Lecture 2

Statistical Methods in cosmology
(lecture 2)

I. Bayesian and frequentist interpretation of probability

- Frequentist defines an event's probability as the limit of its frequency in a large # of trials
- Bayesian defines an event's probability based on the level of belief in a hypothesis

Bayesian: data are fixed and model is repeatable.

Frequentist: model is fixed & the data is repeatable.

Bayes's theorem

likelihood
("Bayesian evidence")

$$\boxed{P(M|D) = \frac{P(D|M) \cdot P(M)}{P(D)}}$$

prior

"posterior probability"

Model comparison:

$$\frac{P(M_0 \mid D)}{P(M_1 \mid D)} = \frac{P(D \mid M_0)\,P(M_0)}{P(D \mid M_1)\,P(M_1)} = B_{01}\,\frac{P(M_0)}{P(M_1)}$$

"Bayes factor"

VI. Likelihoods

Central limit theorem:

$X_1, X_2, \ldots, X_n$

temperature

$$\mathcal{L}(T \mid C_\ell^{th}) \propto \frac{1}{\sqrt{\det(S)}} \exp\left[-\frac{T^t S^{-1} T}{2}\right]$$

covariance

$$S_{ij} = \sum_\ell \frac{(2\ell+1)}{4\pi} C_\ell^{th} P_\ell(\hat{n}_i \cdot \hat{n}_j)$$

$$-2\ln\mathcal{L} = \sum_{\ell} (2\ell+1)\left[\ln\left(\frac{C_\ell^{\text{theory}}}{C_\ell^{\text{data}}}\right) + \left(\frac{C_\ell^{\text{data}}}{C_\ell^{\text{theory}}}\right) - 1\right]$$

VII. Modeling of data & stat inference.

- best fit parameters.
- error estimates of the parameters.
- measure of the goodness of fit.

- goodness of fit, χ^2, confidence regions

D_i

$y(\vec{x}_i | \vec{\alpha})$

$$\chi^2 = \sum_i w_i \left[D - y(\vec{x}_i | \vec{\alpha}) \right]^2$$

$$w_i = \frac{1}{\sigma_i^2} \qquad \frac{\partial \chi^2}{\partial \alpha_i} = 0$$

χ^2 distribution of $n - m$ d.o.f

# of parameters

# of data points

χ^2-distribution:

d.o.f

$$P(Y) = \frac{1}{2^{\nu/2}\,\Gamma(\nu/2)}\, Y^{\frac{\nu}{2}-1}\, e^{-Y/2}$$

$$P(\chi^2 < \hat{\chi}^2, \nu) = \Gamma\left(\nu/2, \frac{\hat{\chi}^2}{2}\right)$$

$$Q = 1 - P$$

Confidence levels: region in some m-dimensional parameter space that contains some portion of the probability distribution.

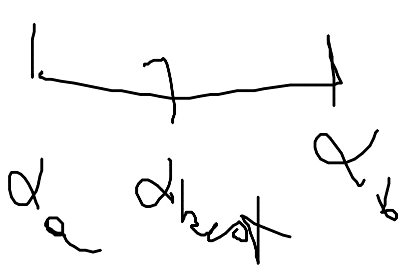
$$\mathcal{L} \propto \exp\left[-\frac{\chi^2}{2}\right]$$

Likelihood ratio:

$$\chi^2 - \chi^2_{min} = -2\ln\left[\frac{\mathcal{L}}{\mathcal{L}_{max}}\right]$$

- Find $\mathcal{L}_{max}$
- Go down the $\mathcal{L}$ until you enclose 68.3% of the total ("1σ")

Find: $\mathcal{L}(a) = \mathcal{L}(b)$



$$\int_a^b \mathcal{L}(x)\,dx = 0.683 \int_{-\infty}^{\infty} \mathcal{L}(x)\,dx$$

68.3% , 95.4% , 99.7%