NanoGrav Signal from Ultra Slow-Roll Inflation?

In collaboration with Hassan Firouzjahi
arXiv: 2307.03164

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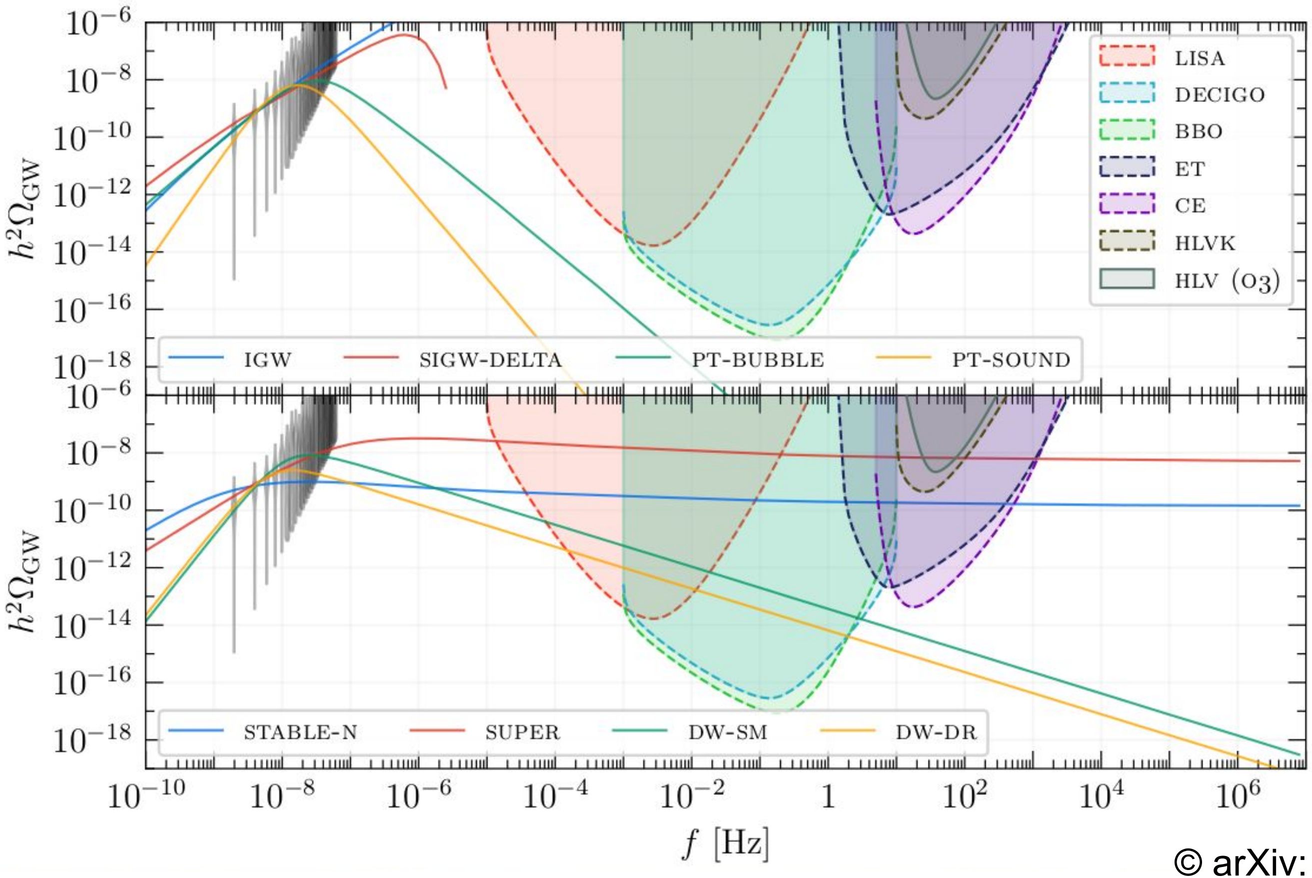
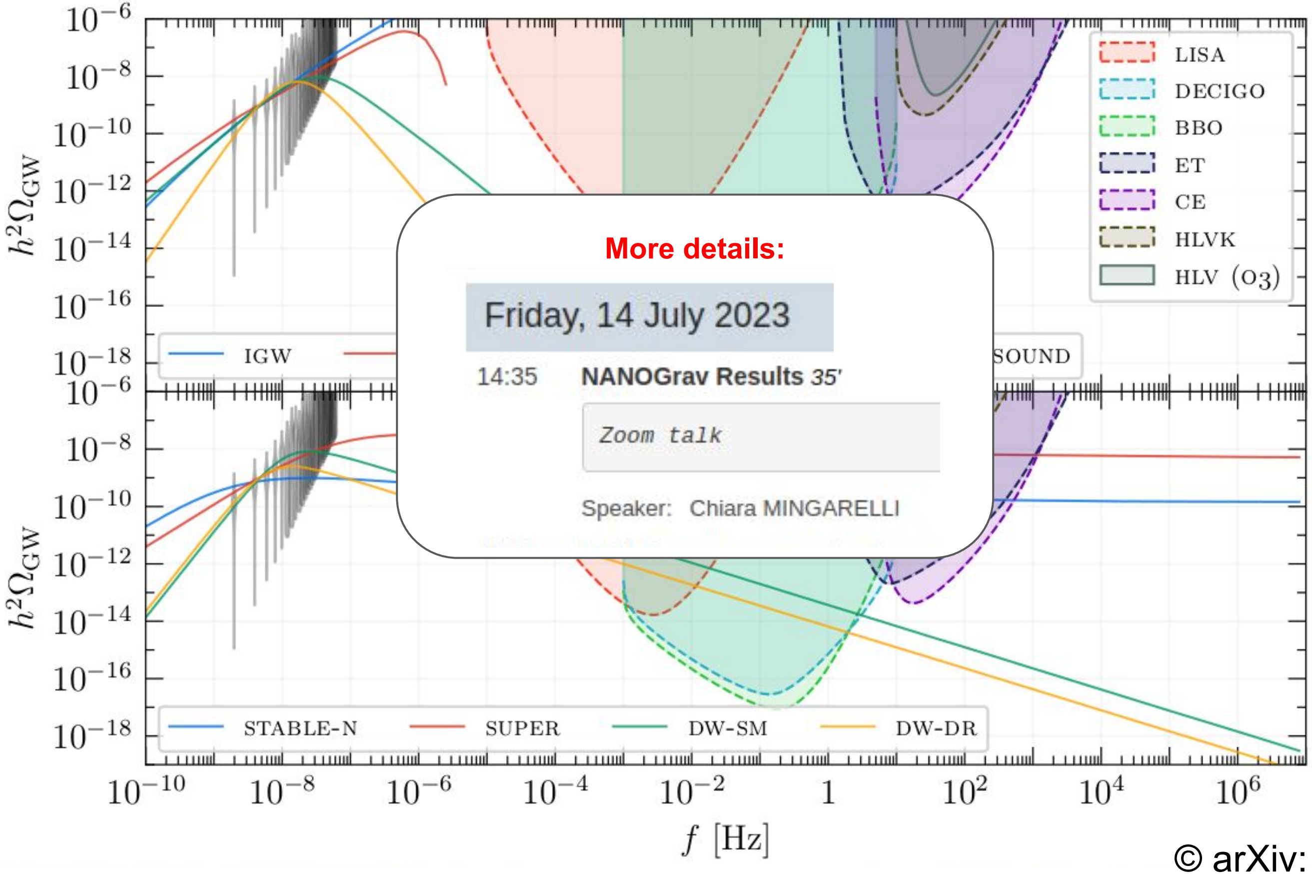
July 11, 2023

NANOGrav 15-year New-Physics Signals



NANOGrav 15-year New-Physics Signals



More details:

Friday, 14 July 2023

14:35 **NANOGrav Results** 35'

`Zoom talk`

Speaker: Chiara MINGARELLI

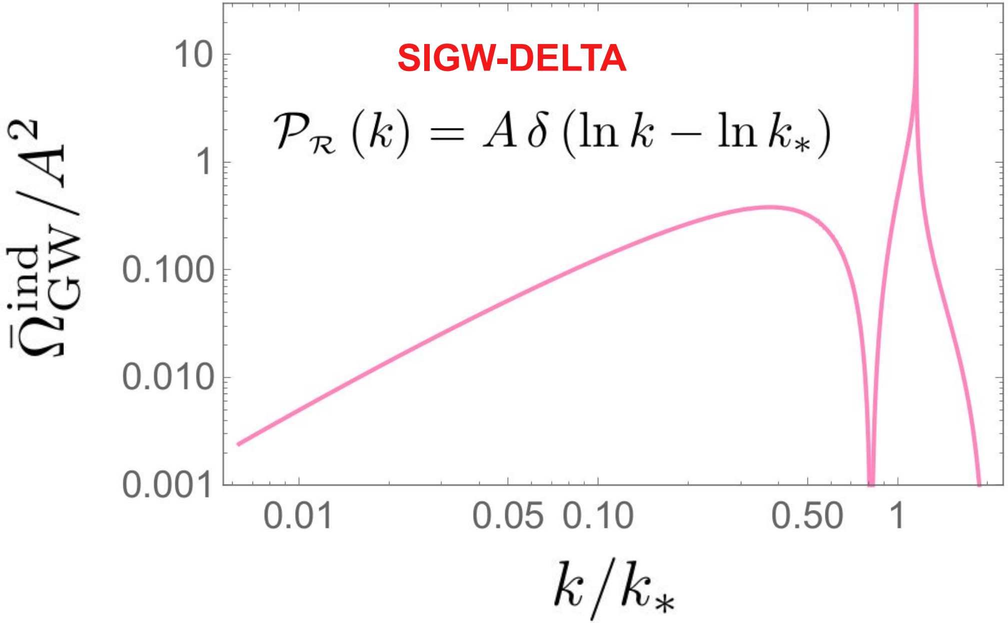
© arXiv: 2306.16219

Scalar-induced gravitational waves (SIGWs)

$$\mathcal{P}_h^{(\mathrm{ind})} \sim \int \mathrm{d}k \int \mathrm{d}k' \left[\int f(k,k',t)\mathrm{d}t \right]^2 \mathcal{P}_{\mathcal{R}}(k)\mathcal{P}_{\mathcal{R}}(k')$$

$\mathcal{R}$ → During **Radiation-dominated** era → **GWs**

$f(k,k',t)$: oscillating function

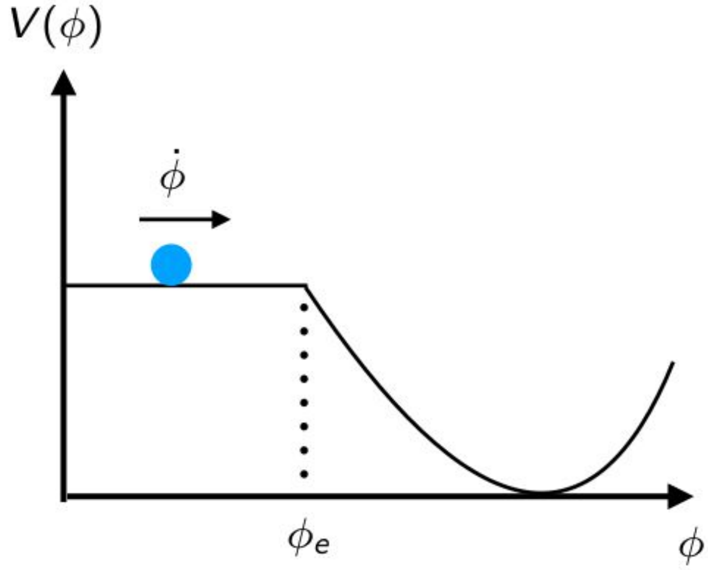


Ultra-Slow-Roll (USR) model

USR inflation is a setup with a flat potential (Kinney 2006)

The background equations are given by

$$\ddot{\phi} + 3H\dot{\phi} = 0\,, \quad 3M_P^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V_0 \simeq V_0\,;$$



A key feature of the USR setup is that $\dot{\phi}$ falls off exponentially:

$$\dot{\phi} \propto a(t)^{-3} \longrightarrow \epsilon \equiv -\frac{\dot{H}}{H^2} \propto a(t)^{-6}$$

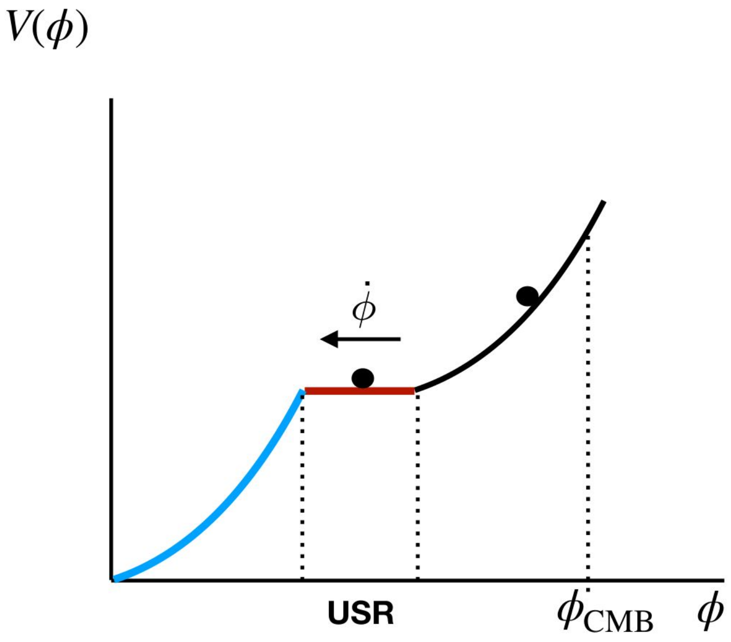
The setup: SR-USR-SR

The setup is a three-phase model of inflation:

$SR \to USR \to SR$

The CMB modes leave the horizon in first SR phase.

The USR modes experience growth: $\mathcal{R} \propto a(t)^3$



USR modes lead to PBHs formation and SIGWs during RD era!

The setup: SR-USR-SR

$$(h, \Delta N) \text{ parameter space}$$

ΔN **: duration of the USR period**

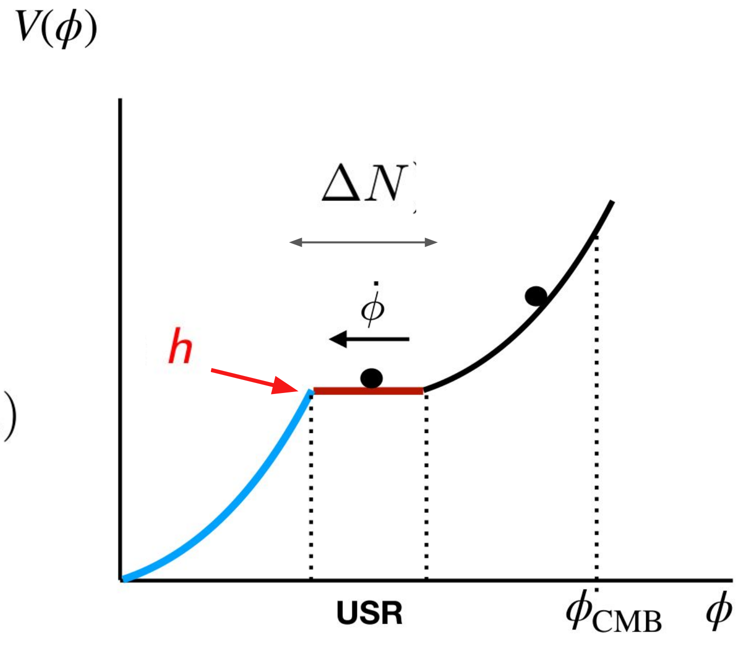
Here h measures the sharpness of the transition:

For a sharp transition $h \ll -1$.
For a mild transition $h \longrightarrow 0$.

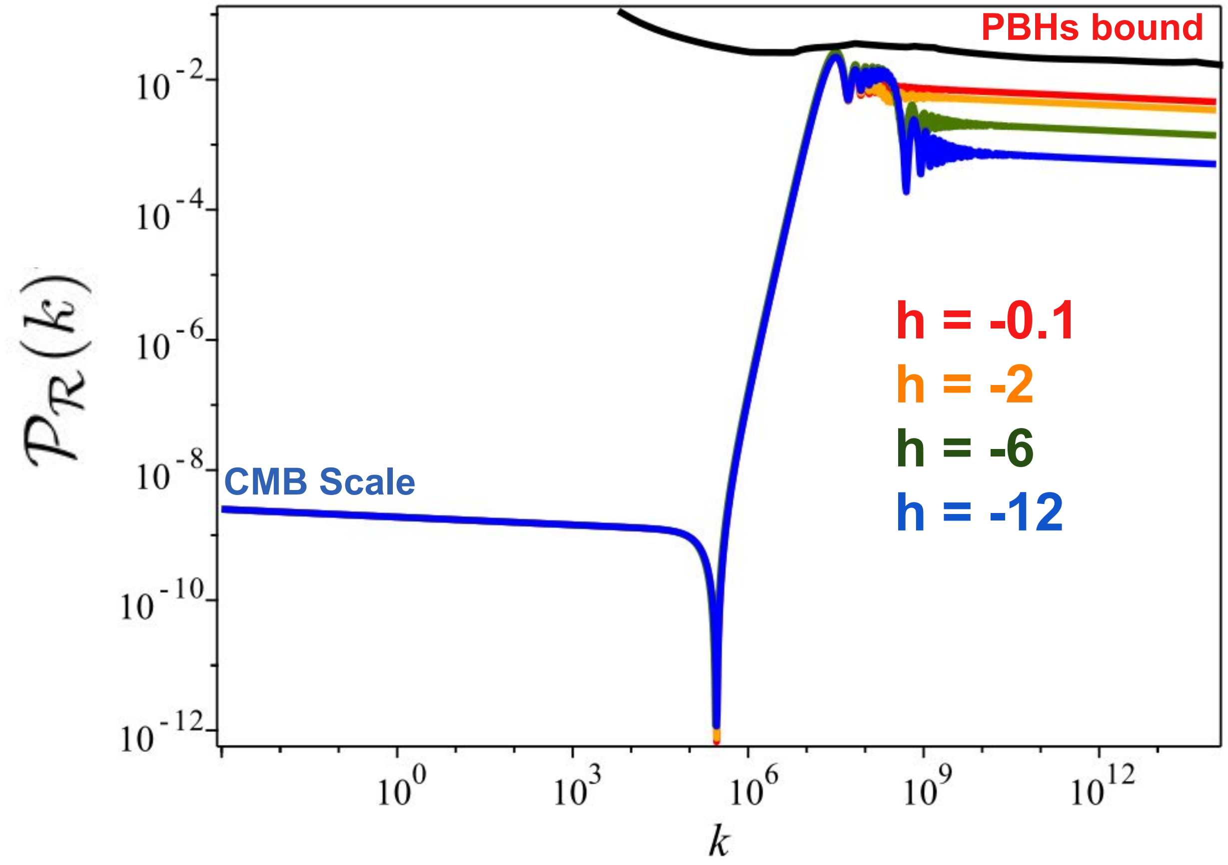
$$\mathcal{P}_{\mathcal{R}}(k, \tau = 0) \simeq \mathcal{P}_{\rm CMB}\; e^{6\Delta N}\; \left(\frac{h-6}{h}\right)^2 g(h, \tau_{\rm i}, \tau_{\rm e})$$

Local-type Non-G:

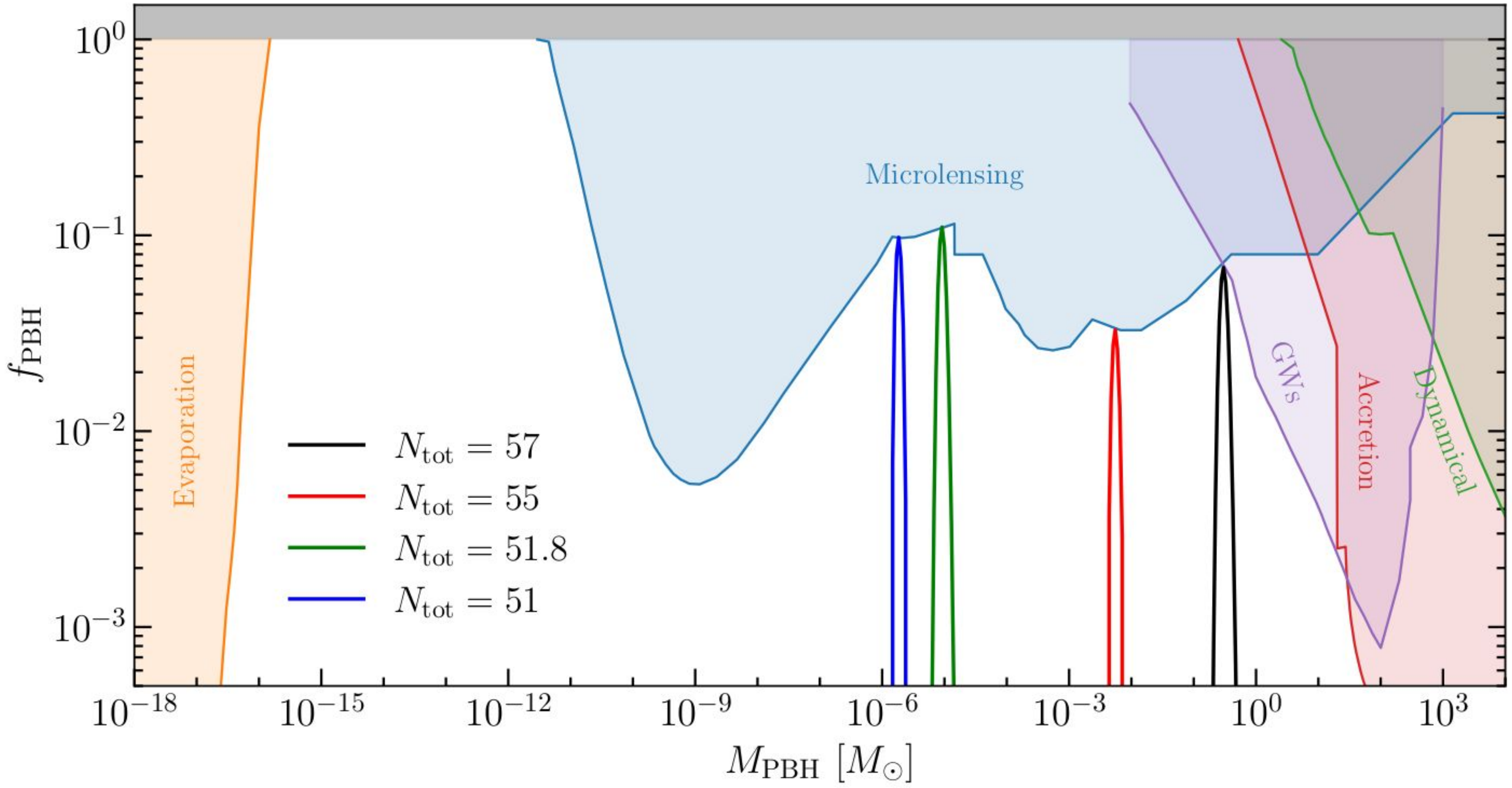
$$f_{NL} = \frac{5h^2}{2(h-6)^2}$$



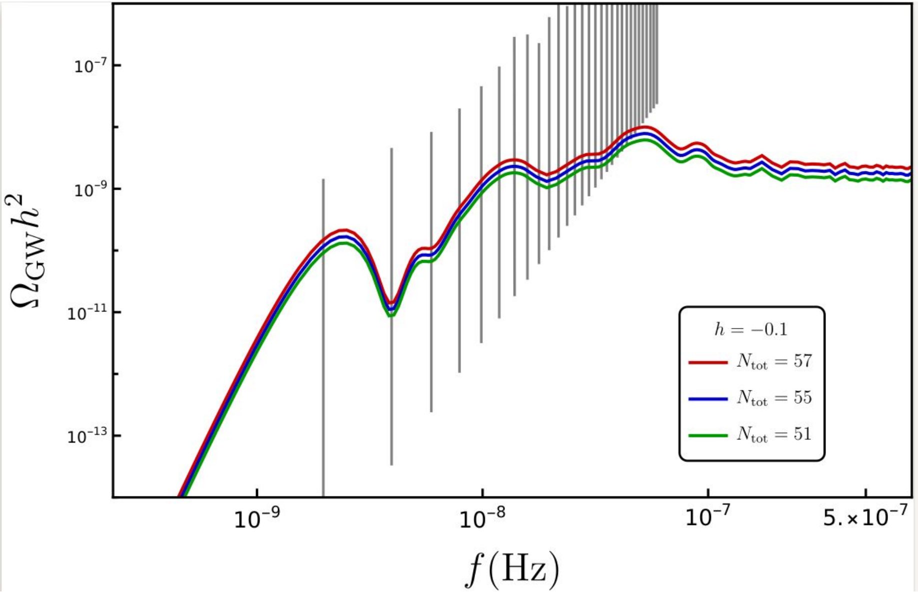
Enhanced Power spectrum

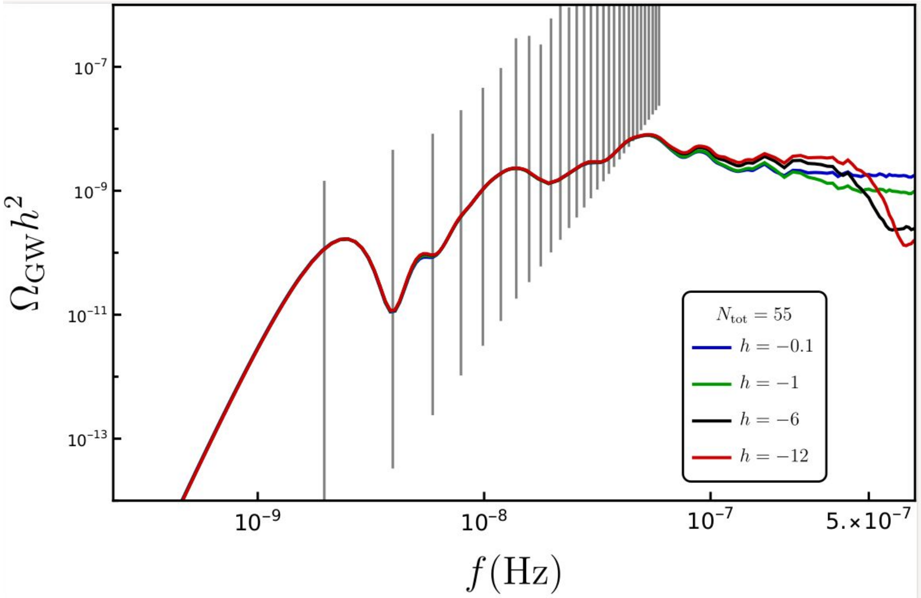


PBH abundance

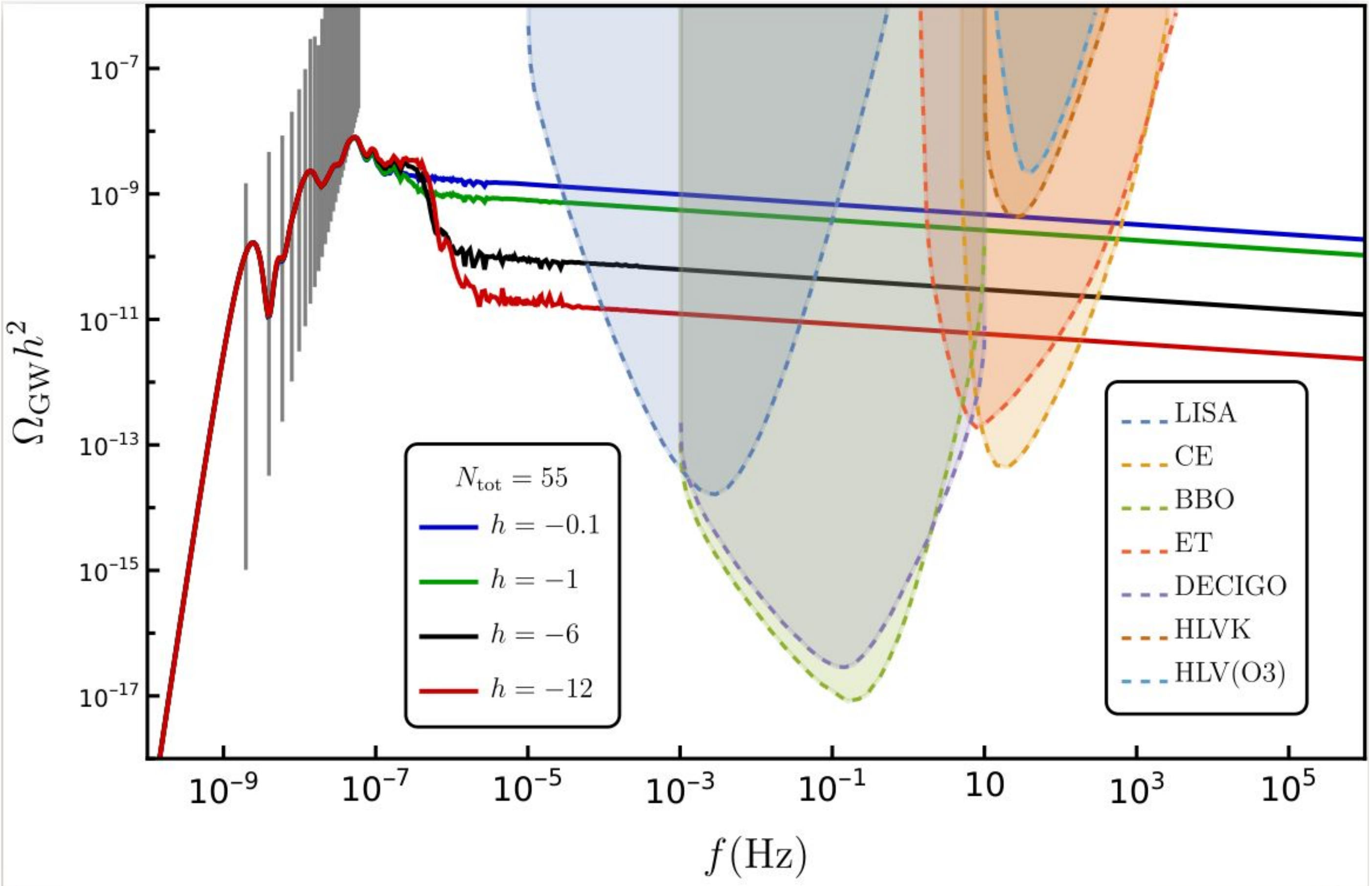


SIGW-USR: NanoGrav signal





SIGW-USR: future observations



Summary:

- The stochastic gravitational wave background (SGWB) detected recently by the pulsar timing arrays (PTAs) observations may have cosmological origins.

- We generated SIGW in nanoHertz frequency from a three-phase model of inflation:

  Slow-Roll > USR > Slow-Roll

  GWs spectrum depends on the sharpness of the transition from the USR phase to the final attractor phase (h) as well as to the duration of the USR period (\Delta N).

- While the model can accommodate the current PTAs data but it has non-trivial predictions for GWs on higher frequency ranges which can be tested by future observations.

- We are doing MCMC simulation to find the best fit!!!

Thank You for Your Attention!

Scalar-induced gravitational waves

$$\mathrm{d}s^2 = -\, a^2 \left[(1+2\Phi)\mathrm{d}\tau^2 + \left((1-2\Psi)\delta_{ij} + \frac{1}{2} h_{ij} \right) \mathrm{d}x^i \mathrm{d}x^j \right]$$

$$\Phi \simeq \Psi$$

$$\Phi_{\mathbf{k}} = \frac{2}{3}\mathcal{T}(\mathbf{k}\tau)\mathcal{R}_{\mathbf{k}}\,.$$

$$h_{\mathbf{k}}^{\lambda\,''}(\eta) + 2\mathcal{H} h_{\mathbf{k}}^{\lambda\,'}(\eta) + k^2 h_{\mathbf{k}}^{\lambda}(\eta) = 4 S_{\mathbf{k}}^{\lambda}(\eta),$$

$$S_{\mathbf{k}}^{\lambda} = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, \varepsilon_{ij}^{\lambda}(\hat{\mathbf{k}})\; q^i q^j \left[2\Phi_{\mathbf{q}}\Phi_{\mathbf{k}-\mathbf{q}} + \left(\mathcal{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}}\right)\left(\mathcal{H}^{-1}\Phi'_{\mathbf{k}-\mathbf{q}} + \Phi_{\mathbf{k}-\mathbf{q}}\right) \right]$$

$$\bar{\Omega}_{\mathrm{GW}}^{\mathrm{ind}}(f) = \int_0^{\infty} \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u\, \mathcal{K}(u,v)\, \mathcal{P}_{\mathcal{R}}(uk)\, \mathcal{P}_{\mathcal{R}}(vk)$$

integration kernel

$$\Omega_{\mathrm{GW}}^{\mathrm{ind}}(f) = \Omega_{\mathrm{r}} \left(\frac{g_*(f)}{g_*^0} \right) \left(\frac{g_{*,s}^0}{g_{*,s}(f)} \right)^{4/3} \bar{\Omega}_{\mathrm{GW}}^{\mathrm{ind}}(f)$$

Primordial Black Holes

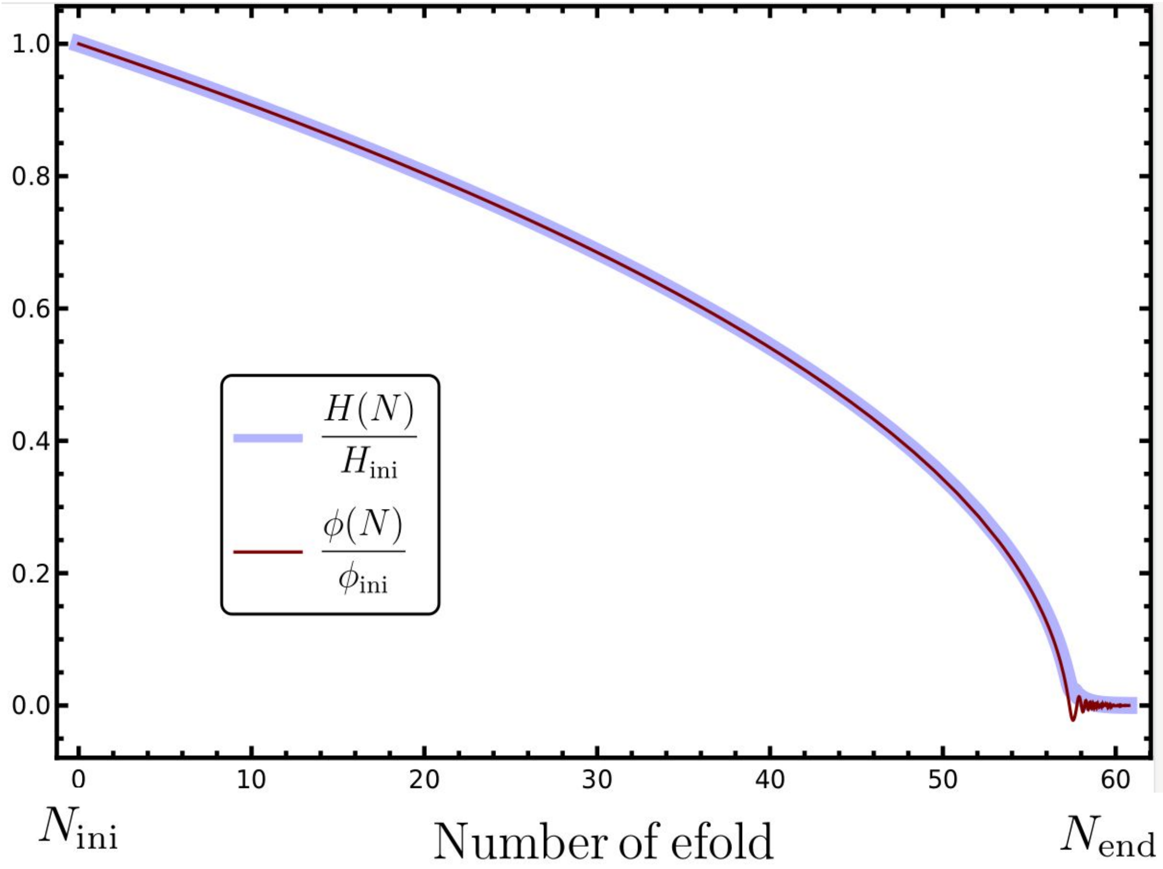
- **Black holes formed in the early Universe (soon after the Big Bang through a non-stellar way)**
  - ❖ Gravitational collapse of the overdense region of inhomogeneities During the radiation dominated era

$$\beta \simeq \int_{\mathcal{R}_c}^{\infty} f_{\mathcal{R}}(x)\, \mathrm{d}x \simeq \frac{1}{2}\mathrm{Erfc}\left(\frac{\mathcal{R}_c}{\sqrt{2\mathcal{P}_{\mathcal{R}}}}\right)$$

$$f_{\mathrm{PBH}}(M_{\mathrm{PBH}}) \simeq 2.7 \times 10^8 \left(\frac{M_{\mathrm{PBH}}}{M_\odot}\right)^{-\frac{1}{2}} \beta(M_{\mathrm{PBH}})$$

$$\frac{M_{\mathrm{PBH}}}{M_\odot} \simeq 10^{-13}\left(\frac{10^{-6} M_{\mathrm{P}}}{H_{\mathrm{inf}}}\right) e^{2(N_{\mathrm{tot}} - N_p - 22.25)}$$

Single (Slow-roll) Inflation



Action:

$$\mathcal{S} = \int \mathrm{d}^4x \, \sqrt{g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} m^2 \phi^2 \right]$$

Slow-roll Eqs.

$$3M_{\rm Pl}^2 H^2 \simeq \frac{1}{2} m^2 \phi^2$$

$$3H\dot{\phi} \simeq -m^2 \phi$$

$$N_{\rm end} \simeq \frac{1}{4} \left(\frac{\phi_{\rm ini}}{M_{\rm Pl}} \right)^2$$

Perturbations: Turn on Quantum Mechanics

$$\phi(t,\mathbf{x}) = \phi(t) + \delta\phi(t,\mathbf{x})$$

Spatially-flat gauge

$$\mathcal{R}(t,\mathbf{x}) = \frac{H(t)}{\dot{\phi}(t)}\delta\phi(t,\mathbf{x})$$

CMB Pivot Scale

$$k_{\rm CMB} \in [0.002, 0.05]\ {\rm Mpc}^{-1}$$

$\mathcal{P}_{\rm COBE} \simeq 2.1 \times 10^{-9}$