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Connecting Inflation to Particle Physics with Next Generation Observations

11.07.2023

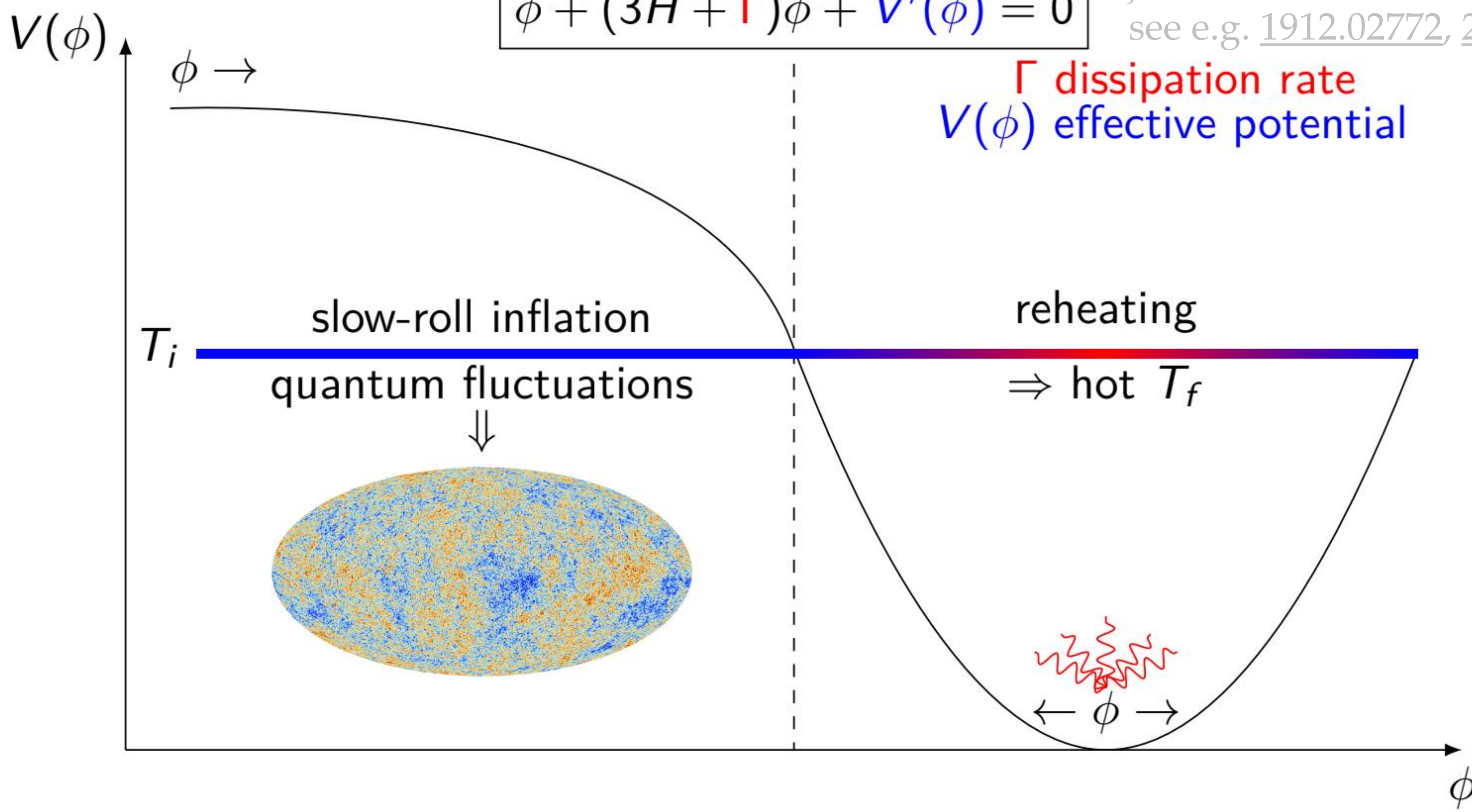
DSU 2023

EAIFR, Kigali, Rwanda

Dissipation during/after inflation

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V'(\phi) = 0$$

justification of Markovain approximation
see e.g. 1912.02772, 2108.00254



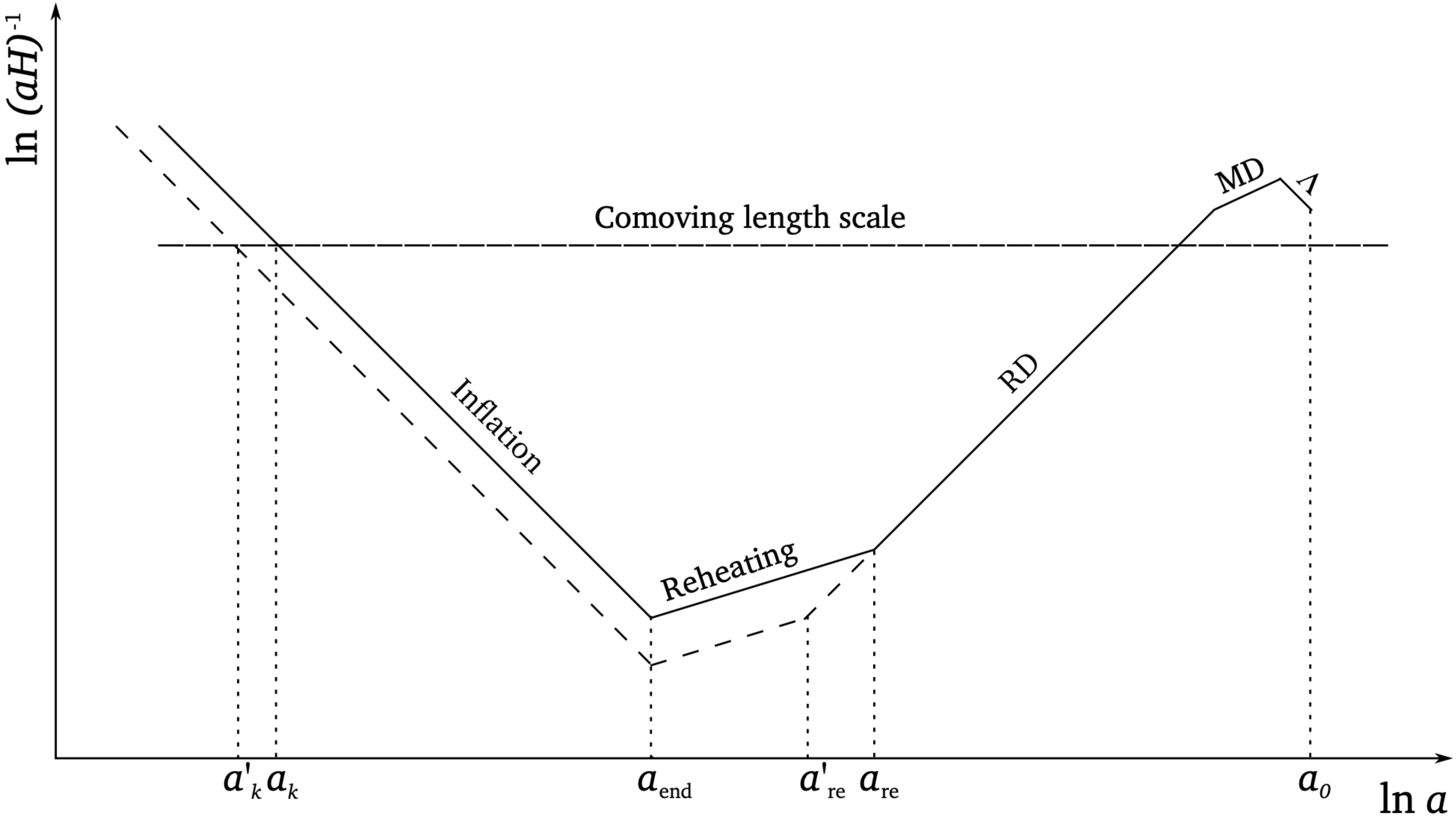
- Inflaton potential determines primordial cosmic perturbations after inflation
- Redshifting of perturbations during reheating affects observed CMB
- CMB sensitive to inflaton coupling to other particles during reheating
- Parameter degeneracies can be avoided for small inflaton coupling

Part I:
Can one in principle constrain the inflaton coupling from CMB data?

In collaboration with: Wenyuan Ai, Gilles Buldgen, Drazen Galvan, Jin U Kang, Ui Ri Mun

[slide added to make online version more structured]

Effect on CMB Modes



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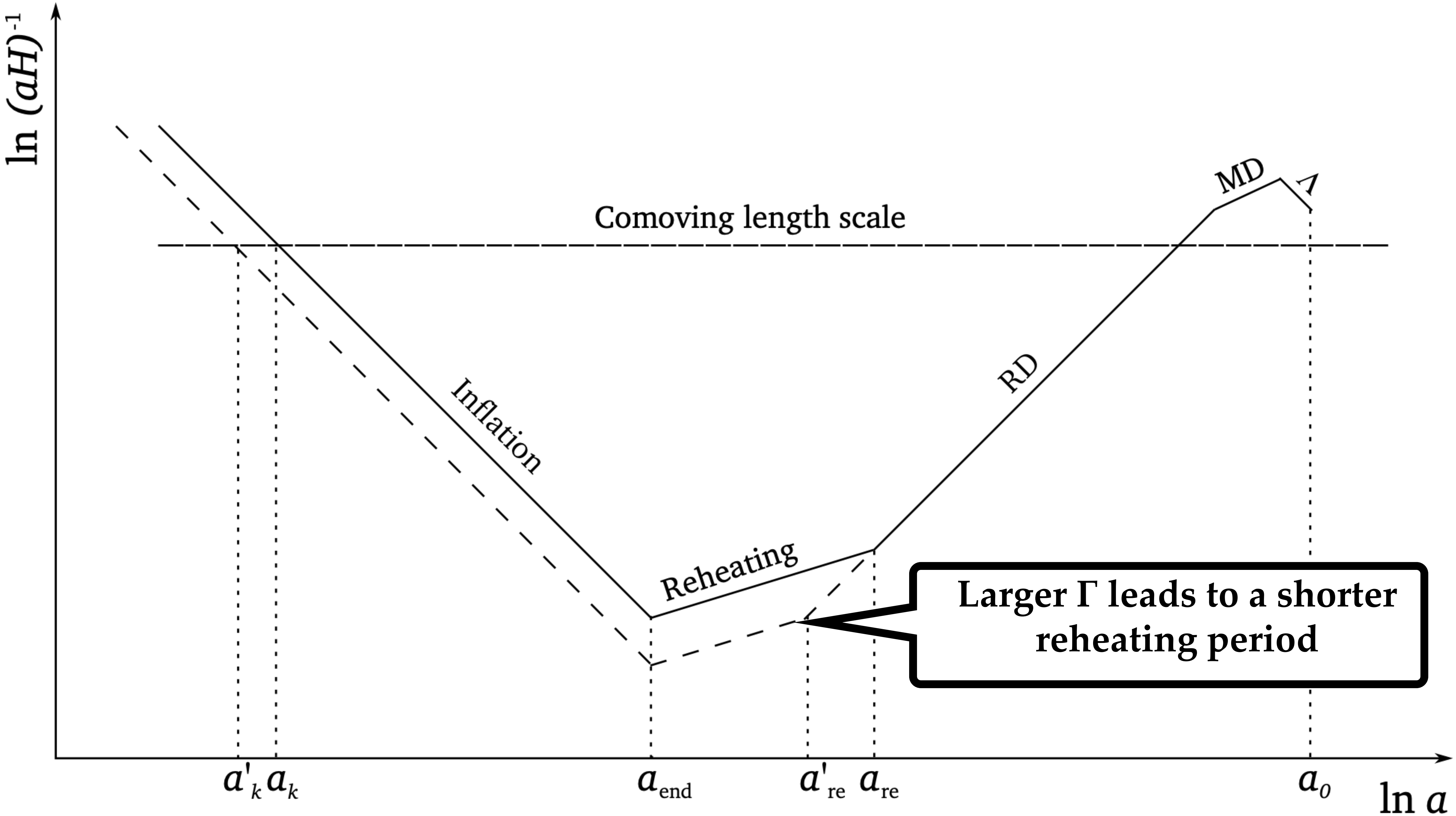

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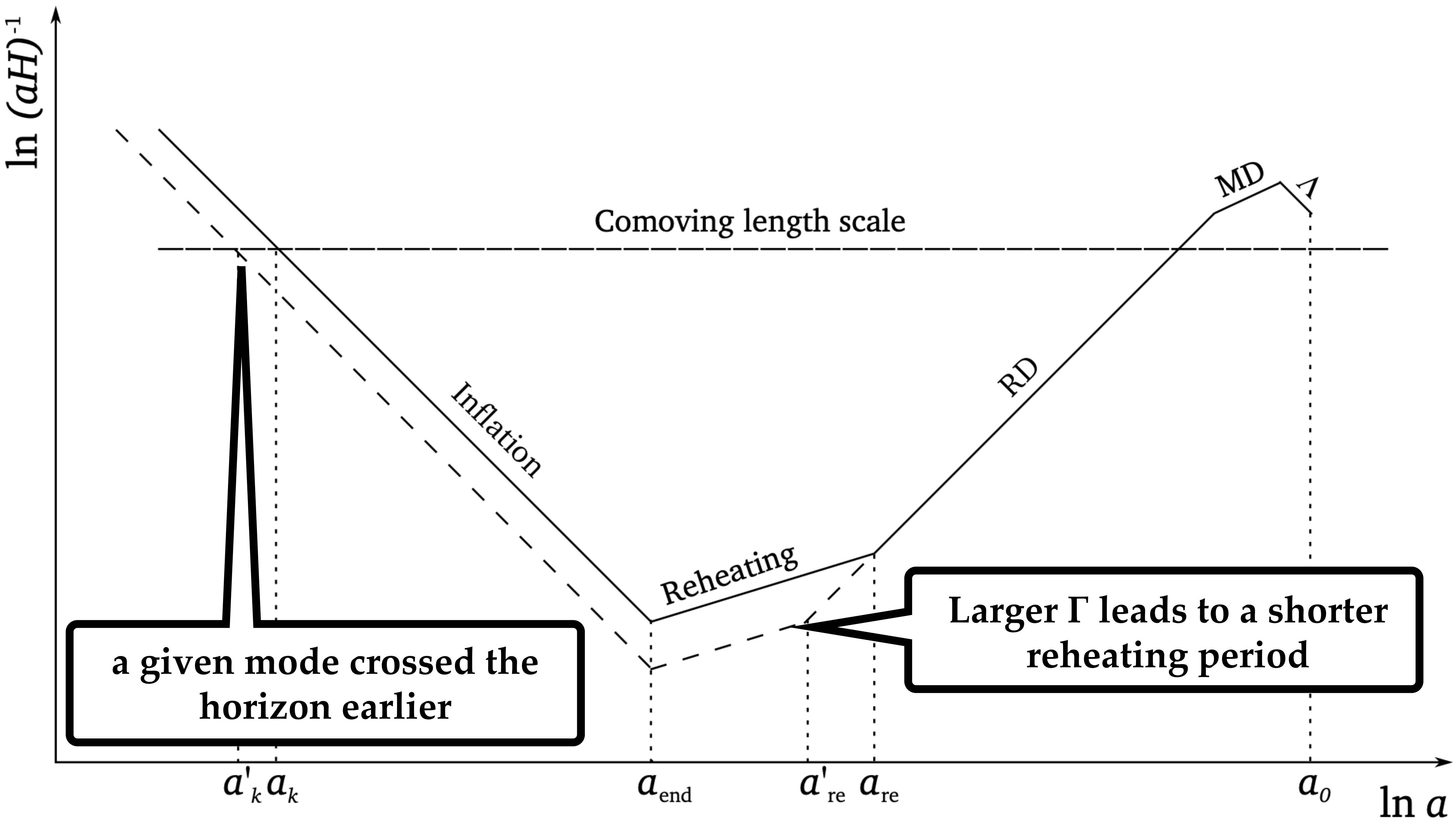
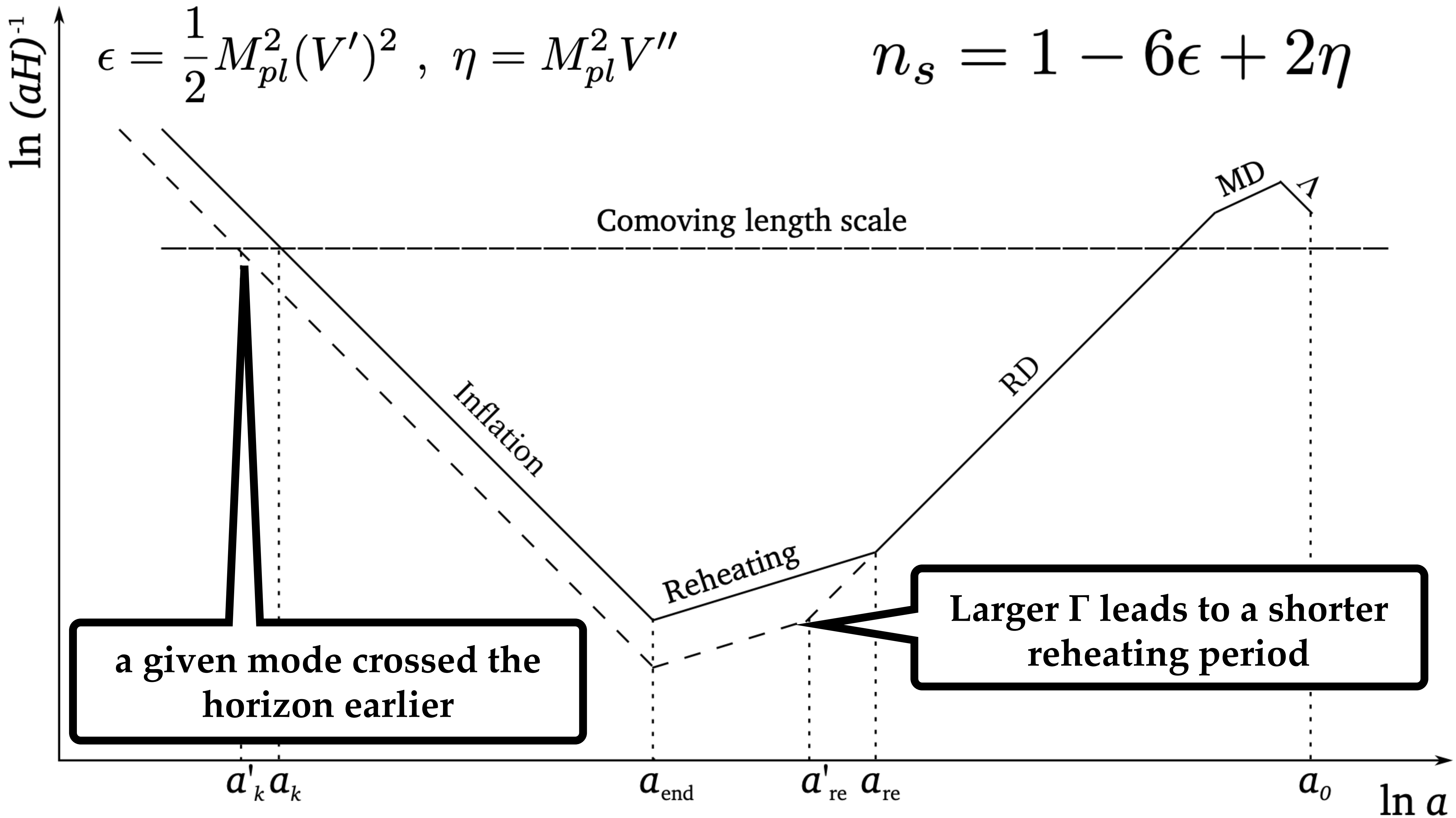


Effect on CMB Modes

$$\epsilon = \frac{1}{2} M_{pl}^2 (V')^2 \ , \ \eta = M_{pl}^2 V''$$

$$n_s = 1 - 6\epsilon + 2\eta$$



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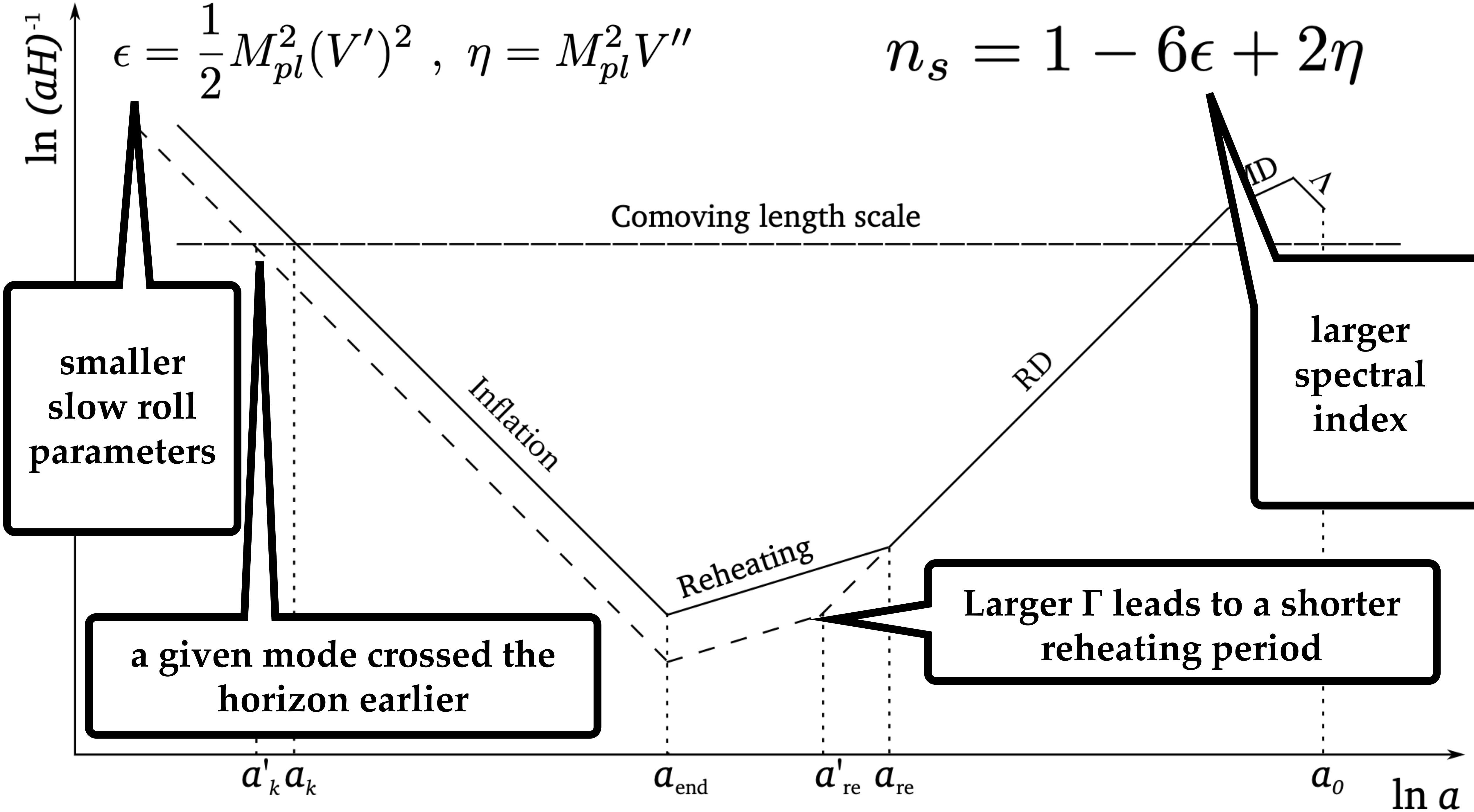
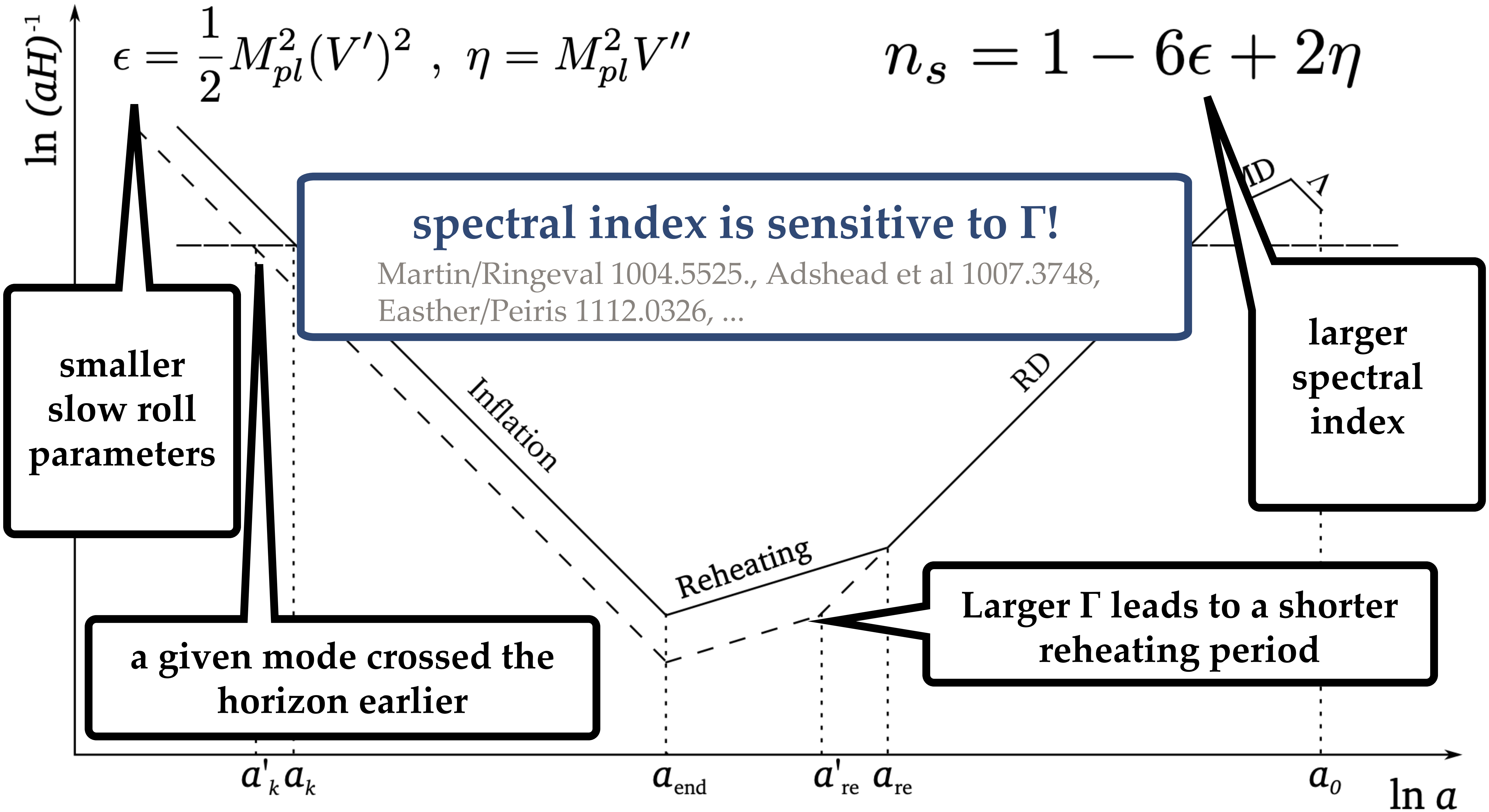


Effect on CMB Modes



What the CMB is sensitive to

- Spectrum of perturbations at end of inflation is given by choice of potential
- Energy density is also determined by the potential

$$\rho_{\rm end} \simeq \frac{4}{3}\mathcal{V}_{\rm end}$$

- Impact of reheating is determined by

  - the averaged equation of state
  (also calculable for a given potential, as inflaton dominates during reheating)

$$\bar{w}_{\rm re} = \frac{1}{N_{\rm re}} \int_0^{N_{\rm re}} w(N)dN$$

  - the duration of the reheating epoch *Nre*.

- Equivalently can use $\rho_{\rm re} = \rho_{\rm end} \exp(-3N_{\rm re}(1+\bar{w}_{\rm re}))$ to obtain reheating temperature:

$$\frac{\pi^2 g_*}{30} T_{\rm re}^4 \equiv \rho_{\rm re}. \qquad T_{\rm re} = \exp\left[-\frac{3(1+\bar{w}_{\rm re})}{4} N_{\rm re}\right] \left(\frac{40\mathcal{V}_{\rm end}}{g_* \pi^2}\right)^{1/4}$$

- *Tre* is the only quantity that is not calculable for a given potential

Connection to Observables

- We use only a small number of observables (A_s, n_s, r)
  (in principle the CMB and LSS contain more bytes, but let's start with this…)
- Need relation between observables and potential parameters and

$$T_{\rm re} = \exp\left[-\frac{3(1+\bar{w}_{\rm re})}{4} N_{\rm re}\right] \left(\frac{40\mathcal{V}_{\rm end}}{g_* \pi^2}\right)^{1/4}$$

N_{re} can be written as

$$N_{\rm re} = \frac{4}{3\bar{w}_{\rm re} - 1}\left[N_k + \ln\left(\frac{k}{a_0 T_0}\right) + \frac{1}{4}\ln\left(\frac{40}{\pi^2 g_*}\right) + \frac{1}{3}\ln\left(\frac{11 g_{s*}}{43}\right) - \frac{1}{2}\ln\left(\frac{\pi^2 M_{pl}^2 r A_s}{2\sqrt{\mathcal{V}_{\rm end}}}\right)\right]$$

where N_k can be obtained from

$$N_k = \ln\left(\frac{a_{\rm end}}{a_k}\right) = \int_{\varphi_k}^{\varphi_{\rm end}} \frac{H d\varphi}{\dot{\varphi}} \approx \frac{1}{M_{pl}^2} \int_{\varphi_{\rm end}}^{\varphi_k} d\varphi \frac{\mathcal{V}}{\partial_\varphi \mathcal{V}}$$

and φ_k is obtained by solving

$$n_s = 1 - 6\epsilon_k + 2\eta_k \ , \qquad r = 16\epsilon_k$$

A subscript k means: evaluated at the moment when the mode k crosses the horizon.

Constraining Microphysics

- Reheating ends when $\Gamma = H$. Using this and $H^2 = \frac{\rho_{\rm end}}{3M_{pl}^2} e^{-3N_{\rm re}(1+\bar{w}_{\rm re})}$ one finds

$$\Gamma|_{\Gamma=H} = \frac{1}{M_{pl}} \left(\frac{\rho_{\rm end}}{3}\right)^{1/2} e^{-3(1+\bar{w}_{\rm re})N_{\rm re}/2}.$$

- Hence we can constrain Γ from observation….
  …and Γ in principle is calculable in terms of microphysical parameters!

Hence, we can not only constrain the reheating temperature, but also the microphysics of reheating…

…which connects inflation to particle physics!

MaD 1511.03280, MaD 1903.09599

Parameters

- We use only a small number of observables (A_s, n_s, r)
- We can therefore only derive a meaningful constraint on microphysical parameters when Γ depends only on a small number of them, ideally only on one.
- We shall distinguish three classes of parameters:

{ vi } the parameters in the inflaton potential *V(φ)* define the "model of inflation".

$$\mathcal{V}(\varphi) = \sum_j \frac{\mathsf{v}_j}{j!} \frac{\varphi^j}{\Lambda^{j-4}}$$

{ gi } the inflaton couplings between the inflaton and other fields connect the "model of inflation" to an underlying "model of particle physics".

{ ai } all other parameters of the "particle physics model", e.g. masses and gauge interactions amongst the produced particles..

Our Goal: Assume that reheating is primarily driven by one interaction with coupling g, identify parameter region where g can be measured without having to specify details of the underlying particle physics model and the *{ai}*

Inflaton Decay

- Reheating ends when $\Gamma = H$. Using this and $H^2 = \frac{\rho_{\rm end}}{3M_{pl}^2} e^{-3N_{\rm re}(1+\bar{w}_{\rm re})}$ one finds

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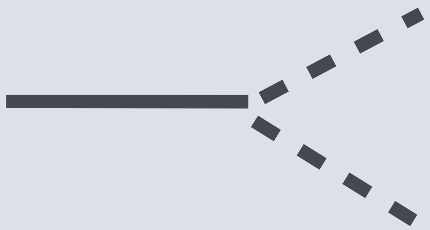
- Hence we can constrain Γ from observation….
  …and Γ in principle is calculable in terms of microphysical parameters!

- If inflaton decays via 1 → 2 or 1 → 3 decays then Γ has the form $\Gamma = \mathrm{g}^2 m_\phi / \#$.
- We assume Yukawa coupling, simple rescaling allows to constrain other interactions

	Yukawa	scalar	axion-like	scalar
interaction	$y\Phi\bar{\psi}\psi$	$g\Phi\chi^2$	$\frac{\sigma}{\Lambda}\Phi F_{\mu\nu}\tilde{F}^{\mu\nu}$	$\frac{h}{3!}\Phi\chi^3$
g	y	$\tilde{g} = g/m_\phi$	$\tilde{\sigma} = \sigma m_\phi/\Lambda$	h
#	8π	8π	4π	$3!64(2\pi)^3$
rescaling factor	1	1	$\frac{1}{\sqrt{2}}$	$8\sqrt{6}\pi$



Parametric Dependencies

- However in general Γ is not a simple function of one single coupling: Once occupation numbers in the plasma reach O[1], feedback effects brig in dependence on the properties of the produced particles, and hence the *{ai}*

- Moreover, other effects can also cause a dependence on the *{ai}* :

  - Unknown value of g* in early universe
  - Non-standard expansion history
  - Plasma equilibration
  - Gravitational waves produced after inflation
  - Radiative corrections to the potential
  - Non-instantaneous inflaton decay
  - Multifield effects
  - Foreground and late time effects

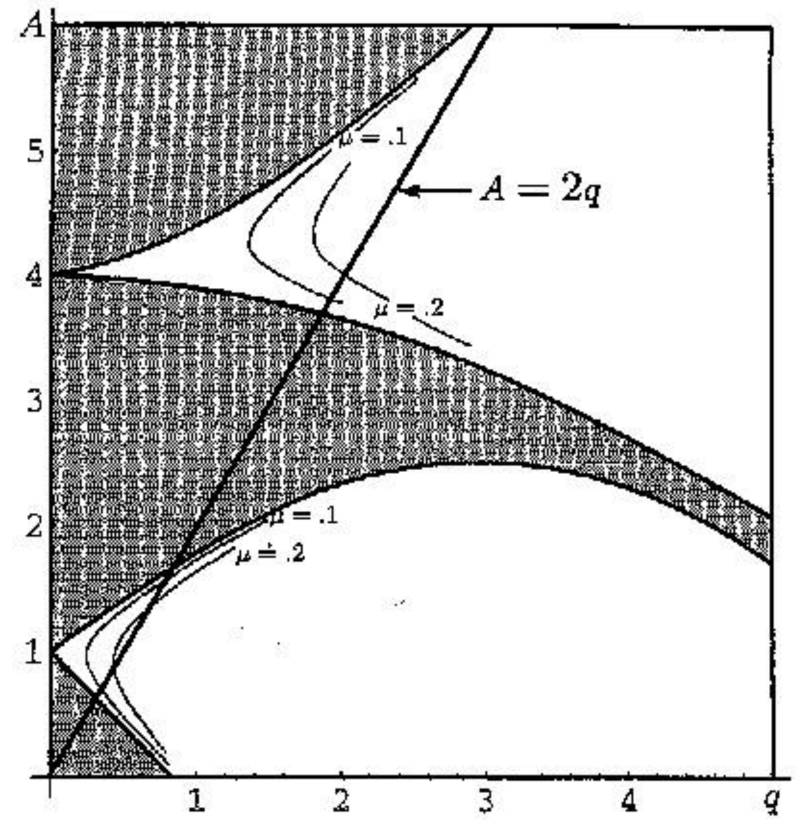
- Within effective single field picture, avoiding feedback from produced particles turns out to be the strongest restriction

MaD 1903.09599

Parametric Resonance

Mode equation for produced particles during harmonic oscillations ***$\varphi(t) = \Phi \cos(\omega t)$*** can are rewritten as **Mathieu equation** with z ~ ωt Kofman/Linde/Starobinski

$$\mathcal{X}_k''(z) + \left[A_k - 2q\cos(2z)\right]\mathcal{X}_k(z) = 0$$

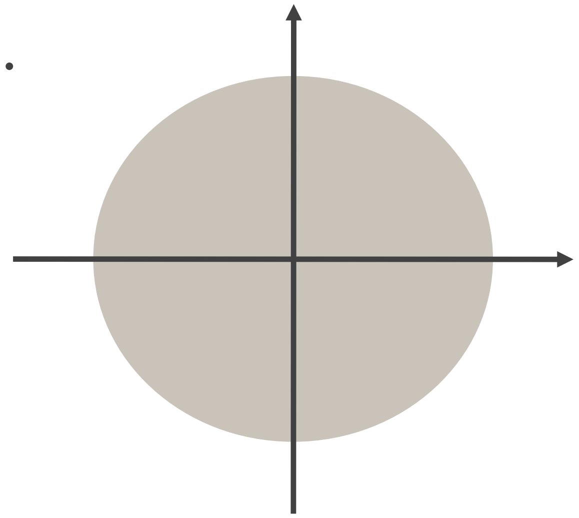


We demand

I) $q < 1$ to avoid "broad resonance"

II) $q^2\, m < H$ to make sure that redshifting avoids "narrow resonance"

Note that redshifting depends only on the model of inflation { vi } because φ dominates during reheating. Avoiding the narrow resonance by rescattering would introduce a dependence on { ai }.



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Kofman/Linde/Starobinski

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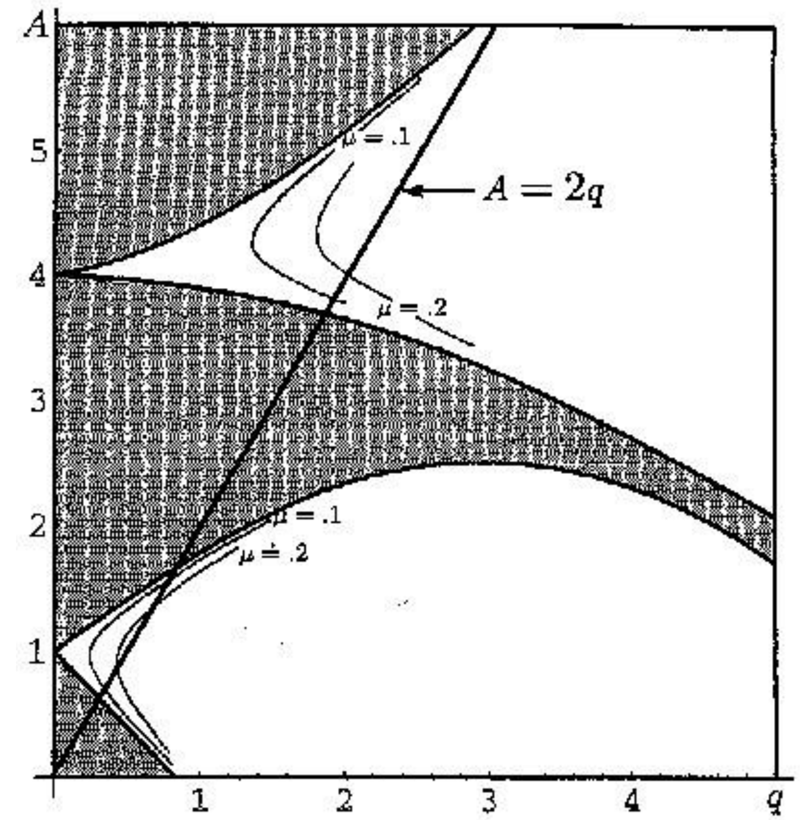

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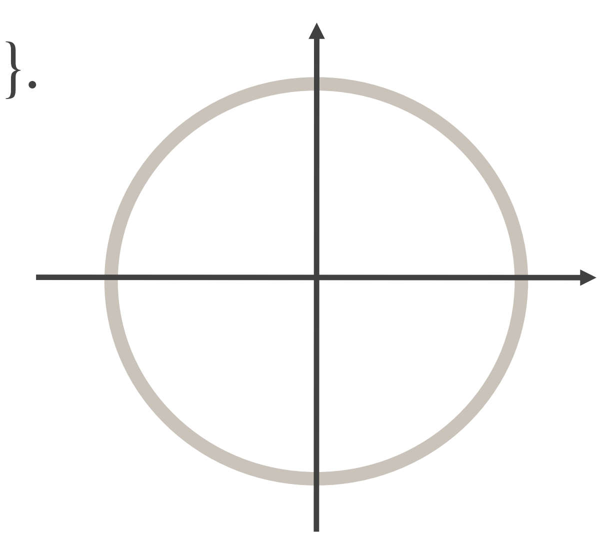

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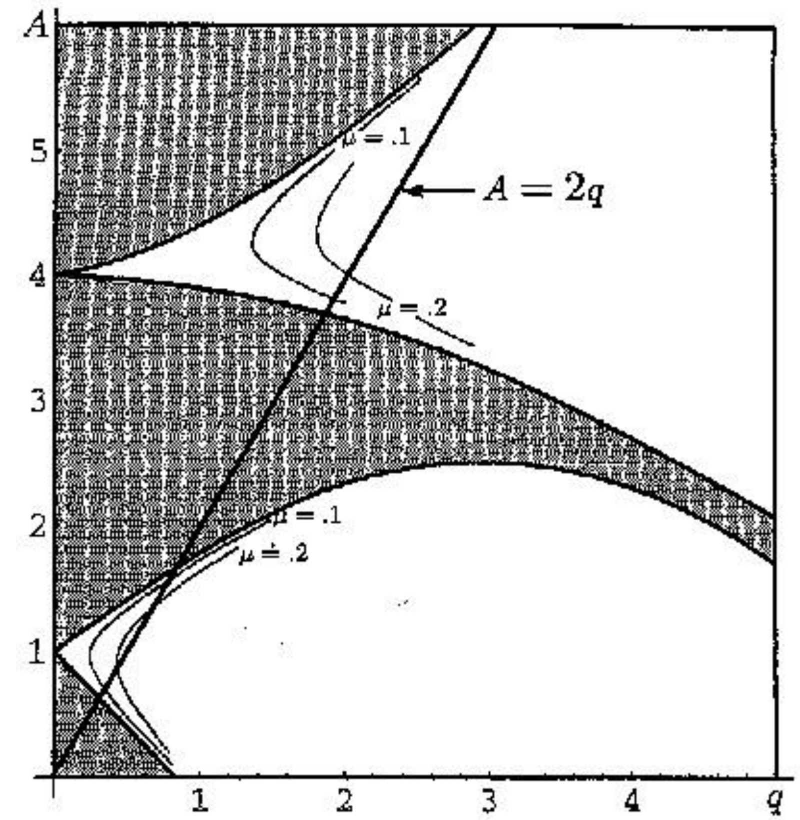


For self-interaction terms this leads to

$$|\mathrm{v}_j| \ll \left(\frac{\omega}{\varphi}\right)^{j-\frac{5}{2}} \min\left(\sqrt{\frac{\omega}{M_{pl}}}, \sqrt{\frac{\omega}{\varphi}}\right) \left(\frac{\omega}{\Lambda}\right)^{4-j},$$

And for couplings to other fields via operator $g\Phi^j\Lambda^{4-D}\mathcal{O}[\{\mathcal{X}_i\}]$

$$|g| \ll \left(\frac{m_\phi}{\varphi_{\rm end}}\right)^{j-\frac{1}{2}} \min\left(\sqrt{\frac{m_\phi}{M_{pl}}}, \sqrt{\frac{m_\phi}{\varphi_{\rm end}}}\right) \left(\frac{m_\phi}{\Lambda}\right)^{4-D}$$

MaD 1903.09599

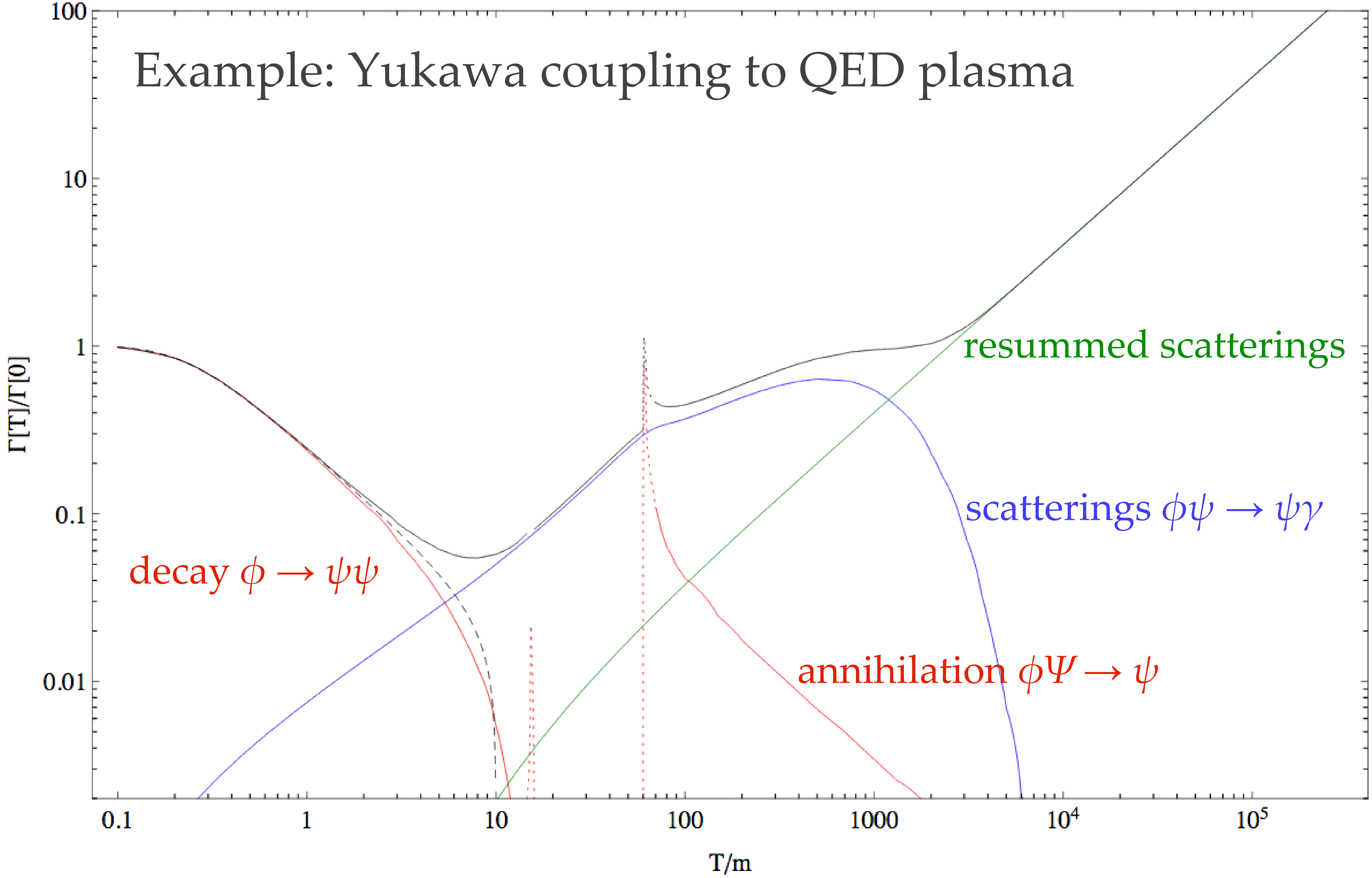
What about thermal feedback?

- Even if there is no resonance, thermal effects can potentially modify Γ

interaction	process	contribution to Γ
$g\Phi\chi^2$	$\varphi \to \chi\chi$	$\frac{g^2}{8\pi M_\phi}\left(1-2M_\chi/M_\phi\right)^{1/2}\left(1+2f_B(M_\phi/2)\right)\theta(M_\phi-2M_\chi)$
$\frac{\kappa}{4}\Phi^2\chi^2$	$\varphi\varphi \to \chi\chi$	$\frac{h^2\varphi^2}{256\pi M_\phi}\left(1-M_\chi/M_\phi\right)^{1/2}\left(1+2f_B(M_\phi)\right)\theta(M_\phi-M_\chi)$
$\frac{\sigma}{\Lambda}\Phi F_{\mu\nu}\tilde{F}^{\mu\nu}$	$\varphi \to \gamma\gamma$	$\frac{\sigma^2}{4\pi}\frac{M_\phi^3}{\Lambda^2}\left(1-2M_\gamma/M_\phi\right)^{1/2}\left(1+2f_B(M_\phi/2)\right)\theta(M_\phi-2M_\gamma)$
$y\Phi\bar{\psi}\psi$	$\varphi \to \psi\bar{\psi}$, $M_\psi \simeq m_\psi$	$\frac{y^2}{8\pi}M_\phi\left(1-2m_\psi/M_\phi\right)^{3/2}\left(1-2f_F(M_\phi/2)\right)\theta(M_\phi-2m_\psi)$
	$\varphi \to \psi\bar{\psi}$, $M_\psi \gg m_\psi$	$\frac{y^2}{8\pi}M_\phi\left(1-2M_\psi/M_\phi\right)^{1/2}\left(1-2f_F(M_\phi/2)\right)\theta(M_\phi-2M_\psi)$

- Prefactor typically depends on a single coupling constant $\in \{ g_i \}$
- Phase space given by "thermal masses" depends on the $\{ \alpha_i \}$, becomes relevant when $T > m_\varphi/\alpha_i$
- Quantum statistical effects are relevant for occupation numbers O[1] ($T > m_\varphi$ for equilibrium distributions), depends on $\{ \alpha_i \}$ because rescatterings determine distribution functions

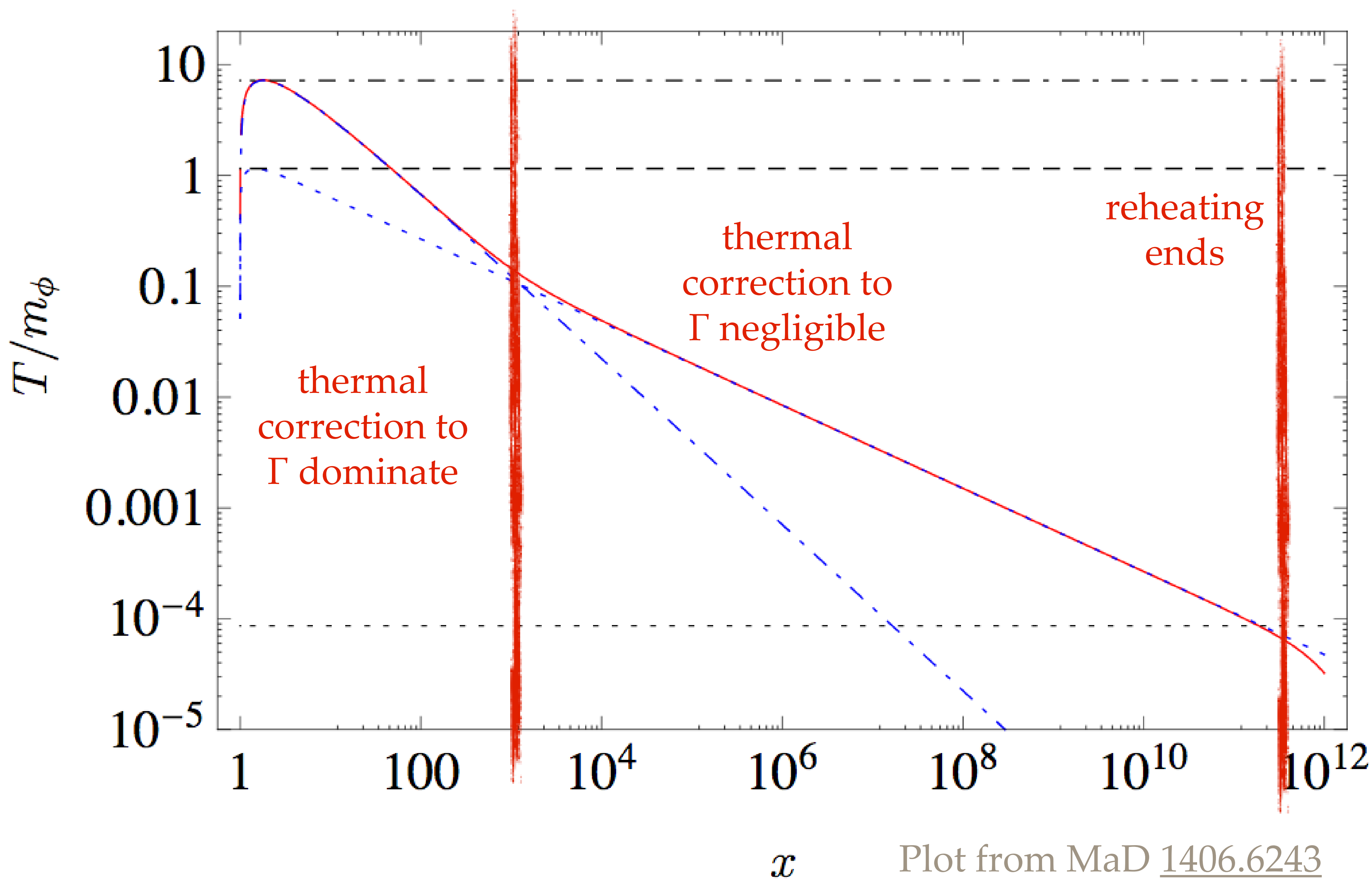
What about thermal feedback?



MaD/Kang 1305.0267

What about thermal feedback?

Thermal corrections modify the thermal history during reheating, but the effect on the expansion history is subdominant within the regime where the previous conditions are fulfilled. MaD 1903.09599



Plot from MaD 1406.6243

No visible effect in CMB!

Summary I

- The inflaton coupling can in principle be "measured" in the CMB if the inflaton self-interactions $\mathcal{V}(\varphi) = \sum_j \frac{\mathsf{v}_j}{j!} \frac{\varphi^j}{\Lambda^{j-4}}$ are smaller than

$$|\mathsf{v}_j| \ll \left(\frac{\omega}{\varphi}\right)^{j-\frac{5}{2}} \min\left(\sqrt{\frac{\omega}{M_{pl}}}, \sqrt{\frac{\omega}{\varphi}}\right) \left(\frac{\omega}{\Lambda}\right)^{4-j},$$

- And the couplings to other fields $\mathsf{g}\Phi^j \Lambda^{4-D} \mathcal{O}[\{\mathcal{X}_i\}]$ are smaller than

$$|\mathsf{g}| \ll \left(\frac{m_\phi}{\varphi_{\rm end}}\right)^{j-\frac{1}{2}} \min\left(\sqrt{\frac{m_\phi}{M_{pl}}}, \sqrt{\frac{m_\phi}{\varphi_{\rm end}}}\right) \left(\frac{m_\phi}{\Lambda}\right)^{4-D}$$

- Practically this restricts us to models where
  - The field elongation at the end of reheating is small enough for oscillations to take place in a mildly non-linear regime
  - The inflaton couplings to other fields are weak (hidden sector inflation)
- For plateau models, the above roughly simplifies to

$$|\mathsf{v}_j| \ll \left(3\pi^2 r A_s\right)^{(j-2)/2} \quad , \quad |\mathsf{g}| \ll \left(3\pi^2 r A_s\right)^{j/2}$$

MaD 1903.09599

Part II:
Can near future observations in practice do the measurement?

In collaboration with: Jin U Kang, Lei Ming, Ui Ri Mun, Isabel Oldengott

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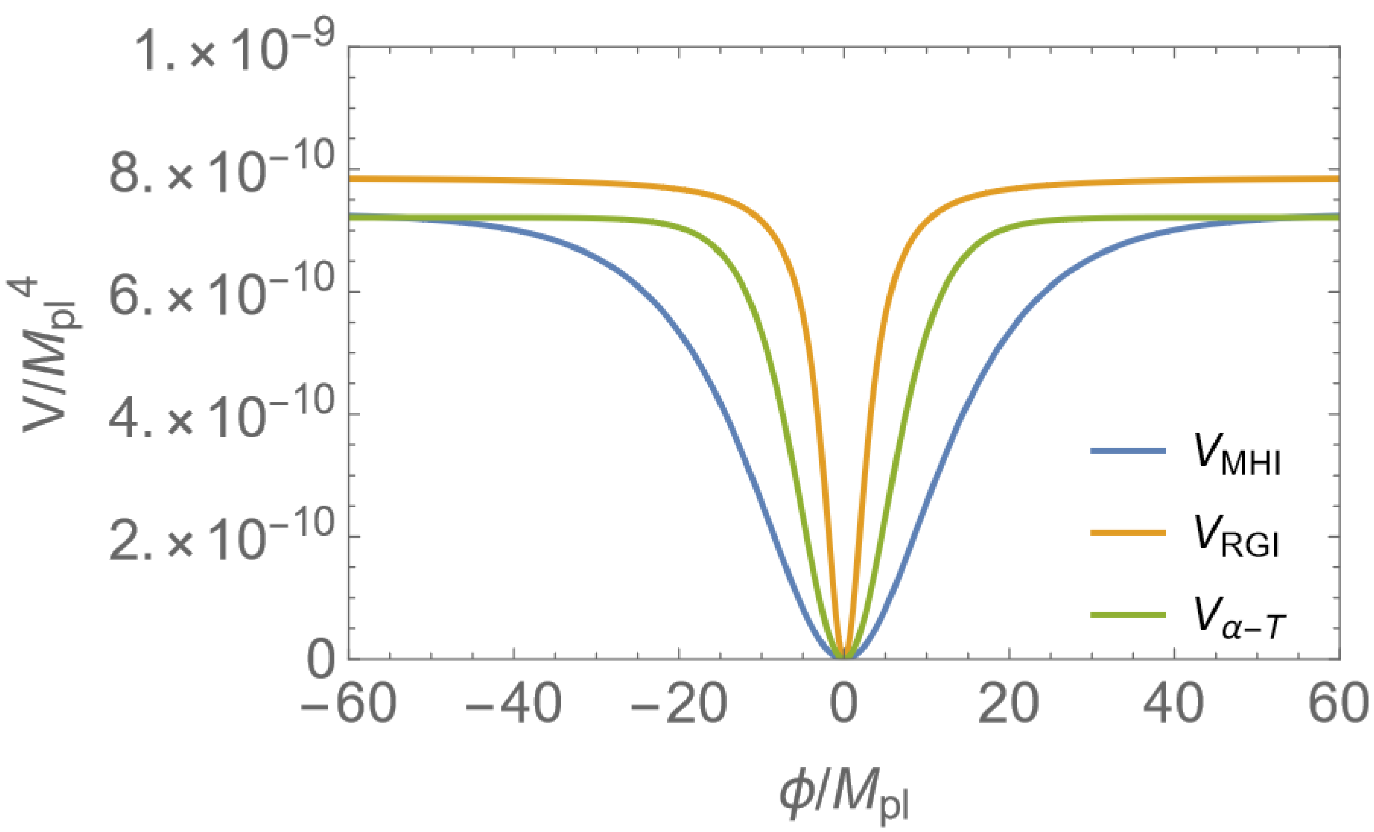
Specific Examples



Mutated Hilltop Inflation (MHI)

$$\mathcal{V}(\varphi) = M^4\left[1 - \frac{1}{\cosh(\alpha\varphi/M_{pl})}\right]$$

Radion Gauge Inflation (RGI)

$$\mathcal{V}(\varphi) = M^4\frac{(\varphi/M_{pl})^2}{\alpha + (\varphi/M_{pl})^2}$$

α -attractor T model (α-T)

$$\mathcal{V}(\varphi) = M^4\tanh^{2n}\left(\frac{\varphi}{\sqrt{6\alpha}M_{pl}}\right)$$

- Potentials have three parameters
  - M determines the scale of inflation
  - α determines the inflaton mass
- Together with the inflaton coupling there are three parameters…
- …and in principle three observables

$$(A_s, n_s, r)$$

Example: RGI Model

Potential

$$\mathcal{V}(\varphi) = M^4 \frac{(\varphi/M_{pl})^2}{\alpha + (\varphi/M_{pl})^2} \quad = \quad \frac{1}{2} m_\phi^2 \varphi^2 + \frac{g_\phi}{3!}\varphi^3 + \frac{\lambda_\phi}{4!}\varphi^4 + \mathcal{O}[\varphi^5]$$

Scale of inflation

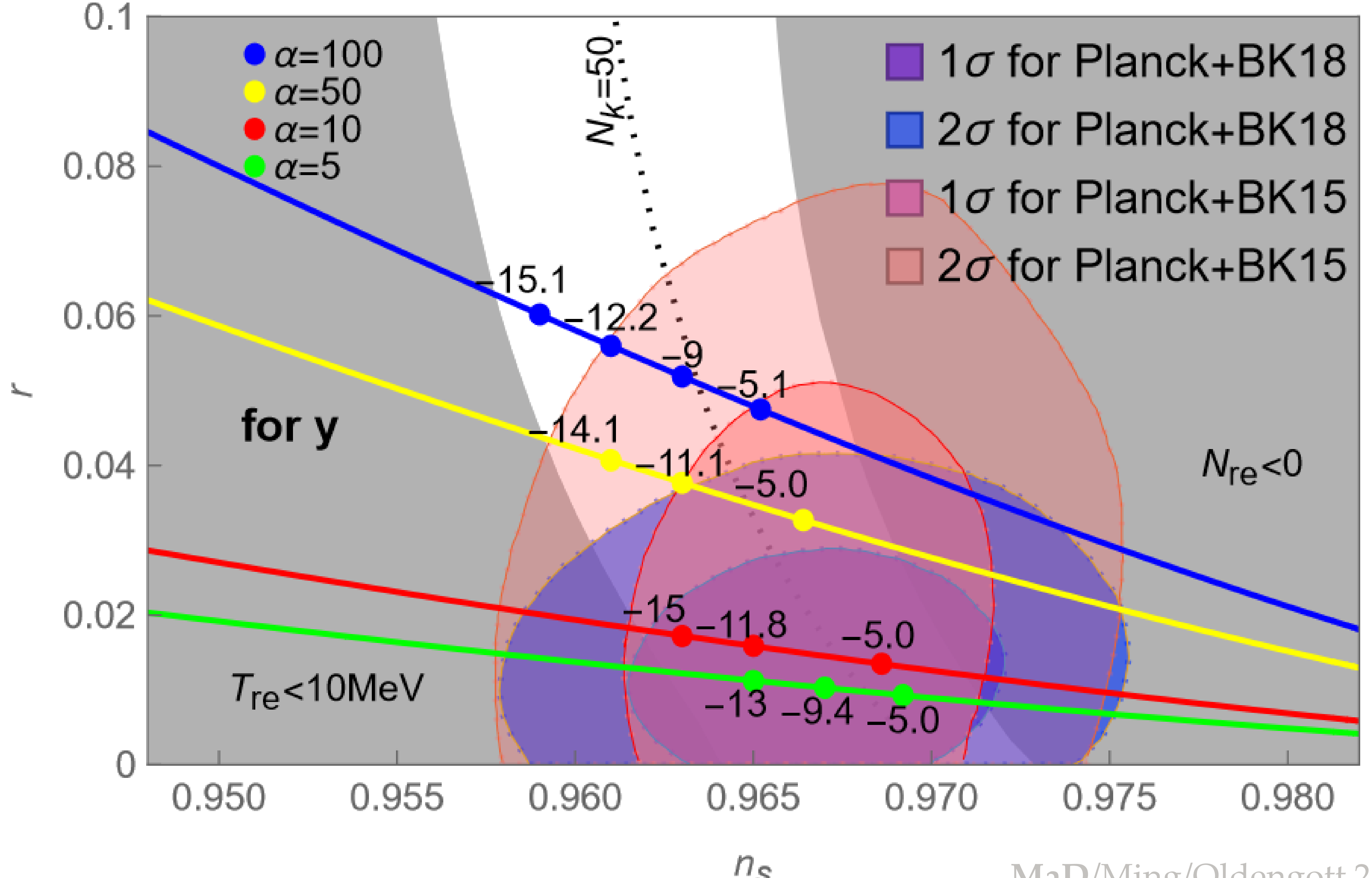
$$M = M_{pl} \left(\frac{3\pi^2}{2} r A_s \left(1 + \alpha \frac{M_{pl}^2}{\varphi_k^2} \right) \right)^{1/4} \qquad \alpha = \frac{432 r^2}{(8(1-n_s)+r)^2 (4(1-n_s)-r)}.$$

Inflaton mass and self-interaction

$$m_\phi = \sqrt{\frac{2}{\alpha}} \frac{M^2}{M_{pl}} \,, \quad g_\phi = 0 \,, \quad \lambda_\phi = -\frac{24 M^4}{\alpha^2 M_{pl}^4}$$

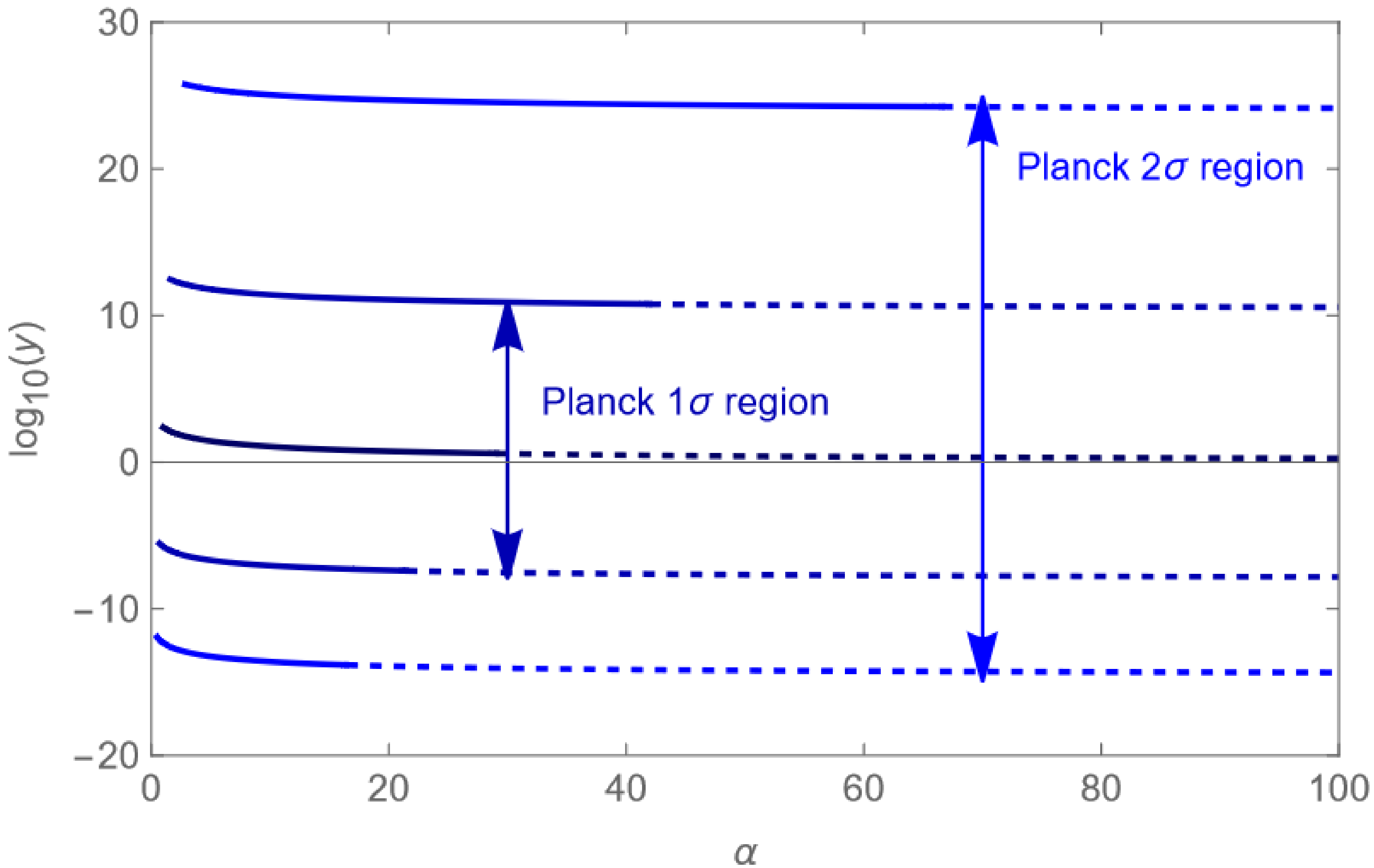
CMB Prediction in RGI Models

Example: reheating through a Yukawa coupling y
(constraints on other couplings can be obtained with rescaling table in Part I)



MaD/Ming/Oldengott 2303.13503

Practical Problems



- Error bar on spectral index is too large to fix all three parameters from observation; here we fix α by hand (e.g. by model building), which defines a family of models
- Range of allowed values for α is restricted by requirement to avoid feedback during reheating (conditions from Part I)

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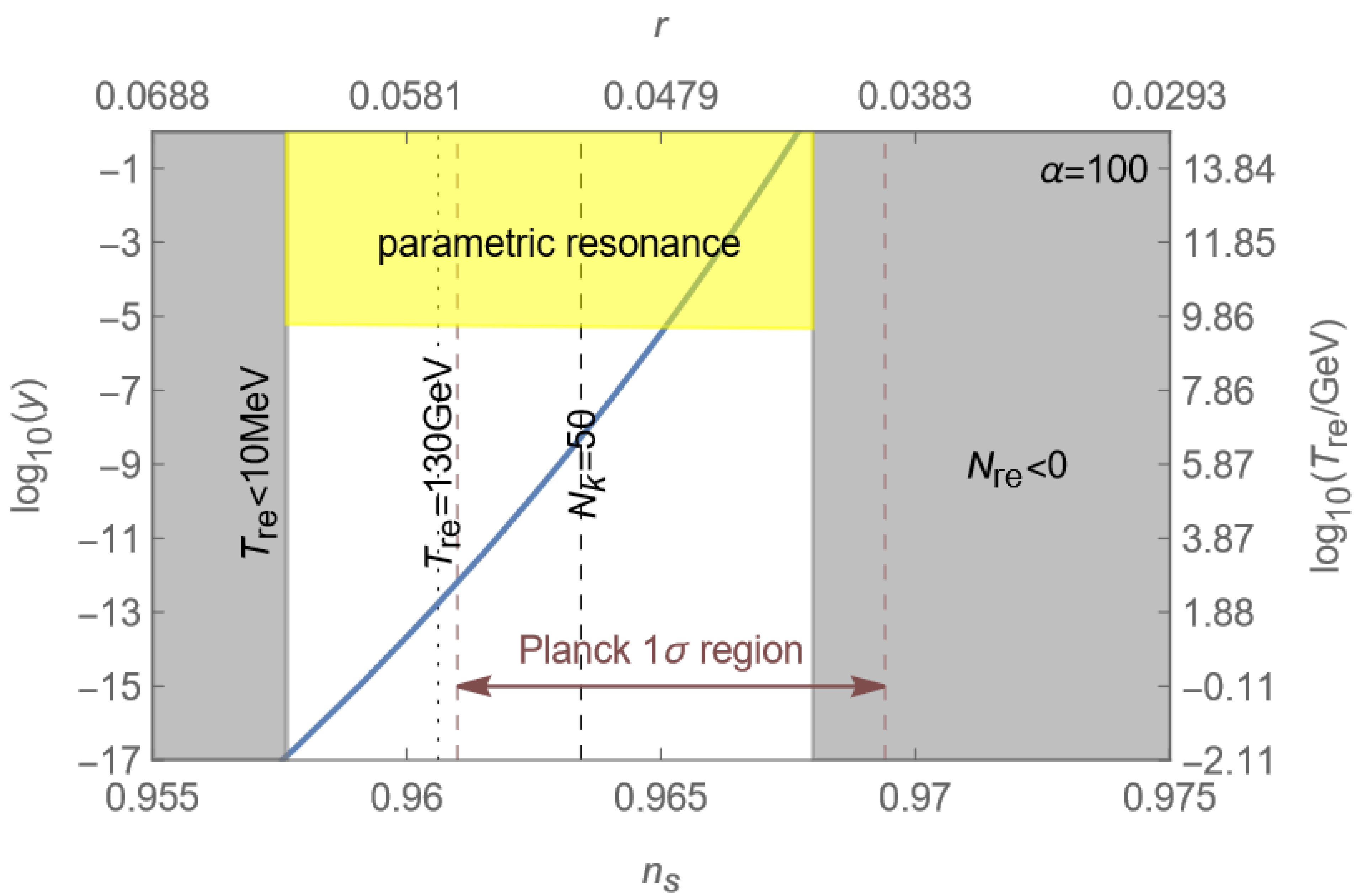
Current Constraints



- Current data does not permit to impose a meaningful constraint

MaD/Ming/Oldengott 2303.13503

(cf. also **MaD**/Kang/Mun 1708.01197)

Future Sensitivities

We employed two methods to estimate the sensitivities of future observations:

- An analytic method that simply assumes a Gaussian likelihood for the sensitivities in n_s and r **MaD**/Ming 2208.07609

- An Forecasts with a modified version of CLASS and MontePython with the free parameters **MaD**/Ming/Oldengott 2303.13503

$$X = \{\omega_{\rm b}, \omega_{\rm cdm}, 100\theta_{\rm s}, \tau_{\rm reio}\} + \log_{10}(y) + M\ ;$$

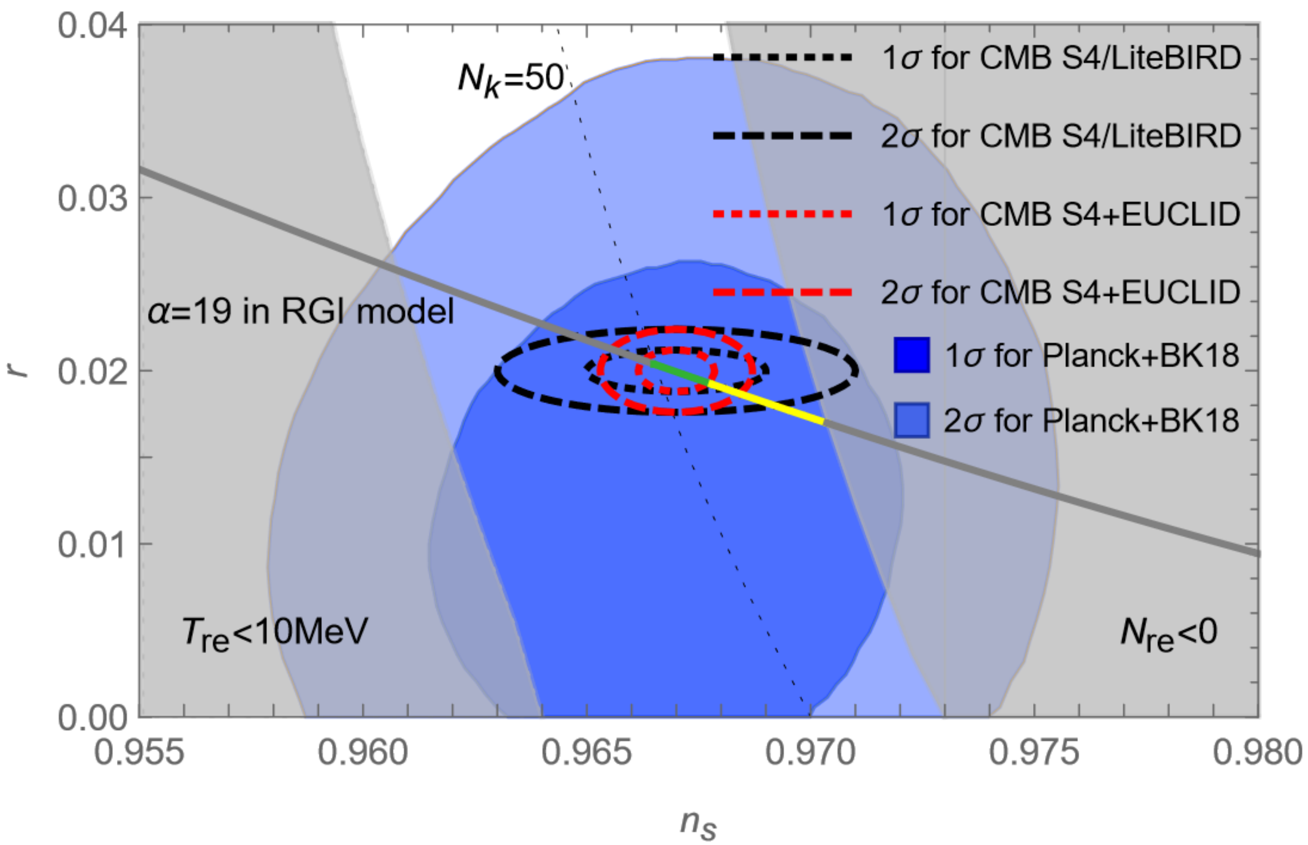
(using build-in functions for LiteBIRD and CMB-S4)

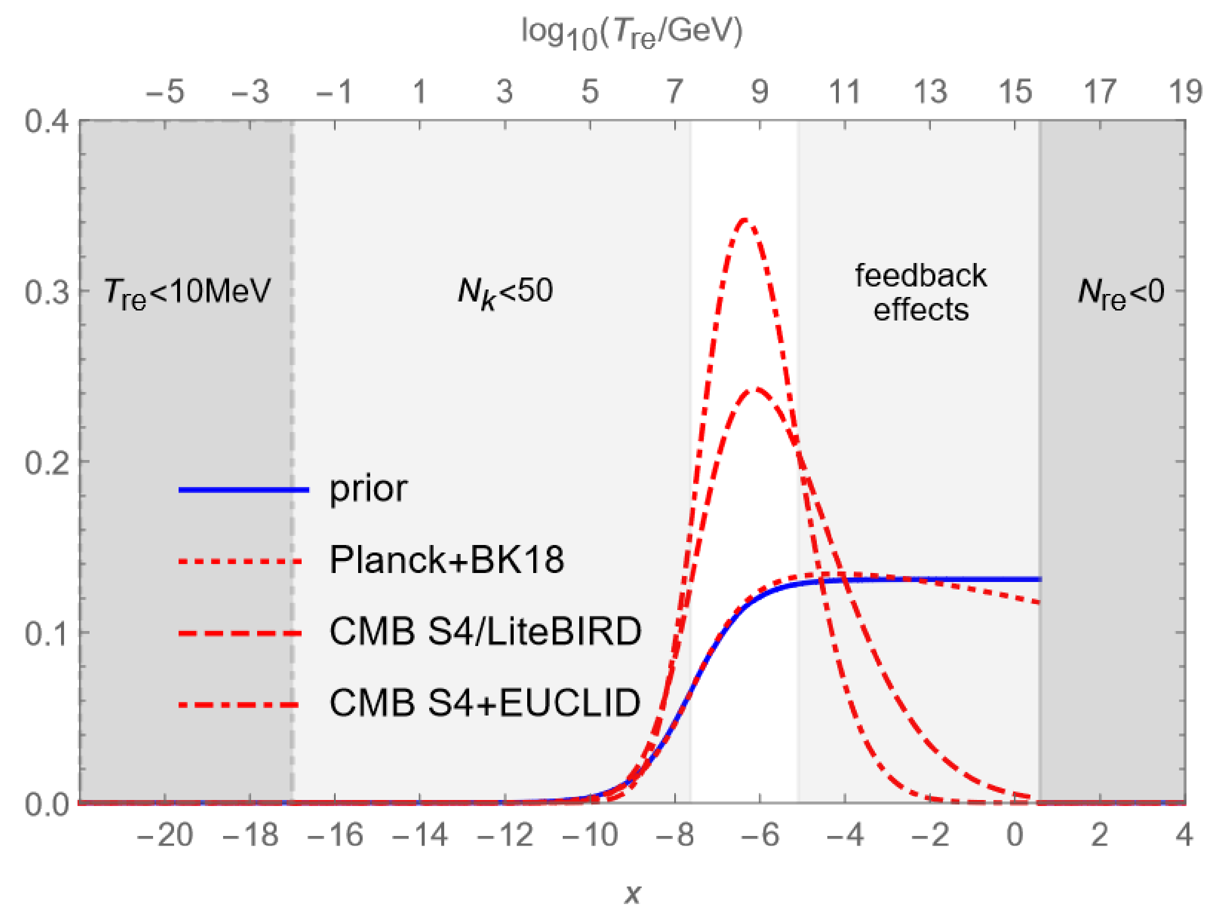
Analytical Method

Define prior in $x = ln(y)$ $$P(\mathcal{D}|x) = c_2 \mathcal{N}(n_s(x), r(x)|\bar{n}_s, \sigma_{n_s}; \bar{r}, \sigma_r)\theta(r)\tilde{\gamma}(x),$$

With 2dimensional Gaussian in n_s and r

$$\mathcal{N}(n_s, r|\bar{n}_s, \sigma_{n_s}; \bar{r}, \sigma_r) = \frac{1}{2\pi\sigma_{n_s}\sigma_r} \exp\left(-\frac{1}{2}\left(\frac{n_s - \bar{n}_s}{\sigma_{n_s}}\right)^2 \left(\frac{r - \bar{r}}{\sigma_r}\right)^2\right).$$

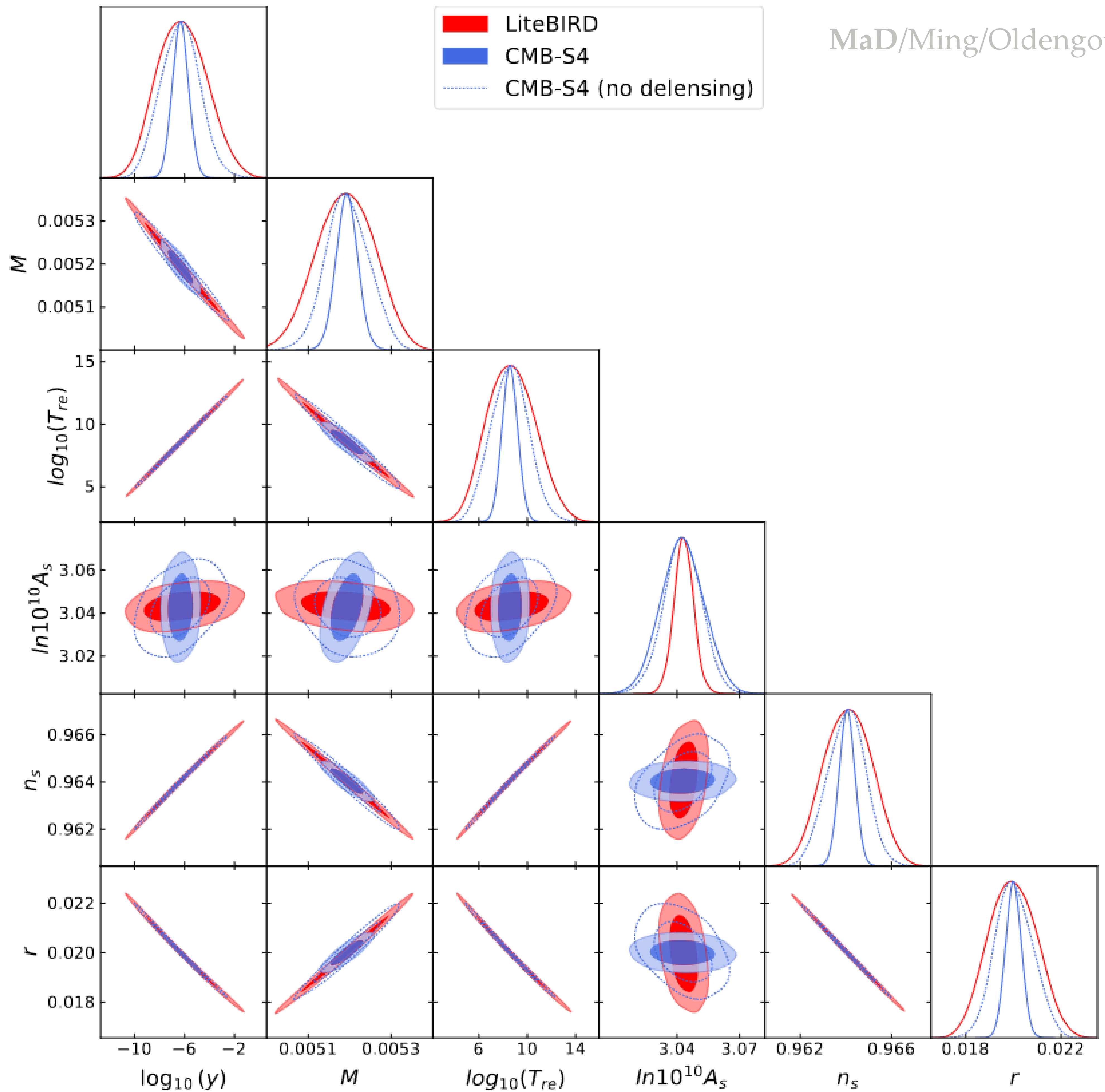




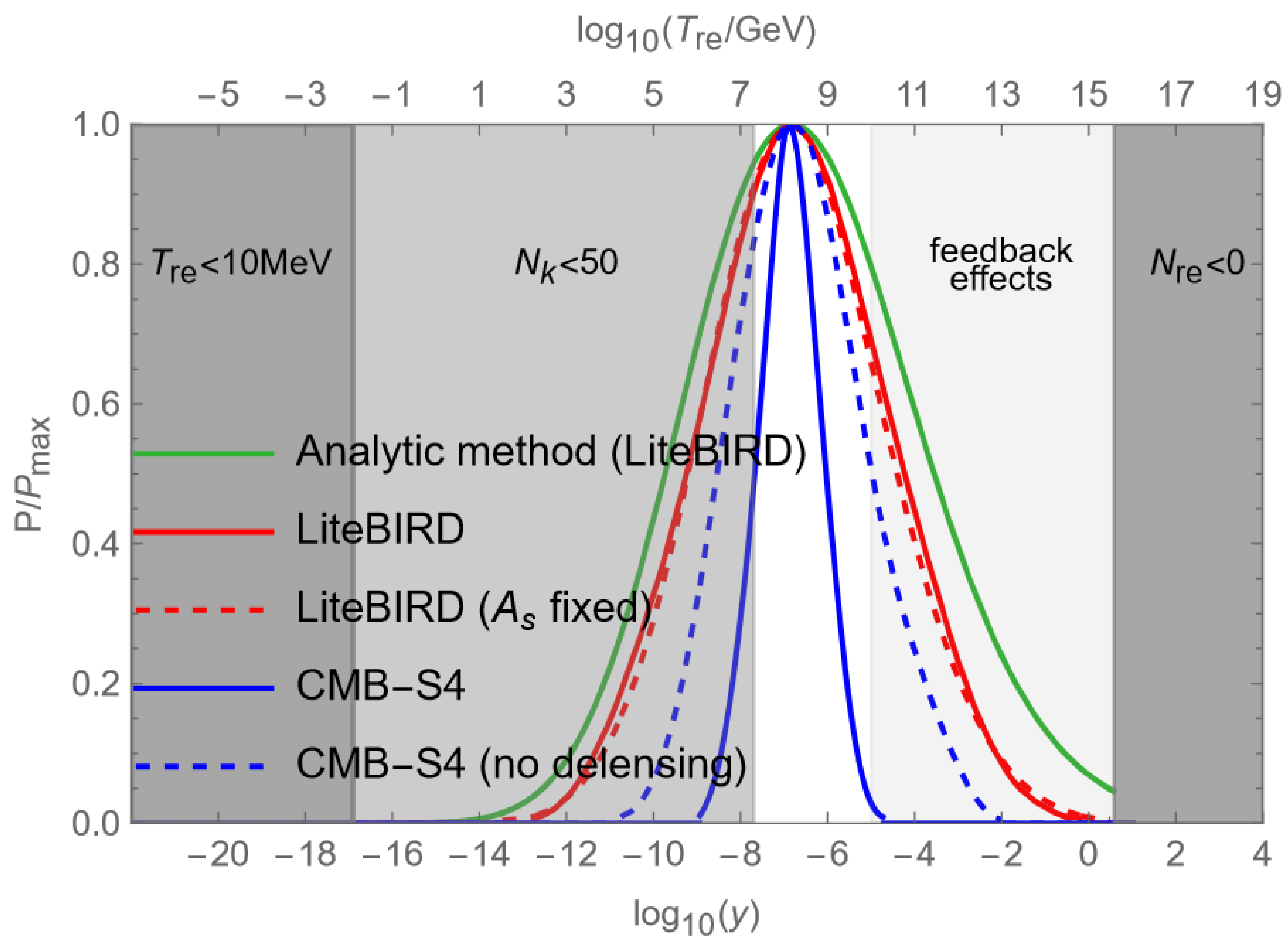
Next generation observations can probe *Tre* and the inflaton coupling!

MaD/Ming 2208.07609

MaD/Ming/Oldengott 2303.13503



Forecast Method: RGI



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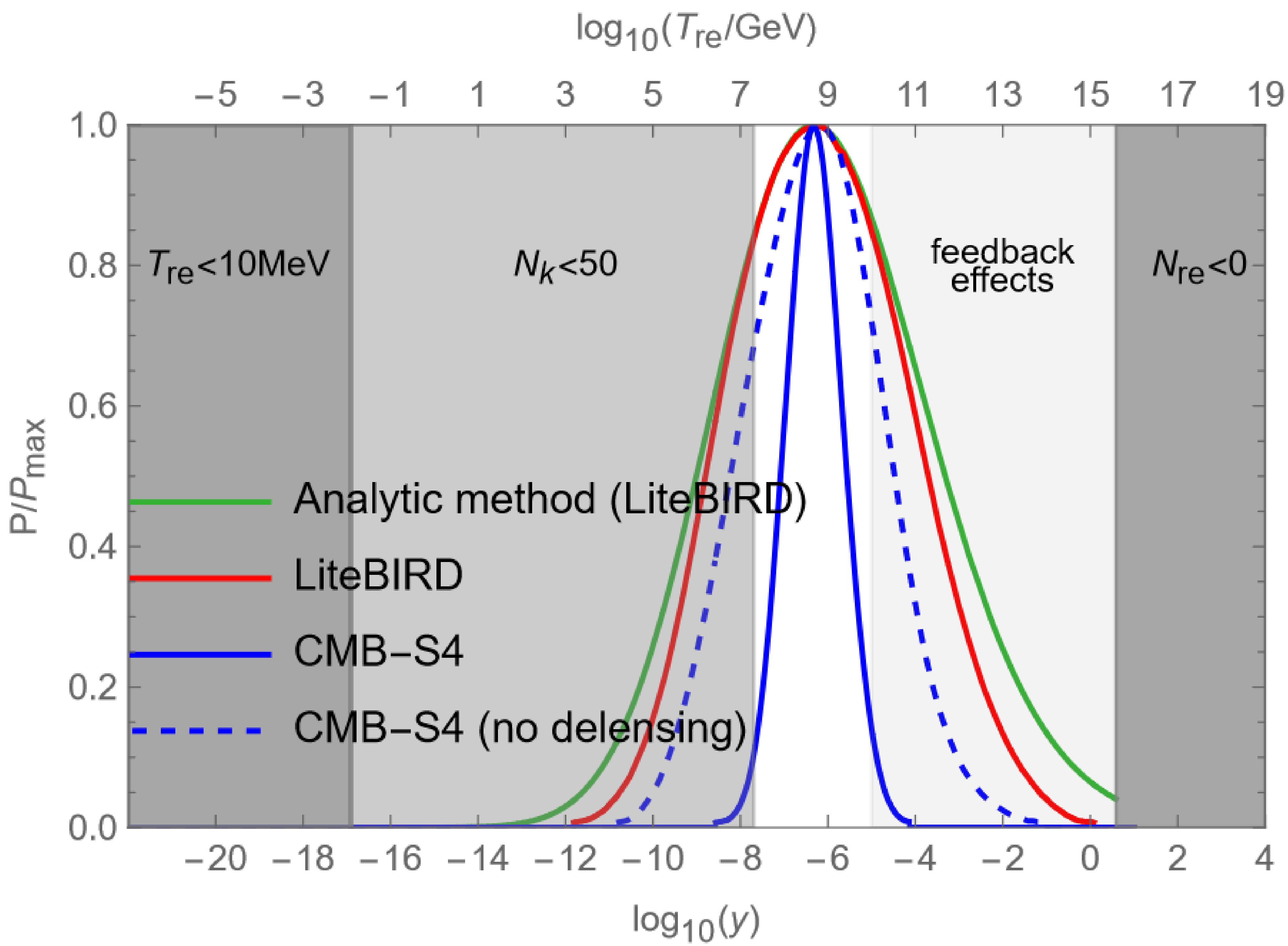
Forecast Method: MHI



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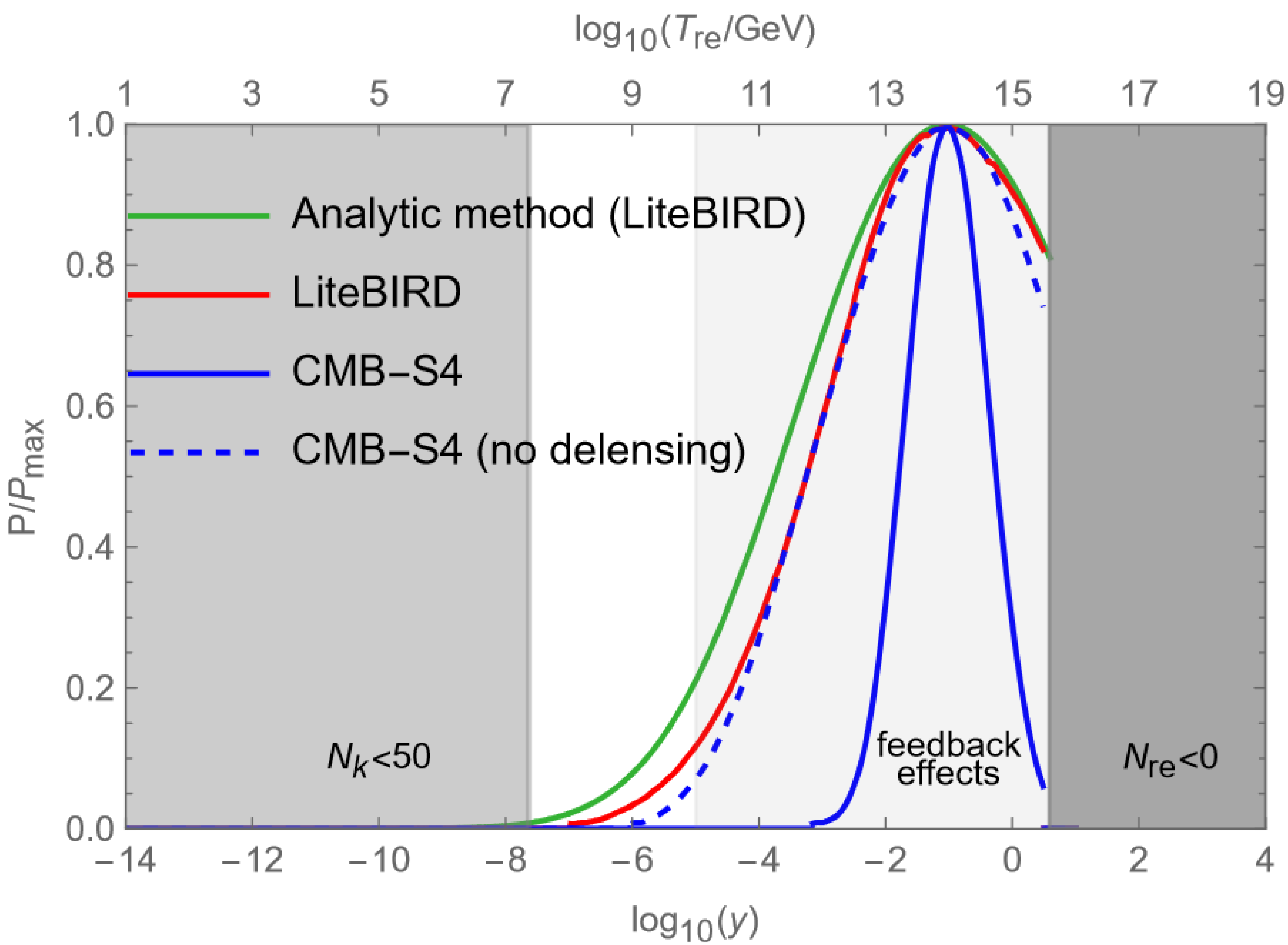
Forecast Method: α-T



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Forecast Method: Summary

LiteBIRD

model	$\log_{10}(y)$	$M[M_{\rm pl}]$	$\log_{10}(T_{\rm re}[{\rm GeV}])$
MHI	-5.39 ± 1.52	0.00516 ± 0.00005	9.49 ± 1.51
RGI	-5.56 ± 1.44	0.00526 ± 0.00005	9.34 ± 1.44
α-T	-1.70 ± 1.43	0.00521 ± 0.00005	13.13 ± 1.43

CMB-S4

model	$\log_{10}(y)$	$M[M_{\rm pl}]$	$\log_{10}(T_{\rm re}[{\rm GeV}])$
MHI	-6.28 ± 0.63 (-5.80 ± 1.19)	0.00519 ± 0.00002 (0.00518 ± 0.00004)	8.59 ± 0.63 (9.08 ± 1.19)
RGI	-6.67 ± 0.57 (-5.97 ± 1.14)	0.00529 ± 0.00002 (0.00527 ± 0.00004)	8.23 ± 0.57 (8.93 ± 1.14)
α-T	-1.03 ± 0.62 (-1.62 ± 1.32)	0.00518 ± 0.00002 (0.00520 ± 0.00005)	13.80 ± 0.62 (13.20 ± 1.32)

MaD/Ming/Oldengott 2303.13503

Summary II

- The inflaton coupling can in practice be "measured" in the CMB if by next generation experiments
- More precisely: In the plateau models considered here, one can simultaneously constrain the scale of inflation and the inflaton coupling (and the reheating temperature)
- Adding information from optical and 21cm surveys will further reduce the error
- One parameter that relates the scale of inflation to the inflaton mass has to be fixed by hand (because the error bar on the spectral index is too large)
- Additional information (e.g. running of the spectral index, non-Gaussianities) may help to break this degeneracy
- Either way, this opens up a new window to probe the connection between inflation and particle physics at very high energies
- Analytic method gives reasonably accurate sensitivity forecasts very quickly