Higgs Spectrum Is Non-thermal after Inflation

Primordial Condensate vs Stochastic Fluctuation

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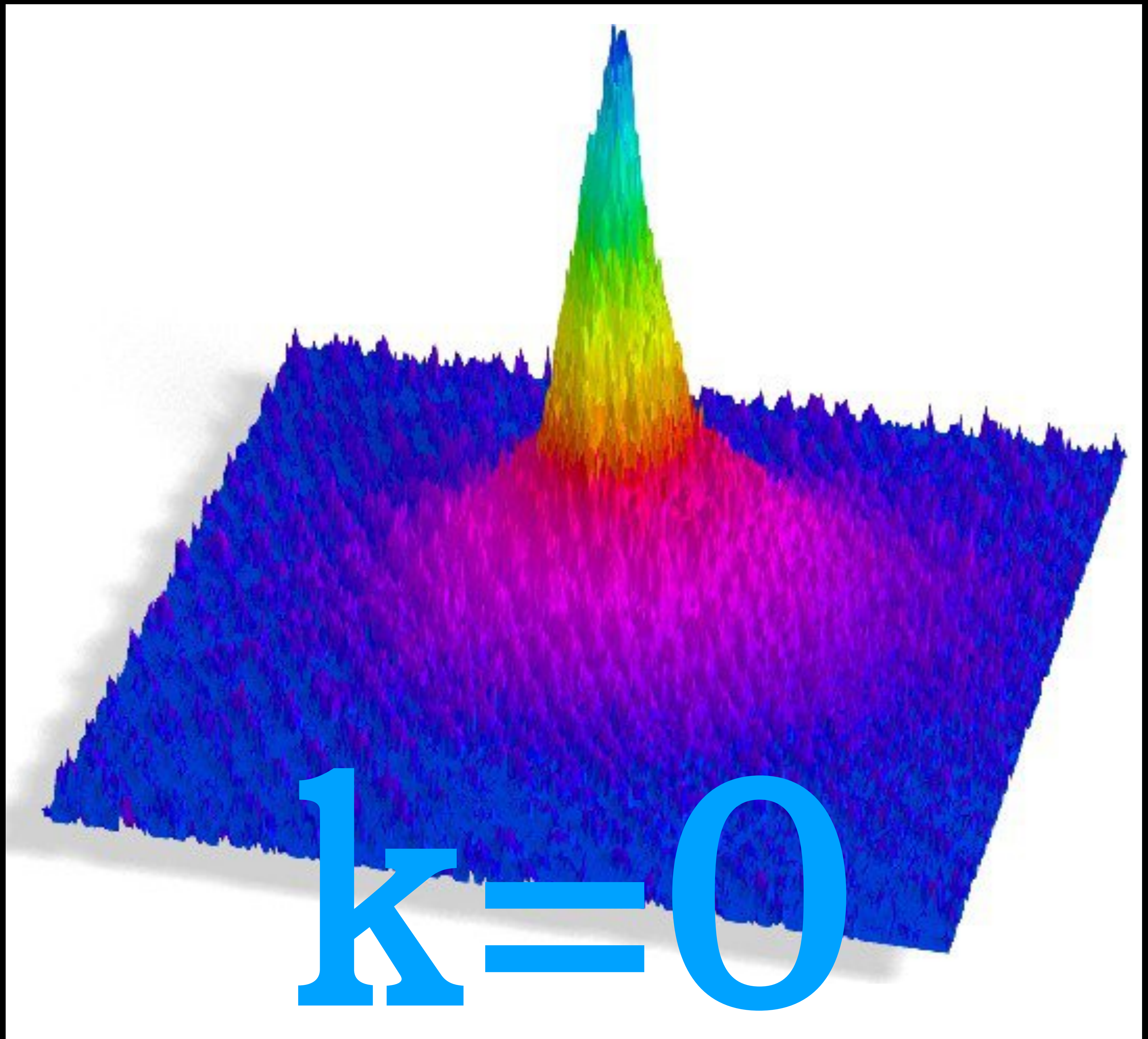
with

Kunio Kaneta (TWCU → Osaka U) [2304.12578].

Also with

Kaneta and **Sung Mook Lee** (CERN, Yonsei U →) JCAP 09 (2022) 018.

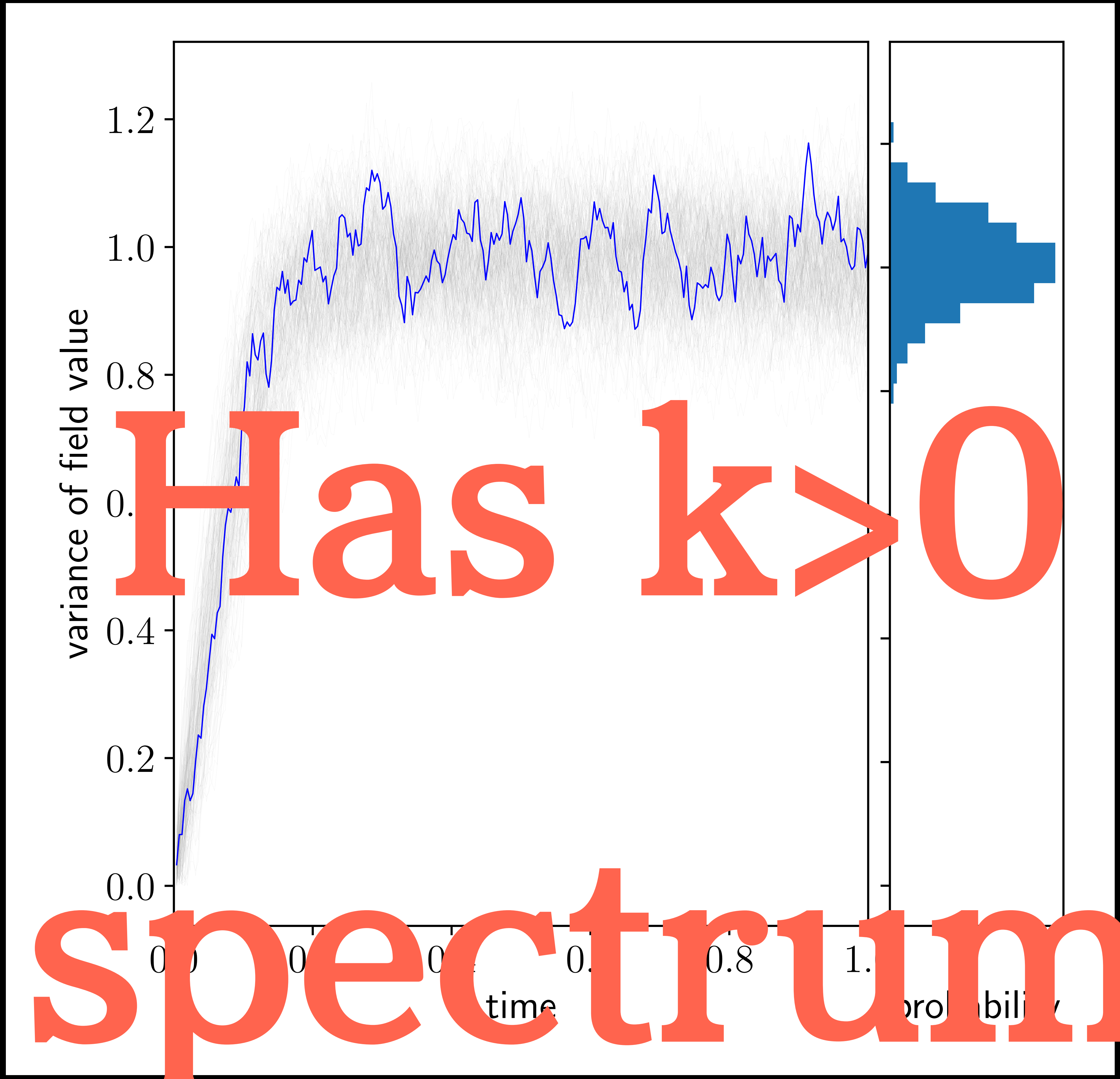
Primordial condensate or stochastic fluctuation?



k=0

[from Zimmermann group at Toront U.]

vs



Has k>0 spectrum

?
←

Is conventional treatment justified?

Messages

- **Higgs** χ and/or **inflaton** ϕ have **non-thermal spectra** after inflation
- Spectrum from **primordial condensate** vs **stochastic fluctuation**
- SF estimated by **analytic formulae** in **Boltzmann** and **Bogoliubov**
- Decay lifetimes of PC and SF almost equal (miraculously?)

Primordial condensate

Primordial condensate

- Slow-roll inflation (loosely) within **chaotic** paradigm
- **In general**, “initial” value for **all** momentum modes for Higgs
- Their physical momenta **red-shifted** during inflation
- After $\ln(M_P/H_{inf}) \sim 10$ e-folds, their phase space distribution:

$$f(\mathbf{p},t) \sim n(t)\ (2\pi)^3\ \delta^3(\mathbf{p})$$

(Barring initial modes with trans-Planckian momenta)

Decay of Higgs primordial condensate

- Trivially,

$$t_{\rm dec} \sim \Gamma_\chi^{-1}$$

- MD evolution due to inflaton oscillation:

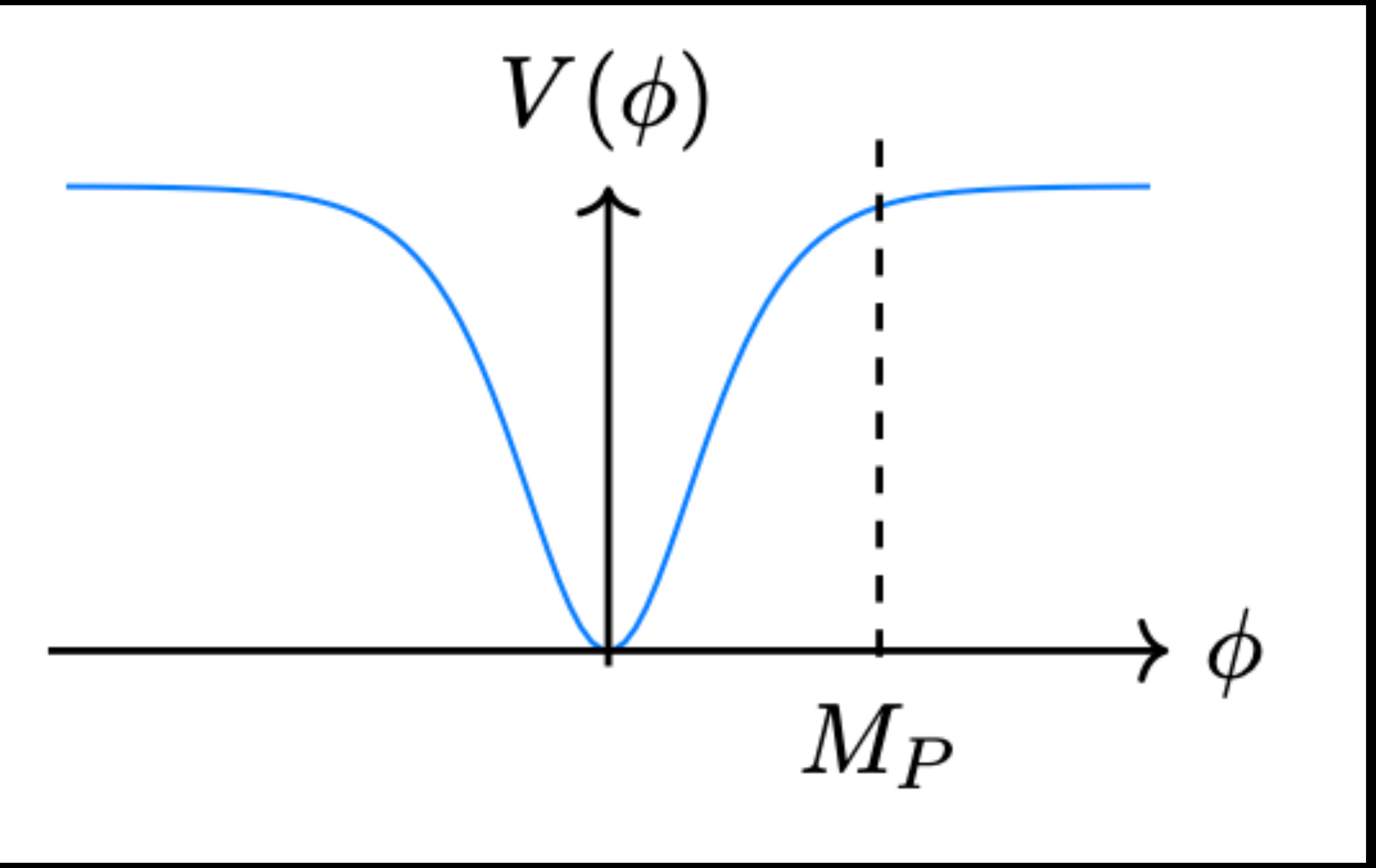
$$a_{\rm dec} \sim (H_e/\Gamma_\chi)^{2/3}\, a_e$$

Stochastic fluctuation

Setup

As a concrete example

- **Inflaton** ϕ (T-model) + SM **Higgs** χ
  - **No coupling** each other
  - Both **minimally** coupled to gravity



- **Criticality** ($\lambda \to 0$ at high scale) NOT assumed for Higgs
- Treat Higgs **decay rate** Γ_χ as parameter

Stochastic fluctuation

- Separate into UV and IR **modes**
- IR-mode evolution affected by **“integrating in”** **UV modes**
  - Their effect as **random noise**
- At the end of the day, we get

$$\langle \chi^2 \rangle \sim \mathrm{H_e}^2 / \sqrt{\lambda}$$

How to estimate spectrum of SF?

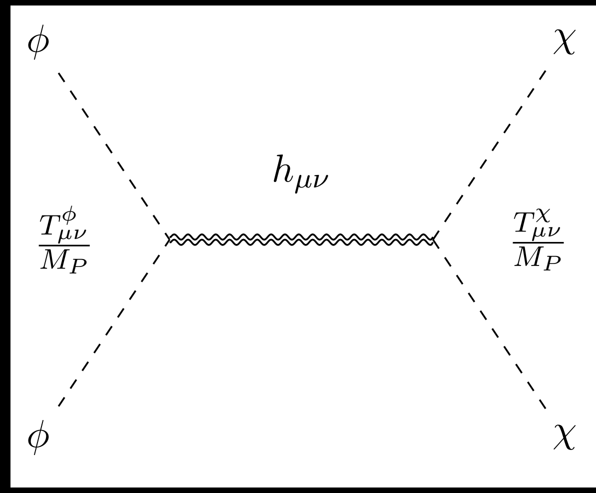
- From Higgs potential,

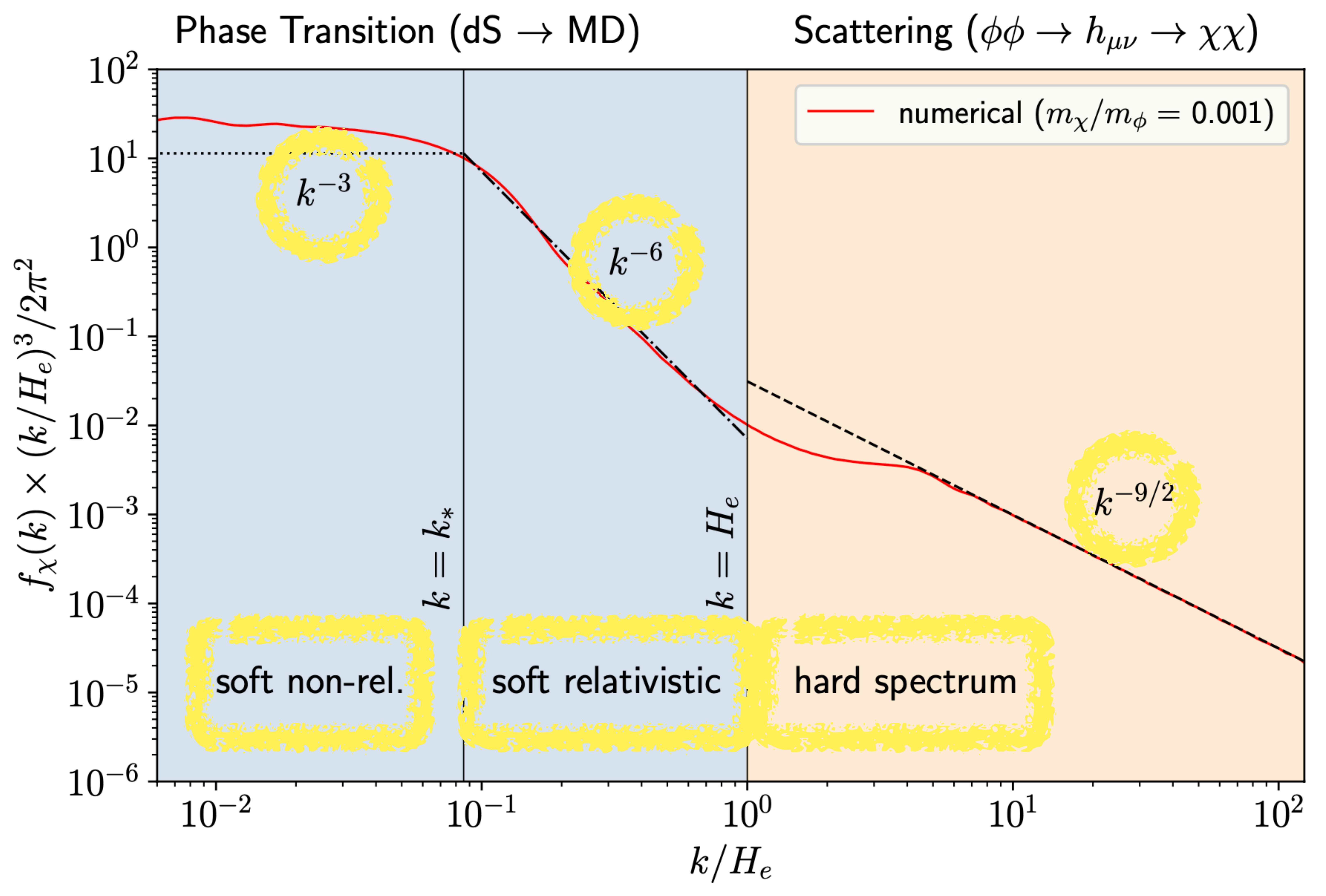
$$m_{\chi\,\mathrm{eff}}{}^2 \sim \lambda \langle \chi^2 \rangle \sim \lambda^{1/2} H_e^2$$

- Put it as **input** for **Bogolioubov** (and **Boltzmann**) approach:
  - Read off **Bogoliubov coefficient**
  - For **Bunch-Davies** → **MD** wave functions

Schematic Bogoliubov approach

Resultant spectrum





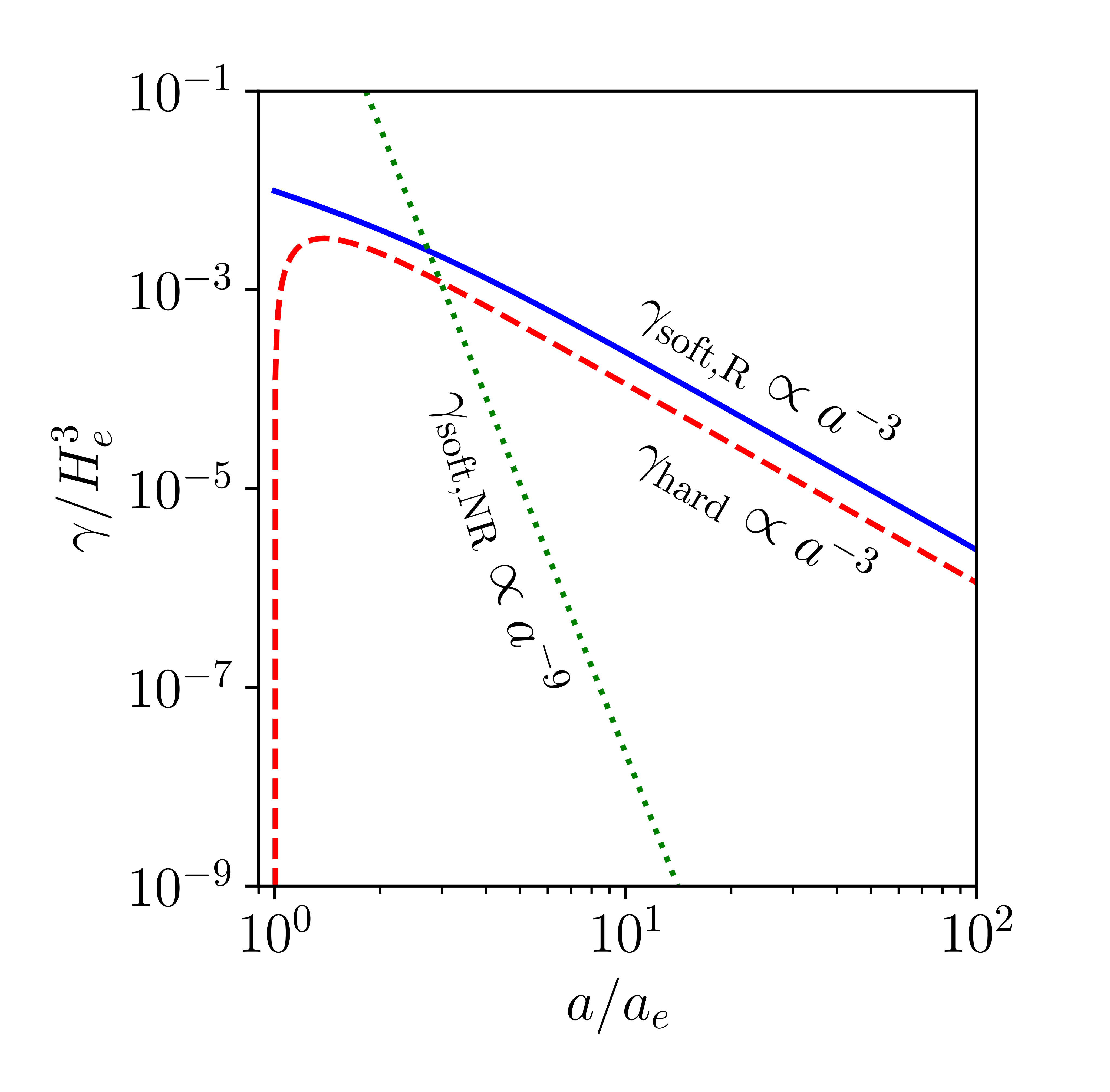
Also via Bogoliubov

Decay lifetime of stochastic fluctuation

- Soft relativistic modes become dominant
- Again, we get

$$a_{\rm dec} \sim (H_e/\Gamma_\chi)^{2/3}\, a_e$$

(Miraculously?)



Summary

- **Higgs** χ and/or **inflaton** ϕ have **non-thermal spectra** after inflation
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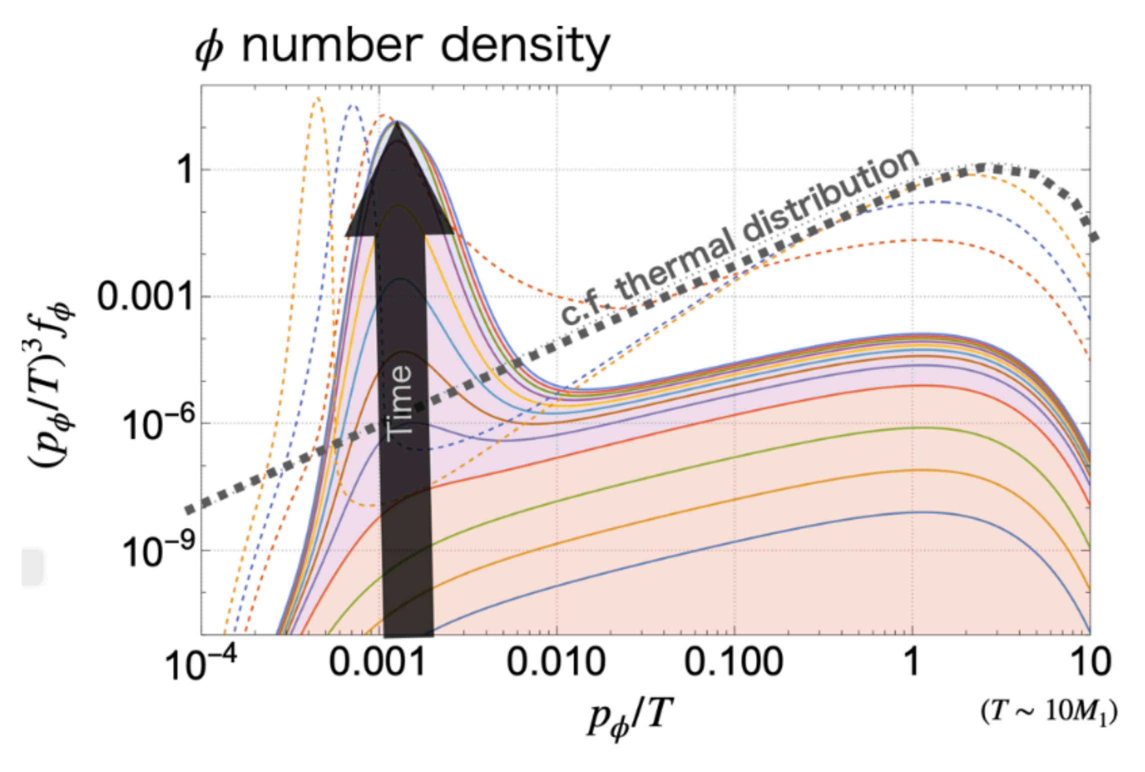
And ⋯

Discussions

- What if we start from **coherent state** (rather than from **BD vacuum**) in **Bogoliubov approach**?

- What if we put $m_{\chi\,\mathrm{eff}}^2 \sim \lambda \langle \chi^2 \rangle$ as **input** for **Bogolioubov approach** for **primordial condensate** too?

- Relation to appearance of Bose-Einstein condensate from Boltzmann eqs?

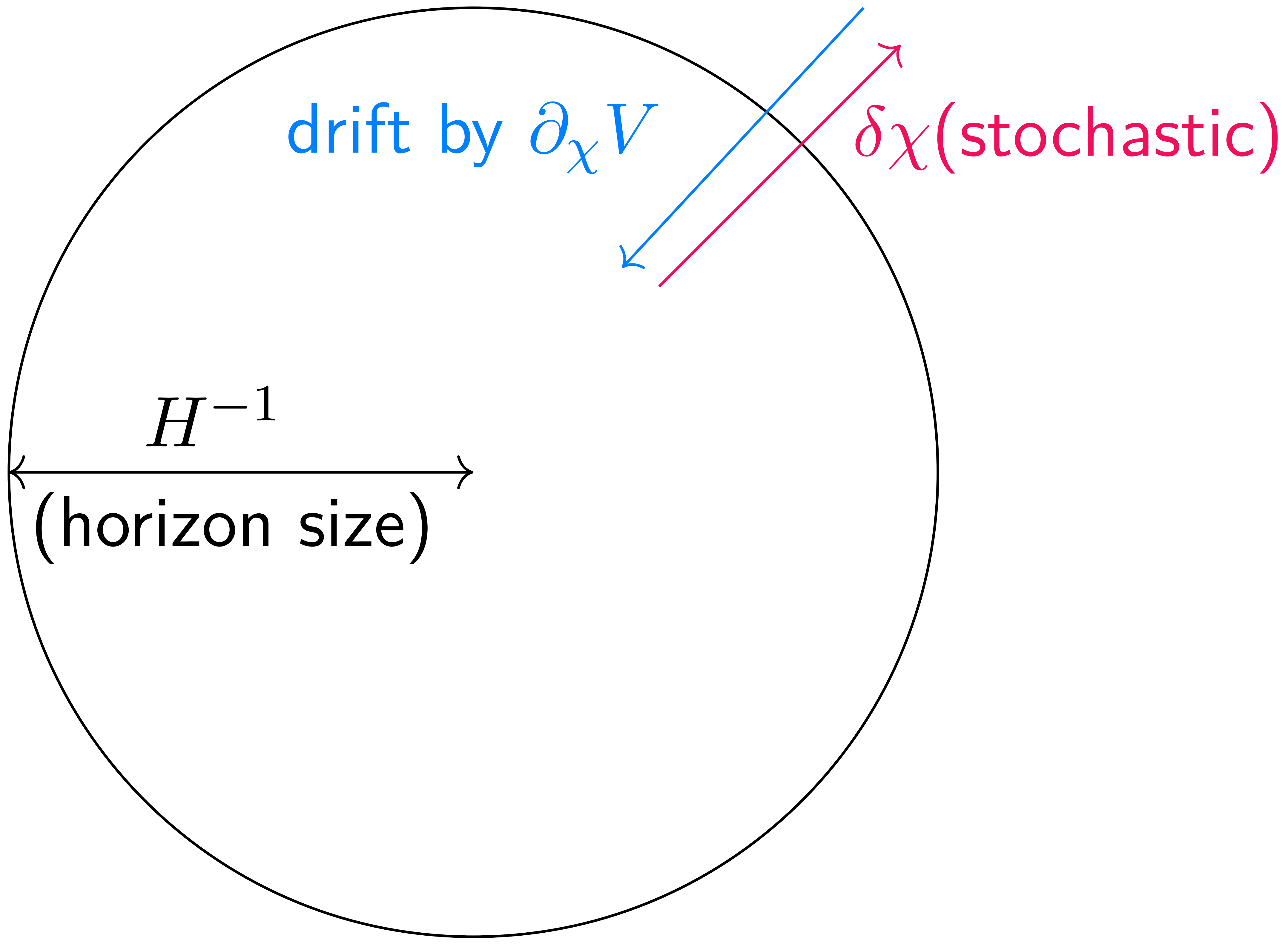


[Wen Yin, HPNP2023]


Thank you!

Backup (from Kunio's slides)

Stochastic Higgs Fluctuation



- Suppose $\chi(t_{\mathrm{ini}}) = 0$ (no condensate)
- Decompose $\chi = \overline{\chi}$ (IR) $+ \delta\chi$ (UV)
- EoM becomes the Langevin Eq.:

$$\dot{\overline{\chi}} \simeq -\frac{\partial_{\overline{\chi}} V(\overline{\chi})}{3H} + f^{\delta\chi}$$

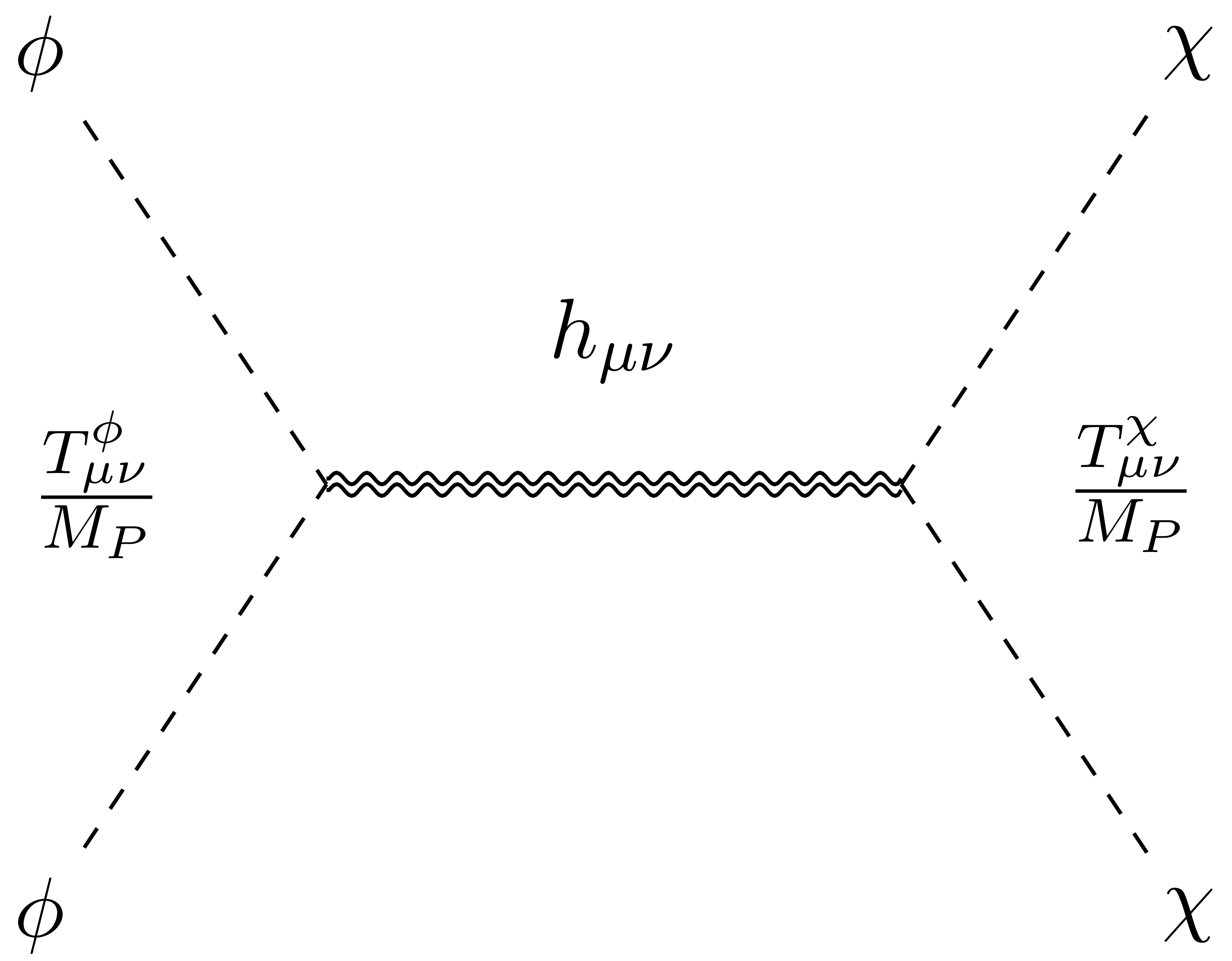
- $\delta\chi$ induces the Gaussian noise $f^{\delta\chi}$:

$$\langle f^{\delta\chi}(t_1) f^{\delta\chi}(t_2)\rangle = \frac{H_e^3}{4\pi^2}\delta(t_1 - t_2)$$

- For a sufficiently long time, $\overline{\chi}$ reaches

$$\langle\overline{\chi}^2\rangle \sim \frac{H_e^2}{\sqrt{\lambda}} \quad \Rightarrow \quad m_{\chi,\mathrm{eff}}^2 \simeq 3\lambda\langle\overline{\chi}^2\rangle$$

Gravitational Particle Production from Inflaton Scattering



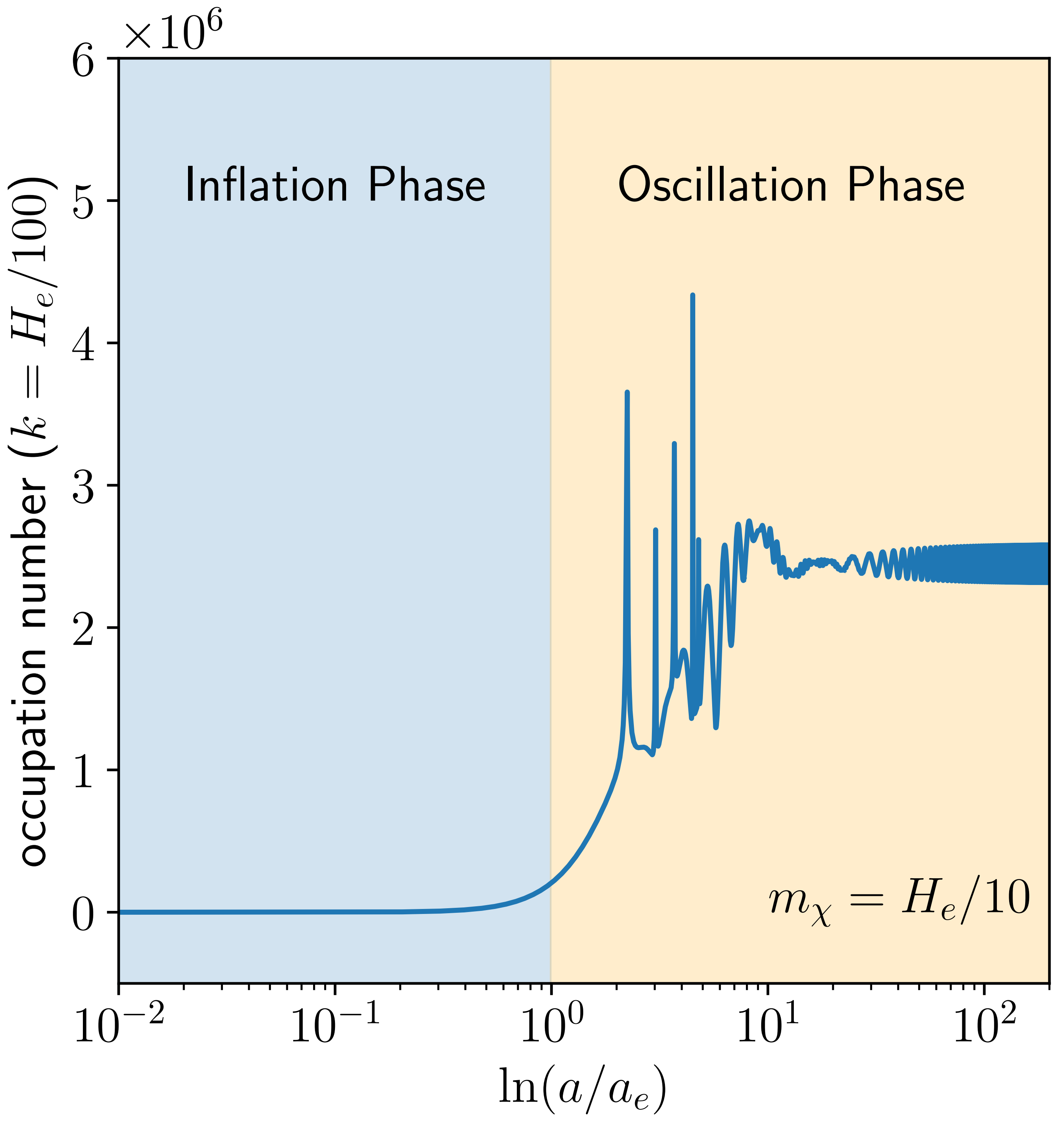
- Gravitational χ production during oscillation phase
- Phase space distribution:

$$f_\chi(k) \simeq \frac{9\pi}{64}\left(\frac{H_e}{m_\phi}\right)^3\left(\frac{m_\phi}{k}\right)^{9/2}$$

$$\begin{cases} k : \text{comoving momentum} \\ p : \text{physical momentum} \\ p = k/a(t) \end{cases}$$

- Energy conservation:
  $p = m_\phi \longleftrightarrow k = am_\phi > m_\phi$

Gravitational Particle Production from Phase Transition



- Particle production through abrupt phase transition (dS $\to$ MD)
- Initial condition: Bunch-Davies vacuum (no particle)
- $\omega_\chi^{(\mathrm{dS})} \neq \omega_\chi^{(\mathrm{MD})}$, producing particles
- Phase space distribution:

$$f_\chi(k) \simeq \begin{cases} \frac{9}{64}\left(\frac{H_e}{k}\right)^6 & (k > k_*) \\ \frac{9}{32}\left(\frac{H_e}{m_{\chi,\mathrm{eff}}}\right)\left(\frac{H_e}{k}\right)^3 & (k < k_*) \end{cases}$$

$$k_* = \left(\frac{H_e^2 m_{\chi,\mathrm{eff}}}{2}\right)^{1/6}$$

- Only $k < H_e$ may be excited

Phase Space Distribution

$\langle|\Phi|^2\rangle \neq 0$ in both cases, but $f_h(k)$ is very different:

Condensate:

$$f_h(p,t) = \frac{\rho_{\mathsf{cond}}(t)}{m_{h,\mathrm{eff}}}(2\pi)^3\delta^3(\vec{p})$$

$$m_{h,\mathrm{eff}} \approx \sqrt{\lambda|\Phi_0|^2}, \quad \rho_{\mathsf{cond}}(t) \approx \begin{cases} \rho_{h,0} & (H > m_{h,\mathrm{eff}}) \\ \rho_{h,0}(a/a_{\mathrm{osc}})^{-4} & (H < m_{h,\mathrm{eff}}) \end{cases}$$

Fluctuation:

$$f_h(p,t) = f_{\mathsf{hard}}(p,t) + f_{\mathsf{sfot,R}}(p,t) + f_{\mathsf{soft,NR}}(p,t)$$

$$\approx \begin{cases} \frac{9\pi}{64}\left(\frac{H_e}{m_\phi}\right)^3\left(\frac{m_\phi}{k}\right)^{9/2} & (k > m_\phi \sim H_e) \\ \frac{9}{64}\left(\frac{H_e}{k}\right)^6 & (k > k_*) \\ \frac{9}{32}\left(\frac{H_e}{m_{\chi,\mathrm{eff}}}\right)\left(\frac{H_e}{k}\right)^3 & (k < k_*) \end{cases}$$

Do they have any phenomenological implication?

Higgs Decay

- Suppose λ is a free parameter $\Rightarrow$ When $m_{h,\rm eff} > T_{\rm RH}$, Higgs is very non-thermal
- If $\sqrt{\lambda} \gtrsim g, y_t$, $h \to WW, ZZ, t\bar{t} \Rightarrow$ Take Γ_h as a free parameter

Condensate:

- $\dot{\rho}_h + 4H\rho_h = -\Gamma_h \rho_h \quad \Rightarrow \quad \rho_h(t) = \rho_{h,0}\left(\frac{a}{a_{\rm osc}}\right)^{-4} e^{-\Gamma_h t}$
- Decays away at $t_{\rm dec} = \Gamma_h^{-1} \quad \Rightarrow \quad \frac{a_{\rm dec}}{a_e} \simeq \left(\frac{3H_e}{2\Gamma_h}\right)^{2/3}$

Higgs Decay

- Suppose λ is a free parameter $\Rightarrow$ When $m_{h,\text{eff}} > T_{\text{RH}}$, Higgs is very non-thermal
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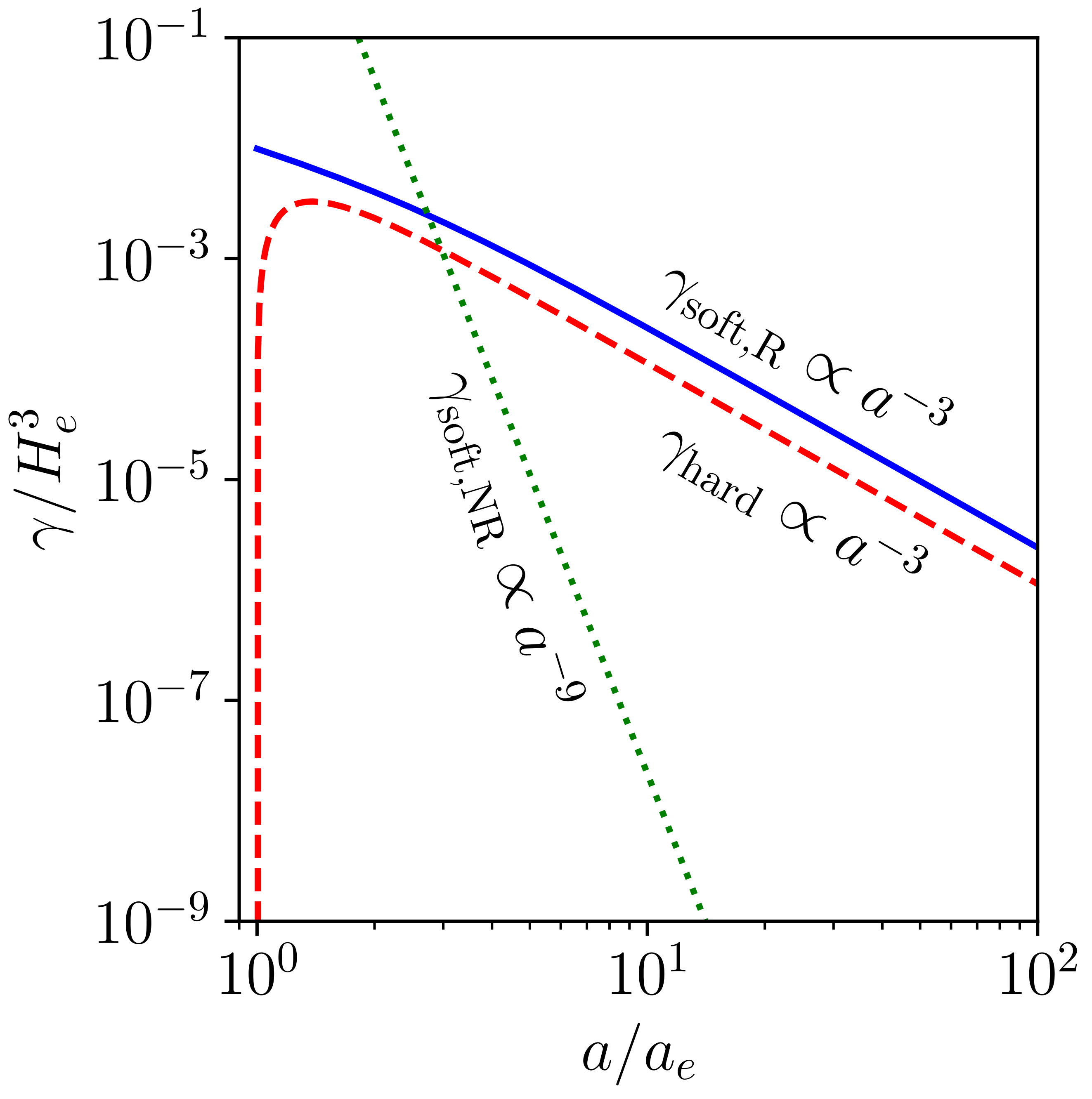
Fluctuation:



- $\dot{n}_h + 3Hn_h = -\gamma\Gamma_h$
$$\gamma = \int \frac{d^3p}{(2\pi)^3} \frac{m_{h,\text{eff}}}{\sqrt{m_{h,\text{eff}}^2 + p^2}} f_h(p,t)$$
- $\gamma_{\text{soft,R}}$ is dominant in decaying
- $n_h(a_{\text{dec}}) = 0 \quad \Rightarrow \quad \dfrac{a_{\text{dec}}}{a_e} \simeq \left(\dfrac{6H_e}{\Gamma_h}\right)^{2/3}$