The ....
Early Universe

DANIEL G. FIGUEROA

IFIC, Valencia, Spain

DSU2023, EAIFR, Kigali, Rwanda, July 10th-14th, 2023

The **Non-Linear** Early Universe

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The Art of simulating the Early Universe

(When things get complicated: non-linear, strong coupling, non-perturbative, etc…)

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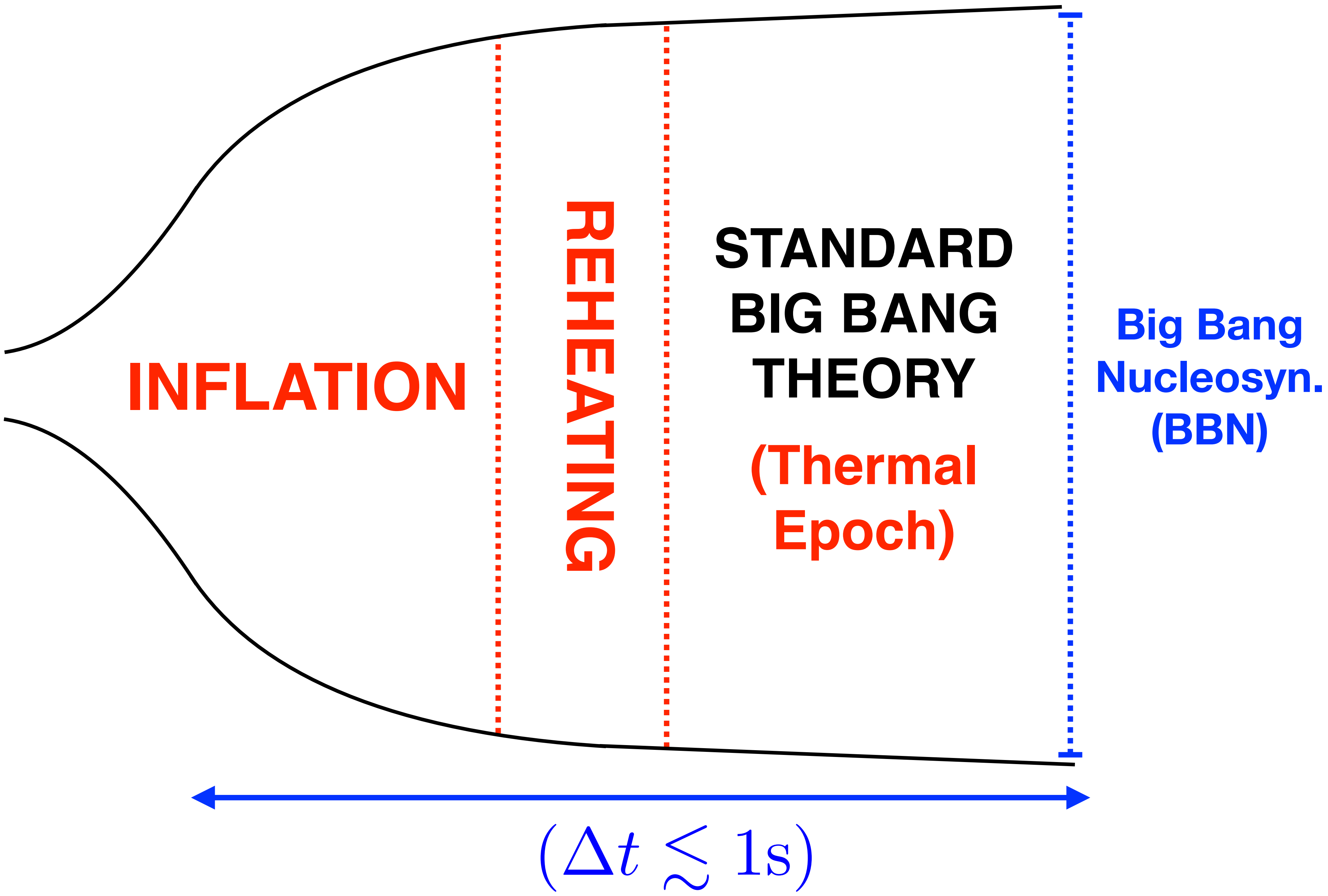
The Early Universe



INFLATION

REHEATING

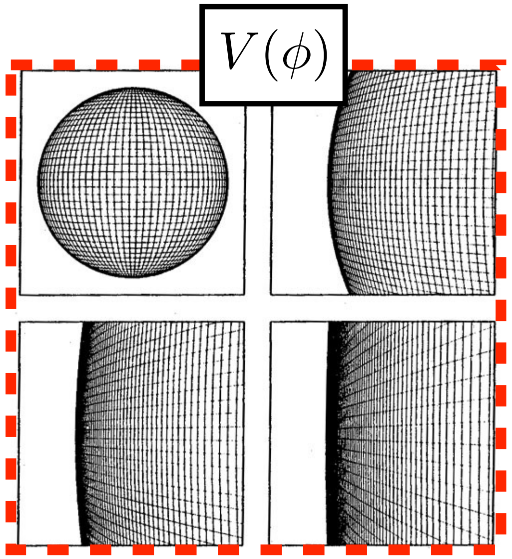
STANDARD
BIG BANG
THEORY

(Thermal
Epoch)

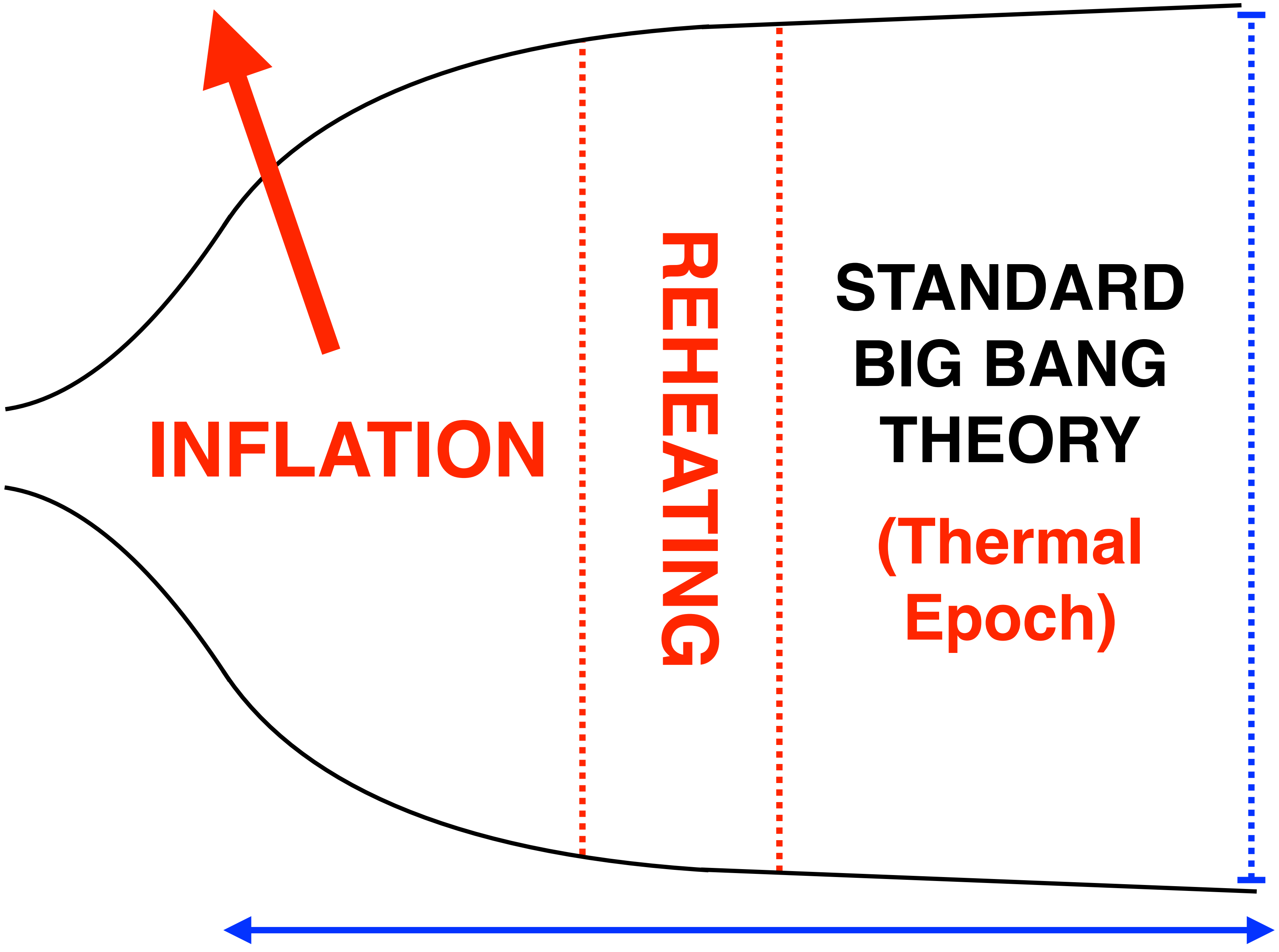
Big Bang
Nucleosyn.
(BBN)

$(\Delta t \lesssim 1\mathrm{s})$

The Early Universe



Accelerated Expansion



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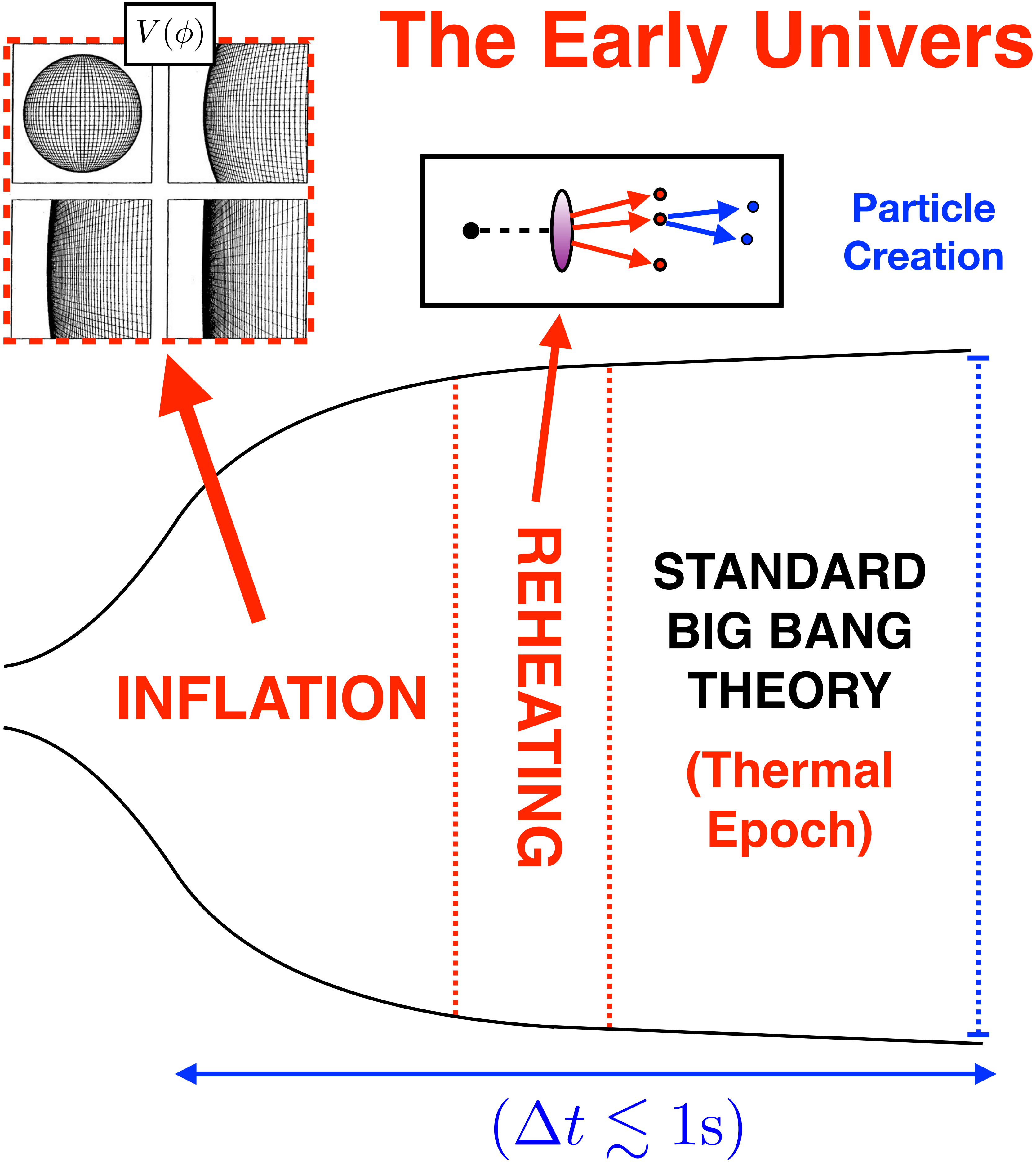

Particle Creation

INFLATION

REHEATING

STANDARD BIG BANG THEORY

(Thermal Epoch)

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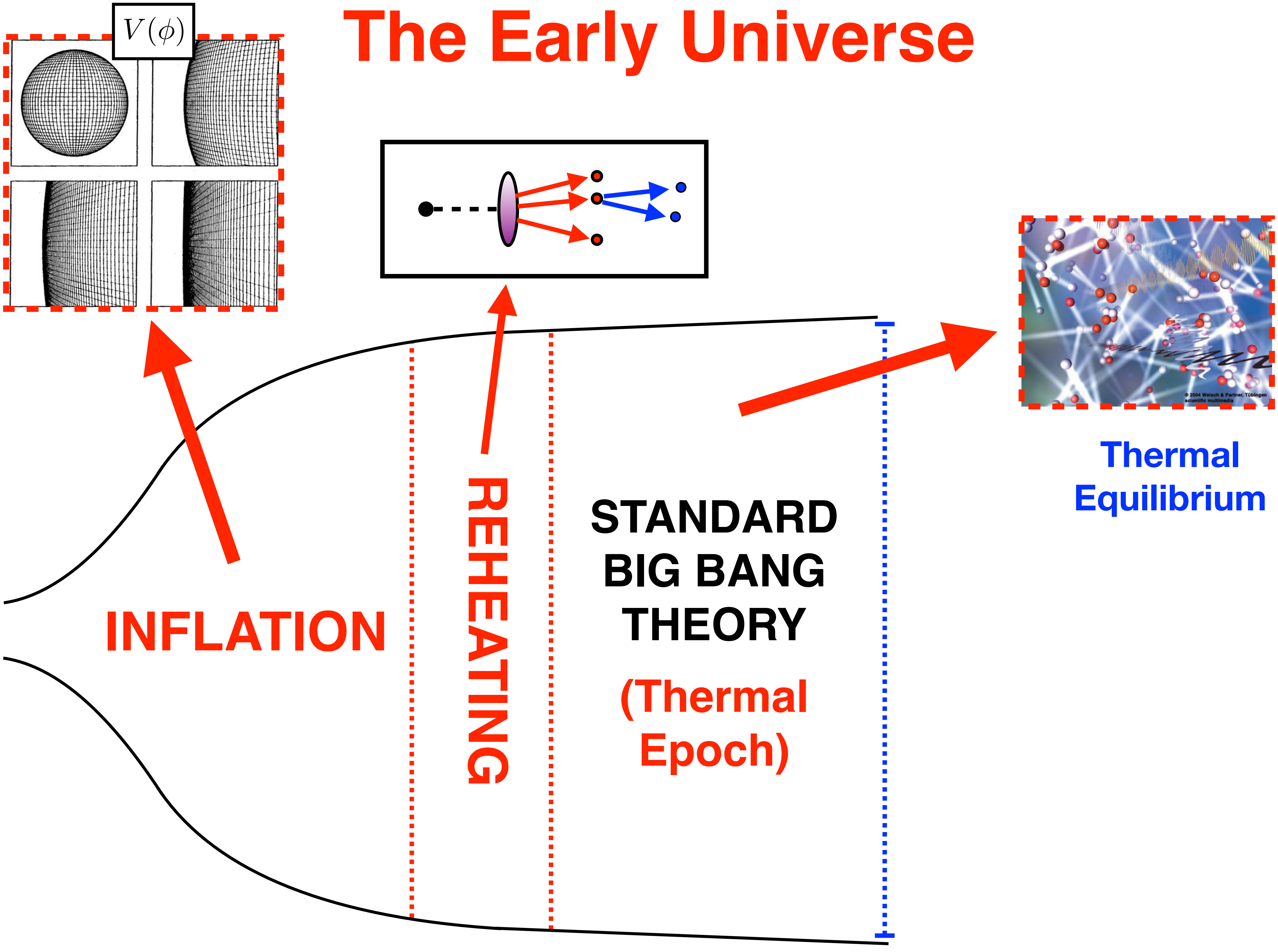


INFLATION

REHEATING

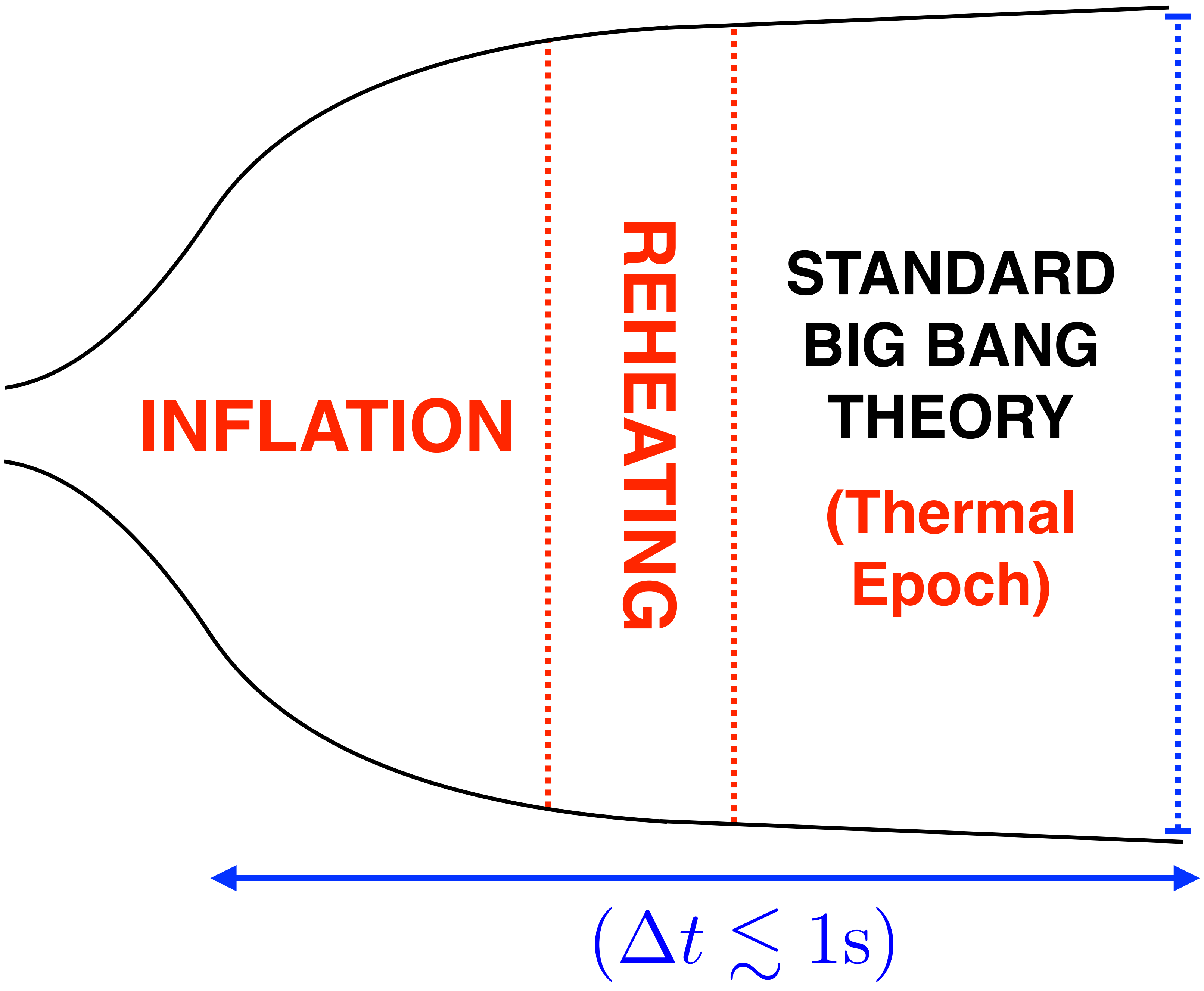
STANDARD BIG BANG THEORY

(Thermal Epoch)

Thermal Equilibrium

$(\Delta t \lesssim 1\mathrm{s})$

The Early Universe: How ?



INFLATION

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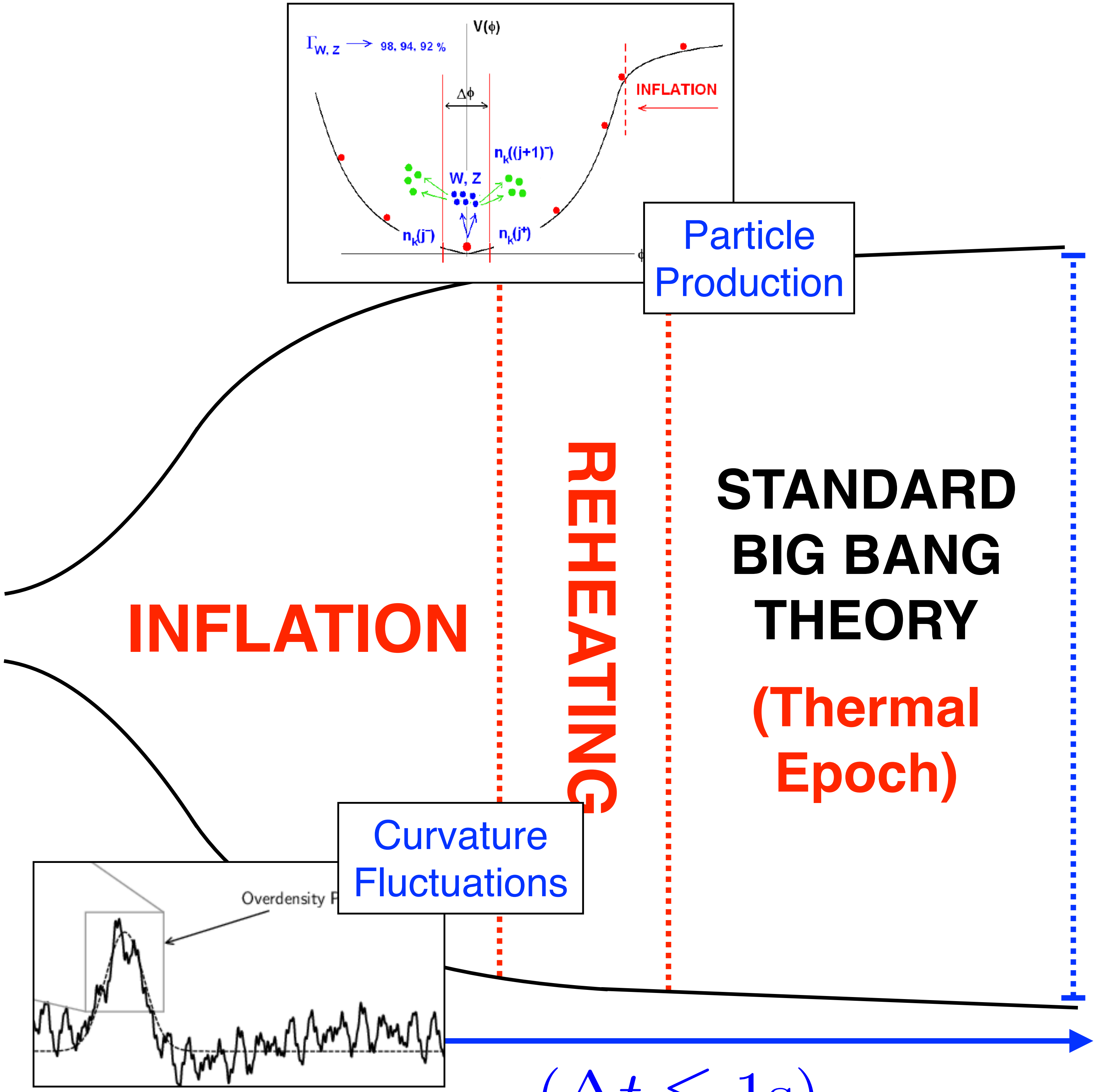


Particle Production

INFLATION

REHEATING

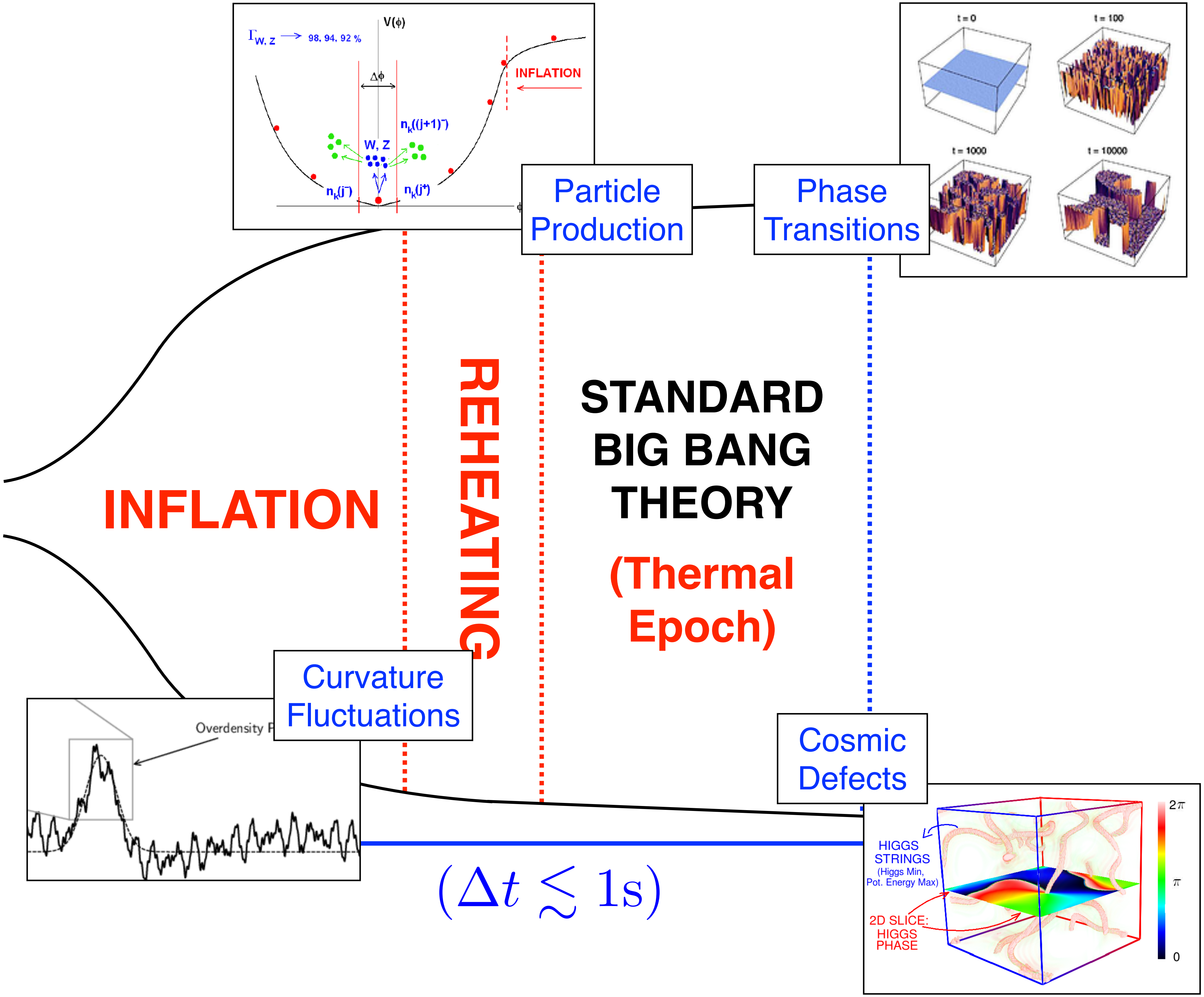
STANDARD BIG BANG THEORY

(Thermal Epoch)

Curvature Fluctuations

$(\Delta t \lesssim 1\mathrm{s})$

The Early Universe: How ?



Particle Production

Phase Transitions

INFLATION

REHEATING

STANDARD BIG BANG THEORY

(Thermal Epoch)

Curvature Fluctuations

Cosmic Defects

$(\Delta t \lesssim 1\mathrm{s})$

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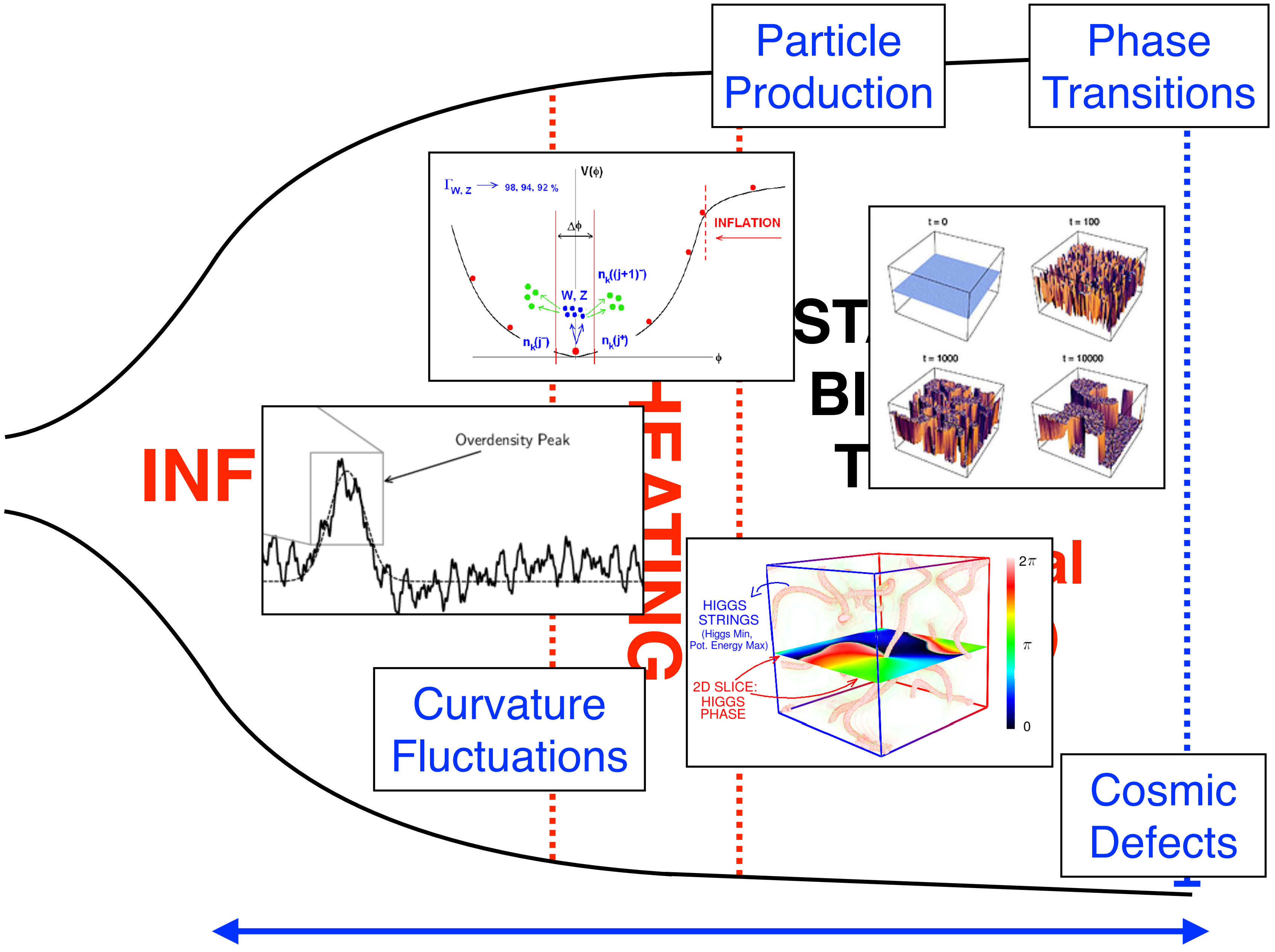

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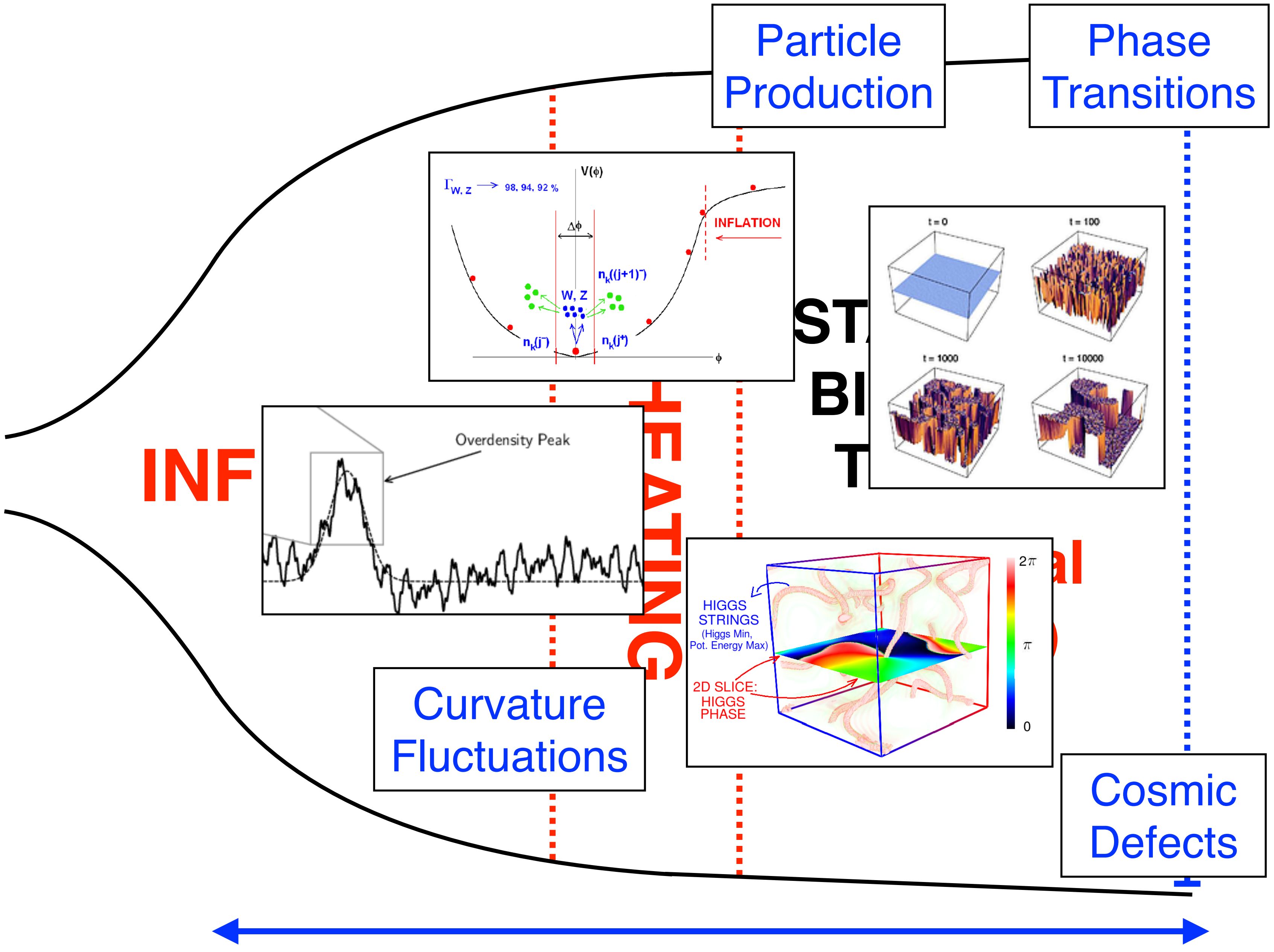

Particle Production

Phase Transitions

Curvature Fluctuations

Cosmic Defects

$$(\Delta t \lesssim 1\mathrm{s})$$

Common Factor

Non-linear Field dynamics

The Early Universe: How ?

Particle Production

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Common Factor

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The Early Universe: How ?

Gravitational waves

Particle Production

Phase Transitions

Baryo-genesis

Non-linear Field dynamics

Magneto-genesis

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

The Early Universe: How ?

Gravitational waves

Particle Production

Phase Transitions

Non-minimal Gravitational Coupling

Baryo-genesis

Non-linear Field dynamics

Non-minimal Kinetic Theories

Magneto-genesis

Turbulence Thermalisation
....

Curvature Fluctuations

Black Hole Formation

Cosmic Defects

The Early Universe: How ?

Gravitational waves

Particle Production

Phase Transitions

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We need NUMERICS !

Non-minimal Kinetic Theories

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CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg

Lattice Theory: arXiv: 2006.15122 (+100 pages)

Code Manual: arXiv: 2102.01031 (+100 pages)

CosmoLattice

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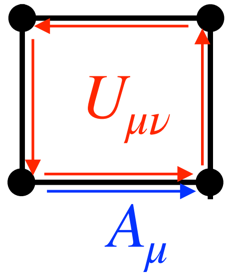
- Simulates **scalar-gauge field dynamics** [w. **self-consistent** expanding background]

[U(1) x SU(2)]

Links & **plaquettes**
(~ **lattice-QCD**)



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- Written in **C++**, with **modular structure** separating physics (CosmoInterface library) and technical details (TempLat library).
- **Parallellized** in multiple spatial dimensions (**but you write in serial !**)
- **Family** of evolution **algorithms**, accuracy ranging from $\delta\mathcal{O}(\delta t^2)$ – $\delta\mathcal{O}(\delta t^{10})$

  [*LeapFrog, Verlet, Runge-Kutta, Yoshida, …*]

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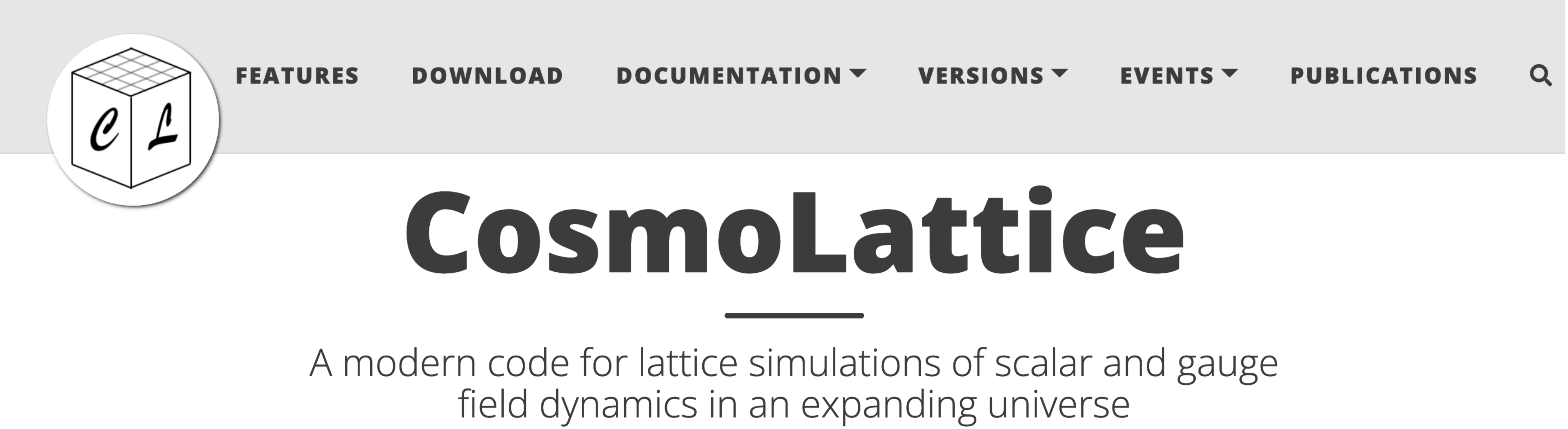

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CosmoLattice

A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

CosmoLattice

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Code Manual
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Field Th. Problem

* Init Conditions
* Eqs. of Motion

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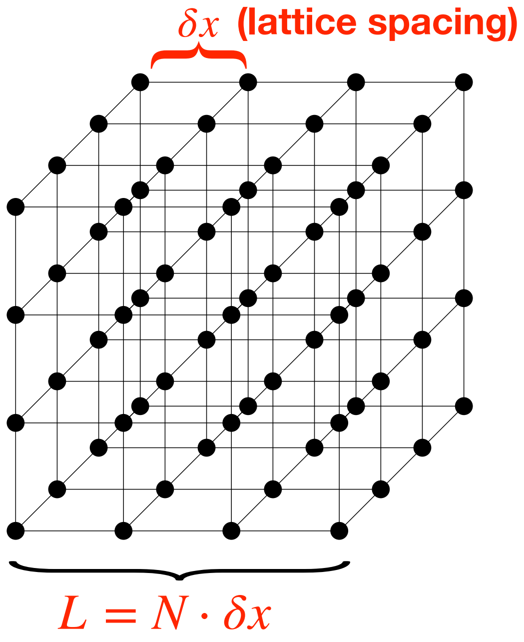
CosmoLattice

Field Th. Problem

- **Init Conditions**
- **Eqs. of Motion**

→

- **Choose Lattice: dt, N, dx**
- **Choose Algorithm** $\mathcal{O}(\delta t^n)$
- **Choose Param: g, m, …**
- **Choose Observables**



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CosmoLattice

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Algorithms

- *Staggered LeapFrog (LF)*
- *Position-Verlet (PV2)*
- *Velocity-Verlet (VV2)*
- *Runge-Kutta (RK2, RK3, RK4)*
- *Yoshida (VV4, VV6, VV8, VV10)*

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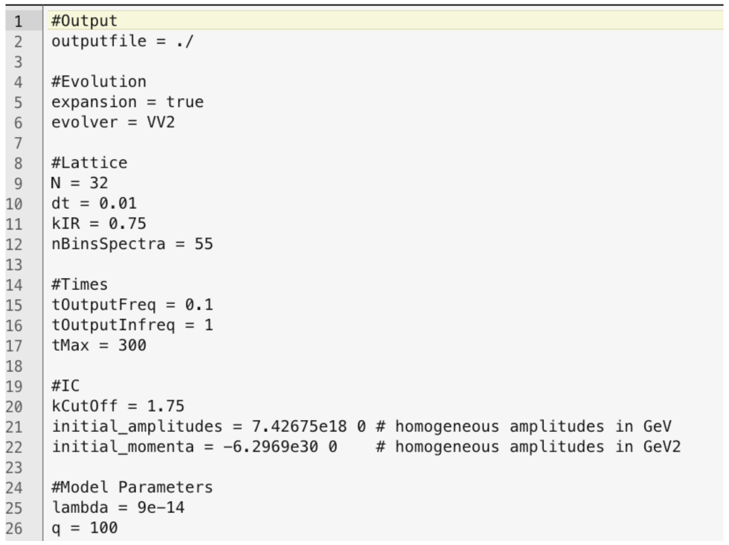
$$\lambda_1\,,\,\lambda_2\,,\,\ldots,\,g_1^2\,,\,g_2^2\,,\,\ldots$$

$$m_\phi^2\,,\,m_\psi^2\,,\,\ldots,v^2,\,\Phi_*,\,\ldots$$



```
#Output
outputfile = ./

#Evolution
expansion = true
evolver = VV2

#Lattice
N = 32
dt = 0.01
kIR = 0.75
nBinsSpectra = 55

#Times
tOutputFreq = 0.1
tOutputInfreq = 1
tMax = 300

#IC
kCutOff = 1.75
initial_amplitudes = 7.42675e18 0 # homogeneous amplitudes in GeV
initial_momenta = -6.2969e30 0    # homogeneous amplitudes in GeV2

#Model Parameters
lambda = 9e-14
q = 100
```

➤ **Parameters** via **input file**
(no need to re-compile !)

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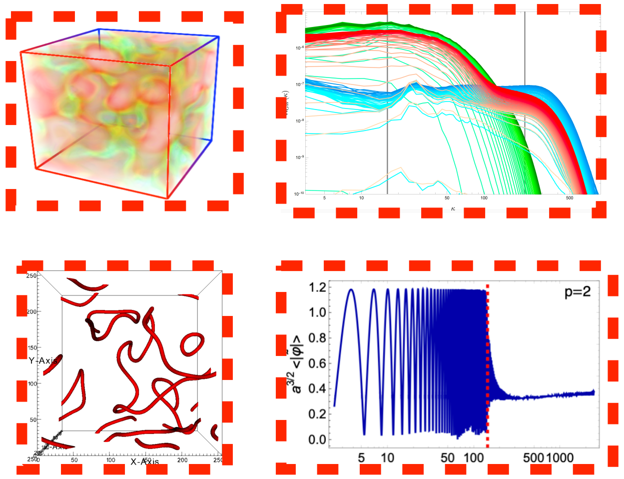

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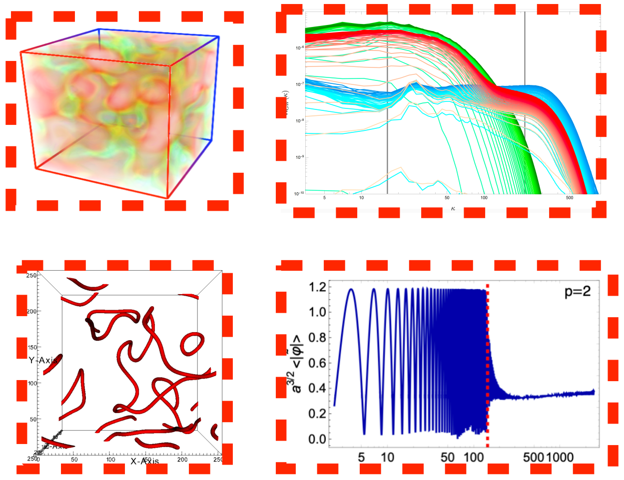

CL is a platform for field theories
You choose the problem to solve !

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CosmoLattice

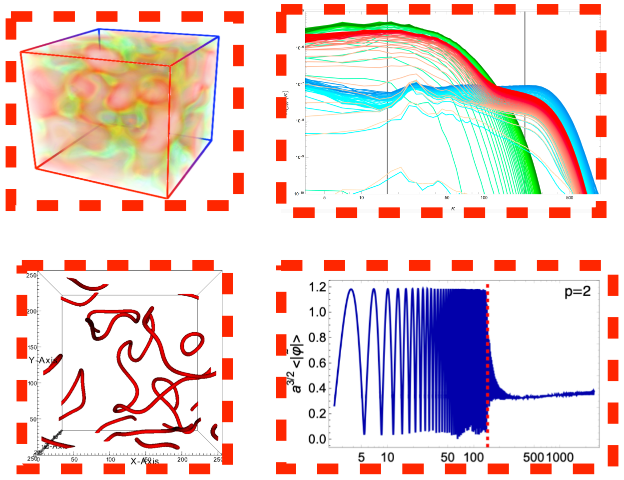
Field Th. Problem

- **Init Co**

New Problem

- **Choose Lattice: dt, N, dx**
- **Choose Algorithm $\mathcal{O}(\delta t^n)$**
- **Choose Param: g, m, …**
- **Choose Observables**

Output



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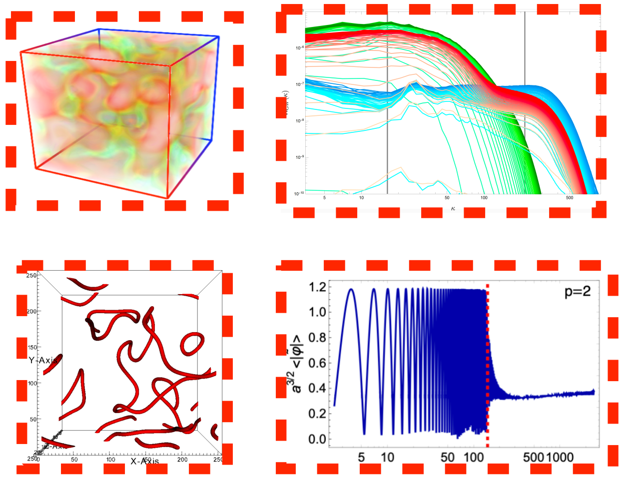

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    - ➤ Global scalar field dynamics
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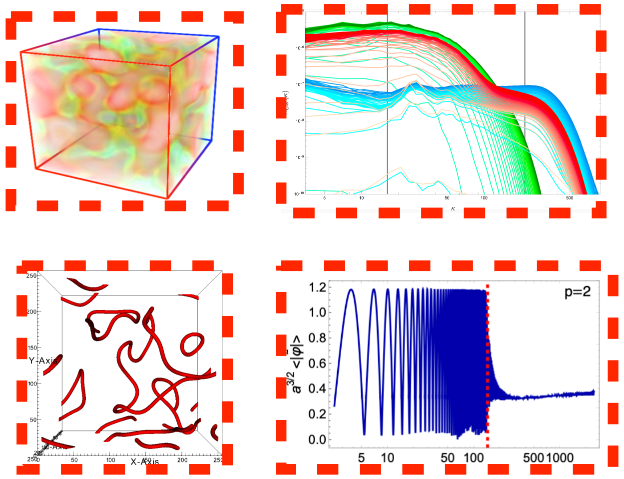

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- **CL update (v2.0, to be released by ~2023)**
  - Gravitational waves $\Box h_{ij} = 2\Pi_{ij}^{\rm TT}$
  - Axion-like couplings $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$
  - Non-minimal coupling $\xi\phi^2 R$
  - Cosmic String Networks

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Field Th. Problem

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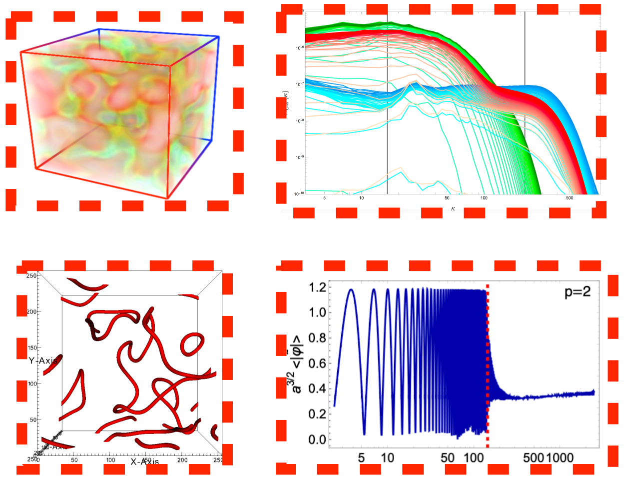

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Released in May 2022 !

Applications

1) Non-linear inflation dynamics (e.g Axion-inflation)

2) GW from non-linear dynamics (e.g Preheating)

3) Preheating & Equation of State after inflation

4) Cosmic string networks (axions, AH, …)

5) Single string loop dynamics

6) Non-minimal gravitational Interactions

7) Phase transitions

….

X) Your project !

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If so …
.. very
briefly

Example I

(Strong Backreaction) Field Dynamics of Axion Inflation

with

J. Lizarraga, A. Urio and J. Urrestilla

(PhD student)

Phys. Rev. Lett. Submitted ; 2303.17436

INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

$$V(\phi) + \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

inflaton ϕ = pseudo-scalar axion

[J. Cook, L. Sorbo (arXiv:1109.0022)]

[N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]

Not the QCD axion;

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Photon:
2 helicities

$$\left[\frac{\partial^2}{\partial\tau^2} + k^2 \pm \frac{2k\xi}{\tau}\right] A_\pm(\tau, k) = 0,$$

$$\boxed{\xi \equiv \frac{\dot{\phi}}{2fH}}$$

Chiral instability

$$A_+ \propto e^{\pi\xi}\,, \quad |A_-| \ll |A_+|$$

A_+ exponentially amplified,

INFLATIONARY MODELS

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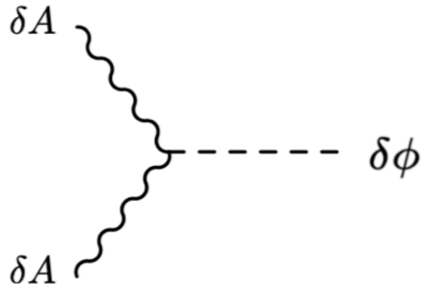

Barnaby, Peloso ’10
Planck '15

INFLATIONARY MODELS

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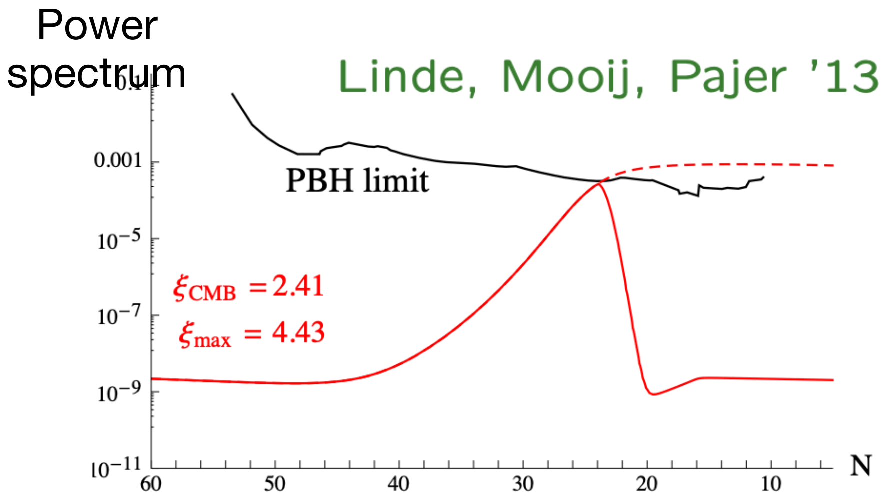
Barnaby, Peloso ’10
Planck ’15

Amplitude $\delta\phi$ must be bounded

Otherwise too many
Primordial Black Holes (PBH) !



Linde, Mooij, Pajer ’13

INFLATIONARY MODELS

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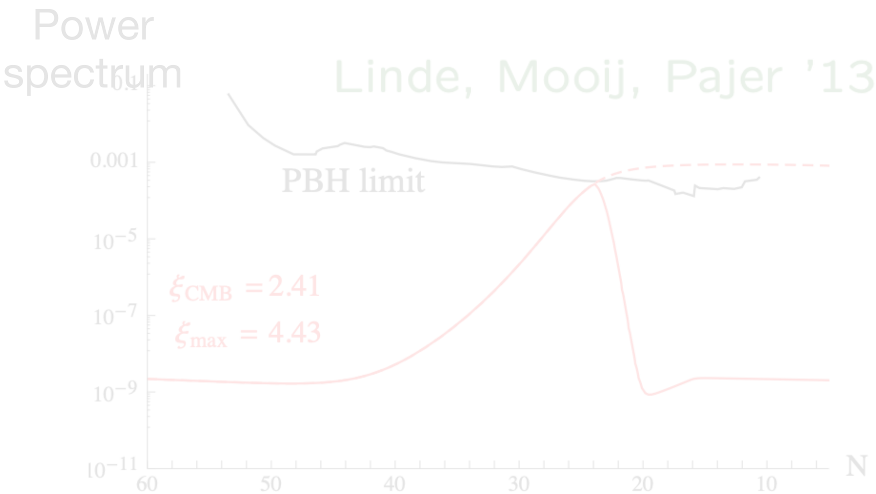


Linde, Mooij, Pajer ’13

Only one chirality of gauge field A_+

then… chiral GWs !

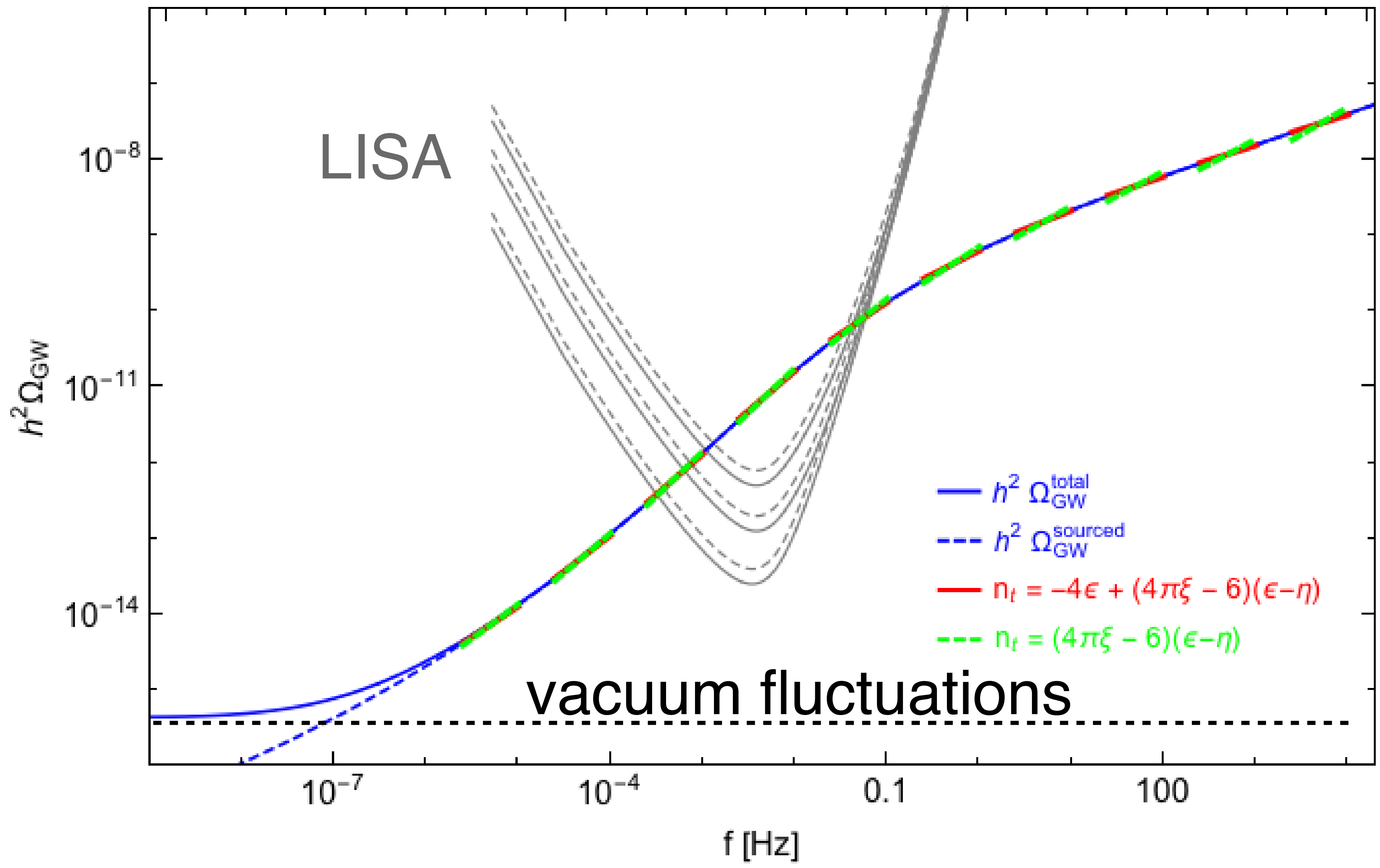
$$\{E_iE_j + B_iB_j\}^{\rm TT}$$

$$h_L \;,\quad \cancel{h_R}$$

INFLATIONARY MODELS

Axion-Inflation

GW energy spectrum today



Blue-Tilted

+ Chiral

+ Non-G

GW background

Bartolo et al ’16, 1610.06481

INFLATIONARY MODELS

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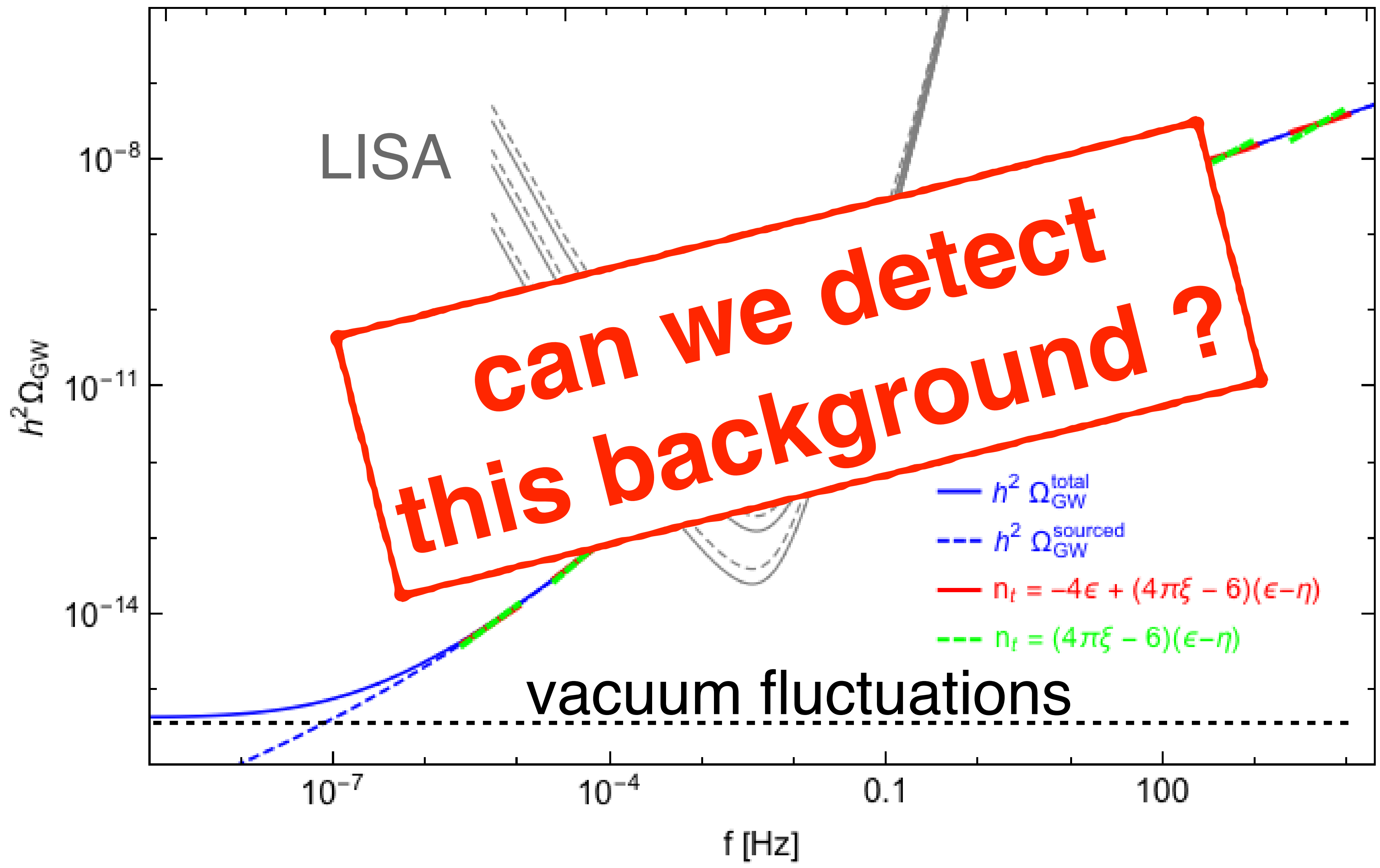

can we detect this background ?

Blue-Tilted

+ Chiral

+ Non-G

GW background

Bartolo et al ’16, 1610.06481

INFLATIONARY MODELS

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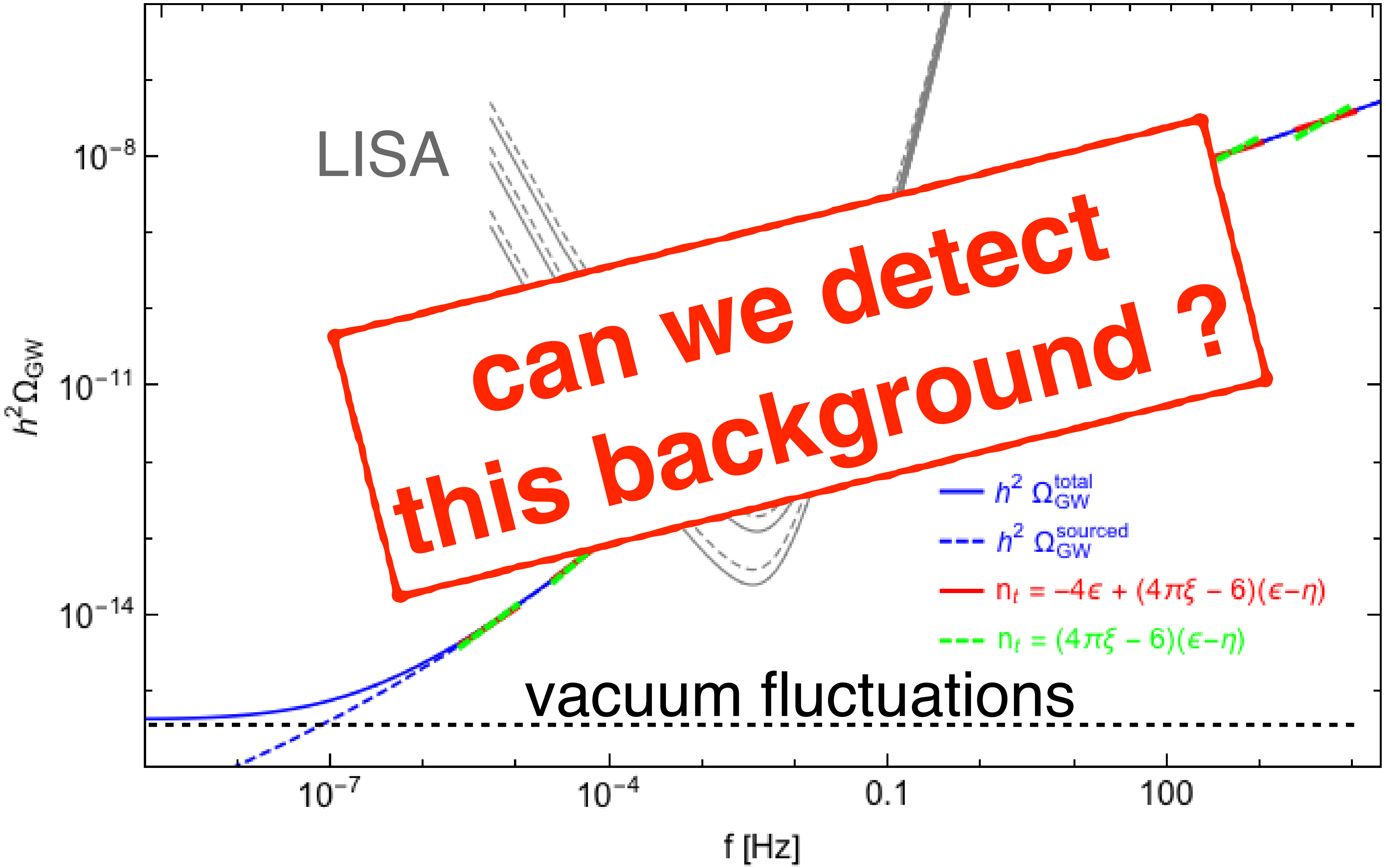

can we detect this background ?

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GW background

As $A_+ \propto e^{\dot{\phi}}$, GWs very sensitive to choice of $V(\phi)$ and calculation details

Bartolo et al '16, 1610.06481

Axion-Inflation

PROBLEM: PNG, GW and PBH ⟶ Approximations !

Axion-Inflation

PROBLEM: PNG, GW and PBH ⟶ Approximations !

$$\pi_\phi \equiv \dot{\phi}\ , \quad E_i \equiv \dot{A}_i\ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde{\pi}_\phi = a^3\pi_\phi\ , \quad \tilde{\vec{E}} = a\vec{E}\ , \ \pi_a \equiv \dot{a}$$

Let's have a look to the full problem !

$$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

$$\pi_\phi \equiv \dot{\phi}\ , \quad E_i \equiv \dot{A}_i\ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde{\pi}_\phi = a^3 \pi_\phi\ , \quad \tilde{\vec{E}} = a\vec{E}\ , \ \ \pi_a \equiv \dot{a}$$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathrm{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B}\ , \\ \\ \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}\ . \end{cases}$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

$$\pi_\phi \equiv \dot{\phi}\ , \quad E_i \equiv \dot{A}_i\ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde{\pi}_\phi = a^3 \pi_\phi\ , \quad \tilde{\vec{E}} = a\vec{E}\ , \ \ \pi_a \equiv \dot{a}$$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B}\ , \\[2em] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}\ . \end{cases}$$

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A} \Big\rangle$$

$$\left(\begin{array}{c} K_\phi \equiv \dfrac{1}{2}\pi_\phi^2 = \dfrac{1}{2a^6}\tilde{\pi}_\phi^2\ , \quad G_\phi \equiv \dfrac{1}{2a^2}\sum(\partial_i\phi)^2\ , \quad V \equiv \dfrac{1}{2}m^2\phi^2\ , \\ K_A \equiv \dfrac{1}{2}\sum_i \dfrac{E_i^2}{a^2} = \dfrac{1}{2}\sum_i \dfrac{\tilde{E}_i^2}{a^4}\ , \quad G_A \equiv \dfrac{1}{2}\sum_i \dfrac{B_i^2}{a^4} \end{array} \right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

$$\pi_\phi \equiv \dot{\phi}\ , \quad E_i \equiv \dot{A}_i\ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde{\pi}_\phi = a^3\pi_\phi\ , \quad \tilde{\vec{E}} = a\vec{E}\ , \ \pi_a \equiv \dot{a}$$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2\phi + \frac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B}\ , \\ \\ \dot{\tilde{\vec{E}}} = -\frac{1}{a}\vec{\nabla}\times\vec{B} - \frac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}\ . \end{cases}$$

Non-Linear dynamics

Hom. EoM (t)

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\langle -2\underbrace{K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}} - \underbrace{K_A}_{\text{(Elec)}} - \underbrace{G_A}_{\text{(Mag)}}\rangle$$

$$\left(\begin{array}{c} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\ , \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\ , \quad V \equiv \frac{1}{2}m^2\phi^2\ , \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i \frac{\tilde{E}_i^2}{a^4}\ , \quad G_A \equiv \frac{1}{2}\sum_i \frac{B_i^2}{a^4} \end{array} \right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

$$\pi_\phi \equiv \dot\phi \;, \quad E_i \equiv \dot A_i \;, \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde\pi_\phi = a^3\pi_\phi \;, \quad \vec{\tilde E} = a\vec E \;, \;\; \pi_a \equiv \dot a$$

EoM

Linear Regime

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde\pi}_\phi = \xcancel{a\vec\nabla^2\phi} - a^3 m^2\phi \xcancel{+ \frac{1}{a\Lambda}\vec{\tilde E}\cdot\vec B} \,, \\ \dot{\vec{\tilde E}} = -\frac{1}{a}\vec\nabla\times\vec B - \overset{\text{Interaction}}{\frac{1}{a^3\Lambda}\tilde\pi_\phi\vec B} + \xcancel{\frac{1}{a\Lambda}\vec\nabla\phi\times\vec{\tilde E}} \,. \end{cases}$$

Hom. EoM $(\mathbf{t})$

$$\dot\pi_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\left\langle -2\underset{\text{(Kin)}}{K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A}\right\rangle$$

$$\left(\begin{array}{c} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde\pi_\phi^2 \,, \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2 \,, \quad V \equiv \frac{1}{2}m^2\phi^2 \,, \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i \frac{\tilde E_i^2}{a^4} \,, \quad G_A \equiv \frac{1}{2}\sum_i \frac{B_i^2}{a^4} \end{array} \right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

$$\pi_\phi \equiv \dot\phi \;, \quad E_i \equiv \dot A_i \;, \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde\pi_\phi = a^3\pi_\phi \;, \quad \tilde{\vec E} = a\vec E \;, \;\; \pi_a \equiv \dot a$$

EoM

Linear Regime

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases} \dot{\tilde\pi}_\phi = \xcancel{a\vec\nabla^2\phi} - a^3m^2\phi\; + \xcancel{\frac{1}{a\Lambda}\tilde{\vec E}\cdot\vec B} \;, \\ \dot{\tilde{\vec E}} = -\frac{1}{a}\vec\nabla\times\vec B - \frac{1}{a^3\Lambda}\tilde\pi_\phi\vec B + \xcancel{\frac{1}{a\Lambda}\vec\nabla\phi\times\tilde{\vec E}} \;. \end{cases}$$

Interaction

Hom. EoM $(\mathbf{t})$

$$\dot\pi_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} \xcancel{\underset{\text{(Elec)}}{-K_A} \underset{\text{(Mag)}}{-G_A}} \Big\rangle$$

$$\left(\begin{array}{c} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde\pi_\phi^2 \;, \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2 \;, \quad V \equiv \frac{1}{2}m^2\phi^2 \;, \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i \frac{\tilde E_i^2}{a^4} \;, \quad G_A \equiv \frac{1}{2}\sum_i \frac{B_i^2}{a^4} \end{array} \right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

$$\pi_\phi \equiv \dot{\phi}\ , \quad E_i \equiv \dot{A}_i\ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde{\pi}_\phi = a^3\pi_\phi\ , \quad \tilde{\vec{E}} = a\vec{E}\ , \ \pi_a \equiv \dot{a}$$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3m^2\phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B}\ , \\[2ex] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\ . \end{cases}$$

→ **Back-Reaction**

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left\langle -\underset{\text{(Kin)}}{2K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A}\right\rangle$$

$$\left(\begin{array}{c} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\ , \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\ , \quad V \equiv \frac{1}{2}m^2\phi^2\ , \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i\frac{\tilde{E}_i^2}{a^4}\ , \quad G_A \equiv \frac{1}{2}\sum_i\frac{B_i^2}{a^4} \end{array}\right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

$$\pi_\phi \equiv \dot\phi\ , \quad E_i \equiv \dot A_i\ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde\pi_\phi = a^3\pi_\phi\ , \quad \tilde{\vec E} = a\vec E\ ,\ \pi_a \equiv \dot a$$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde\pi}_\phi = \xcancel{a\vec\nabla^2\phi} - a^3m^2\phi + \dfrac{1}{a\Lambda}\langle\tilde{\vec E}\cdot\vec B\ \rangle \\ \dot{\tilde{\vec E}} = -\dfrac{1}{a}\vec\nabla\times\vec B - \dfrac{1}{a^3\Lambda}\tilde\pi_\phi\vec B + \xcancel{\dfrac{1}{a\Lambda}\vec\nabla\phi\times\tilde{\vec E}}\ . \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot\pi_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V}\big\rangle - \big\langle \underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A}\big\rangle\Big)$$

$$\left(\begin{array}{c} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde\pi_\phi^2\ , \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\ , \quad V \equiv \frac{1}{2}m^2\phi^2\ , \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i\frac{\tilde E_i^2}{a^4}\ , \quad G_A \equiv \frac{1}{2}\sum_i\frac{B_i^2}{a^4} \end{array}\right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

$$\left\langle \vec{E}\vec{B} \right\rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi}k^3\frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\left\langle K_A + G_A \right\rangle \equiv \left\langle \frac{E^2+B^2}{2} \right\rangle = \frac{1}{a^4}\int \frac{dk}{4\pi^2}k^2\left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2\left|A_{-\lambda}(\tau,\vec{k})\right|^2\right) \quad ;$$

$$\lambda = \pm \begin{cases} \nearrow \ \lambda = +\,,\,\text{if}\,\phi > 0 \\ \searrow \ \lambda = -\,,\,\text{if}\,\phi < 0 \end{cases}$$

EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3m^2\phi + \dfrac{1}{a\Lambda}\left\langle \tilde{\vec{E}}\cdot\vec{B} \right\rangle \\[2ex] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM (t)

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\big\langle -2K_\phi + V \big\rangle - \big\langle K_A + G_A \big\rangle\Big)$$

(Kin) (Pot) (Elec) (Mag)

$$\left(\begin{gathered} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\,, \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\,, \quad V \equiv \frac{1}{2}m^2\phi^2\,, \\ K_A \equiv \frac{1}{2}\sum_i\frac{E_i^2}{a^2} = \frac{1}{2}\sum_i\frac{\tilde{E}_i^2}{a^4}\,, \quad G_A \equiv \frac{1}{2}\sum_i\frac{B_i^2}{a^4} \end{gathered} \right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

$$\left\langle \vec{E}\vec{B} \right\rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi} k^3 \frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\left\langle K_A + G_A \right\rangle \equiv \left\langle \frac{E^2+B^2}{2} \right\rangle = \frac{1}{a^4}\int \frac{dk}{4\pi^2} k^2 \left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2 \left|A_{-\lambda}(\tau,\vec{k})\right|^2 \right) \quad ;$$

$$\begin{tikzcd} & \lambda = + \,, \text{if} \phi > 0 \\ \lambda = \pm \arrow[ur] \arrow[dr] & \\ & \lambda = - \,, \text{if} \phi < 0 \end{tikzcd}$$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\left\langle \tilde{\vec{E}} \cdot \vec{B} \right\rangle \\[2ex] \dfrac{d^2 A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau,\vec{k}) = 0 \;\cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left(\left\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} \right\rangle - \left\langle \underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A} \right\rangle \right)$$

DallAgata et al 2019, Domcke et 2020 → **Elaborated Iterative scheme !**

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

$$\left\langle \vec{E}\vec{B} \right\rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi} k^3 \frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\left\langle K_A + G_A \right\rangle \equiv \left\langle \frac{E^2+B^2}{2} \right\rangle = \frac{1}{a^4}\int \frac{dk}{4\pi^2} k^2 \left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2 \left|A_{-\lambda}(\tau,\vec{k})\right|^2 \right) \quad ;$$

$$\begin{tikzcd} & \lambda = +\,,\text{if}\phi > 0 \\ \lambda = \pm \arrow[ur] \arrow[dr] & \\ & \lambda = -\,,\text{if}\phi < 0 \end{tikzcd}$$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\left\langle \tilde{\vec{E}} \cdot \vec{B} \right\rangle \\[2em] \dfrac{d^2 A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau,\vec{k}) = 0 \; \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \tilde{\vec{E}}} \; . \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left(\left\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} \right\rangle - \left\langle \underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A} \right\rangle \right)$$

Gorbar et al 2021 → **Correlator Gradient Expansion**

Axion-Inflation

PROBLEM: PNG, GW and PBH ⟶ Approximations !

Can we do better than homogeneous backreaction ?

EoM

Back-Reaction (Homog. Approx.)

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\langle \tilde{\vec{E}} \cdot \vec{B} \rangle \\[2ex] \dfrac{d^2 A_\pm(\tau, \vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau, \vec{k}) = 0 \; \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \tilde{\vec{E}}} \; . \end{cases}$$

Hom. (t) Approx.

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big(\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} \rangle - \langle \underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A} \rangle\Big)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B} \\[2em] \dfrac{d^2 A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau,\vec{k}) = 0 \; \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}} \, . \end{cases}$$

Back-Reaction (Source InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A} \right\rangle$$

Axion-Inflation

PROBLEM: **PNG, GW and PBH** → **Approximations !**

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2\phi + \frac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B} \\ \dot{\tilde{\vec{E}}} = -\frac{1}{a}\vec{\nabla}\times\vec{B} - \frac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \cancel{\frac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\,. \end{cases}$$

Back-Reaction (Source InHom.)

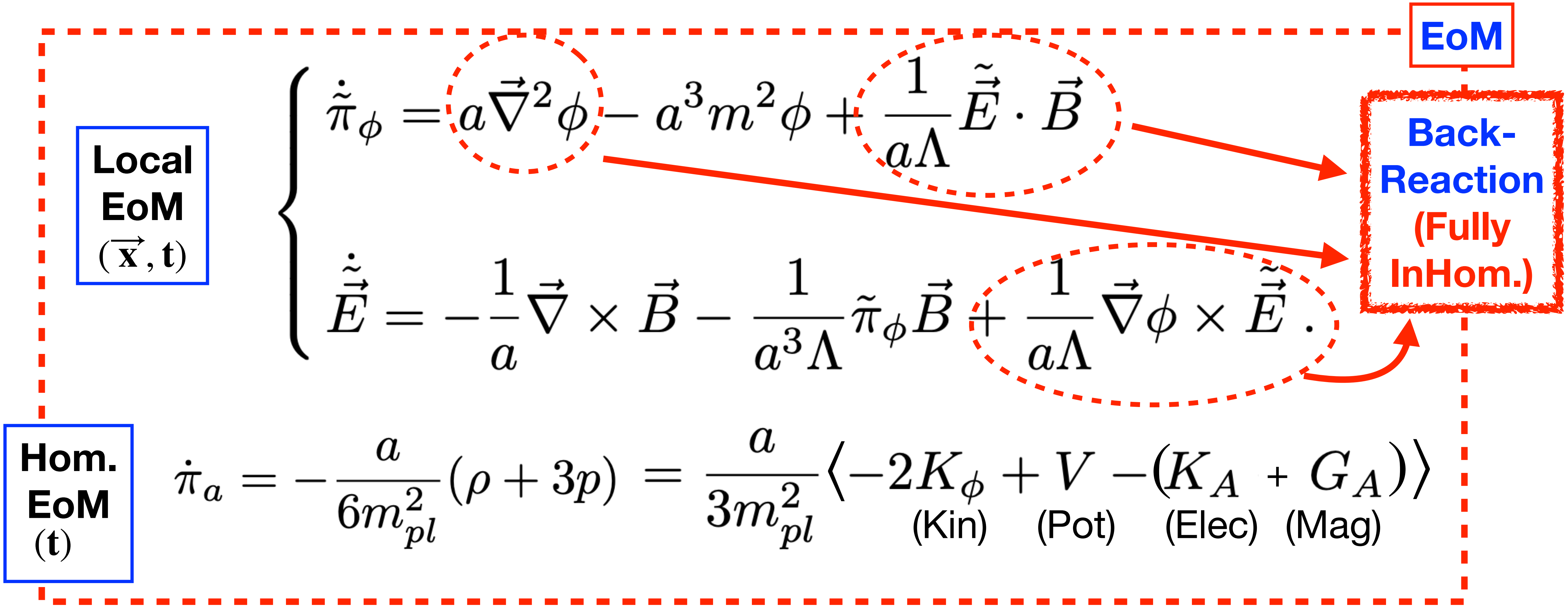
Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\left\langle -2\underset{\text{(Kin)}}{K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A}\right\rangle$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

Yes, we need a full lattice approach



EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3m^2\phi + \frac{1}{a\Lambda}\vec{\tilde{E}}\cdot\vec{B} \\[2ex] \dot{\vec{\tilde{E}}} = -\frac{1}{a}\vec{\nabla}\times\vec{B} - \frac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{\tilde{E}}\,. \end{cases}$$

Back-Reaction (Fully InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - (\underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A})\rangle$$

Axion-Inflation

PROBLEM: **PNG, GW and PBH** → **Approximations !**

DGF + Canivete/Shaposhnikov + Lizarraga/Urio/Urrestilla **2017-2023**
Caravano, Komatsu, Lozanov, Weller **2021-2022**

EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2\phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B} \\ \\ \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}\,. \end{cases}$$

Back-Reaction (Fully InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - (\underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A})\Big\rangle$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Approximations !**

DGF + Canivete/Shaposhnikov + Lizarraga/Urio/Urrestilla 2017-2023
Caravano, Komatsu, Lozanov, Weller 2021-2022

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2\phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B} \\[2ex] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}\,. \end{cases}$$

Back-Reaction (Fully InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - \big(\underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A}\big)\Big\rangle$$

The strong backreaction regime of axion inflation

(DGF, Lizarraga, Urio & Urrestilla, 2303.17436)

(PhD)

$$\left(\frac{m_p}{\Lambda} \geq 15\right)$$

The strong backreaction regime of axion inflation

(DGF, Lizarraga, Urio & Urrestilla, 2303.17436)

(PhD)

$$\left(\frac{m_p}{\Lambda} \geq 15\right)$$

The strong backreaction regime of axion inflation

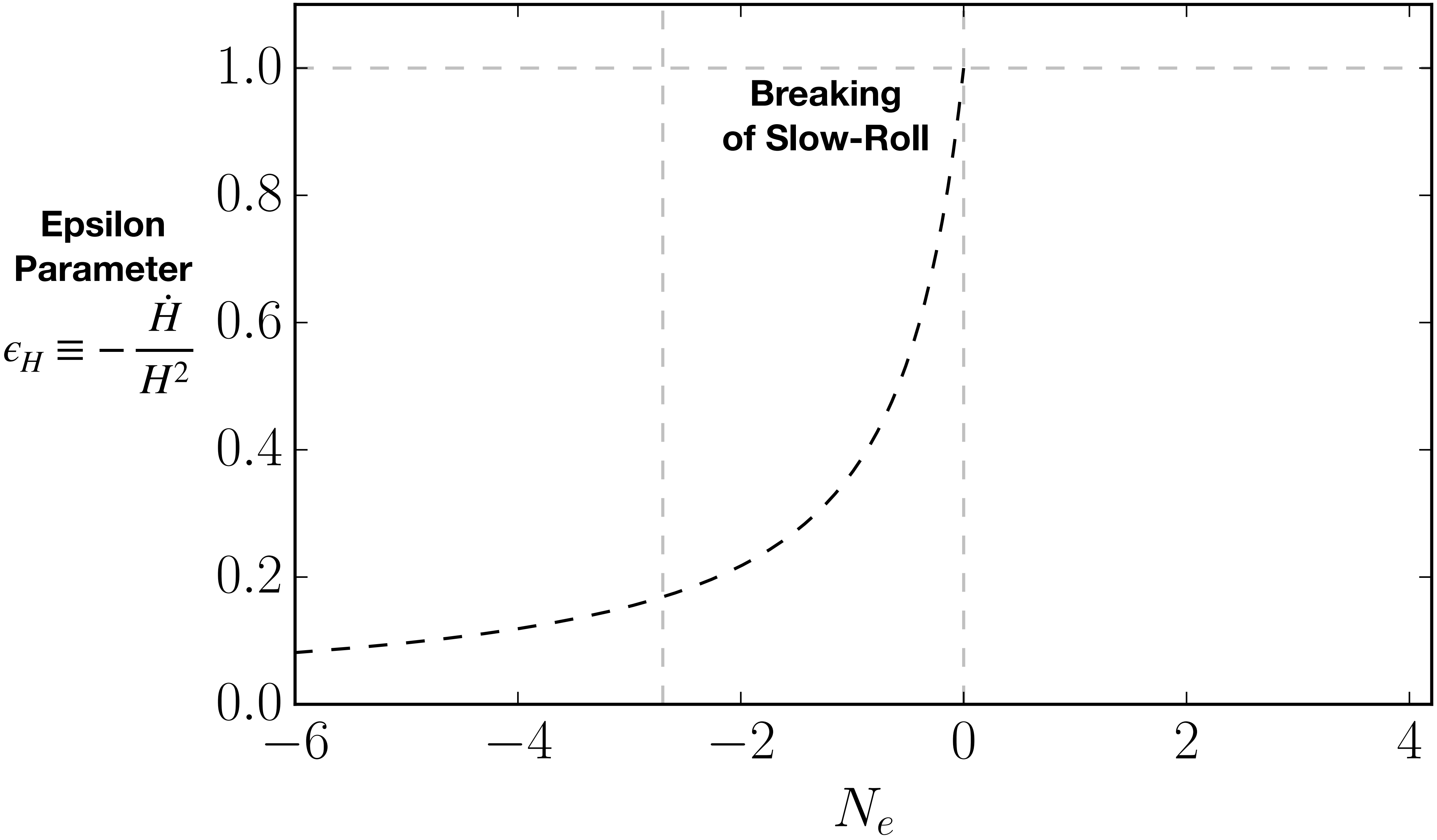
(DGF, Lizarraga, Urio & Urrestilla, 2303.17436)

(PhD)

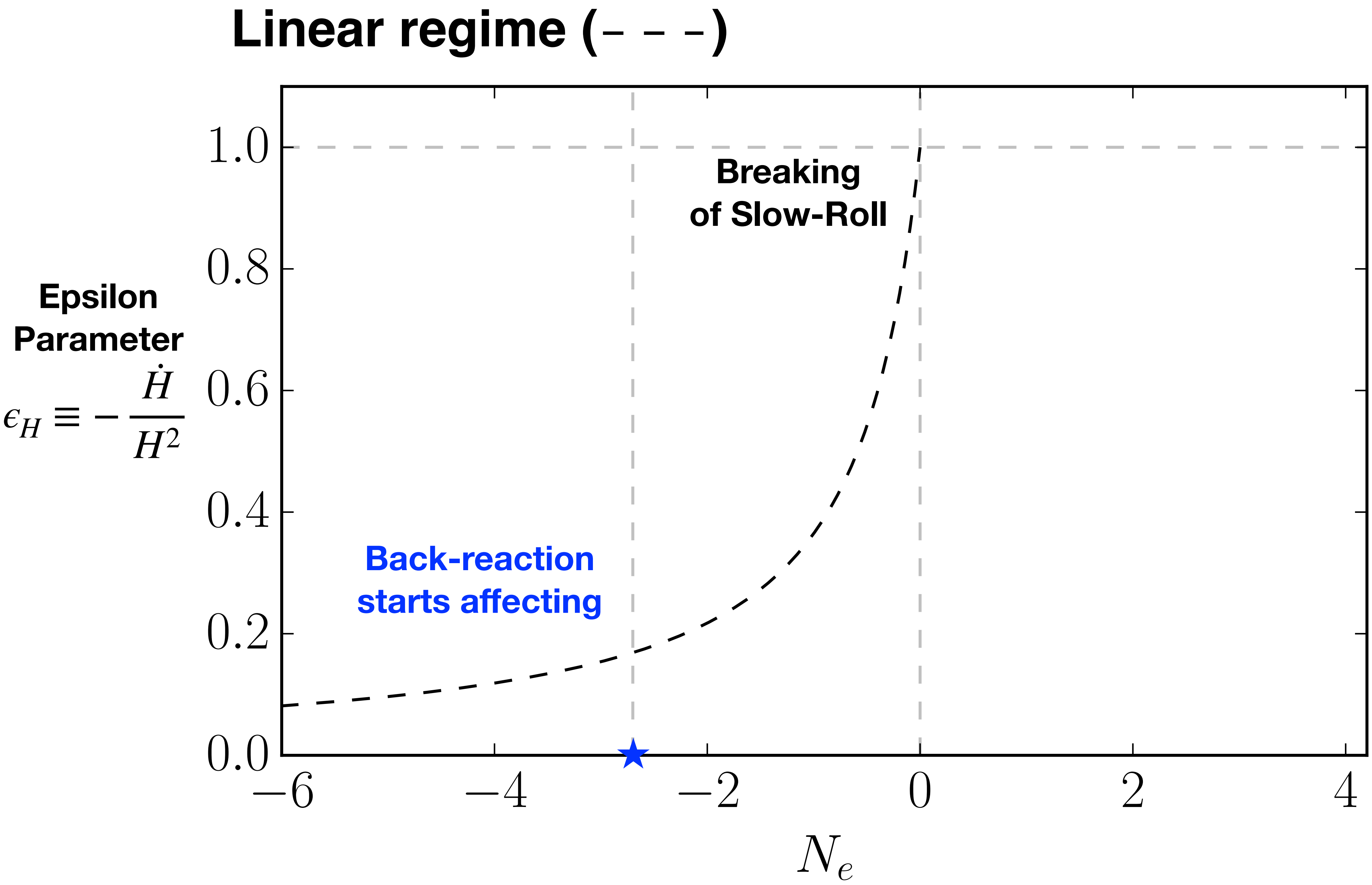
$$V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \frac{m_p}{\Lambda} = 18$$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{18}$)

Linear regime (- - -)



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{18}$)



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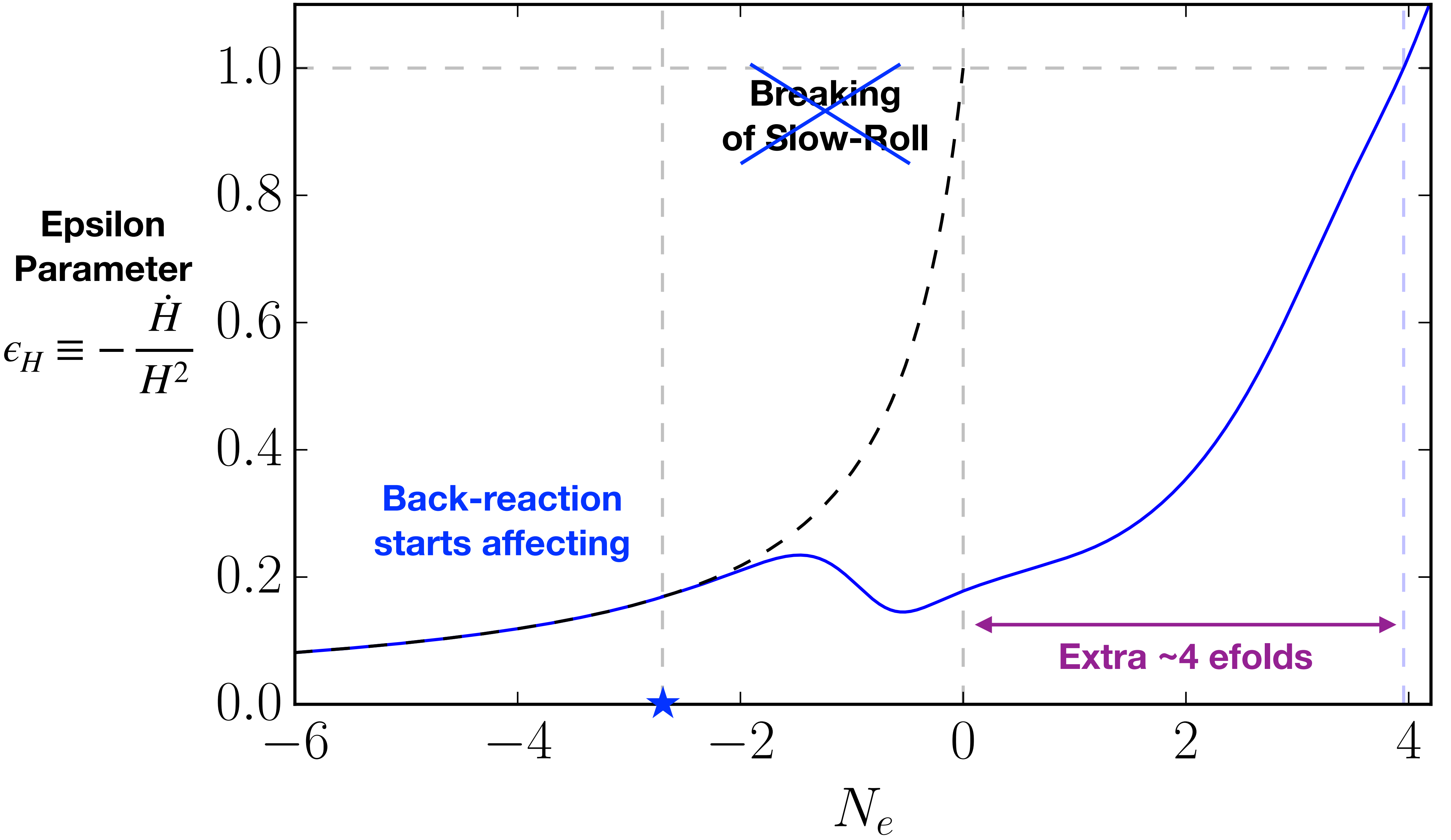
Linear regime (– – –) **Non-Linear regime (——)**



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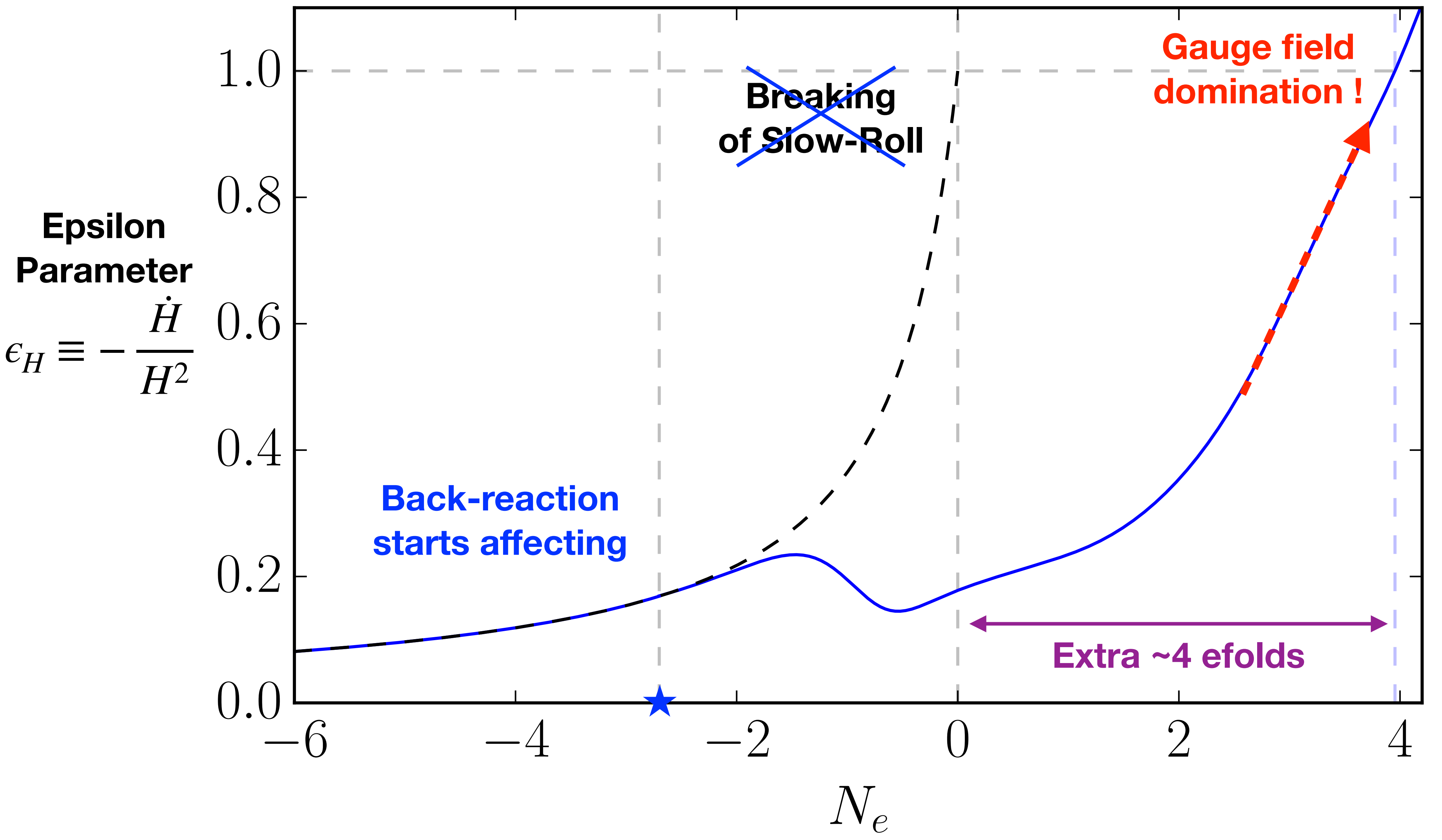
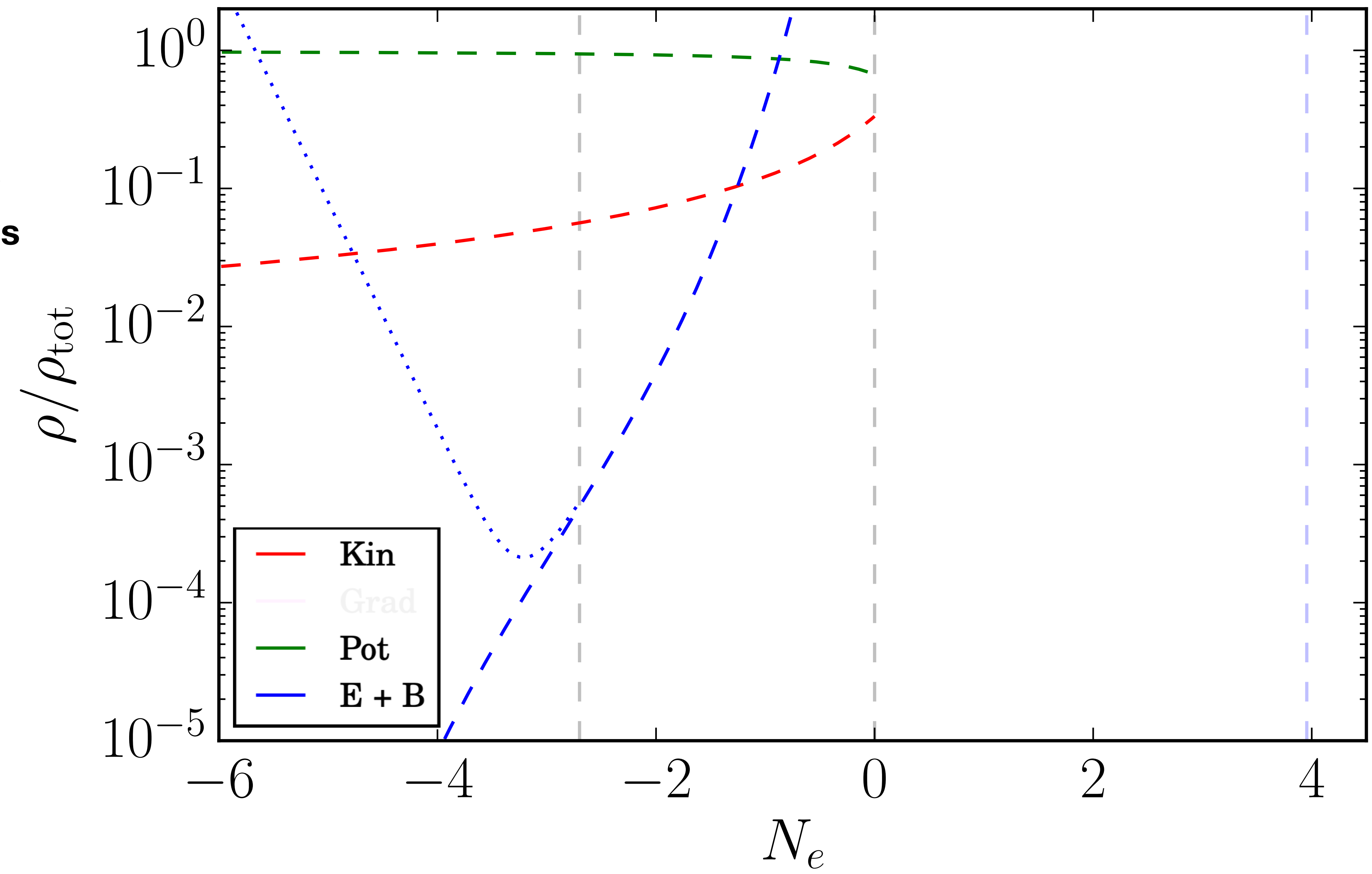
Linear regime (- - -) **Non-Linear regime (——)**



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{18}$)

Linear (– – –)

Energy Densities



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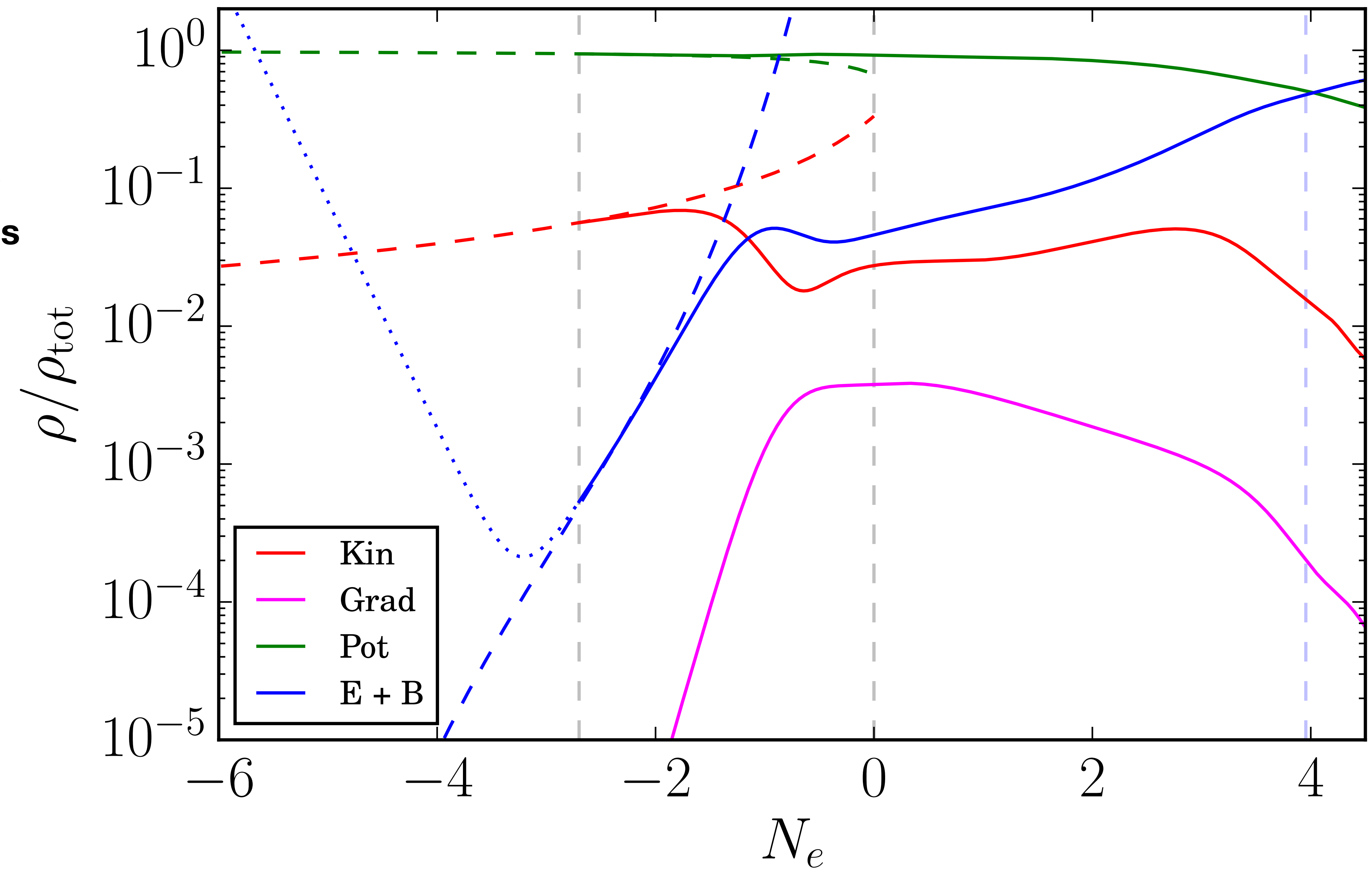
Linear (- - -) **Non-Linear (——)**

Energy Densities



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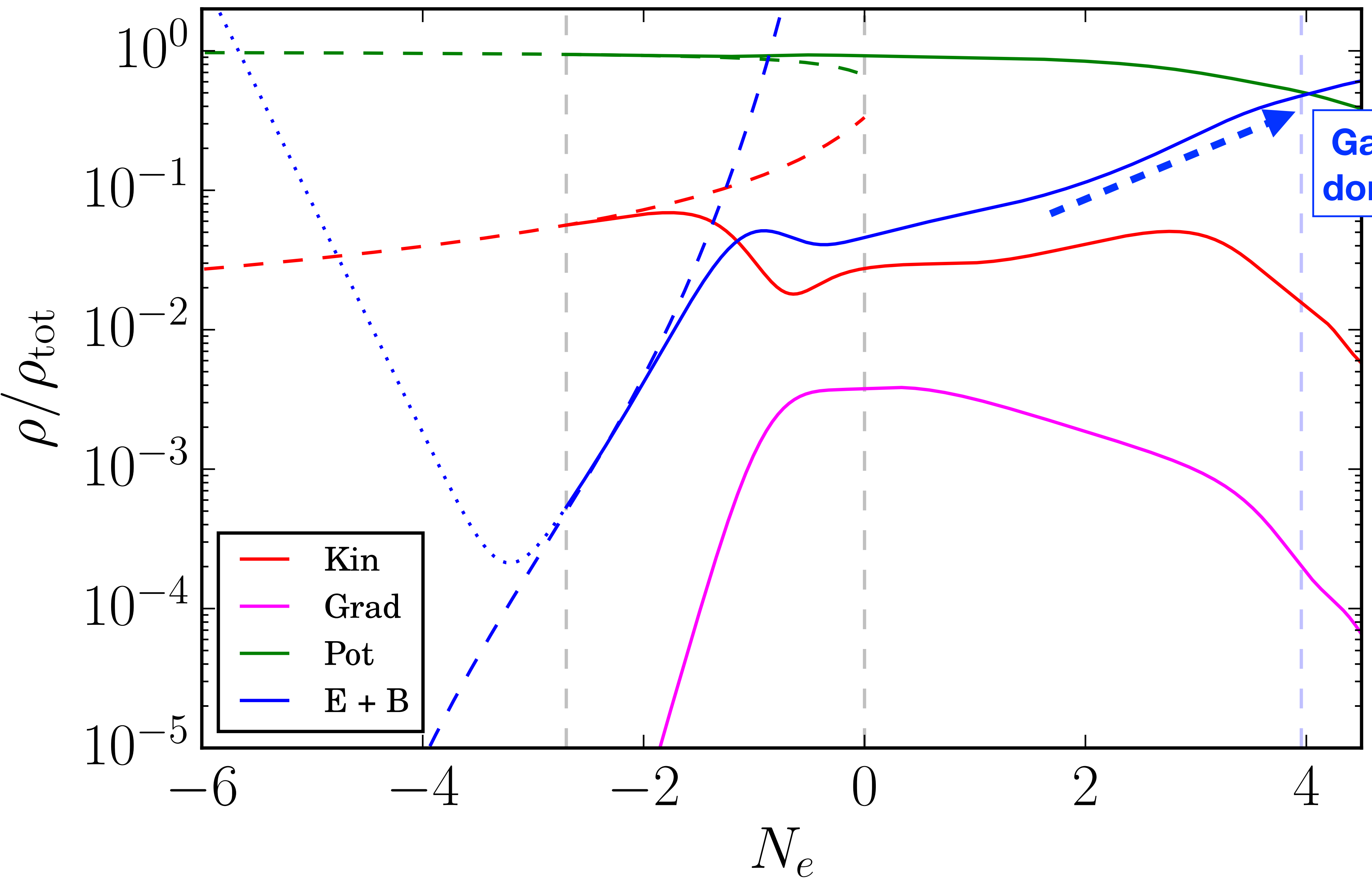
Linear (- - -) **Non-Linear (——)**

Energy Densities

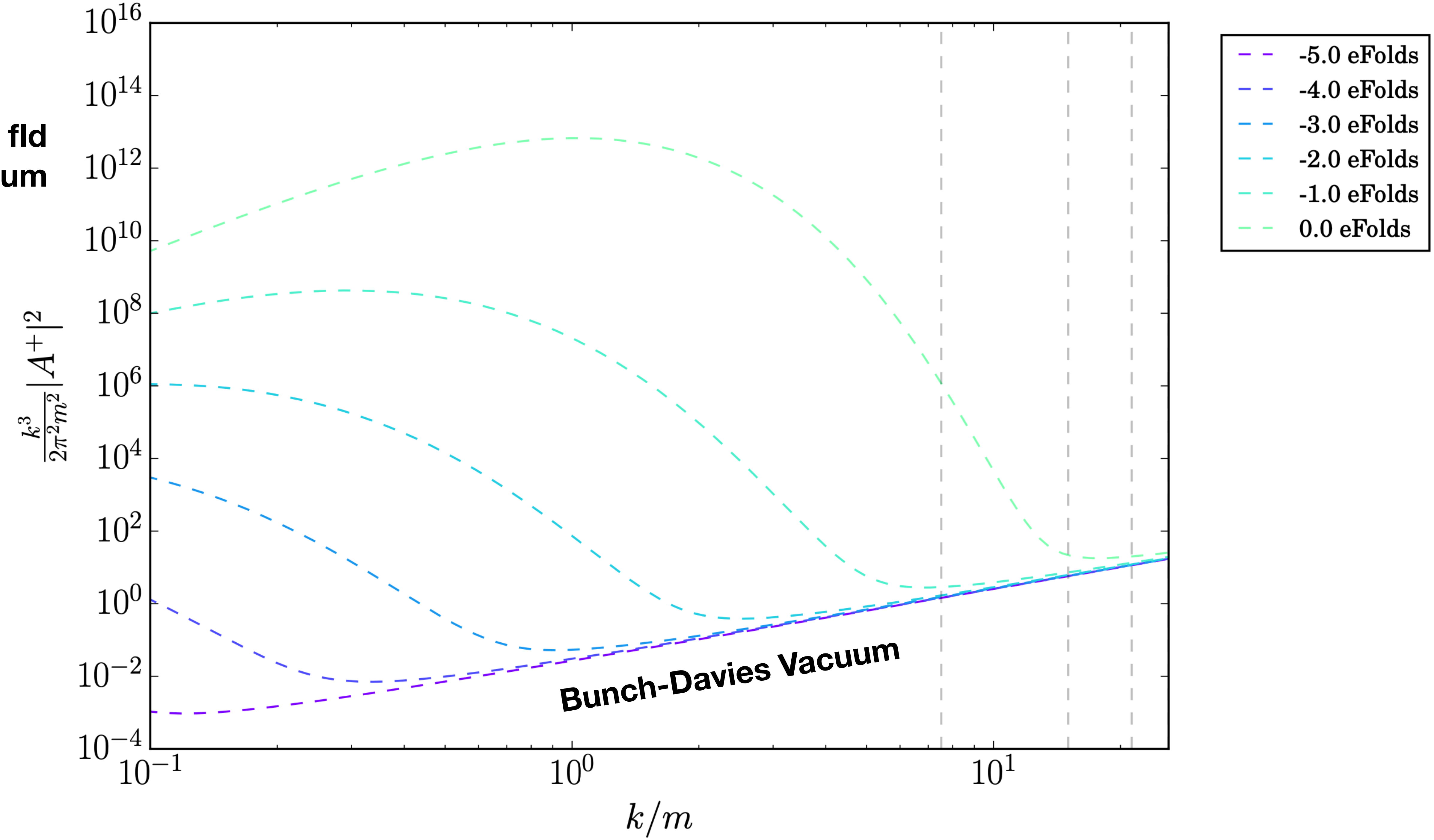


Gauge field domination !

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{18}$)

Linear regime (≡ ≡ ≡)

Gauge fld Spectrum



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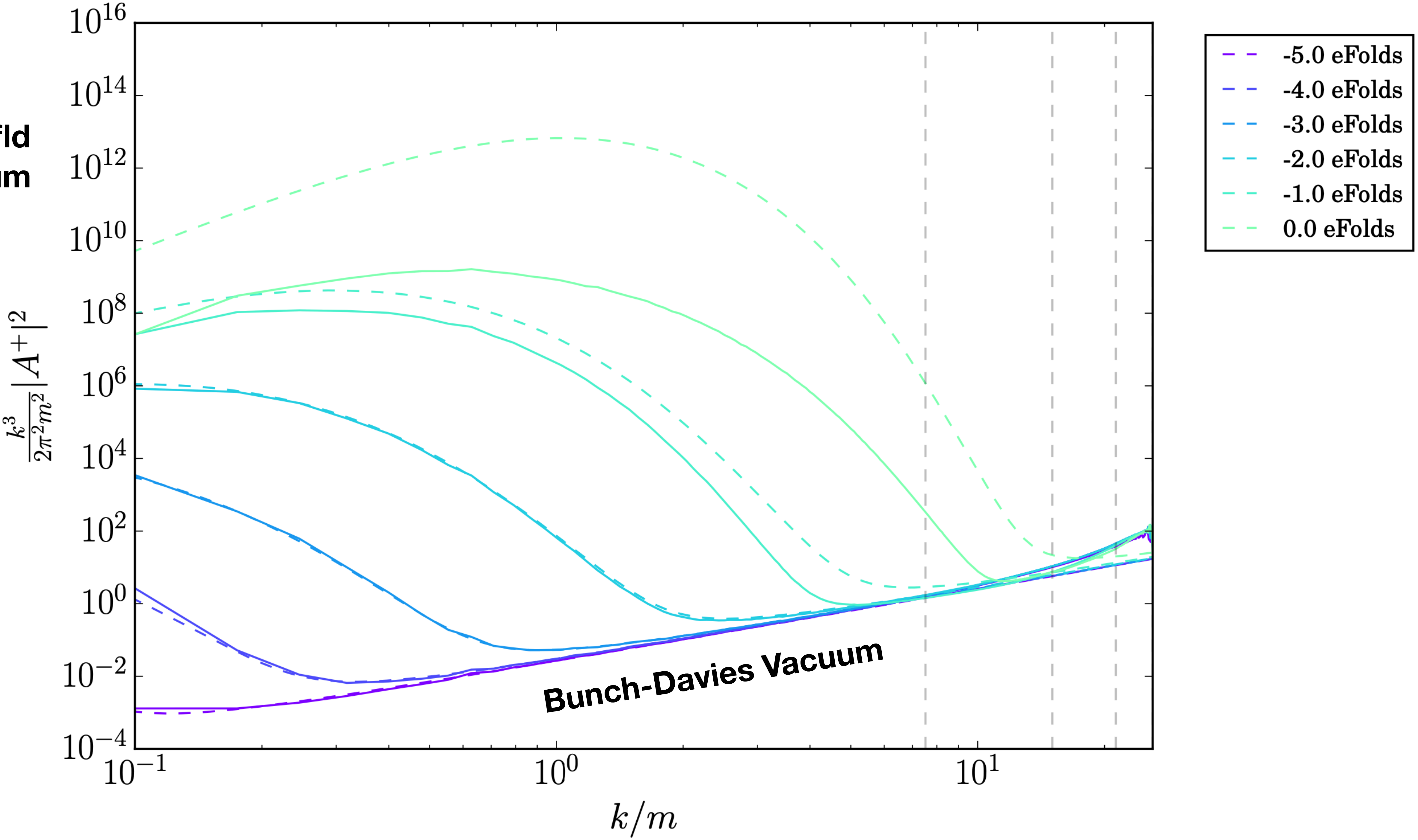
Linear regime (- - -) **Non-Linear regime (——)**

Gauge fld Spectrum



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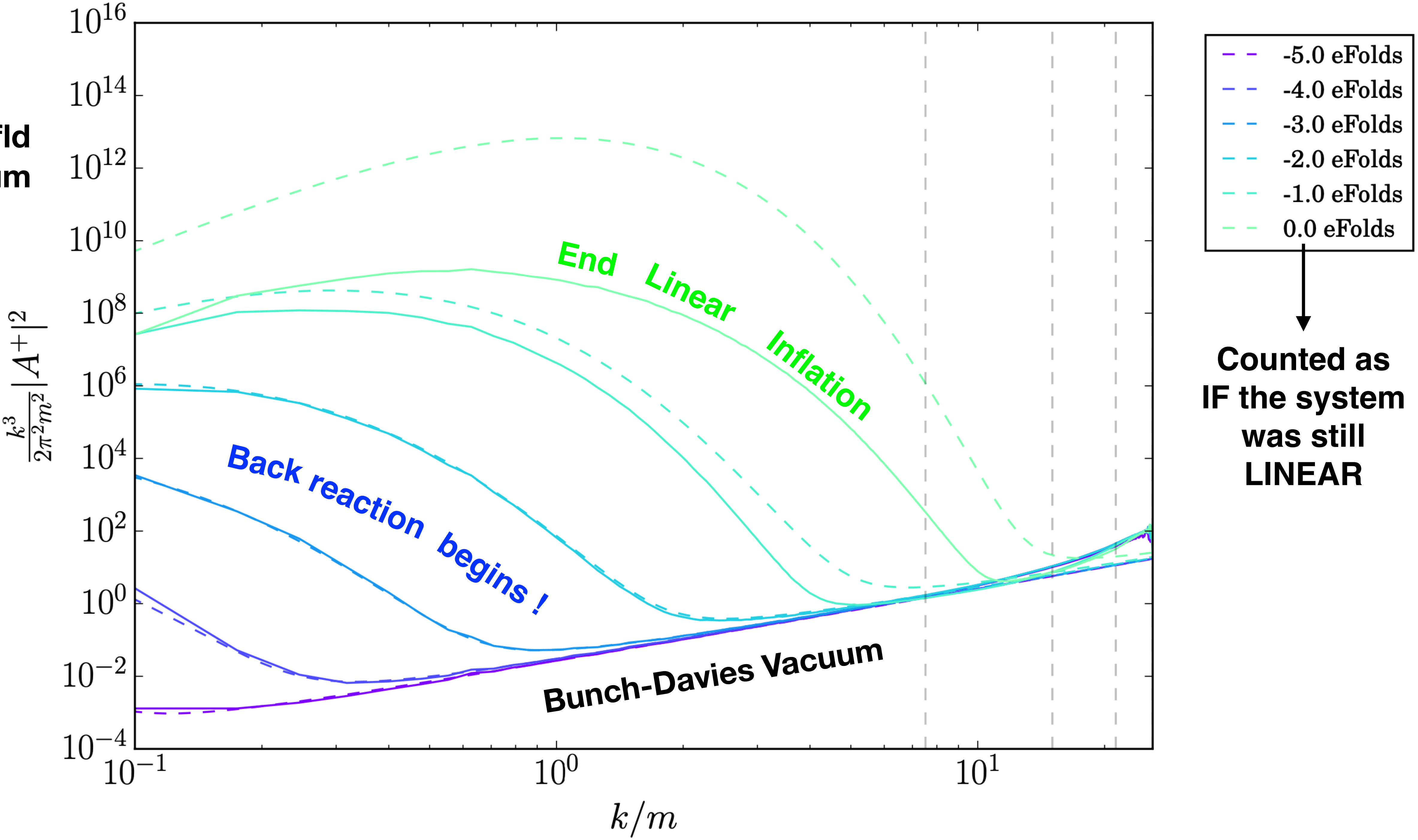
Linear regime (- - -) Non-Linear regime (—)

Gauge fld Spectrum



Counted as IF the system was still LINEAR

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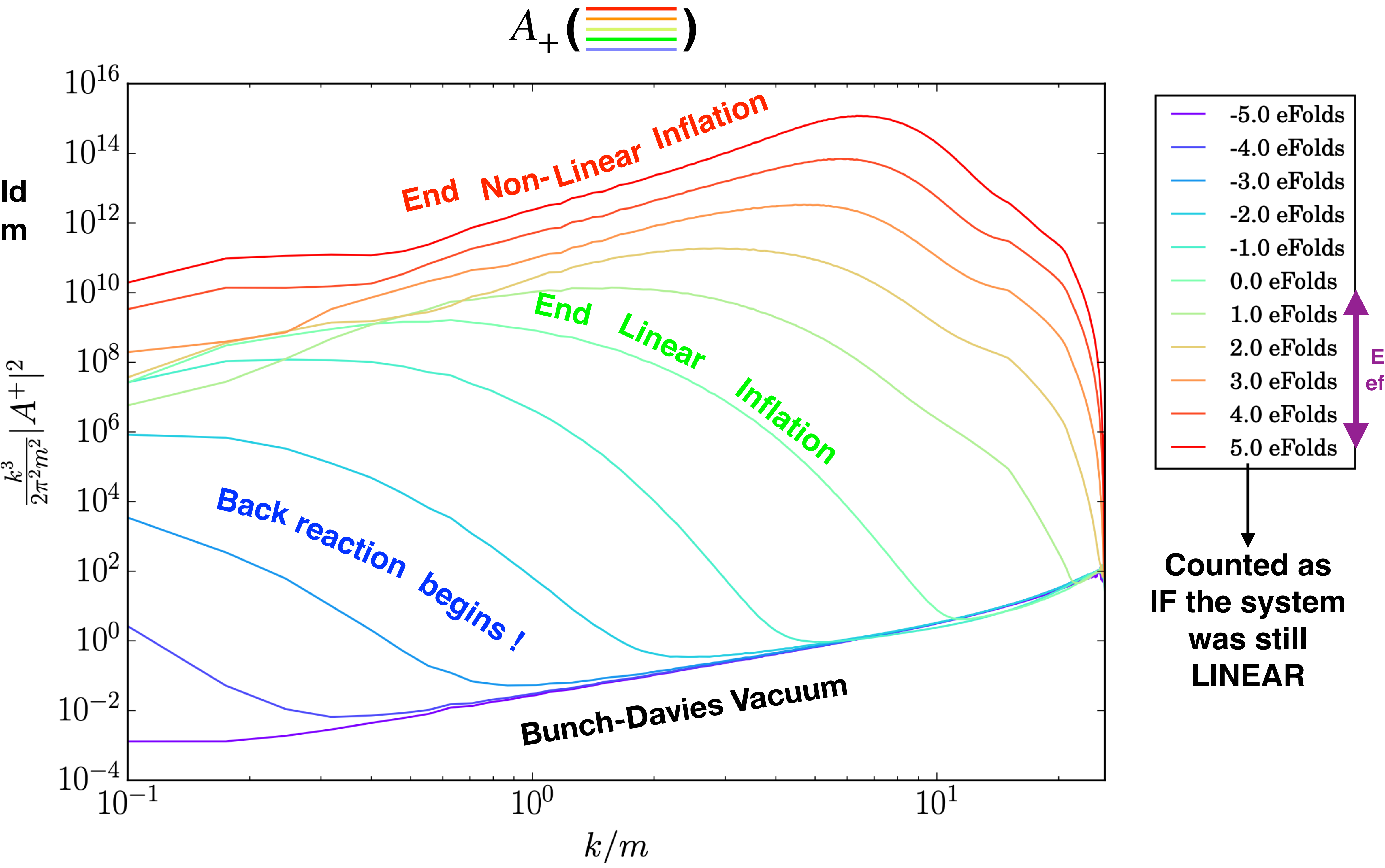
Gauge fld Spectrum



Extra efolds

Counted as IF the system was still LINEAR

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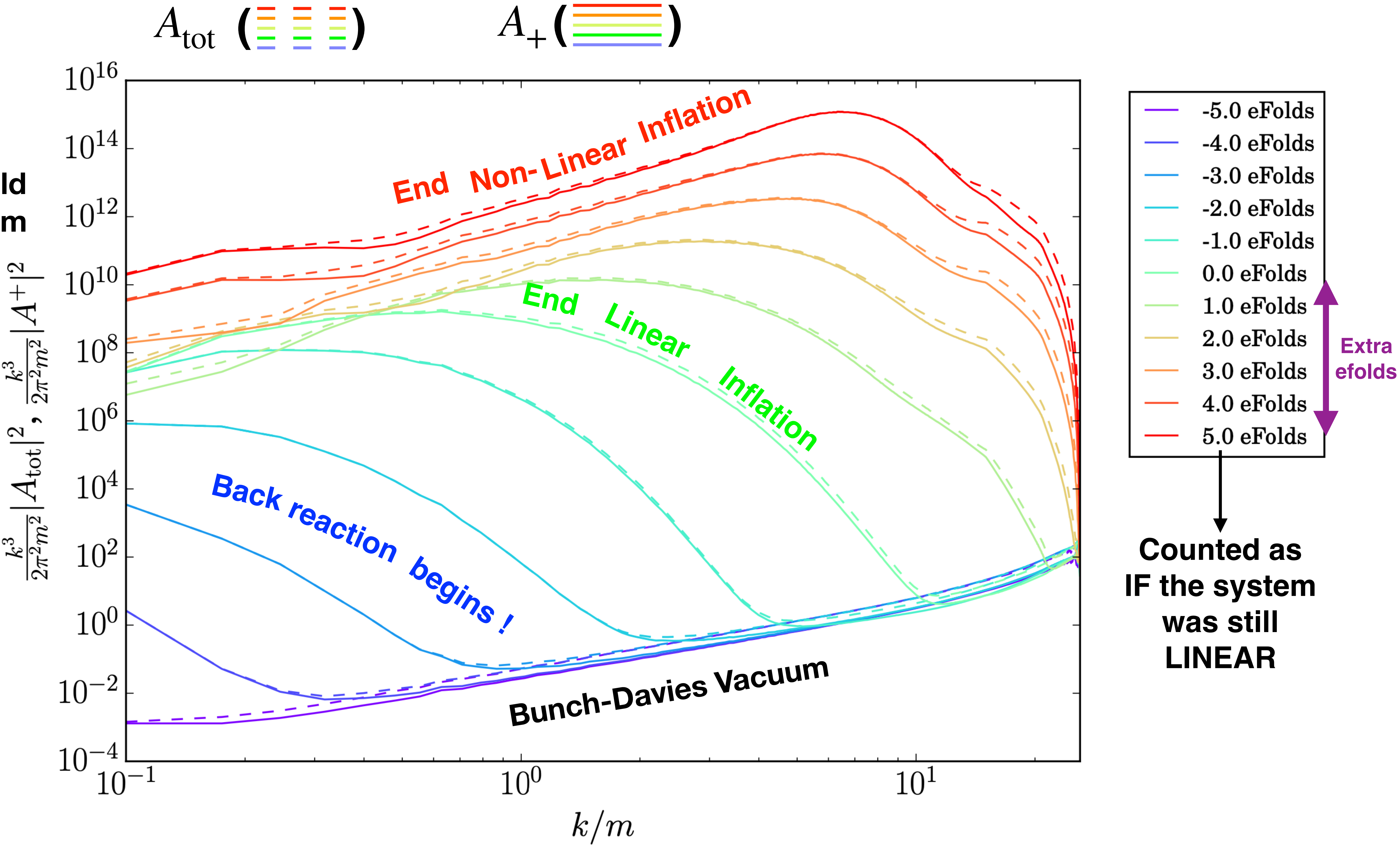
Gauge fld Spectrum



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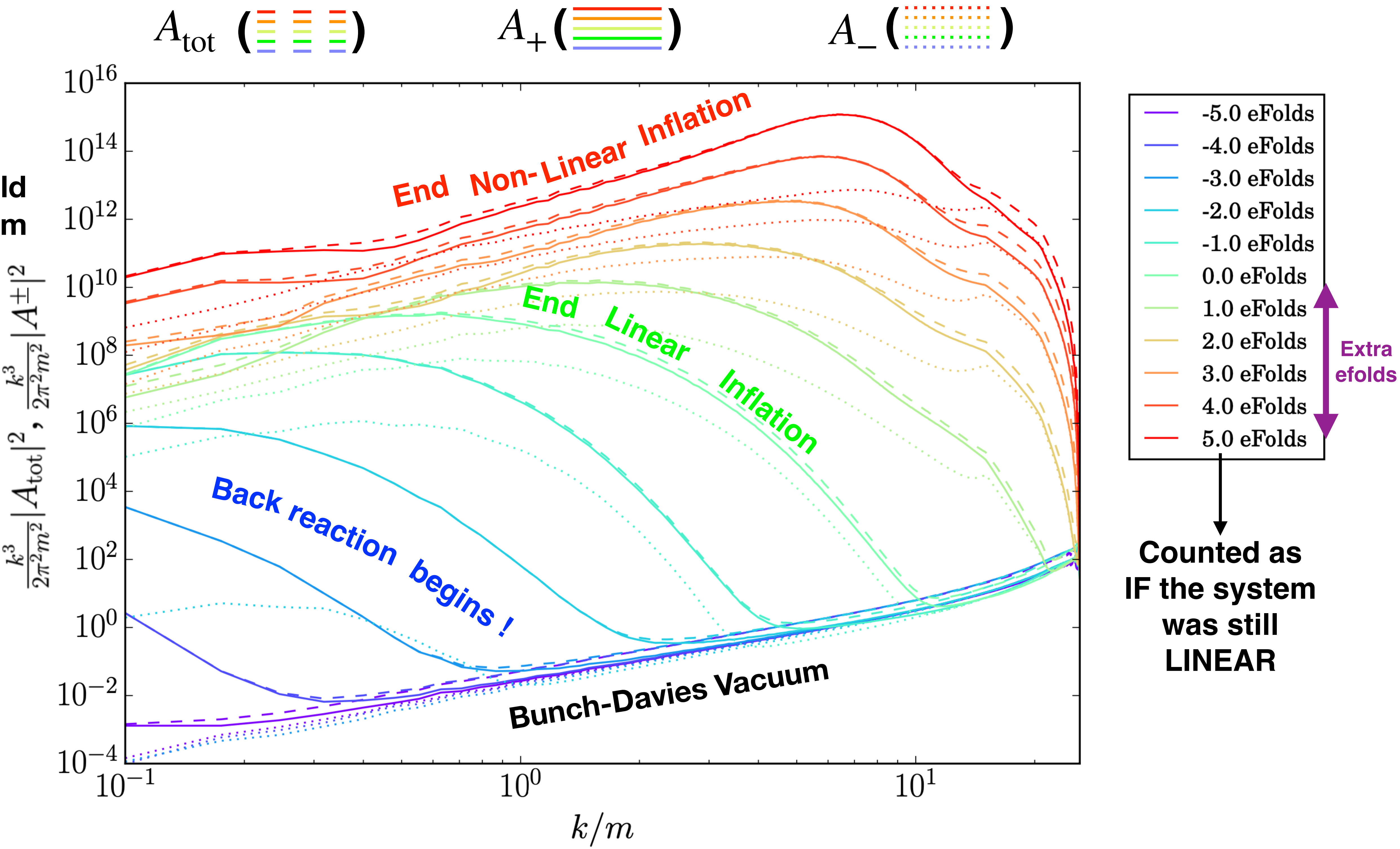

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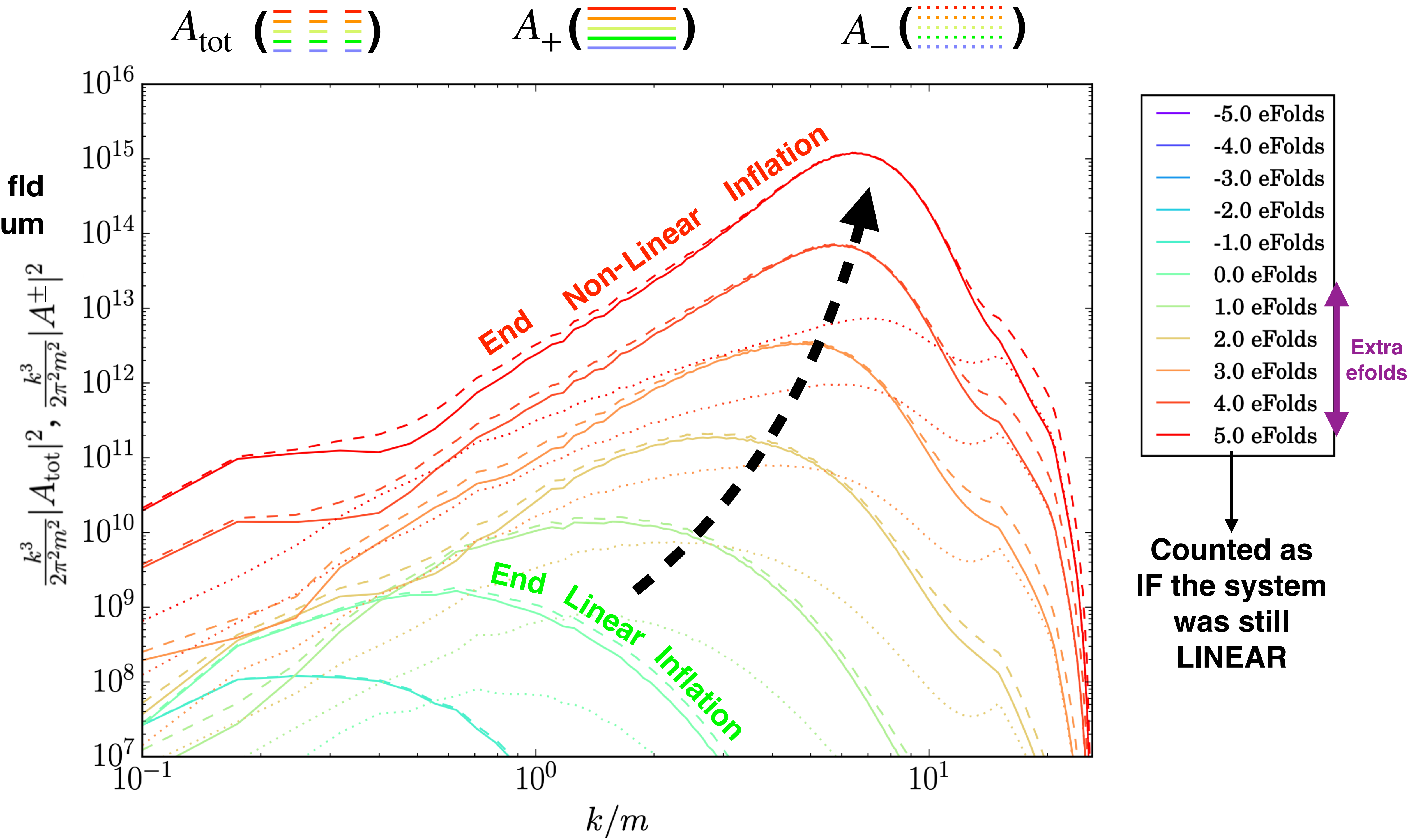

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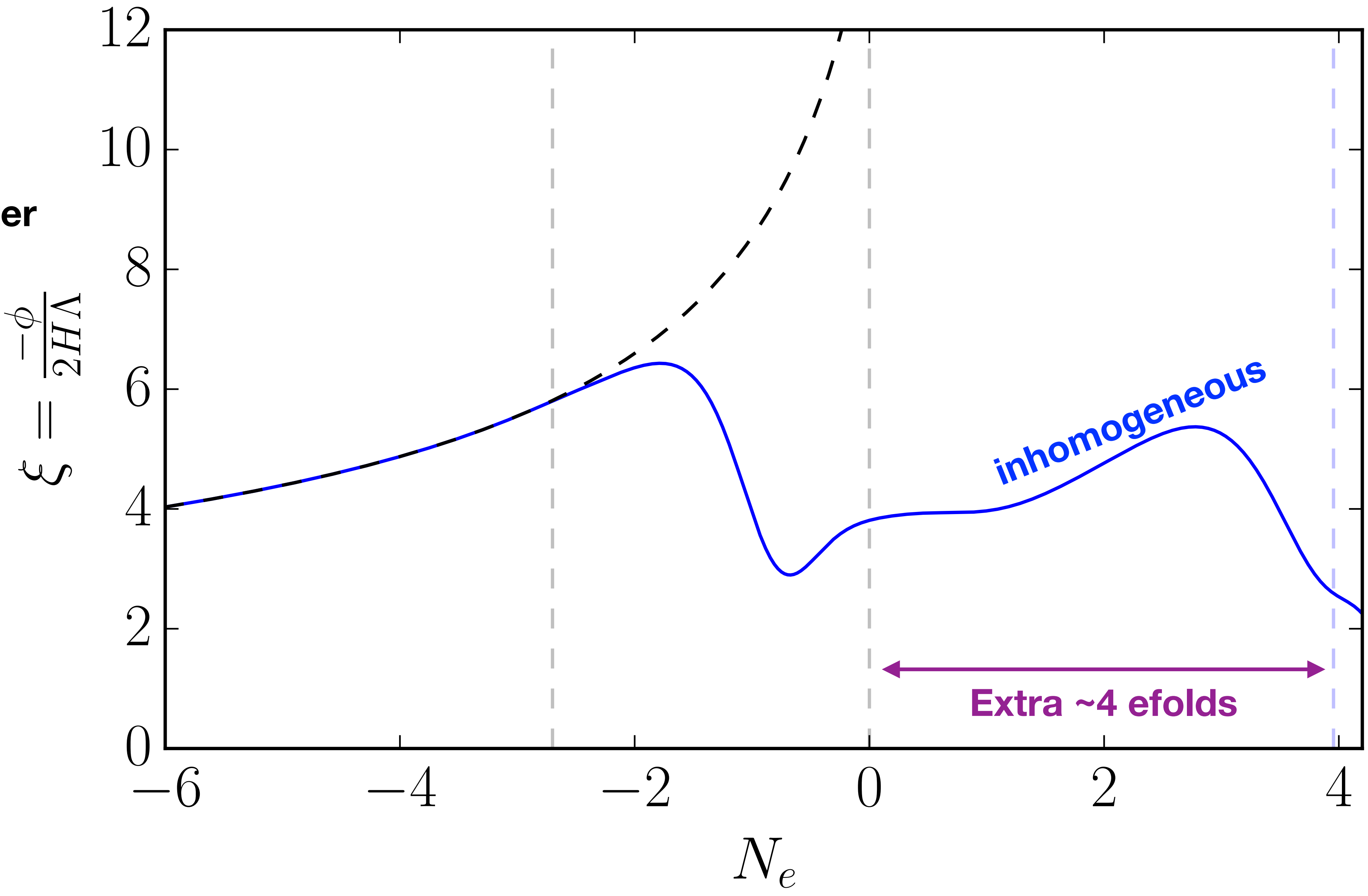
Linear (- - -) **Non-Linear (——)**

Xi parameter



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Linear (- - -) **Non-Linear (——)**

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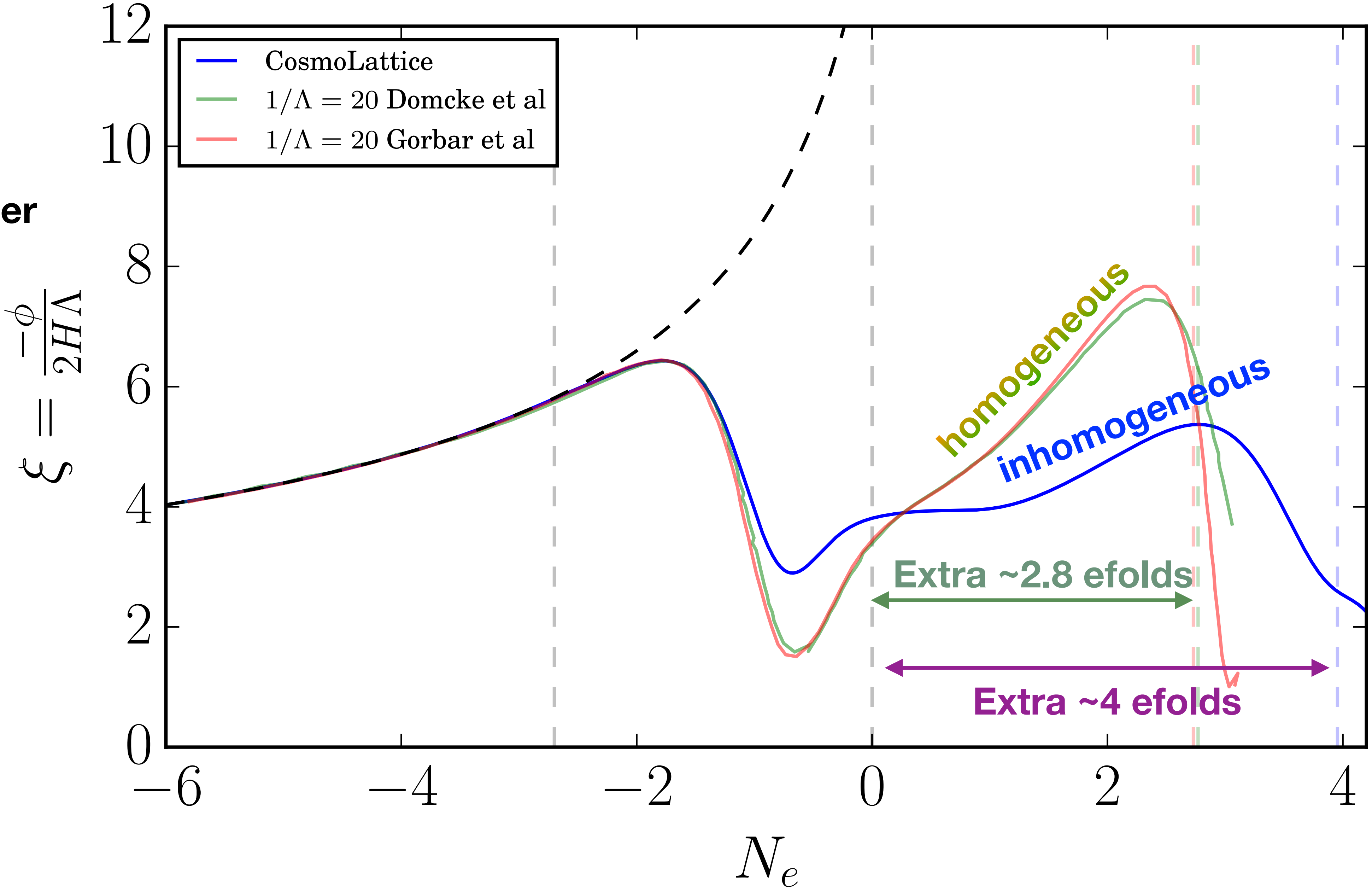
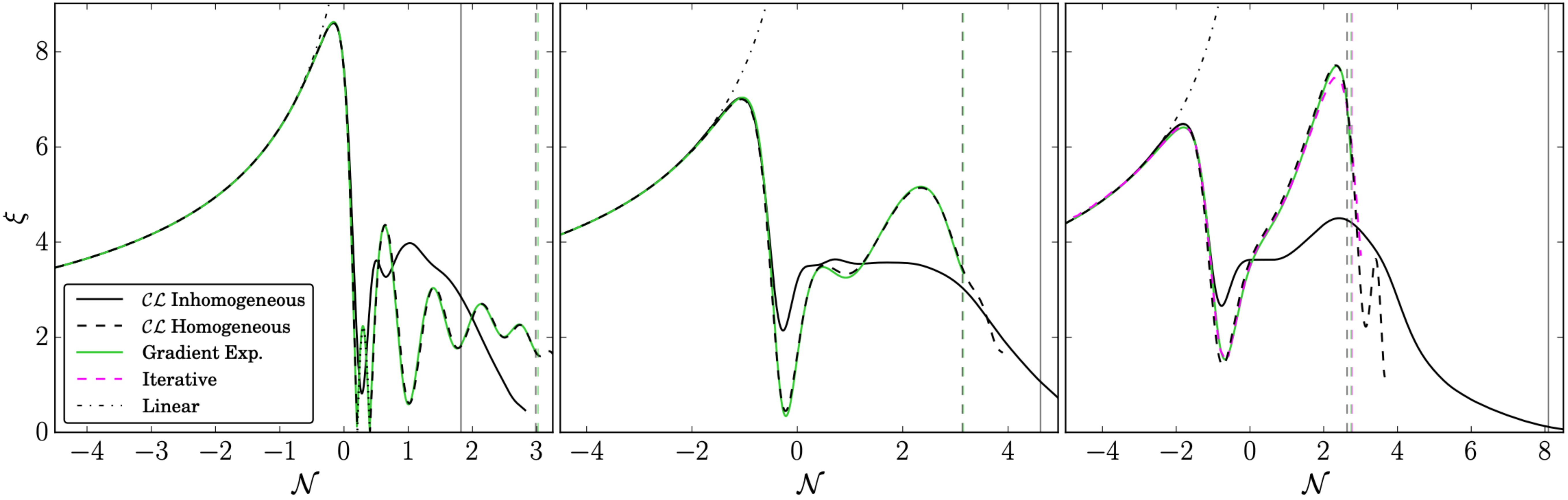


Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{\alpha}\right)$

$(\alpha = 15, 18, 20)$

$\alpha_\Lambda = 15$ $\alpha_\Lambda = 18$ $\alpha_\Lambda = 20$



$\Delta\mathcal{N}_{Hom} \simeq 3$ $\Delta\mathcal{N}_{Hom} \simeq 3$ $\Delta\mathcal{N}_{Hom} \simeq 3$

$\Delta\mathcal{N}_{InH} \simeq 1.8$ $\Delta\mathcal{N}_{InH} \simeq 4.5$ $\Delta\mathcal{N}_{InH} \simeq 8$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{X}$)

$(X = 15, 20, 25)$

Summary

* ξ Controls the Gauge field excitation

* Linear change in ξ : exponential response in A_μ

* Predictions/constraints (PNG, PBH and GWs) depend crucially on ξ : we will re-assess real observability !

* Adding Schwinger pair production easy via $\overrightarrow{J} = \sigma \overrightarrow{E}$

* Other phenomena: BAU, Magnetogenesis, …

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \Lambda = \frac{m_p}{X}\right)$

$(X = 15, 20, 25)$

Summary

Soon in the ArXiv !

- ξ Controls the Gauge field excitation

- Linear change in ξ : exponential response in A_μ

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- Adding Schwinger pair production easy via $\overrightarrow{J} = \sigma \overrightarrow{E}$

- Other phenomena: BAU, Magnetogenesis, …

Example II

Particle coupling reconstruction with gravitational waves

with

A. Florio, N. Loayza and M. Pieroni

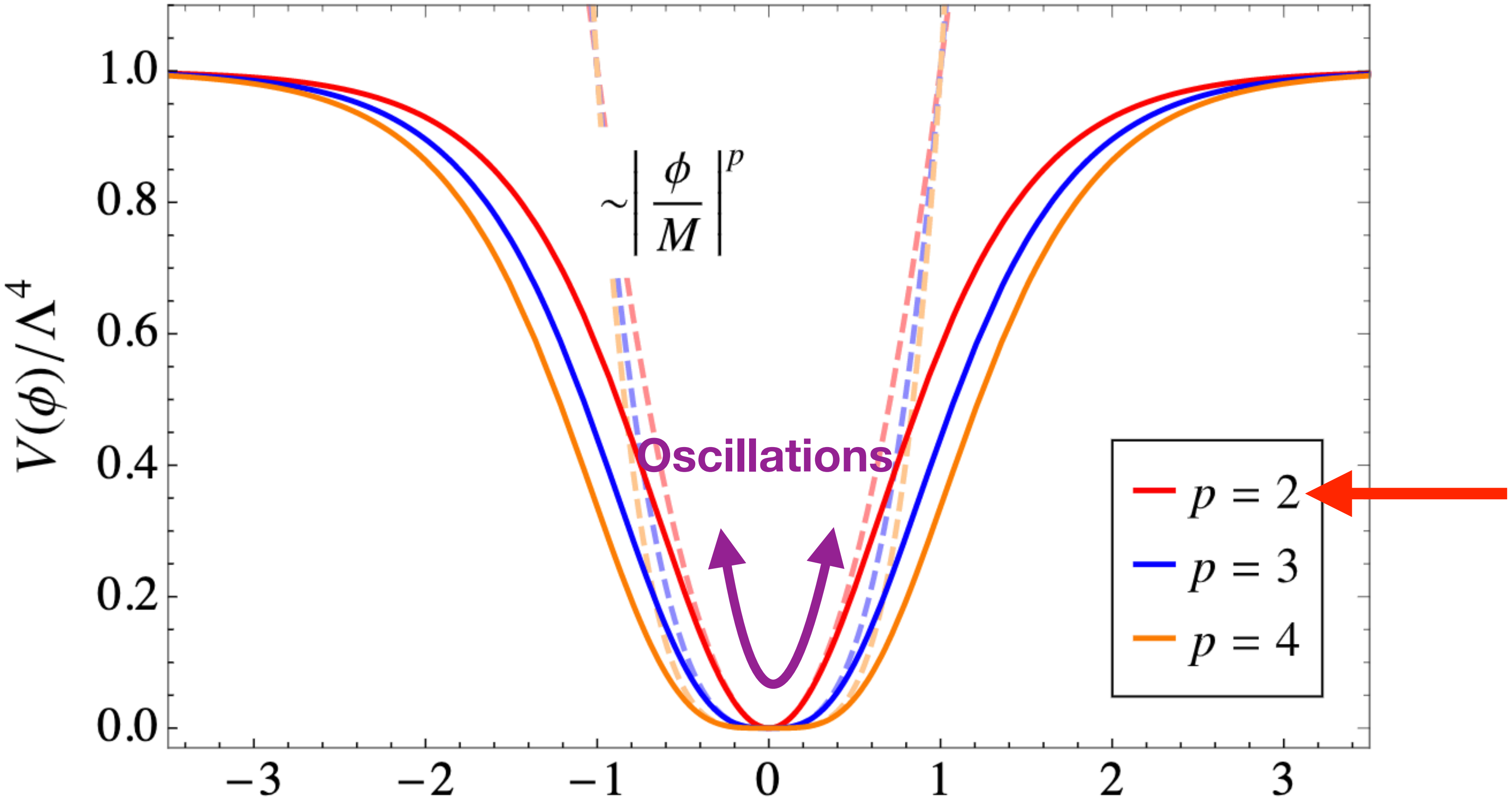
Phys. Rev. D 106 (2022) 6, 063522 ; [2202.05805]

Buchalter Cosmology 2022
— Third Prize —

SCALAR (P)REHEATING

$$V(\phi,\chi) = \frac{1}{2}\Lambda^4 \tanh\left(\frac{\phi}{M}\right)^2 + \frac{1}{2}g^2\chi^2\phi^2$$

(*e.g.* α – attractors, Kallosh, Linde 2013)

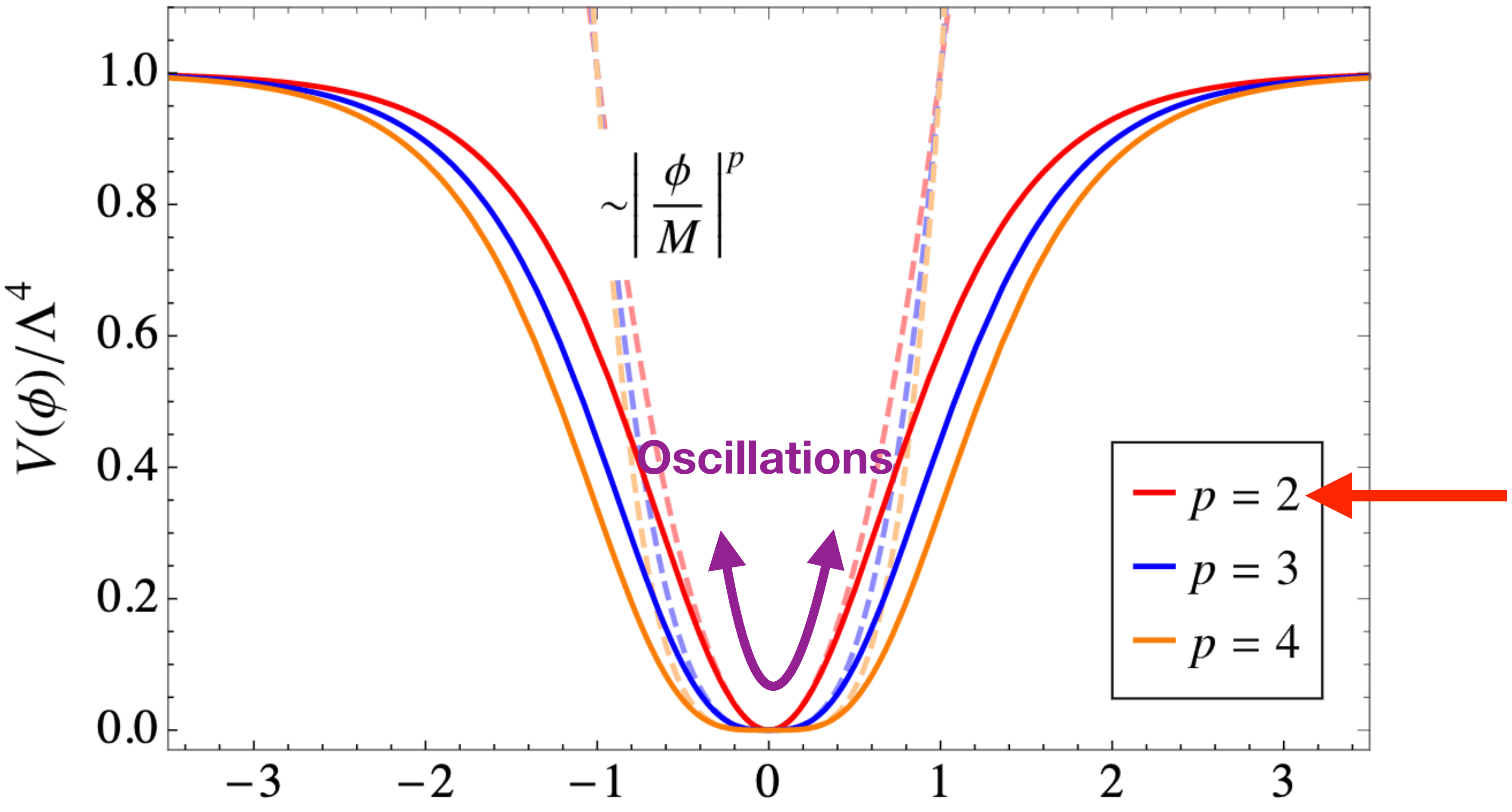


SCALAR (P)REHEATING

$$\boxed{V(\phi,\chi) = \frac{1}{2}\Lambda^4 \tanh\left(\frac{\phi}{M}\right)^2 + \frac{1}{2}g^2\chi^2\phi^2}$$

$$\chi_k'' + [\kappa^2 + m^2(\phi)]\chi_k = 0$$

(Daughter fld excitation)



SCALAR (P)REHEATING

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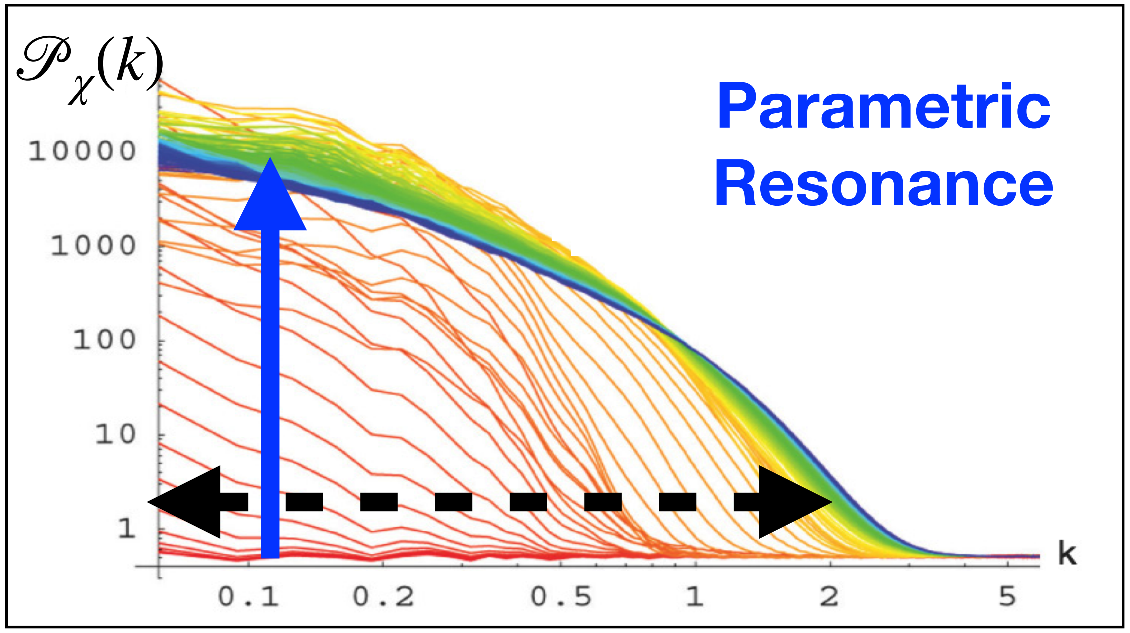
χ **Particle Production**



$$\mathscr{P}_\chi(k) = \frac{k^3}{2\pi^2}\langle|\chi_k|^2\rangle$$

SCALAR (P)REHEATING

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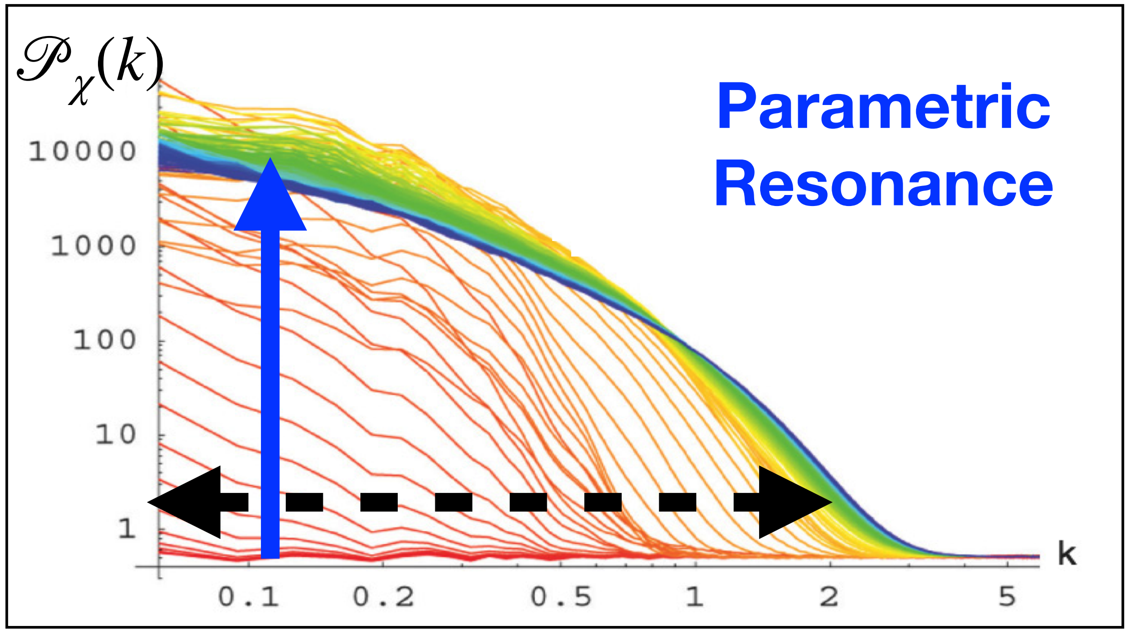
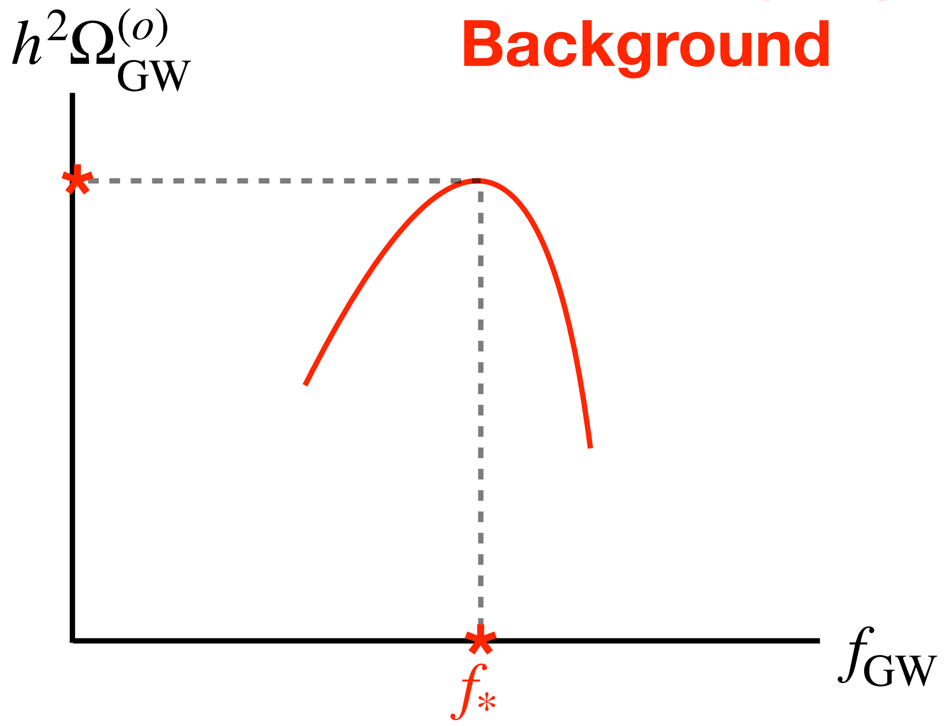
χ Particle Production



$$\mathscr{P}_\chi(k) = \frac{k^3}{2\pi^2}\langle|\chi_k|^2\rangle$$

Grav. Wave (GW) Background



Khlebnikov, Tkachev ’97
Easther, Giblin, Lim ’06-’08
DGF, G -Bellido, et al ’07-’10
Kofman, Dufaux et al ’07-’09

INFLATIONARY PREHEATING

Non - linear dynamics

↓

Lattice Simulations

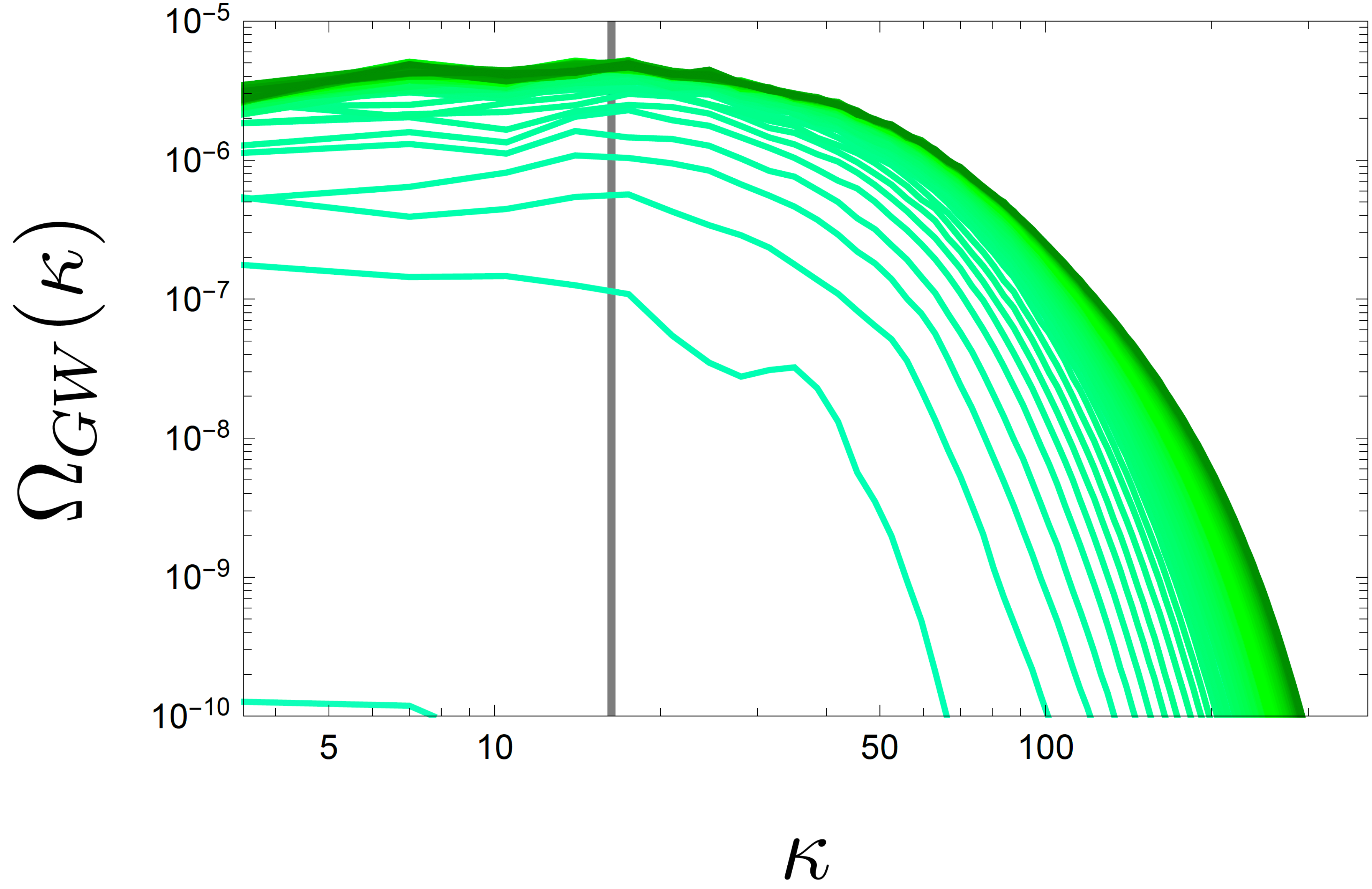
w/ Grav. Wave CL module

(Baeza-Ballesteros, DGF, Loayza)

GW Spectroscopy

Parameter Dependence (Peak amplitude**)**

$$V(\phi) + \frac{1}{2}g_1^2\phi^2\chi_1^2$$

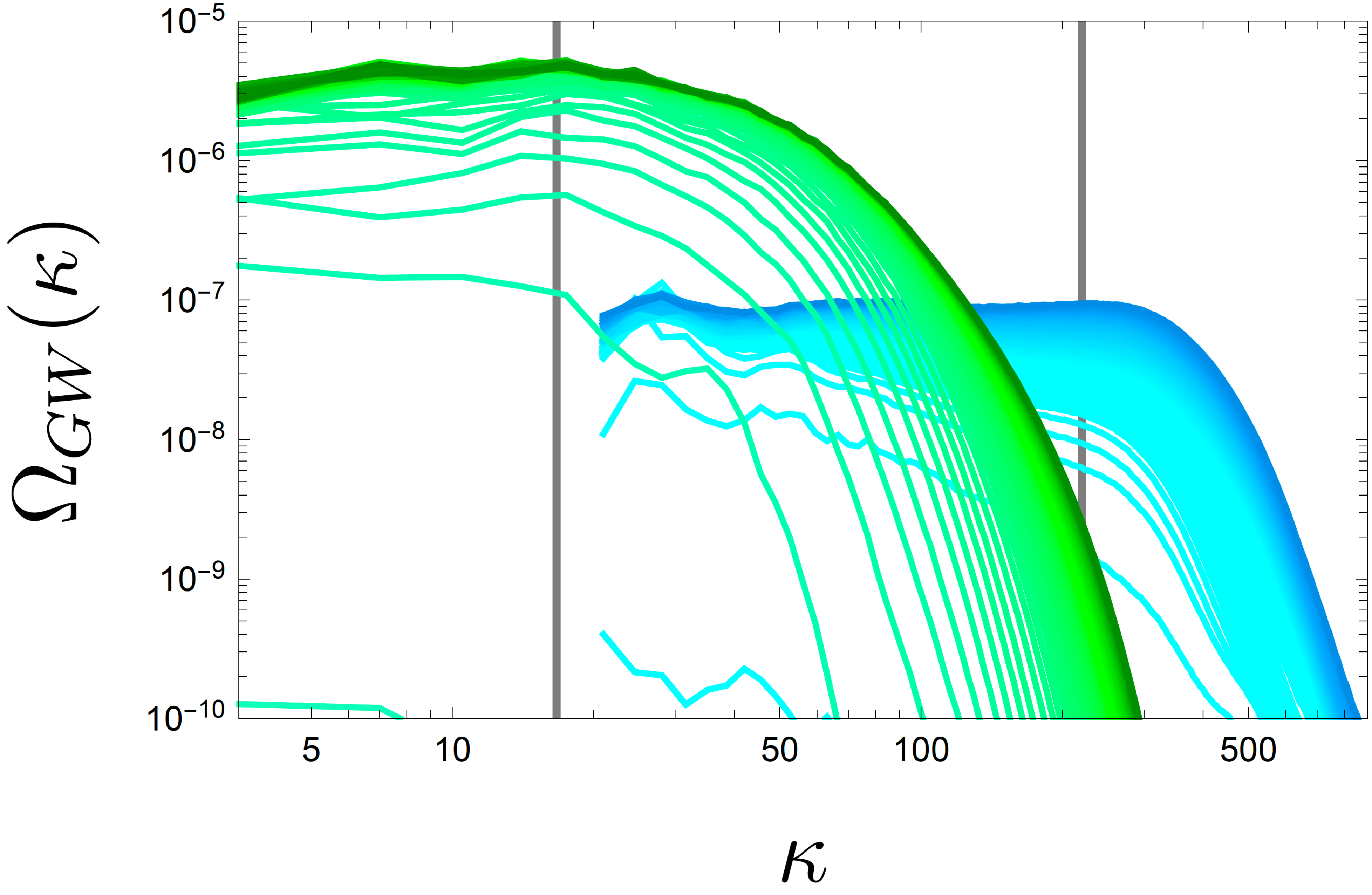


DGF, Florio, Loayza, Pieroni, 2202.05805

GW Spectroscopy

Parameter Dependence (Peak amplitude**)**

$$V(\phi) + \frac{1}{2}g_1^2\phi^2\chi_1^2 \quad ; \quad V(\phi) + \frac{1}{2}g_2^2\phi^2\chi_2^2$$

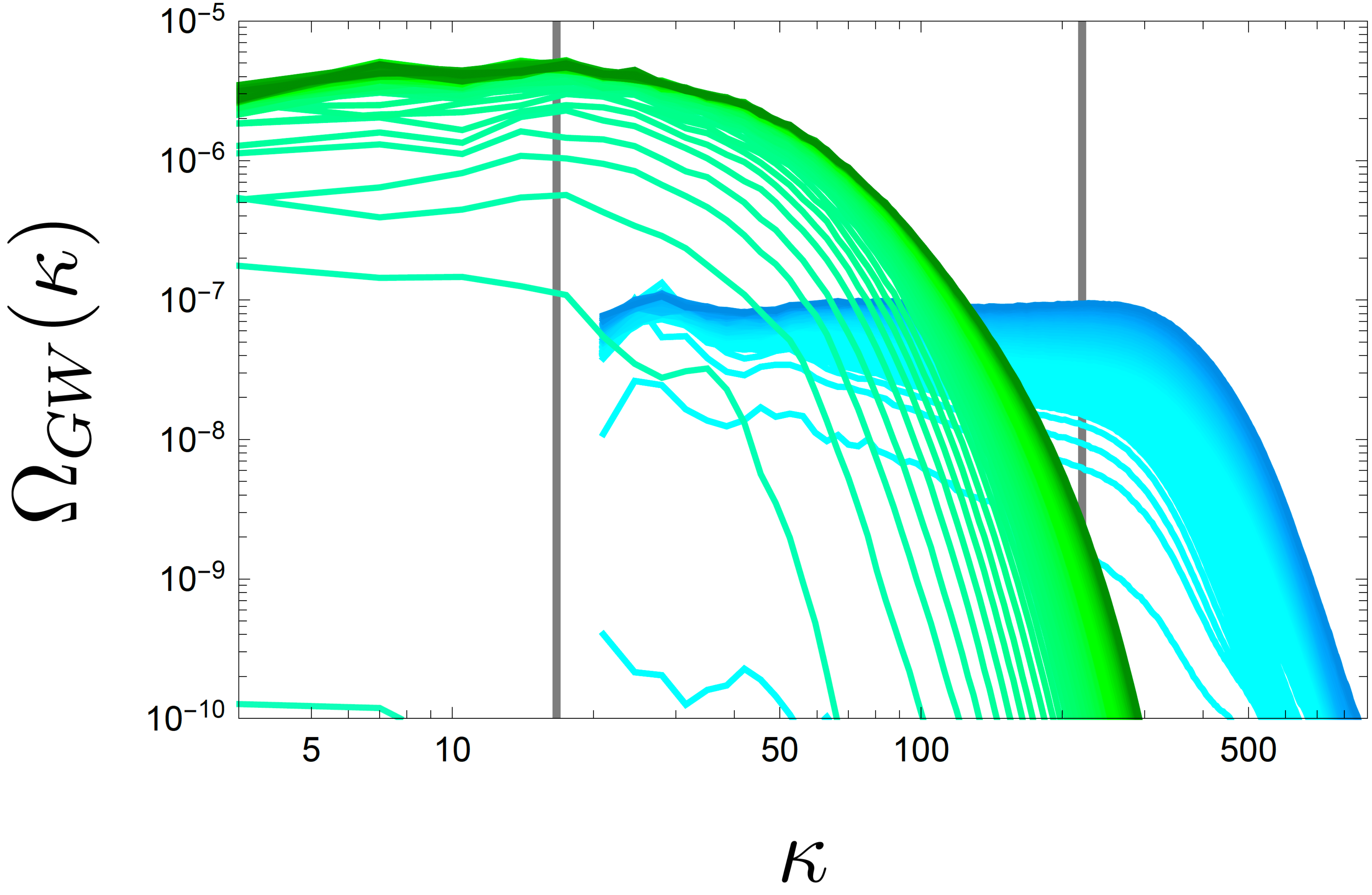


DGF, Florio, Loayza, Pieroni, 2202.05805

GW Spectroscopy

Parameter Dependence (Peak amplitude)

$$V(\phi) + \frac{1}{2}g_1^2\phi^2\chi_1^2 + \frac{1}{2}g_2^2\phi^2\chi_2^2 \quad ?$$



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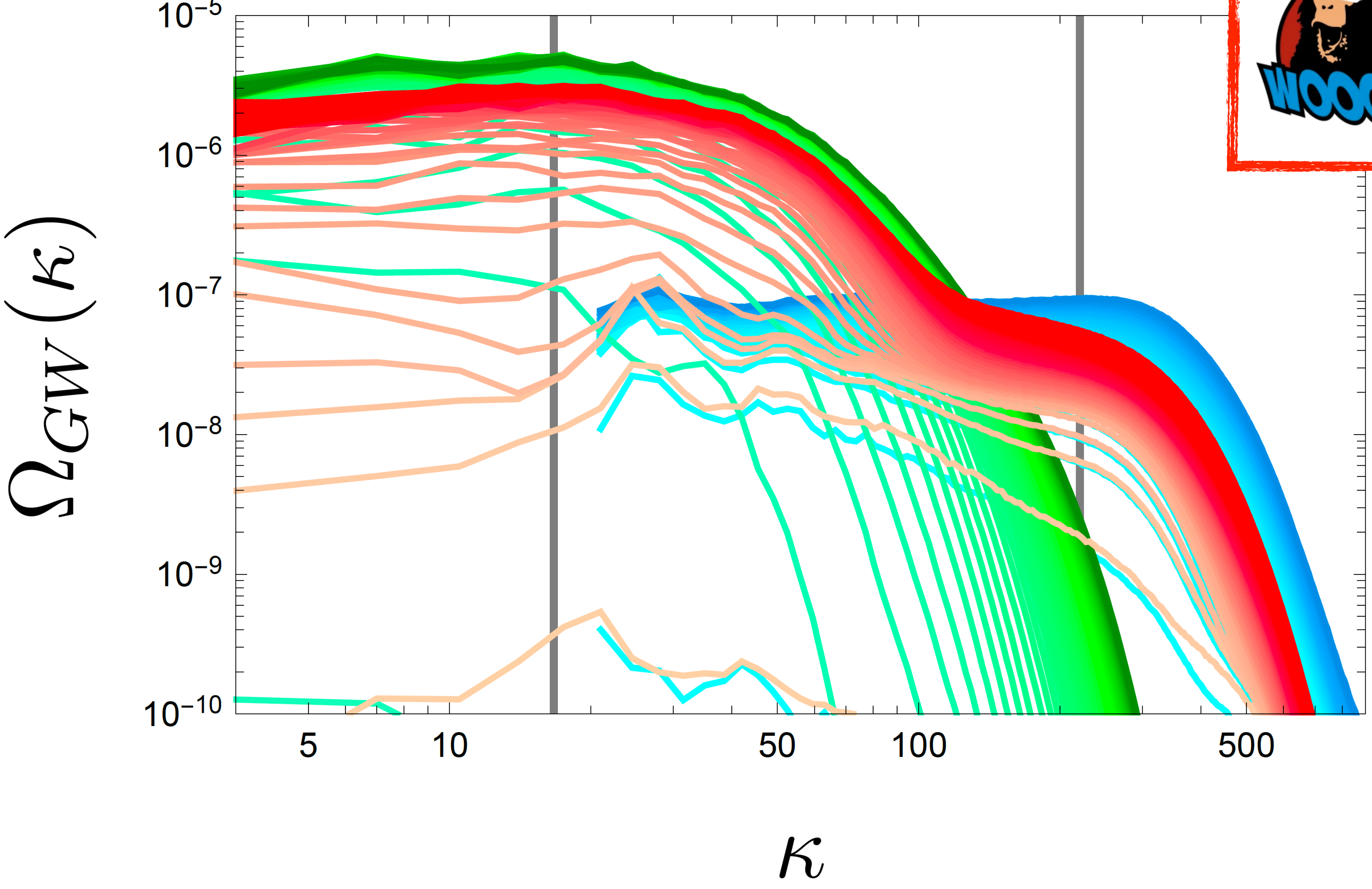

DGF, Florio, Loayza, Pieroni, 2202.05805

GW Spectroscopy

Parameter Dependence (Peak amplitude**)**

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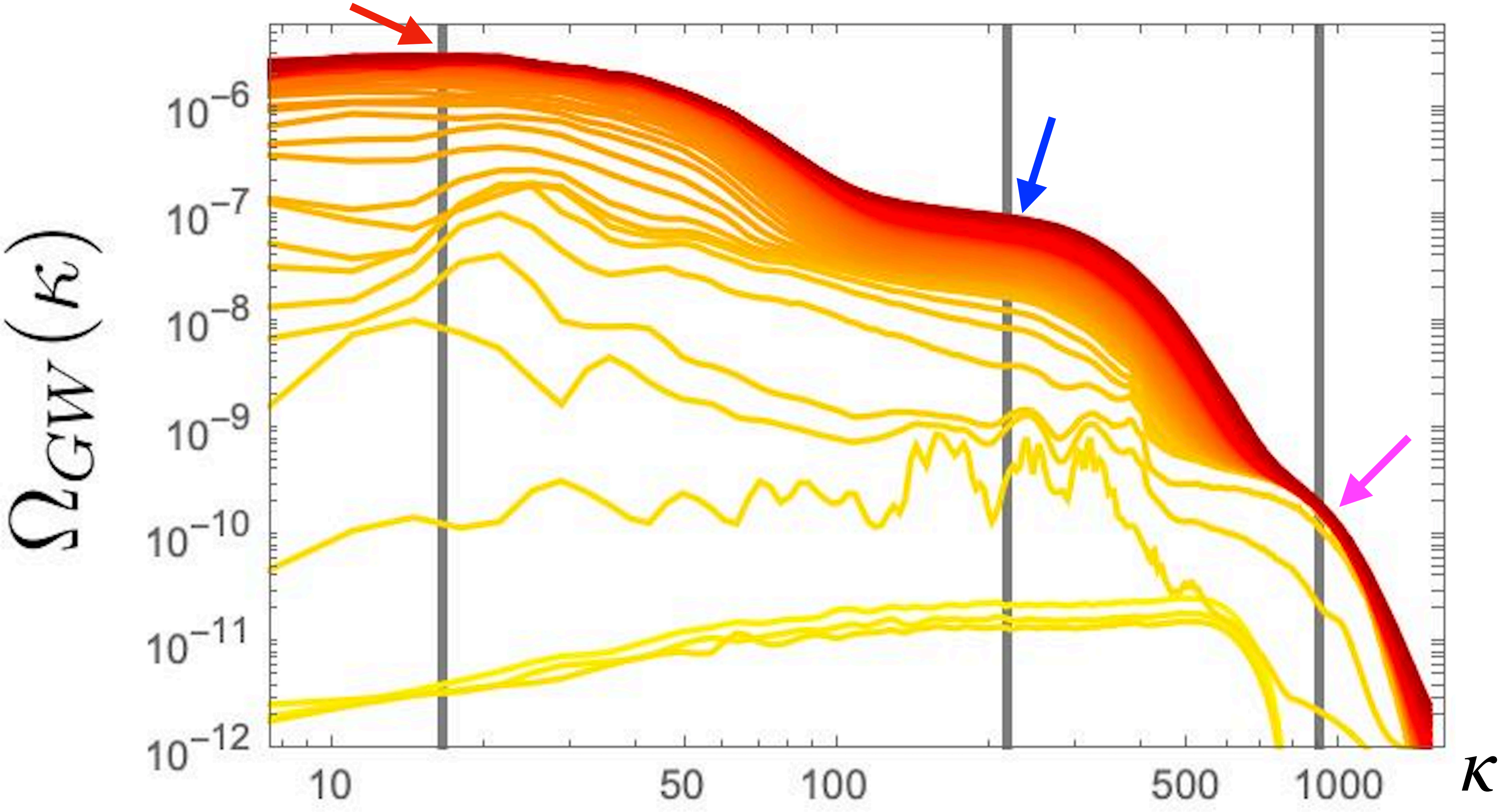
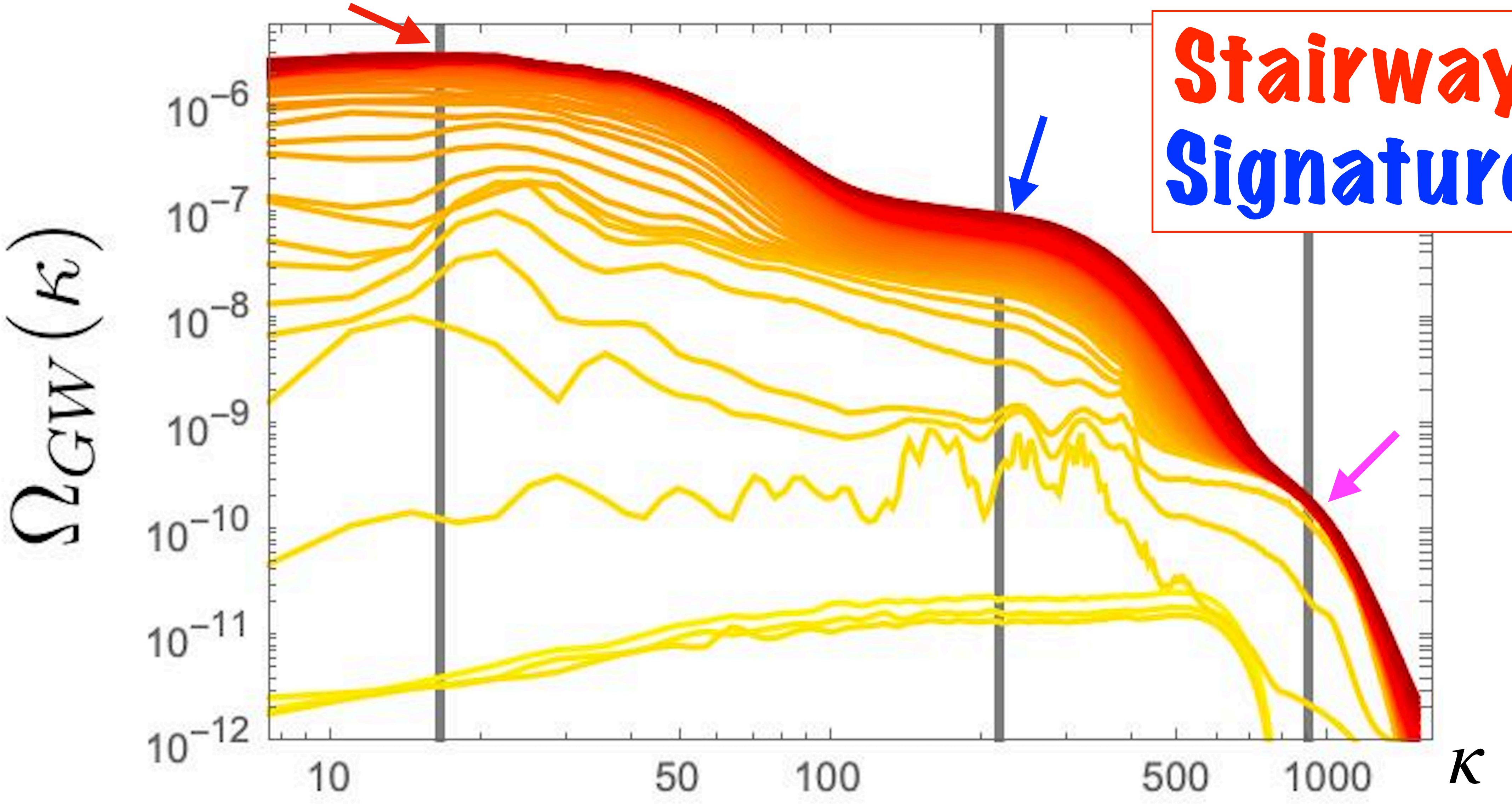


DGF, Florio, Loayza, Pieroni, 2202.05805

GW Spectroscopy

Parameter Dependence (Peak amplitude**)**

$$V(\phi) + \frac{1}{2}\mathbf{g}_1^2\phi^2\chi_1^2 + \frac{1}{2}\mathbf{g}_2^2\phi^2\chi_2^2 + \frac{1}{2}\mathbf{g}_3^2\phi^2\chi_3^2$$



DGF, Florio, Loayza, Pieroni, 2202.05805

GW Spectroscopy

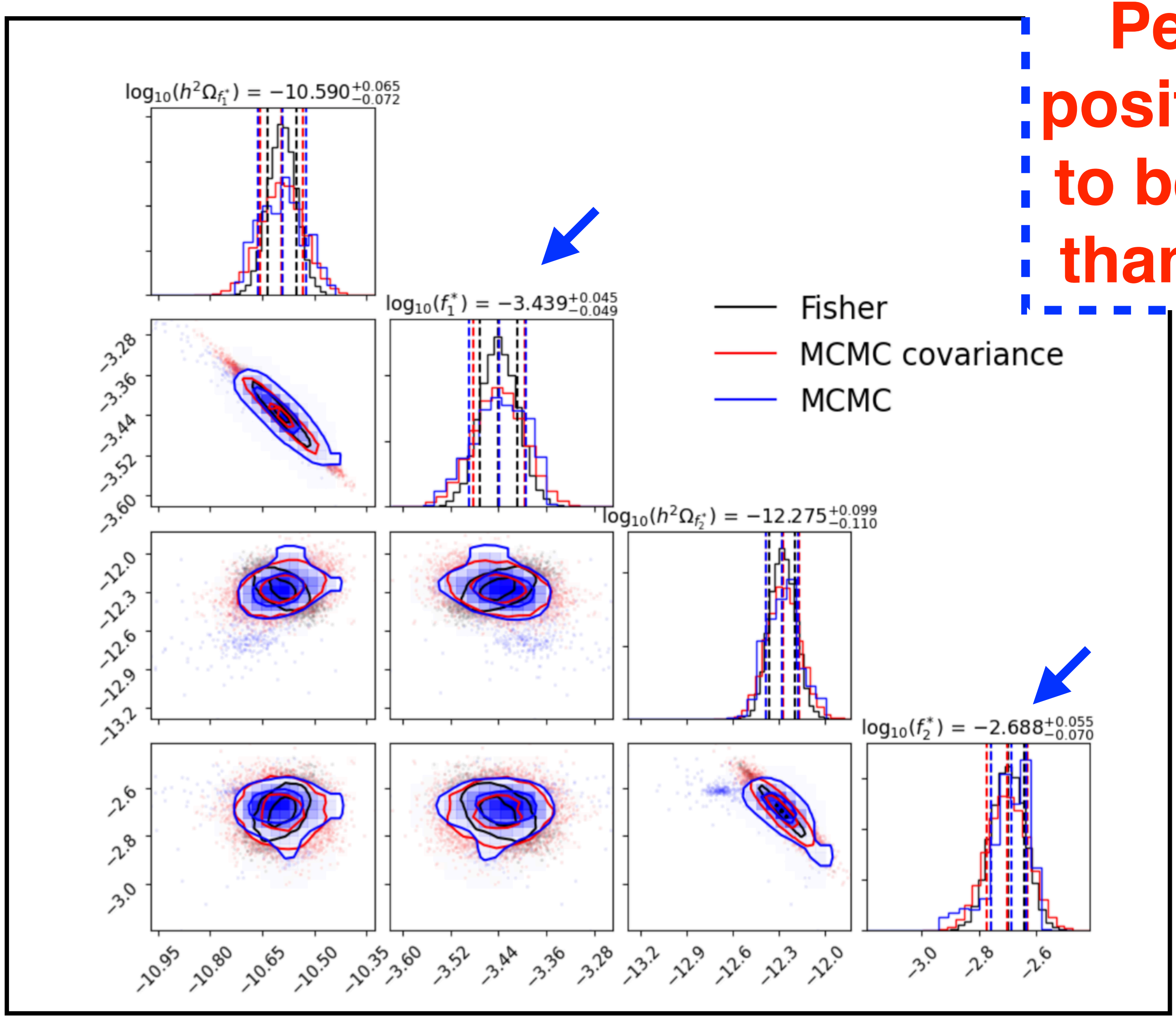
Reconstruction (2-peak signal)

@ LISA

GW Spectroscopy

Reconstruction (2-peak signal)

@ LISA



Peak positions to better than 1%

Phys.Rev.D 106 (2022) 6, 063522, **2202.05805**

GW Spectroscopy

Reconstruction (2-peak signal)

@ LISA

Coupling Reconstruction !

Theoretical	LISA	BBO
$g_1 = 1.16 \cdot 10^{-3}$	$1.66^{+4.01}_{-0.55} \cdot 10^{-3}$	$0.76^{+0.73}_{-0.18} \cdot 10^{-3}$
$g_2 = 8.2 \cdot 10^{-3}$	$4.39^{+14.3}_{-1.53} \cdot 10^{-3}$	$7.64^{+21.8}_{-2.61} \cdot 10^{-3}$

Phys.Rev.D 106 (2022) 6, 063522, **2202.05805**

GW Spectroscopy

Our example serves as proof of principle !

Possible new door to particle physics interactions with GW backgrounds !

GW Spectroscopy

Our example serves as proof of principle !

Possible new door to particle physics interactions with GW backgrounds !

Multi-peak Stairway signatures expected at:

low scale (p)reheating

phase transitions

.....

High-Freq GW Detection ?

Example III

String Loop Dynamics + Grav. Wave emission

with *J. Baeza-Ballesteros, E. Copeland, J. Lizarraga*

(PhD student)

String Loop Dynamics + GW emission

GOAL

Dynamics of an isolated loop and its GW emission

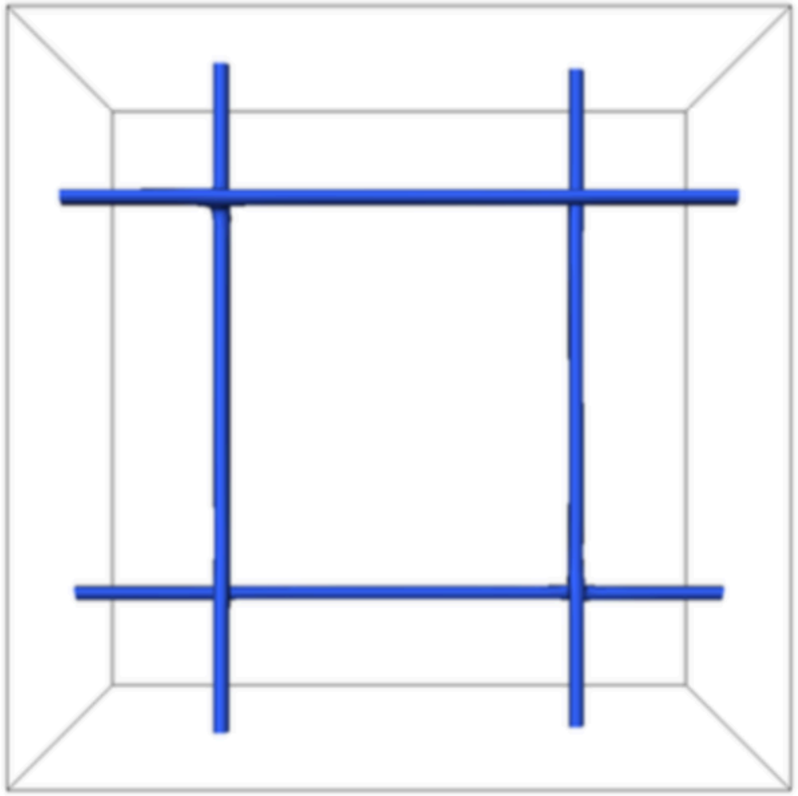
String Loop Dynamics + GW emission

GOAL

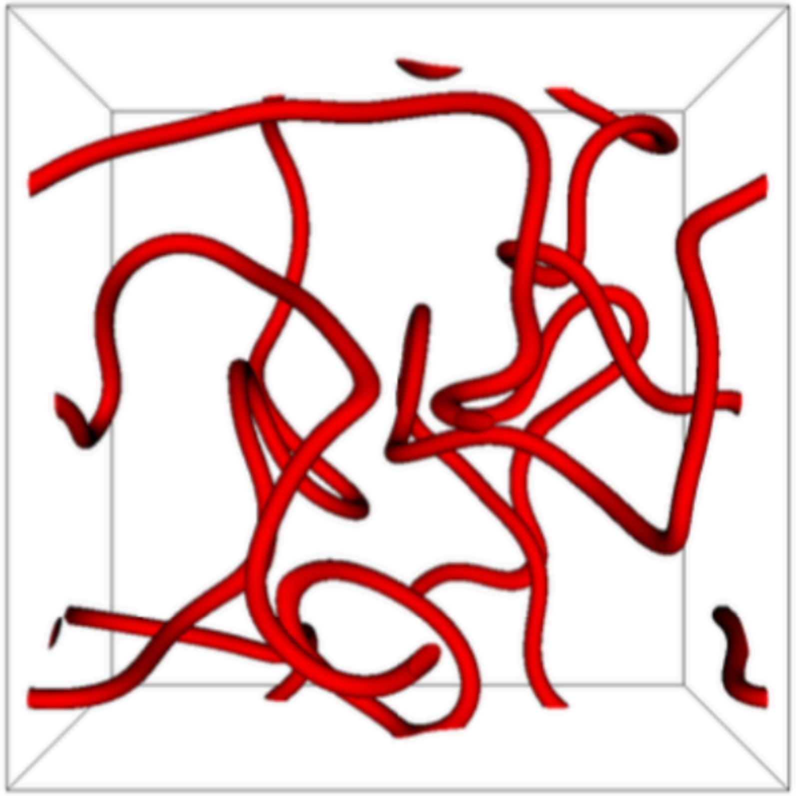
Dynamics of an isolated loop and its GW emission

Case I : Nielsen-Olesen

Case II : Network



(following Vachaspati et al 2020)



(following Lizarraga et al 2020/21)

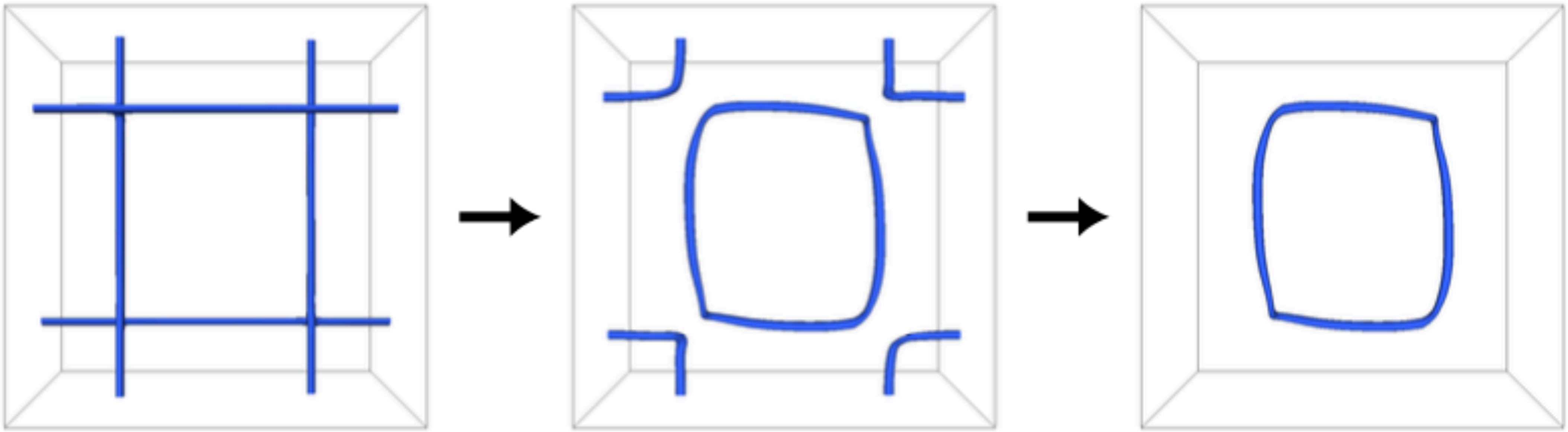
String Loop Dynamics + GW emission

GOAL

Dynamics of an isolated loop and its GW emission

Case I

— Isolate the inner loop —



String Loop Dynamics + GW emission

GOAL

Dynamics of an isolated loop and its GW emission

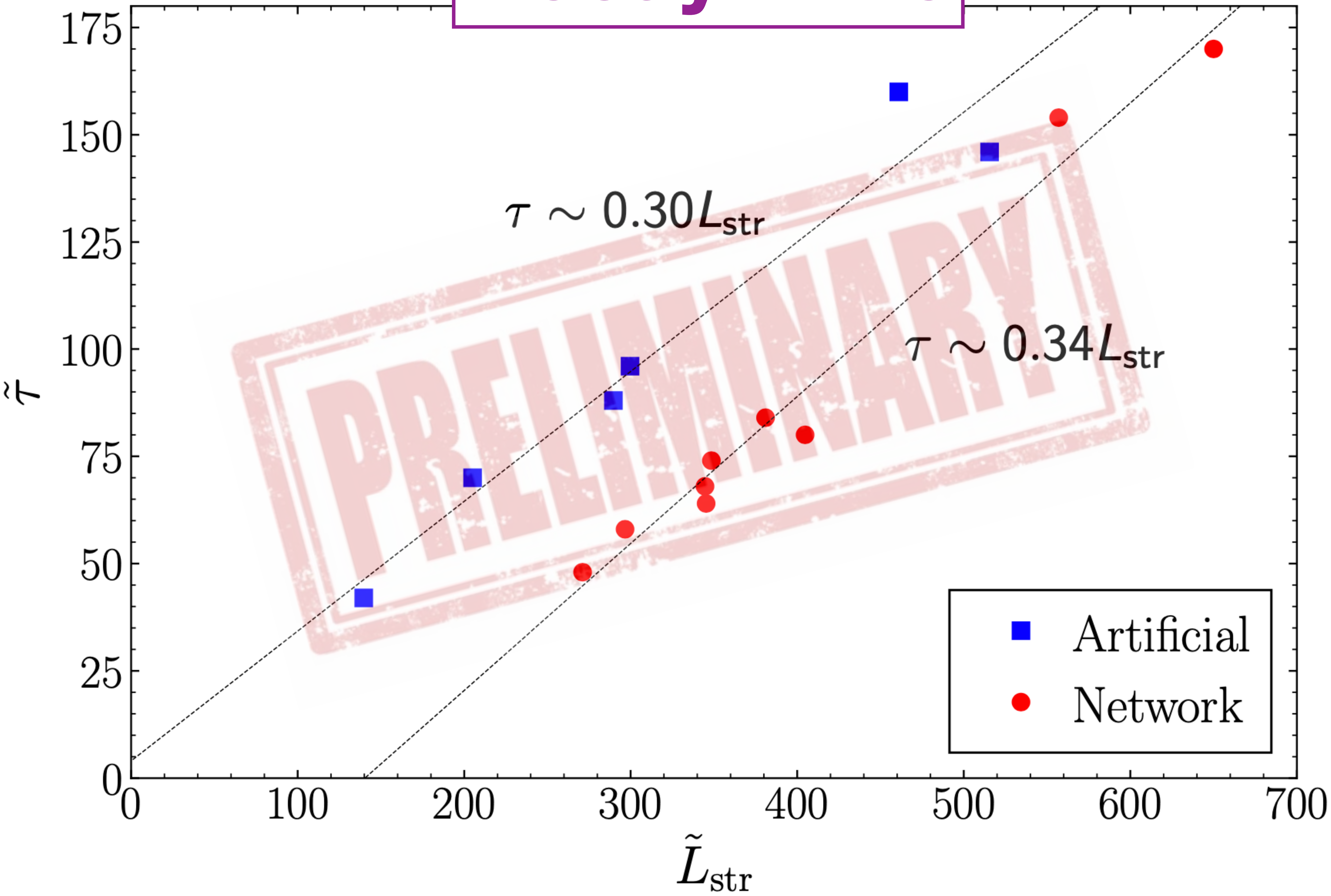
Case II

— Only one loop remains eventually —

String Loop Dynamics + GW emission

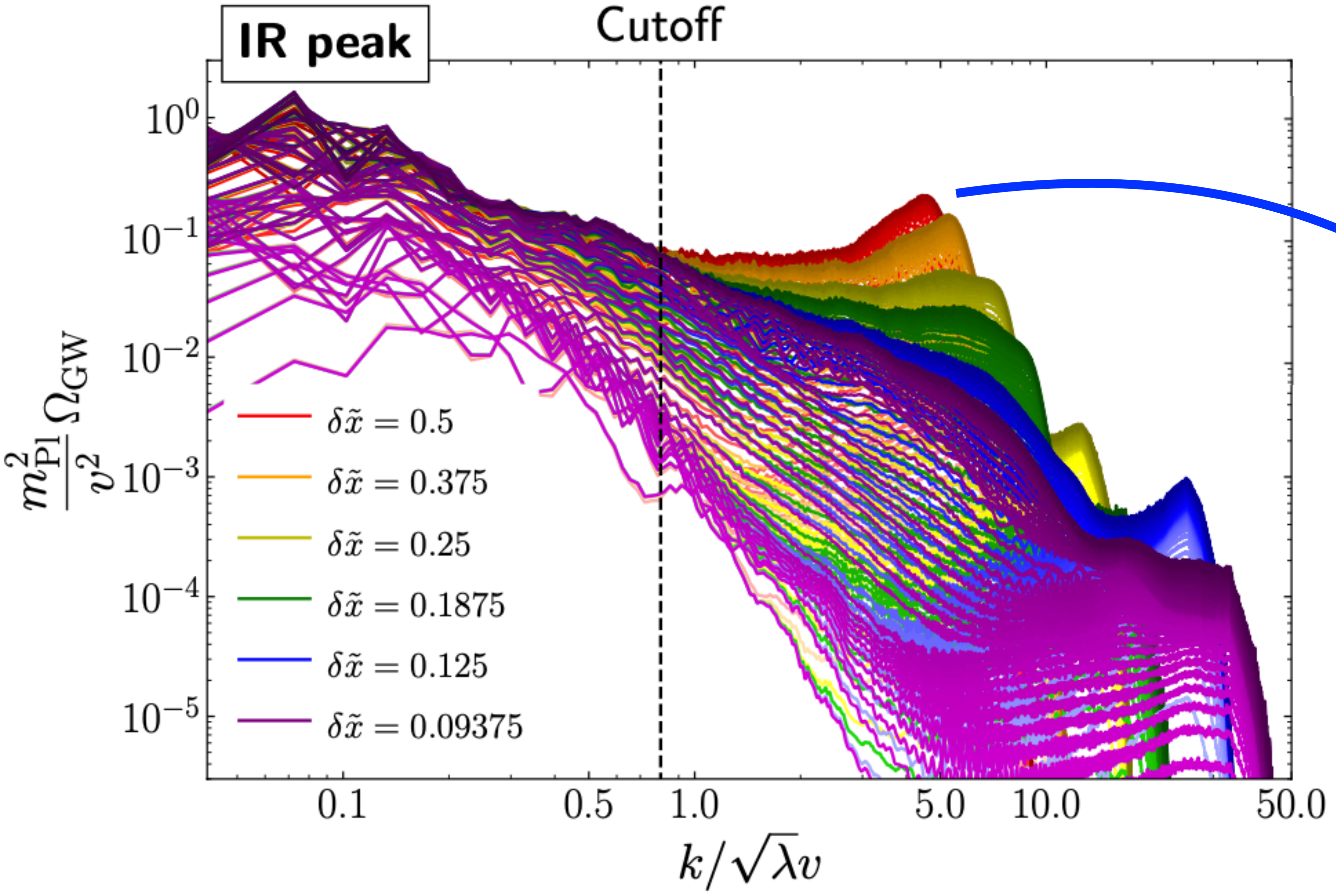
$$\boxed{\tilde{x}^\mu = \sqrt{\lambda}\, v\, x^\mu}$$

Decay Time



String Loop Dynamics + GW emission

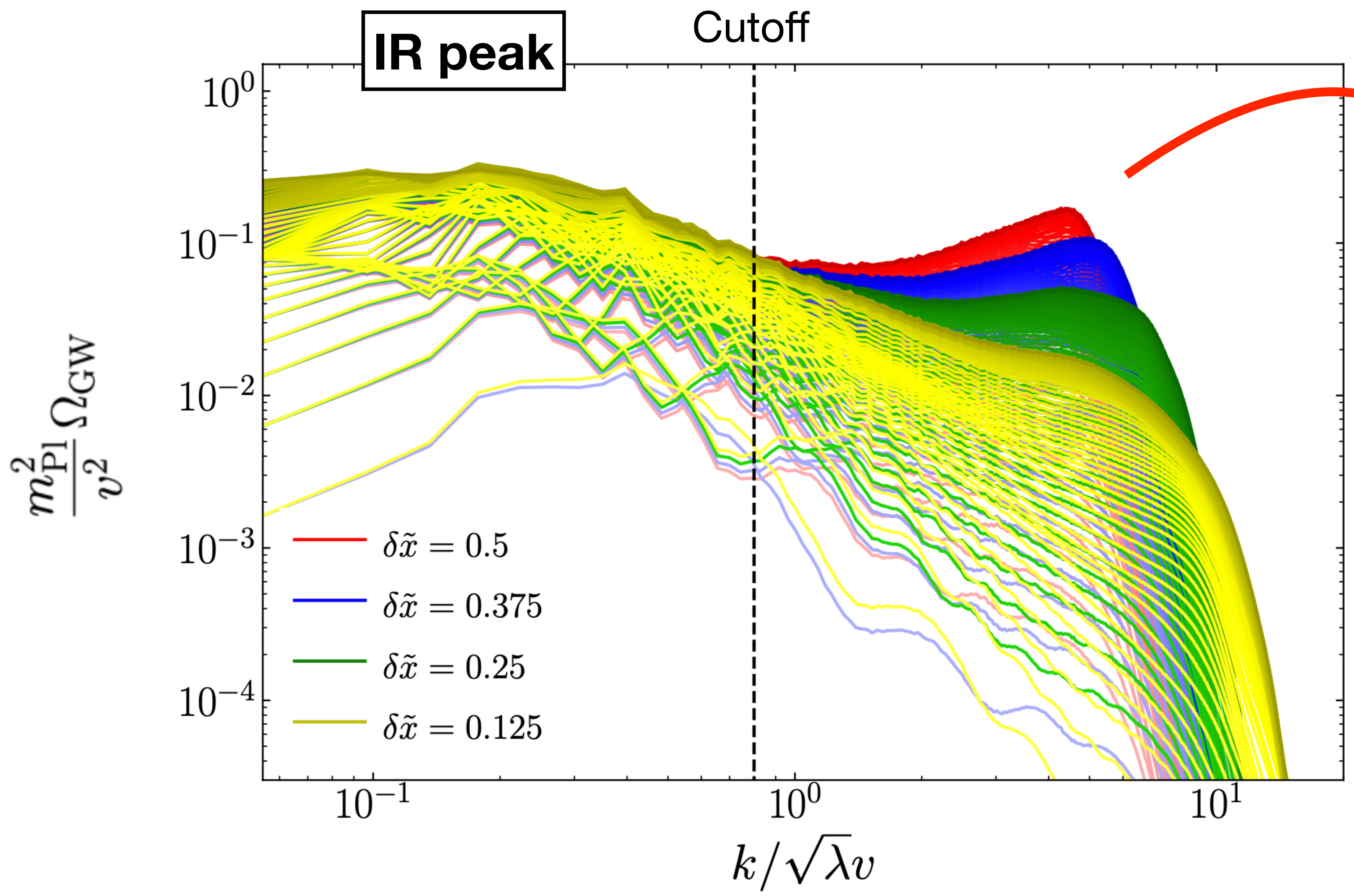
GW energy density power spectrum (Case I)



UV artifact !

String Loop Dynamics + GW emission

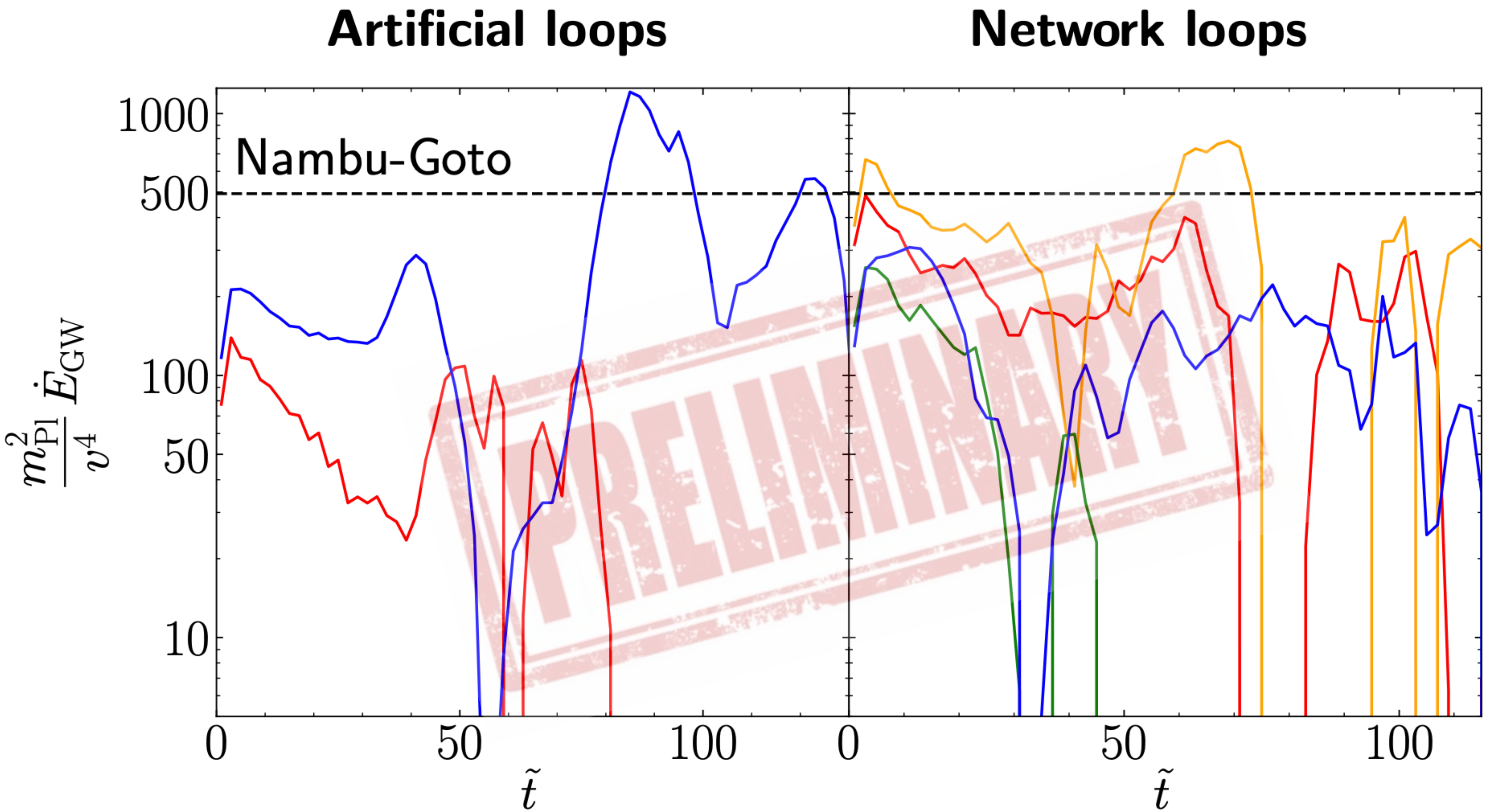
GW energy density power spectrum (Case II)



UV artifact !

String Loop Dynamics + GW emission

GW Power Emitted



String Loop Dynamics + GW emission

Impact of our Study

Real evaluation of GW emission

Re-evaluation of PTA constraints

(Pulsar Time Array)

String Loop Dynamics + GW emission

Impact of our Study

Real evaluation of GW emission

Re-evaluation of PTA constraints

(Pulsar Time Array)

Implications for

Dark Matter Axion string network

Local (Abelian-Higgs) string network

Comparison with Nambu-Goto

GUT models

….

String Loop Dynamics + GW emission

Soon in the ArXiv !

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….

If you want to know more …

If you want to learn how to
"latticesize" your problems ...

CosmoLattice

1st CL School 2022: Sept 5-8

@Valencia:

If you want to learn how to
"latticesize" your problems ...

CosmoLattice

2nd CL School 2023: Sept 25-29

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If you want to learn how to
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2nd CL School 2023: Sept 25-29

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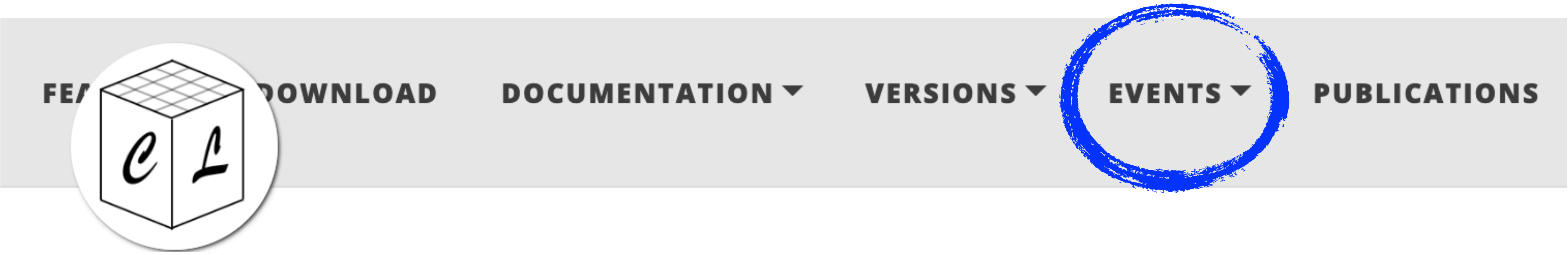

If you want to learn how to "latticesize" your problems …

CosmoLattice

2nd CL School 2023: Sept 25-29

Thanks for your attention !

Back
Slides

CosmoLattice

What Field theory ?

- **Matter content:**

$$\boxed{S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + (D_\mu^A\varphi)^*(D^\mu_A\varphi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{1}{2}\mathrm{Tr}\{G_{\mu\nu}G^{\mu\nu}\} + V(\phi,|\varphi|,|\Phi|)\right\}}$$

End

What Field theory ?

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$$\boxed{\phi \in \mathscr{R}e}$$

Scalar sector

$$\boxed{\begin{aligned} \varphi &\equiv \frac{1}{\sqrt{2}}(\varphi_0 + i\varphi_1) \\ D^{\mathrm{A}}_\mu &\equiv \partial_\mu - iQ^{(\varphi)}_A g_{_A} A_\mu \\ F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu \end{aligned}}$$

U(1) gauge sector

$$\boxed{\begin{aligned} \Phi &= \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_0 + i\varphi_1 \\ \varphi_2 + i\varphi_3\end{pmatrix} \\ D_\mu &\equiv \mathscr{I}D^{\mathrm{A}}_\mu - ig_BQ_BB^a_\mu\,T_a \\ G_{\mu\nu} &\equiv \partial_\mu B_\nu - \partial_\nu B_\mu - i[B_\mu, B_\nu] \end{aligned}}$$

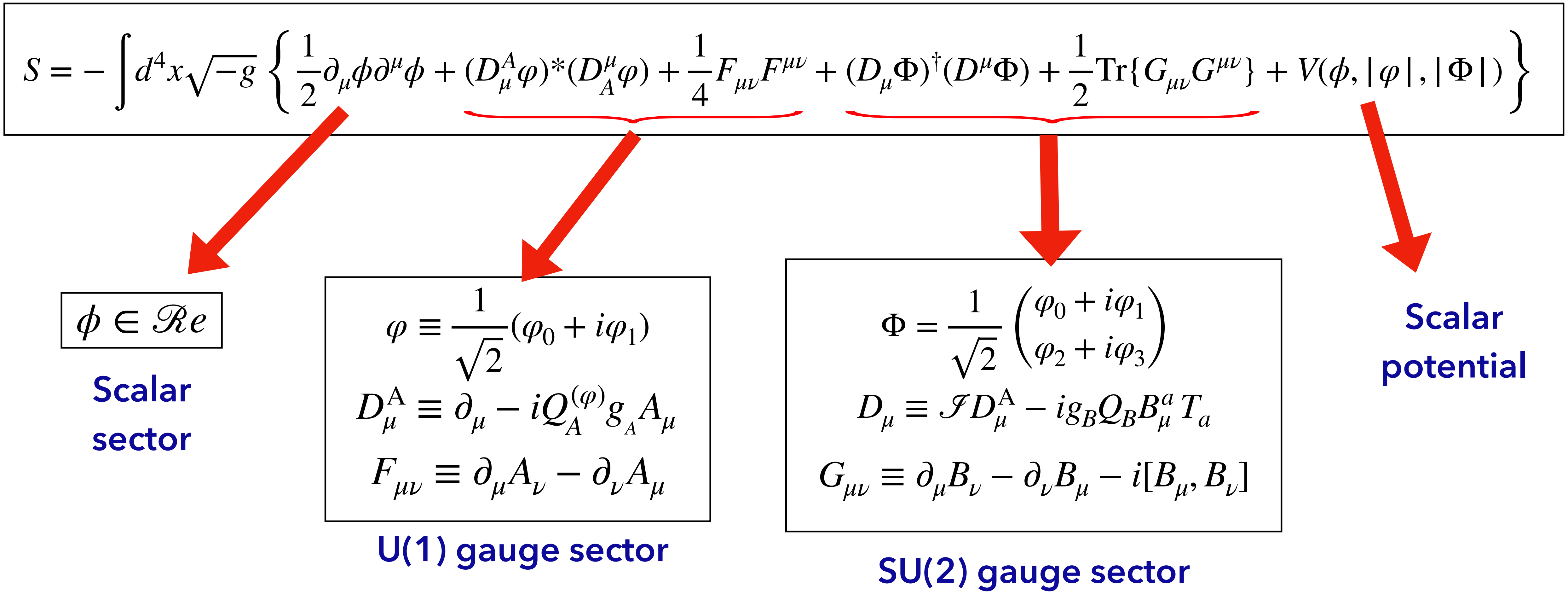
SU(2) gauge sector

End

Transparencies worked out together with Paco Torrentí

What Field theory ?

- **Matter content:**



$$S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \underbrace{(D_\mu^{\mathrm{A}}\varphi)^*(D_{\mathrm{A}}^\mu\varphi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}} + \underbrace{(D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{1}{2}\mathrm{Tr}\{G_{\mu\nu}G^{\mu\nu}\}} + V(\phi,|\varphi|,|\Phi|)\right\}$$

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SU(2) gauge sector

Scalar potential

End

What Field theory ?

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$$\boxed{\phi \in \mathscr{Re}}$$

Scalar sector

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SU(2) gauge sector

Scalar potential

- **Background Metric:**

$$ds^2 = -dt^2 + a^2(t)\,\delta_{ij}\,dx^i dx^j$$

- **Self-consistent expansion** (Friedmann equations)
- **Fixed power-law background** $a(t) \sim t^{\frac{2}{3(1+w)}}$

End

Transparencies worked out together with Paco Torrentí

Lattice Equations

- **Hamiltonian scheme**: coupled first-order differential equations

- **Scalar fld example**

$$\phi'' - \frac{1}{a^2}\nabla^2\phi + \frac{3}{a}\frac{da}{dt}\frac{d\phi}{dt} = -\frac{\partial V}{\partial \phi} \xrightarrow{\pi_\phi \equiv \phi' a^{3-\alpha}}$$

KICK: $(\pi_\phi)' = -\, a^{3+\alpha}\frac{\partial V}{\partial \phi} + a^{1+\alpha}\nabla^2\phi$

DRIFT: $\phi' \equiv \pi_\phi a^{\alpha-3}$

End

Transparencies worked out together with Paco Torrentí

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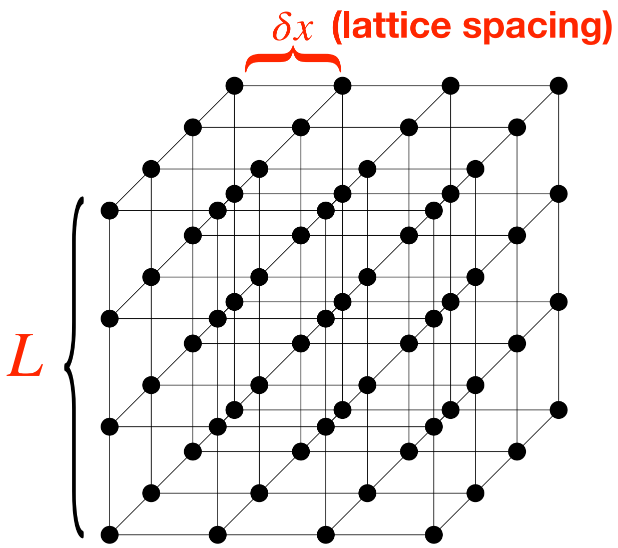
- **Scalar Fields and momenta** are defined in the **lattice sites**



N : number of points/dimension

$L = N \cdot \delta x$: length side

δt : time step

$\rightarrow$

Minimum and maximum momenta:

$$k_{\rm min} = \frac{2\pi}{L} \qquad k_{\rm max} = \frac{\sqrt{3}}{2} N k_{\rm min}$$

End

Lattice Equations

- **Hamiltonian scheme**: coupled first-order differential equations

- **Scalar fld example**

$$\phi'' - \frac{1}{a^2}\nabla^2\phi + \frac{3}{a}\frac{da}{dt}\frac{d\phi}{dt} = -\frac{\partial V}{\partial \phi} \xrightarrow{\pi_\phi \equiv \phi' a^{3-\alpha}}$$

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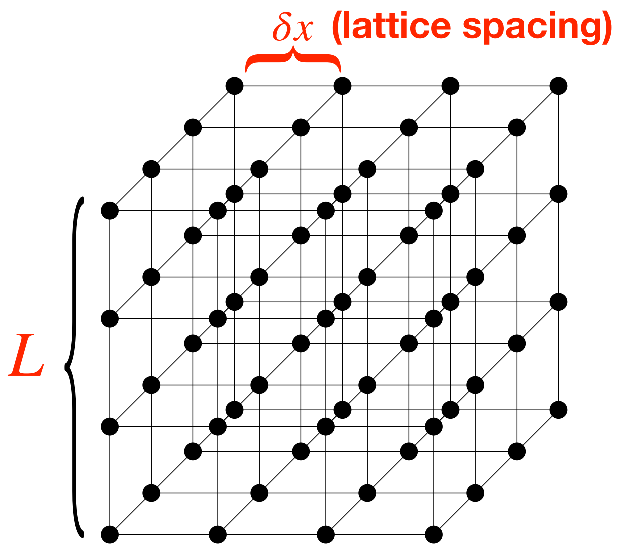

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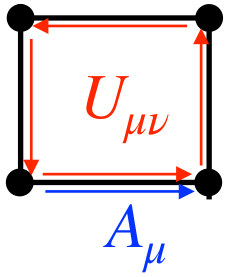
Minimum and maximum momenta:

$$k_{\rm min} = \frac{2\pi}{L} \qquad k_{\rm max} = \frac{\sqrt{3}}{2} N k_{\rm min}$$

- **Gauge fields** introduced via **links** and **plaquettes** (like in **lattice-QCD**)



End

Writing a model

- Equations solved in (dimensionless) ***program variables*:**

Choose:

$$\left\{\alpha, \omega_*, f_*\right\} \longrightarrow \boxed{\begin{aligned} d\tilde{\eta} &\equiv a^{-\alpha}\omega_* dt \\ d\tilde{x}^i &\equiv \omega_* dx^i \end{aligned}}$$

Space and time

$$\boxed{\tilde{\phi} = \frac{\phi}{f_*} \qquad \tilde{\varphi} = \frac{\varphi}{f_*} \qquad \tilde{\Phi} = \frac{\Phi}{f_*}}$$

Scalar fields

$$\boxed{\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \qquad \widetilde{B}^a_\mu = \frac{B^a_\mu}{\omega_*}}$$

Gauge fields

End

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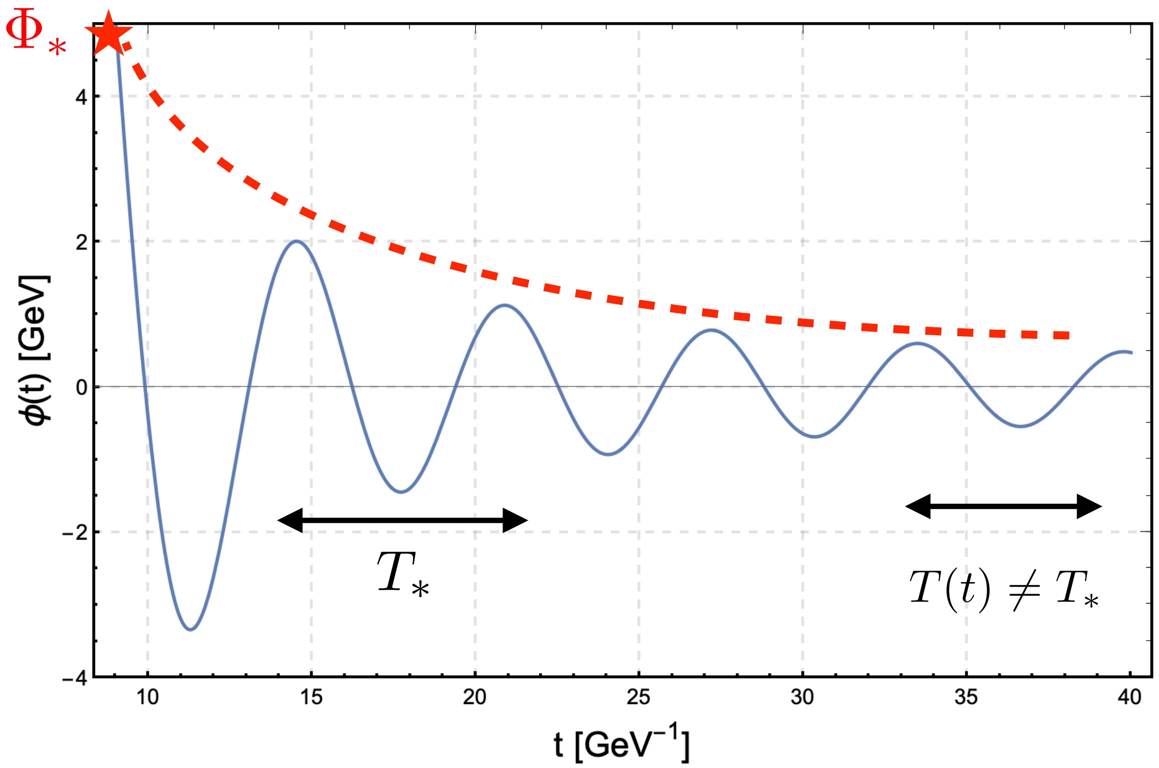
Gauge fields

Example: $\phi(t) \simeq \Phi_* \times f_{\rm osc}(t)$



End

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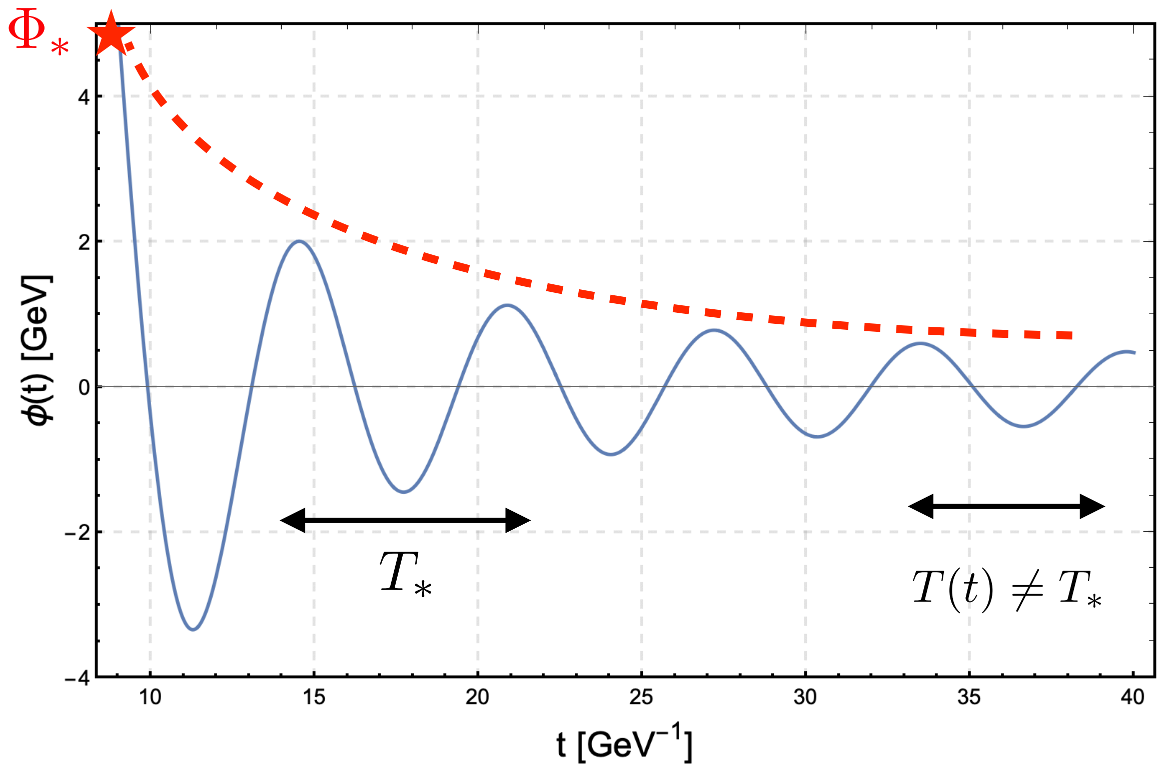

$$\Longrightarrow \begin{cases} f_* = \Phi_* \\ \omega_* = 1/T_* \\ \alpha \longrightarrow \text{Make period constant in } \tilde{\eta} \end{cases}$$

End

Transparencies worked out together with Paco Torrentí

Writing a model

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Gauge fields

- Write **scalar potential** and first and second derivatives in **one file** (*model.h*)

$$\boxed{\tilde{V}(\tilde{\phi}, |\tilde{\varphi}|, |\widetilde{\Phi}|) \equiv \frac{1}{f_*^2\omega_*^2} V(f_*\tilde{\phi}, f_*|\tilde{\varphi}|, f_*|\widetilde{\Phi}|)} \longrightarrow \frac{\partial\tilde{V}}{\partial\tilde{\phi}}, \frac{\partial\tilde{V}}{\partial|\tilde{\varphi}|}, \frac{\partial\tilde{V}}{\partial|\tilde{\Phi}|}, \frac{\partial^2\tilde{V}}{\partial\tilde{\phi}^2}, \frac{\partial^2\tilde{V}}{\partial|\tilde{\varphi}|^2}, \frac{\partial^2\tilde{V}}{\partial|\tilde{\Phi}|^2},$$

End

Transparencies worked out together with Paco Torrentí

Writing a model

- Equations solved in (dimensionless) ***program variables*:**

Choose:

$$\{\alpha, \omega_*, f_*\}$$

→

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$$\tilde{\phi} = \frac{\phi}{f_*} \qquad \tilde{\varphi} = \frac{\varphi}{f_*} \qquad \tilde{\Phi} = \frac{\Phi}{f_*}$$

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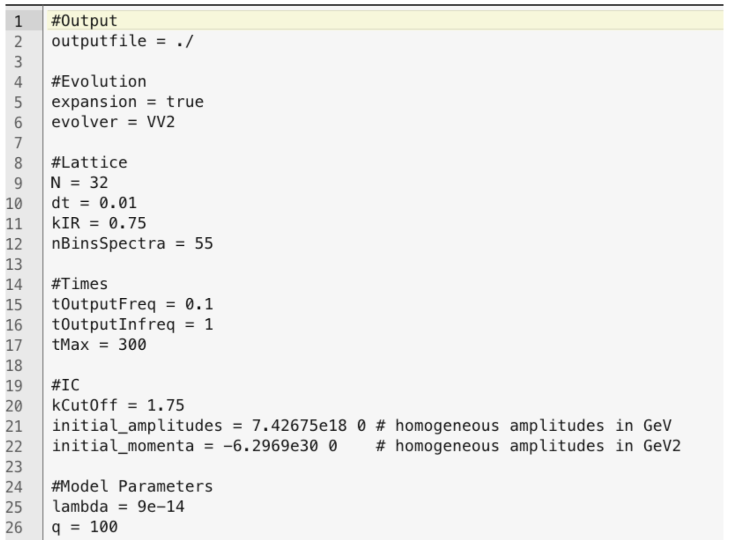
→

$$\frac{\partial\tilde{V}}{\partial\tilde{\phi}}, \quad \frac{\partial\tilde{V}}{\partial|\tilde{\varphi}|}, \quad \frac{\partial\tilde{V}}{\partial|\tilde{\Phi}|}, \quad \frac{\partial^2\tilde{V}}{\partial\tilde{\phi}^2}, \quad \frac{\partial^2\tilde{V}}{\partial|\tilde{\varphi}|^2}, \quad \frac{\partial^2\tilde{V}}{\partial|\tilde{\Phi}|^2},$$

- **Parameters** passed via **one file** (input.txt)

  (no need to re-compile !)

→



```
#Output
outputfile = ./

#Evolution
expansion = true
evolver = VV2

#Lattice
N = 32
dt = 0.01
kIR = 0.75
nBinsSpectra = 55

#Times
tOutputFreq = 0.1
tOutputInfreq = 1
tMax = 300

#IC
kCutOff = 1.75
initial_amplitudes = 7.42675e18 0 # homogeneous amplitudes in GeV
initial_momenta = -6.2969e30 0    # homogeneous amplitudes in GeV2

#Model Parameters
lambda = 9e-14
q = 100
```

End

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Self-consistent Expansion

- Algorithms use **second Friedmann equation** to **evolve the scale factor**.
- The **first Friedmann equation** is used to check the accuracy of the simulation.

$$\frac{a''}{a} = \frac{a^{2\alpha}}{3m_p^2}\Big\langle(\alpha-2)(K_\phi+K_\varphi+K_\Phi)+\alpha(G_\phi+G_\varphi+G_\Phi)+(\alpha+1)V + (\alpha-1)(K_{U(1)}+G_{U(1)}+K_{SU(2)}+G_{SU(2)})\Big\rangle$$

$$\left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{3m_p^2}\langle K_\phi+K_\varphi+K_\Phi+G_\phi+G_\varphi+G_\Phi+K_{U(1)}+G_{U(1)}+K_{SU(2)}+G_{SU(2)}+V\rangle$$

$\langle ... \rangle$ represents volume averaging

End

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$\langle\ldots\rangle$ represents volume averaging

$$\begin{aligned} K_\phi &= \tfrac{1}{2a^{2\alpha}}\phi'^2 \\ K_\varphi &= \tfrac{1}{a^{2\alpha}}(D_0^A\varphi)^*(D_0^A\varphi) \\ K_\Phi &= \tfrac{1}{a^{2\alpha}}(D_0\Phi)^\dagger(D_0\Phi) \end{aligned} \;;\quad \begin{aligned} G_\phi &= \tfrac{1}{2a^2}\textstyle\sum_i(\partial_i\phi)^2 \\ G_\varphi &= \tfrac{1}{a^2}\textstyle\sum_i(D_i^A\varphi)^*(D_i^A\varphi) \\ G_\Phi &= \tfrac{1}{a^2}\textstyle\sum_i(D_i\Phi)^\dagger(D_i\Phi) \end{aligned} \;;\quad \begin{aligned} K_{U(1)} &= \tfrac{1}{2a^{2+2\alpha}}\textstyle\sum_i F_{0i}^2 \\ K_{SU(2)} &= \tfrac{1}{2a^{2+2\alpha}}\textstyle\sum_{a,i}(G_{0i}^a)^2 \\ G_{U(1)} &= \tfrac{1}{2a^4}\textstyle\sum_{i,j<i}F_{ij}^2 \\ G_{SU(2)} &= \tfrac{1}{2a^4}\textstyle\sum_{a,i,j<i}(G_{ij}^a)^2 \end{aligned}$$

(Kinetic-Scalar) (Gradient-Scalar) (Electric & Magnetic)

End

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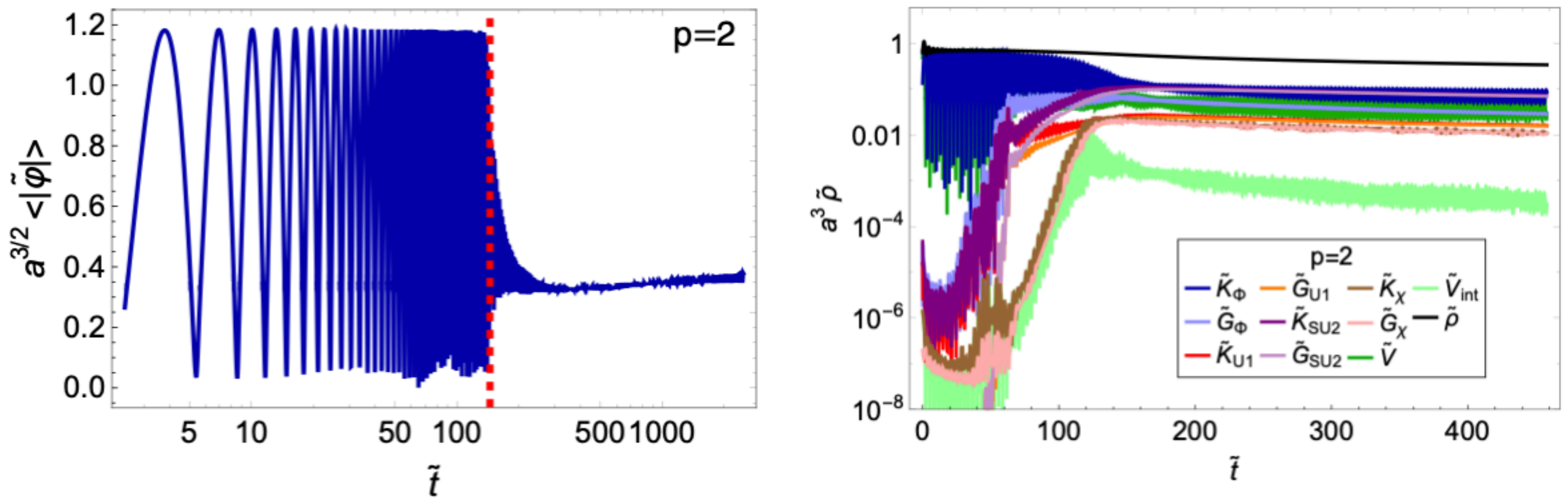
Output from your Run

Output Types

Volume averages: variance, energies, etc



End

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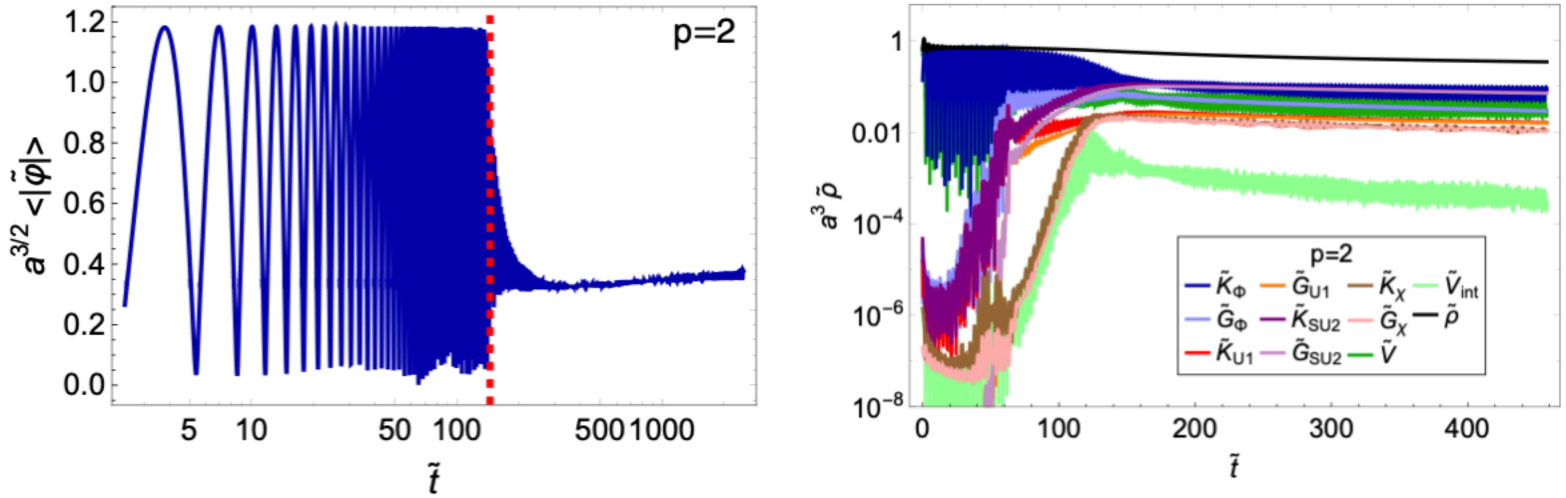
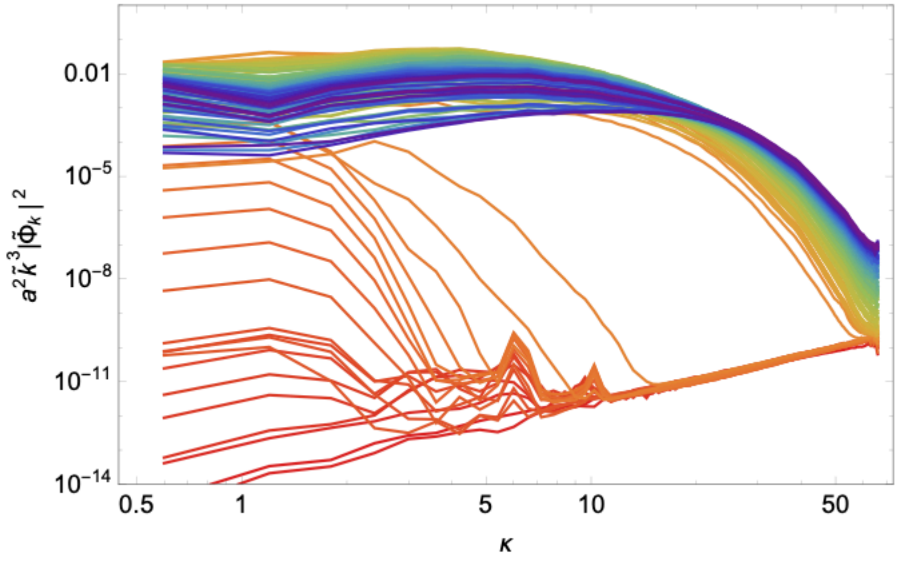


Fld Spectra: Raw/Binned



End

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Output from your Run

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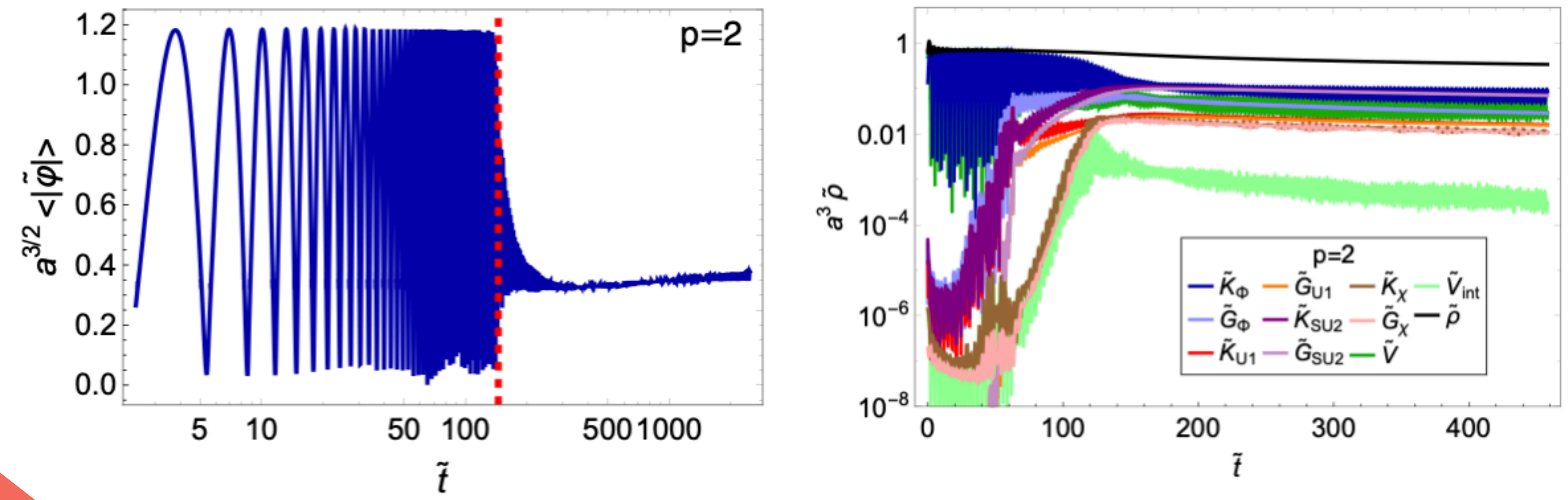
Volume averages: variance, energies, etc



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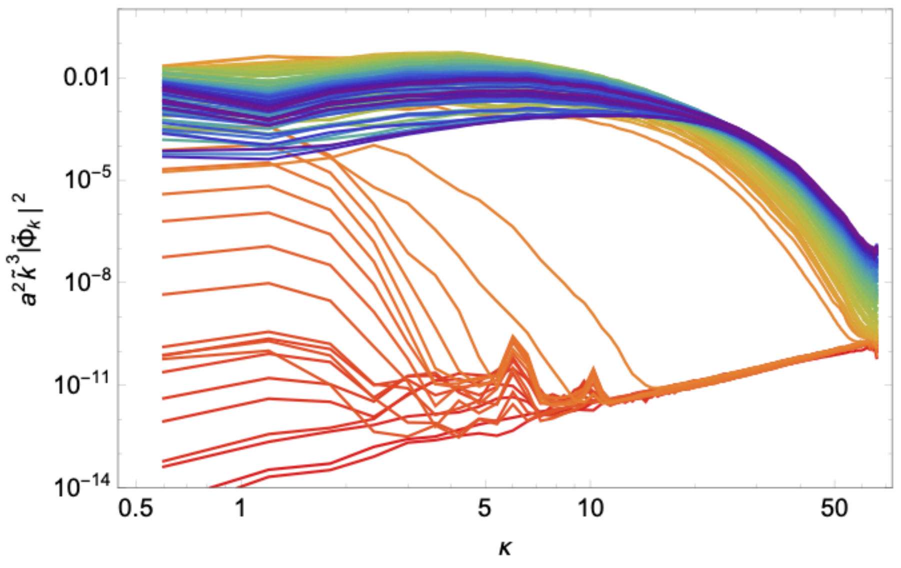
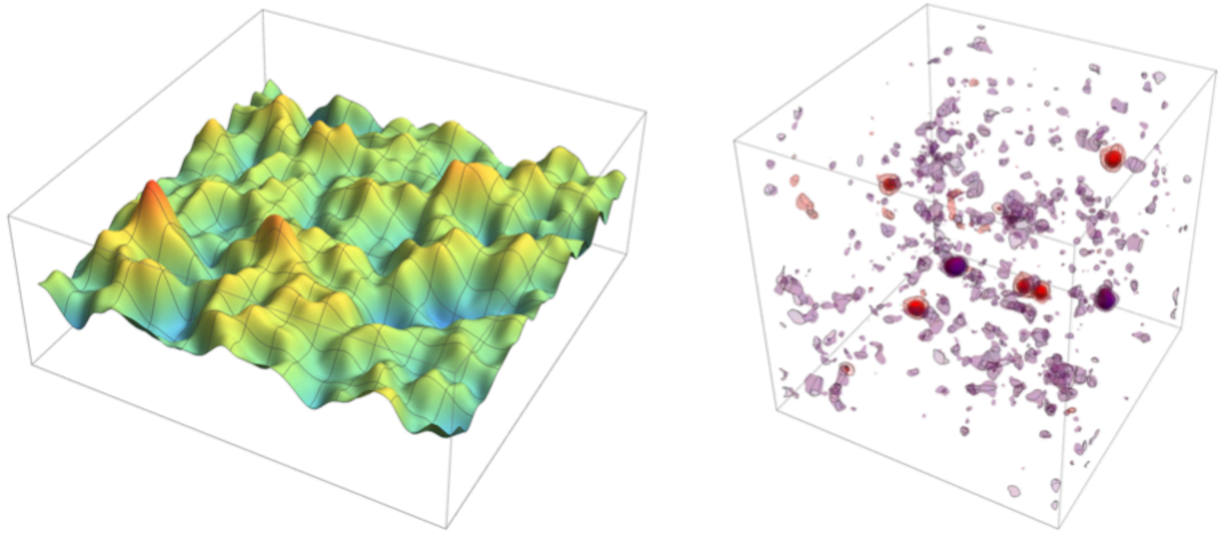


Snapshots: 2D/3D distribution



End

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Constraints

Energy conservation

- The **first Friedmann equation** is used to check the accuracy of the simulation.

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 \simeq \frac{\rho}{3m_p^2}}$$

$$\Delta_e \equiv \frac{\langle \mathrm{LHS} - \mathrm{RHS} \rangle}{\langle \mathrm{LHS} + \mathrm{RHS} \rangle}$$

End

Transparencies worked out together with Paco Torrentí

(back)

Energy conservation

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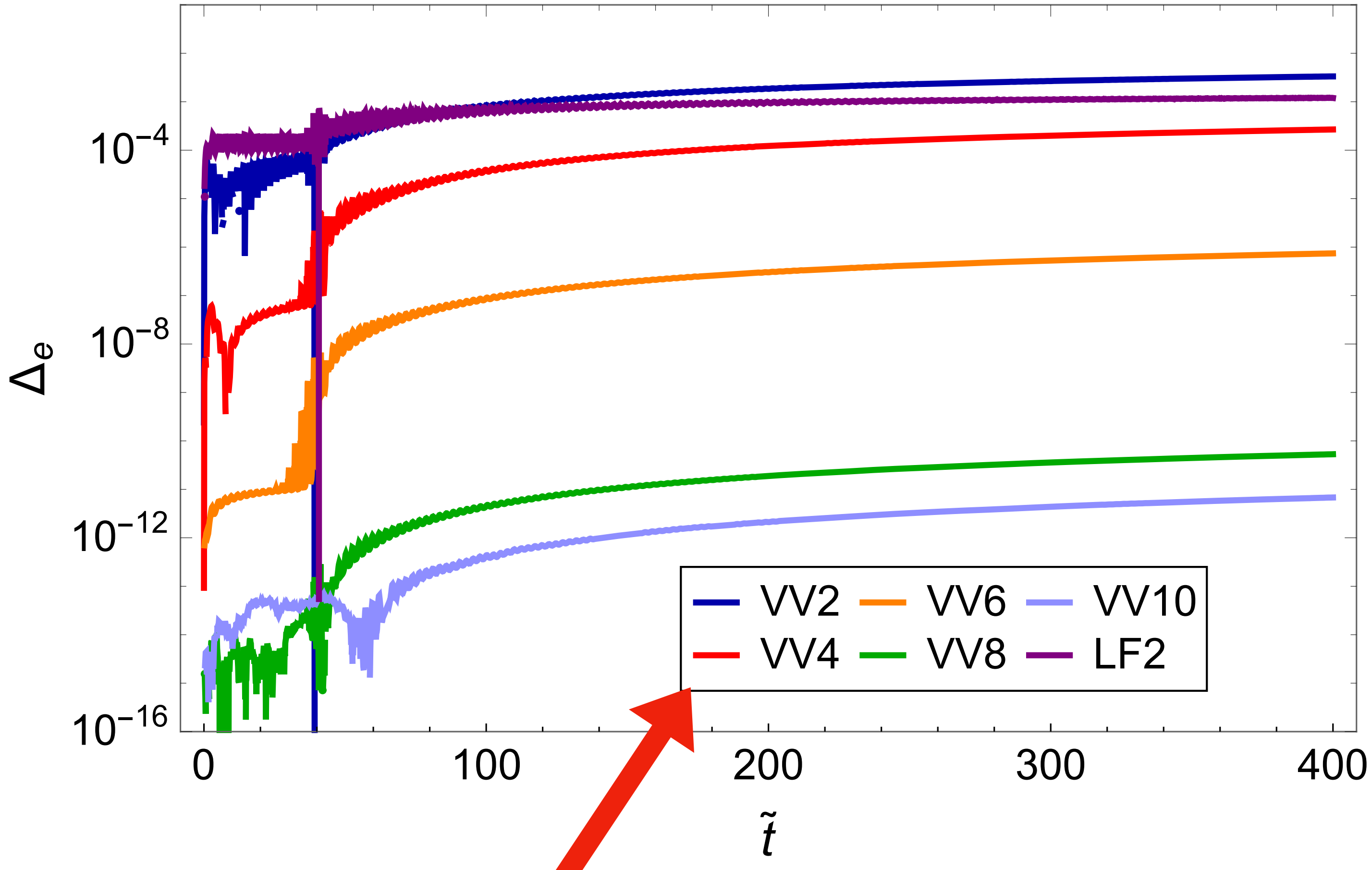

Evolution algorithms:

- **VVn:** Velocity-verlet of accuracy order O(dt^n)
- **LF2:** Staggered leapfrog, accuracy order O(dt^2)

End

Transparencies worked out together with Paco Torrentí

(back)

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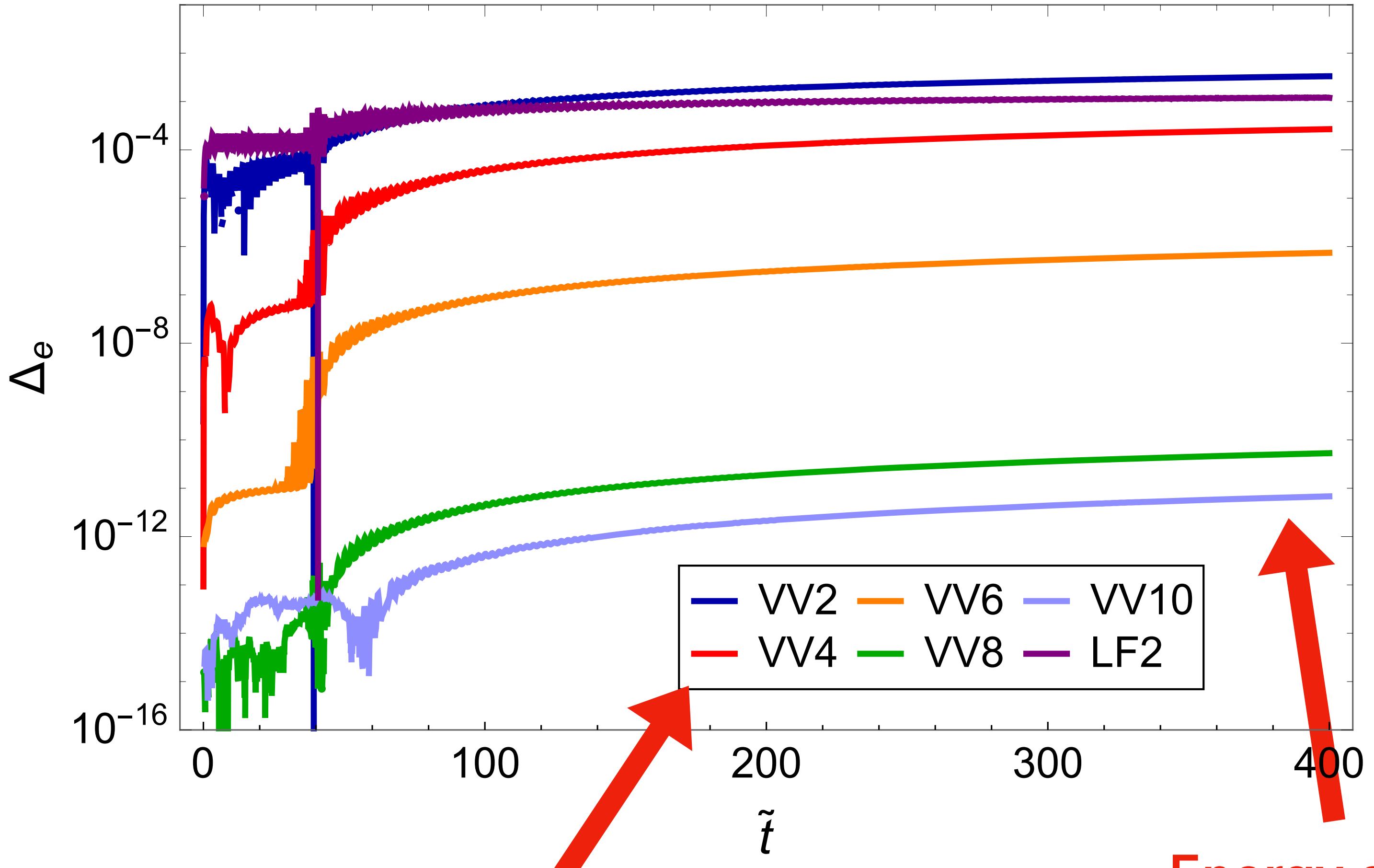


Evolution algorithms:

- ➤ **VVn:** Velocity-verlet of accuracy order O(dt^n)
- ➤ **LF2:** Staggered leapfrog, accuracy order O(dt^2)

Energy conserved up to machine precision for VV10!

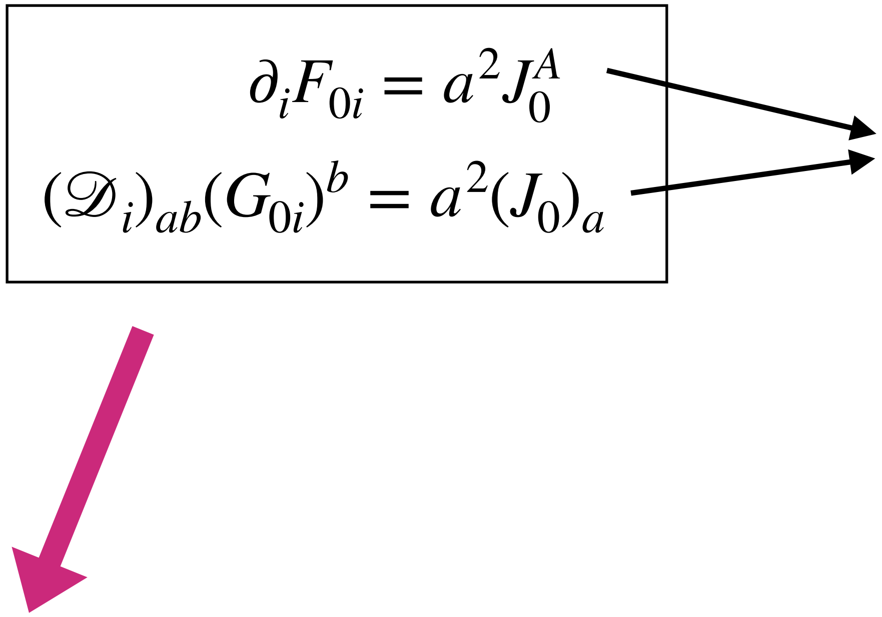
End

Transparencies worked out together with Paco Torrentí

(back)

Gauge theories: Gauss constraint

- Preservation of U(1) & SU(2) **Gauss constraints** (for all integrators!)



$$\boxed{\begin{aligned} \partial_i F_{0i} &= a^2 J_0^A \\ (\mathscr{D}_i)_{ab}(G_{0i})^b &= a^2 (J_0)_a \end{aligned}}$$

Gauge charges

$$\Delta_g \equiv \frac{\langle \sqrt{(\mathrm{LHS} - \mathrm{RHS})^2} \rangle}{\langle \sqrt{(\mathrm{LHS} + \mathrm{RHS})^2} \rangle}$$

End

Transparencies worked out together with Paco Torrentí

(back)

Gauge theories: Gauss constraint

- Preservation of U(1) & SU(2) **Gauss constraints** (for all integrators!)

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$$(\mathcal{D}_i)_{ab}(G_{0i})^b = a^2 (J_0)_a$$

Gauge charges

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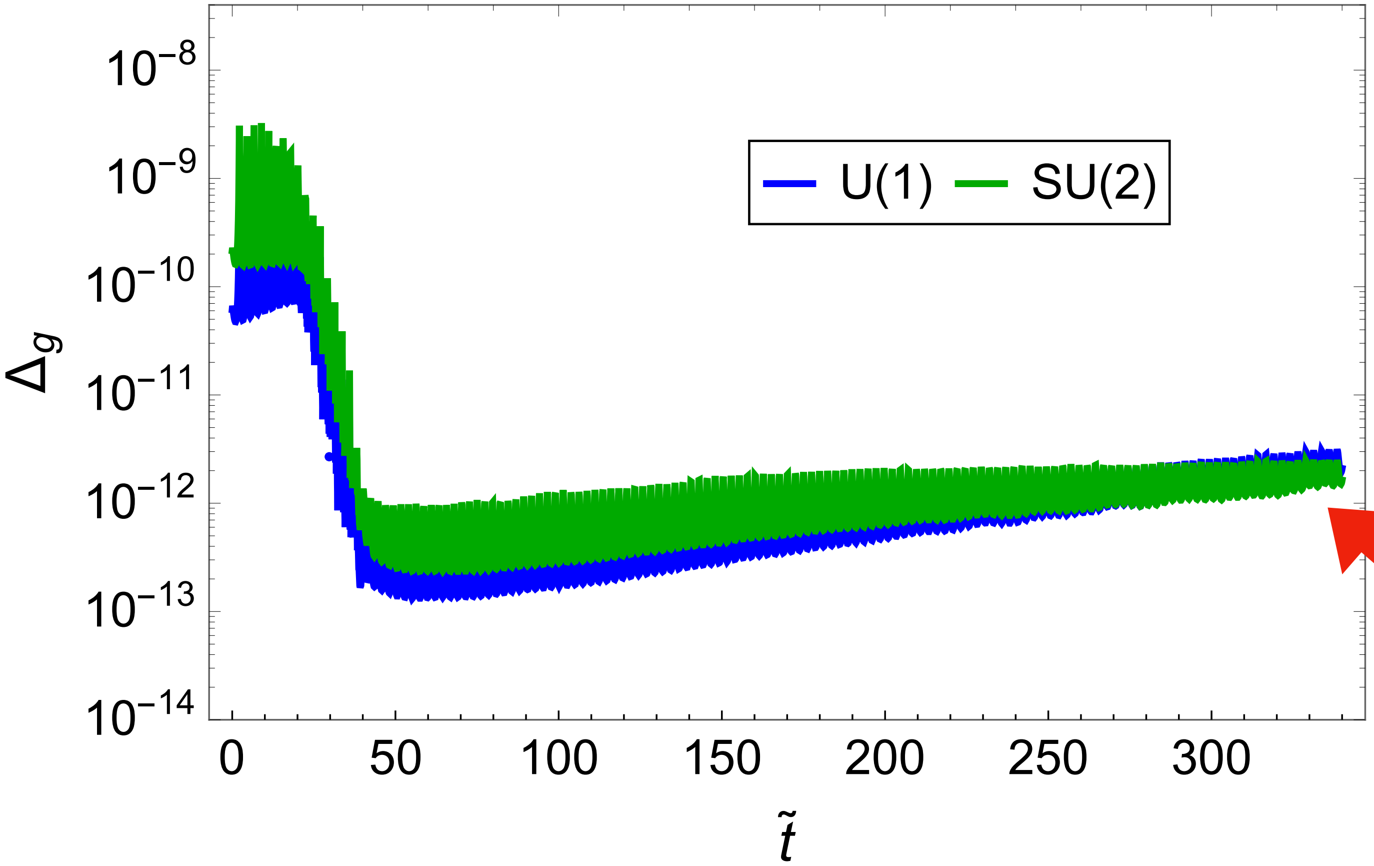

Gauss constraint preserved up to machine precision

End

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(back)

More on
Axion-Inflation

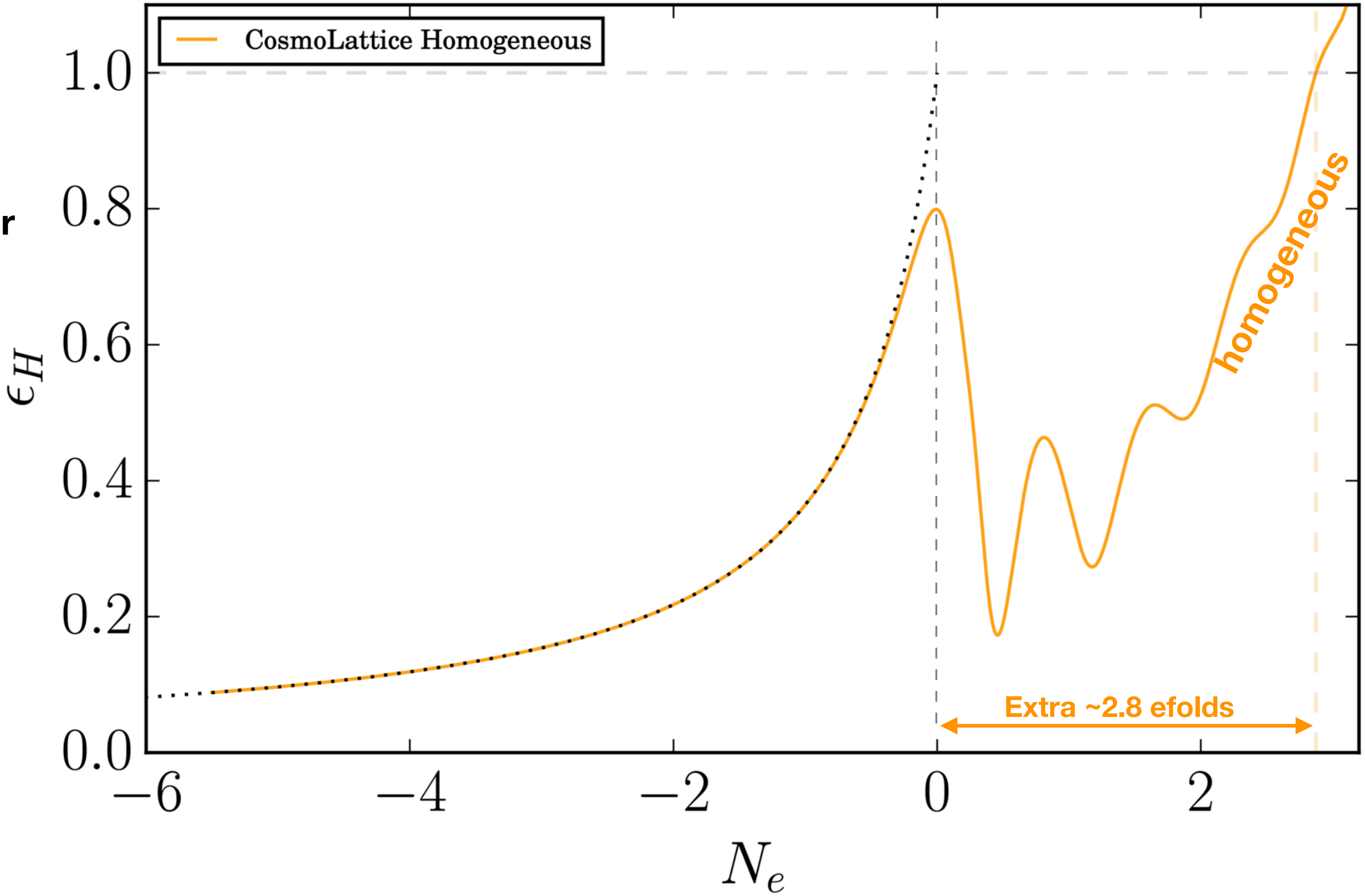
End

$$V(\phi) = \frac{1}{2}m^2\phi^2 \;;\; \frac{\phi}{4\Lambda}F\tilde{F} \;;\; \boxed{\Lambda = \frac{m_p}{15}}$$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{15}$)

Linear (········) **Non-Linear (———)**

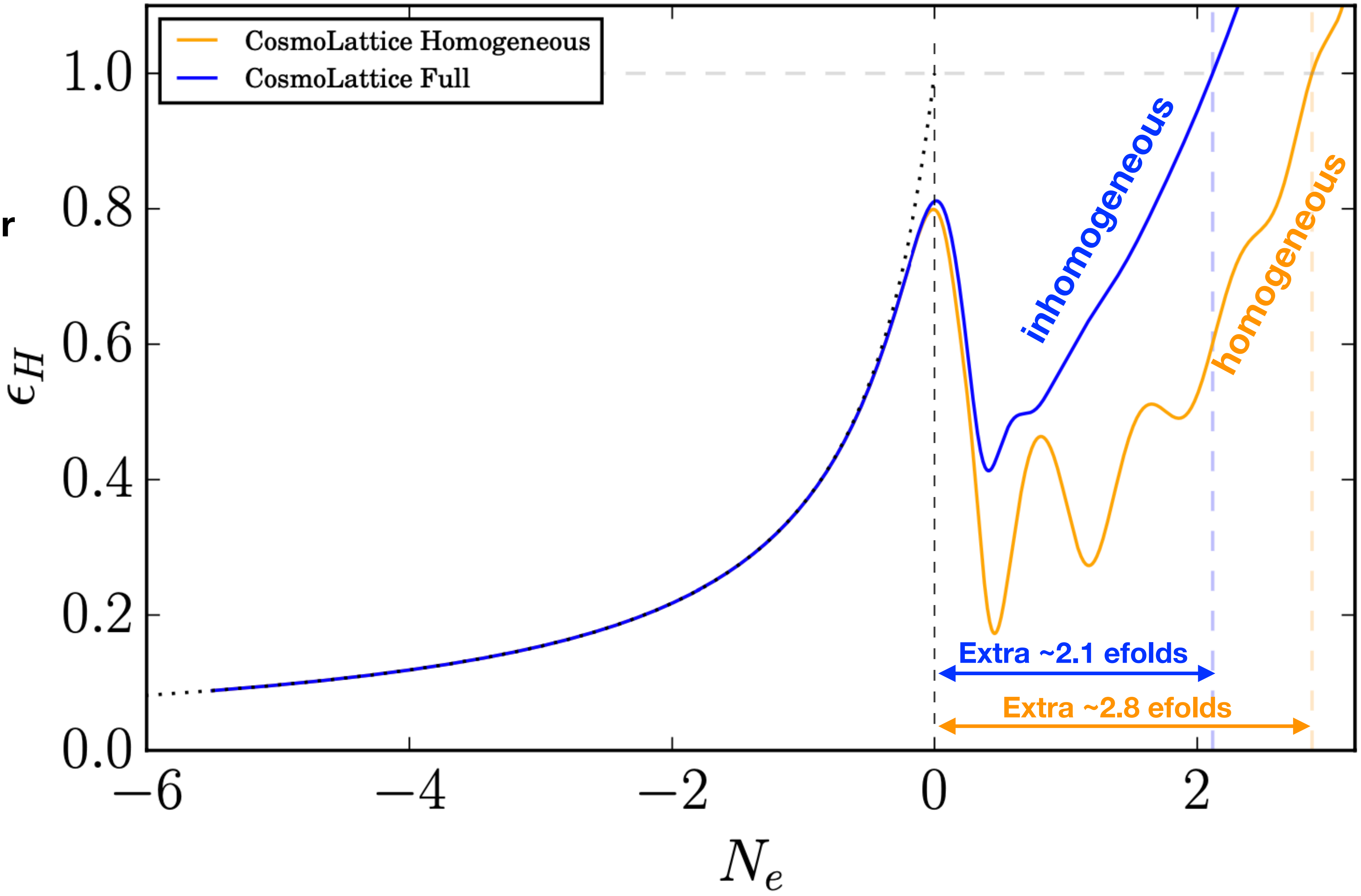
Epsilon Parameter



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{15}$)

Linear (········) **Non-Linear (——)**

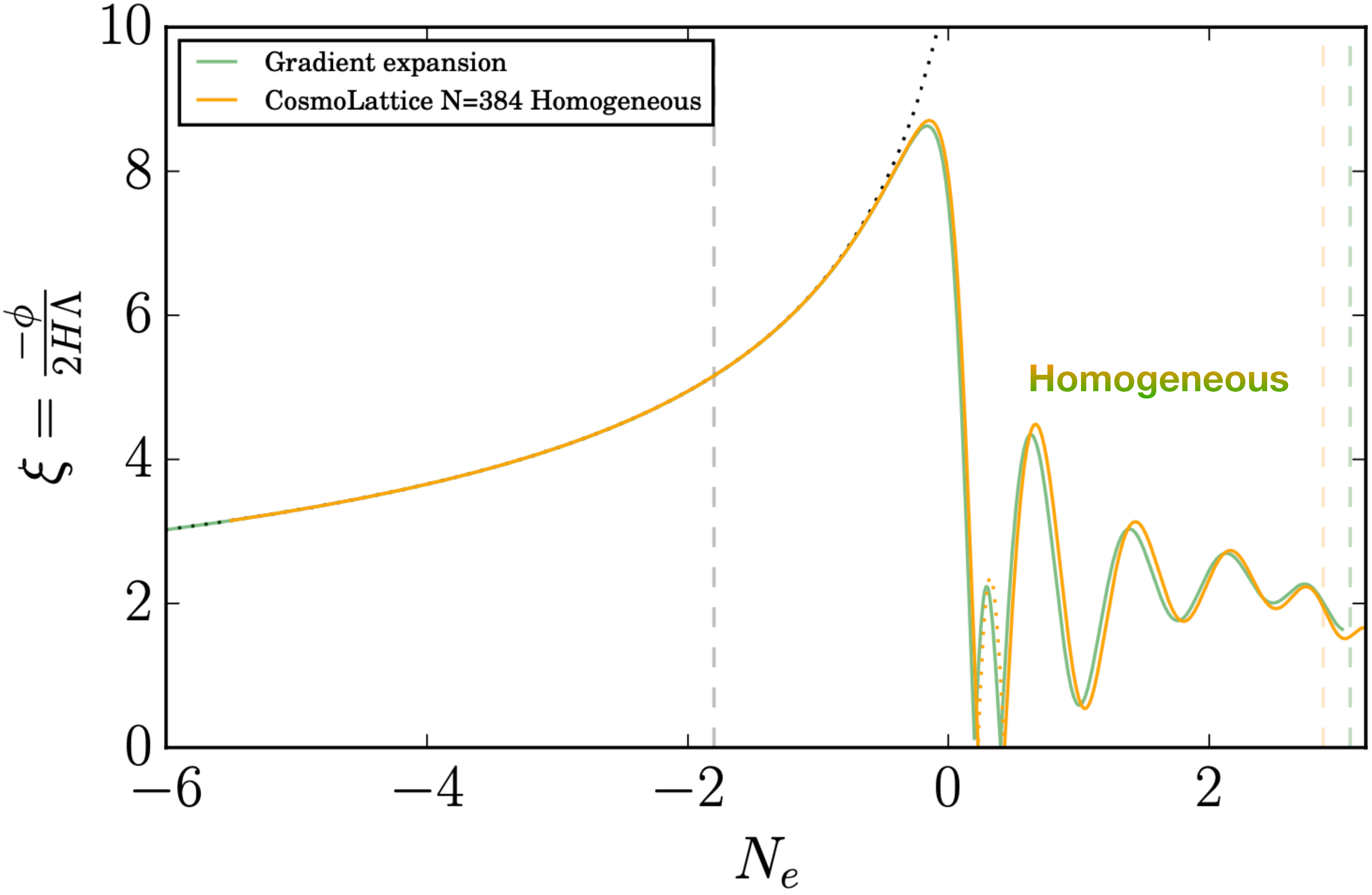
Epsilon Parameter



Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{15}\right)$

Homogeneous (🟧🟩)

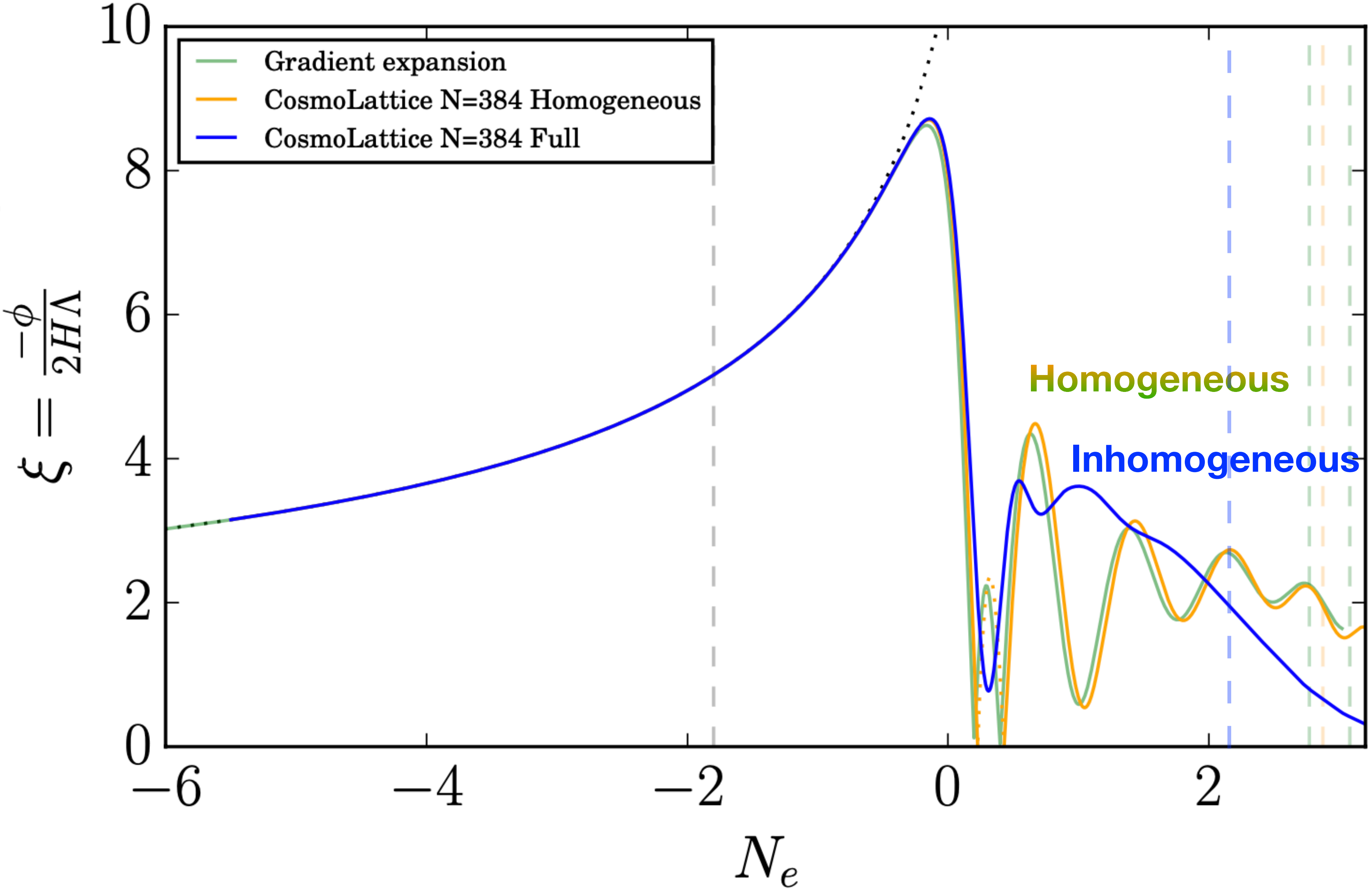
Xi parameter



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{15}$)

Homogeneous (▬) **In-Homogeneous (▬)**

Xi parameter



Example IV

Non-minimally coupled Scalar fields in the Jordan Frame

with

A. Florio, T. Opferkuch and B. Stefanek

SciPost, accepted ; 2112.08388 [astro-ph.CO]

Non-minimally coupled Scalars

Set-up

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \mathcal{L}_\chi + \mathcal{L}_{\text{inf}} + \frac{1}{2}\xi_\phi\phi^2 R\right]$$

or $\frac{1}{2}\xi_\chi\chi^2 R$

- Inflaton ϕ

$$\mathcal{L}_{\text{inf}} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_{\text{inf}}(\phi)$$

- Spectator field χ

$$\mathcal{L}_\chi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi,\phi)$$

- Non minimal coupling to gravity ξ_ϕ or ξ_χ
- Stay in Jordan frame

End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Set-up

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \mathcal{L}_\chi + \mathcal{L}_{\rm inf} + \cancel{\frac{1}{2}\xi_\phi\phi^2 R}\right]$$

or $\frac{1}{2}\xi_\chi\chi^2 R$ **Spectator fld non-min. Coupled**

- Inflaton ϕ

$$\mathcal{L}_{\rm inf} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_{\rm inf}(\phi)$$

- Spectator field χ

$$\mathcal{L}_\chi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi,\phi)$$

- Non minimal coupling to gravity ξ_ϕ or ξ_χ

- Stay in Jordan frame

End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

EoM

$$\nabla_\mu \nabla^\mu \chi + \frac{\partial V}{\partial \chi} + \xi_\chi \chi R = 0$$

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Non-minimally coupled Scalars

EoM

$$\nabla_\mu \nabla^\mu \chi + \frac{\partial V}{\partial \chi} + \xi_\chi \chi R = 0$$

$$R = F(\chi)\Big((1 - 6\xi)\,\langle \partial^\mu \chi \partial_\mu \chi \rangle$$

$$+4\left(\langle V \rangle - \tfrac{3\xi}{2} \langle \chi V_{,\chi} \rangle \right) - \langle \rho_m \rangle - 3\langle p_m \rangle \Big)$$

$$F(\chi) = \frac{1}{M_P^2\left[1 + (6\xi - 1)\xi \langle \chi^2 \rangle / M_P^2\right]}$$

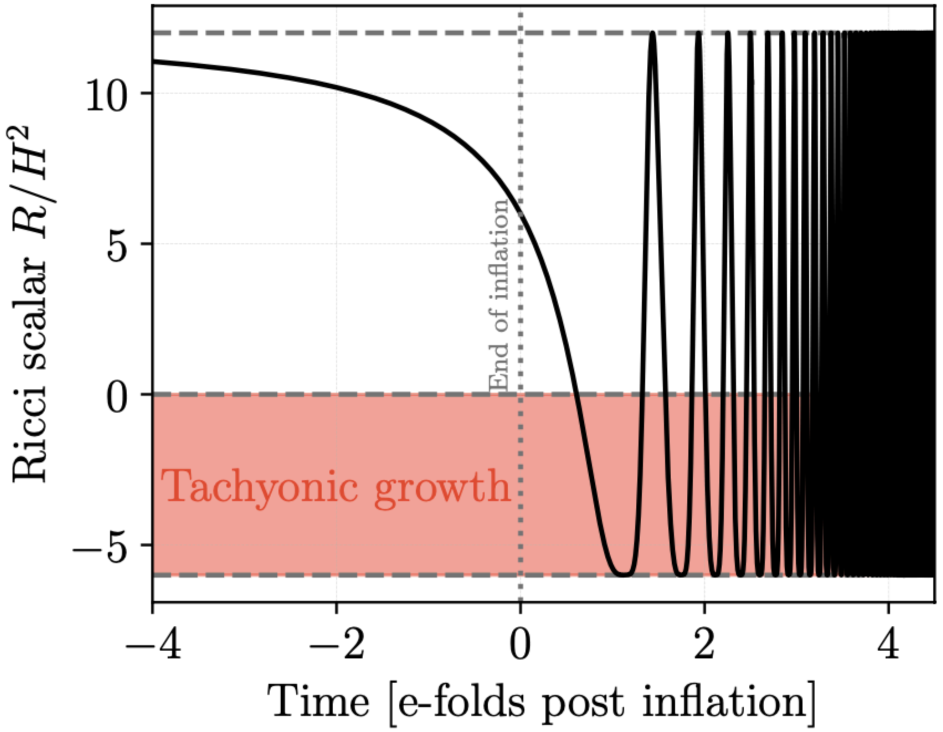
End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

- Standard inflaton

$V_{inf} \propto \tanh^4(\tilde{\phi})$



Curvature Oscillates !

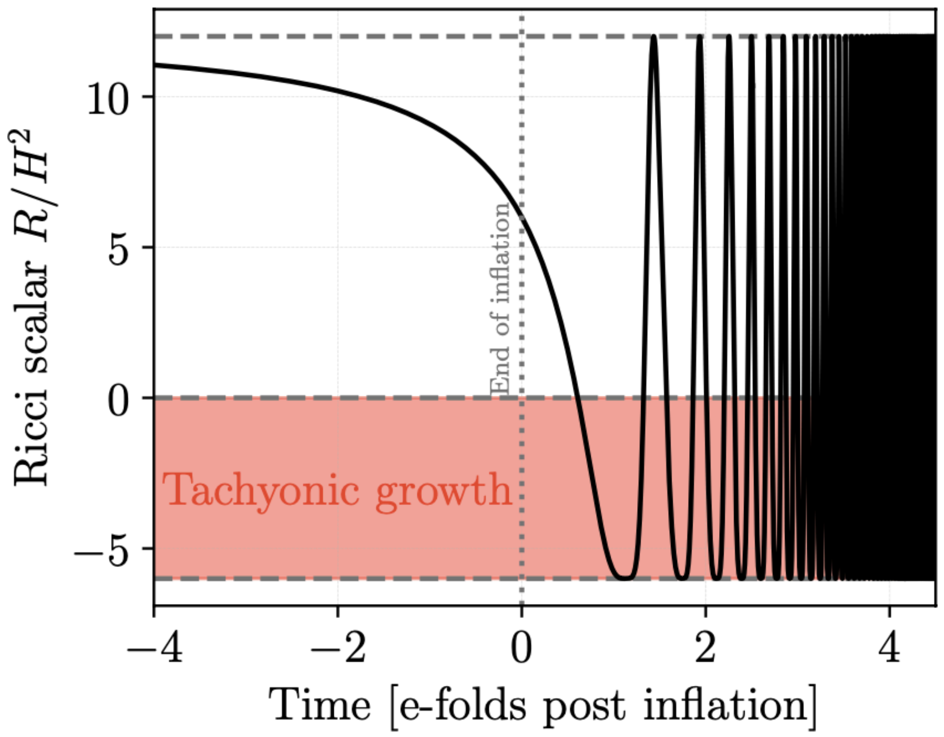
(sourced by Inflaton Oscillations)

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

- Standard inflaton

$V_{inf} \propto \tanh^4(\tilde{\phi})$



Geometric Preheating

[Basset & Liberati '99]

$\xi \chi^2 R \quad , \xi = 10{,}50{,}100$

The preheat field is excited exponentially

Curvature Oscillates !

(sourced by Inflaton Oscillations)

End

Transparency worked out with Adrien Florio

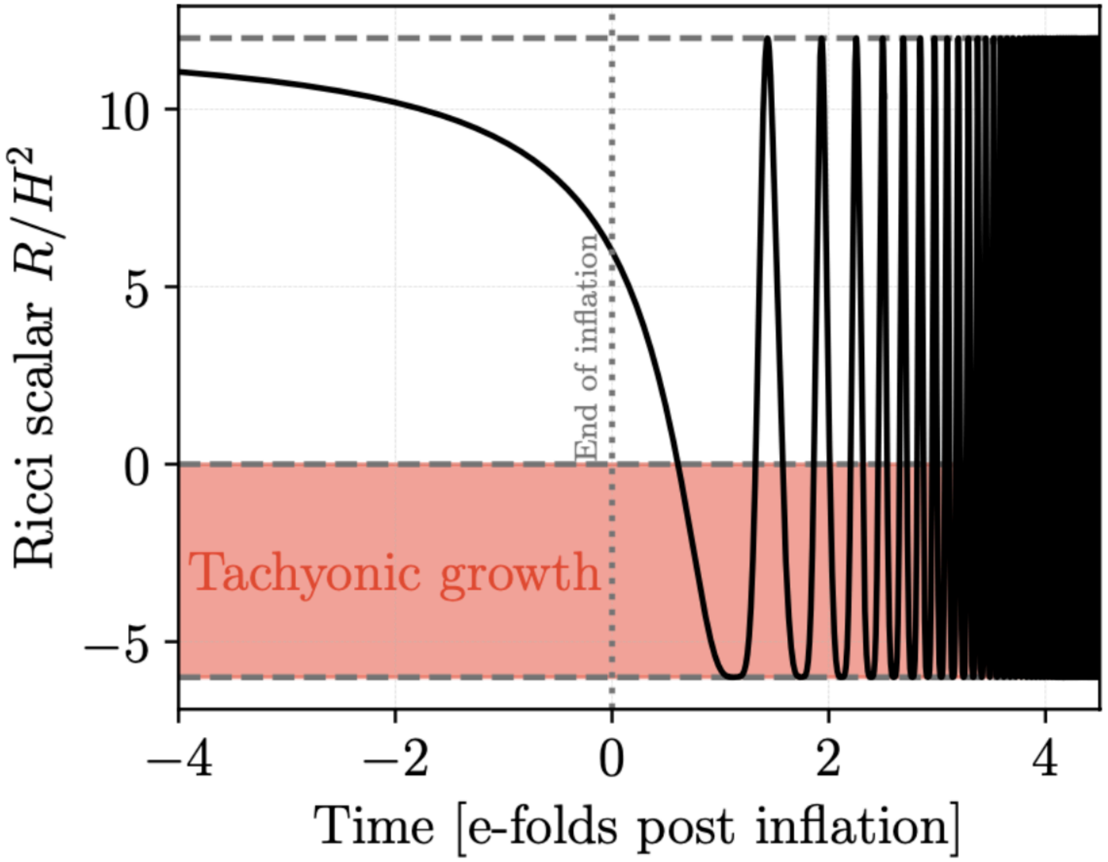
Non-minimally coupled Scalars

Geometric Preheating

[Basset & Liberati '99]

$$\xi \chi^2 R \quad , \xi = 10,50,100$$

How is the preheat field excited?



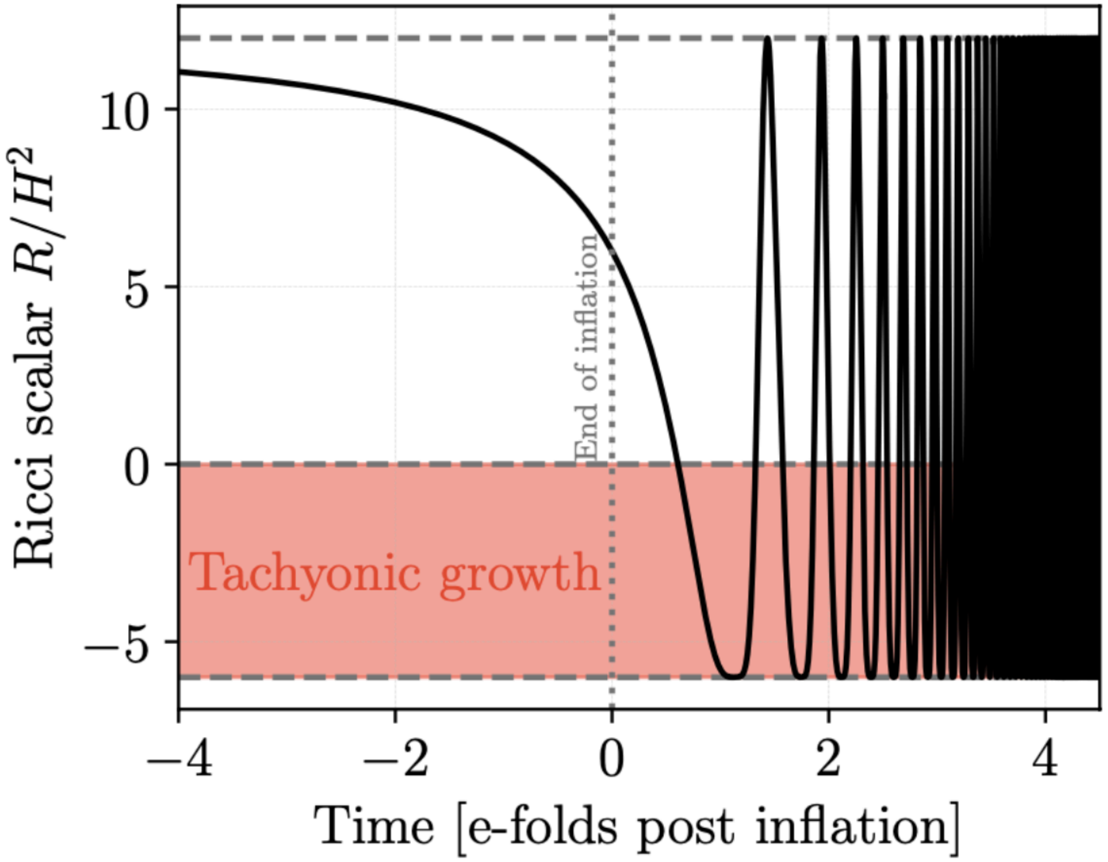
Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Geometric Preheating

[Basset & Liberati '99]

$$\xi\chi^2 R \quad , \xi = 10{,}50{,}100$$



How is the preheat field excited?

$$m_\chi^2 \propto \left(\frac{\partial^2 V}{\partial\chi^2} + \xi\, R\right)$$

Tachyonic term every time R < 0

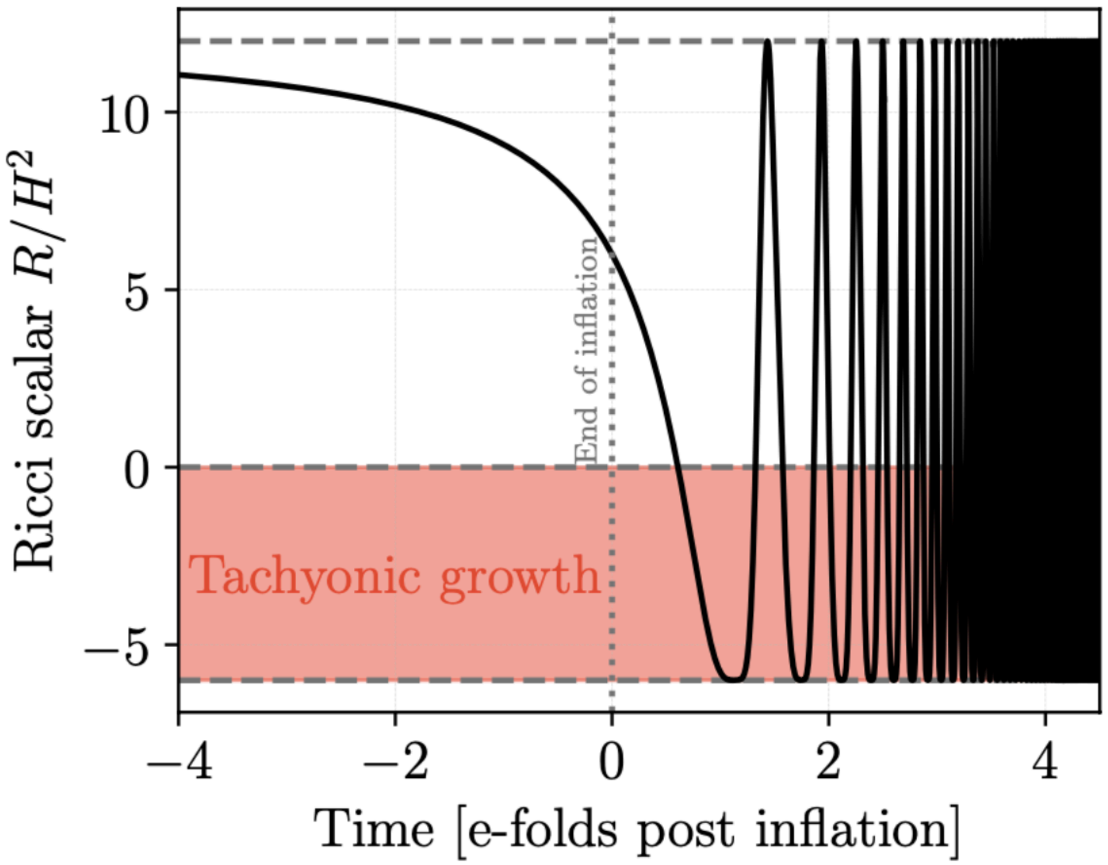
Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Geometric Preheating

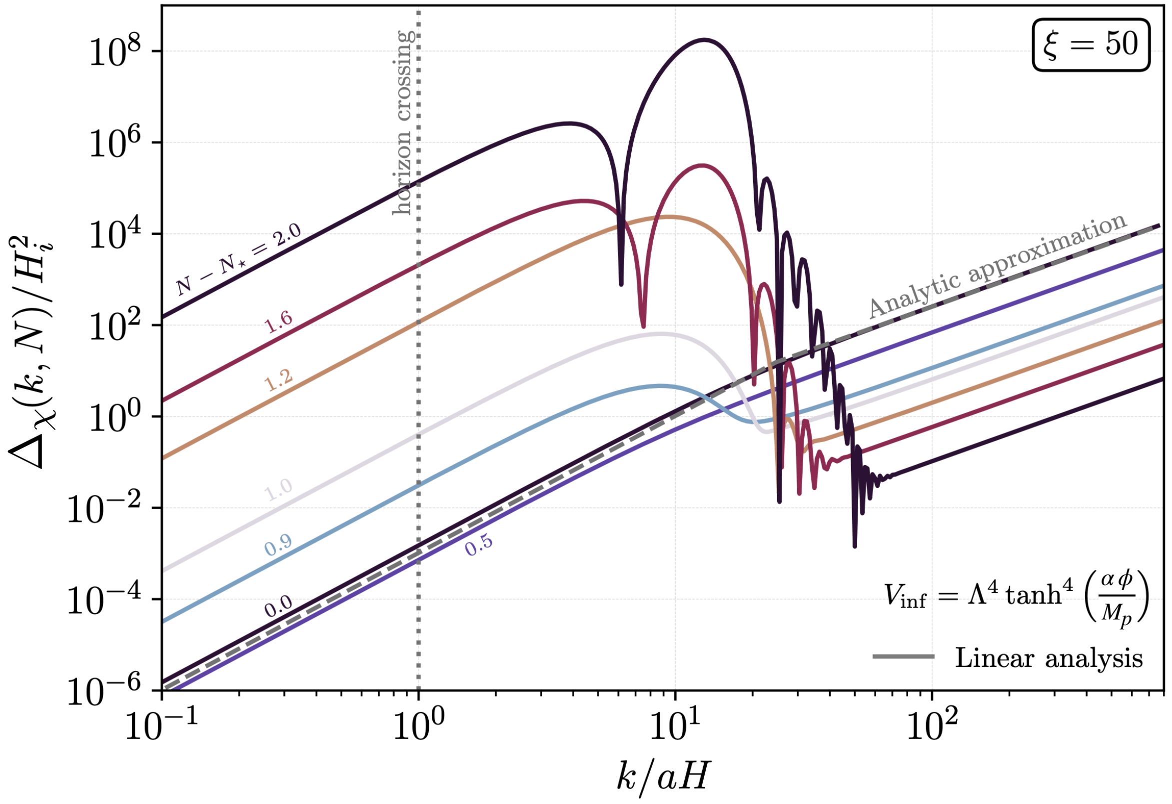
[Basset & Liberati ’99]

$$\xi \chi^2 R \quad , \xi = 10,50,100$$



How is the preheat field excited?

Linear Regime



End

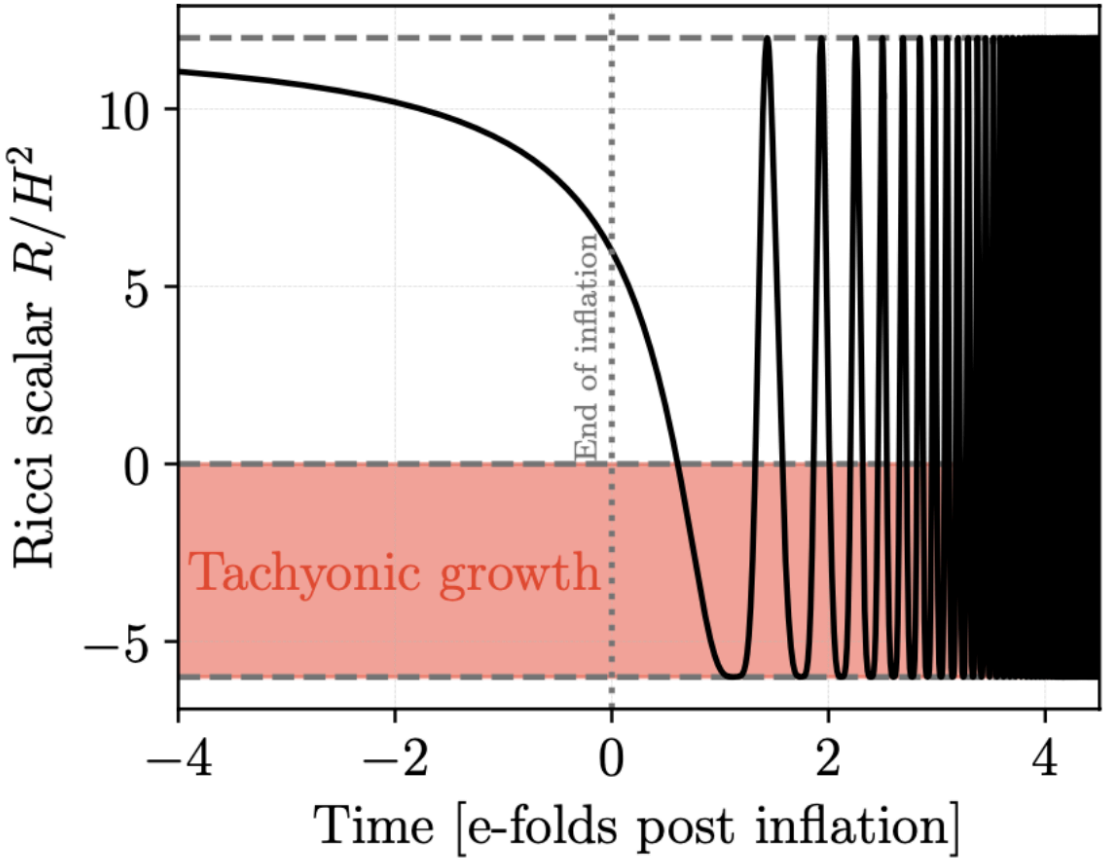
Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Geometric Preheating

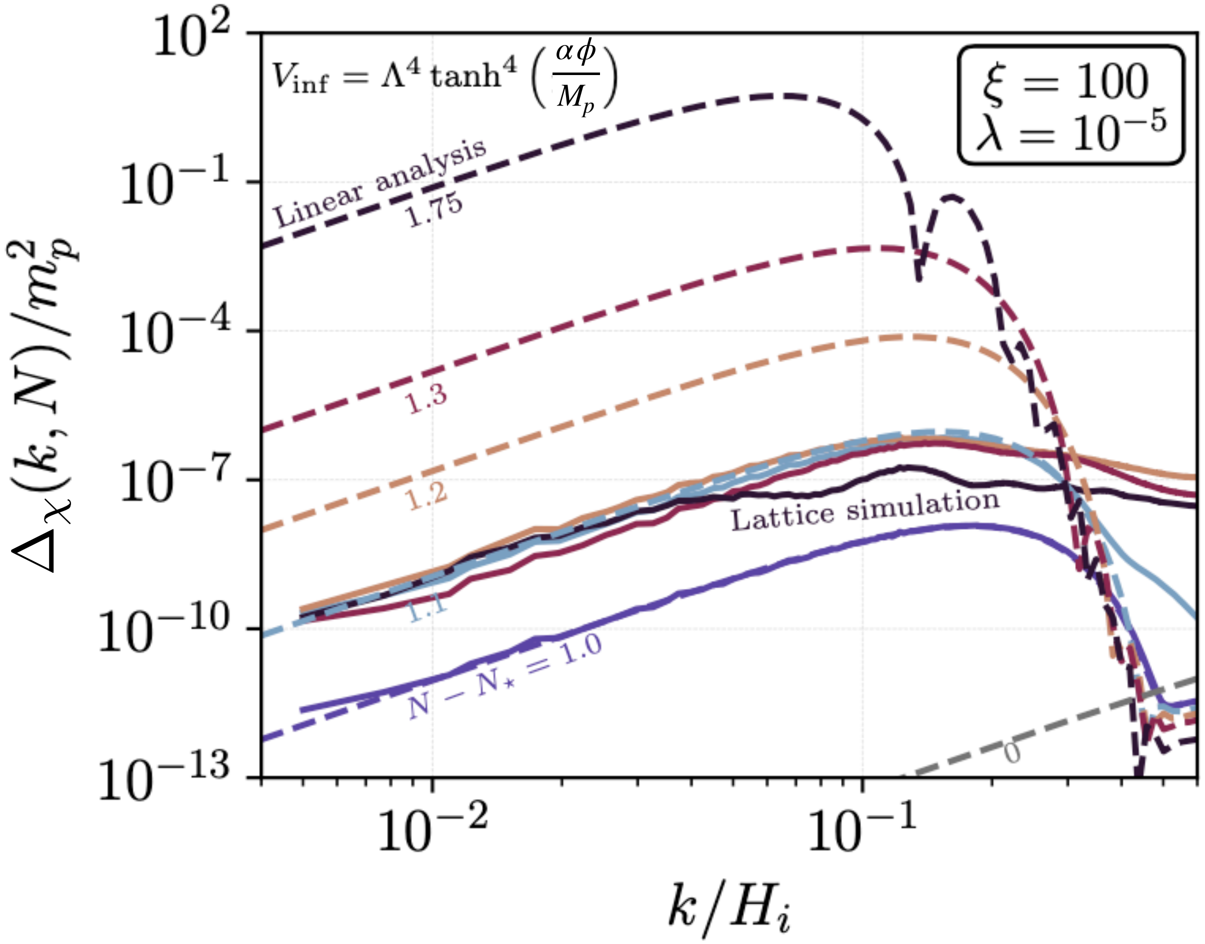
[Basset & Liberati ’99]

$$\xi \chi^2 R \quad , \xi = 10{,}50{,}100$$



How is the preheat field excited?

Non-Linear Regime



End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

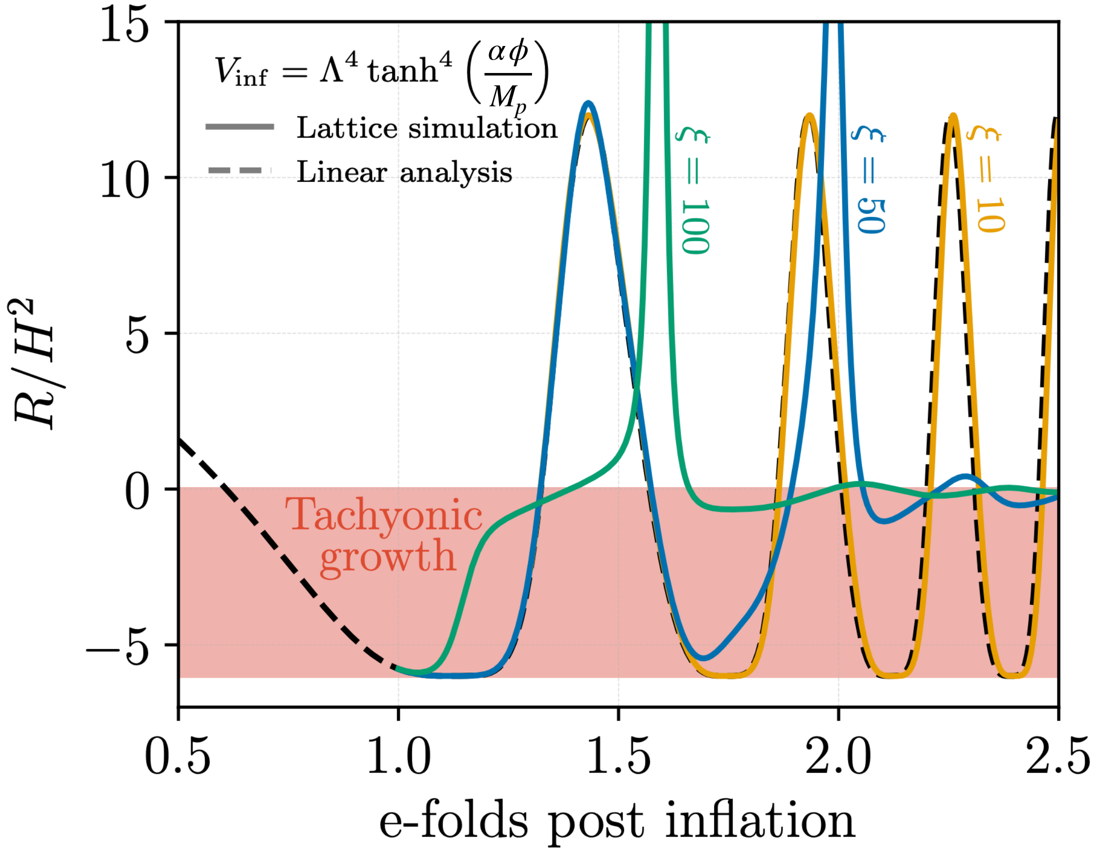
Geometric Preheating

[Basset & Liberati ’99]

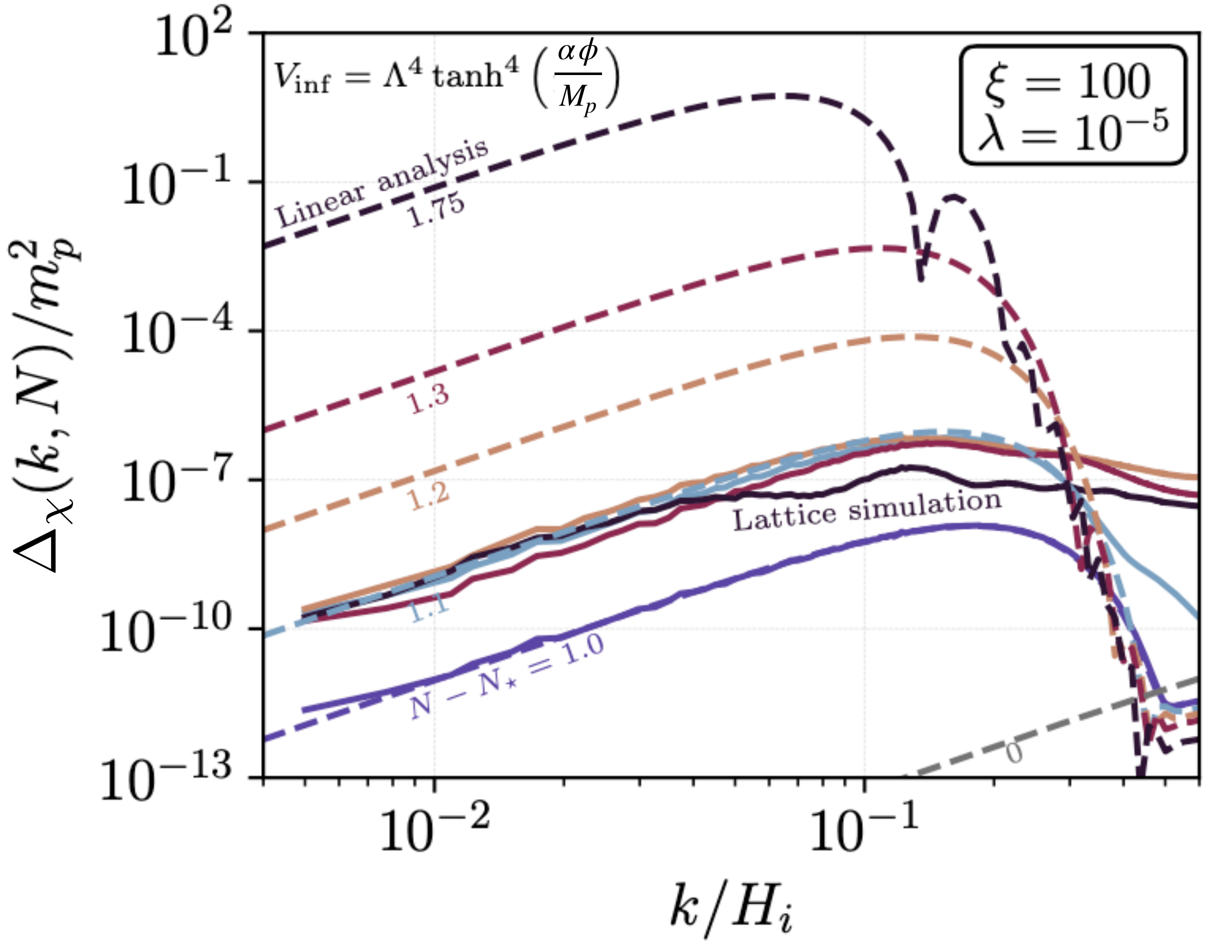
$\xi \chi^2 R \quad , \xi = 10{,}50{,}100$

How is the preheat field excited?

Back-reaction



Non-Linear Regime



Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

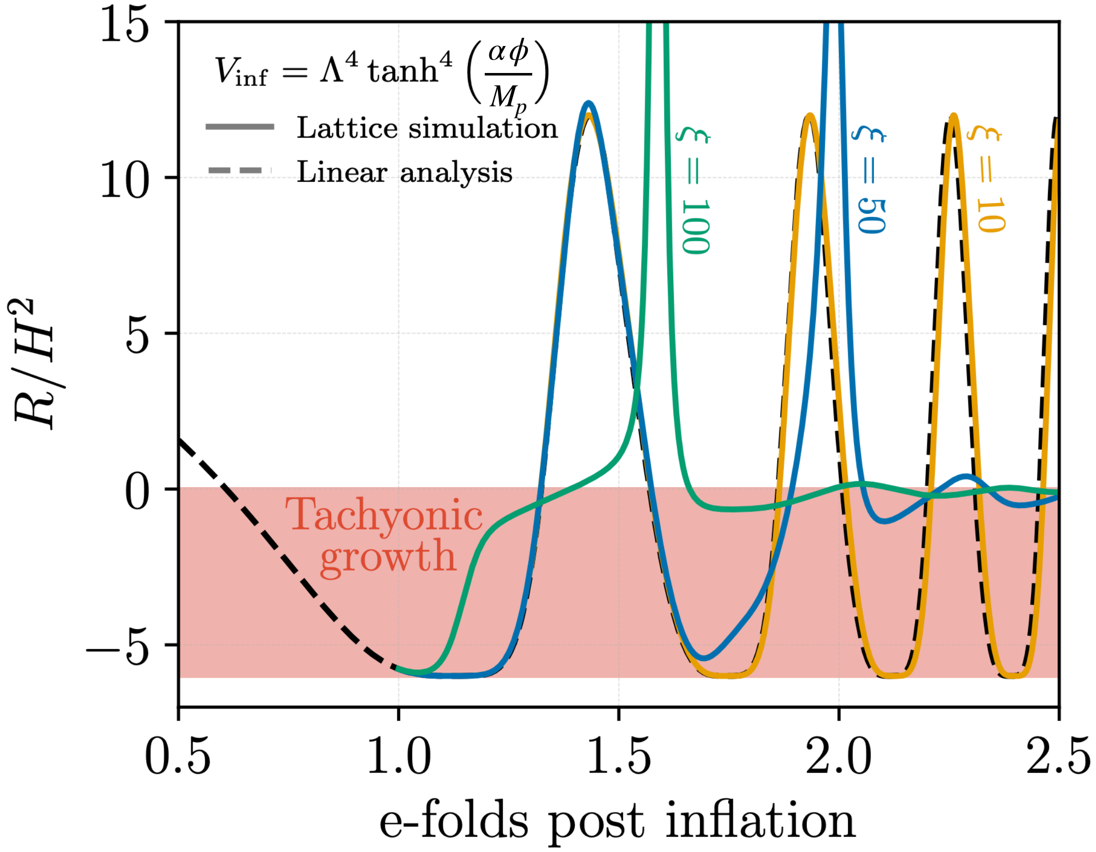
Geometric Preheating

[Basset & Liberati ’99]

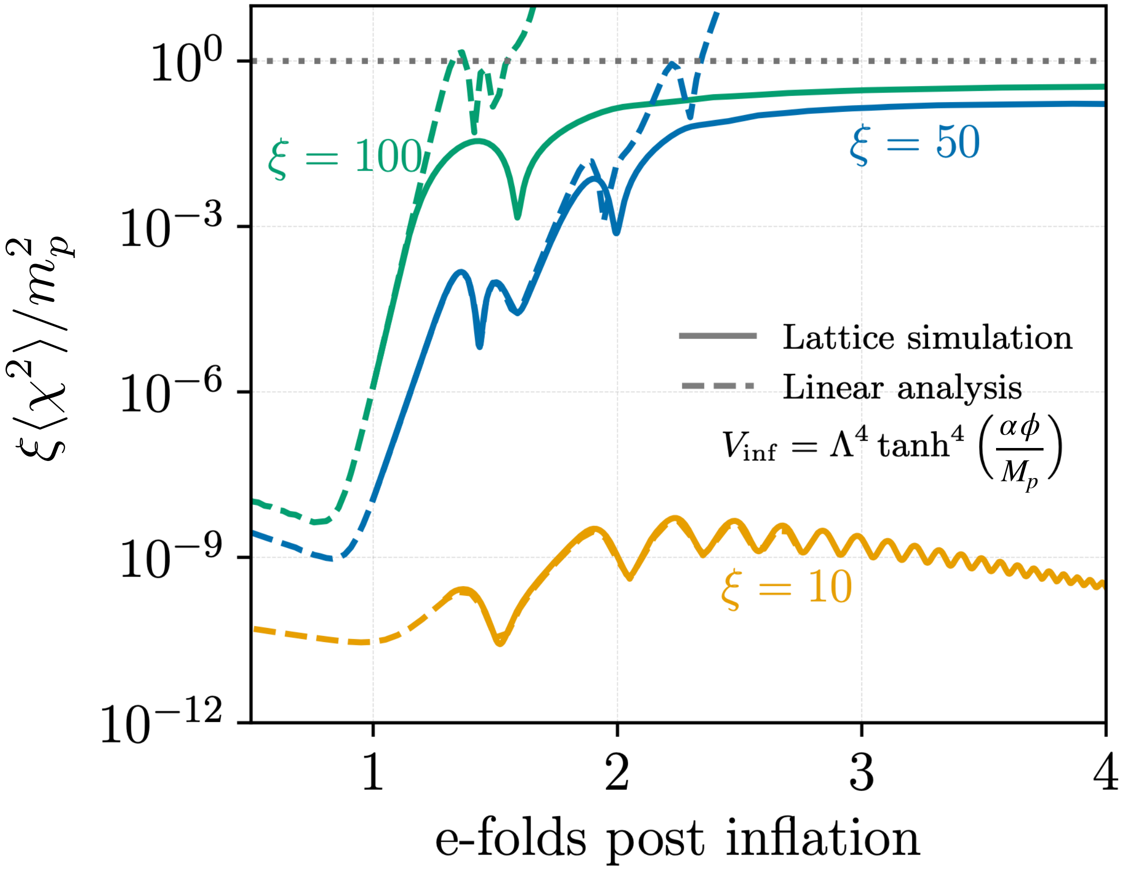
$$\xi \chi^2 R \quad , \xi = 10{,}50{,}100$$

How is the preheat field excited?

Back-reaction



Non-Linear Regime

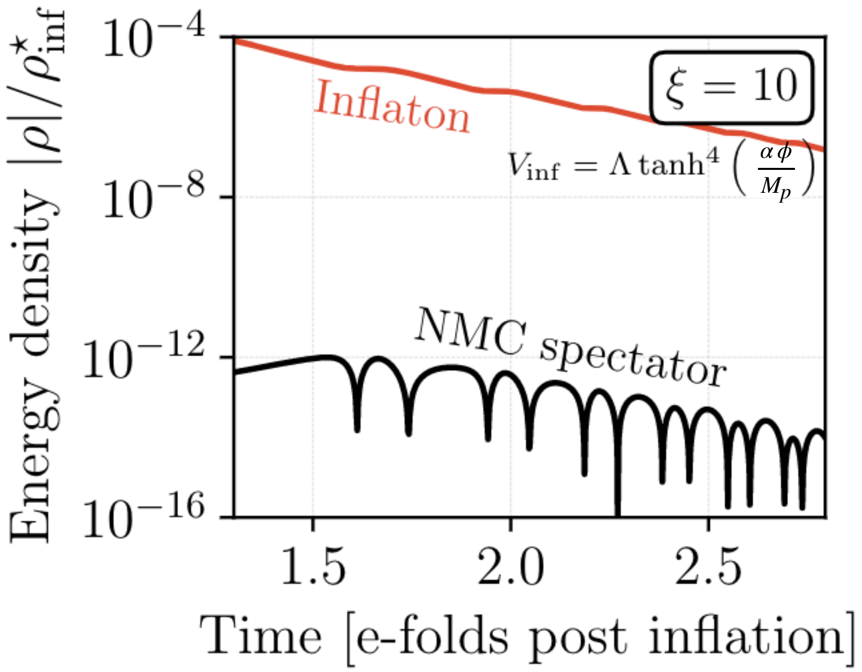


End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

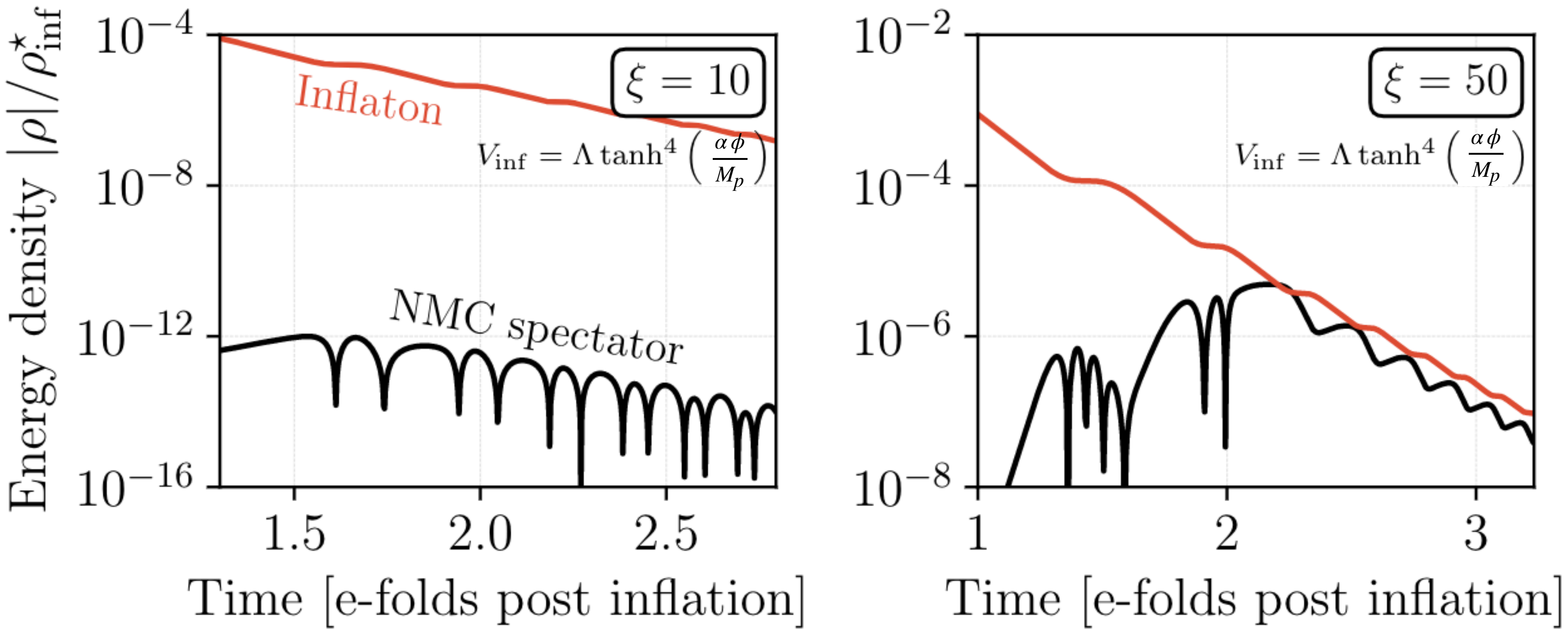
Geometric Preheating excitation (e.g. p = 4)



Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Geometric Preheating excitation (e.g. p = 4)

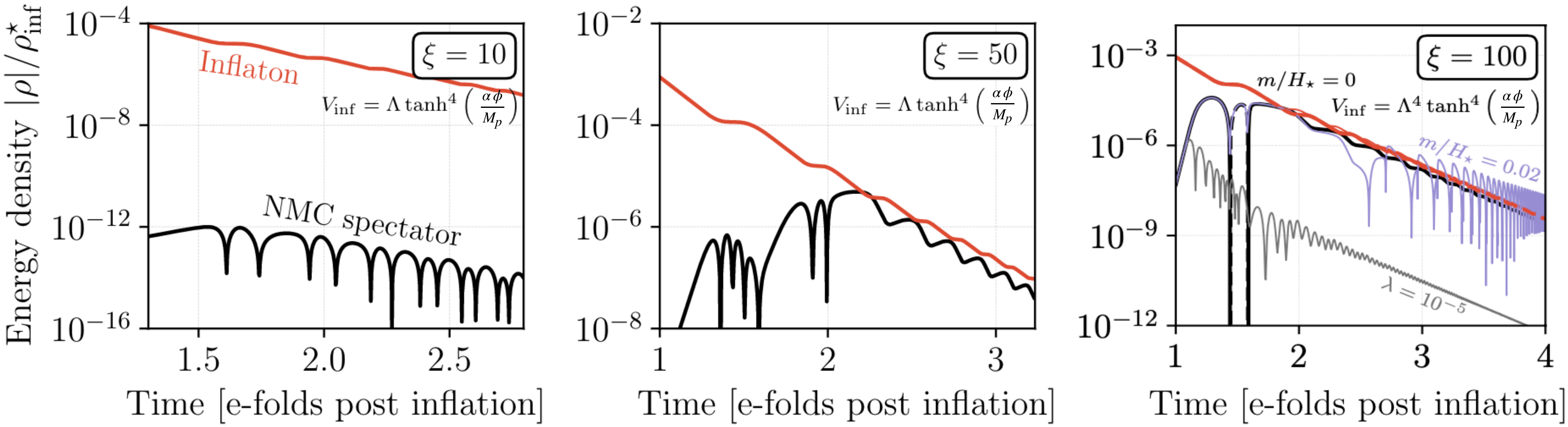


End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

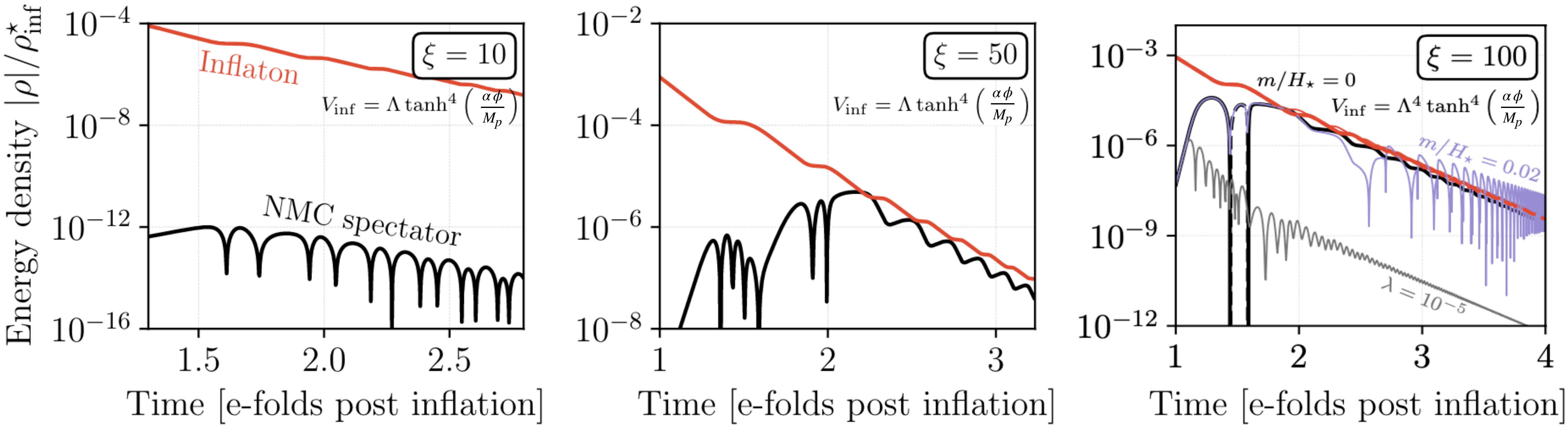
Geometric Preheating excitation (e.g. p = 4)



Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Geometric Preheating excitation (e.g. p = 4)



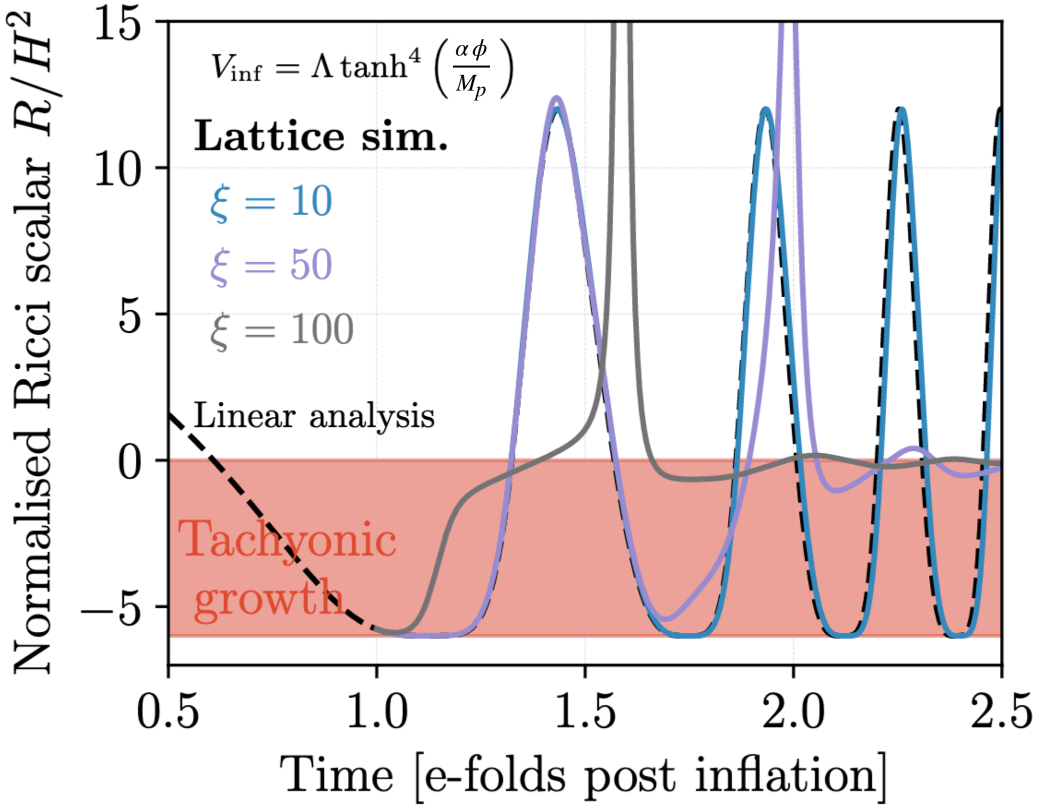
We can do it for any **p**

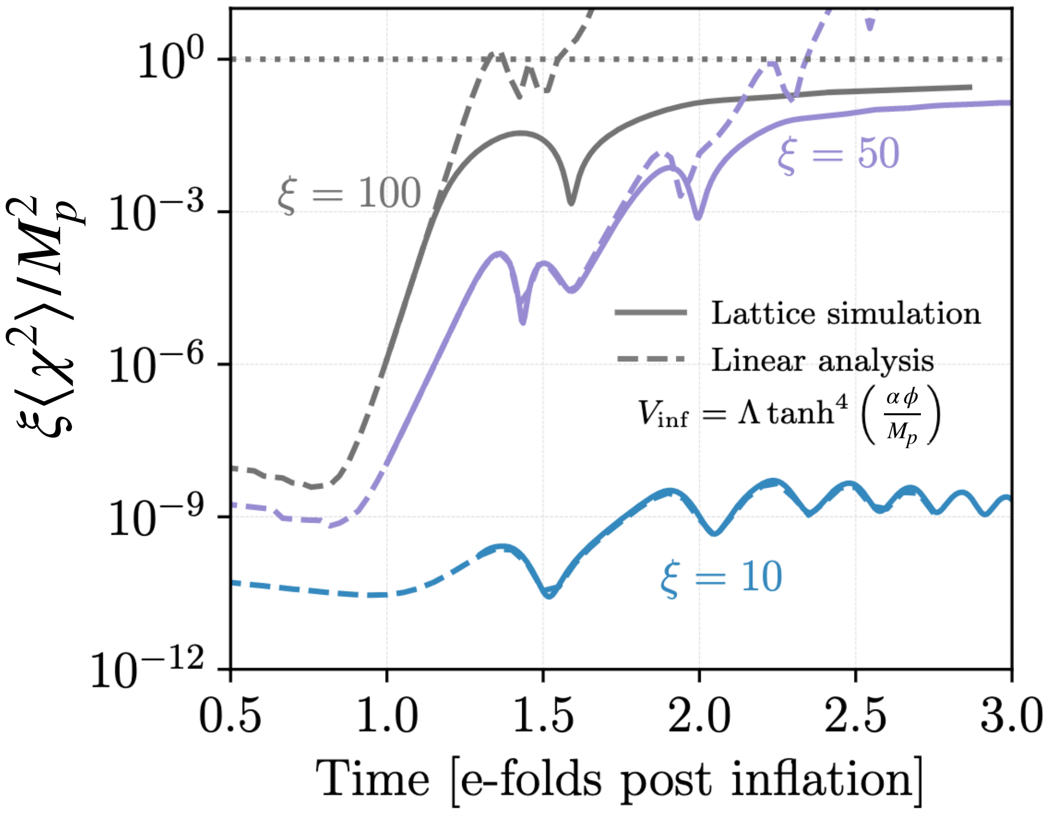
$V(|\phi|^p) \propto |\phi|^p$

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Full non-linear Geometric Preheating



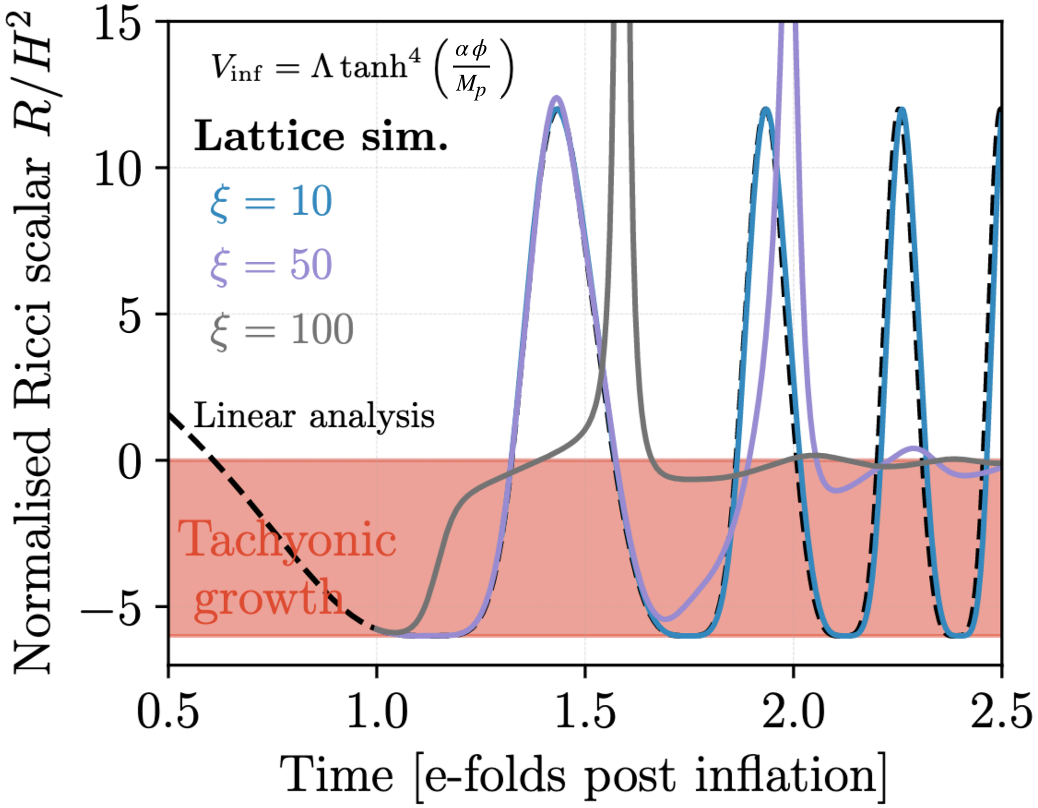


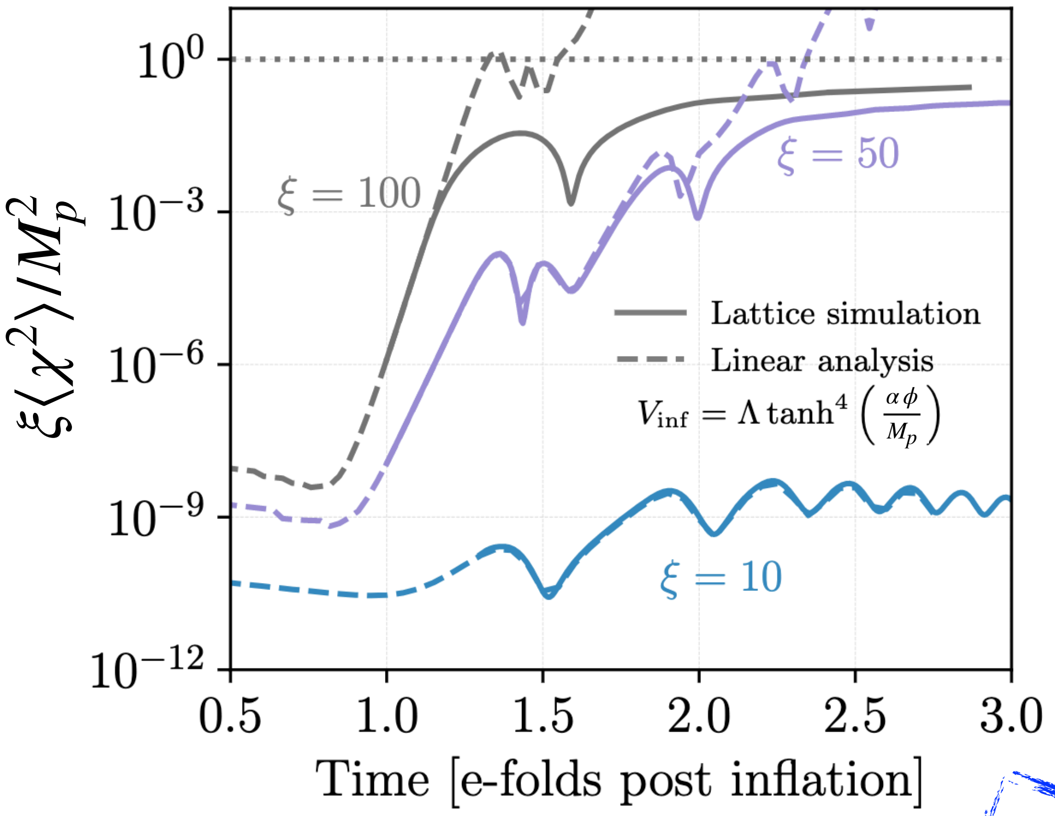
End

Transparency worked out with Adrien Florio

Non-minimally coupled Scalars

Full non-linear Geometric Preheating





Work in progress

We can compare Jordan vs Einstein frames

End

Transparency worked out with Adrien Florio