Sachs-Wolfe term

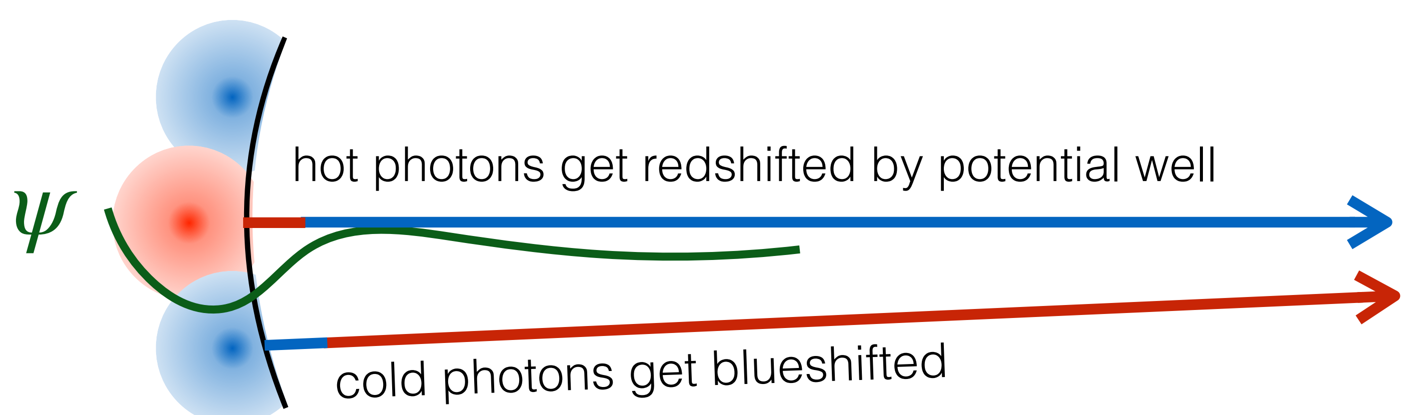
$$\Theta_l(\eta_0,\vec{k}) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \, \{ g \, (\Theta_0 + \psi) \; j_l(k(\eta_0 - \eta))$$
$$+ \, g \, k^{-1} \theta_{\rm b} \; j_l'(k(\eta_0 - \eta))$$
$$+ \, e^{-\tau} (\phi' + \psi') \, j_l(k(\eta_0 - \eta)) \}$$

Neglecting reionization: $g(\eta)$ very peaked at $\eta_{\rm dec}$

$\Rightarrow$ effect takes place only on last scattering sphere

$\Rightarrow$ mode k project to $\ell = k(\eta_0 - \eta_{\rm dec})$

$\Theta_0(\eta_{\rm dec}, \vec{k}) + \psi(\eta_{\rm dec}, \vec{k})$ = intrinsic fluctuation + gravitational Doppler shift



(super-Hubble modes with adiabatic IC: $\psi = -2\Theta_0$, Sachs-Wolfe effect wins, negative picture of last scattering sphere !)

Doppler term

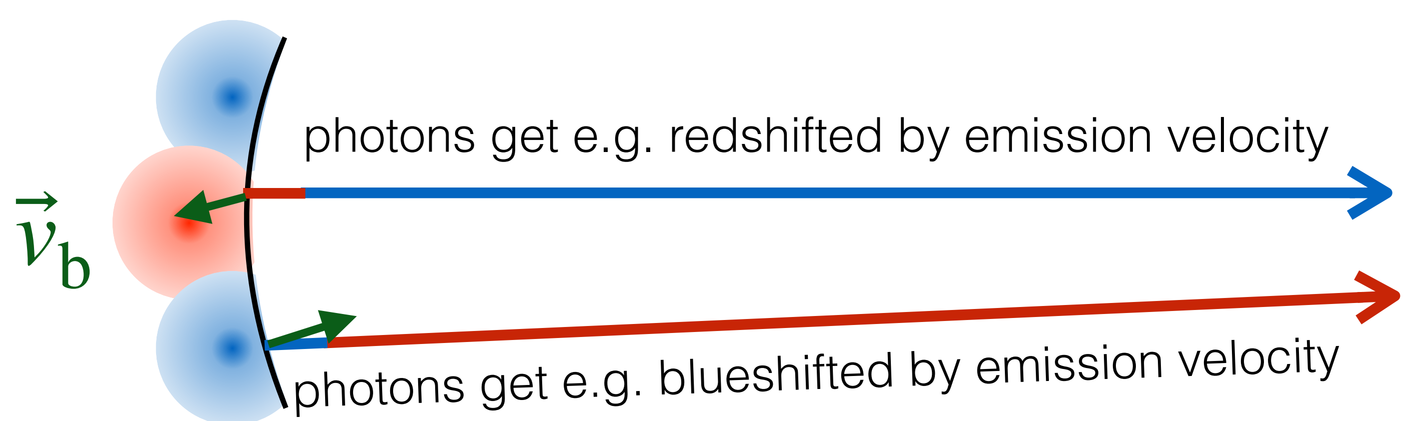
$$\Theta_l(\eta_0,\vec{k}) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta\,\{g\,(\Theta_0+\psi)\;j_l(k(\eta_0-\eta))$$
$$+\,g\,k^{-1}\theta_{\rm b}\;j_l'(k(\eta_0-\eta))$$
$$+\;e^{-\tau}(\phi'+\psi')\,j_l(k(\eta_0-\eta))\}$$

Neglecting reionization: $g(\eta)$ very peaked at $\eta_{\rm dec}$

$\Rightarrow$ effect takes place only on last scattering sphere

$\Rightarrow$ mode k project to $\ell = k(\eta_0 - \eta_{\rm dec})$

$\hat{n}\cdot\vec{v}_{\rm b}^{\rm scalar} \rightarrow k^{-1}\theta_{\rm b}$ = velocity Doppler shift (j_ℓ' from a gradient)



Integrated Sachs-Wolfe (ISW) term

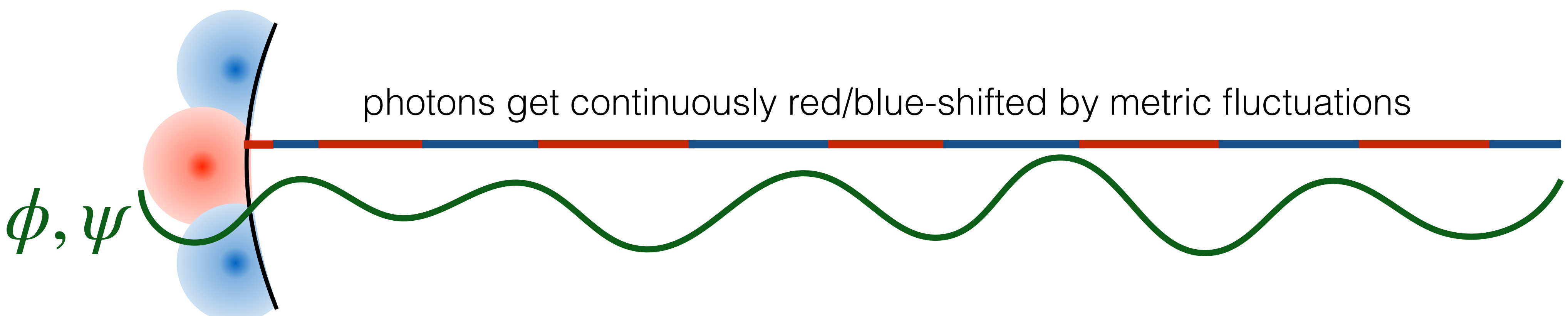
$$\Theta_l(\eta_0,\vec{k}) = \ \dots \ + e^{-\tau}(\phi' + \psi')\, j_l(k(\eta_0 - \eta))$$

Neglecting reionization: $e^{-\tau}$ negligible before $\eta_{\rm dec}$, $\simeq 1$after

$\Rightarrow$ effect takes place at all times $\eta > \eta_{\rm dec}$ along each line of sight

$\Rightarrow$ mode k projects from each sphere to $\ell = k(\eta_0 - \eta)$

$\partial_\eta\{\phi(\eta,\vec{k}) + \psi(\eta,\vec{k})\}$ comes from dilation + gravitational Doppler effects



- ϕ, ψ static: no dilation, gravitational Doppler effect is conservative: only $(\psi_{\rm dec} - \psi_{\rm obs})$
- ϕ, ψ time-dependent: net effect (e.g. net redshift when crosses deepening potential wells)

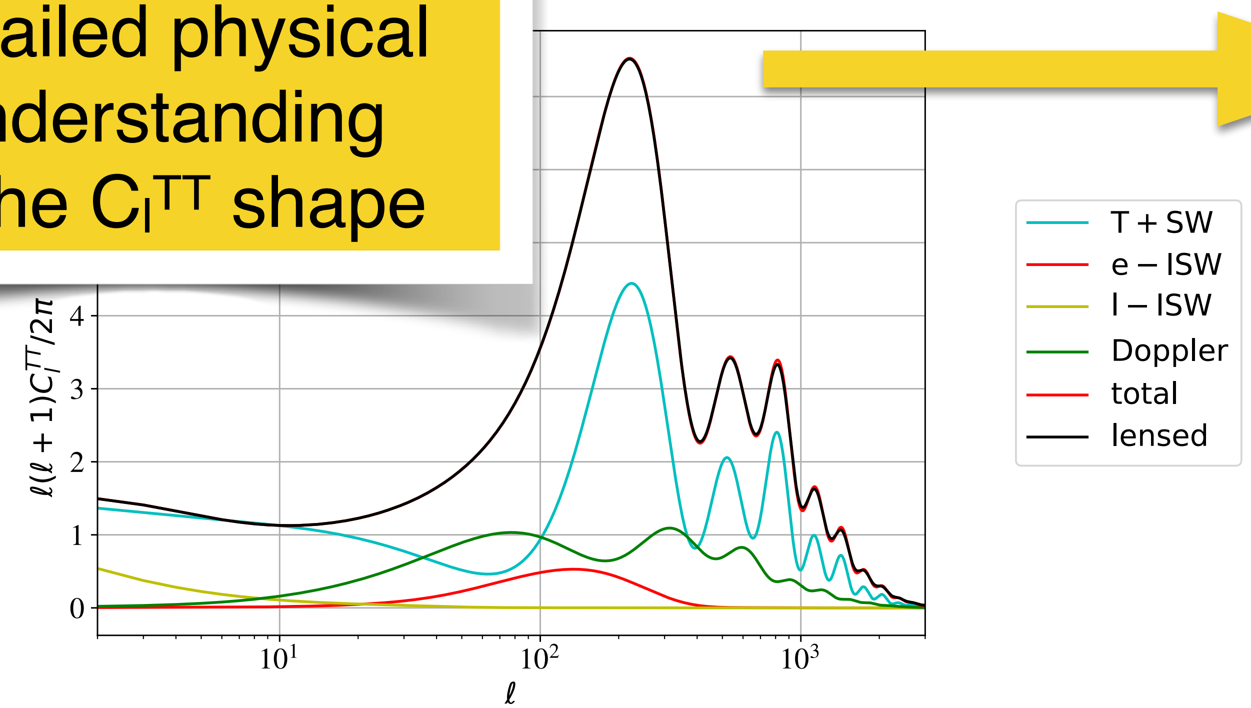
Summary

Final goal: compute $C_\ell = \langle a_{lm} a^*_{lm} \rangle = \frac{2}{\pi} \int dk\, k^2 \Theta^2_\ell(\eta_0, k)\;\; P_{\mathcal{R}}(k)$

with transfer functions

$$\Theta_l(\eta_0,k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \, \{ g \, (\Theta_0 + \psi) \; j_l(k(\eta_0 - \eta)) + g \, k^{-1}\theta_{\rm b} \; j_l'(k(\eta_0 - \eta)) + \, e^{-\tau} (\phi' + \psi') \, j_l(k(\eta_0-\eta)) \}$$

Detailed physical understanding of the C_l^{TT} shape



behaviour of

$\Theta_0(\eta_{\rm dec}, k)$

$\theta_{\rm b}(\eta_{\rm dec})$

$\psi(\eta \geq \eta_{\rm dec}, k) \simeq \phi$

Tight-Coupling Approximation (TCA)

When $\Gamma_\gamma \gg \dfrac{a'}{a}$:
tightly-coupled baryon-photon fluid:

$$\begin{cases} \Theta_0 = \frac{1}{4}\delta_\gamma = \frac{1}{3}\delta_b \\ 3k\Theta_1 = \theta_\gamma = \theta_b \\ \Theta_{l\geq 2} = 0 \end{cases}$$

from thermal equilibrium

from efficient Thomson scattering

⇒ photon Boltzmann hierarchy + baryon fluid equations —> single TCA equation:

$$\boxed{\Theta_0'' + \underbrace{\frac{R}{1+R}\frac{a'}{a}\Theta_0'}_{\text{baryon damping}} + \underbrace{k^2 c_\mathrm{s}^2 \Theta_0}_{\text{pressure force}} = \underbrace{-\frac{k^2}{3}\psi}_{\text{gravity force}} + \underbrace{\frac{R}{1+R}\frac{a'}{a}\phi'}_{\text{local baryon damping}} + \underbrace{\phi''}_{\text{dilation}}}$$

Squared sound speed / baryon-to-photon ratio: $c_\mathrm{s}^2 = \dfrac{1}{3(1+R)}$, $\quad R \equiv \dfrac{3\bar{\rho}_\mathrm{b}}{4\bar{\rho}_\gamma} \propto a$

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Tight-coupling equation

$$\Theta_0'' + \frac{R}{1+R}\frac{a'}{a}\Theta_0' + k^2 c_s^2 \Theta_0 = -\frac{k^2}{3}\psi + \frac{R}{1+R}\frac{a'}{a}\phi' + \phi''$$

baryon damping — pressure force — gravity force — local baryon damping — dilation

Squared sound speed / baryon-to-photon ratio: $c_s^2 = \frac{1}{3(1+R)}\,, \quad R \equiv \frac{4\bar{\rho}_b}{3\bar{\rho}_\gamma} \propto a$

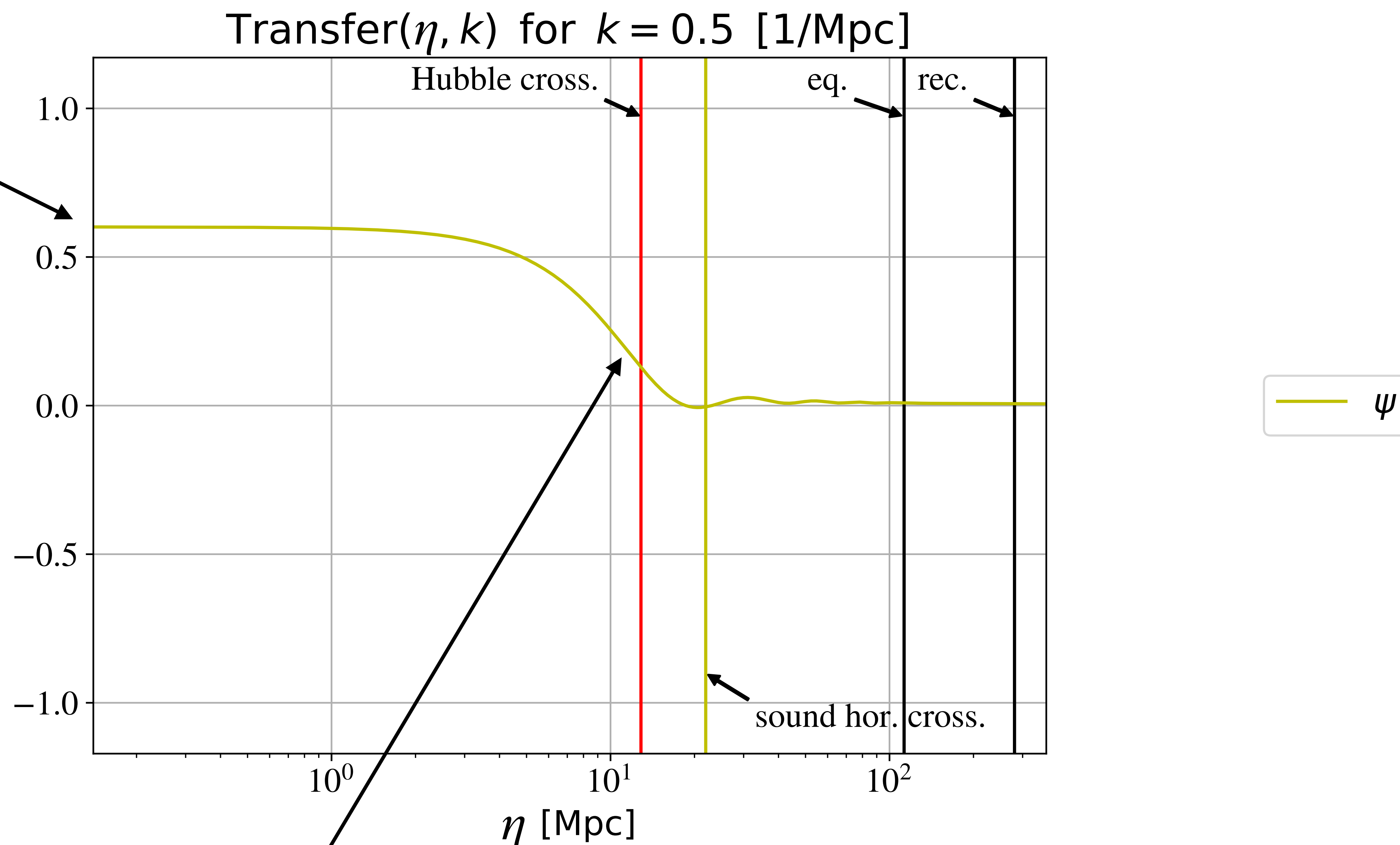
Equilibrium point neglecting metric time derivatives: $\Theta_0^{\mathrm{equi.}} = -\frac{1}{3c_s^2}\psi = -(1+R)\psi$

WKB TCA solution " " " : $\Theta_0 = A(1+R)^{-1/4}\cos\left(k \int c_s(\eta)d\eta\right) - (1+R)\psi$

Very good approximation up to gravity boost + (Silk) damping/diffusion effects

Evolution for one mode with given k

Adiabatic IC
($\mathcal{R}$=1, ψ= 2/3)



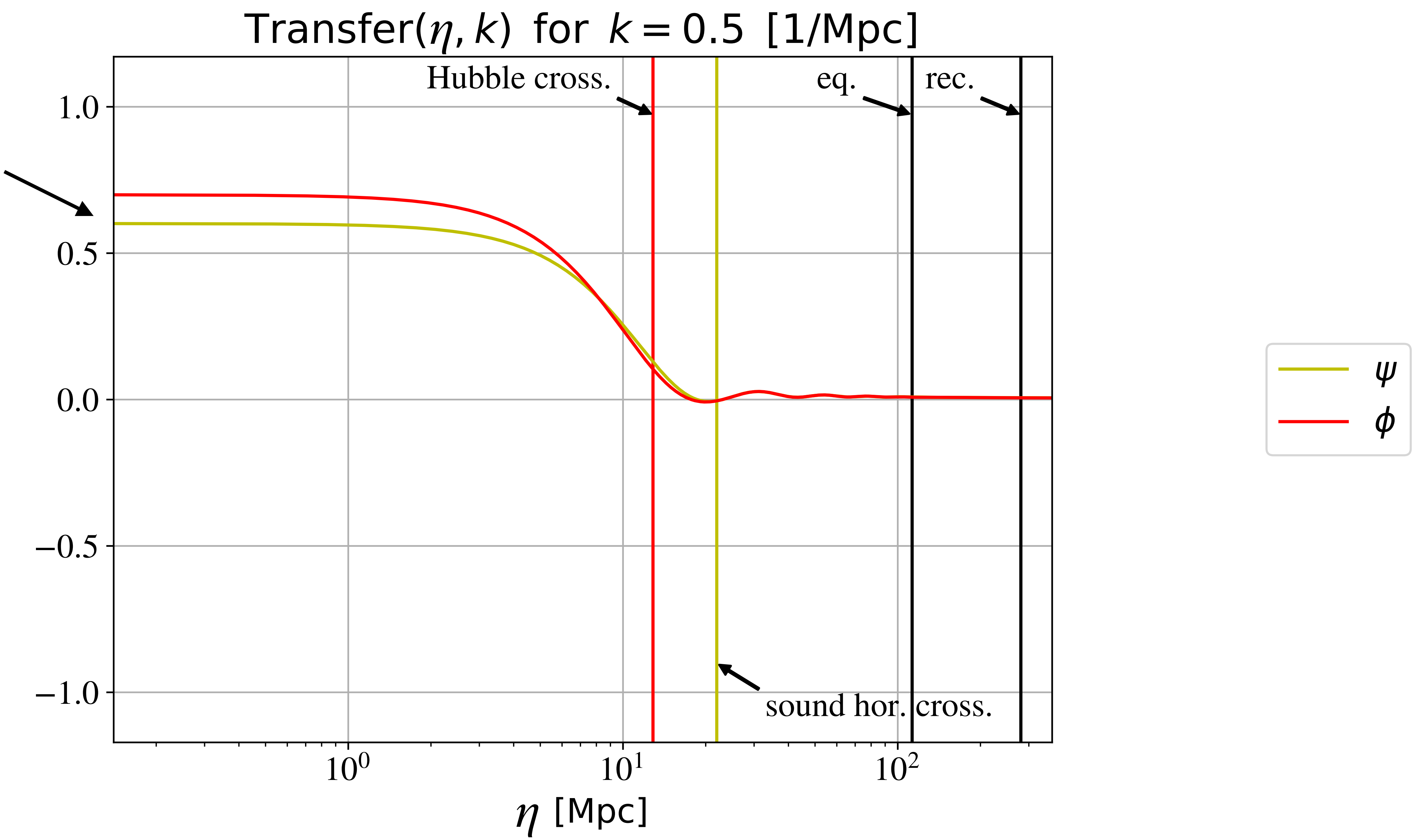
Metric damped near Hubble crossing during RD

—> photon pressure, Poisson: $-k^2\phi = 4\pi G\, a^2\, \delta\rho_r \propto a^2 \rho_r\, \delta_r \sim a^{2-4+0} \sim a^{-2}$

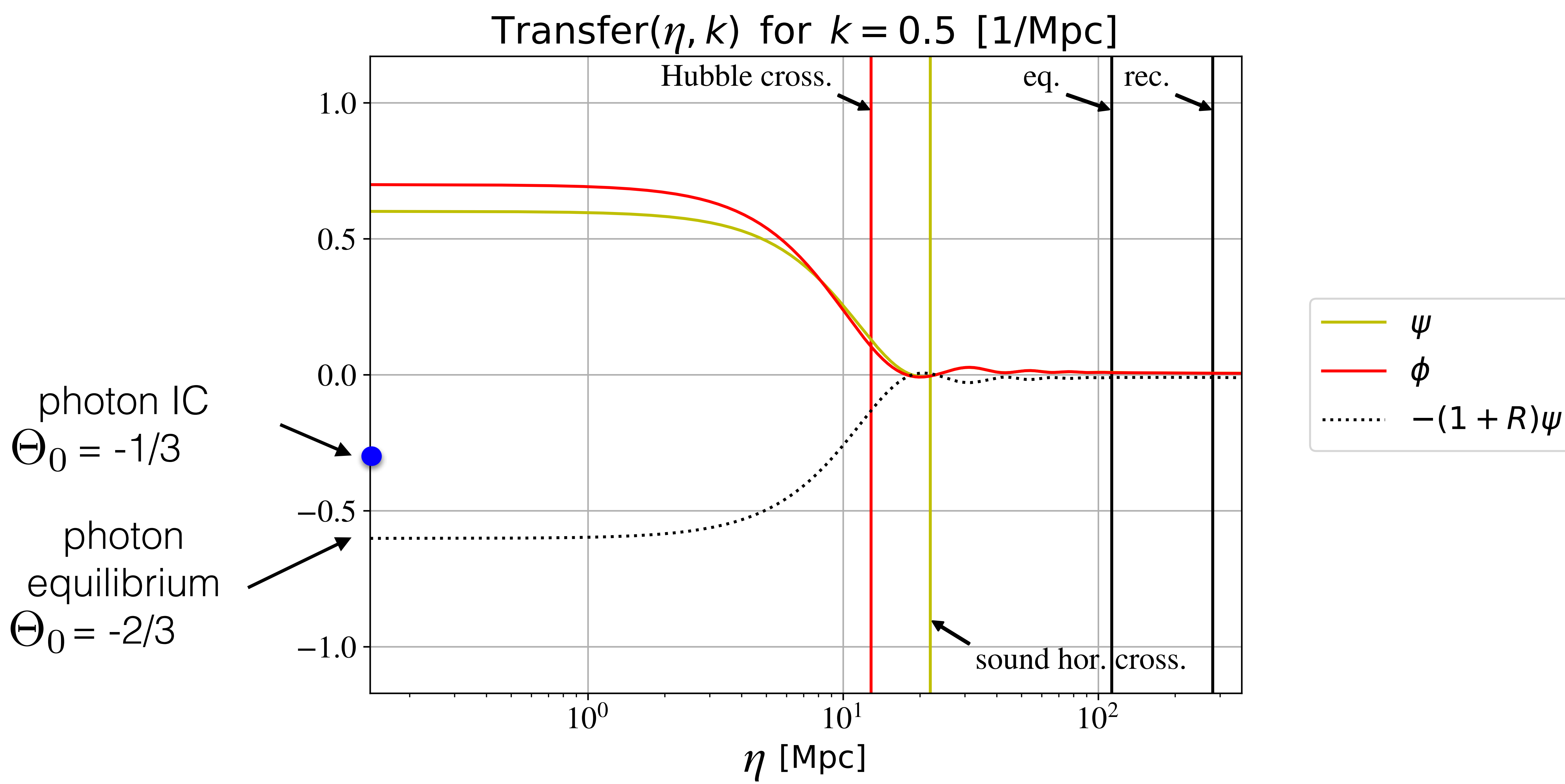
—> very different from MD: $-k^2\phi = 4\pi G\, a^2\, \delta\rho_m \propto a^2 \rho_m\, \delta_m \sim a^{2-3+1} \sim \mathrm{constant}$

Evolution for one mode

neutrino shear
(unimportant)



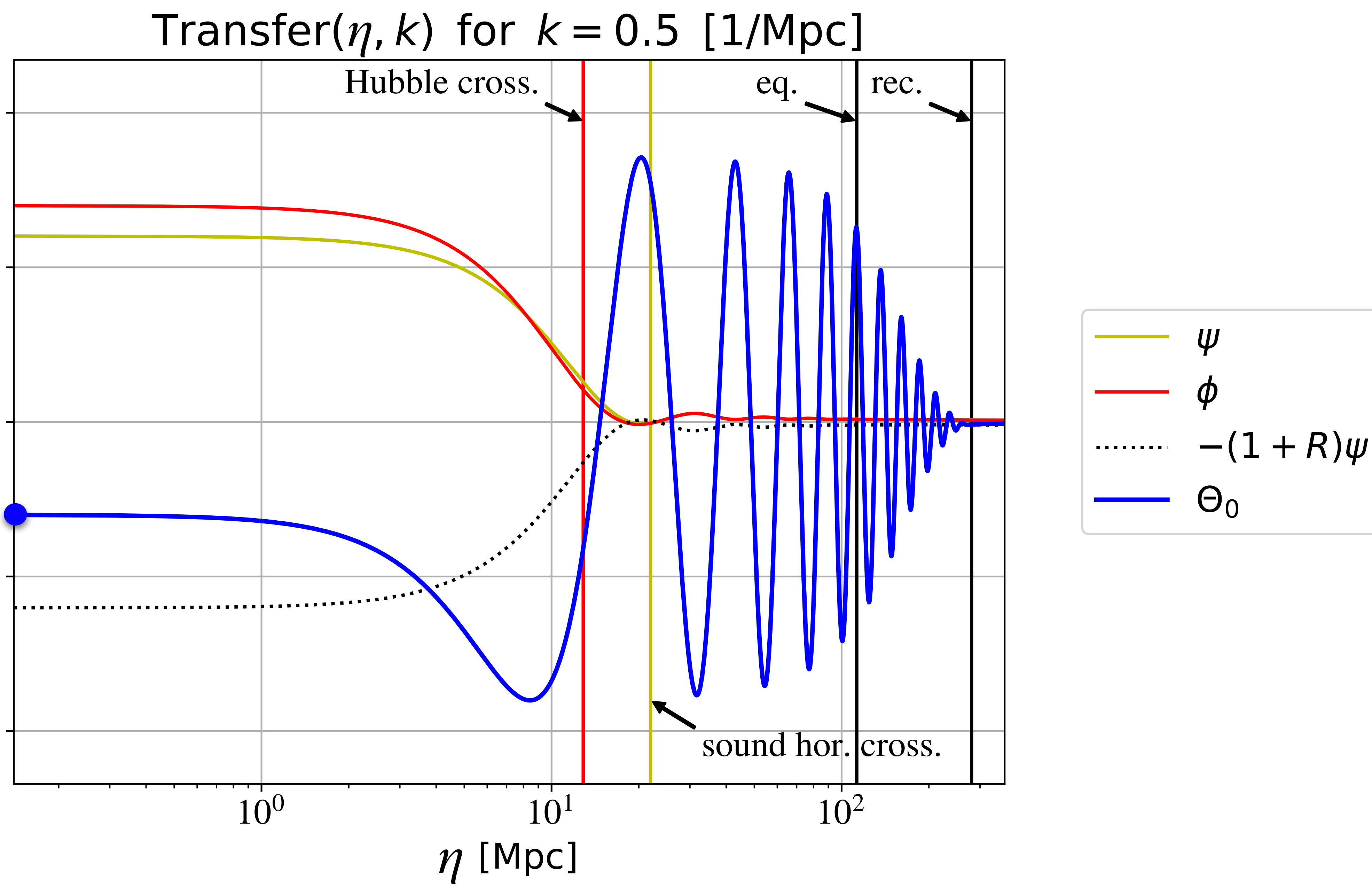
Evolution for one mode



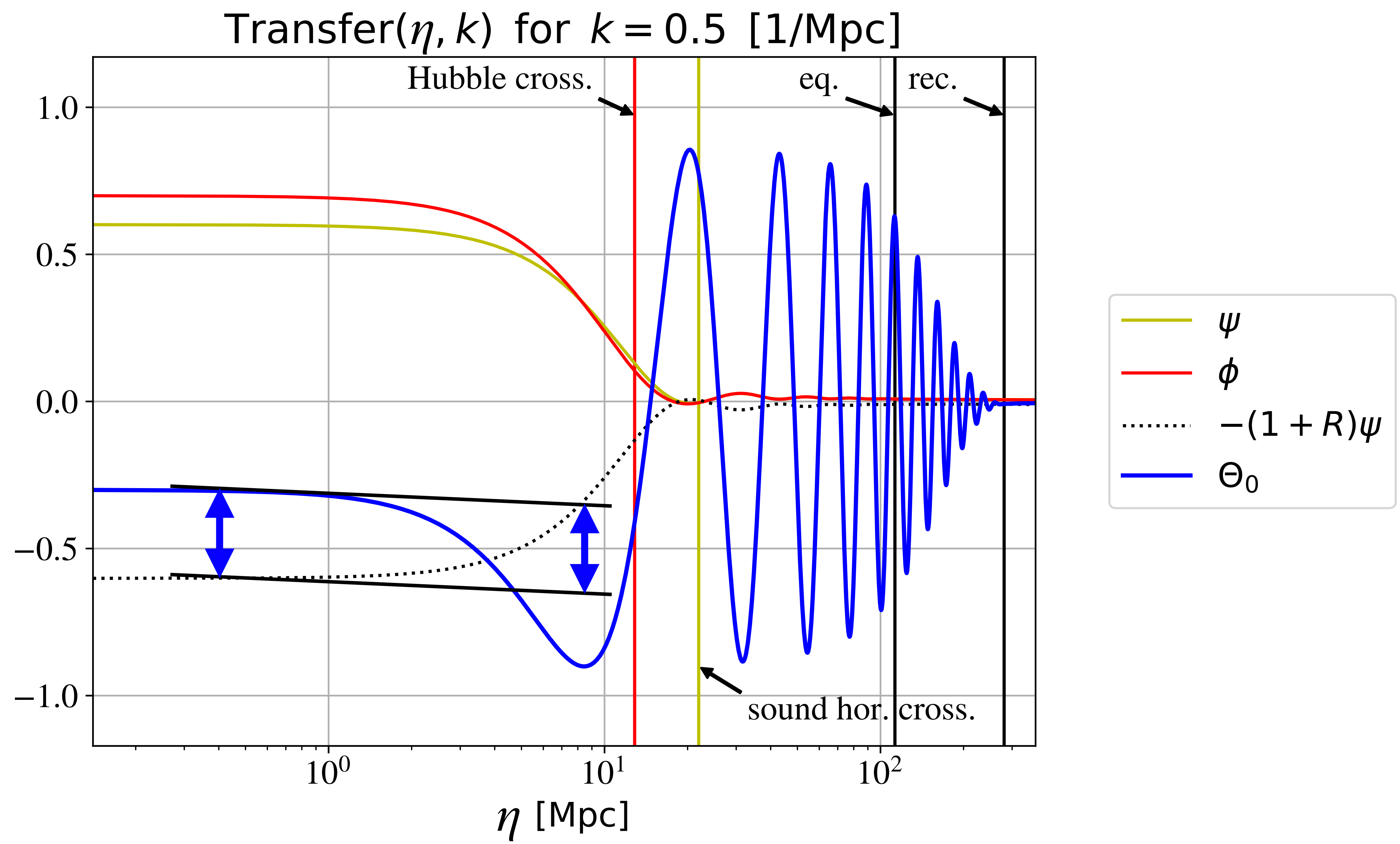
Evolution for one mode

photon IC
Θ_0 = -1/3

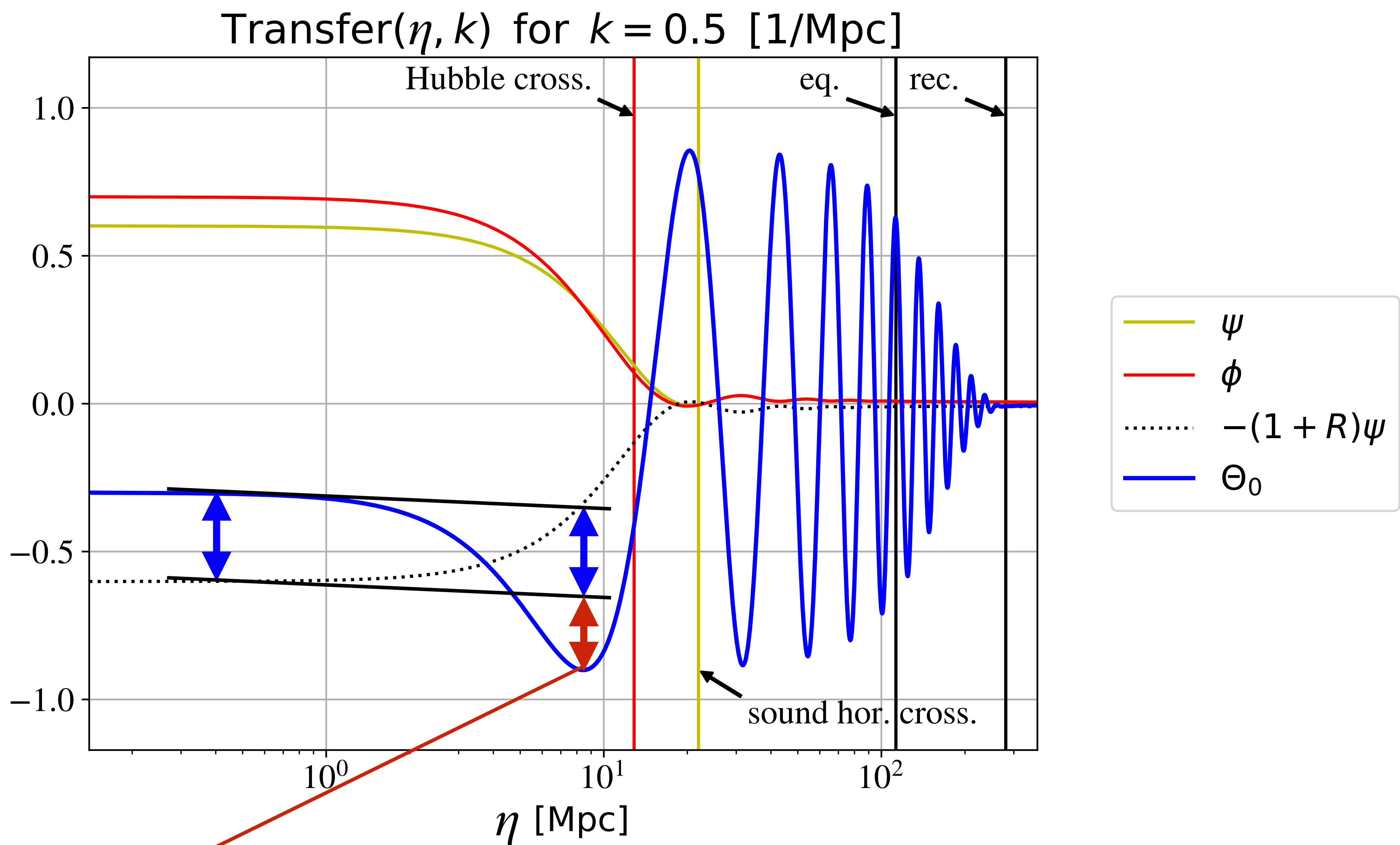
photon
equilibrium
Θ_0 = -2/3



Evolution for one mode



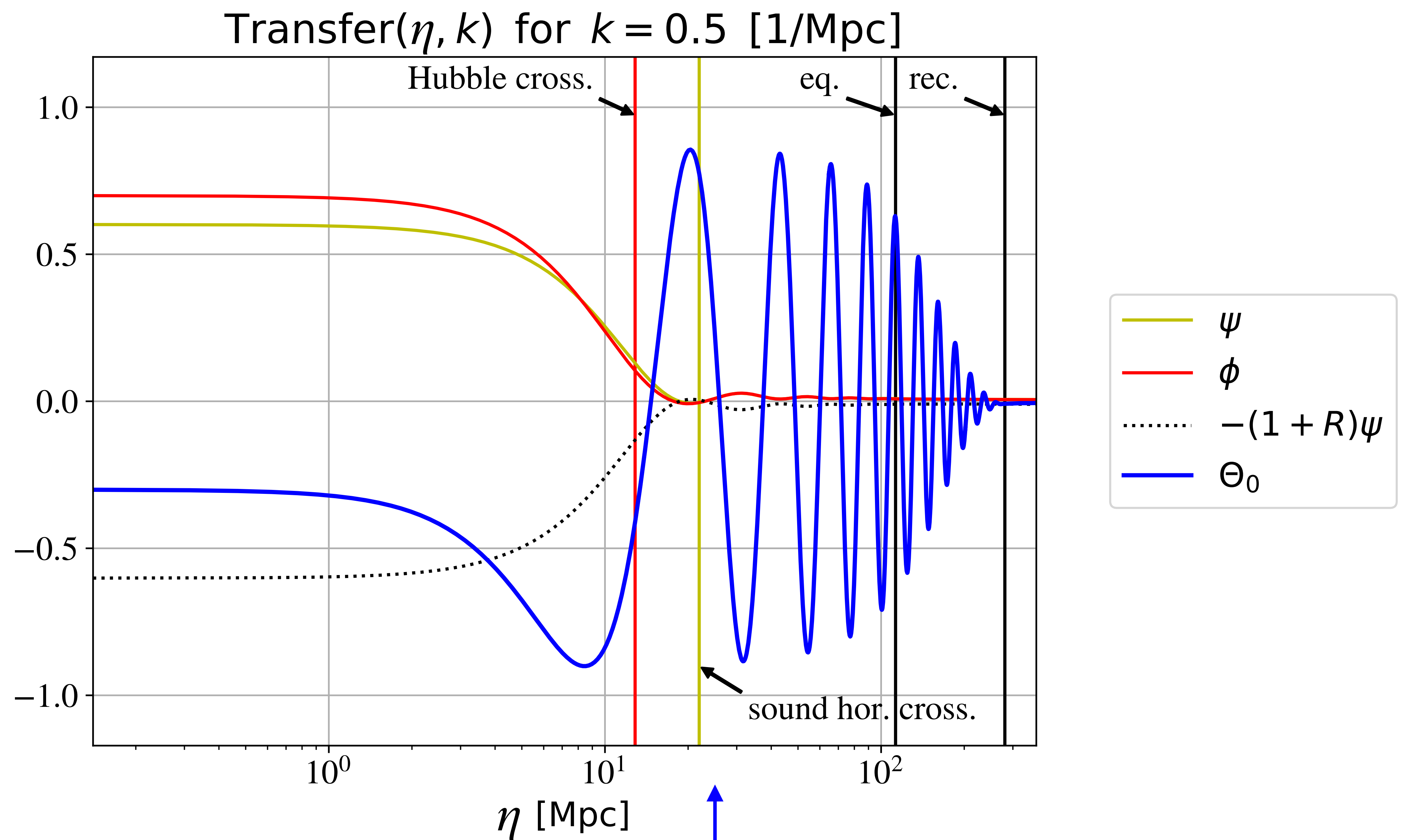
Evolution for one mode



Gravity boost effect from $\frac{R'}{1+R}\phi' + \phi''$

Will be important for effect of neutrinos, DR…

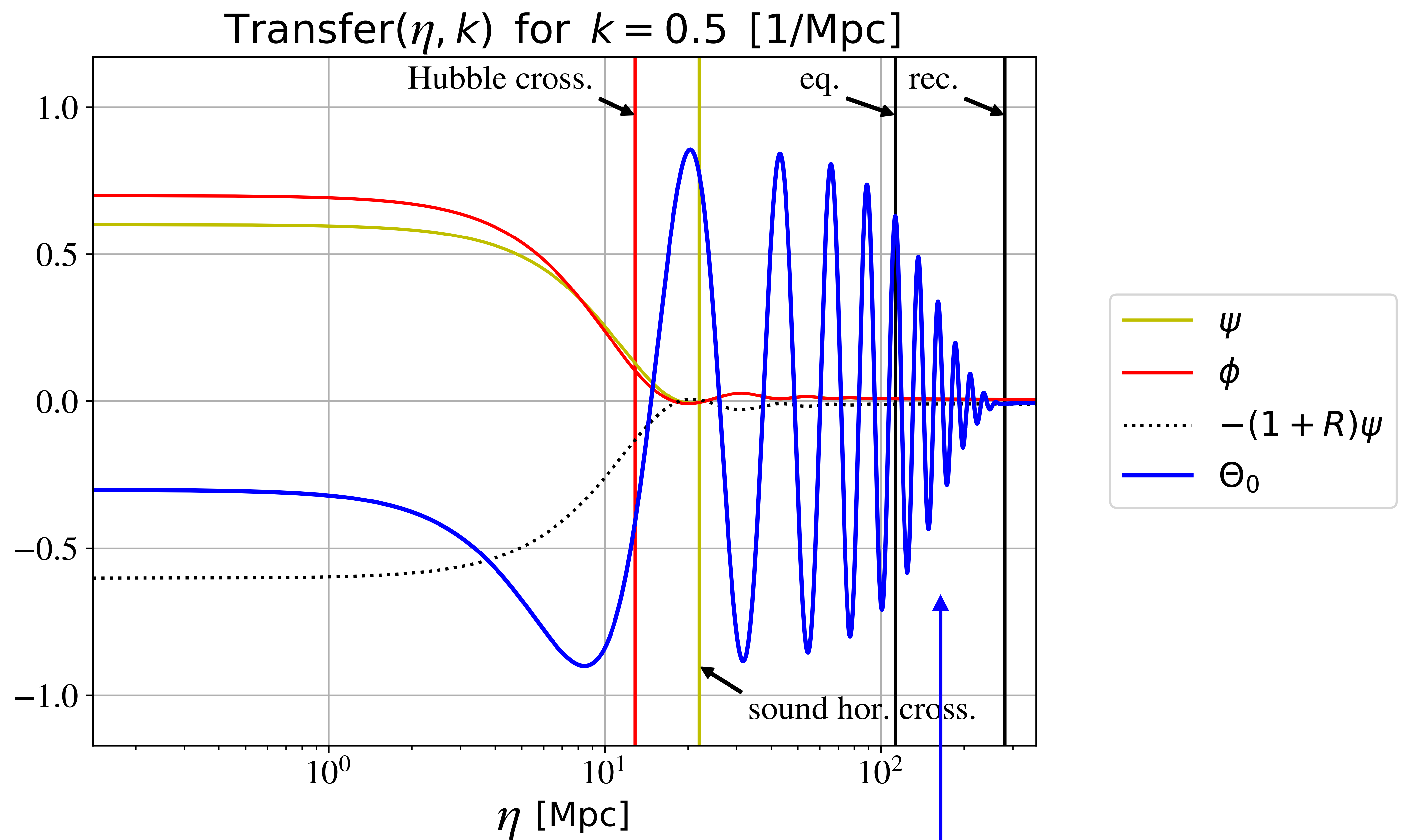
Evolution for one mode



symmetric and stationary oscillation

(deep sub-Hubble, deep DR)

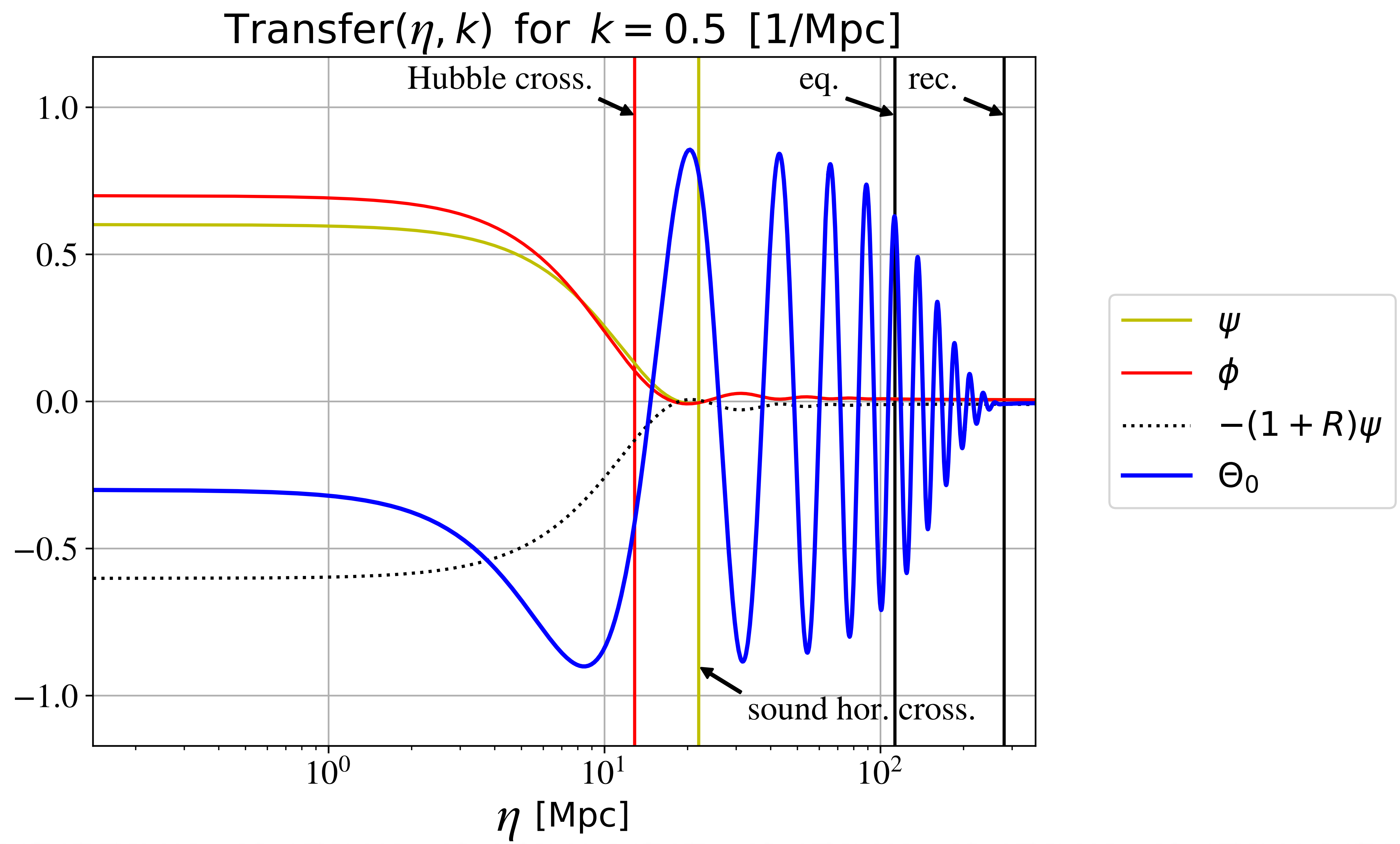
Evolution for one mode



exponentially damped oscillations

(approaching recombination)

Evolution for one mode



Final goal:

(MZ's line-of-sight integral)

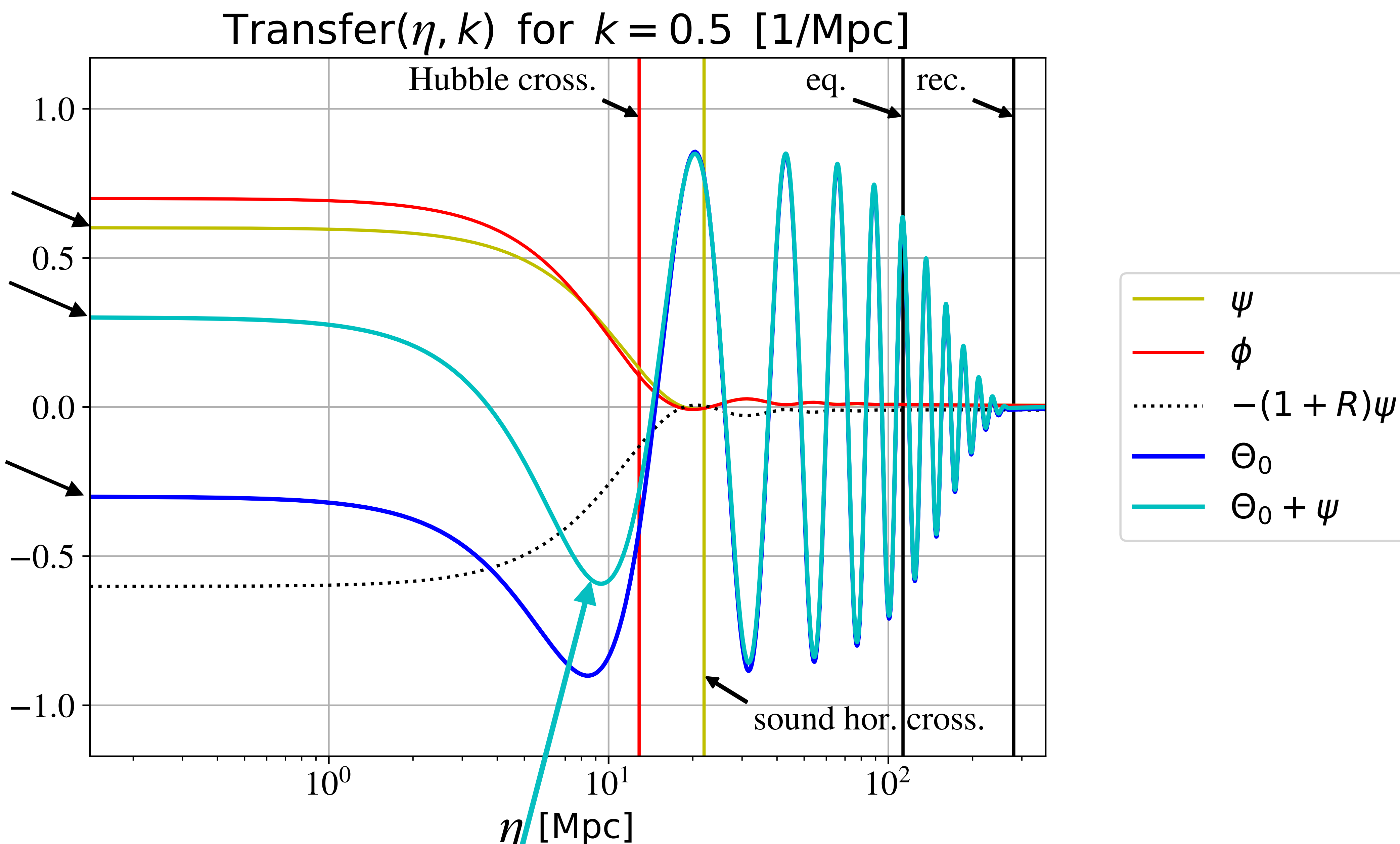
$$\Theta_l(\eta_0,k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \left(\underbrace{g\,(\Theta_0+\psi)}_{\rm SW} + \underbrace{\left(g\,k^{-2}\theta_{\rm b}\right)'}_{\rm Doppler} + \underbrace{e^{-\tau}(\phi'+\psi')}_{\rm ISW} \right) j_l(k(\eta_0-\eta))$$

Evolution for one mode

Adiabatic IC
($\mathcal{R}$=1, ψ= 2/3)

$\Theta_0 + \psi = 1/3$

photon IC
Θ_0 = -1/3



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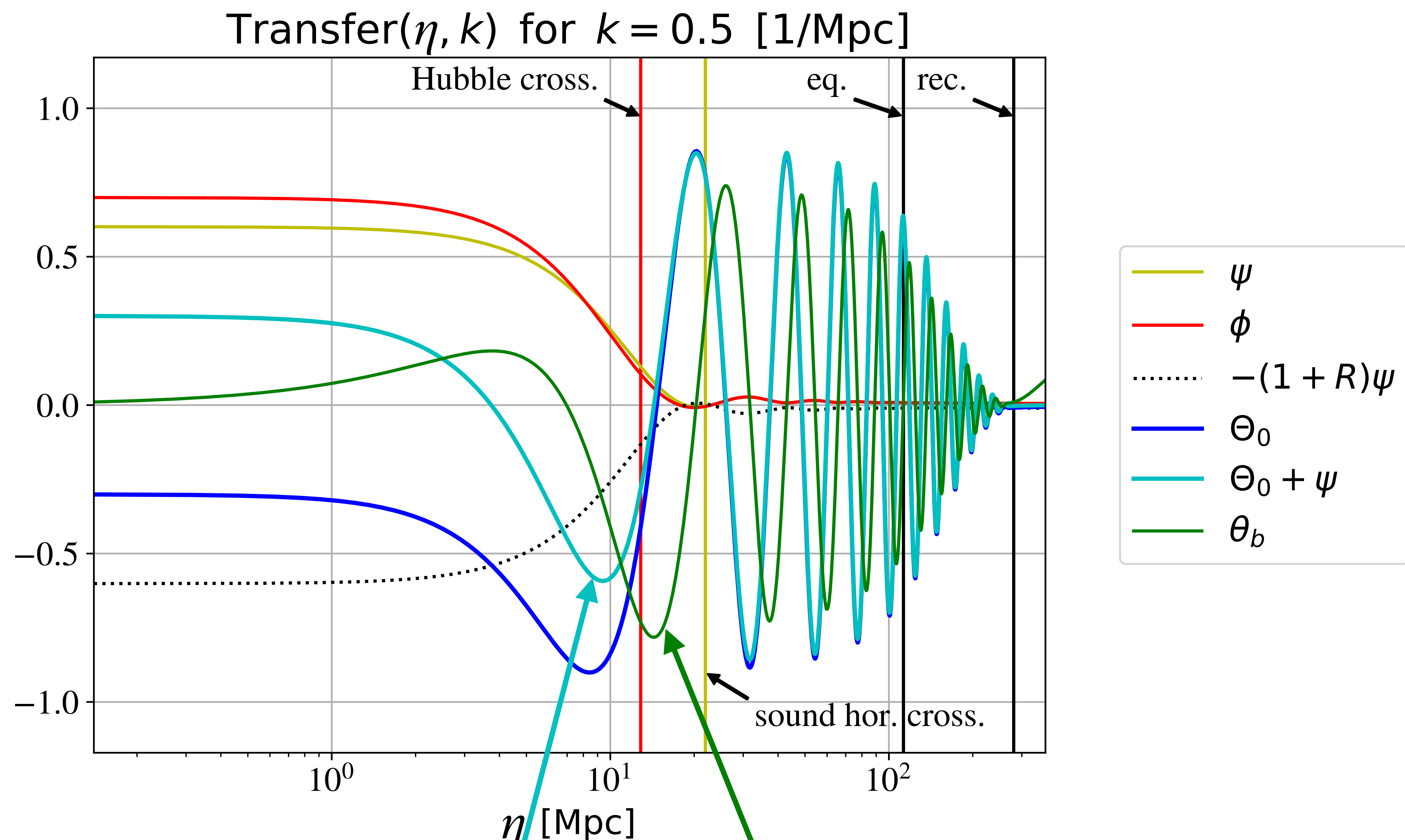
Evolution for one mode



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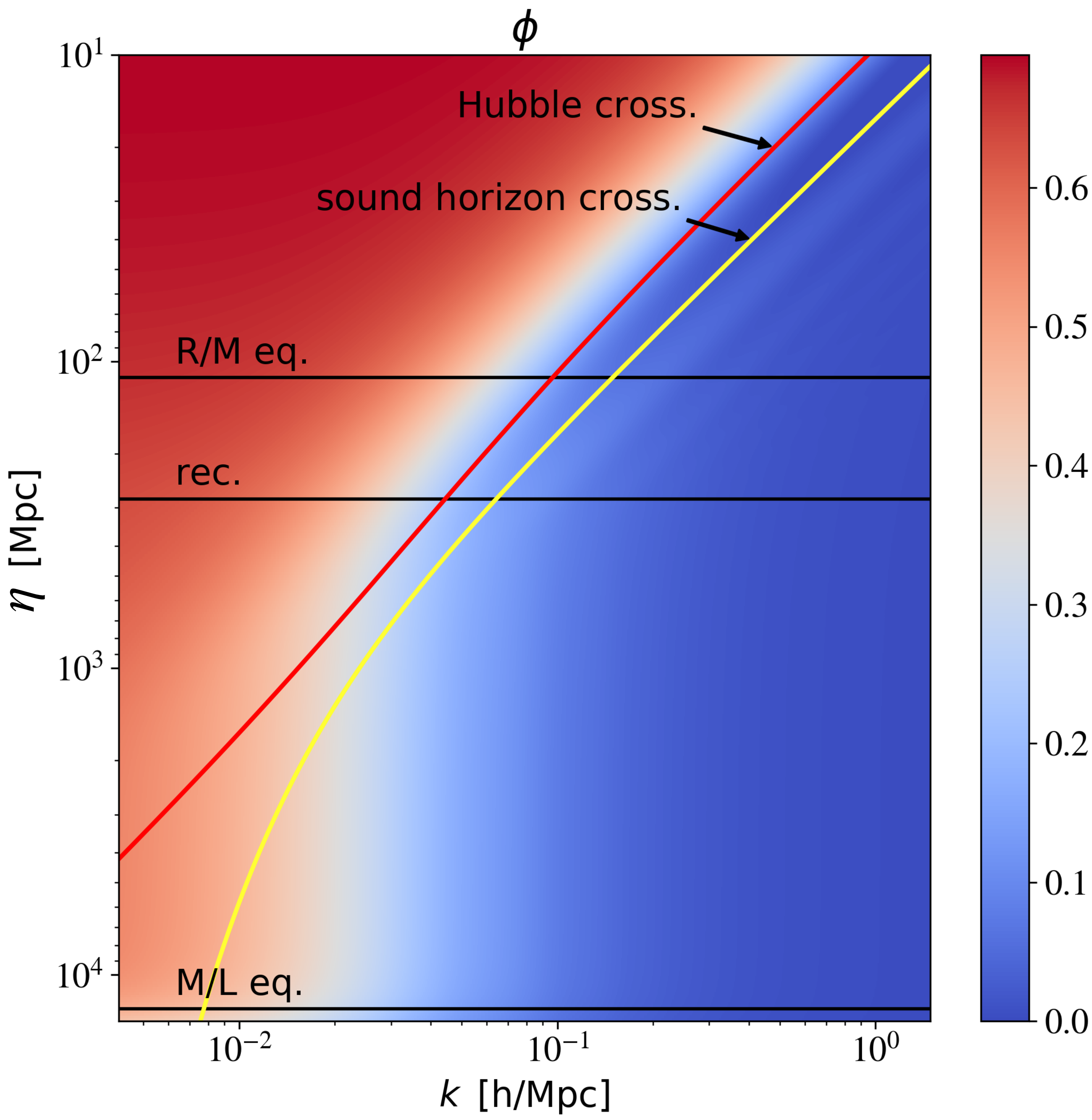
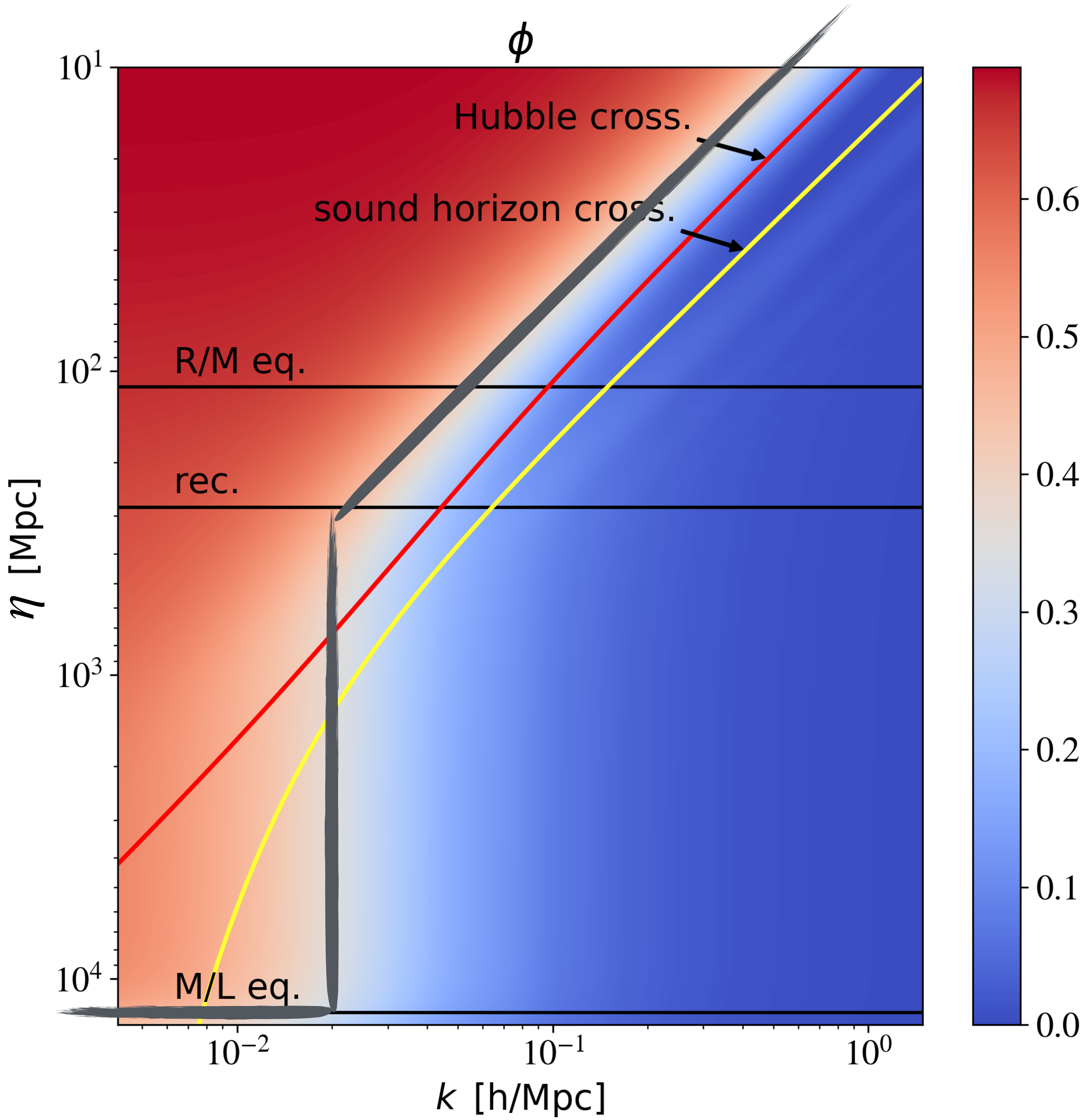
Evolution for all wavenumbers

Metric $\phi(\eta, k)$:



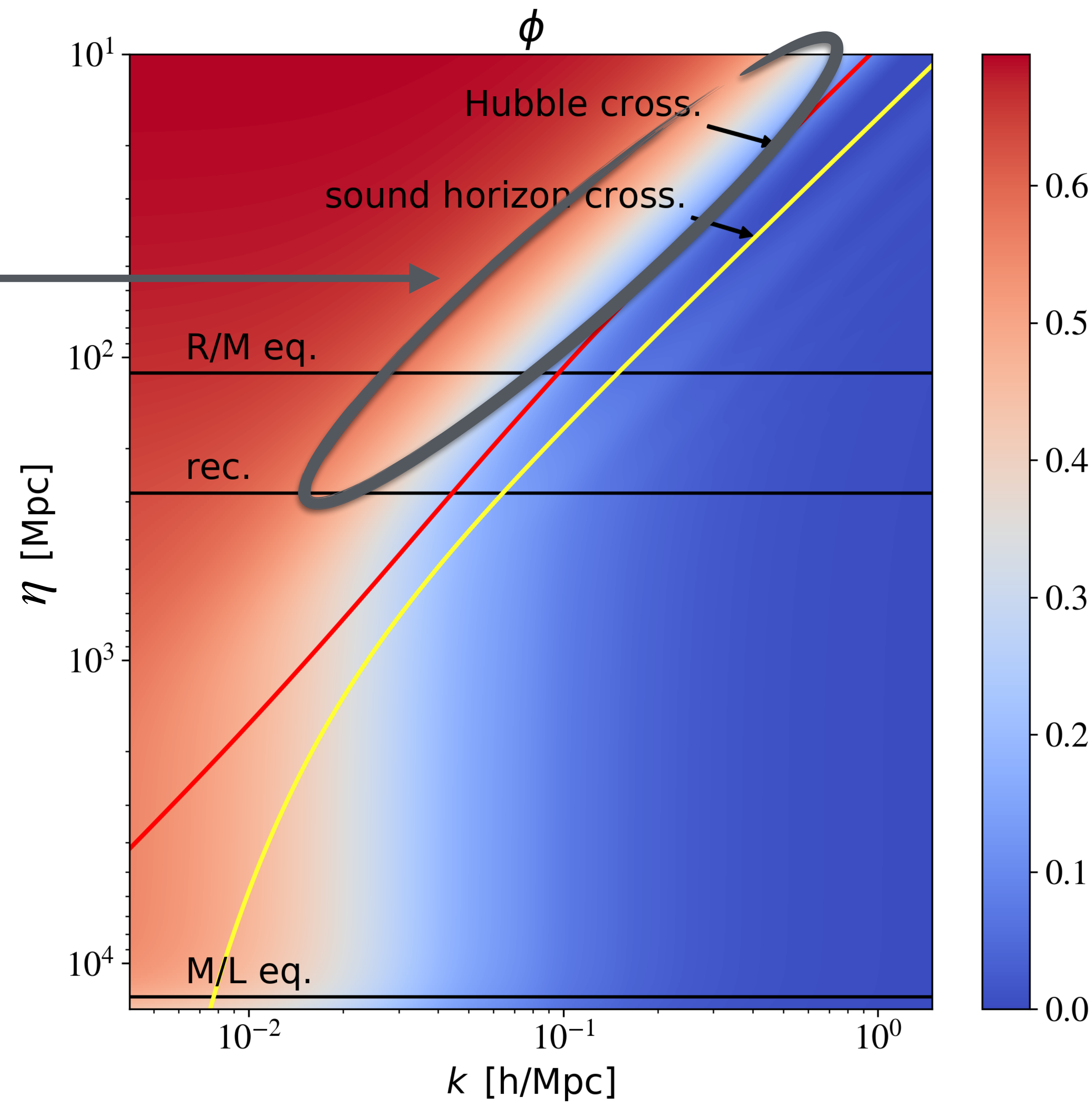
Evolution for all wavenumbers

Metric $\phi(\eta, k)$:



Evolution for all wavenumbers

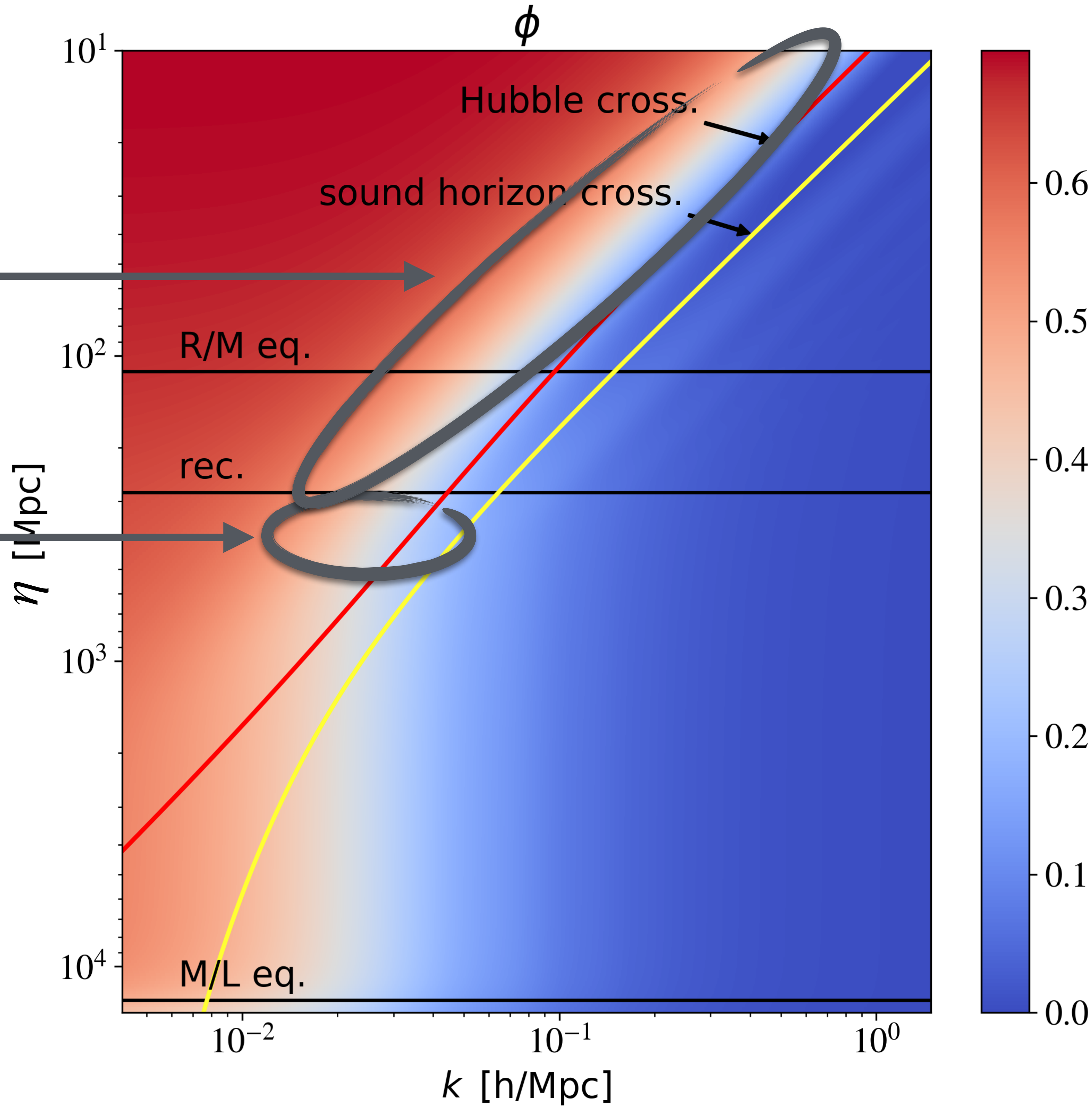
$$\frac{R'}{1+R}\phi' + \phi'' \Rightarrow \text{gravity boost}$$



Evolution for all wavenumbers

$$\frac{R'}{1+R}\phi' + \phi''$$ gravity boost

$$\underbrace{e^{-\tau}(\phi' + \psi')}_{\text{ISW}}$$ early ISW

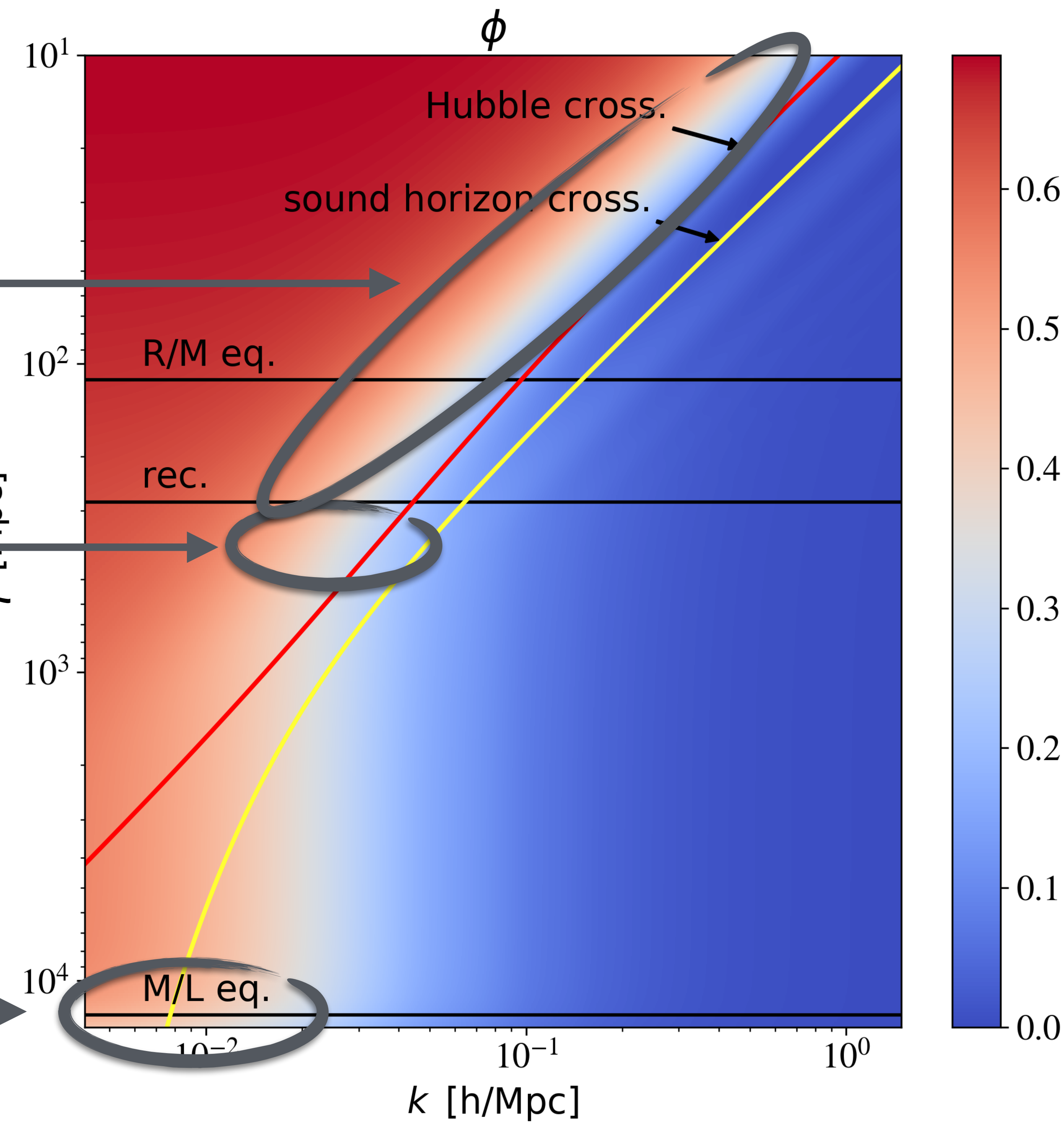


Evolution for all wavenumbers

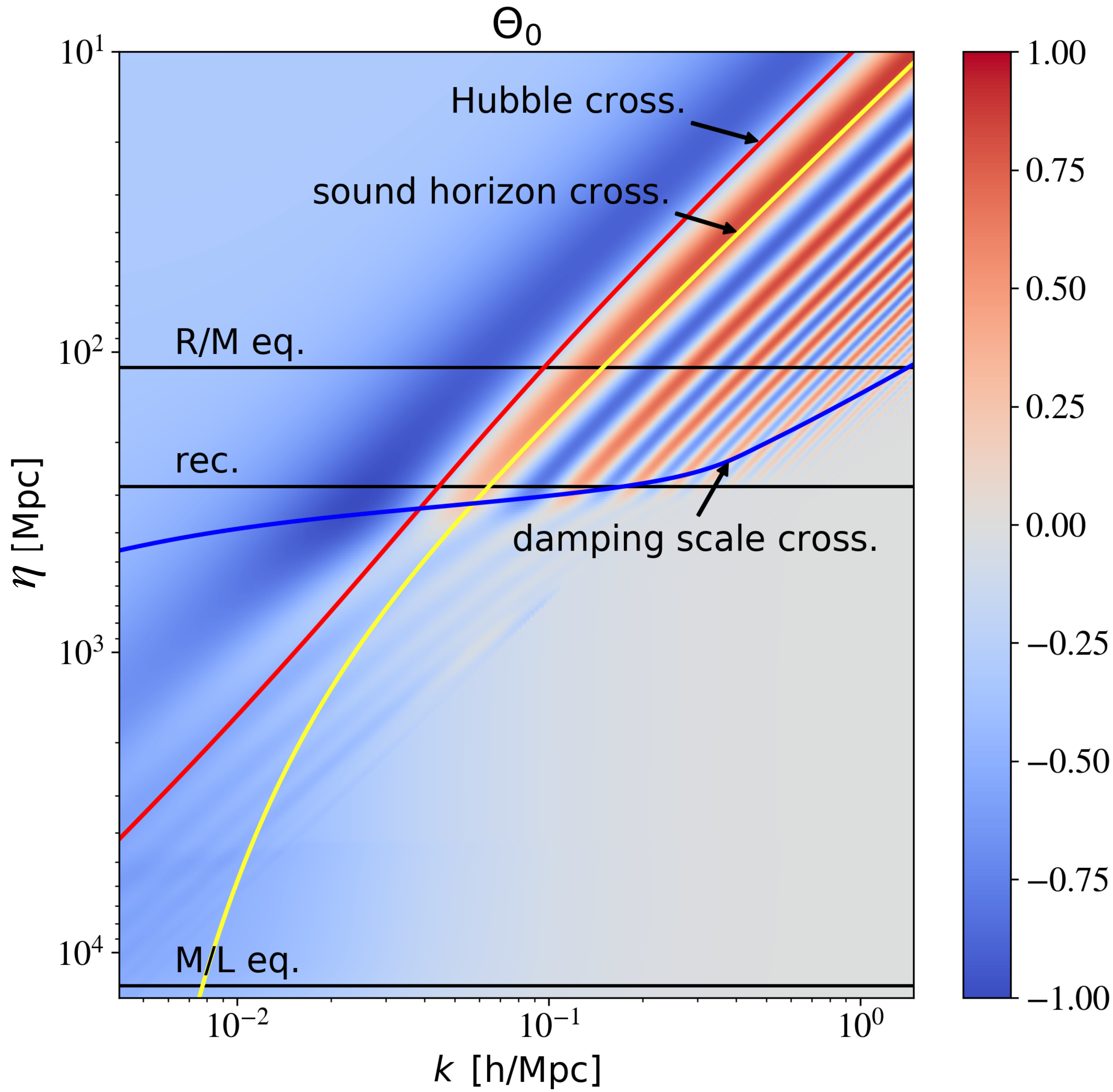
$$\frac{R'}{1+R}\phi' + \phi''$$ gravity boost

$$\underbrace{e^{-\tau}(\phi' + \psi')}_{\text{ISW}}$$ early ISW

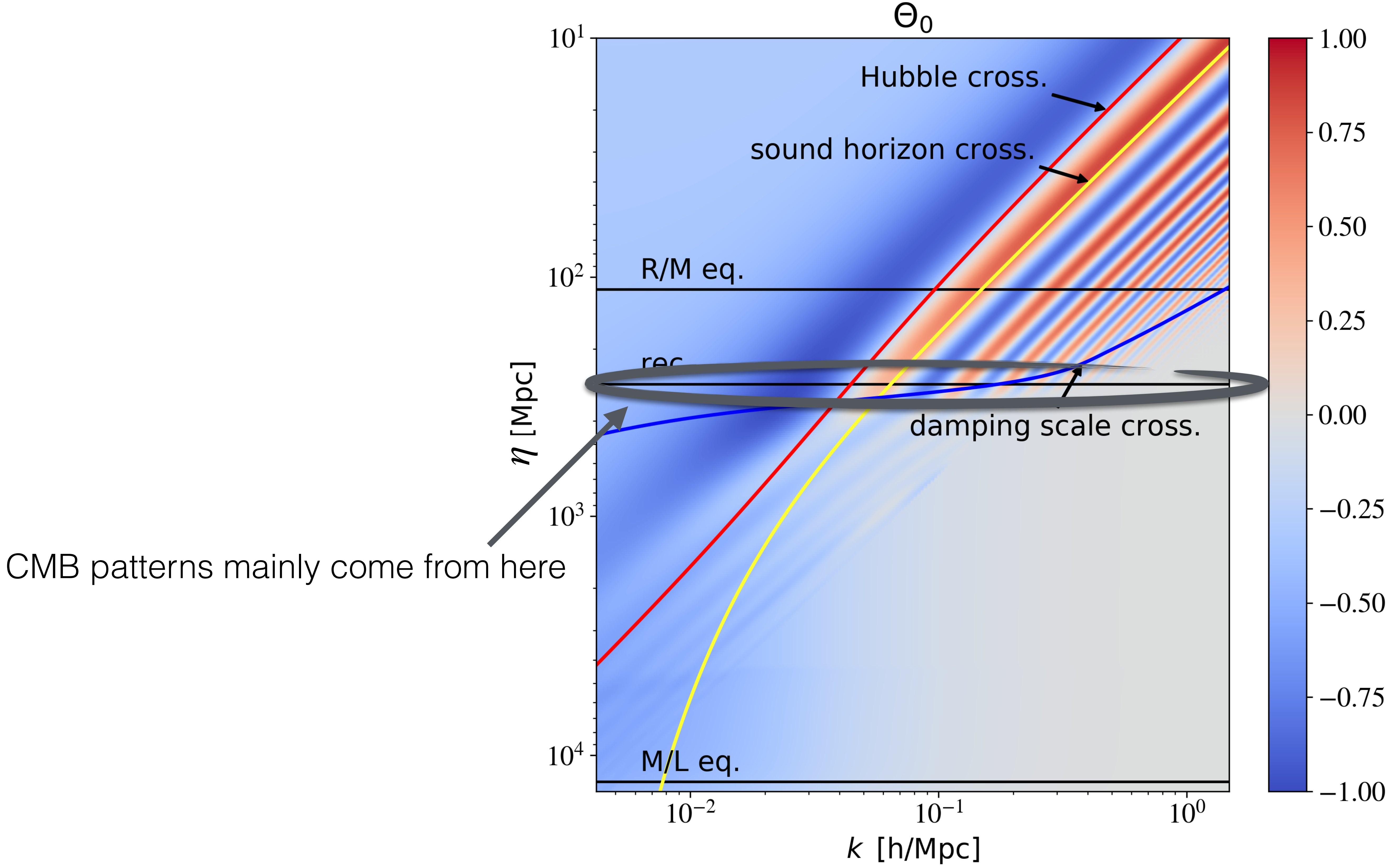
$$\underbrace{e^{-\tau}(\phi' + \psi')}_{\text{ISW}}$$ late ISW



Evolution for all wavenumbers



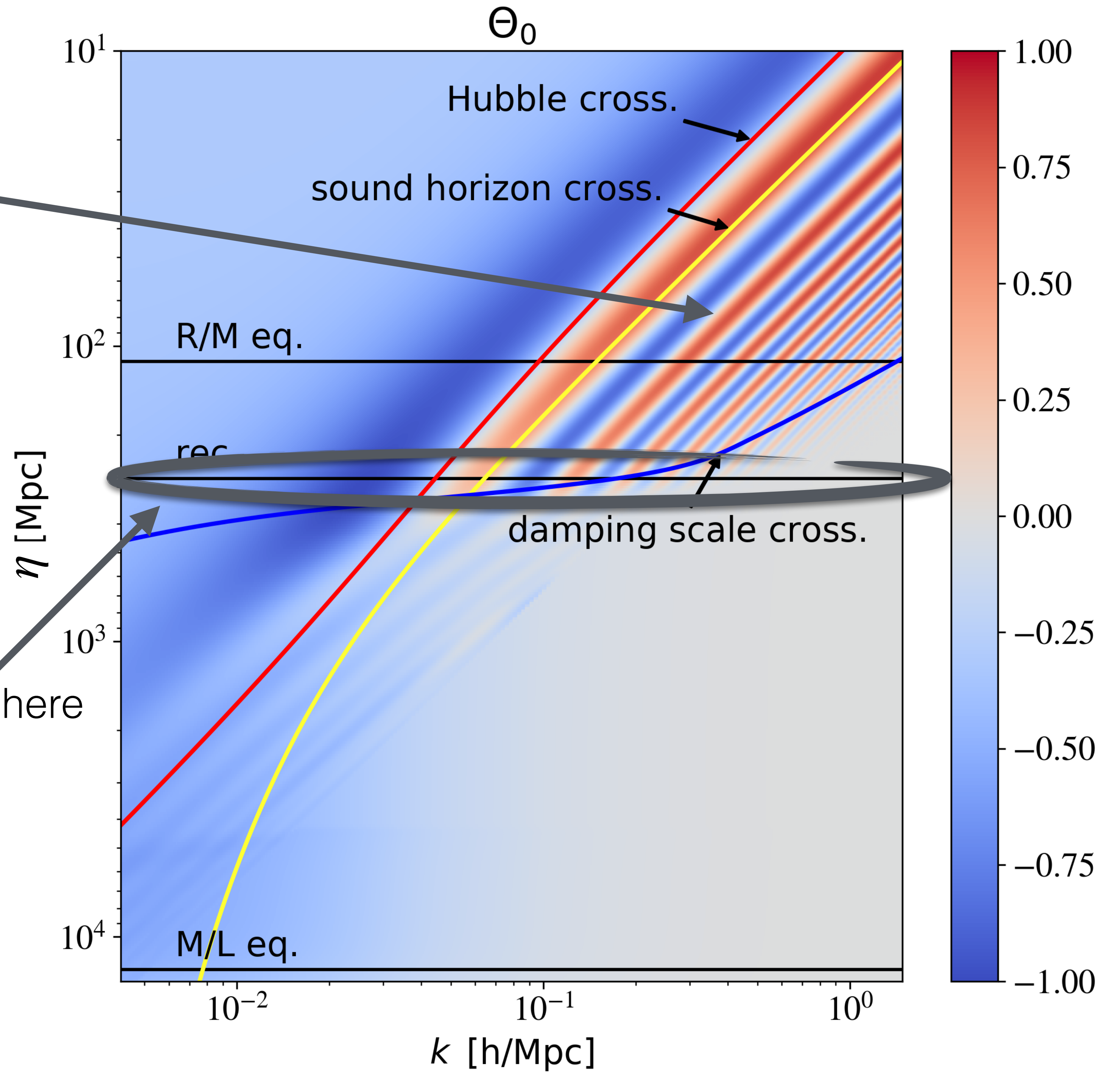
Evolution for all wavenumbers



Evolution for all wavenumbers

45° symmetry comes from

$$\cos(kc_s\eta)$$

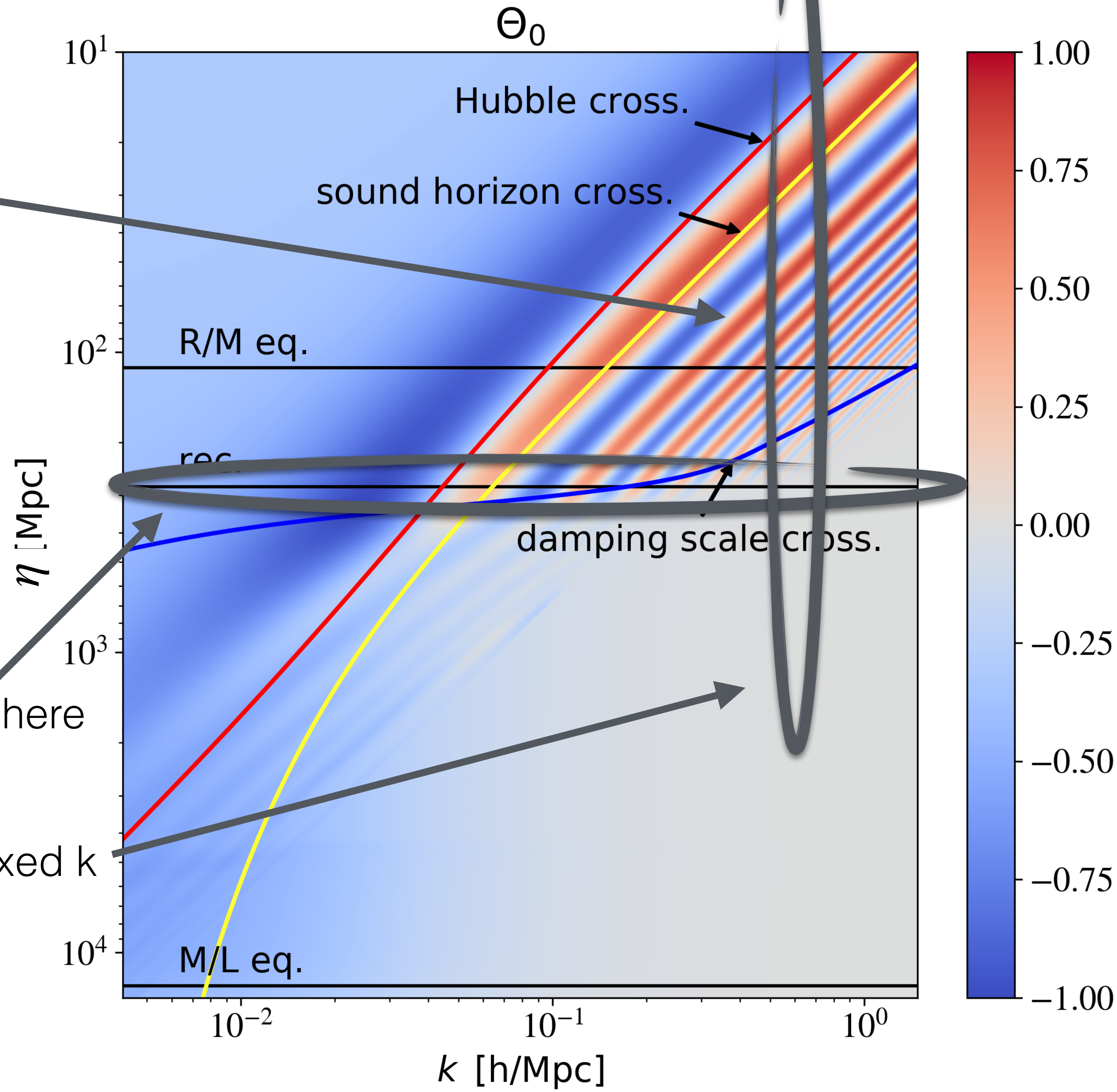


CMB patterns mainly come from here

Evolution for all wavenumbers

45° symmetry comes from

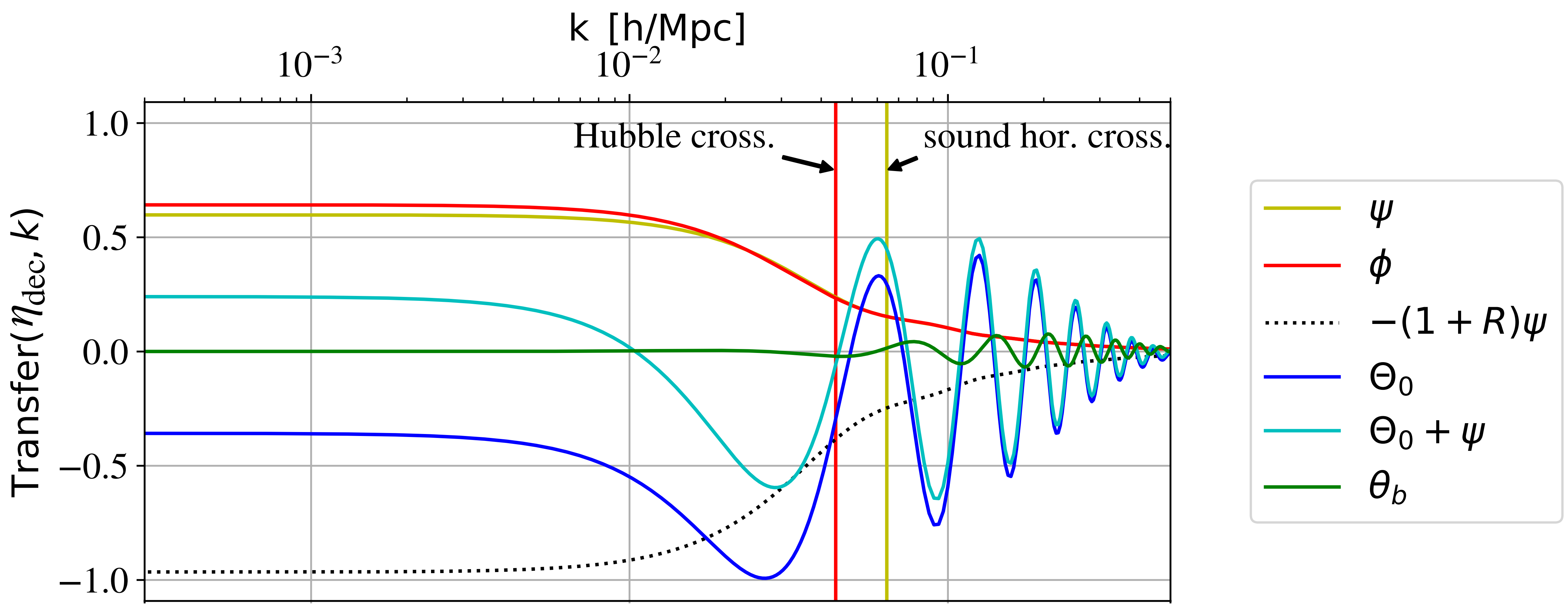
$$\cos(kc_s\eta)$$

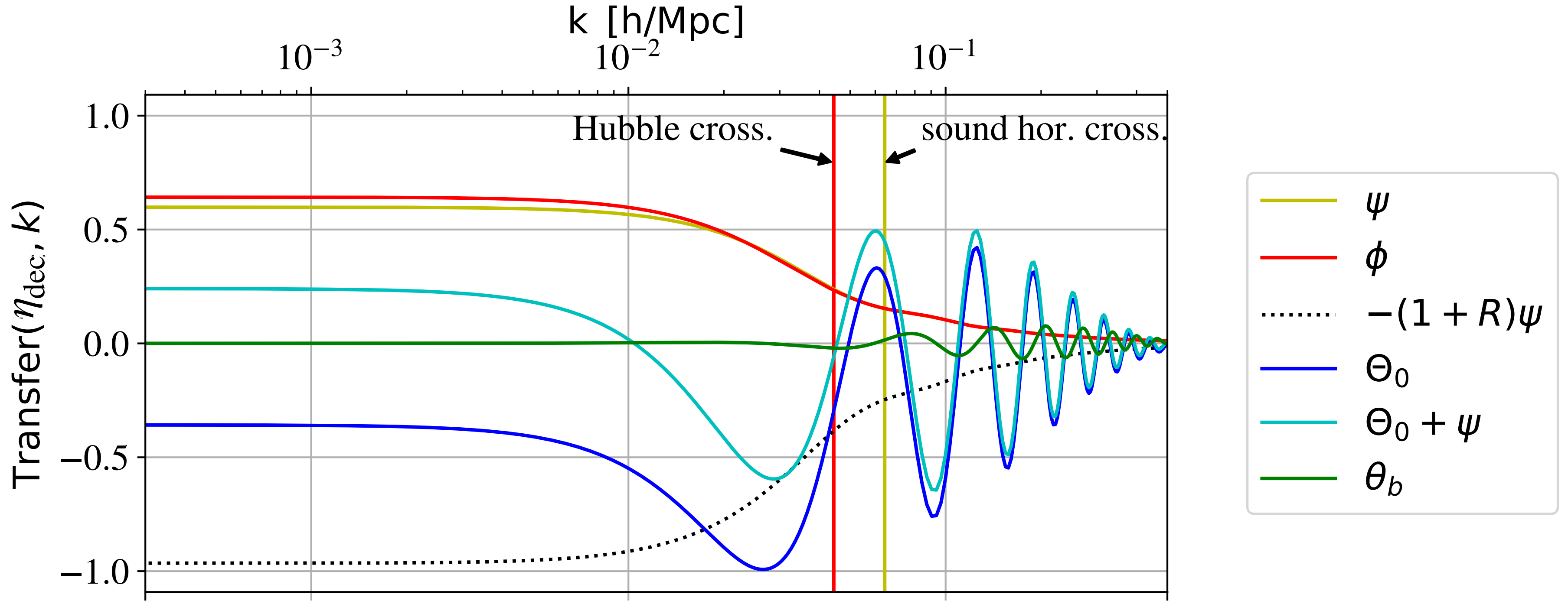


CMB patterns mainly come from here

fixed time will look like previous fixed k

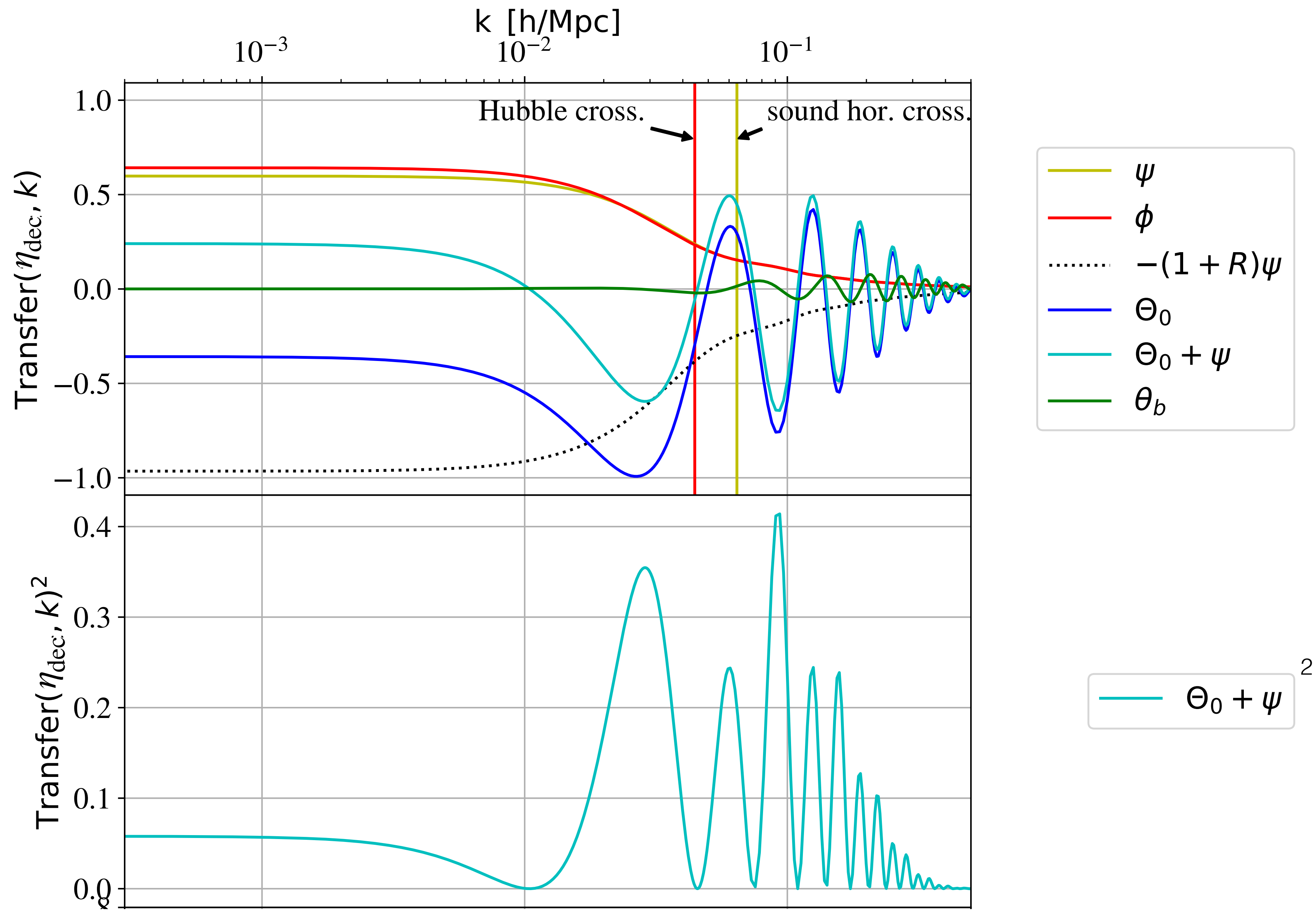
Transfer functions at recombination/decoupling



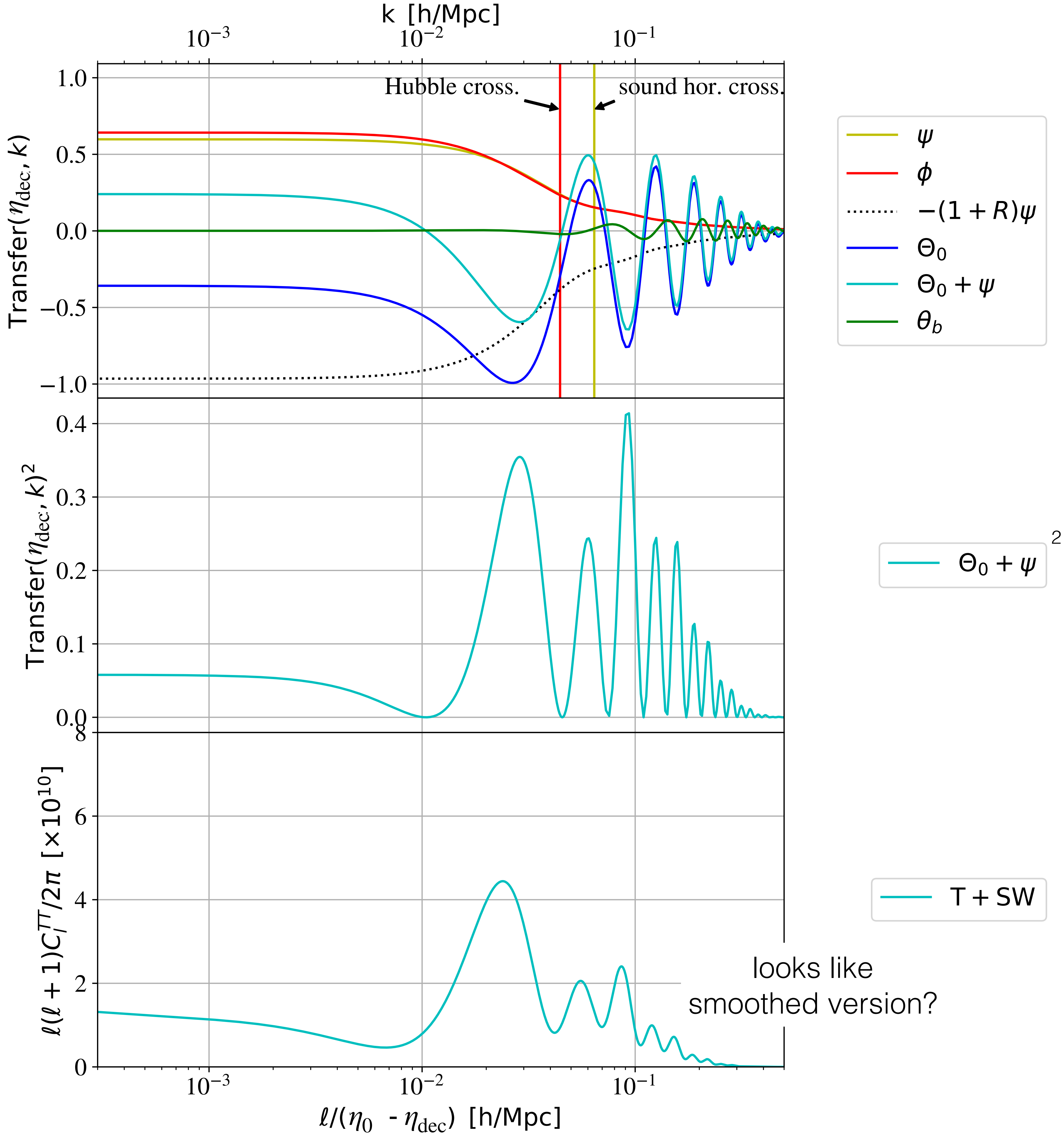


from transfer
to C_ℓ :

from transfer to C_ℓ :



from transfer to C_ℓ :



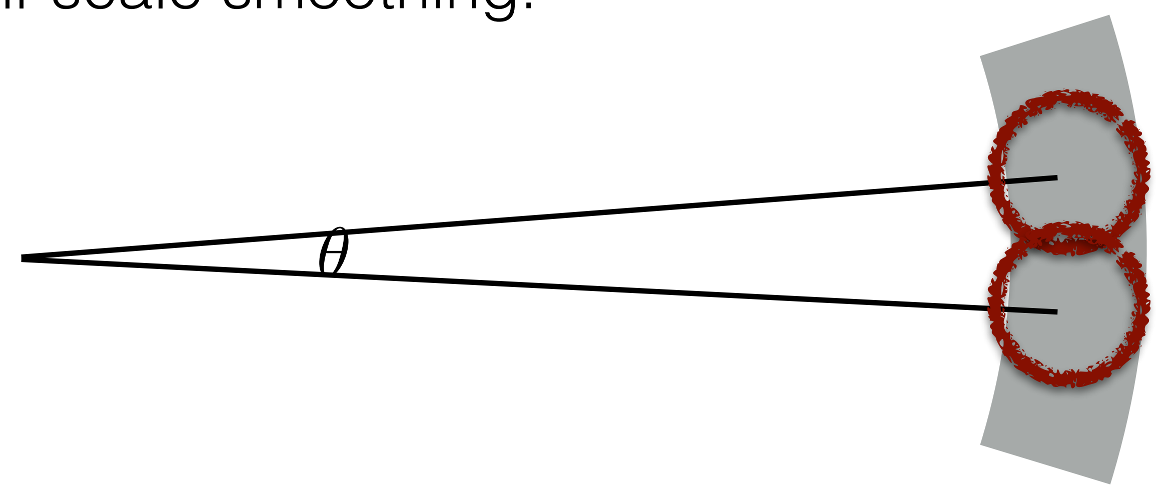
looks like smoothed version?

$\Theta_0(\eta_{\mathrm{dec}}, k) + \psi(\eta_{\mathrm{dec}}, k)$ independent of k would give $l(l+1)C_l =$ constant

Projection effects

- Thickness of l.s.s produces small-scale smoothing:

observed photons could carry temperature from wherever inside circles with radius $\lambda_D(\eta_{\mathrm{dec}})$



- Mathematically, two types of smoothing Kernels:

$$\Theta_l(\eta_0,k) = \int_{\eta_{\mathrm{ini}}}^{\eta_0} d\eta \left(g\,(\Theta_0 + \psi) + \ldots \right) j_l(k(\eta_0 - \eta))$$

$$C_l \equiv \langle |a_{lm}|^2 \rangle = \frac{1}{2\pi^2} \int \frac{dk}{k} \Theta_l^2(\eta_0,k) \mathcal{P}_{\mathcal{R}}(k)$$

—> contribution of wide range of *times* and *wavenumber* to single C_l

from transfer to C_ℓ :