

# Similarities & Halo-to-Halo Scatter in the 'Observed' DM Phase-Space of Milky Way like Galaxies



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Dark Side of the Universe  
ICTP-EAIFR, Kigali, 10 July 2023



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# DM Phase-Space in halos...

**Density:** Despite hierarchical formation, N-body simulations have shown that haloes exhibit a degree of universality → NFW profile.

**Velocity:** Extend the universality of density prof to the velocity distribution functions (VDFs) of dark matter particles.

**Two points –**

1. Hierarchical nature of structure formation could result in haloes having different VDFs due to the variations in the merger history and other factors such as tidal stripping & heating.
2. Process of violent relaxation (Lynden-Bell, 1967) → near-equilibrium distributions.  
→ the Standard Halo Model (SHM), King model, the double power-law model, and the Tsallis model, are all variants of the Maxwell–Boltzmann distribution.

# More motivations for studying DM VDFs ...

1. DM phase-space distribution in dark matter halos motivate a study of the VDF.

Just for a theoretical understanding of the phase-space distribution in dark matter halos

2. DM VDF affects DM detection :

Implications for direct (and indirect) DM detection limits

3. A well parameterized VDF :

Understand relations between the VDF and other physical quantities of the halos, such as mass, density profile, shape, and formation history.

Mao et al. (2013), among others, used cosmological N-body simulations finds

degree of universality in the VDFs of haloes of similar mass and formation history →

→ well approximated by a universal functional fitting form.

→ which depends primarily on  $r/r_s$ , (i.e radius by NFW scale radius).

→ halo to halo scatter in the simulated halos

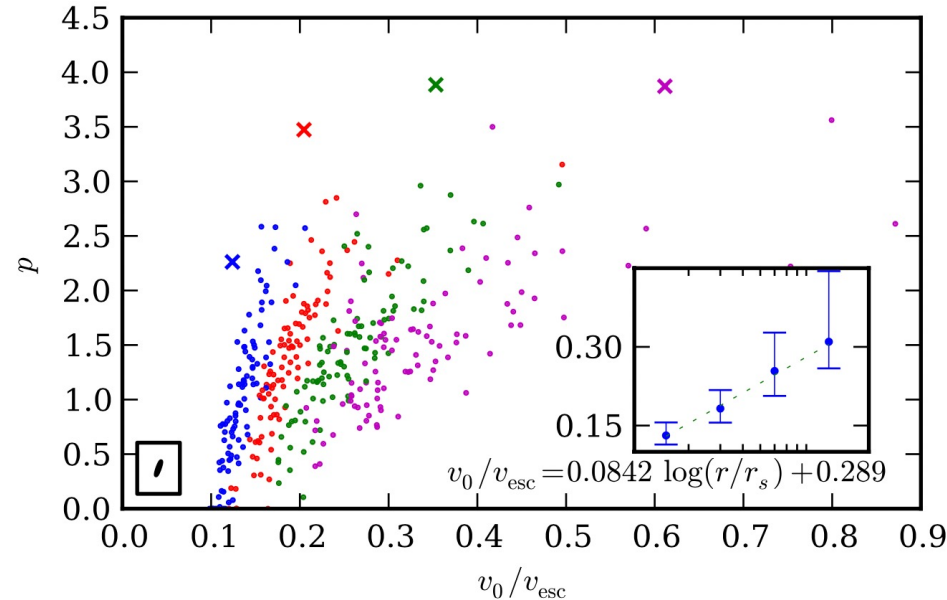
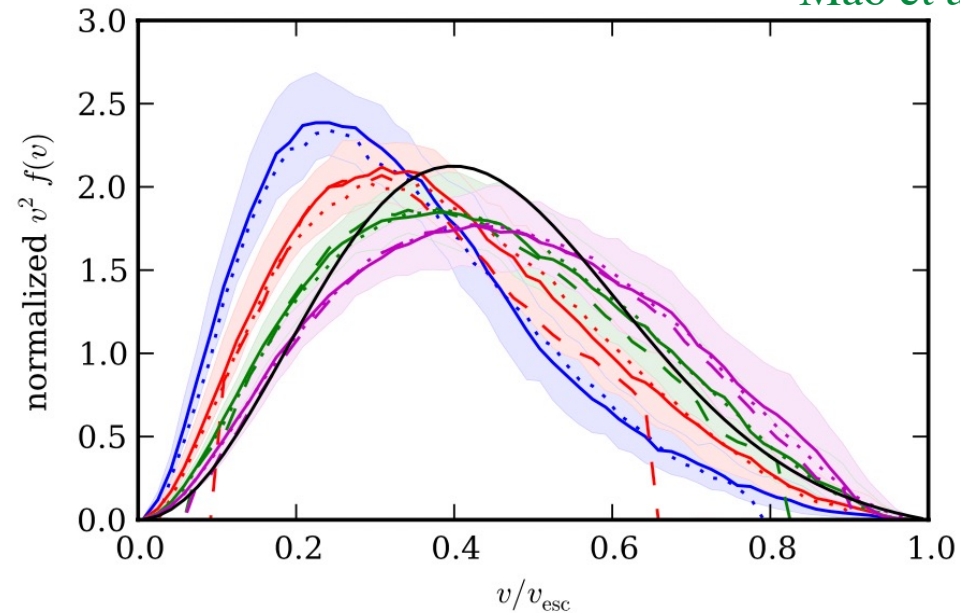
→ the most significant uncertainty in the VDF of the Milky Way arises from the unknown

$R_\odot/r_s$



# Similarity & halo-to-halo scatter in simulated DM only halos...

Mao et al. (2013)



$$f(|\mathbf{v}|) = \begin{cases} A \exp(-|\mathbf{v}|/v_0) (v_{esc}^2 - |\mathbf{v}|^2)^p, & 0 \leq |\mathbf{v}| \leq v_{esc} \\ 0, & \text{otherwise,} \end{cases}$$

$r/r_s$  : 0.15 (blue), 0.3 (red), 0.6 (green), 1.2 (magenta)

Solid lines = stacked velocity distribution for 96 halos in Rhapsody sims.

Dashed and dotted lines are for Bolshoi sims halos of  $M_{vir} \sim 10^{12}$  and  $10^{13} M_{\odot} h^{-1}$

Weak dependence on conc, shape, formation time, local DM density slope

# All good, but one caveat – NO Baryons!

We have not yet investigated the impact of baryons; we expect that adiabatic contraction of dark matter halos would raise the velocity but preserve the shape of the VDF.

Baryonic effects in isolated halos have been studied in the context of dark matter detection (Bruch et al. 2009; Ling et al. 2010); however, **simulating a statistical sample of halos similar to what we consider here with both sufficient resolution and realistic baryonic physics is not yet tractable.**

Note: For isolated halos, Tsallis model provides better fit to simulations with baryons (Ling et al. 2010)

**So, we cannot study similarities and halo-to-halo scatter in realistic MW like halos from simulations yet.**

**Now we have enough data to do it from observations  
aka Real Galaxies**

(albeit with its own caveats)



# An ensemble of real galaxies ...

1. High-quality HI/H $\alpha$  Rotn Curves of the nearby  $\sim$  **175 galaxies** from the SPARCs catalog.

2. Most SPARC galaxies are **disk galaxies** with a rotationally supported stellar disks dominating the central part and extended gas disks – like **MILKY WAY**

3. Choose only galaxies with extended “high quality” RC with data up to the flat outer part  
→ **129 galaxies**

4. Build the multi-component mass model : Bayesian analysis on all the RCs.

$$V_c^2 = V_{\text{DM}}^2 + V_{\text{gas}} |V_{\text{gas}}| + \Upsilon_{\text{disk}} V_{\text{disk}}^2 + \Upsilon_{\text{bul}} V_{\text{bul}}^2; \quad V_{\text{DM}} = \sqrt{GM(r)/r}$$

a)  $\log(M_{200}/M_{\odot})$  and  $c_{200}$  as the two free parameters of NFW.

-- lognormal prior from Dutton and Maccio (2014)

b)  $\Upsilon_{\text{disk}}$  and  $\Upsilon_{\text{bul}}$  are free parameters

--Population synthesis models  $\rightarrow$  mass-light ratios to be  $0.5 - 0.7 M_{\odot}/L_{\odot}$

c) Plus, “reliability cuts” using millions of mock RCs.

-- (ref: Manju and Majumdar 2023a, tbs, JCAP)

→ **106 galaxies**

# An ensemble of real Milky Way lookalikes ...

1. We employ criteria for choosing MW-like galaxies similar to simulations (Bozorgnia 2017)  
Milky Way mass range of  $7 \times 10^{11} < M_{200}/M_{\odot} < 3 \times 10^{12}$

→ 32 galaxies

2. Stellar mass:  $4.5 \times 10^{10} < M_{\text{star}}/M_{\odot} < 8.3 \times 10^{10}$  within  $3\sigma$  observed value of MW,
3. Define the angular circular velocity ( $\omega = V_c(r)/r$ )

$$\chi^2 = \sum_{r_i < 2\text{kpc}}^{< 30\text{kpc}} \frac{[\omega_{\text{MW}}(r_i) - \omega_{\text{RC}}(r_i)]^2}{[\Delta\omega_{\text{MW}}(r_i)]^2}$$

We implement a cut-off condition of  $\text{sqrt}[\chi^2/(N - 1)] < 90$  to choose MW-like RCs.

Note: Bozorgnia (2017) chose this  $< 300$ .

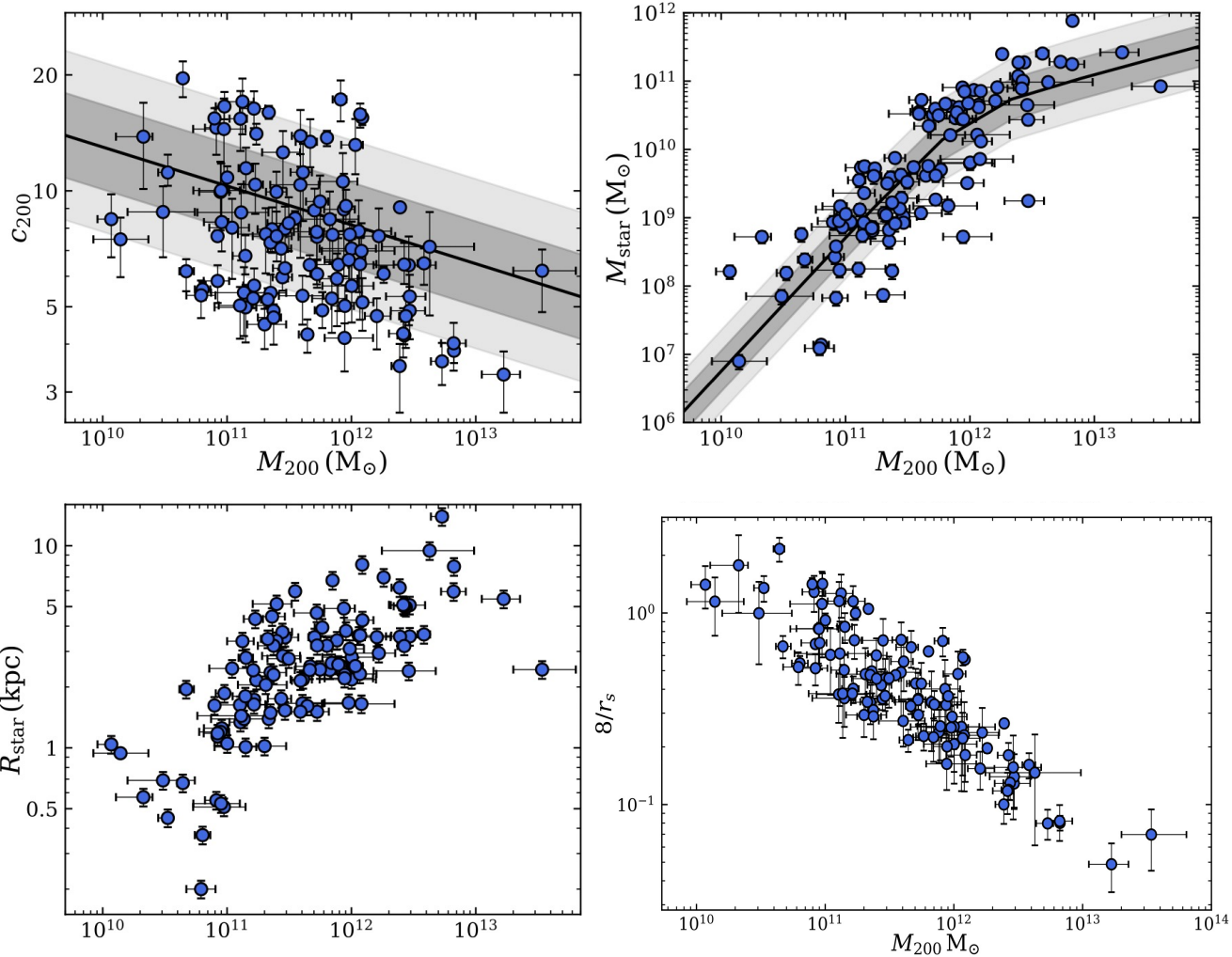
Our results very mildly depend on this choice

→ 8 Milky Way galaxies



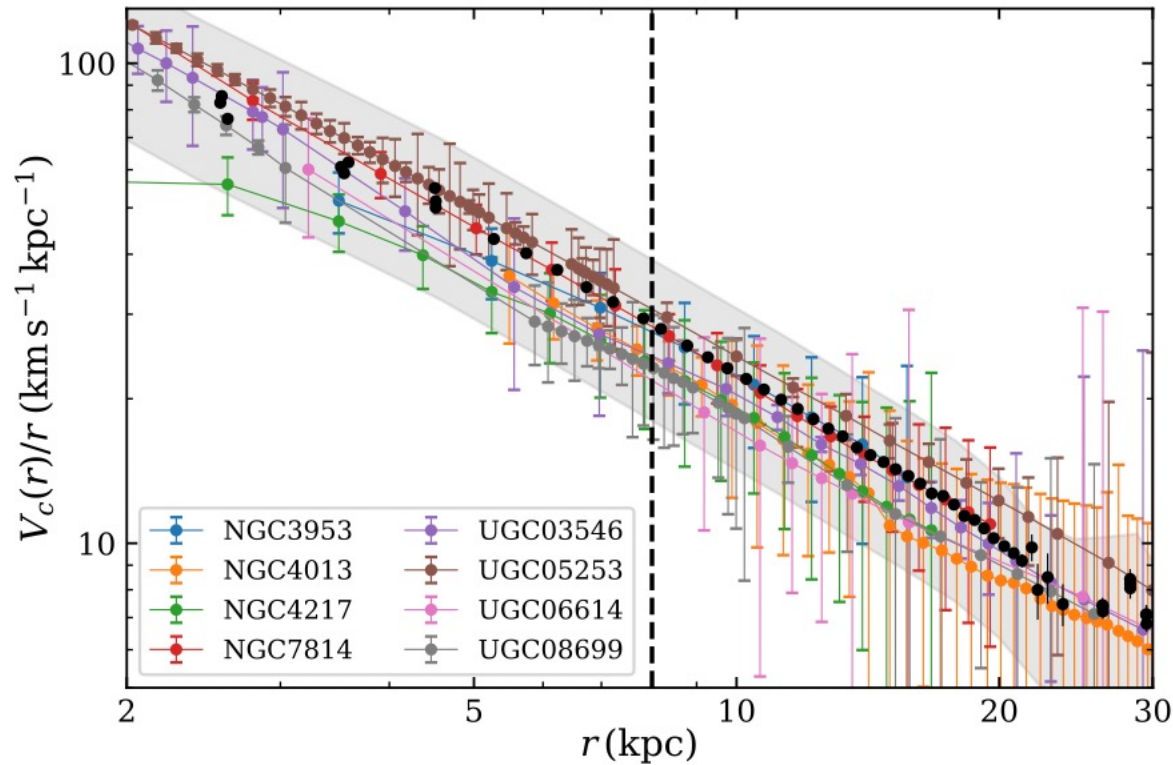
# 175 galaxies to 8 Milky Ways...

Manju & Majumdar 2023a



# An ensemble of Milky Ways ...

Manju & Majumdar 2023b



1. MW  $c_{200} = 4 - 16$  in literature: not well constrained.
2. circular velocity  $V_c(R_\odot) \sim 200 \text{ km/s}$  at solar radius also uncertain  $\rightarrow$  determines the DM VDF peak velocity  $\rightarrow$  significant impact on the direct detection results.
3. Baryon dominated central region of all the 8 MW-like galaxies.  
[exception UGC05253]
4. The MW local DM  $\rho_\odot = 0.2 - 0.8 \text{ GeV/cc}$ . All MW-like galaxies falls in this range.



# Simulated vs Observed Ensemble ...

Halo Name	$M_* [\times 10^{10} M_\odot]$	$M_{200} [\times 10^{12} M_\odot]$	D/T	$\chi^2/(N-1)$
E1	5.88	14.26	0.18	105.54
E2	7.12	9.48	0.33	324.31
E3	5.77	5.16	0.35	190.97
E4	5.14	3.24	0.18	44.70
E5	5.18	5.57	0.14	160.27
E6	5.05	5.42	0.10	266.83
E7	7.02	4.32	0.25	160.58
E8	4.65	4.06	0.13	55.07
E9	5.31	3.50	0.45	74.55
E10	4.85	3.30	0.36	59.65
E11	5.48	2.76	0.46	220.96
E12	4.87	2.99	0.25	45.45
A1	4.88	1.64	0.70	221.27
A2	4.48	2.15	0.50	51.04

Bozorgnia 2017

Manju & Majumdar 2023b

Galaxy	$\log(M_{200}/M_\odot)$	$c_{200}$	$\log(M_{\text{star}}/M_\odot)$	$R_{\text{star}}$ (kpc)	$V_c(R_\odot)$ (km s <sup>-1</sup> )	$V_{\text{esc}}(R_\odot)$ (km s <sup>-1</sup> )	$\rho_\odot$ (GeV cm <sup>-3</sup> )	$\frac{M_{\text{star}}(R_\odot)}{M_{\text{DM}}(R_\odot)}$	$\sqrt{\chi^2/(N-1)}$
MW	12.1	8.7	10.8	3.0	232.1	578.1	0.33	2.10	-
NGC3953	11.93 ± 0.14	8.95 ± 2.53	10.91	4.89 ± 0.49	199.5	523.9	0.24	2.96	87.5
NGC4013	12.20 ± 0.10	4.73 ± 0.74	10.71	3.53 ± 0.35	201.8	581.5	0.19	3.26	41.0
NGC4217	12.22 ± 0.17	7.63 ± 1.42	10.91	2.94 ± 0.29	261.8	638.6	0.30	3.36	42.7
NGC7814	12.03 ± 0.10	13.16 ± 2.23	10.87	2.54 ± 0.25	284.4	611.3	0.40	2.44	7.6
UGC03546	11.96 ± 0.04	9.14 ± 0.97	10.85	3.79 ± 0.38	227.8	542.7	0.27	2.36	23.8
UGC05253	12.09 ± 0.01	15.49 ± 0.70	10.85	8.07 ± 0.81	243.1	631.9	0.51	0.88	25.6
UGC06614	12.42 ± 0.10	4.26 ± 0.63	10.89	5.10 ± 0.51	230.0	690.0	0.21	4.20	55.5
UGC08699	11.98 ± 0.12	7.70 ± 1.63	10.67	3.09 ± 0.31	207.0	521.1	0.23	2.38	45.7

# Masses to velocities ...

$f(\vec{r}, \vec{v})$  is the DM phase-space distribution function.

Eddington's Inversion method makes use of the Jeans theorem  $\rightarrow$  steady-state solution of the collisionless Boltzmann equation depends on the phase-space coordinates **only through the integrals of motion**.

Now, we have Poisson's eqn  $\nabla^2 \Psi = -4\pi \dot{G} \rho$

Introducing the relative energy,  $\mathcal{E} = \Psi - \frac{1}{2}v^2$

**Very importantly:**  $\Psi = \Psi_{\text{DM}} + \Psi_{\text{Bary}}$

One can arrive at the solution, self consistent solution of density and velocity:

$$\rho(r) = 4\pi \int_0^{\Psi} d\mathcal{E} f(\mathcal{E}) \sqrt{2(\Psi - \mathcal{E})}$$

$$f(\mathcal{E}) = \frac{1}{\sqrt{8\pi^2}} \left[ \int_0^{\mathcal{E}} \frac{d\Psi}{\sqrt{\mathcal{E} - \Psi}} \frac{d^2\rho}{d\Psi^2} + \frac{1}{\sqrt{\mathcal{E}}} \left( \frac{d\rho}{d\Psi} \right)_{\Psi=0} \right]$$

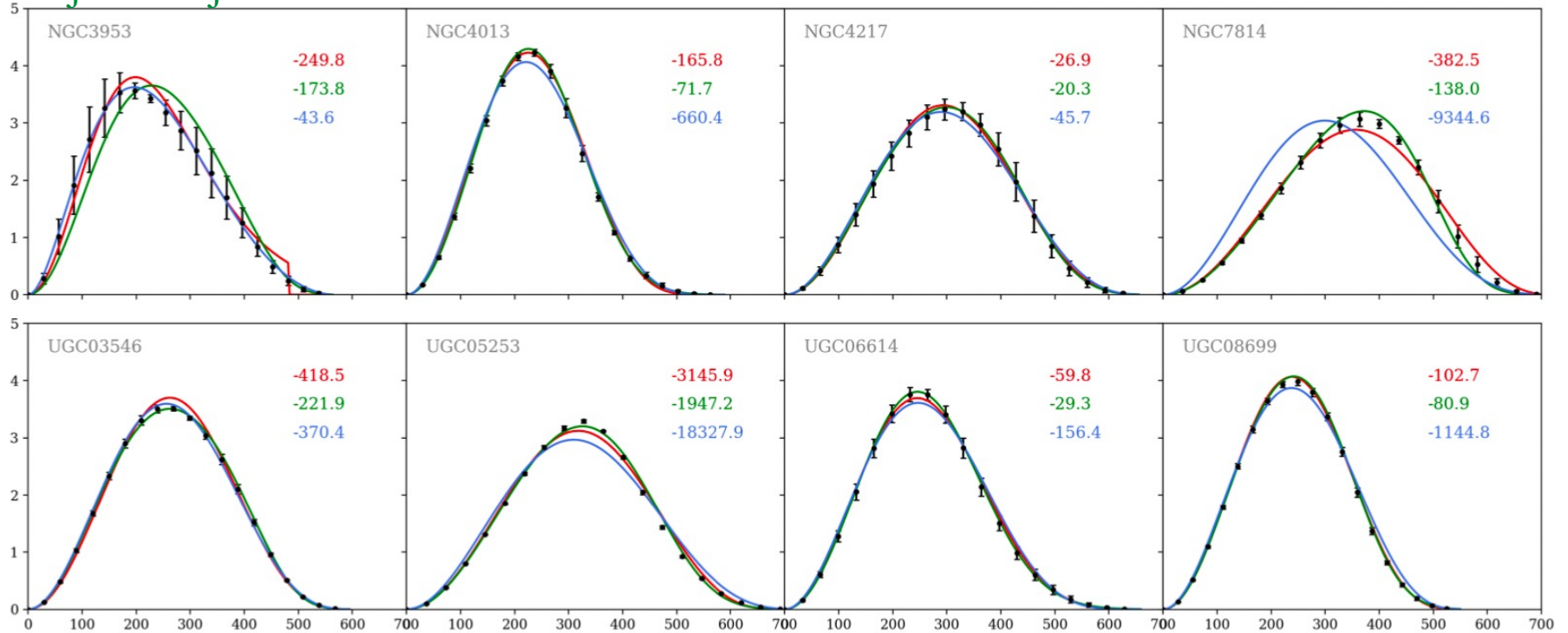
And finally the DM VDF is given by  $f_{\vec{r}}(\vec{v}) = f(\mathcal{E})/\rho(r)$

# “Observed” VDF’s & a Better Fit...

Manju & Majumdar 2023b

$r/r_s = 0.15$

— Tsallis fit — Mandal fit — Mao fit



$r / r_s = 0.15$

1. Mao et al empirical fit **does not work** for real galaxies!

2. A better empirical fit  
(Mandal, SM etal 2019)

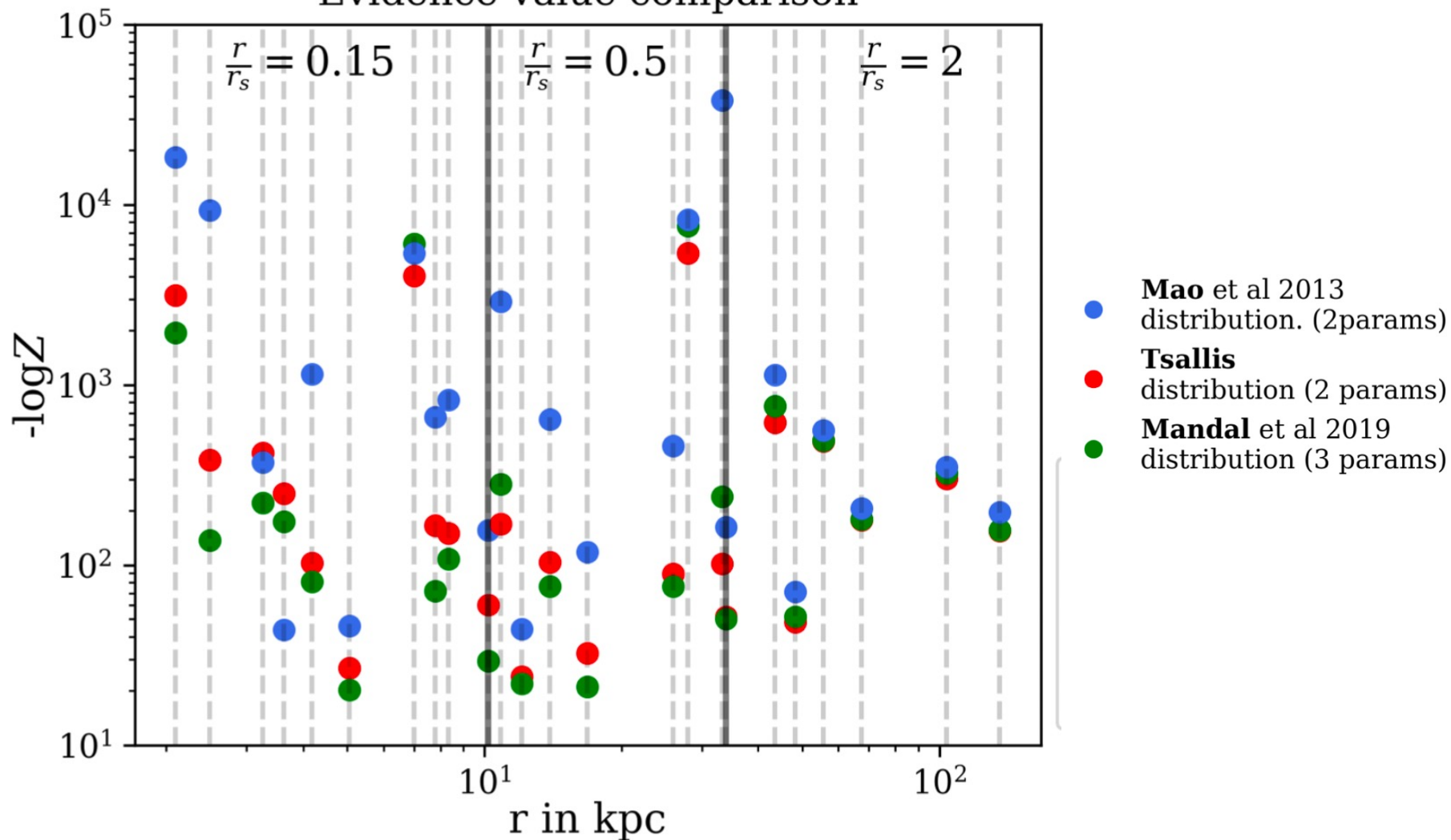
$$f_{\text{obs}}(\mathbf{v}) \approx B(\zeta(\beta) - \zeta(\beta_{\text{max}}))$$

$$\zeta(x) = (1 + x)^k \exp[-x^{(1-p)}], \quad \beta = v^2/v_{\star}^2 \quad \beta_{\text{max}} = v_{\text{esc}}^2/v_{\star}^2.$$

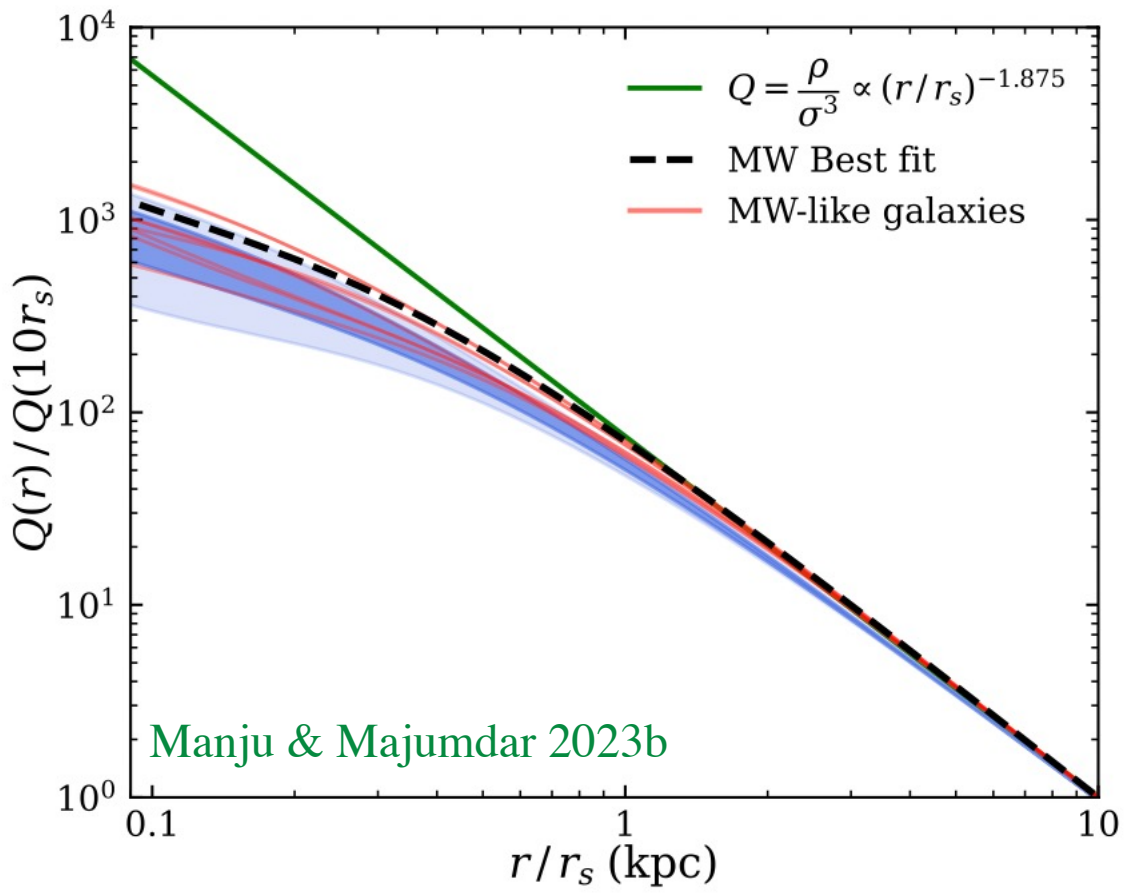


# A better fit at all radii (Bayesian evidence)..

Evidence value comparison

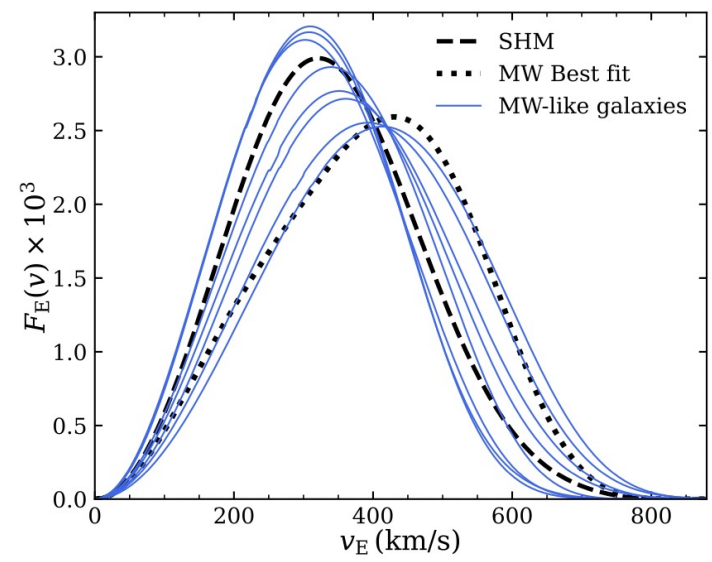


# Its no surprise that DM only galaxies are not like real galaxies : The pseudo phase-space



The MW pseudo phase-space and  
The sample variance from  
structure formation

$V_s$   
The NFW DM only case



# DM Direct Detection ...

1. Direct detection of DM aims to measure the recoil of atomic targets imparted from collisions with the galactic “wind” of DM particles resulting from the combined motion of Sun, Earth and the local VDF of DM.

2. Neglecting the annual motion of Earth around the Sun, the unmodulated differential recoil rate given by the product of the number density and velocity averaged differential scattering cross-section is

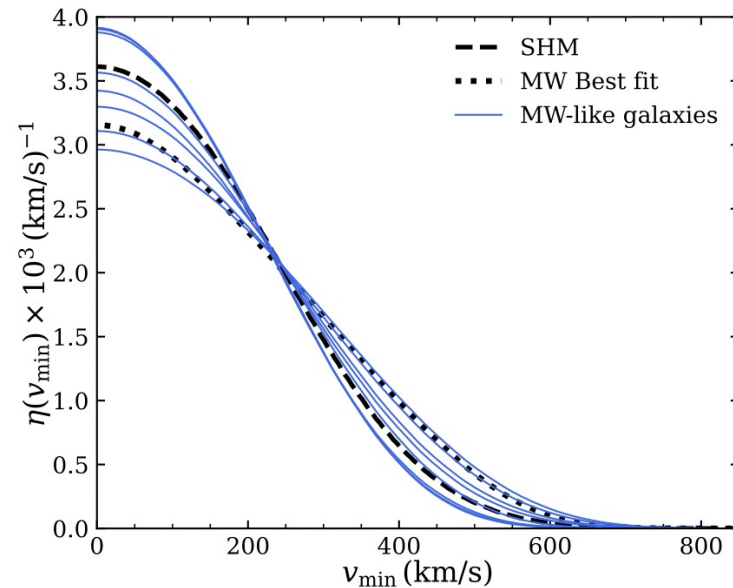
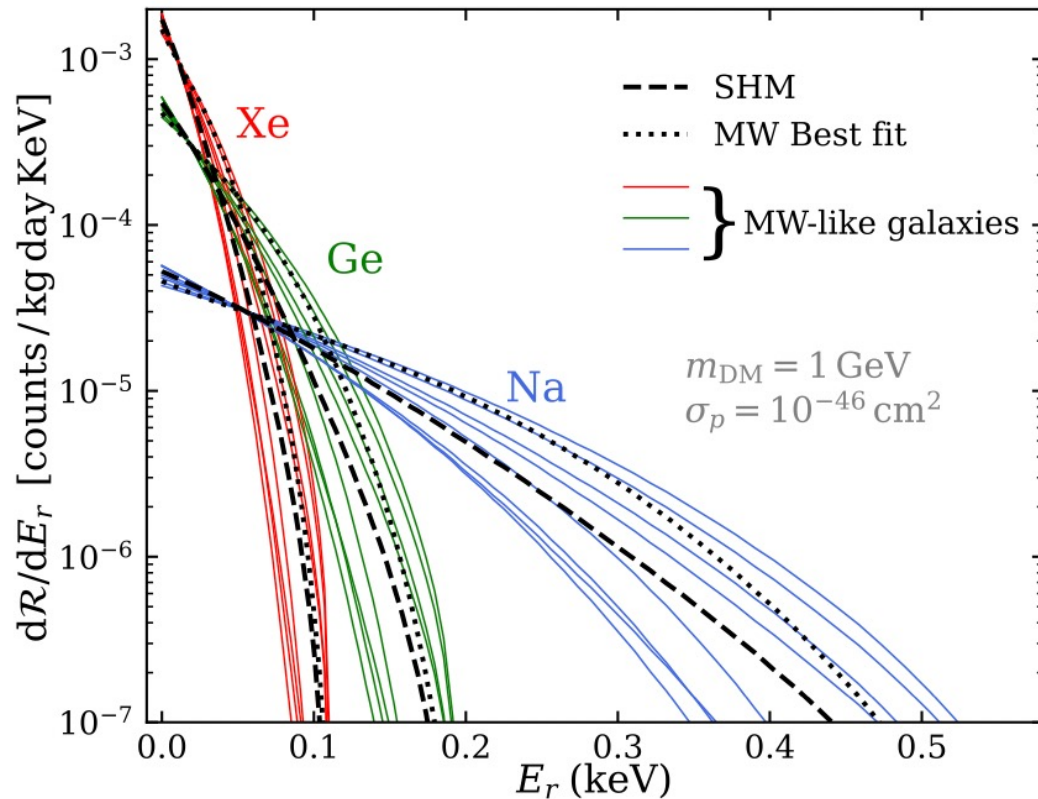
$$\frac{dR}{dE_r} = \sigma_p \frac{\rho_\odot}{2m_{\text{DM}}\mu_N^2} A^2 F^2(E_r) \eta, \quad \text{where } \eta(E_r) = \int_{v_{\min}(E_r)}^{v_{\text{esc}}} dv \frac{F_E(v)}{v}$$

- a)  $v_{\min}$  is the minimum velocity of the incoming DM particles in Earth’s reference frame,
- b)  $E_r$  is the imparted recoil energy to the target nucleus,
- c)  $F_E$  is the 1-D local speed distribution of DM particles in Earth’s reference frame

The escape velocity  $v_{\text{esc}} = 544$  km/s and  $V_c(R_\odot) = 220$  km/s



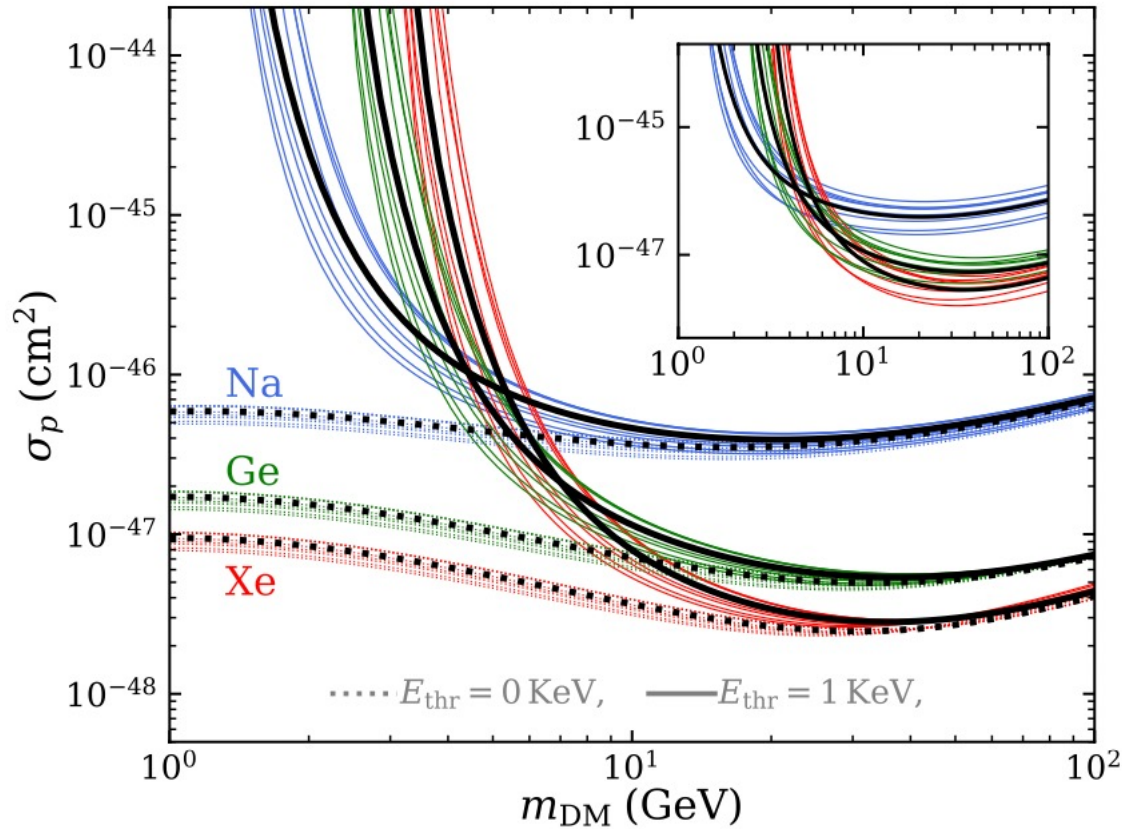
# DM vel & recoil rates: MW ensemble



The differential recoil rate for three different target materials:  
 $\text{Xe}^{132}$  (in red),  $\text{Ge}^{74}$  (in green) and  $\text{Na}^{23}$  (blue)

For DM of mass 10 GeV and interaction strength,  $\sigma_p = 10^{-46} \text{ cm}^2$

# DM exclusion limits: MW ensemble ..



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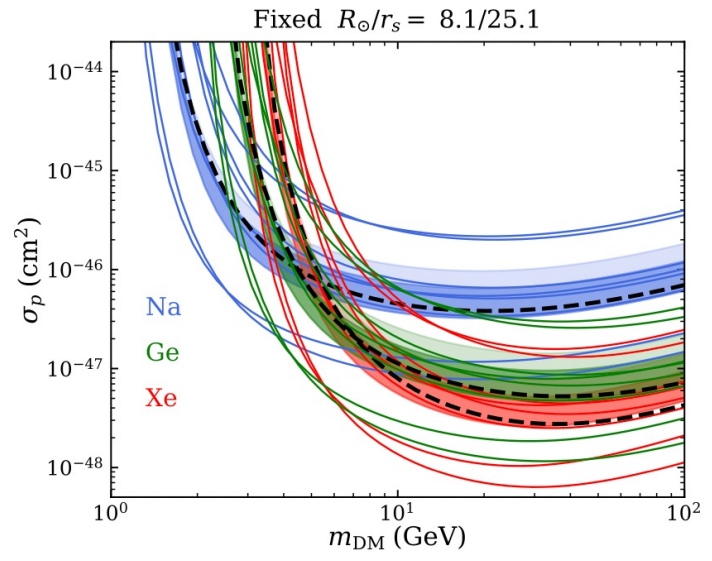
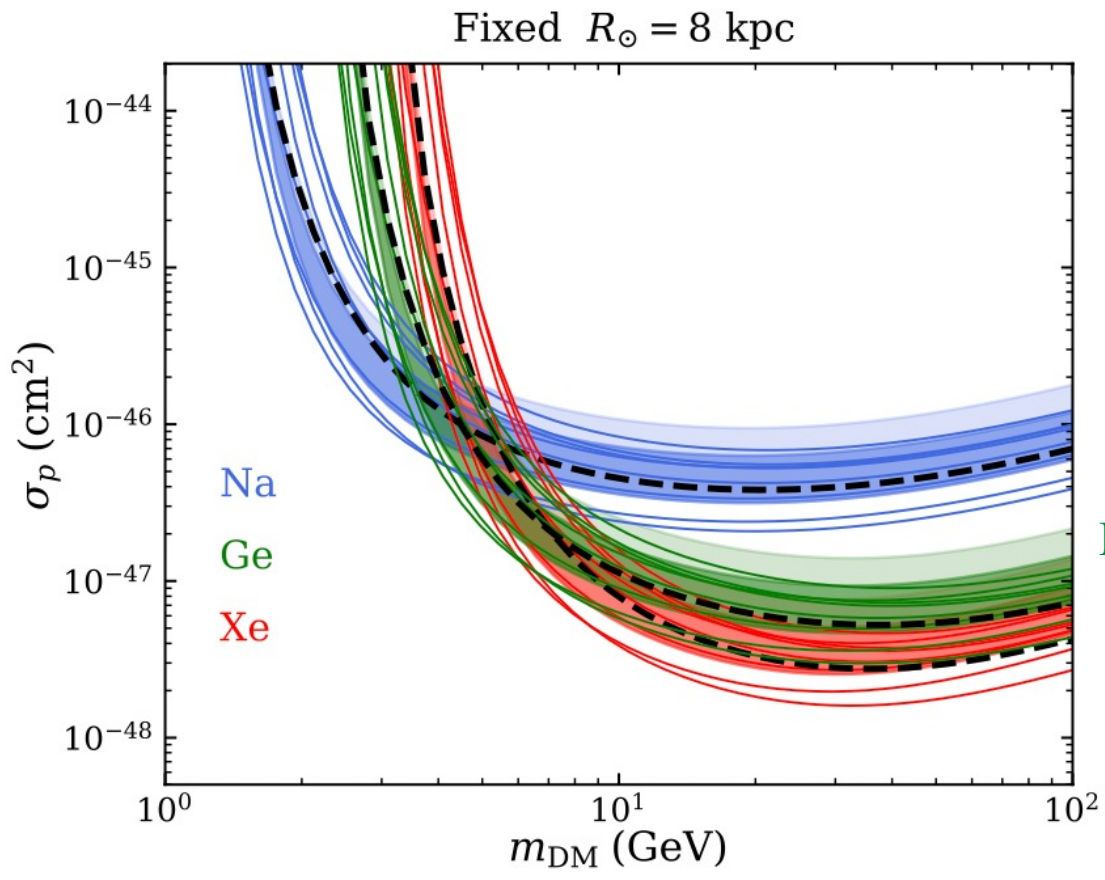
## General comments:

$v_{\text{min}}$  for low mass will be closer to  $v_{\text{esc}}$  and leading to a higher fractional change in  $\eta$ . Thus, The effect of change in VDF is more pronounced for low mass region.

A decrease (increase) in the total number of events  $\rightarrow$  tighter (weaker) constrain on  $\sigma_p$ .

Change in  $\rho_{\odot}$  will scale the limit on  $\sigma_p$  up or down over entire mass range.

# The cosmological sample variance in DM exclusion limits ...

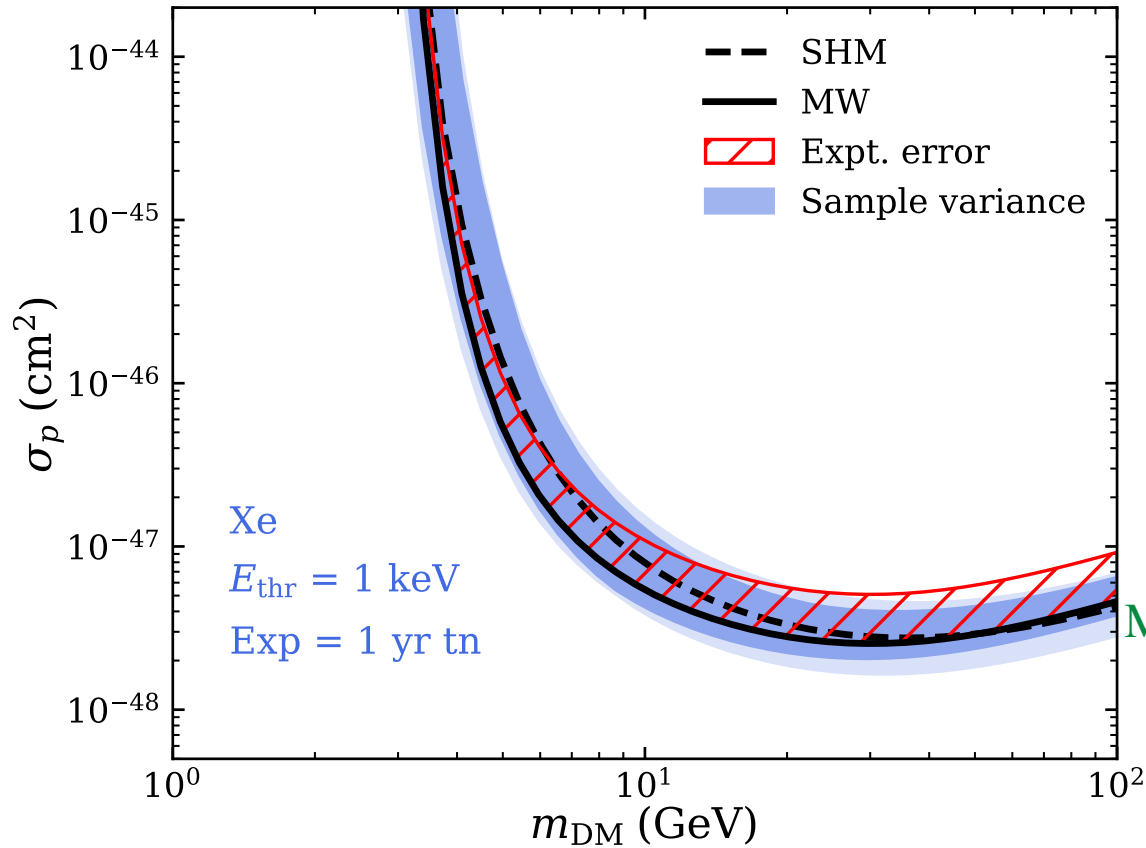


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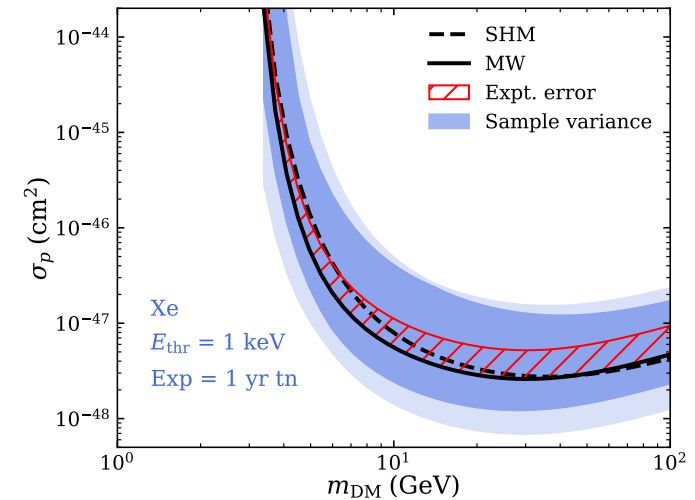


# The irreducible error (shown for Xe)...

Fixed  $R_{\odot} = 8$  kpc



Fixed  $R_{\odot}/r_s = 8.1/25.1$



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Better density, better VDF, and better expts will reduce error bars on limits  
But you cannot do better than the sample variance

# The take away mesg...

Simulations and Observations both have their pros and cons. However, simple DM only simulation based inferences are not enough for DM studies.

We present a gold sample of galaxies from the SPARCs database for DM studies. The first observed ensemble of Milky Way like galaxies.

We identify an empirical relation that better fits observed MW like galaxies (compared to previous universal empirical fit to DM only halos in simulations).

We show how the presence of baryons, below the scale radii, change the pseudo phase-space away from the power law expected in DM only NFW halos.

Finally, we show the observed sample variance error on the DM direct detection exclusion plots, and argue this to be the minimum error that will always be present.

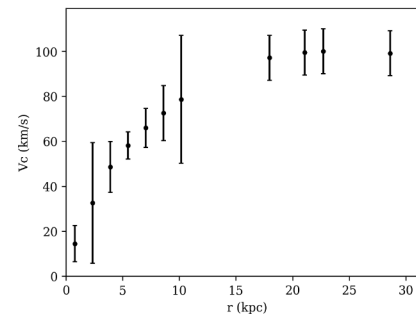
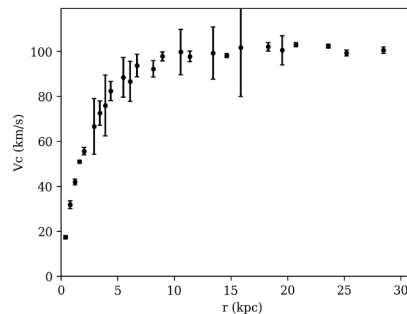
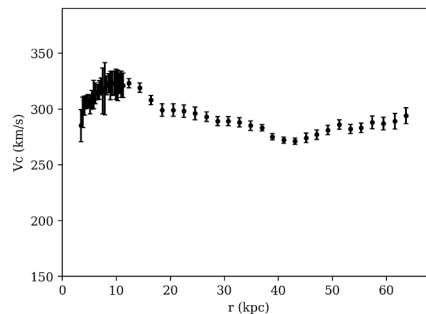
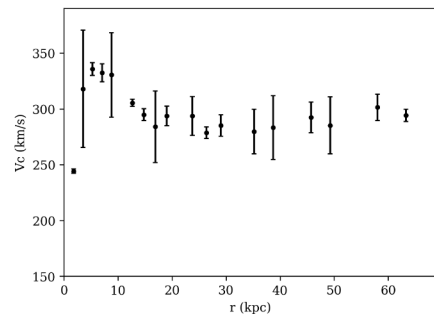
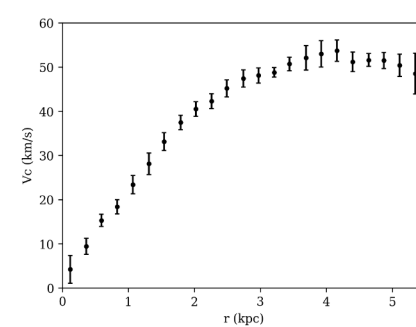
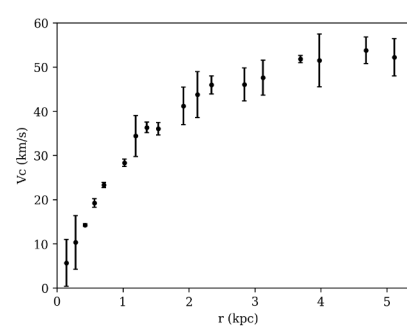
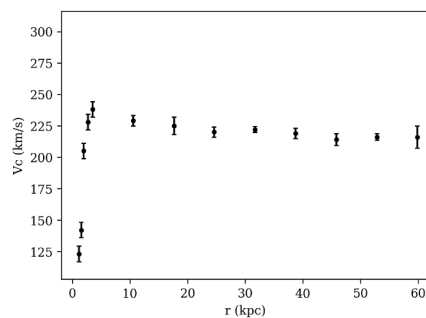
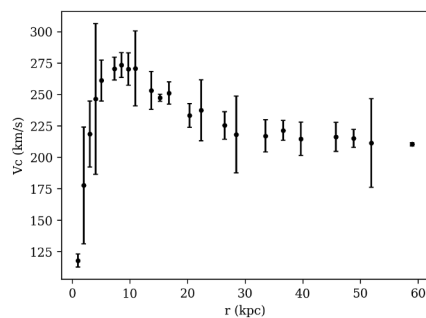
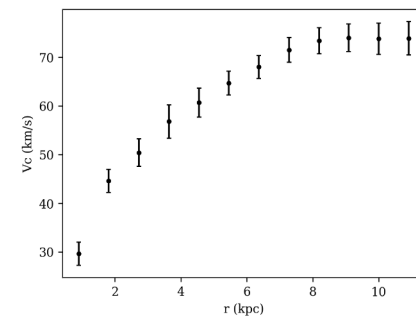
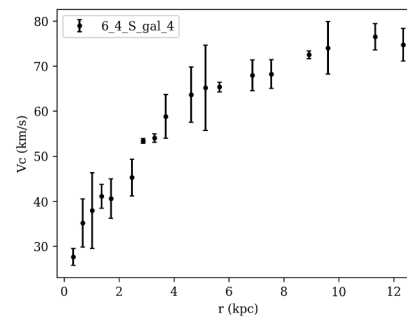
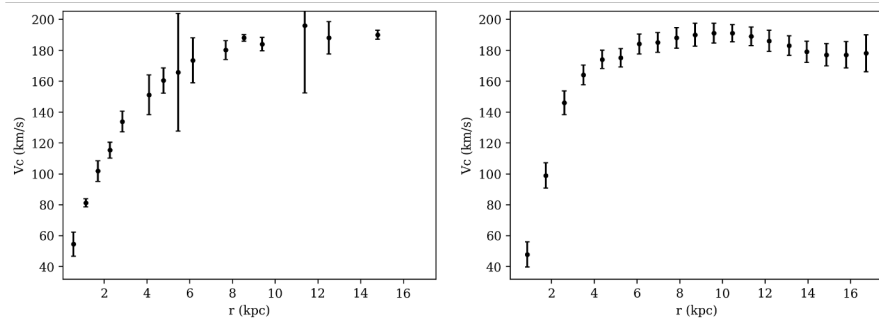
This, to my knowledge, is the first full phase-space study of an ensemble of observed Galaxies (of which the MW look alike is a subsample).

**--Stay tuned for follow-up works**

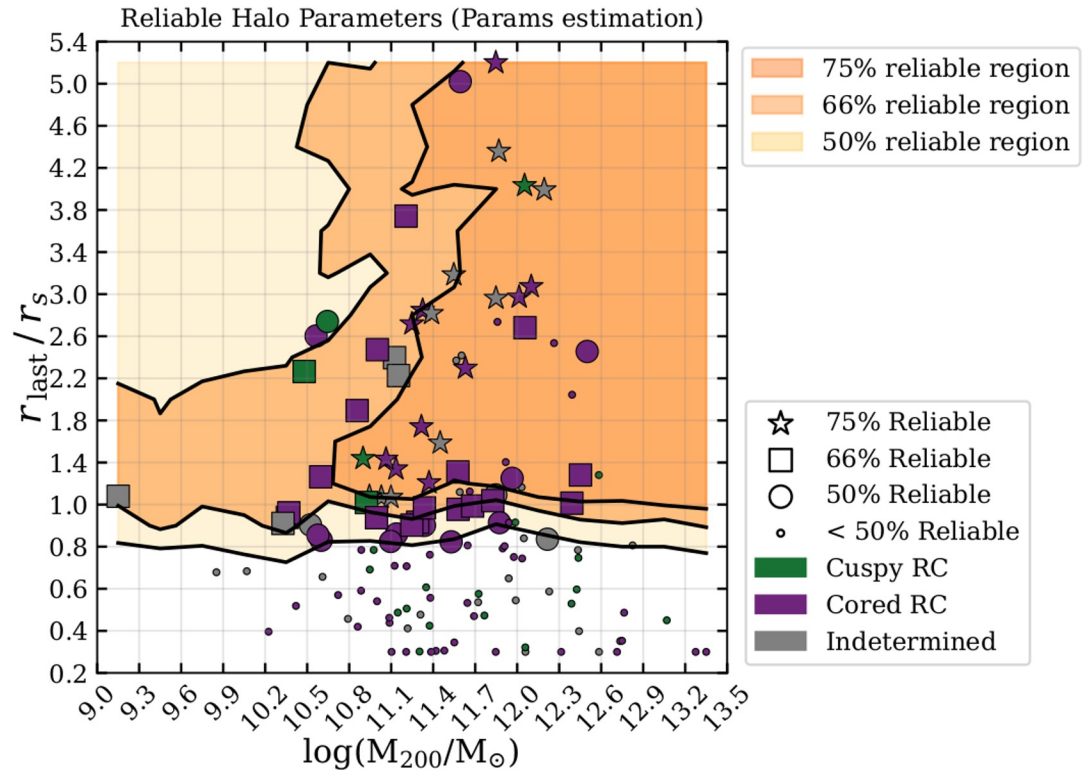
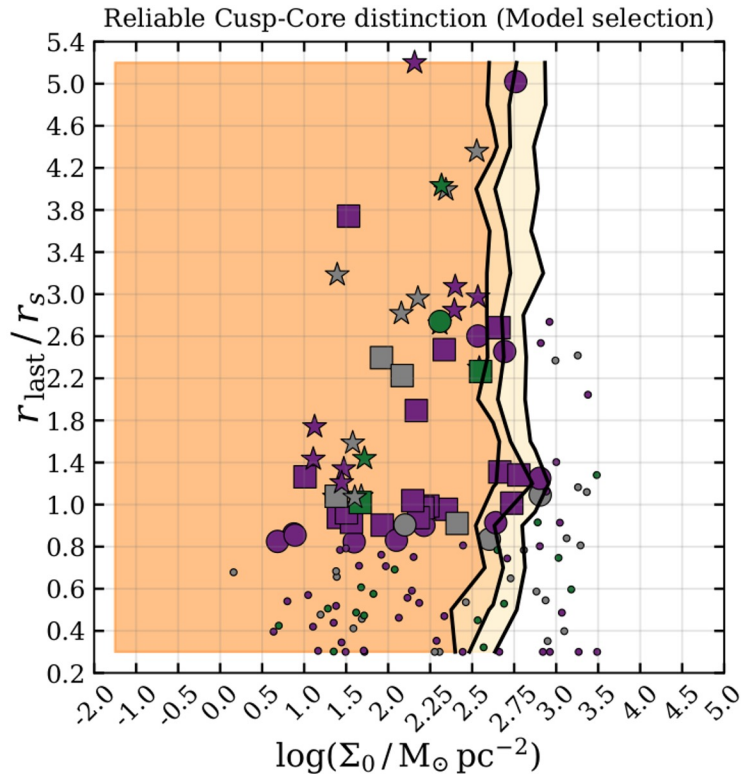
# Extras...



# Rotation Curves of Nearby galaxies: SpArcs observations and Realistic mocks

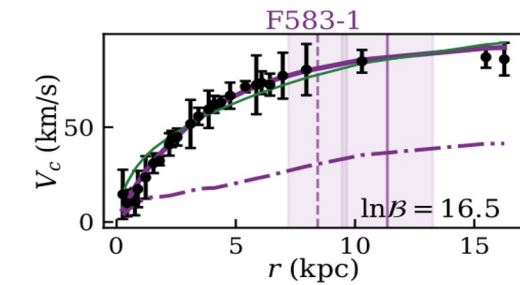
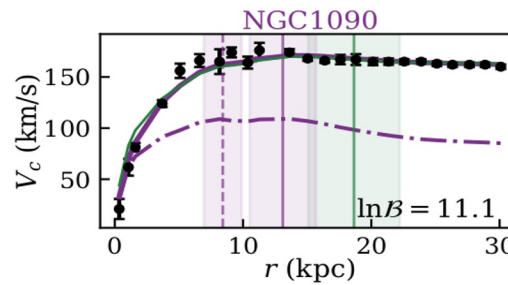
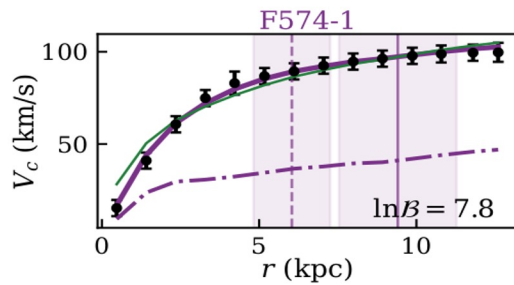
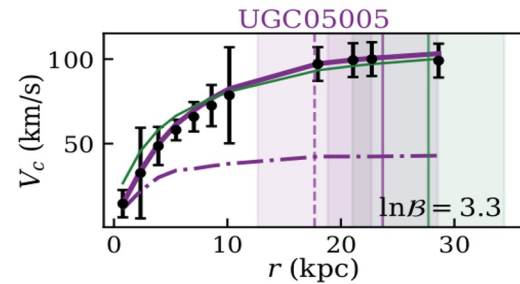
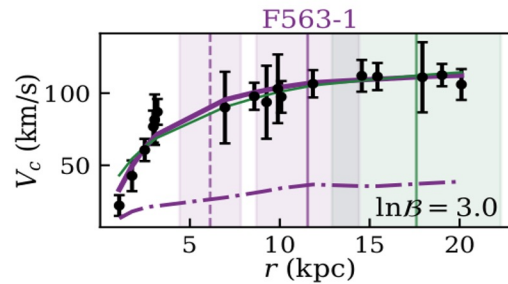
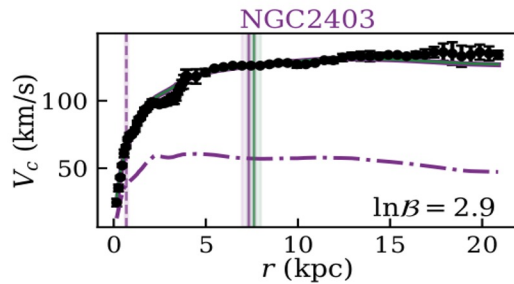
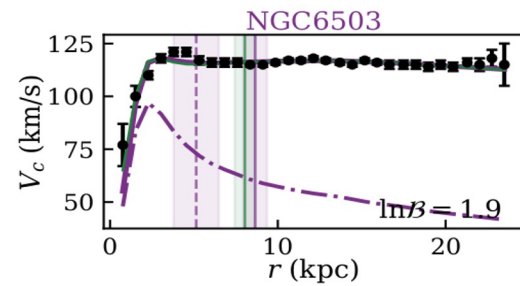
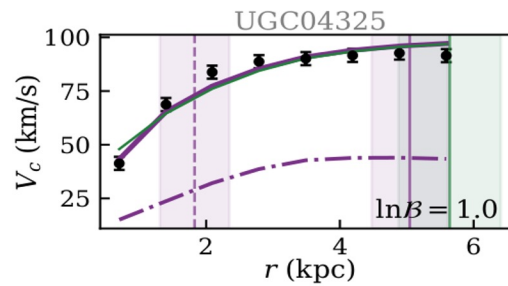
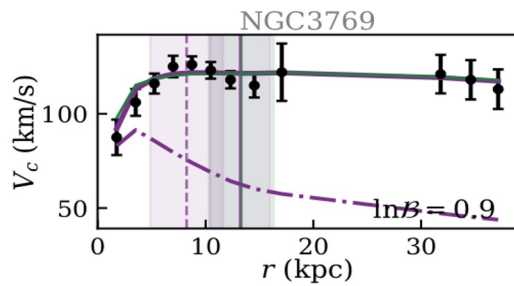


# The importance of reliability



75% reliable means, 3 out of 4 mock RCs in that grid is reliable (satisfies our condition for checking reliability, i.e, successful model selection / parameter estimation)

# Gold Sample of reliable rotation curves



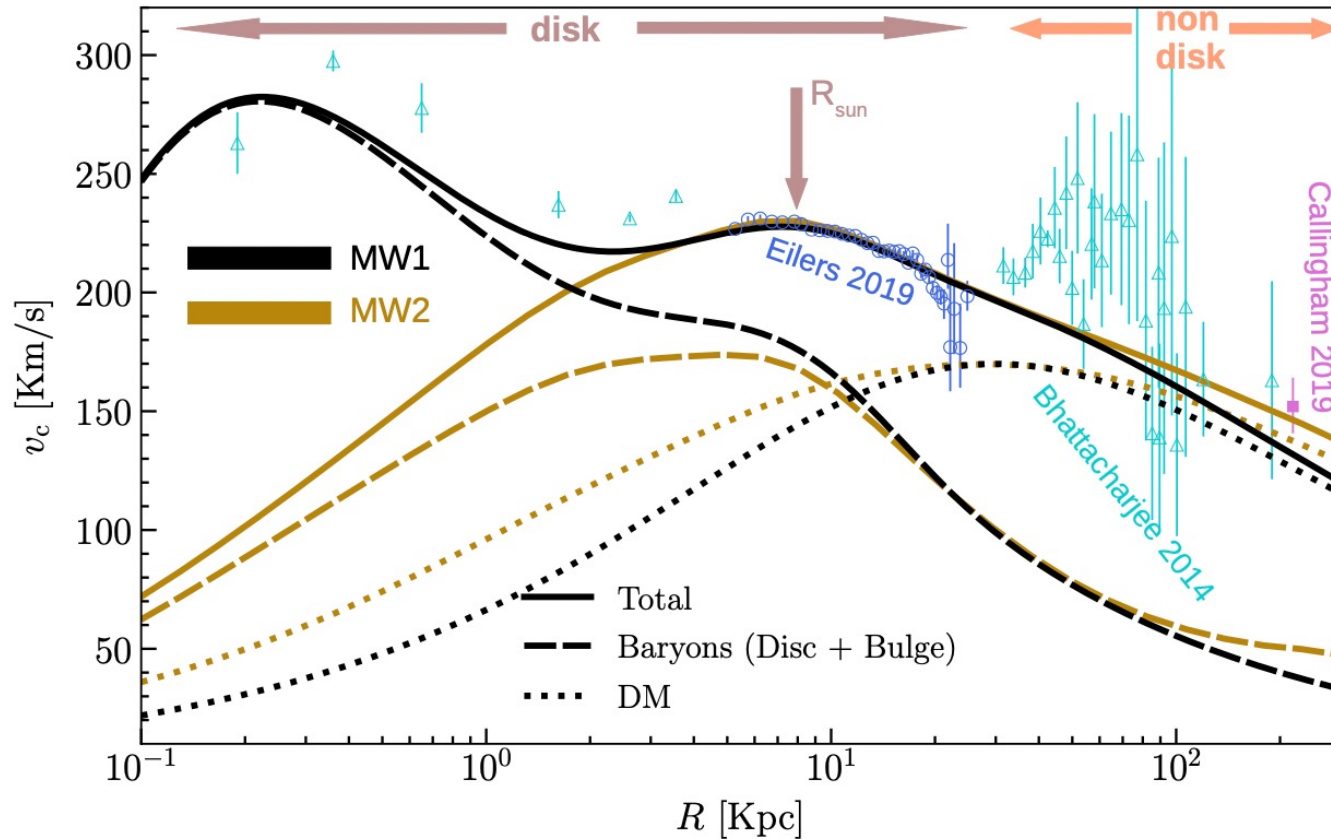
Green => cusp model; Purple => cored model; Grey => model couldn't be determined

Vertical lines: green solid =  $r_s$  of NFW (cusp)

purple solid =  $r_s$  of core

Purple dashed = rc of core ( $r < r_c$  == burkert and  $r > r_c$  == NFW)

# Milky Way RC post GAIA



Only GAIA disk RC needs fixed VM (Eilers et al) to give decent DM.  
If VM is left free, it prefers a very low DM density  $< 0.1 \text{ gv/cc}$  and VM density  $> 100 \text{ M}_\odot/\text{pc}^2$

Adding inner and outer radii RC from pre GAIA is needed

# On different DM VDFs ...

The dark matter velocity distribution in halos is set by a sequence of mergers and accretion. The process of violent relaxation (Lynden-Bell 1967) may be responsible for the resulting near-equilibrium distributions observed in dark matter halos and in galaxies. These near-equilibrium distributions explain why existing VDF models (see, e.g., Frandsen et al. 2012), including the Standard Halo Model (SHM), King model, the double power-law model, and the Tsallis model, are all variants of the Maxwell–Boltzmann distribution. Recent studies have shown that the widely used SHM is inconsistent with the VDF found in a handful of individual simulations (Stiff & Widrow 2003; Vogelsberger et al. 2009; Kuhlen et al. 2010; Purcell et al. 2012) and in the study of rotation curve data (Bhattacharjee et al. 2012). The double power-law model was proposed to suppress the tail of the distribution, by raising the SHM to the power of a parameter  $k$  (Lisanti et al. 2011). The Tsallis model replaces the Gaussian in Maxwell–Boltzmann distribution with a  $q$ -Gaussian, which approaches to a Gaussian as  $q \rightarrow 1$  (Vergados et al. 2008). It was argued that the Tsallis model provides better fit to simulations with baryons (Ling et al. 2010)