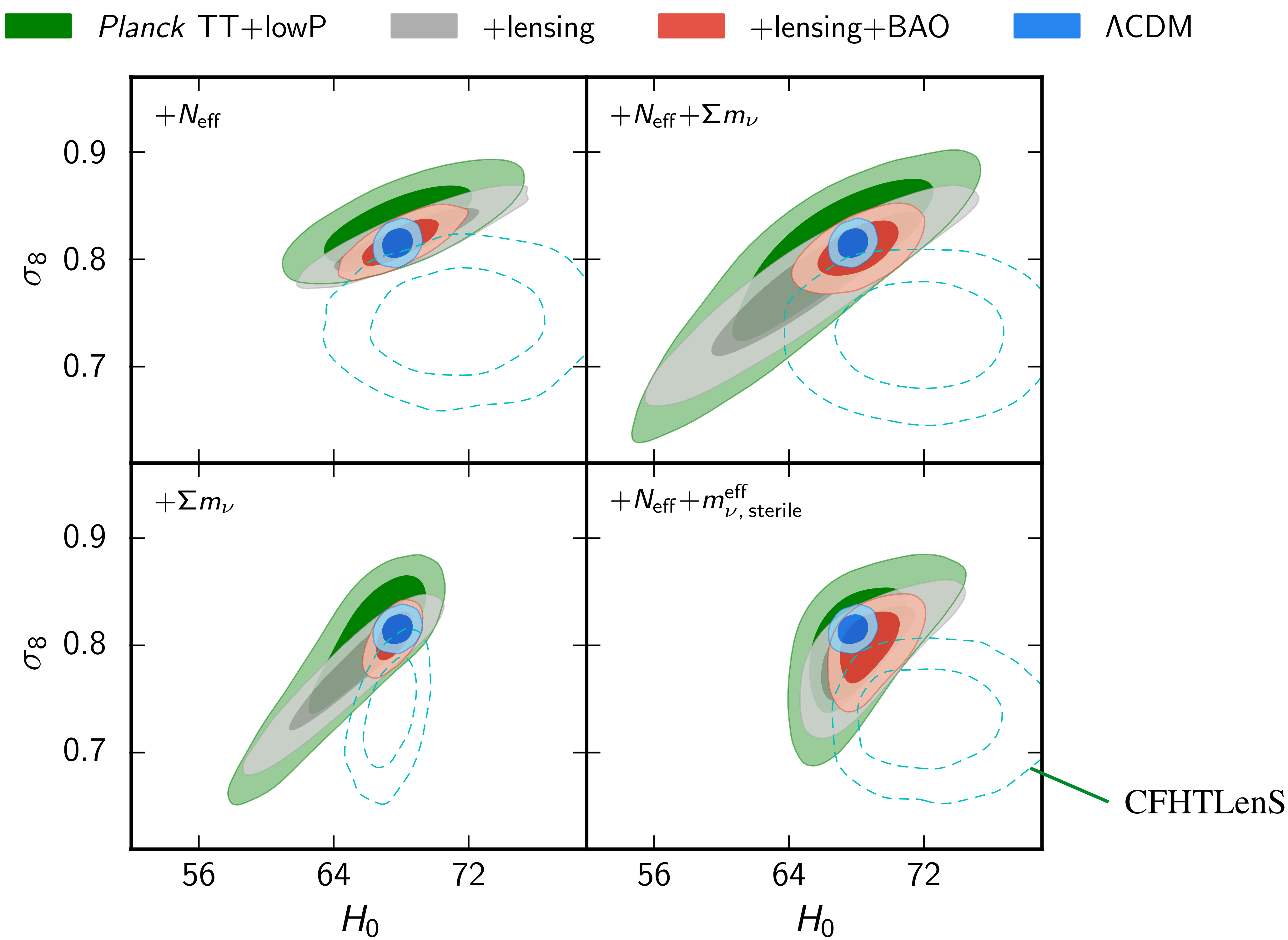
RWTH AACHEN UNIVERSITY

ΛCDM + N_{eff}

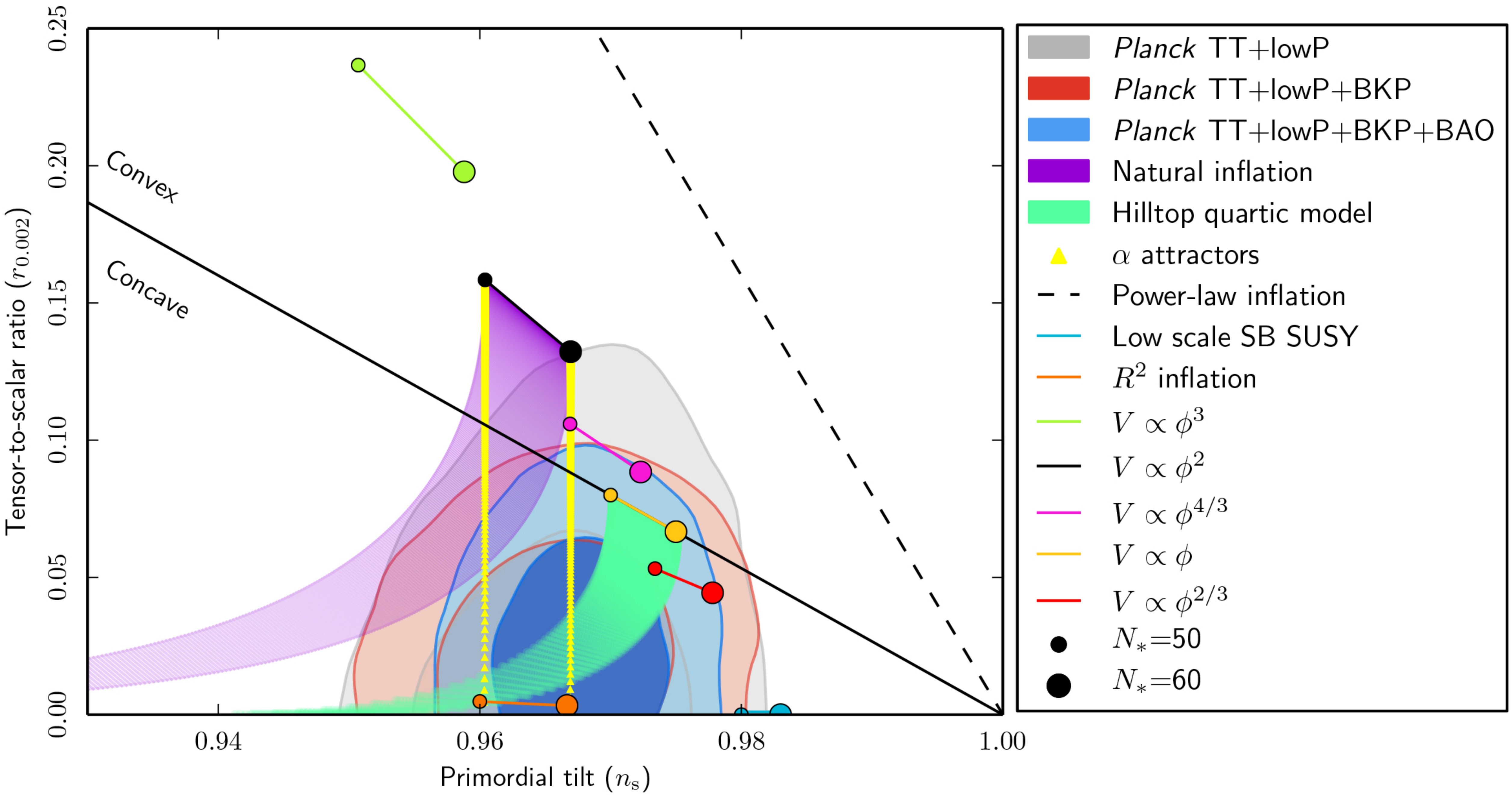
$$
\begin{aligned}
N_{\rm eff} &= 3.13 \pm 0.32 \quad \textit{Planck}\ \text{TT+lowP}\,; \\
N_{\rm eff} &= 3.15 \pm 0.23 \quad \textit{Planck}\ \text{TT+lowP+BAO}\,; \\
N_{\rm eff} &= 2.99 \pm 0.20 \quad \textit{Planck}\ \text{TT, TE, EE+lowP}\,; \\
N_{\rm eff} &= 3.04 \pm 0.18 \quad \textit{Planck}\ \text{TT, TE, EE+lowP+BAO}
\end{aligned}
$$



ΛCDM + N_{eff}

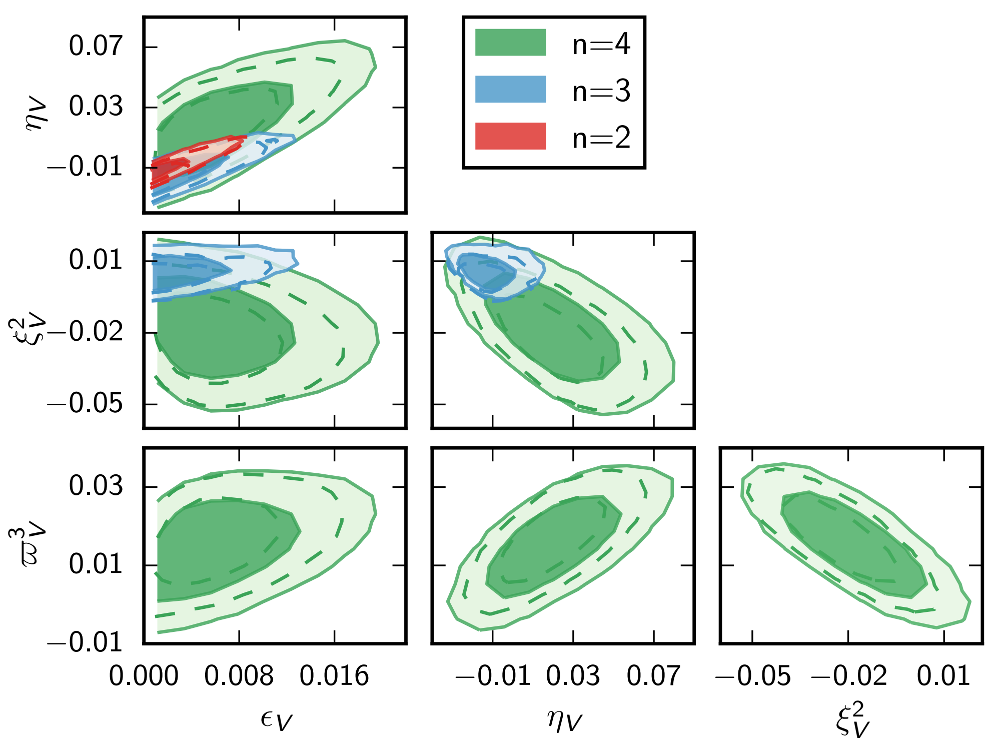


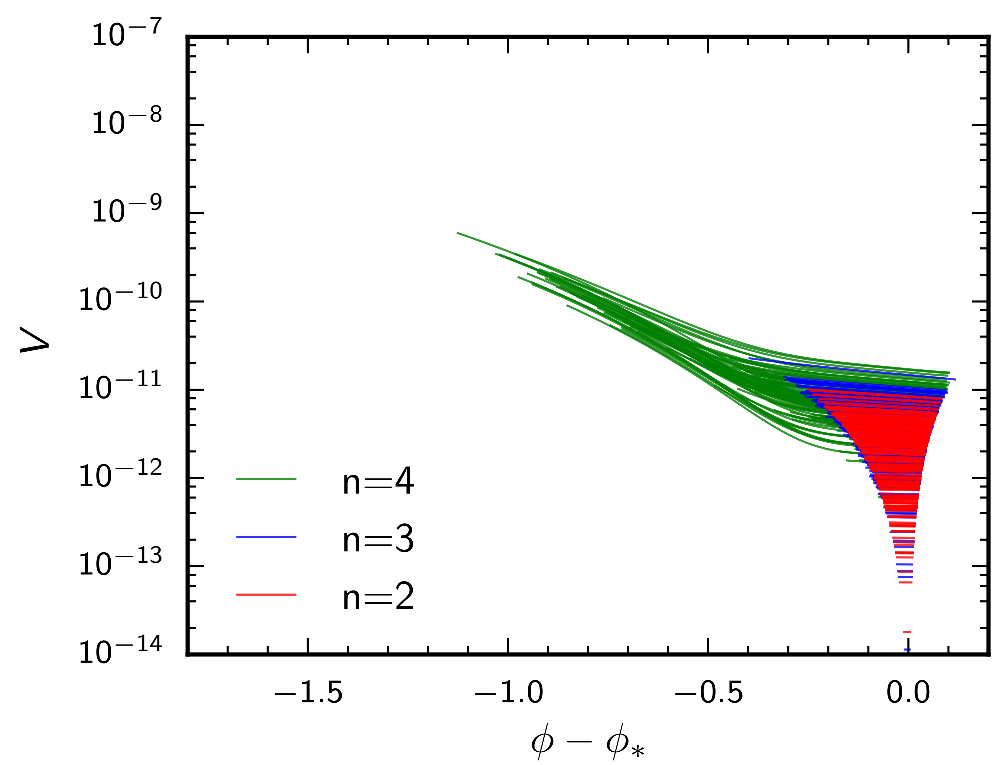
ΛCDM + r



Planck 2015 XX constraints on inflation 1502.02114

ΛCDM + r
ΛCDM + r + running
ΛCDM + r + running + running of the running
ΛCDM + r + primordial spectra with parametrised features
ΛCDM + r + binned primordial spectra
ΛCDM + r + parametrised inflation potential
ΛCDM + r + isocurvature modes
etc.





Planck 2015 XX constraints on inflation 1502.02114

Tensor modes

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu})$$

Bardeen scalars (spin-0)

$$h_{\mu\nu} = \begin{pmatrix} -2\psi & 0 & 0 & 0 \\ 0 & -2\phi & 0 & 0 \\ 0 & 0 & -2\phi & 0 \\ 0 & 0 & 0 & -2\psi \end{pmatrix}$$

(Newtonian gauge)

Bardeen tensors (spin-2)

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_1 & h_2 & 0 \\ 0 & h_2 & -h_1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{-i\omega x^3}$$

(for GWs along x^3)

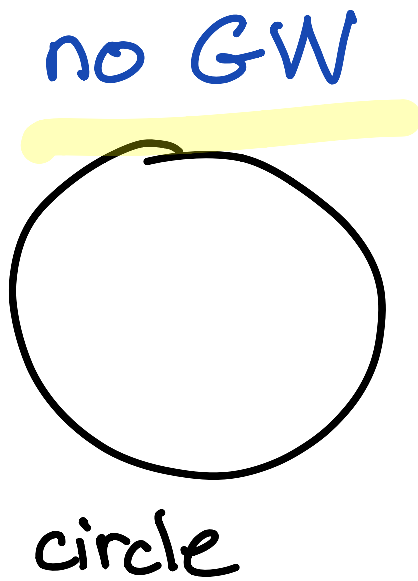
Boltzmann: $\Theta' + \hat{n}\cdot\vec{\nabla}\Theta = \hat{n}\cdot\vec{\nabla}\psi - \phi' + [\mathrm{Thomson}]$

grav. Dop. dilation

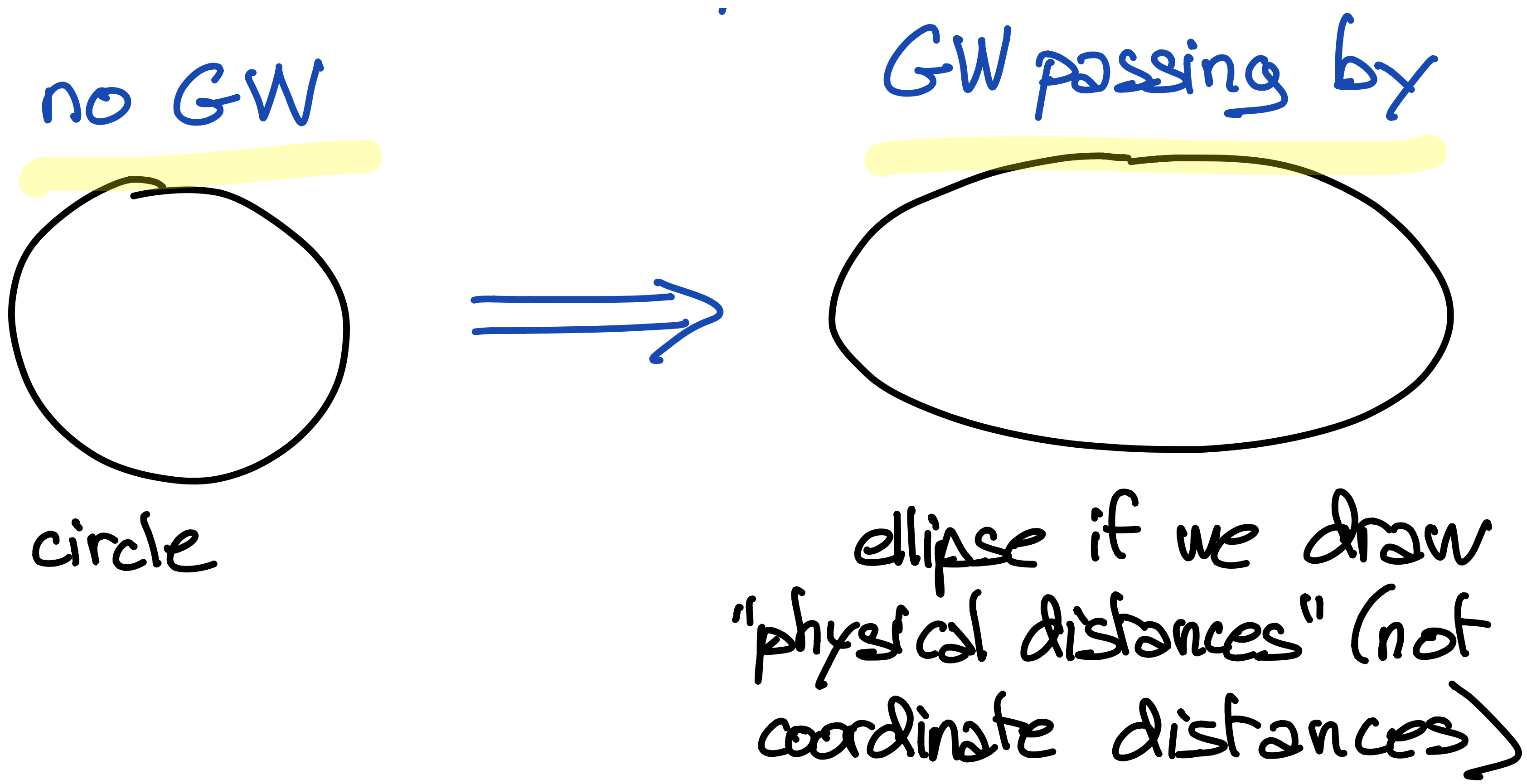
$$\Theta' + \hat{n}\cdot\vec{\nabla}\Theta = -\frac{1}{2}h'_{ij}\hat{n}^i\hat{n}^j + \frac{1}{2}\partial_i h_{00}\hat{n}^i - h'_{0i}\hat{n}^i + [\mathrm{Thomson}]$$

Relevant for tensors

Tensor modes

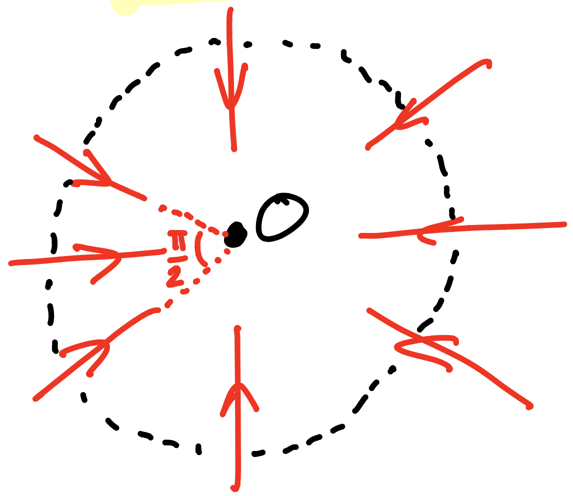


Tensor modes



Tensor modes

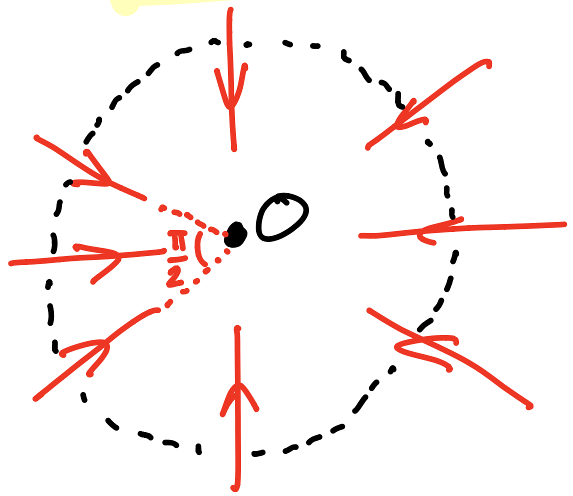
no GW



perfectly isotropic photons
(no perturbations, $\Theta = 0$,
just homogeneous/isotropic
photon background)

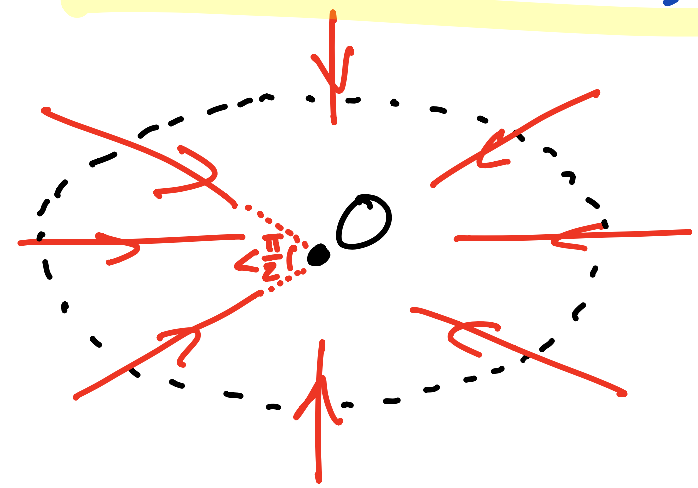
Tensor modes

no GW



perfectly isotropic photons (no perturbations, $\Theta = 0$, just homogeneous/isotropic photon background)

GW passing by:



GW produces deformation of space time such that observer sees more photons per solid angle coming from left/right and less from top/bottom: a quadrupole Θ_2 has been generated!

Tensor modes

Two sectors decoupled and statistically independent: split in two

$$\Theta = \Theta^{(s)} + \Theta^{(t)}$$

each obeying

$$\Theta' + \hat{n} \cdot \vec{\nabla}\Theta = -\frac{1}{2} h'_{ij} \hat{n}^i \hat{n}^j + \frac{1}{2} \partial_i h_{00} \hat{n}^i - h'_{0i} \hat{n}^i + [\mathrm{Thomson}]$$

with either curvature perturbations or GWs in the metric.

All tensor modes vanish before recombination (h'=0 + scattering)

Then $\Theta_2^{(t)}$ —> other temperature multipoles

—> polarisation

+ ISW-like contribution to temperature (***h'*** along line of sight)

Tensor modes

Finally

$$C_l^{XY} = 4\pi \sum_m \int dk\ k^2\ \Delta_l^{X(m)}(k)\ \Delta_l^{Y(m)}(k)\ \mathcal{P}_{(m)}(k)$$

$$X,Y \in \{T,E,B\} \qquad m \in \{s,t\} \qquad \mathcal{P}_{(s)}(k) = \mathcal{P}_{\mathcal{R}}(k) \qquad \mathcal{P}_{(t)}(k) = \mathcal{P}_h(k)$$

with the line-of-sight integrals

$$\Delta_l^{T(t)}(k) \sim \int d\tau \{g\Theta_2^{(t)} + e^{-\kappa'} h'_\lambda + ...\}(...)j_l(k(\tau_0 - \tau_{\rm rec}))$$

$$\Delta_l^{E,B(t)}(k) \sim \int d\tau \{g\Theta_2^{(t)} + ...\}(...)j_l(k(\tau_0 - \tau_{\rm rec}))$$

Scalar versus tensor spectra