Large Scale Structure II

(precision frontier)

Lecture 1: introduction, context, SPT

Lecture 2: one-loop power spectrum

Lecture 3: EFTofLSS, IR resummation

Lecture 4: biased tracers, RSD, application to BOSS

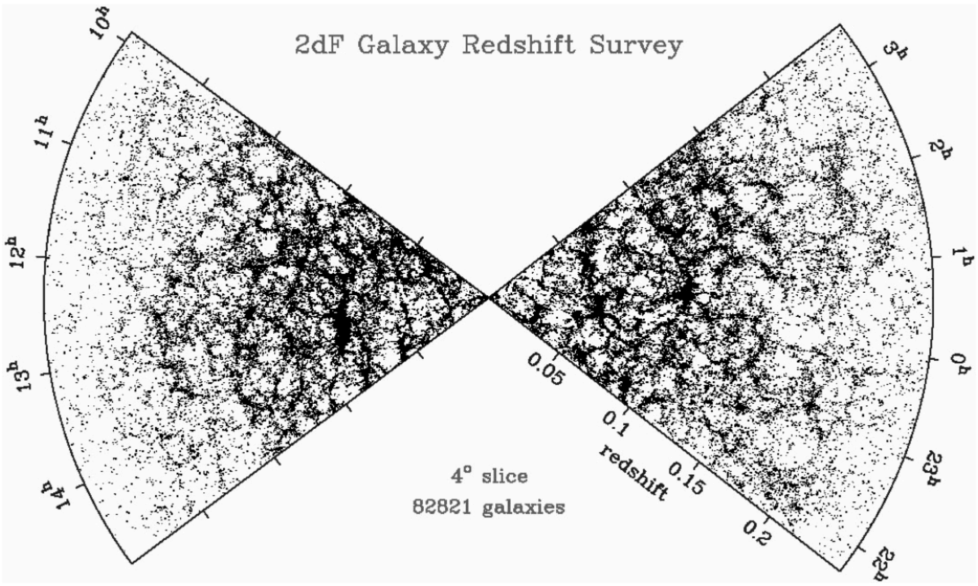
Literature

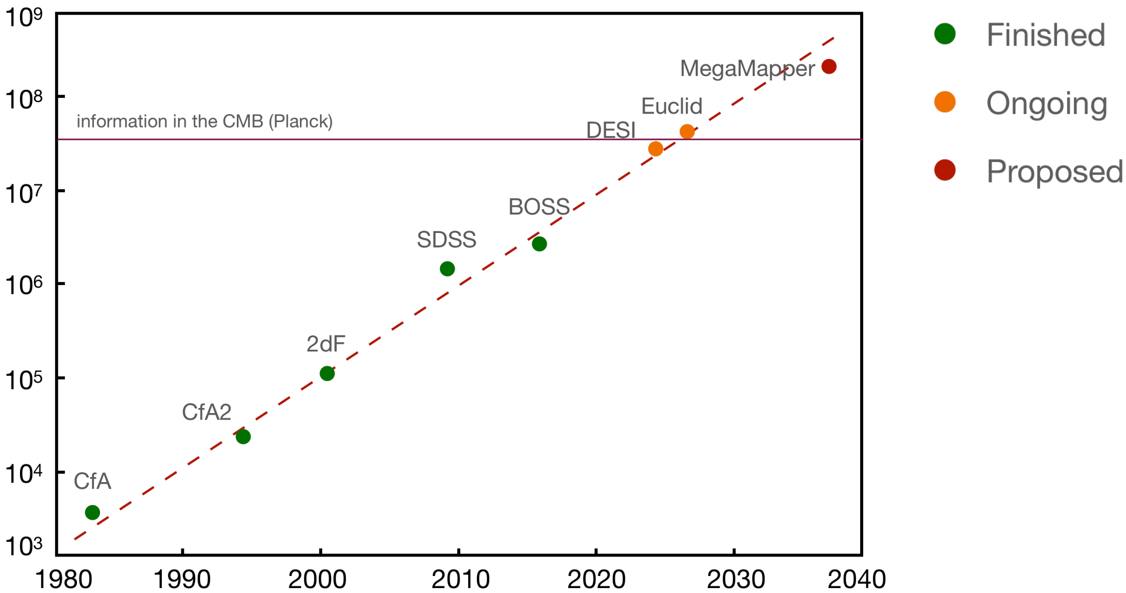
Effective Field Theory of Large-Scale Structure by Tobias Baldauf

Lectures on the Effective Field Theory of Large-Scale Structure by Leonardo Senatore

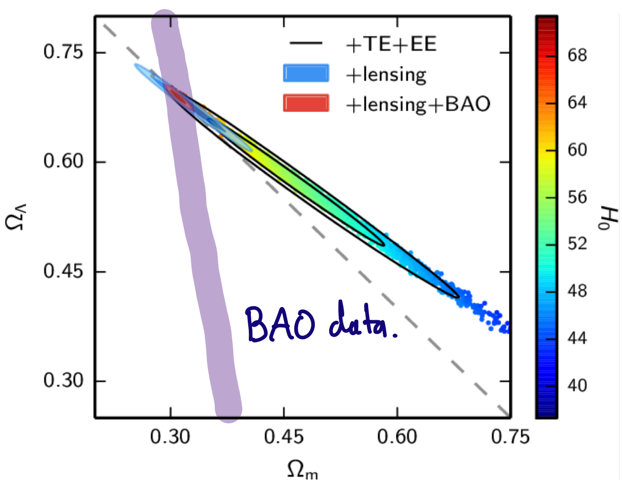
Effective Field Theory for Large-Scale Structure by Mikhail Ivanov

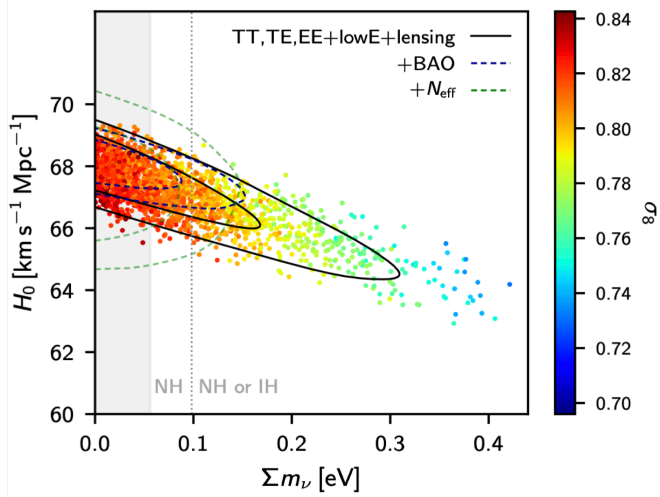
Lecture 1


2dF Galaxy Redshift Survey
4° slice
82821 galaxies
redshift
0.05
0.1
0.15
0.2


information in the CMB (Planck)
CfA
CfA2
2dF
SDSS
BOSS
DESI
Euclid
MegaMapper
Finished
Ongoing
Proposed

Why not just CMB?





- Information in the CMB is limited $N_{pix} \sim \ell_{max}$
  $\ell_{max} \sim 3000$
- Beyond ΛCDM there are strong degeneracies that CMB alone cannot break

What do we hope to learn?

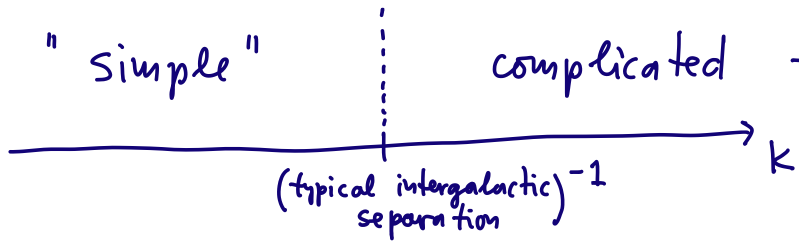
1) Distribution of galaxies remembers the initial conditions

Single "clock"? Speed of inflaton fluctuations less than 1?
"Spectroscopy" of massive/higher spin particles?
Primordial features in the power spectrum?

2) Everything gravitates

Sum of neutrino masses. Other massive (but light) relics? Ultralight axions?
Spatial curvature, dark energy?
New energy components in early or late universe?
Probing dark sector, new long-range interactions?

Why focusing on large scales?



$\rightarrow$ needs a very good knowledge of astrophysics

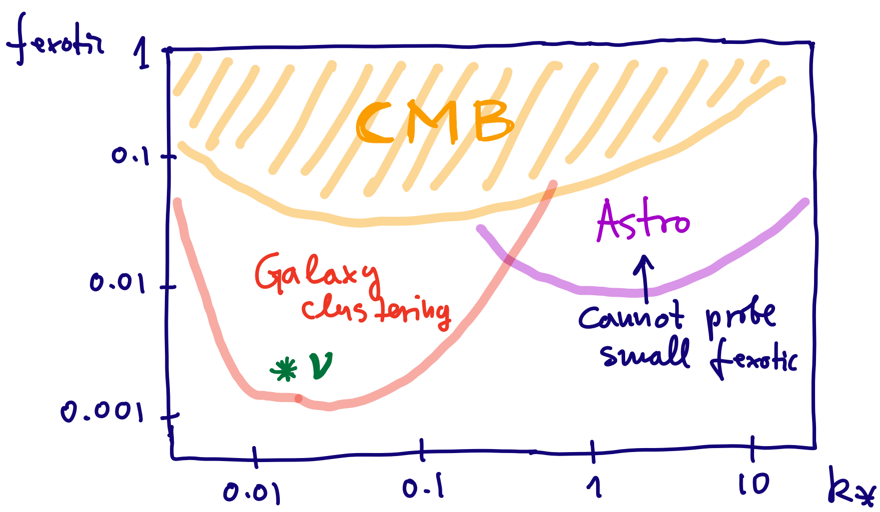
Example: A fraction of DM is exotic

Two parameters: f_{exotic}, k_*

f_{exotic} ↓ fraction

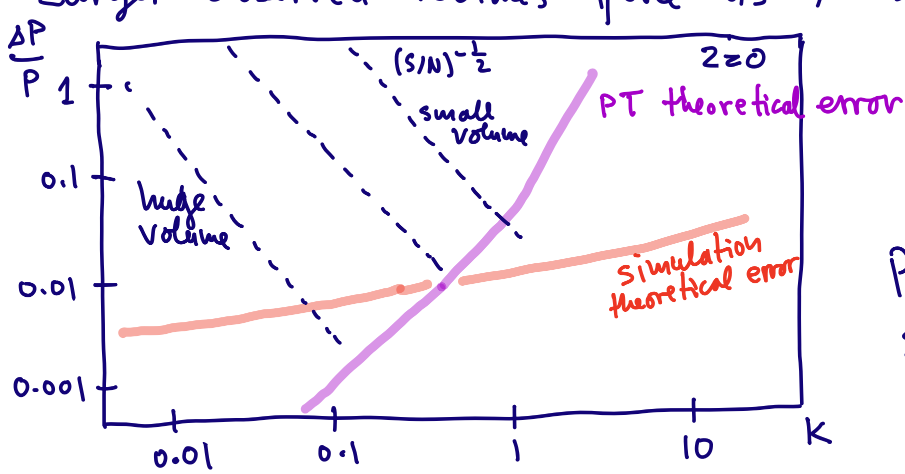
k_* ↓ relevant scale

Neutrinos: $f_{exotic} \sim 0.002$, $k_* \sim 0.01\ h/\mathrm{Mpc}$



Unique parameter space for small f_{exotic} and small k_*!

Larger observed volumes force us to stay on large scales



$(S/N) \sim V k_{max}^3$

$\frac{\Delta P}{P} \sim (S/N)^{-\frac{1}{2}}$

Precision on large scales pays off!

Nonlinear dark matter

Naively, we say that DM is preassureless ideal fluid

We will see that this is wrong!

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}_r (\rho \cdot \vec{V}) = 0 \qquad \text{continuity eq.}$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla}_r) \vec{V} = - \vec{\nabla} \Phi \qquad \text{Euler eq.}$$

$$\nabla^2 \Phi = 4\pi G \rho \qquad \text{Poisson eq.}$$

- Split the fields in homogeneous part and fluctuations

$$\delta(\vec{r}, t) \equiv \frac{\rho(\vec{r}, t) - \bar{\rho}(t)}{\bar{\rho}(t)} \Rightarrow \rho = \bar{\rho}(1+\delta)$$

$$\vec{V} = \underset{\substack{\shortparallel \\ H\vec{r}}}{\vec{V}_0} + \delta\vec{V} \qquad \text{and} \qquad \Phi = \bar{\Phi} + \phi$$

- Go to comoving coordinates $\vec{x} = \frac{\vec{r}}{a}$ $\quad \vec{v} \equiv \frac{\delta \vec{V}}{a}$

$$\vec{\nabla}_r \to \frac{1}{a} \vec{\nabla}_x$$

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - H \vec{x} \cdot \vec{\nabla}_x$$

- Use conformal time τ as a time variable $\quad ' \equiv \frac{\partial}{\partial \tau}$

Exercise: Derive e.o.m. for fluctuations $\quad ' \equiv \frac{\partial}{\partial \tau}$

$$\delta' + \vec{\nabla} \cdot \left((1+\delta) \vec{v} \right) = 0$$

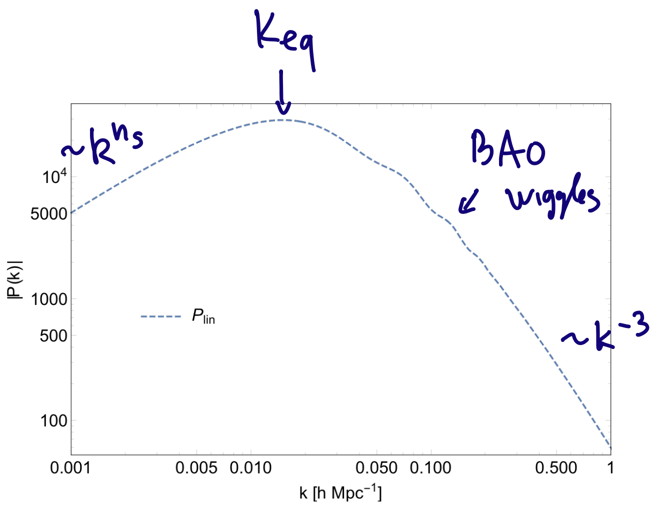
$$v_i' + \mathcal{H} v_i + v_j \nabla_j v_i = - \nabla_i \phi$$

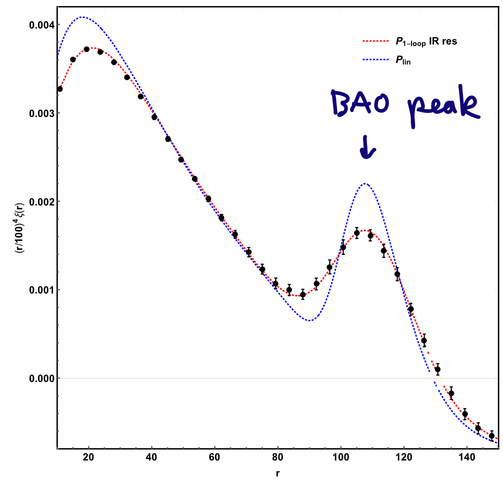
$$\nabla^2 \phi = \frac{3}{2} \mathcal{H}^2 \Omega_m \cdot \delta$$

Initial conditions

δ is a Gaussian random field with the power spectrum $P_{lin}(k)$

Velocities are zero.





How large is a typical fluctuation of δ on a scale R?

$$\delta_R(\vec{x}) = \int d^3\vec{x}'\, W_R(\vec{x}-\vec{x}')\, \delta(\vec{x}')$$

$$\hookrightarrow W_R(\vec{x}-\vec{x}') \sim e^{-\frac{1}{2}\frac{(\vec{x}-\vec{x}')^2}{R^2}}$$

$\langle \delta_R(\vec{x})\rangle = 0$ by definition.

$$\mathrm{var}(\delta_R) = \langle \delta_R^2(\vec{x})\rangle - \underbrace{\langle \delta_R(\vec{x})\rangle^2}_{0} = \int \frac{d^3\vec{k}}{(2\pi)^3} P(k)\, W_R^2(k)$$

At redshift $z=0$, $\mathrm{var}(\delta_R) \simeq 1$ when $R \simeq$ few Mpc/h

We define a parameter $\Delta^2(k)$:

$$\boxed{\Delta^2(k) \equiv \int^{k} \frac{d^3\vec{q}}{(2\pi)^3} P(q)}$$

approximation $\Delta^2(k) \approx \frac{k^3}{2\pi^2} P(k)$

When $\Delta^2(k) < 1$, fluctuations are small.

Perturbative solution

$$\delta' + \vec{\nabla}\cdot\left((1+\delta)\vec{v}\right) = 0$$

$$v_i' + \mathcal{H} v_i + v_j \nabla_j v_i = -\nabla_i \phi$$

$$\nabla^2 \phi = \frac{3}{2}\mathcal{H}^2 \Omega_m \cdot \delta$$

We want to solve these e.o.m. for $\Delta^2(k) \lesssim 1$.

When $\Delta^2(k) \ll 1$ fluctuations are very small
We can use the linear theory.

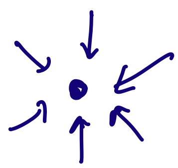
$$\delta' + \vec{\nabla}\cdot\vec{v} = 0 \qquad\qquad \vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp}$$

$$v_i' + \mathcal{H} v_i = -\vec{\nabla}\phi \qquad\qquad \vec{\nabla}\cdot\vec{v}_{\perp} = 0 \qquad \vec{\nabla}\times\vec{v}_{\parallel} = 0$$

$$\nabla^2\phi = \frac{3}{2}\mathcal{H}\Omega_m\delta \qquad\qquad \theta \equiv \vec{\nabla}\cdot\vec{v} \qquad \vec{\omega} = \vec{\nabla}\times\vec{v}$$

$\vec{\omega}' + \mathcal{H}\vec{\omega} = 0 \Rightarrow \vec{\omega}$ decays with time.

$$\delta' + \theta = 0$$



Positive $\delta \Rightarrow$ negative $\vec{\nabla}\cdot\vec{v}$

$$\delta'' + \mathcal{H}\delta' - \frac{3}{2}\Omega_m \mathcal{H}\delta = 0$$

$$\Omega_m = 1 \Rightarrow D_+ = a \qquad D_- = a^{-3/2}$$

1) At the nonlinear level:

$$\vec{\omega}' + \mathcal{H}\vec{\omega} + \vec{\nabla} \times (\vec{v} \times \vec{\omega}) = 0$$

we will neglect $\vec{\omega}$ from now on.

2) Finally we focus on

$$\delta' + \vec{\nabla} \cdot ((1+\delta)\vec{v}) = 0$$

$$v_i' + \mathcal{H} v_i + v_j \nabla_j v_i = -\nabla_i \phi \quad \text{with} \quad \vec{\nabla} \cdot \vec{v} = \theta$$

$$\nabla^2 \phi = \frac{3}{2} \mathcal{H}^2 \Omega_m \cdot \delta$$

Fourier conventions:

$$\delta(\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3} \delta(\vec{k}) e^{i\vec{k}\vec{x}} \qquad \int_{\vec{q}} \equiv \int \frac{d^3\vec{q}}{(2\pi)^3}$$

Exercise: Derive the eom in Fourier space.

$$\delta'(\vec{k}) + \theta(k) = -\int_{\vec{q}_1, \vec{q}_2} (2\pi)^3 \delta^D(\vec{k} - \vec{q}_1 - \vec{q}_2) \alpha(\vec{q}_1, \vec{q}_2) \theta(\vec{q}_1) \delta(\vec{q}_2)$$

$$\theta'(\vec{k}) + \mathcal{H}\theta(\vec{k}) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta(\vec{k})$$

$$= -\int_{\vec{q}_1, \vec{q}_2} (2\pi)^3 \delta^D(\vec{k} - \vec{q}_1 - \vec{q}_2) \beta(\vec{q}_1, \vec{q}_2) \theta(\vec{q}_1) \theta(\vec{q}_2)$$

$$\alpha(\vec{q}_1, \vec{q}_2) = \frac{\vec{q}_1 (\vec{q}_1 + \vec{q}_2)}{q_1^2} \qquad \beta(\vec{q}_1, \vec{q}_2) = \frac{1}{2} \frac{(\vec{q}_1 + \vec{q}_2)^2 \, \vec{q}_1 \cdot \vec{q}_2}{q_1^2 q_2^2}$$