Muon $g-2$ anomaly in a minimal left-right model with an inverse seesaw mechanism

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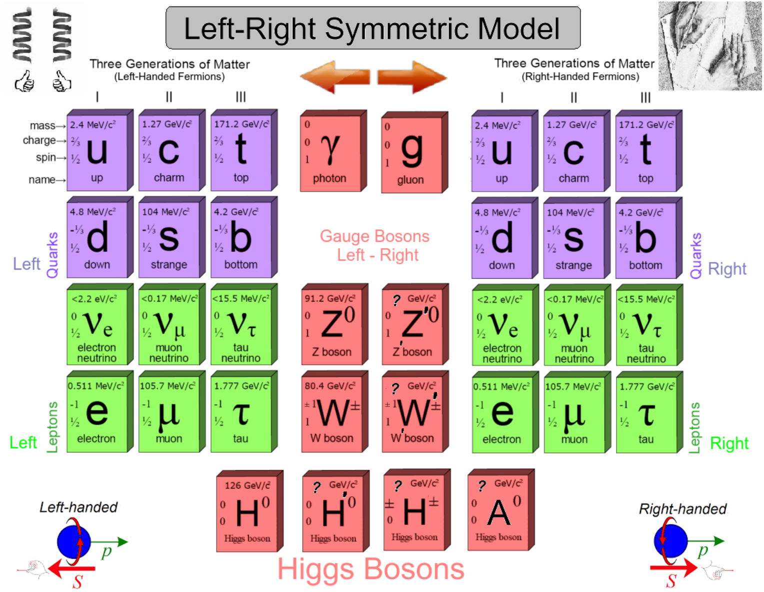
Left-Right Symmetric Model
Three Generations of Matter (Left-Handed Fermions)
Three Generations of Matter (Right-Handed Fermions)
I II III
mass→ charge→ spin→ name→
2.4 MeV/c² ⅔ ½ u up
1.27 GeV/c² ⅔ ½ c charm
171.2 GeV/c² ⅔ ½ t top
4.8 MeV/c² -⅓ ½ d down
104 MeV/c² -⅓ ½ s strange
4.2 GeV/c² -⅓ ½ b bottom
<2.2 eV/c² 0 ½ νe electron neutrino
<0.17 MeV/c² 0 ½ νμ muon neutrino
<15.5 MeV/c² 0 ½ ντ tau neutrino
0.511 MeV/c² -1 ½ e electron
105.7 MeV/c² -1 ½ μ muon
1.777 GeV/c² -1 ½ τ tau
Left Quarks
Left Leptons
Quarks Right
Leptons Right
0 0 1 γ photon
0 0 1 g gluon
Gauge Bosons
Left - Right
91.2 GeV/c² 0 1 Z0 Z boson
? GeV/c² 0 1 Z'0 Z' boson
80.4 GeV/c² ±1 1 W± W boson
? GeV/c² ±1 1 W'± W' boson
126 GeV/c² 0 0 H0 Higgs boson
? GeV/c² 0 0 H'0 Higgs boson
? GeV/c² ± 0 H± Higgs boson
? GeV/c² 0 0 A0 Higgs boson
Higgs Bosons
Left-handed
p
S
Right-handed
p
S

Fields	$SU(3)_C \times SU(2)_L \times SU(2)_R \times U_{B-L}$	$\mathbb{Z}_2$
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3}, \mathbf{2}, \mathbf{1}, \frac{1}{3})$	$+1$
$Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$	$(\mathbf{3}, \mathbf{1}, \mathbf{2}, \frac{1}{3})$	$+1$
$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, -1)$	$+1$
$L_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, -1)$	$+1$
S_1	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, -2)$	-1
S_2	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, 2)$	$+1$
$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, 0)$	$+1$
$\chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}$	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, 1)$	$+1$

Table 1: The LRIS particle Content quantum numbers.

- The Higgs potential is [6]

$$
\begin{aligned}
V(\phi,\chi_R) &= \mu_1 \mathsf{Tr}(\phi^\dagger\phi) + \mu_2[\mathsf{Tr}(\tilde{\phi}\phi^\dagger) + \mathsf{Tr}(\tilde{\phi}^\dagger\phi)] + \lambda_1(\mathsf{Tr}(\phi^\dagger\phi))^2 \\
&+ \lambda_2[(\mathsf{Tr}(\tilde{\phi}\phi^\dagger))^2 + (\mathsf{Tr}(\tilde{\phi}^\dagger\phi))^2] + \lambda_3 \mathsf{Tr}(\tilde{\phi}\phi^\dagger)\mathsf{Tr}(\tilde{\phi}^\dagger\phi) \\
&+ \lambda_4 \mathsf{Tr}(\phi\phi^\dagger)(\mathsf{Tr}(\tilde{\phi}\phi^\dagger) + \mathsf{Tr}(\tilde{\phi}^\dagger\phi)) + \mu_3(\chi_R^\dagger\chi_R) + \rho_1(\chi_R^\dagger\chi_R)^2 \\
&+ \alpha_1 \mathsf{Tr}(\phi^\dagger\phi)(\chi_R^\dagger\chi_R) + \alpha_2(\chi_R^\dagger\phi^\dagger\phi\chi_R) + \alpha_3(\chi_R^\dagger\tilde{\phi}^\dagger\tilde{\phi}\chi_R) \\
&+ \alpha_4(\chi_R^\dagger\phi^\dagger\tilde{\phi}\chi_R + h.c.). \qquad (1)
\end{aligned}
$$

- The Yukawa Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathsf{Y}} = \sum_{i,j=1}^{3} &\bar{L}_{L,i}\big(\phi y_{ij}^L + \tilde{\phi}\tilde{y}_{ij}^L\big)L_{R,j} + \bar{Q}_{L,i}\big(\phi y_{ij}^Q + \tilde{\phi}\tilde{y}_{ij}^Q\big)Q_{R,j} \\
&+\bar{L}_{R,i}\tilde{\chi}_R y_{ij}^s S_{2,j}^c + H.c. \ . \qquad (2)
\end{aligned}
$$

- Spntaneous symmetry breaking (SSB) occurs via the vevs

$$\langle\phi\rangle = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \sim \mathcal{O}(\text{GeV}), \quad \langle\chi\rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix} \sim \mathcal{O}(\text{TeV}). \tag{3}$$

and $t_\beta = \tan\beta = k_1/k_2, v = \sqrt{k_1^2 + k_2^2} = 246$ GeV.

- After SSB, the IS neutrino masses Lagrangian is [16, 17, 12, 18]

$$\mathcal{L}_m^\nu = M_D \bar{\nu}_L \nu_R + M_R \bar{\nu}_R^c S_2 + \mu_s \bar{S}_2^c S_2 + h.c., \tag{4}$$

where $M_D = v(y^L s_\beta + \tilde{y}^L c_\beta)/\sqrt{2}$ is the neutrino Dirac mass matrix and $M_R = y^s v_R/\sqrt{2}$.

- In the basis (ν_L^c, ν_R, S_2), the neutrino mass matrix is

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_R \\ 0 & M_R^T & \mu_s \end{pmatrix}. \tag{5}$$

- The physical light and heavy neutrino states ν_{ℓ_i}, ν_{h_j}, have masses

$$m_{\nu_{\ell_i}} = M_D M_R^{-1} \mu_s (M_R^T)^{-1} M_D^T, \quad i = 1 \dots 3, \tag{6}$$
$$m^2_{\nu_{h_j}} = M_R^2 + M_D^2, \quad j = 1 \dots 6. \tag{7}$$

- The inverse relation of Eq. (6) is

$$M_D = U_{\text{PMNS}} \sqrt{m_{\nu_\ell}} \mathcal{R} \sqrt{(\mu^s)^{-1}} M_R, \tag{8}$$

  $\mathcal{R}$ is an orthogonal matrix and U_{PMNS} is the 3×3 light neutrino mixing [7, 1, 9].
- Choices of $\mu_s \sim \mathcal{O}(10^{-7})$ GeV, and $v_R \sim \mathcal{O}(10^3)$ GeV So for $y^s \sim \mathcal{O}(10^{-3})$ we need $M \sim \mathcal{O}(10)$ TeV gives the experimental light neutrino masses.

- The symmetric mass matrix of the charged Higgs bosons $(\phi_1^{\pm}, \phi_2^{\pm}, \chi_R^{\pm})$ is

$$M_{H^{\pm}}^2 = \frac{\alpha_{32}}{2} \begin{pmatrix} \frac{v_R^2 s_\beta^2}{c_{2\beta}} & \frac{v_R^2 s_{2\beta}}{2c_{2\beta}} & -v v_R s_\beta \\ . & \frac{v_R^2 c_\beta^2}{c_{2\beta}} & -v v_R c_\beta \\ . & . & v^2 c_{2\beta} \end{pmatrix}, \quad (9)$$

- Only one physical charged Higgs boson with mass are

$$m_{H^{\pm}}^2 = \frac{\alpha_{32}}{2} \left(\frac{v_R^2}{c_{2\beta}} + v^2 c_{2\beta} \right), \quad (10)$$

where $\alpha_{32} = \alpha_3 - \alpha_2$.

- For $v_R \gtrsim \mathcal{O}(\text{TeV})$, the physical charged Higgs boson is [10]

$$H^{\pm} \approx -(s_\beta \phi_1^{\pm} + c_\beta \phi_2^{\pm}). \quad (11)$$

- The relevant $H^{\pm}$-fermions couplings are

$$\Gamma^{H^{\pm}}_{\bar{u}_i d_j} = C_{ij} P_L + D_{ij} P_R, \quad \Gamma^{H^{\pm}}_{\bar{\nu}_k \ell} = \xi_{k\ell} P_L + \zeta_{k\ell} P_R. \tag{12}$$

and the couplings

$$C_{ij} \simeq \frac{\sqrt{2}}{v c_{2\beta}} \sum_{a=1}^{3} V^*_{ja} \big(V M_d V^\dagger - s_{2\beta} M_u \big)^*_{ai}, \tag{13}$$

$$D_{ij} \simeq \frac{\sqrt{2}}{v c_{2\beta}} \sum_{a=1}^{3} V_{ja} \big(s_{2\beta} V M_d V^\dagger - M_u \big)_{ia}, \tag{14}$$

$$\xi_{k\ell} \simeq \frac{\sqrt{2}}{v c_{2\beta}} \sum_{i=1}^{3} U^*_{k,i+3} \big(s_{2\beta} M_{\mathsf{lp}} - M_D \big)_{\ell i}, \quad k = 4, \ldots, 9, \tag{15}$$

$$\zeta_{k\ell} \simeq \frac{\sqrt{2}}{v c_{2\beta}} \sum_{i=1}^{3} U_{ki} \big(M_{\mathsf{lp}} - s_{2\beta} M_D \big)_{i\ell}, \qquad k = 1, 2, 3, \tag{16}$$

V and U are the quark CKM and IS neutrino mixing matrices.

Outline

- Recent experimental results indicate a possible 4.2σ difference between the measured value of the anomalous magnetic moments of muons a_μ and the SM expectations [11, 14, 13, 2], namely

$$\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (2.51 \pm 0.59) \times 10^{-9}. \tag{17}$$

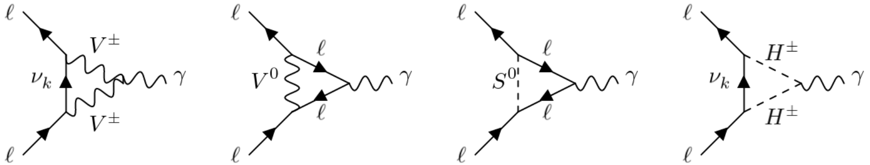

Figure 1: LRIS one-loop Feynman diagrams contributions to lepton $g_\ell - 2$ via massive neutrinos, $V^\pm = W, W'$, $V^0 = Z, Z'$, $S^0 = h, A$ and the charged Higgs boson $H^\pm$.

- The charged Higgs $H^\pm$ contribution to a_μ is given by

$$a_\ell^{H^\pm} = G_\mathsf{F}^\ell \, \Gamma_\gamma^{H^\pm} \sum_{k=1}^{9} \Big(|{\zeta'}_{k\ell}|^2 \, \mathcal{F}_2(x_{H^\pm}^{\nu_k}) + 2\mathrm{Re}[{\zeta'}_{k\ell}{\xi'}^*_{k\ell}] \, \mathcal{F}_1(x_{H^\pm}^{\nu_k}) \Big), \quad (18)$$

where the couplings ${\zeta'}_{k\ell} = \frac{v}{m_{\nu_k}}\zeta_{k\ell}$ and ${\xi'}_{k\ell} = \frac{v}{m_\ell}\xi_{k\ell}$.

- The charged Higgs boson interaction coupling with photons is

$$\Gamma_\gamma^{H^\pm} \simeq \frac{1}{6e}\Big(g_L U_{21}^0 + g_R U_{31}^0 \Big), \quad (19)$$

U^0 is the neutral gauge bosons mixing [10].

- The loop functions $\mathcal{F}_k$ $(k = 1, 2)$ in Eq. (18) are given by

$$\mathcal{F}_k(y) = \frac{y\mathcal{P}_k(y)}{(y-1)^{k+1}} - \frac{6y^{k+1}\log(y)}{(y-1)^{k+2}}, \quad k = 1, 2, \quad (20)$$

$$\mathcal{P}_1(y) = 3y + 3, \quad (21)$$

$$\mathcal{P}_2(y) = 2y^2 + 5y - 1. \quad (22)$$

- The charged Higgs boson contribution to the a_μ anomaly Eq. (18) can be approximated to

$$a_\ell^{H^\pm} \simeq 2G_{\mathsf{F}}^{\ell}\, \Gamma_\gamma^{H^\pm} \sum_{k=4}^{9} \mathrm{Re}[{\zeta'}_{k\ell}{\xi'}^{*}_{k\ell}]\, \mathcal{F}_1(x_{H^\pm}^{\nu_k}) \lesssim \frac{3\Gamma_\gamma^{H^\pm}}{8\pi^2}\, m_\ell \sum_{k=4}^{9} \frac{\zeta_{k\ell}\xi_{k\ell}}{m_{\nu_k}}. \quad (23)$$

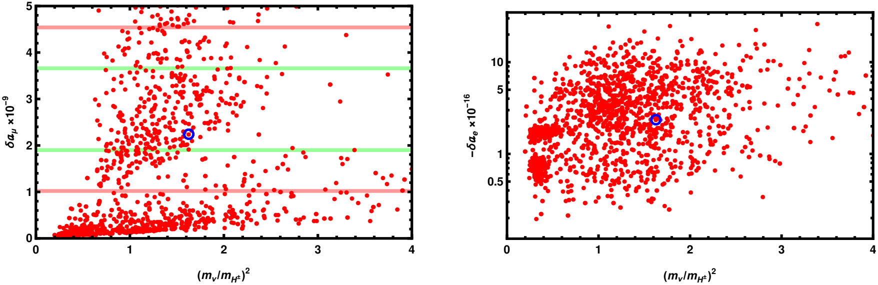

Figure 2: (Left/right) $\delta a_{\mu,e}$ with $x_{H^\pm}^{\nu_5} = m_{\nu_5}^2/m_{H^\pm}^2$. The 1σ and 2σ standard errors of measurements of a_μ are included in green and red borders. BP is encircled.

α_{32}	t_β	v_R	$Z^{H^\pm}_{31}$	$Z^{H^\pm}_{32}$	$Z^{H^\pm}_{33}$	$m_{H^\pm}$
0.0058	0.1	10000	-0.099	-0.994	0.024	545

Table 2: BP and $H^\pm$ mixing and mass for $y^s = \text{diag}(1.53\times10^{-2}, 9.76\times10^{-1}, 2.05\times10^{-1})$ and $\mu^s = \text{diag}(1.01\times10^{-5}, 3.82\times10^{-9}, 5.49\times10^{-6})$. Finally, the nonvanishing elements of the orthogonal matrix $\mathcal{R}$ are $\mathcal{R}_{13} = \mathcal{R}_{21} = \mathcal{R}_{32} = 1$.

m_{ν_1}	m_{ν_2}	m_{ν_3}	m_{ν_4}	m_{ν_5}	m_{ν_6}	m_{ν_7}	m_{ν_8}	m_{ν_9}
1.0×10^{-13}	8.5×10^{-12}	5.0×10^{-11}	108	695	1449	108	695	1449

Table 3: BP neutrino mass spectrum in GeVs.

δa_μ	$-\delta a_e$	BR$(\mu\to e\gamma)$
2.5×10^{-9}	8.1×10^{-17}	3.4×10^{-13}

Table 4: BP Observalbles $g_{\mu(e)}-2$, BR$(\mu\to e\gamma)$ of the BP given in Tab. 3.

Outline

- Experimentally, BR$(\mu \to e\gamma) \lesssim 4.2 \times 10^{-13}$ (90%CL) [5]. LRIS $H^\pm$

$$\text{BR}(\mu \to e\gamma)_{\text{LRIS}} \simeq \frac{\alpha_w^3 s_w^2}{256\pi^2}\frac{m_\mu}{\Gamma_\mu}(x_W^\mu)^2 \sum_{k=4}^{9}\left|\left({\zeta'}_{k,e}{\xi'}^*_{k,\mu} + {\xi'}_{k,e}{\zeta'}^*_{k,\mu}\right)\ \mathcal{F}_1(x_{H^\pm}^{\nu_k})\right|^2$$
$$\lesssim \frac{9\alpha_{\text{em}}}{256\pi^4}\frac{m_\mu^5}{\Gamma_\mu}\sum_{k=4}^{9}\frac{1}{m_{\nu_k}^2}\left(\frac{\zeta_{k,e}\xi_{k,\mu}}{m_\mu} + \frac{\xi_{k,e}\zeta_{k,\mu}}{m_e}\right)^2. \quad (24)$$

- Finally, we check experimental limits on $H^\pm$ contributions to the μ-e conversion on a nucleus (A). Experiments make the upper bounds $R^{\text{Ti}}_{\mu\to e} \leqslant 10^{-18}$, $R^{\text{Al}}_{\mu\to e} \leqslant 10^{-16}$, $R^{\text{Au}}_{\mu\to e} \leqslant 7\times 10^{-13}$ [15].
- The $H^\pm$ contribution to the μ-e conversion is [3, 8]

$$R^A_{\mu\to e} = \frac{32G_{\text{F}}^2 m_\mu^5}{\Gamma^A_{\text{capt}}}\left[\left|\widetilde{C}^{pp}_{V,R}V_A^{(p)} + \widetilde{C}^{nn}_{V,R}V_A^{(n)} + \frac{1}{4}C_{D,L}D_A\right|^2 + \{L \leftrightarrow R\}\right]. \quad (25)$$

$\Gamma^A_{\text{capt}} \sim \mathcal{O}(1-10)\times 10^6\ s^{-1}$ is the rate for the muon to transform to a neutrino by capture on the nucleus (A). The nuclear "overlap integrals" $V_A^{(p)}, V_A^{(n)}, D_A \sim \mathcal{O}(10^{-2}-10^{-1})$ for $A =$ Al, Ti, Au [15].

- In LRIS, the nucleon-dependent Wilson coefficients are given by

$$C_{D,L} = \frac{8G_{\mathsf{F}}\alpha_{\mathsf{em}}}{\pi s_w^2\sqrt{2}} \sum_{k=1}^{9}\sum_{j=1}^{3}\sum_{q,q'=u,d,q\neq q'} \left(U_{k,e}^{*}U_{k,\mu}|V_{q',q_j}|^2\right) B_2(x_W^{\nu_k}, x_W^{q_j}), \quad (26)$$

$$\widetilde{C}_{V,R}^{pp} = \frac{1}{8\pi^2 m_{H^\pm}^2} \sum_{k=1}^{9}\sum_{j=1}^{3}\sum_{q,q'=u,d,q\neq q'} \left(\zeta_{k,e}\xi_{k,\mu} + \zeta_{k,\mu}\xi_{k,e}\right)$$
$$\times \left(C_{q',q_j}^2 + D_{q',q_j}^2\right) B_2(x_{H^\pm}^{\nu_k}, x_{H^\pm}^{q_j}), \quad (27)$$

$$\widetilde{C}_{V,R}^{nn} = \frac{1}{4\pi^2 m_{H^\pm}^2} \sum_{k=1}^{9}\sum_{j=1}^{3}\sum_{q,q'=u,d,q\neq q'} \left(\zeta_{k,e}\zeta_{k,\mu} + \xi_{k,e}\xi_{k,\mu}\right)$$
$$\times \left(C_{q',q_j} D_{q',q_j}\right) B_1(x_{H^\pm}^{\nu_k}, x_{H^\pm}^{q_j})\sqrt{x_{H^\pm}^{\nu_k} x_{H^\pm}^{q_j}}, \quad (28)$$

- The loop functions are

$$J_k(x) = \frac{1}{1-x} + \frac{x^k \log(x)}{(1-x)^2}, \quad (29)$$

$$B_k(x,y) = \frac{J_k(x) - J_k(y)}{x-y}, \quad k = 1, 2. \quad (30)$$

$\text{BR}(\mu \to e\gamma)$	$R^{\text{Al}}_{\mu\to e}$	$R^{\text{Ti}}_{\mu\to e}$	$R^{\text{Au}}_{\mu\to e}$
2.10×10^{-13}	4.10×10^{-51}	3.80×10^{-50}	4.10×10^{-49}

Table 5: LFV observables BP given in Table 3 in LRIS.

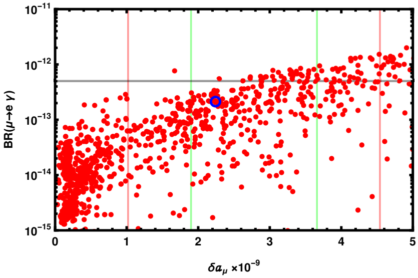

Figure 3: $\text{BR}(\mu \to e\gamma)$ versus δa_μ in LRIS. BP is encircled.

- All BPs are tested and found to satisfy the μ-e conversion experimental limits as in Table 5.

Outline

- We have analyzed a_μ in a minimal left-right symmetric model with an inverse seesaw mechanism.
- We found that a large region of the parameter space of the model is consistent with the observed a_μ anomaly.
- BP satisfy the δa_e limits and the BR$(\mu \to e\gamma)$ and the $\mu \to e$-conversion rates limits.

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Thank you!
Questions?