Lecture 3

We have to smooth all fields and find the e.o.m. for the long-wavelength fields.

$$\frac{\partial f}{\partial \tau} + \frac{\vec{p}}{ma}\frac{\partial f}{\partial \vec{x}} - am\vec{\nabla}\phi \frac{\partial f}{\partial \vec{p}} = 0$$

We still want to find moments:

$$\rho(\vec{x},\tau) \equiv \frac{m}{a^3}\int d^3\vec{p}\; f(\vec{x},\vec{p},\tau)$$

$$\pi^i(\vec{x},\tau) \equiv \frac{1}{a^4}\int d^3\vec{p}\cdot p^i f(\vec{x},\vec{p},\tau)$$

...

$$\vec{v} \equiv \frac{\vec{\pi}}{\rho}$$

We want to split each field: $Y = Y_e + Y_s$

$$Y_e(\vec{x},\tau) \equiv \int d^3\vec{x}'\; W_R(\vec{x}-\vec{x}')\, Y(\vec{x}',\tau)$$

$$W_R \sim e^{-\frac{1}{2}\frac{(\vec{x}-\vec{x}')^2}{R^2}}$$

Find the e.o.m. for the long-wavelength fields

$$\delta_e' + \nabla_i\left((1+\delta_e) v_e^i\right) = 0$$

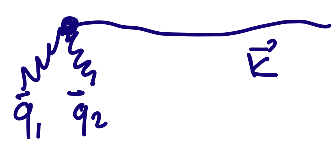
$$v_{i,e}' + \mathcal{H} v_{i,e} + v_e^j \nabla_j v_{i,e} + \nabla_i \phi = -\frac{1}{1+\delta_e}[\tau_{ij}]_R$$

effective stress-tensor

Additional terms are a consequence of smoothing

$$(fg)_R = f_e \cdot g_e + R^2 \vec{\nabla} f_e \cdot \vec{\nabla} g_e + \cdots + (f_s g_s)_R$$

stochastic



$\sim k^2$

$[\tau_{ij}]_R$ is expanded in $\delta_e, \vec{v}_e$ and stochastic terms

The coefficients in this expansion are unknown, they depend on small scale physics

At leading order:

$$[\tau_{ij}]_R = A(\tau)\nabla_i\nabla_j\phi + B(\tau)\underset{\delta^K_{ij}}{\nabla^2\phi} + \tau_{ij}^{\text{stoch.}}$$

We can now plug this in the e.o.m.

We keep as many terms as necessary

Expansion in perturbations: $\Delta^2(k) \simeq \frac{k^3}{2\pi^2} P(k)$

and derivatives: $\frac{\nabla^i}{k_{NL}}$

This is the EFT of LSS

New e.o.m.

$$\delta_\ell'(\vec{k}) + \theta_\ell(k) = - \int_{\vec{q}_1, \vec{q}_2} (2\pi)^3 \delta^D(\vec{k} - \vec{q}_1 - \vec{q}_2)\, \alpha(\vec{q}_1, \vec{q}_2)\, \theta_\ell(\vec{q}_1)\, \delta_\ell(\vec{q}_2)$$

$$\theta_\ell'(\vec{k}) + \mathcal{H}\theta_\ell(\vec{k}) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_\ell(\vec{k})$$

$$= - \int_{\vec{q}_1, \vec{q}_2} (2\pi)^3 \delta^D(\vec{k} - \vec{q}_1 - \vec{q}_2)\, \beta(\vec{q}_1, \vec{q}_2)\, \theta_\ell(\vec{q}_1)\, \theta_\ell(\vec{q}_2)$$

$$\underbrace{- \tilde{c}_s^{\,2}(\tau)\, \nabla^2 \delta_\ell - J}_{\text{counter terms}} \underset{\substack{\downarrow \\ \text{higher order / derivative} \\ \text{terms.}}}{+ \cdots} \quad \left(J \equiv \nabla^i \nabla^j \tau_{ij}^{\text{stoch.}}\right)$$

This is how small scales affect large scales at leading order.

The perturbative solution is now modified.

Exercise: Show that:

$$P_{NL}(k) = P_{lin}(k) + P_{22}(k) + P_{13}(k) - 2c_s^2 k^2 P_{lin}(k) + \alpha P_{JJ}(k)$$

free parameters encode small-scale physics

c_s^2 small-scale effects correlated with δ_e

α small-scale effects uncorrelated with δ_e
$\Rightarrow$ stochastic noise

$$P_J = \frac{1}{k_{NL}^3}\left(\frac{k}{k_{NL}}\right)^4$$, follows from mass and momentum conservation.

Leading UV dependence of the one-loop:

$$P_{13}(k) = 6 P_{lin}(k) \int_{\vec{q}}^{\Lambda} F_3(\vec{q}, -\vec{q}, \vec{k}) P_{lin}(q)$$

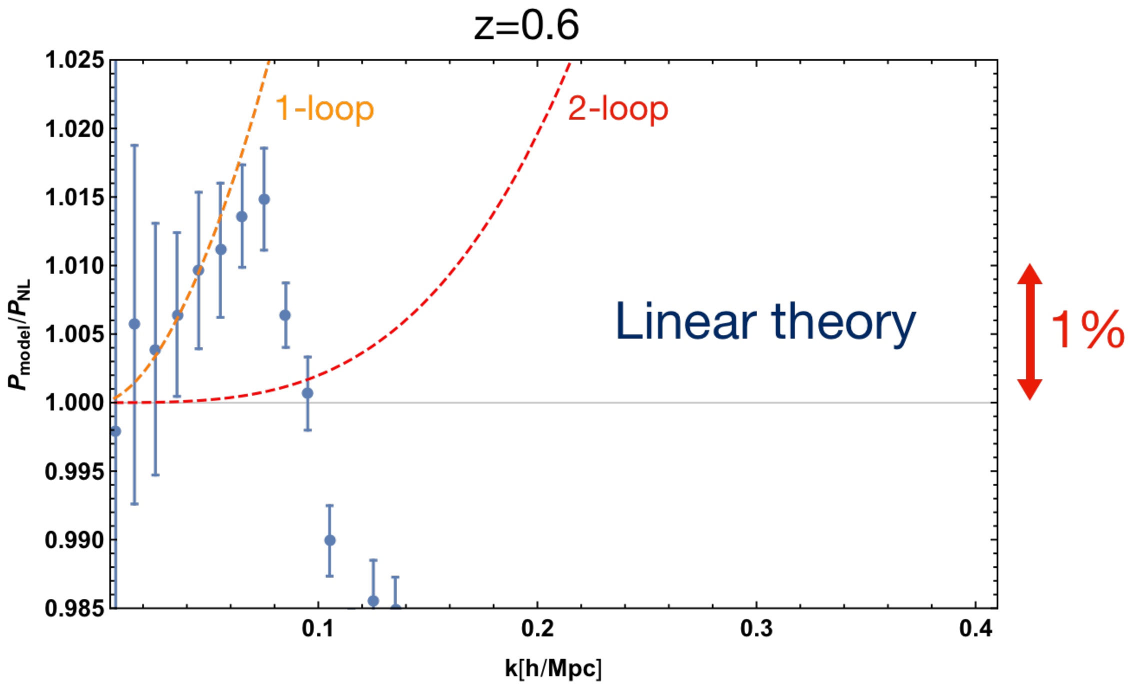
$$\Downarrow$$

$$q \gg k \rightarrow \frac{k^2}{q^2}$$

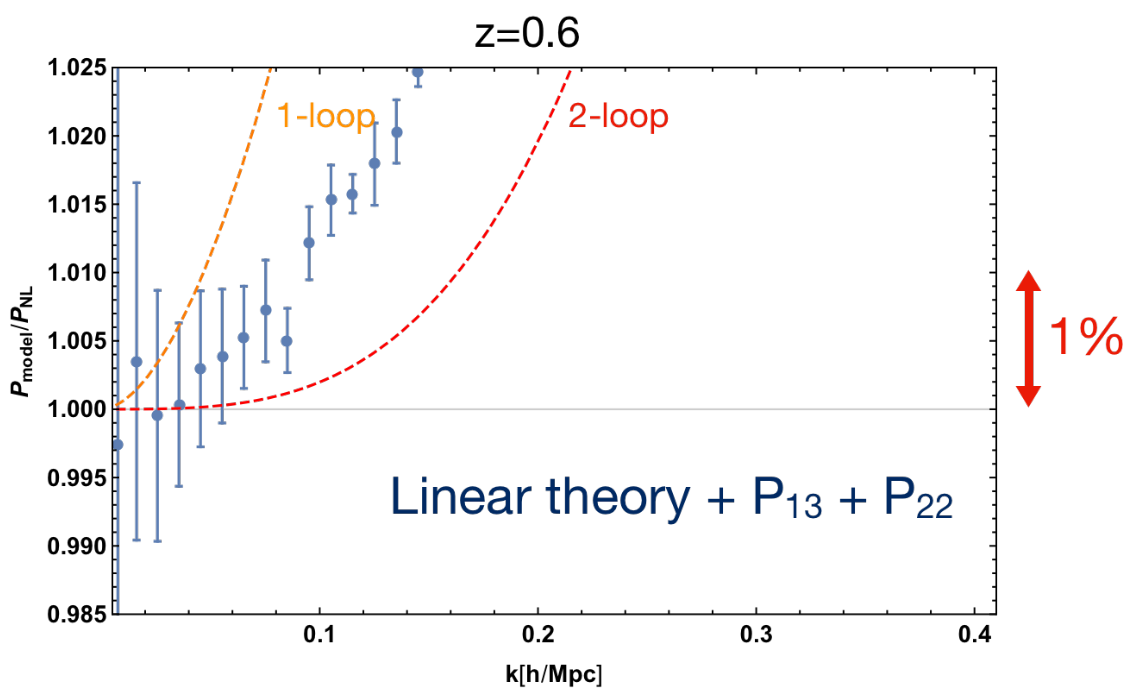
$c_s^2 k^2 P_{lin}$ absorbs all UV dependence!

How well does this work in practice?

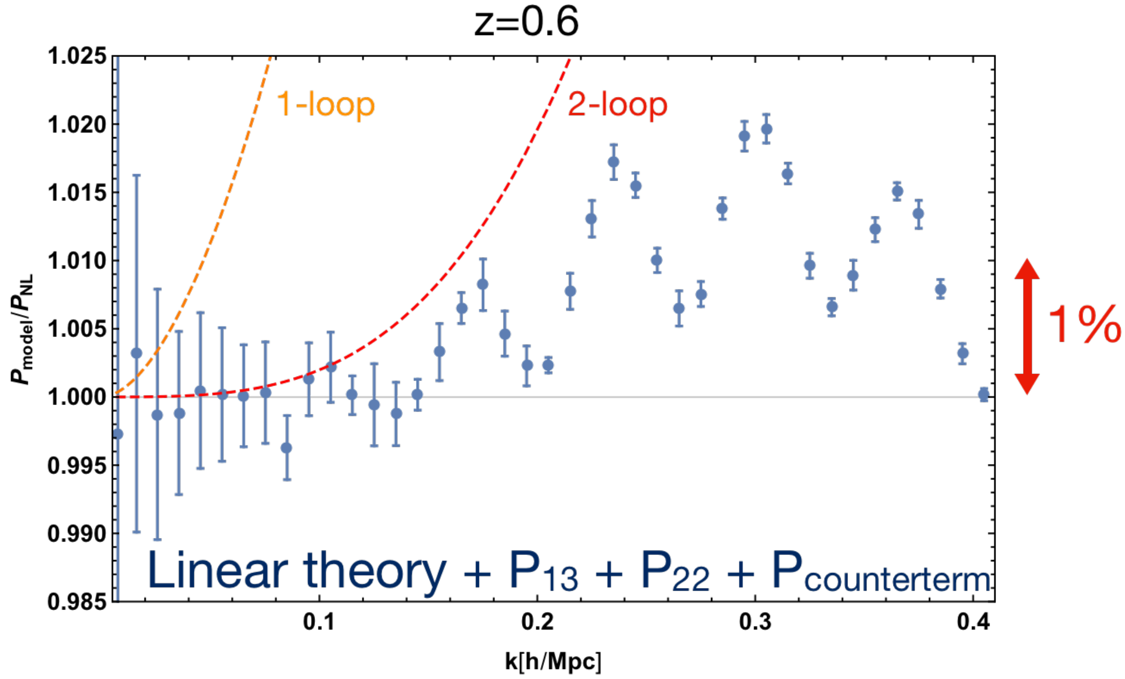
Everything makes sense, the difference w.r.t. the truth is ~ 1-loop



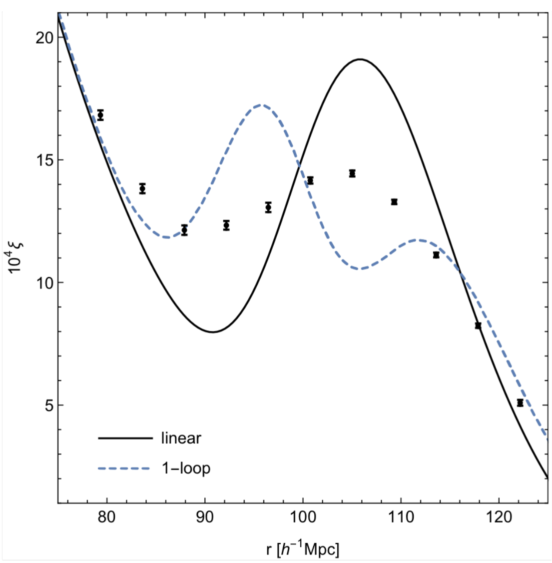
There is a problem: the difference bigger than ~ 2-loop contribution



Now everything makes sense again!



The BAO peak is still wrong!

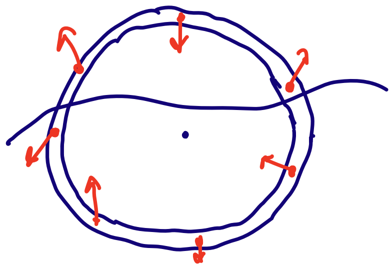


This is not a UV problem!

$\Delta^2(k)$ is not the only relevant parameter!

Infrared resummation.

What causes the spread of the BAO peak?



To estimate the size of bulk flows

$$\delta' + \theta = 0$$

$$\delta' + \vec{\nabla}\cdot\vec{v} = 0 \Rightarrow \delta = -\vec{\nabla}\underbrace{\int d\tau\, \vec{v}}_{\vec{\Psi}_1}$$

$$\vec{\Psi}_1 = -\frac{\vec{\nabla}}{\nabla^2}\delta$$

Variance of the displacement:

$$\langle \Psi_1^2 \rangle = \left\langle \int_{\vec{k}_1}\int_{\vec{k}_2} \frac{-i\vec{k}_1}{k_1^2}\cdot\frac{-i\vec{k}_2}{k_2^2} \underbrace{\delta(\vec{k}_1)\delta(\vec{k}_2)}_{2\pi^3 \delta^D(\vec{k}_1+\vec{k}_2)} e^{i\vec{k}_1\vec{x}} e^{i\vec{k}_2\vec{x}} \right\rangle$$

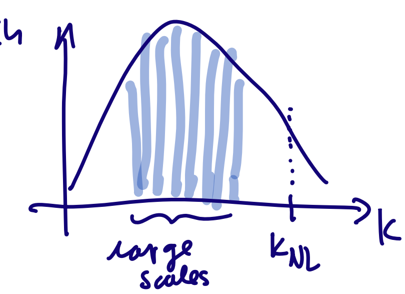
$$= \frac{1}{2\pi^2}\int dk.\, P(k)$$

This is very different from $\Delta^2(k) = \frac{1}{2\pi^2}\int^k dk'.\, k'^2 P_{lin}(k')$

$$\langle \Psi_1^2 \rangle = \frac{1}{2\pi^2}\int^k dk'.\, P_{lin}(k')$$

picks up most of the contribution around $k \sim k_{eq}$.

This is why we call this infrared effects.



In the PT kernels:

$$F_2(\vec{q},\vec{k}) \sim \frac{1}{2}\frac{\vec{q}\cdot\vec{k}}{q^2}$$

$$F_3(\vec{q}_1,\vec{q}_2,\vec{k}) \sim \frac{1}{6}\frac{\vec{q}_1\cdot\vec{k}}{q_1^2}\frac{\vec{q}_2\cdot\vec{k}}{q_2^2}$$

$\langle \vec{\Psi}_1^2 \rangle \simeq 10\text{-}15$ Mpc/h at $z=0$.

A significant part of Ψ_1 in correlation functions cancels

$$B(\vec{k}_1, \vec{k}_2, \vec{q}) \xrightarrow{q \ll k_i} \frac{1}{2}\frac{\vec{q}\cdot\vec{k}_1}{q^2} P_{lin}(q) \underbrace{P_{lin}(k_1)} + \frac{1}{2}\frac{\vec{q}\cdot\vec{k}_2}{q^2} P_{lin}(q) \underbrace{P_{lin}(k_2)}$$

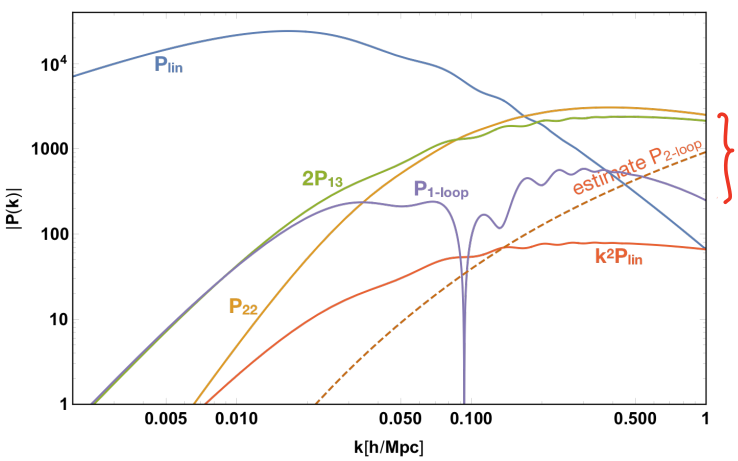
$$\vec{k}_1 = \vec{k} - \frac{\vec{q}}{2} \qquad \vec{k}_1 + \vec{k}_2 + \vec{q} = 0$$
$$\vec{k}_2 = -\vec{k} - \frac{\vec{q}}{2}$$

$$B(\vec{k}_1, \vec{k}_2, \vec{q}) \xrightarrow{q \ll k_i} \frac{1}{2}\frac{\vec{k}\cdot\vec{q}}{q^2} P_{lin}(q) \Big[\underbrace{P_{lin}\left(\vec{k} - \frac{\vec{q}}{2}\right) - P_{lin}\left(\vec{k} + \frac{\vec{q}}{2}\right)}_{\text{can we expand?}} \Big]$$

If $P_{lin}(k)$ is smooth, then YES.

Careful with features in $P_{lin}(k)$!

Similarly, these effects cancel in the loops.



$$|P_{22}| \sim |P_{13}| \gg P_{1\text{-loop}}$$

$$P_{22}(k \to \infty) \approx -P_{13}(k \to \infty)$$

Bulk flows make $P_{22}(k\to\infty)$ and $P_{13}(k\to\infty)$ large, but the sum is much smaller.

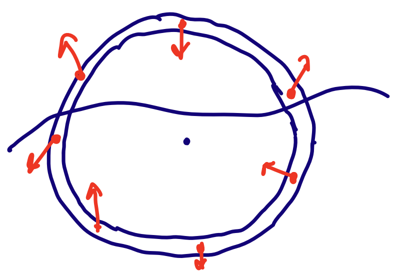
However, the cancellation is exact only if we can allways expand $P_{lin}(\vec{k}+\vec{q}) = P_{lin}(\vec{k}) + \vec{q}\cdot\vec{\nabla}_k P_{lin}(\vec{k}) + \dots$

This is not the case if we have features.

$$P_{lin}(k) = P_{lin}^{smooth}(k) + P_{lin}^{wiggly}(k)$$

$$P_{lin}^{wiggly}(k) = A^{smooth}(k)\cdot \sin(k\,\ell_{BAO}) \qquad \ell_{BAO} \simeq 110\ \mathrm{Mpc}/h$$

If $k \gg q > \ell_{BAO}^{-1}$ we cannot expand $P_{lin}(k)$!



Modes with $q \gtrsim \ell_{BAO}^{-1}$ have effect.

Modes with $q \ll \ell_{BAO}^{-1}$ do not have effect.

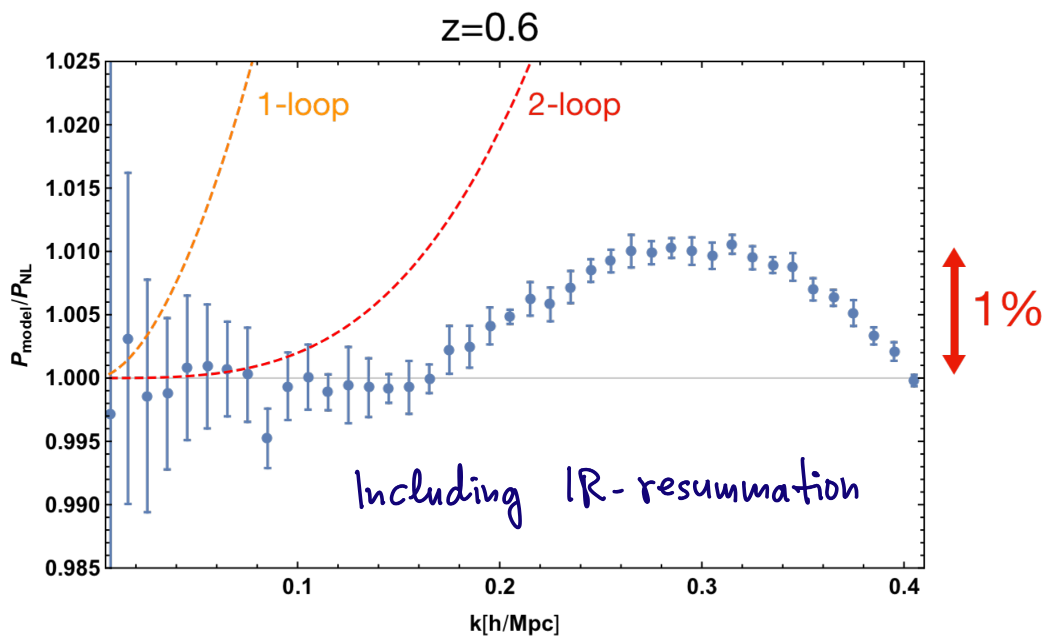
Large bulk flows are easy to understand and we can include them : resum all terms in PT with $\frac{\vec{q}_i\cdot\vec{k}}{q_i^2}\dots$

IR-resummation:

$$P_{NL}^{IR}(k) = P_{lin}^{smooth}(k) + P_{1\text{-}loop}^{smooth}(k) + (1+\Sigma_\epsilon^2 k^2)\,e^{-\Sigma_\epsilon^2 k^2} P_{lin}^{W} + e^{-\Sigma_\epsilon^2 k^2} P_{1\text{-}loop}^{W}$$

$$\Sigma_\epsilon^2(k) = \frac{1}{6\pi^2}\int_0^{\epsilon k} dq\, P_{lin}(q)\left[1 - j_0(q\,\ell_{BAO}) + 2 j_2(q\,\ell_{BAO})\right]$$

The 2-loop terms can fit a big part of the smooth residual



The IR-resummation solves the problem of the BAO peak