

# Muon $g-2$ , DM, and lepton flavor violation in SUSY-GUT theories

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*Colaboration with A. Tiwari, Q. Shafi, C. Ün, J. Ellis, S. Lola, Ruiz de Austri, JHEP 07 (2020) 07, 096 JHEP 09 (2020), Eur. Phys. J. C (2022) 82:561 + work in progress.*

# OUTLINE

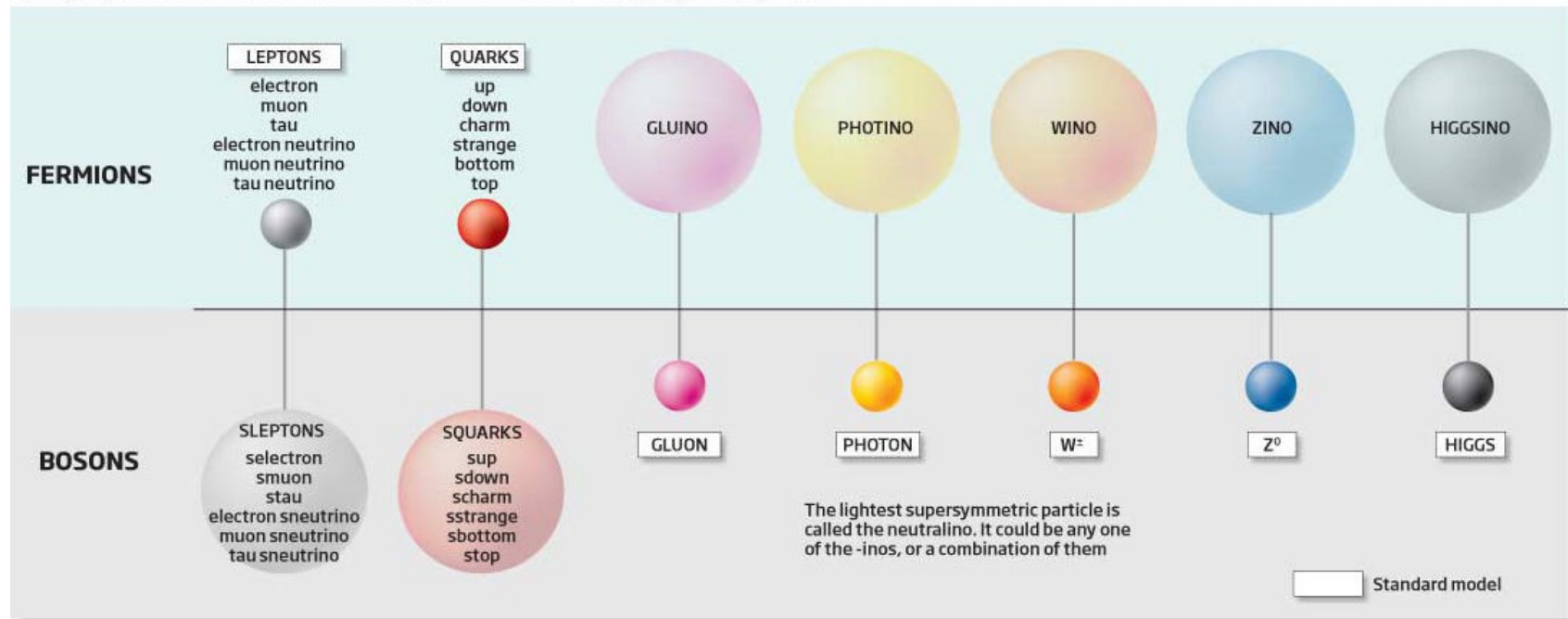
- GUT's and SUSY.
- PS:  $SU(4) \times SU(2) \times SU(2)$
- Fitting muon  $(g-2)$  in  $SU(4) \times SU(2) \times SU(2)$  models
- *LFV constraints and prospects.*
- *Neutralino relic density .*
- *LHC vs LFV.*

*Conclusions.*

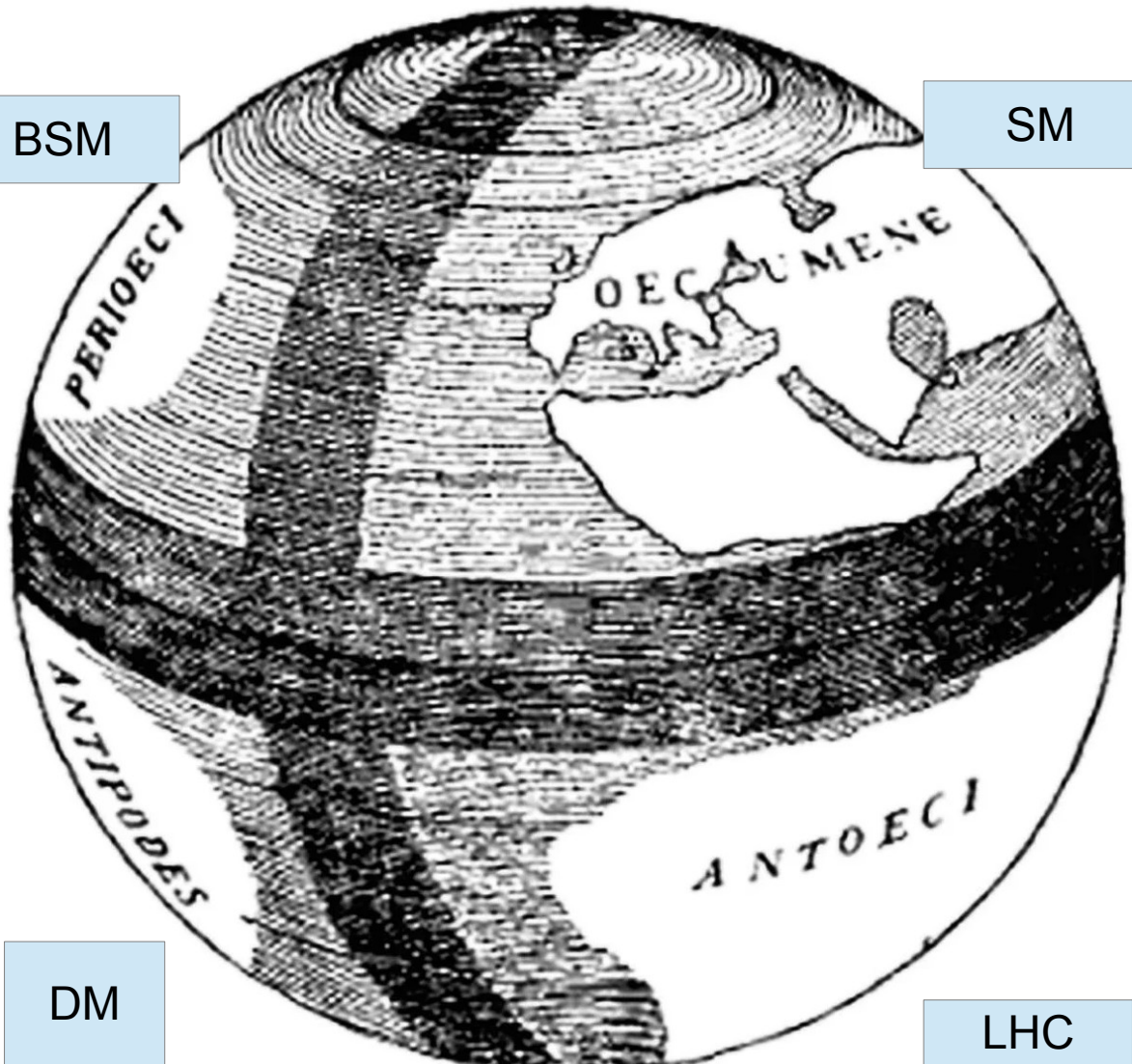
# Particle zoo

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Particles are divided into two families called bosons and fermions. Among them are groups known as leptons, quarks and force-carrying particles like the photon. Supersymmetry doubles the number of particles, giving each fermion a massive boson as a super-partner and vice versa. The LHC is expected to find the first supersymmetric particle



# Crates of Mallus 150

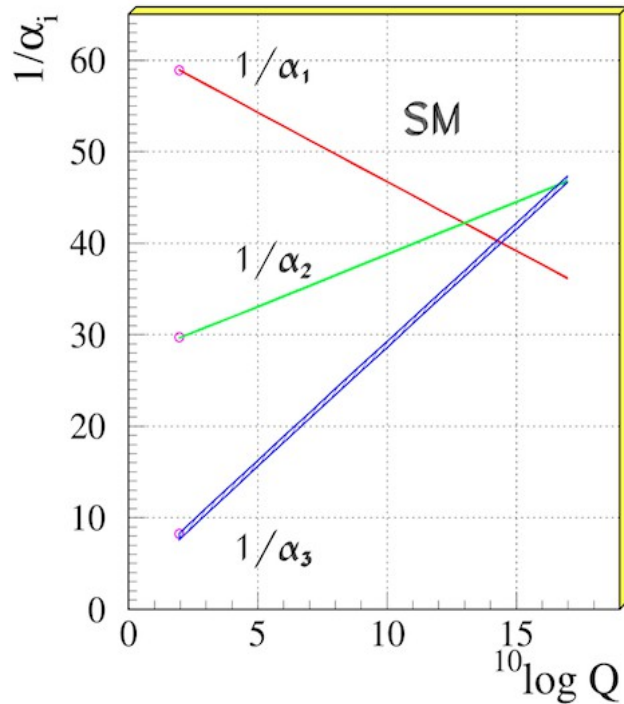


**Antipodes theory:**  
Popular debate in the Middle age.

Probably referred in all explation trip proposals until it was eliminated by direct observation.



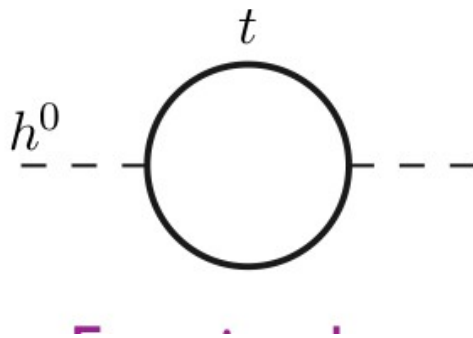
# Hierarchy Problem



Almost unification at  $\sim 10^{14}$  GeV

$$\mu \frac{d\alpha_i(\mu)}{d\mu} = -\frac{1}{2\pi} \left[ b_i + \frac{1}{4\pi} \sum b_{ij} \alpha_j(\mu) \right] \alpha_i^2(\mu)$$

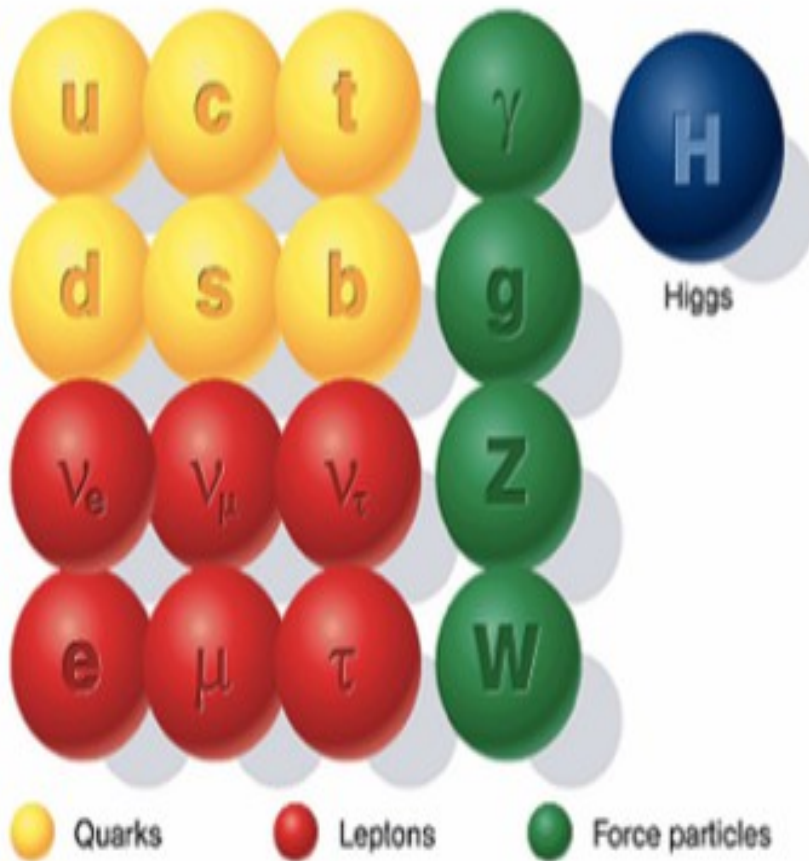
$$b_i = (0, -22/3, -11) + N_F(4/3, 4/3, 4/3) + N_H(1/10, 1/6, 0)$$



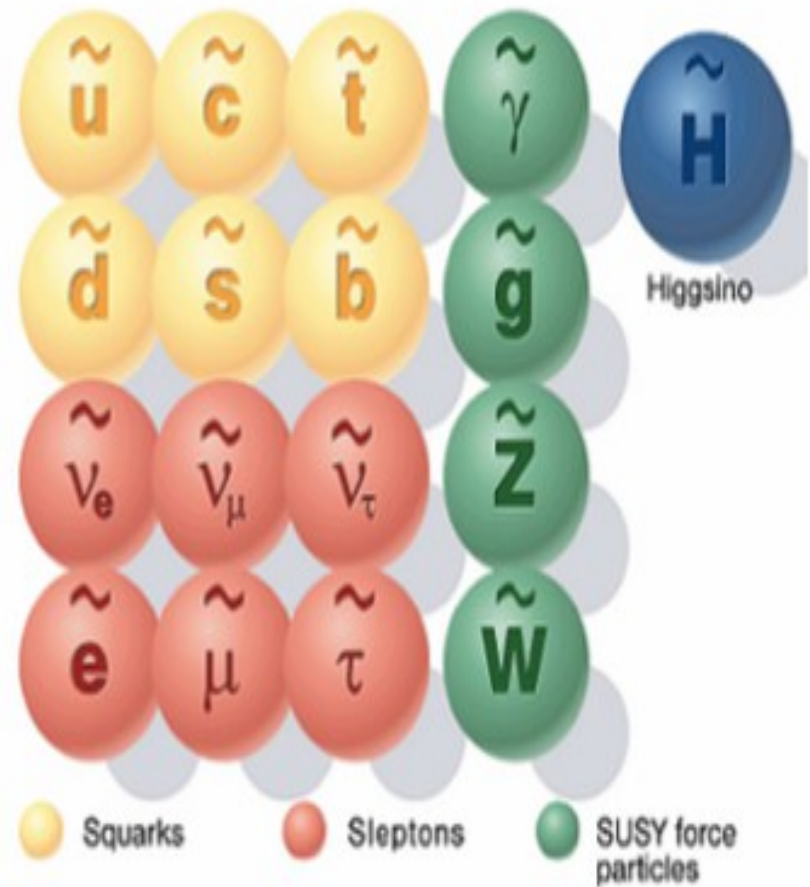
Divergent contribution to Higgs mass  $\uparrow$  with  $(m_{\text{scale}})^2$



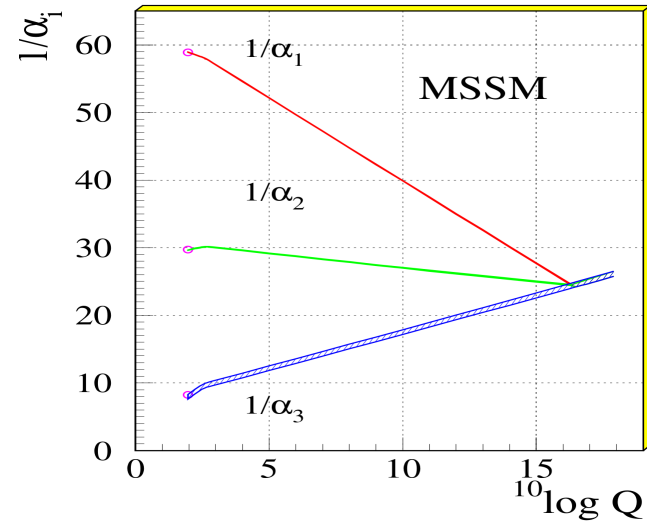
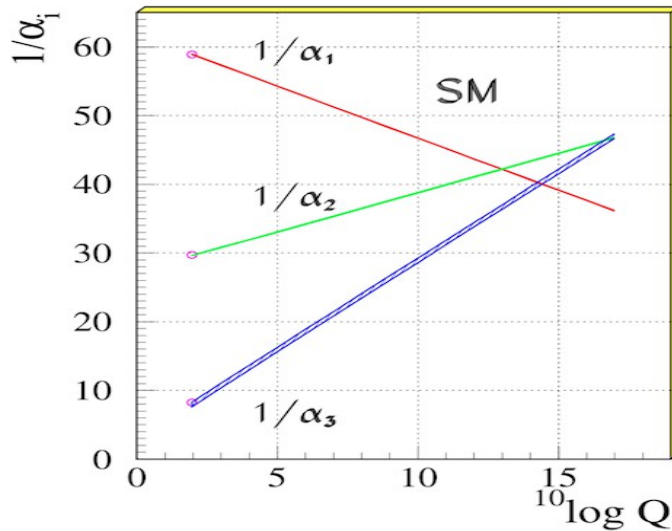
## Standard particles



## SUSY particles

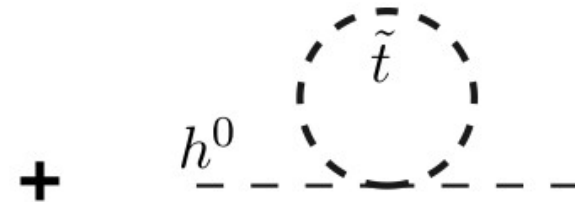
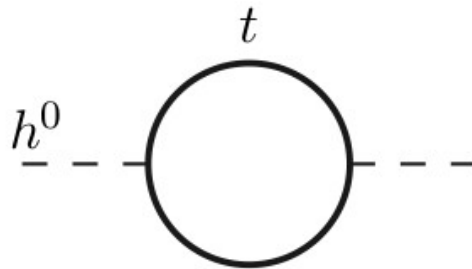


# Hierarchy Problem



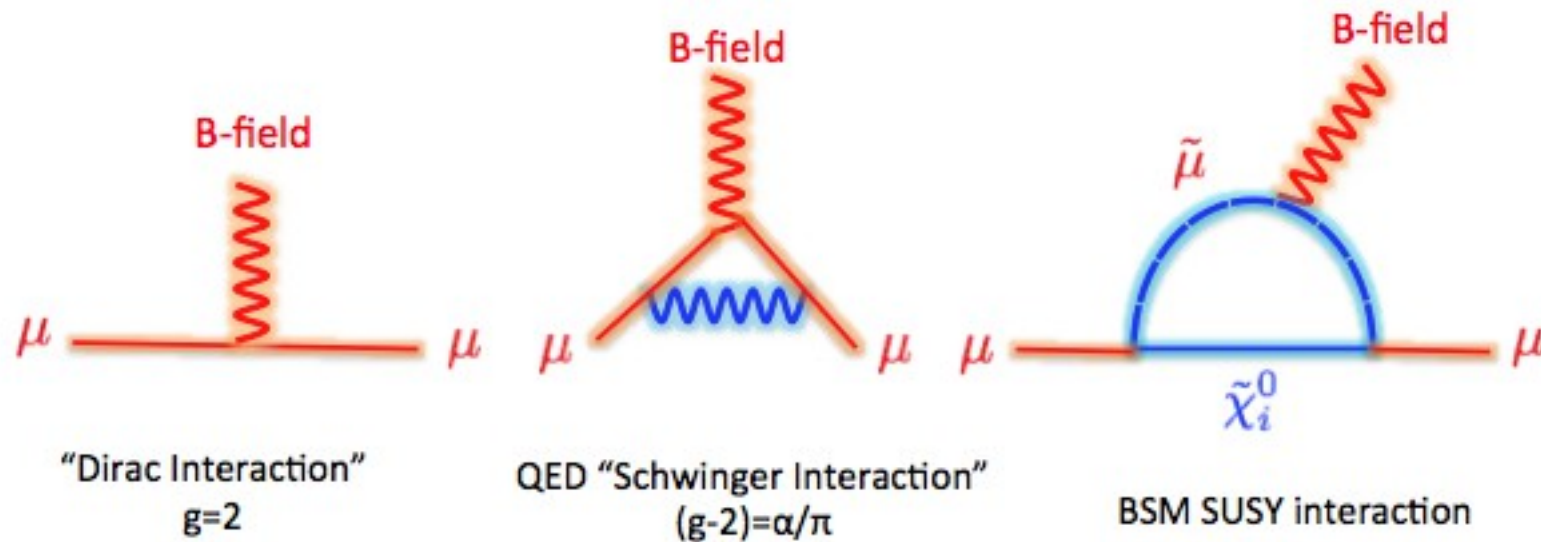
Unification at  $\sim 10^{16}$  GeV

$$b_i = (0, -6, -9) + N_F(2, 2, 2) + N_H(3/10, 1/2, 0)$$



Cancelation of quadratic divergences

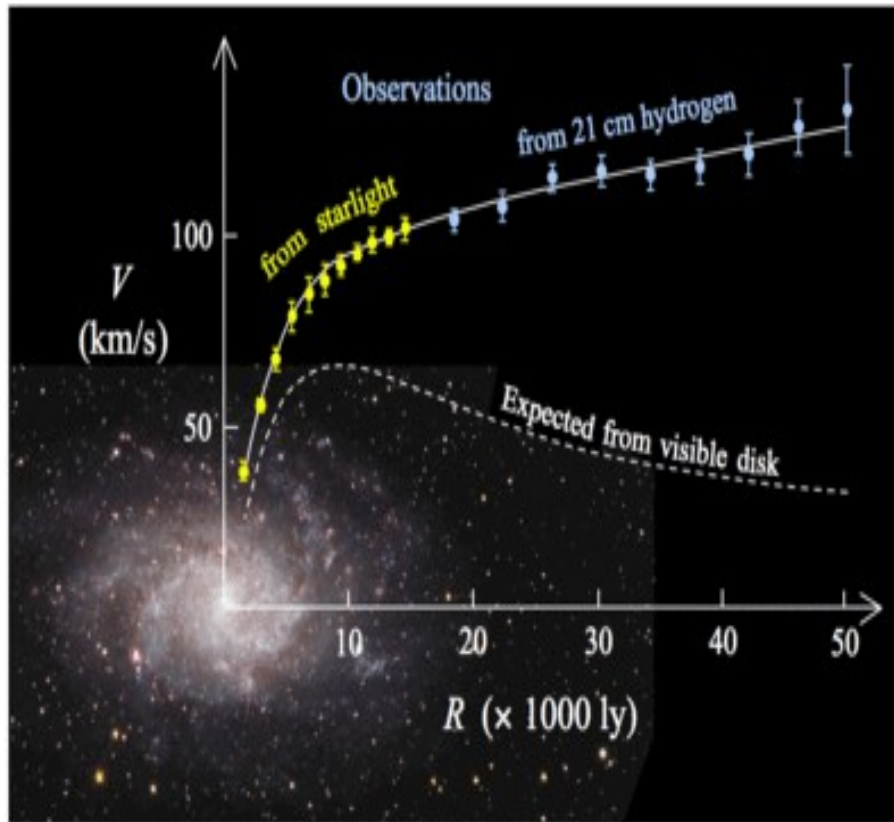
# One loop contribution to SM proc.



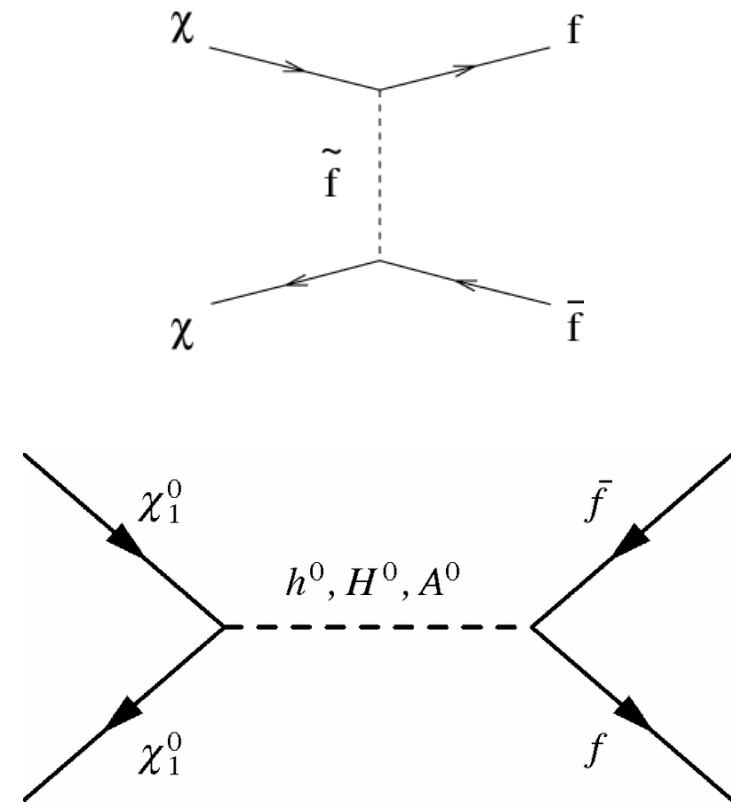
SM vs Experiment Discrepancies  
anomalous magnetic dipole moment ( $g_\mu - 2$ )



# Dark Matter problem

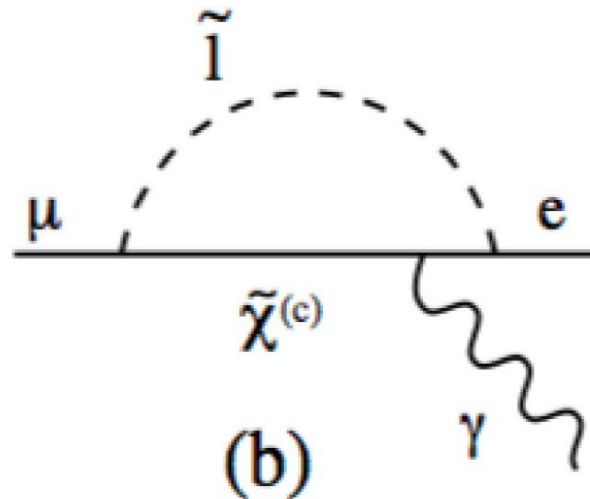
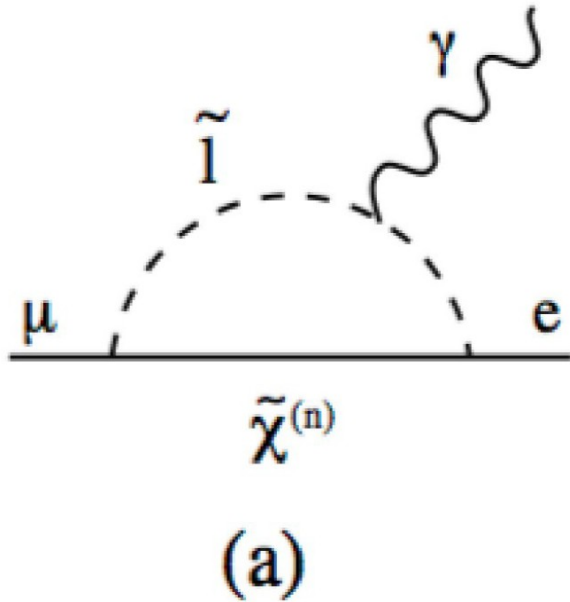


Rotation curve of spiral galaxy M 33 (yellow and blue points with error bars), and a predicted one from distribution of the visible matter (white line). The discrepancy between the two curves can be accounted for by adding a dark matter halo surrounding the galaxy



$$\Omega h^2 \sim \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma_{\text{ann}} v_{\text{Mol}} \rangle}$$

In SUSY flavor mixing lepton-slepton vertices can induce LFV diagrams:



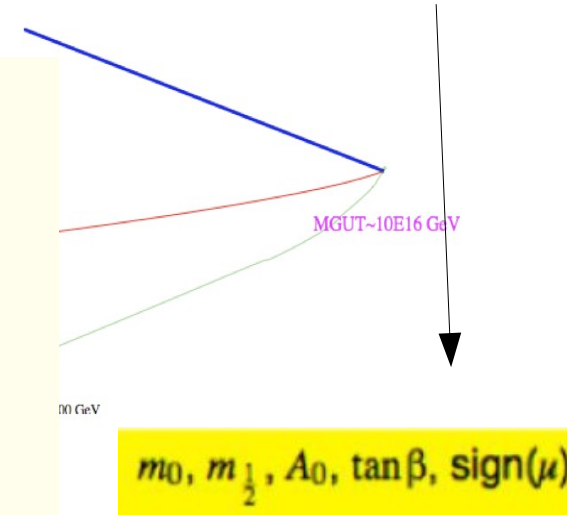
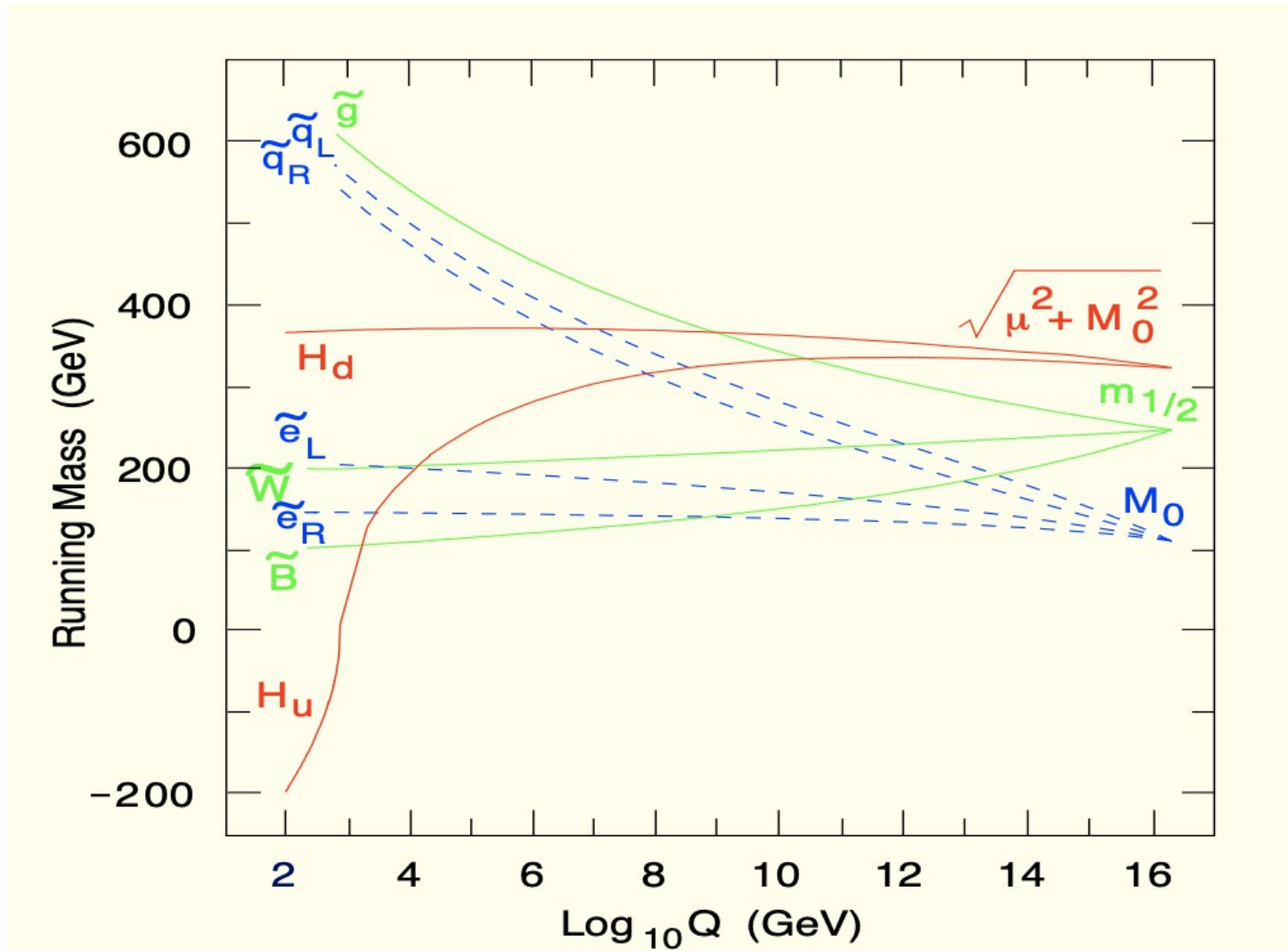
Lepton-slepton flavor mixing is very constrained by the experimental limits:

$$BR(\mu \rightarrow e \gamma) < 5.7 \times 10^{-13}$$

$$BR(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}$$

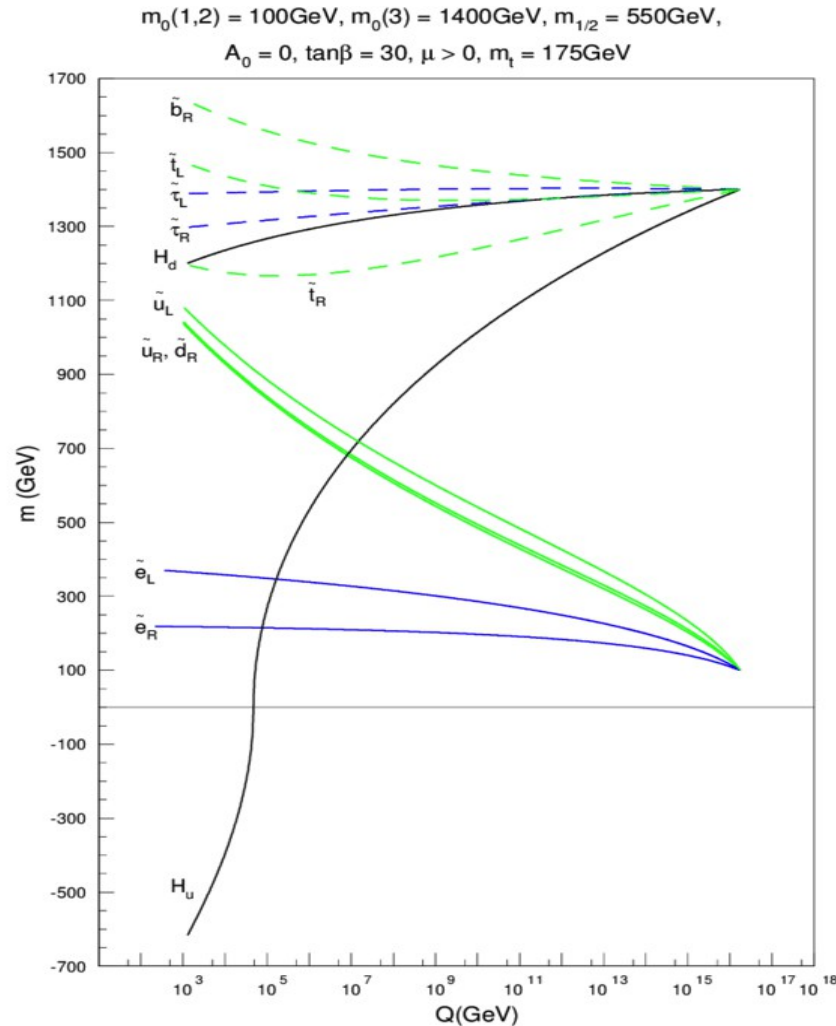
$$BR(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8}$$

# GUT initial conditions

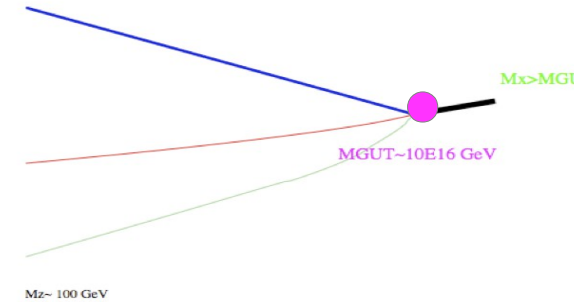


Gunion,  
Int. J. Mod. Phys. A 2010

# Non Universal scenarios



Baer et al., HEP 06 (2004) 044



## CMSSM choice:

- $m_0$  Universal soft masses.
- $m_{1/2}$  Universal gaugino masses.
- $A_0$  Universal Trilinear terms.

## Representation-dependent choice

$$m_r = x_r m_0$$

$$A_r = Y_r A_0, \quad A_0 = a_0 m_0$$



SM

MSSM+neutrino masses

GUT Sacale

Family Symmetries  
Additional Fields .....

Planck Sacale





# PATI-SALAM Unification

$$G_{PS} \equiv SU(4) \times SU(2)_L \times SU(2)_R$$

$4_c 2_L 2_R$

**MATTER FIELDS**

$$F_r \quad \begin{pmatrix} d_r & -u_r \\ e_r & -\nu_r \end{pmatrix} \quad (4, 2, 1)$$

$$F_r^c \quad \begin{pmatrix} u_r^c \\ d_r^c \end{pmatrix}, \begin{pmatrix} \nu_r^c \\ e_r^c \end{pmatrix} \quad (\bar{4}, 1, 2)$$

$$\langle \tilde{\nu}_H^c \rangle = \langle \tilde{\nu}_H^c \rangle \sim M$$

**HIGGS FIELDS**

$$H^c \quad \begin{pmatrix} u_H^c \\ d_H^c \end{pmatrix}, \begin{pmatrix} \nu_H^c \\ e_H^c \end{pmatrix} \quad (\bar{4}, 1, 2)$$

$$\bar{H}^c \quad \begin{pmatrix} \bar{u}_H^c & \bar{d}_H^c \\ \bar{\nu}_H^c & \bar{e}_H^c \end{pmatrix} \quad (4, 1, 2)$$

$$h \quad \begin{pmatrix} h_2^+ & h_1^0 \\ h_2^0 & h_1^- \end{pmatrix} \quad (1, 2, 2)$$

$$G_{PS} \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

$$M_4 = M_3; M_2 = y_{LR} M_{2R}$$

$$M_4 = M_3; M_2 = y_{LR} M_{2R}$$

Condition for gaugino masses.

$$m_{H_{u,d}}^2 = m_{10}^2 \mp 2M_D^2$$



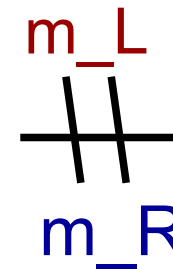
PS(4-2-1) *LR Asymmetry*

**MATTER FIELDS**

---

$$F_r \quad \begin{pmatrix} d_r & -u_r \\ e_r & -\nu_r \end{pmatrix} \quad (4, 2, 1)$$

$$F_r^c \quad \begin{pmatrix} u_r^c \\ d_r^c \end{pmatrix}, \begin{pmatrix} \nu_r^c \\ e_r^c \end{pmatrix} \quad (\bar{4}, 1, 2)$$



Gaugino Masses

$$M_1 = \frac{3}{5}M_{2R} + \frac{2}{5}M_4 ,$$

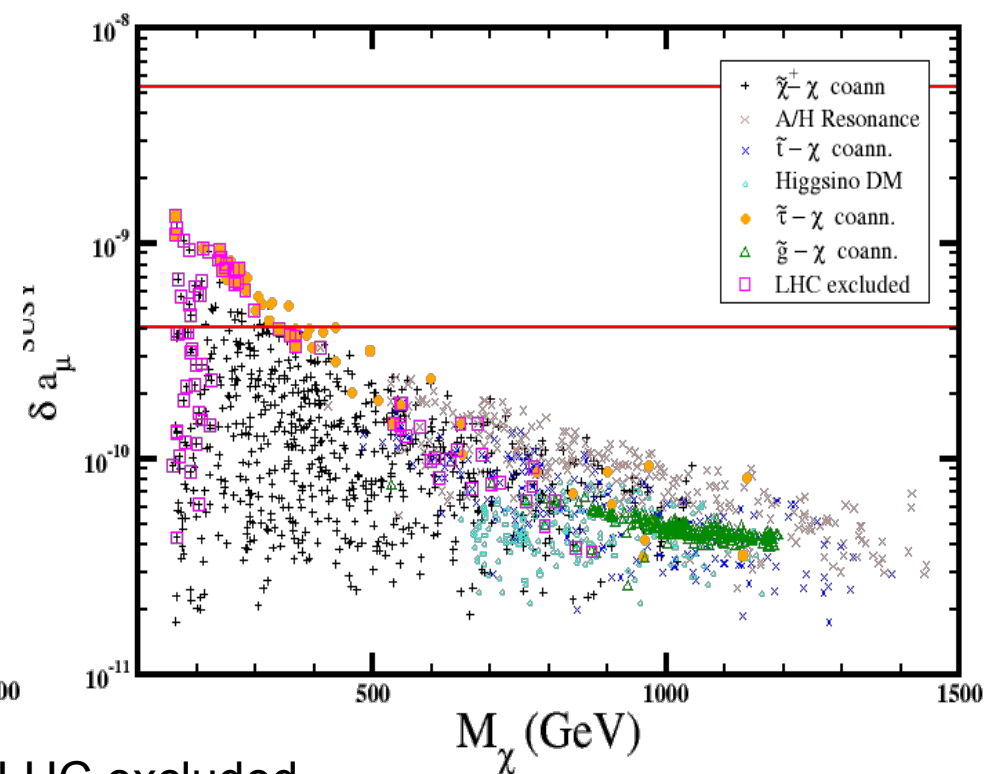
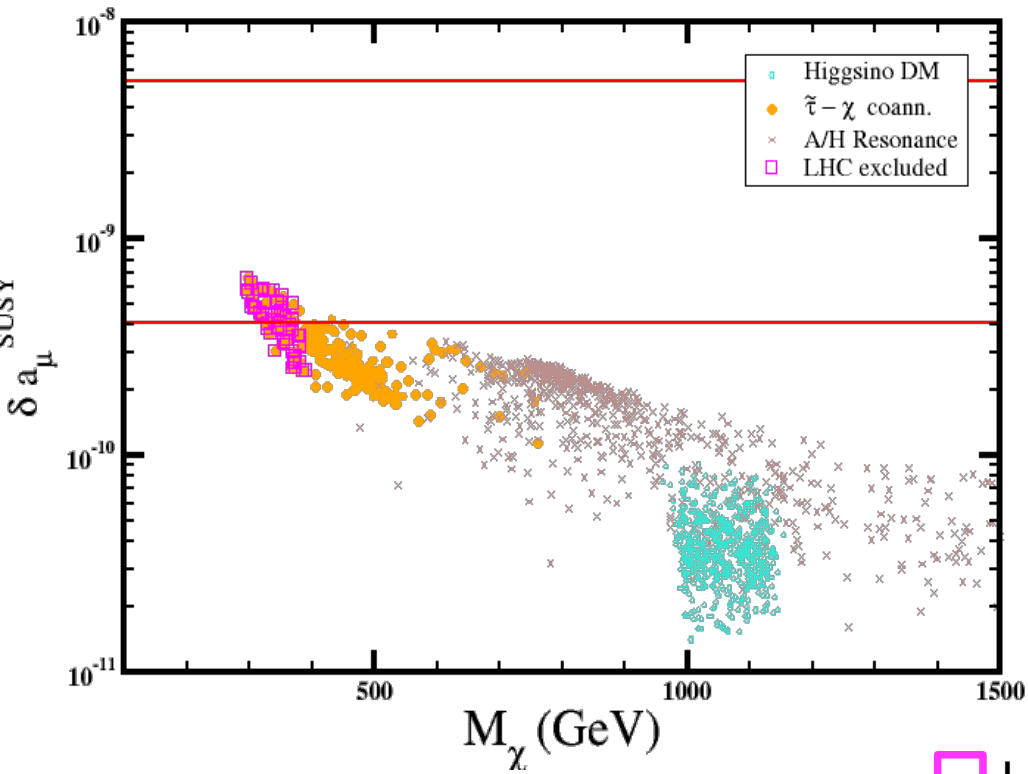
$$M_4 = M_3 ; M_2 = y_{LR} M_{2R}$$

New Parameter

$$X_{LR} = \frac{m_R}{m_L}$$

# SO(10)


# PS(4-2-2)



 LHC excluded.

Higgsino DM

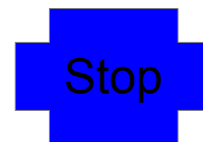
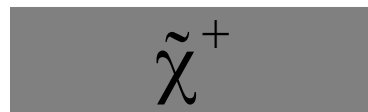
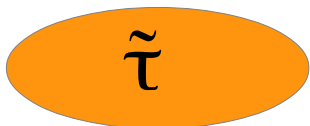
A/H Resonances

$h_f > 0.1, |m_A - 2m_\chi| > 0.1 m_\chi.$  

$|m_A - 2m_\chi| \leq 0.1 m_\chi$  

$h_f \equiv |N_{13}|^2 + |N_{14}|^2,$

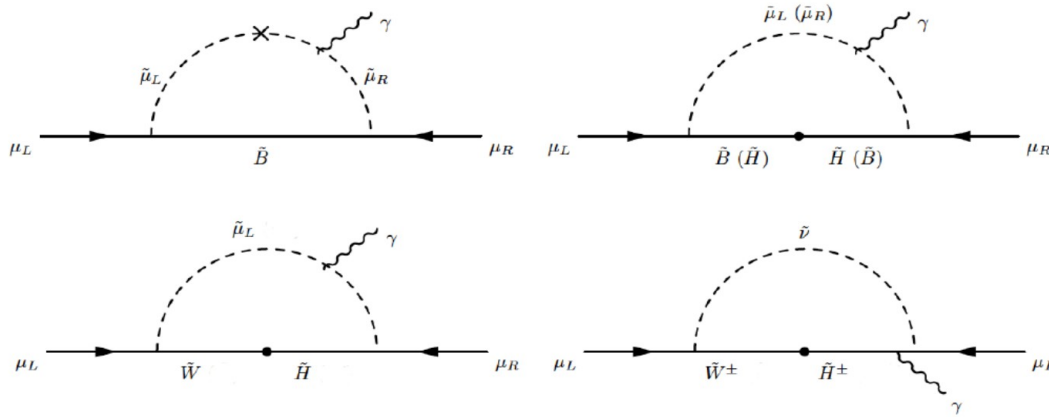
Coannihilations:  $(m_z - m_{LSP}) < 0.1 m_{LSP}$



# Muon g-2 combining Fermilab + BNL data

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.1 \pm 5.9) \times 10^{-10} .$$

## SUSY Contribution to Muon g-2



Low Scale	GUT Scale
$m_{\tilde{\mu}_L}, m_{\tilde{\nu}}$	$m_L$
$m_{\tilde{\mu}_R}$	$m_R$
$M_{\tilde{B}}$	$M_1$
$M_{\tilde{W}}$	$M_2$
$\mu$	$m_{H_u}, m_{H_d}$
$A_\mu$	$A_0$
$\tan \beta$	$\tan \beta$

$$m_h = 123 - 127 \text{ GeV}$$

$$m_{\tilde{g}} \geq 2.1 \text{ TeV (800 GeV if it is NLSP)}$$

$$0.8 \times 10^{-9} \leq \text{BR}(B_s \rightarrow \mu^+ \mu^-) \leq 6.2 \times 10^{-9} (2\sigma)$$

$$2.99 \times 10^{-4} \leq \text{BR}(B \rightarrow X_s \gamma) \leq 3.87 \times 10^{-4} (2\sigma)$$

$$0.114 \leq \Omega_{\text{CDM}} h^2 \leq 0.126 .$$

$$0 \leq m_L \leq 5 \text{ TeV}$$

$$0 \leq M_{2L} \leq 5 \text{ TeV}$$

$$-3 \leq M_3 \leq 5 \text{ TeV}$$

$$-3 \leq A_0/m_L \leq 3$$

$$1.2 \leq \tan \beta \leq 60$$

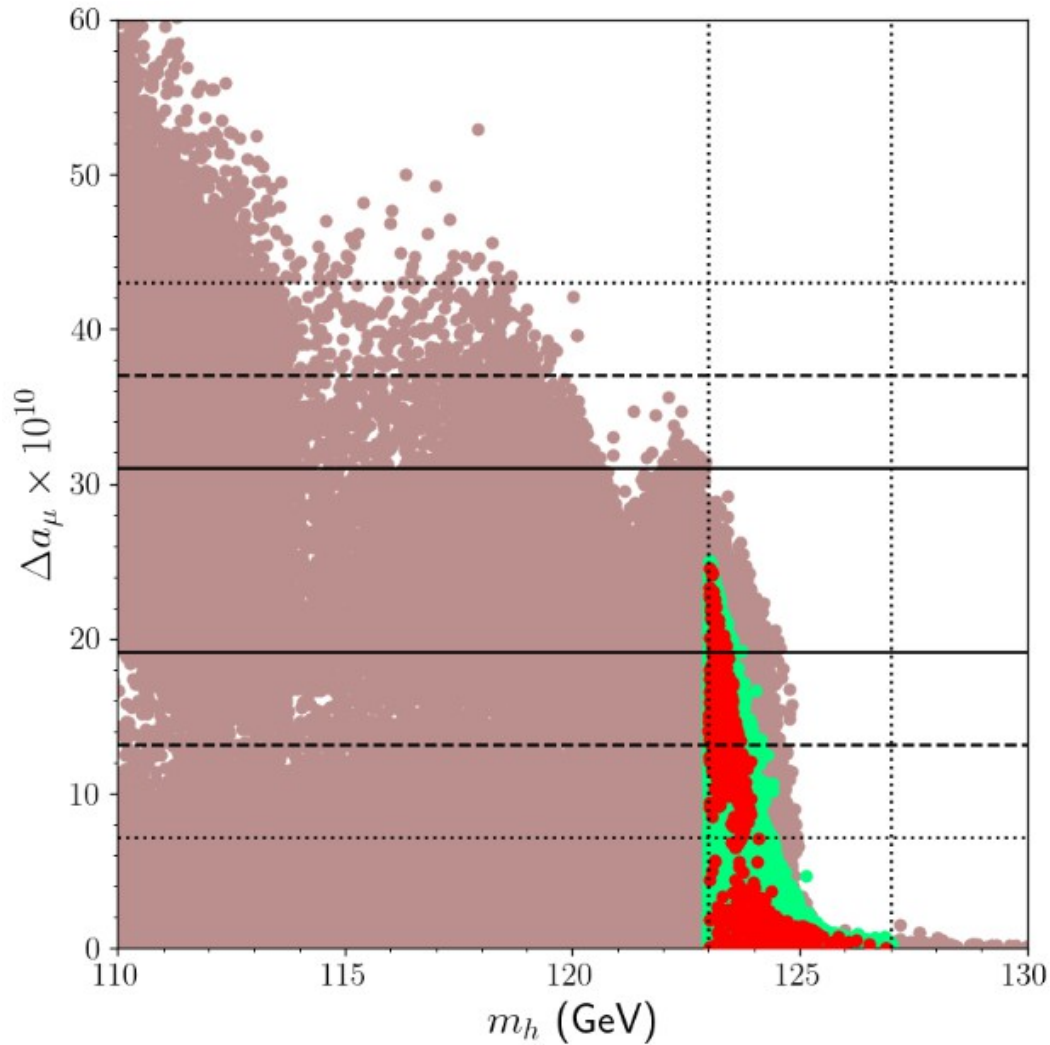
$$0 \leq x_{\text{LR}} \leq 3$$

$$-3 \leq y_{\text{LR}} \leq 3$$

$$0 \leq x_{\text{d}} \leq 3$$

$$-1 \leq x_{\text{u}} \leq 2 .$$

## Important contribution is in tension with the Higgs mass



$$\Delta a_\mu^{\tilde{B}\tilde{\mu}_L\tilde{\mu}_R} \simeq \frac{g_1^2}{16\pi^2} \frac{m_\mu^2 M_{\tilde{B}} (\mu \tan \beta - A_\mu)}{m_{\tilde{\mu}_L}^2 m_{\tilde{\mu}_R}^2} F_N \left( \frac{m_{\tilde{\mu}_L}^2}{M_{\tilde{B}}^2}, \frac{m_{\tilde{\mu}_R}^2}{M_{\tilde{B}}^2} \right)$$

$$\Delta m_h^2 \simeq \frac{m_t^4}{16\pi^2 v^2 \sin^2 \beta} \frac{\mu A_t}{M_{\text{SUSY}}^2} \left[ \frac{A_t^2}{M_{\text{SUSY}}^2} - 6 \right]$$

$$\begin{aligned}
\frac{dm_{\tilde{l}_{R_i}}^2}{dt} &= -\frac{1}{16\pi^2} \left[ 2(Y_l^i)^2 P_{\tilde{l}}^i + g_1^2 \text{Tr}(Y m^2) - 4g_1^2 M_1^2 \right] \\
\frac{dm_{\tilde{L}_i}^2}{dt} &= -\frac{1}{16\pi^2} \left[ (Y_l^i)^2 P_{\tilde{l}}^i - \frac{1}{2} g_1^2 \text{Tr}(Y m^2) - (g_1^2 M_1^2 + 3g_2^2 M_2^2) \right] \\
\frac{dm_{\tilde{d}_{R_i}}^2}{dt} &= -\frac{1}{16\pi^2} \left[ 2(Y_d^i)^2 P_{\tilde{d}}^i + \frac{1}{3} g_1^2 \text{Tr}(Y m^2) - \left( \frac{4}{9} g_1^2 M_1^2 + \frac{16}{3} g_3^2 M_3^2 \right) \right] \\
\frac{dm_{\tilde{u}_{R_i}}^2}{dt} &= -\frac{1}{16\pi^2} \left[ 2(Y_u^i)^2 P_{\tilde{u}}^i - \frac{2}{3} g_1^2 \text{Tr}(Y m^2) - \left( \frac{16}{9} g_1^2 M_1^2 + \frac{16}{3} g_3^2 M_3^2 \right) \right] \\
\frac{dm_{\tilde{Q}_i}^2}{dt} &= -\frac{1}{16\pi^2} \left[ (Y_u^i)^2 P_{\tilde{u}}^i + (Y_d^i)^2 P_{\tilde{d}}^i + \frac{1}{6} g_1^2 \text{Tr}(Y m^2) - \left( \frac{1}{9} g_1^2 M_1^2 + 3g_2^2 M_2^2 + \frac{16}{3} g_3^2 M_3^2 \right) \right]
\end{aligned}$$

$$m_{\tilde{u}_{iL}}^2 = m_{\tilde{q}_{i0}}^2 + m_{u_i}^2 + \tilde{\alpha}_G \left( \frac{8}{3} f_3 m_3^2 + \frac{3}{2} f_2 m_2^2 + \frac{1}{30} f_1 m_1^2 \right)$$

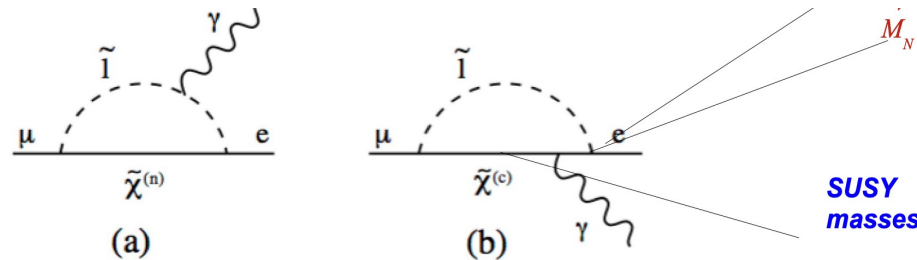
$$m_{\tilde{u}_{iR}}^2 = m_{\tilde{u}_{i0}}^2 + m_{u_i}^2 + \tilde{\alpha}_G \left( \frac{8}{3} f_3 m_3^2 + \frac{8}{15} f_1 m_1^2 \right)$$

$$m_{\tilde{e}_{iL}}^2 = m_{\tilde{\ell}_{i0}}^2 + m_{e_i}^2 + \tilde{\alpha}_G \left( \frac{3}{2} f_2 m_2^2 + \frac{3}{10} f_1 m_1^2 \right)$$

$$m_{\tilde{e}_{iR}}^2 = m_{\tilde{e}_{i0}}^2 + m_{e_i}^2 + \tilde{\alpha}_G \frac{6}{5} f_1 m_1^2.$$

## LFV violating through soft masses.

- LFV can be induced by a misalignment of leptons and sleptons



- Flavor dependence on soft terms can be induced:
  - Below GUT,** Radiatively generated by the same mechanism that explain neutrino oscillations.
  - Above GUT,** by extra fields needed to explain fermion hierarchy.



# MSSM extended by seesaw mechanism

- The superpotential for MSSM-Seesaw I can be written as

$$W = W_{\text{MSSM}} + Y_{\nu}^{ij} \epsilon_{\alpha\beta} H_2^{\alpha} N_i^c L_j^{\beta} + \frac{1}{2} M_N^{ij} N_i^c N_j^c, \quad (5)$$

- The full set of soft SUSY-breaking terms is given by,

$$\begin{aligned} -\mathcal{L}_{\text{soft,SI}} = & -\mathcal{L}_{\text{soft}} + (m_{\tilde{\nu}}^2)_j^i \tilde{\nu}_{Ri}^* \tilde{\nu}_R^j + \left( \frac{1}{2} B_{\nu}^{ij} M_N^{ij} \tilde{\nu}_{Ri}^* \tilde{\nu}_{Rj}^* \right. \\ & \left. + A_{\nu}^{ij} h_2 \tilde{\nu}_{Ri}^* \tilde{l}_{Lj} + \text{h.c.} \right), \end{aligned} \quad (6)$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^{D^T} & M_R \end{pmatrix}$$

“See-Saw” explanation for tiny masses.

• The physical masses are:

1.  $m_1 \equiv m_{light} \simeq \frac{(m_\nu^D)^2}{M_R}$
2.  $m_2 \simeq M_R$

• For  $(m_\nu^D)_{33} \approx (200 \text{ GeV})$  ( $\lambda_\nu \approx \lambda_t$ ) and  $M_{N_3} \approx O(10^{14} \text{ GeV})$ ,  $m_{eff} \approx 0.05 \text{ eV}$

$$W = W_{\text{MSSM}} + \frac{1}{2} (Y_\nu L H_2)^T M_N^{-1} (Y_\nu L H_2).$$

$$m_{\text{eff}} = -\frac{1}{2} v_u^2 Y_\nu \cdot M_N^{-1} \cdot Y_\nu^T, \quad m_\nu^\delta = U^T m_{\text{eff}} U$$

# Slepton flavor mixings

$$(m_{\tilde{L}}^2)_{ij} \sim \frac{1}{16\pi^2} (6m_0^2 + 2A_0^2) (Y_\nu^\dagger Y_\nu)_{ij} \log \left( \frac{M_{\text{GUT}}}{M_N} \right)$$

$$(m_{\tilde{e}}^2)_{ij} \sim 0$$

$$(A_l)_{ij} \sim \frac{3}{8\pi^2} A_0 Y_{li} (Y_\nu^\dagger Y_\nu)_{ij} \log \left( \frac{M_{\text{GUT}}}{M_N} \right)$$

Orthogonal matrix

$$Y_\nu = \frac{\sqrt{2}}{v_u} \sqrt{M_R^\delta} R \sqrt{m_\nu^\delta} U^\dagger$$

Casas + Ibarra

Diagonal Universal  
1E14 GeV

Limit case of degenerate MR

Order 1

$$Y_\nu^\dagger Y_\nu = \frac{2}{v_u^2} M_R U m_\nu^\delta U^\dagger$$

Using neutrino data  
LFV depends is  
controlled by MR

# Slepton flavor mixing above GUT.

Generation of Yukawa textures using family symmetries, for example Abelian U(1)'s

$$\Phi_i \Phi_J^c h \frac{\theta^{(q_i - q_j + q_h)}}{M}, \quad \epsilon = \frac{\langle \theta \rangle}{M} \longrightarrow Y_{IJ} \Phi_I \Phi_J^c h$$

$$Y_{IJ} \sim \epsilon^{(q_i - q_j + q_h)}$$

Froggatt-Nilsen 1979

Soft terms: In SUGRA models, redefinition of fields due to flavons results in non universal soft masses.

$$m_f^2 = \begin{pmatrix} 1 & \epsilon^{q^{12}} & \epsilon^{q^{13}} \\ \epsilon^{q^{12}} & 1 & \epsilon^{q^{23}} \\ \epsilon^{q^{12}} & \epsilon^{q^{23}} & 1 \end{pmatrix} \times m_{f0}^2,$$

S. F. King et al 2005,  
Olive+Velasco-Sevilla 2005  
Das et al 2017

LFV depends on  
the value of

$\epsilon$



GUT

$$m_f^2 = \begin{pmatrix} 1 & \epsilon^{q_L^{12}} & \epsilon^{q_L^{13}} \\ \epsilon^{q_L^{12}} & 1 & \epsilon^{q_L^{23}} \\ \epsilon^{q_L^{12}} & \epsilon^{q_L^{23}} & 1 \end{pmatrix} \times m_L^2,$$

$$m_{fc}^2 = \begin{pmatrix} 1 & \epsilon^{q_R^{12}} & \epsilon^{q_R^{13}} \\ \epsilon^{q_R^{12}} & 1 & \epsilon^{q_R^{23}} \\ \epsilon^{q_R^{12}} & \epsilon^{q_R^{23}} & 1 \end{pmatrix} \times m_R^2,$$



M\_SUSY scale

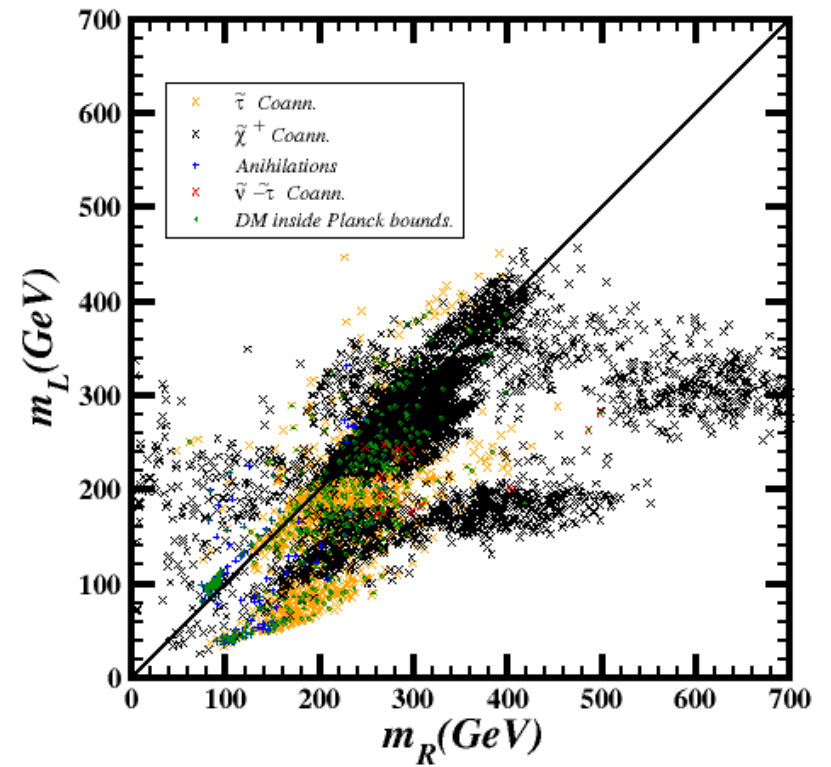
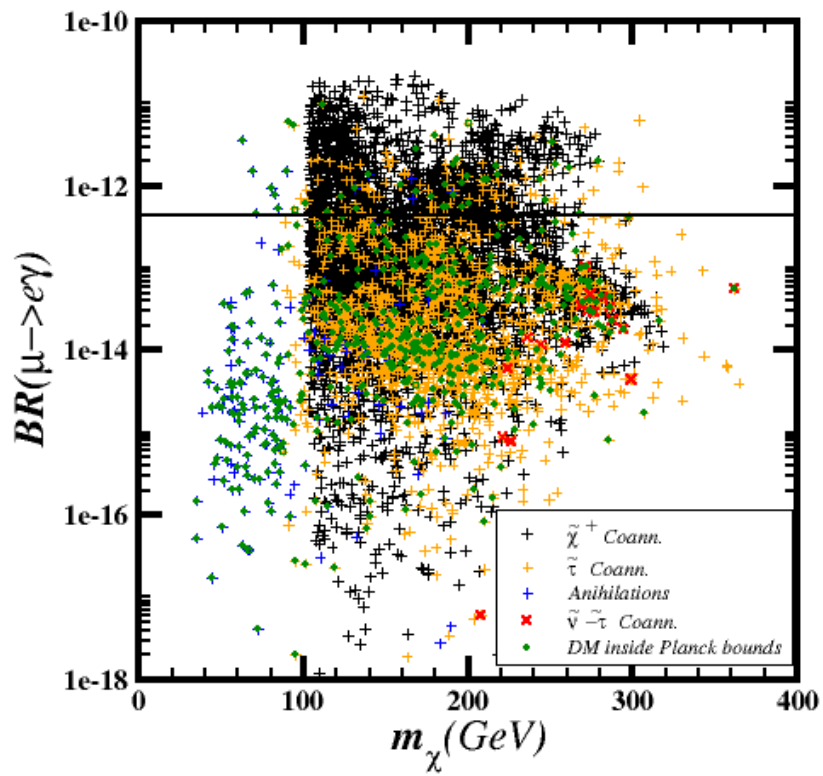
$$m_{\tilde{Q}}^2 \sim m_L^2 + (k_3 \cdot M_3^2 + k_2 \cdot M_1^2 + \frac{1}{36} k_1 \cdot M_1^2) \times I,$$

$$m_{\tilde{U}}^2 \sim m_R^2 + (k_3 \cdot M_3^2 + \frac{4}{9} k_1 \cdot M_1^2) \times I,$$

$$m_{\tilde{D}}^2 \sim m_R^2 + (k_3 \cdot M_3^2 + \frac{1}{9} k_1 \cdot M_1^2) \times I,$$

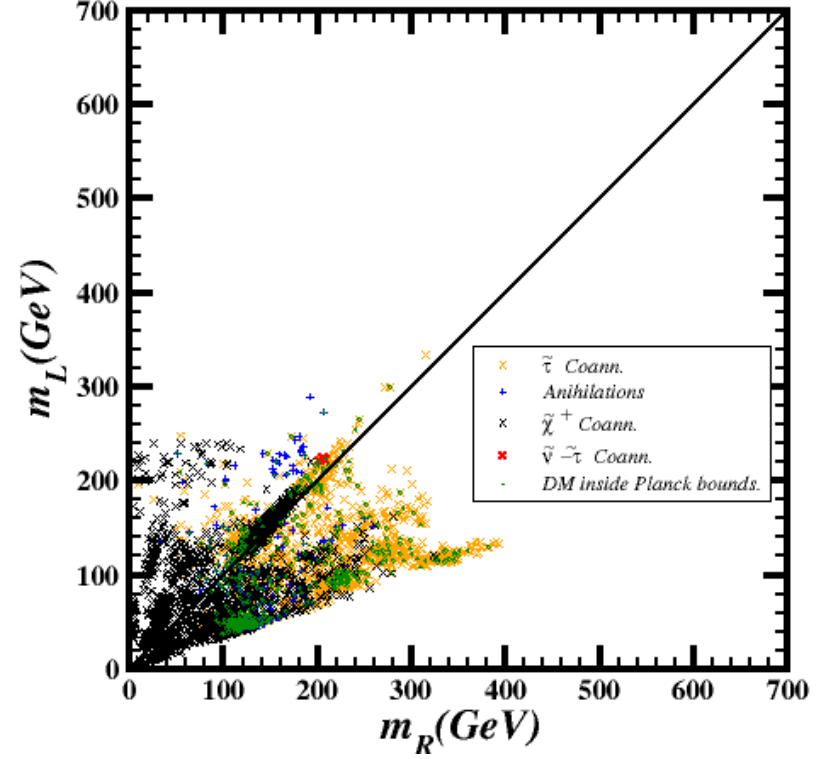
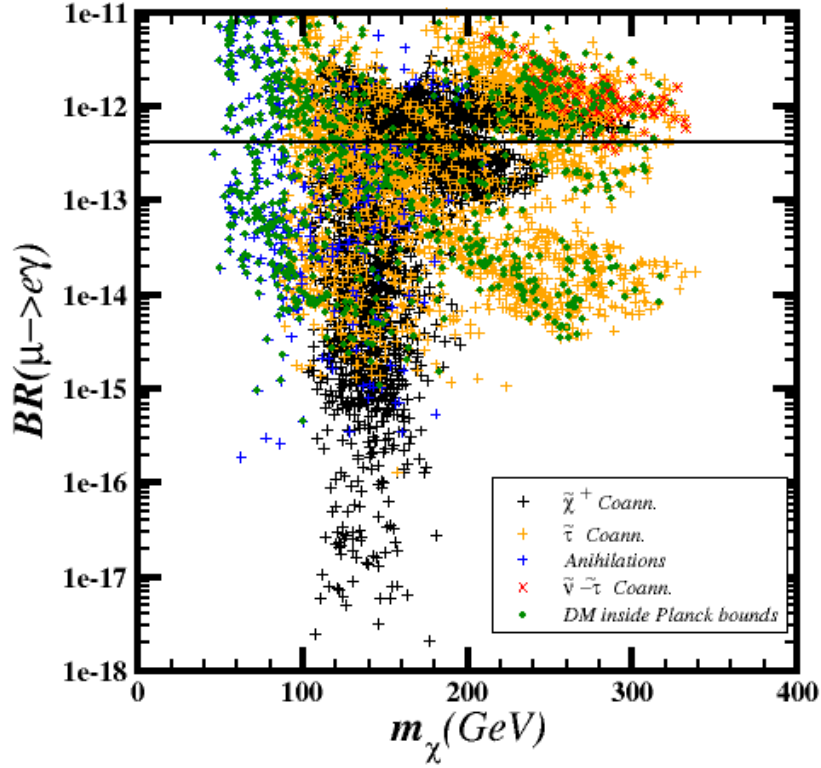
$$m_{\tilde{e}_L}^2 \sim m_L^2 + (k_2 \cdot M_2^2 + \frac{1}{4} k_1 \cdot M_1^2) \times I,$$

$$m_{\tilde{e}_R}^2 \sim m_R^2 + (k_1 \cdot M_1^2) \times I.$$



See-Saw with MR scale at  $2.5 \cdot 10^{12}$  GeV

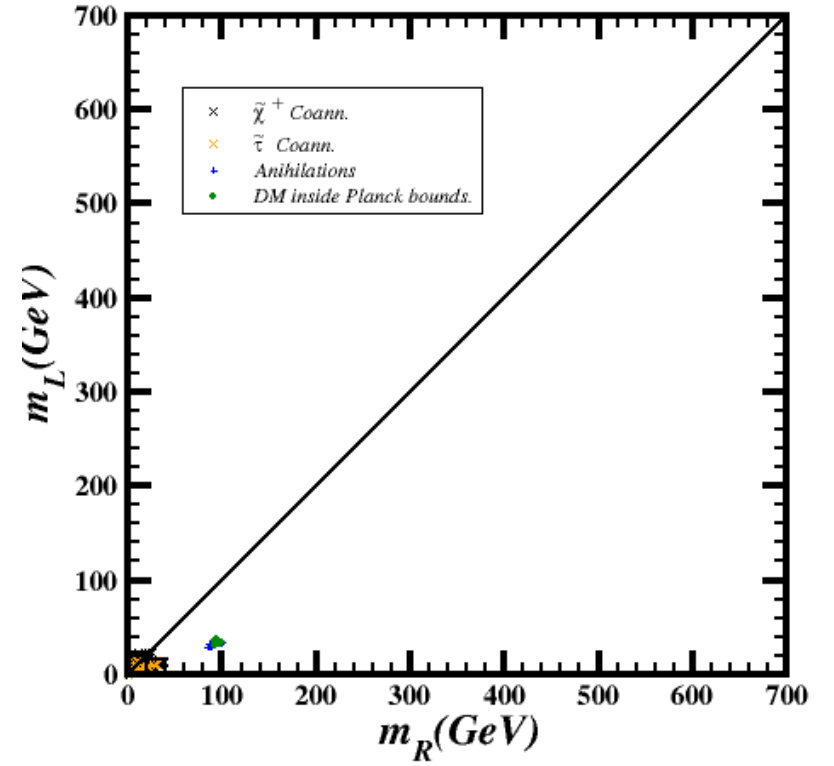
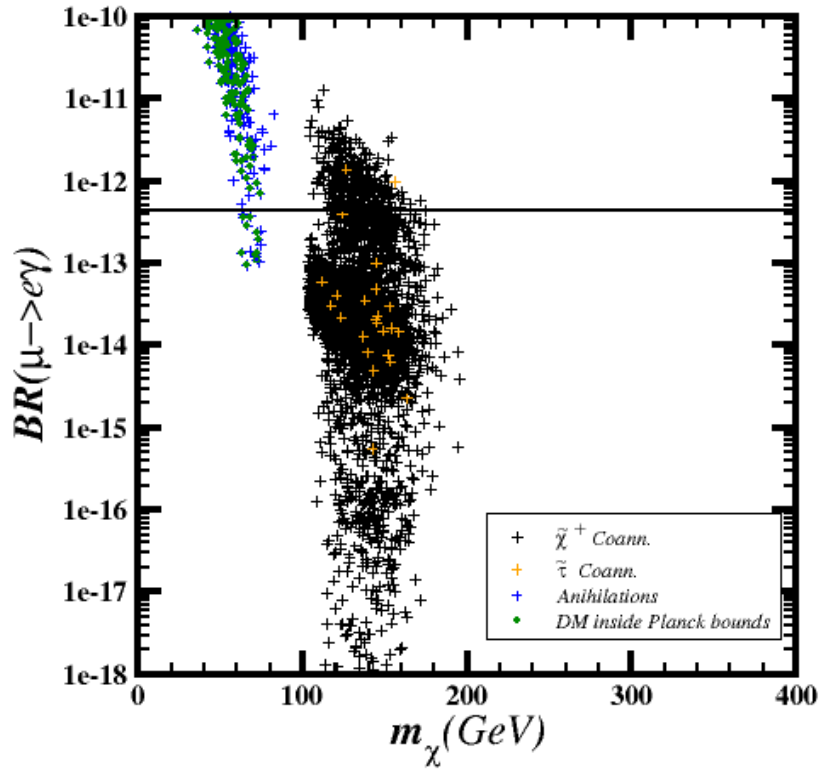




$$m_f^2 = \begin{pmatrix} 1 & \epsilon^{q_L^{12}} & \epsilon^{q_L^{13}} \\ \epsilon^{q_L^{12}} & 1 & \epsilon^{q_L^{23}} \\ \epsilon^{q_L^{12}} & \epsilon^{q_L^{23}} & 1 \end{pmatrix} \times m_L^2,$$

$$m_{fc}^2 = \begin{pmatrix} 1 & \epsilon^{q_R^{12}} & \epsilon^{q_R^{13}} \\ \epsilon^{q_R^{12}} & 1 & \epsilon^{q_R^{23}} \\ \epsilon^{q_R^{12}} & \epsilon^{q_R^{23}} & 1 \end{pmatrix} \times m_R^2,$$

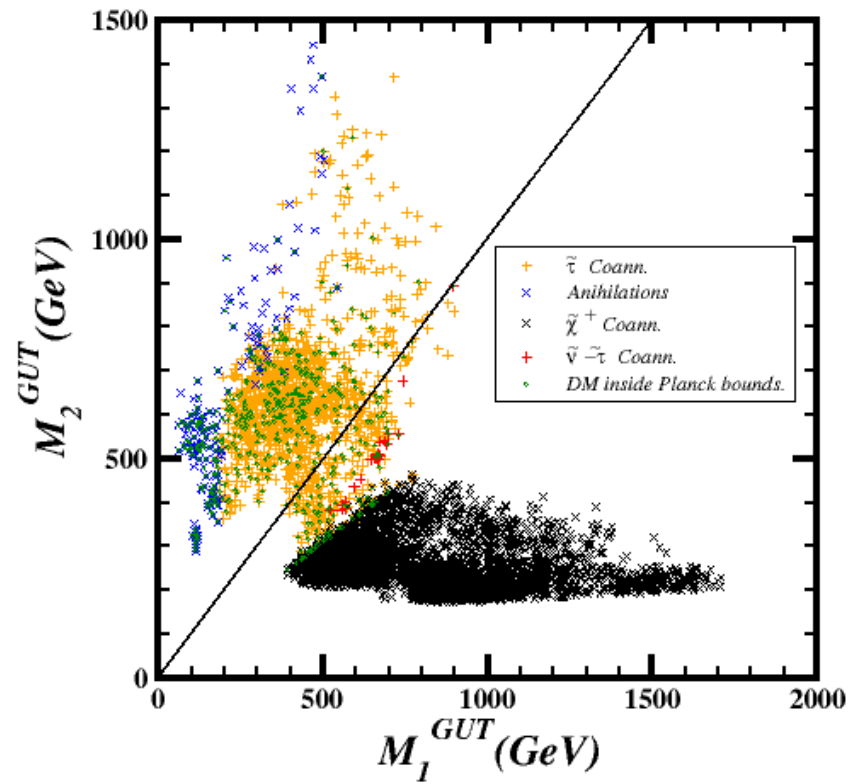
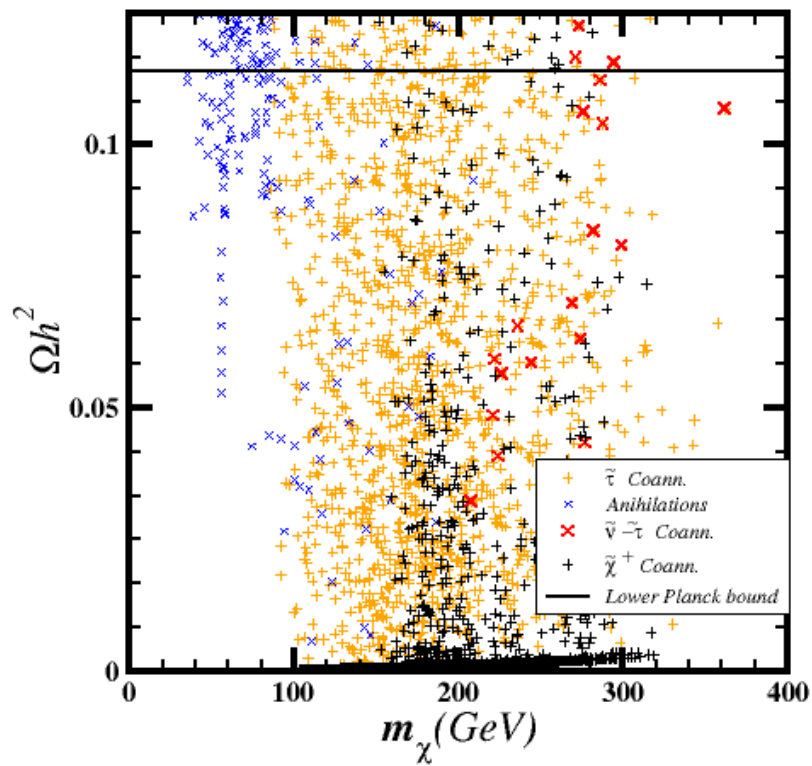
$$\epsilon = 0.05$$



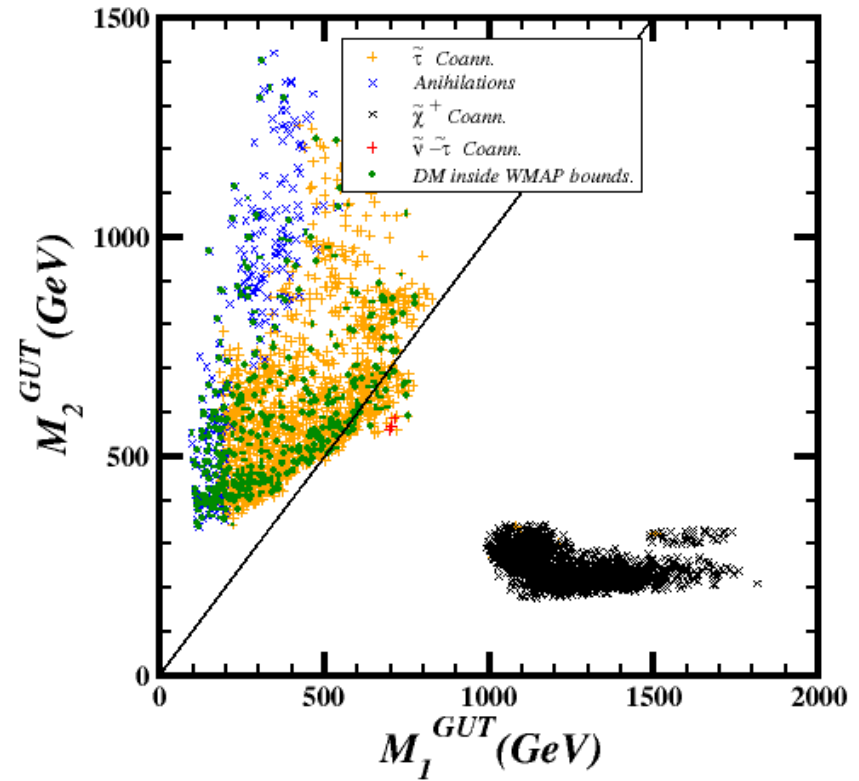
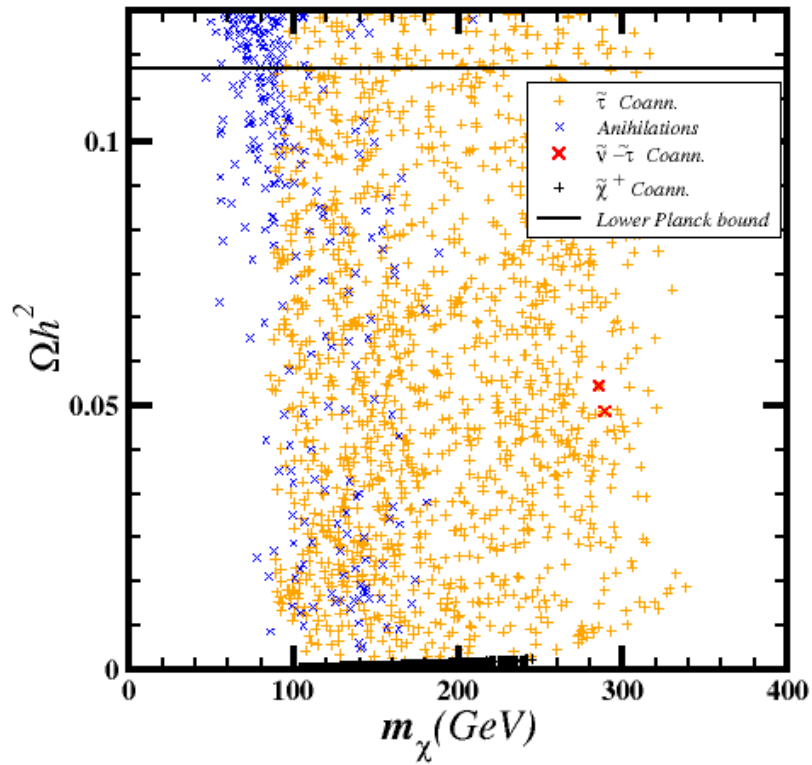
$$m_f^2 = \begin{pmatrix} 1 & \epsilon^{q_L^{12}} & \epsilon^{q_L^{13}} \\ \epsilon^{q_L^{12}} & 1 & \epsilon^{q_L^{23}} \\ \epsilon^{q_L^{12}} & \epsilon^{q_L^{23}} & 1 \end{pmatrix} \times m_L^2,$$

$$m_{fc}^2 = \begin{pmatrix} 1 & \epsilon^{q_R^{12}} & \epsilon^{q_R^{13}} \\ \epsilon^{q_R^{12}} & 1 & \epsilon^{q_R^{23}} \\ \epsilon^{q_R^{12}} & \epsilon^{q_R^{23}} & 1 \end{pmatrix} \times m_R^2,$$

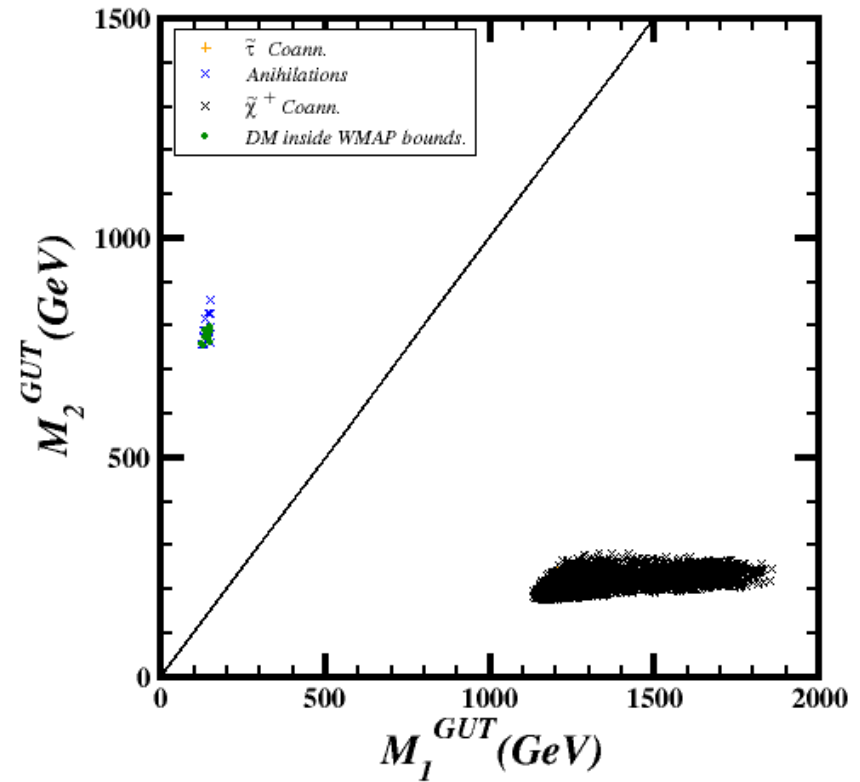
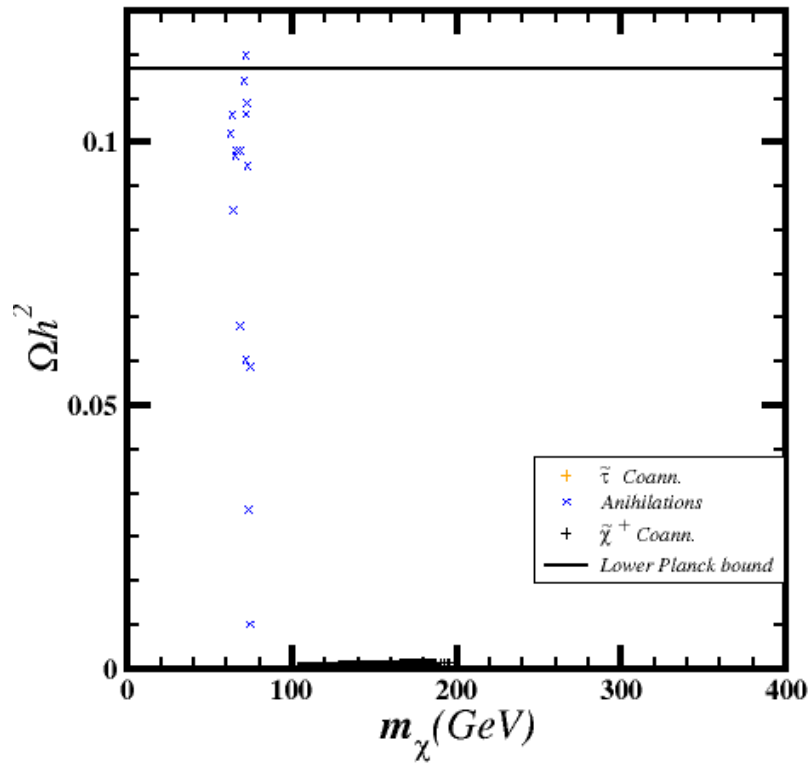
$$\epsilon = 0.2$$



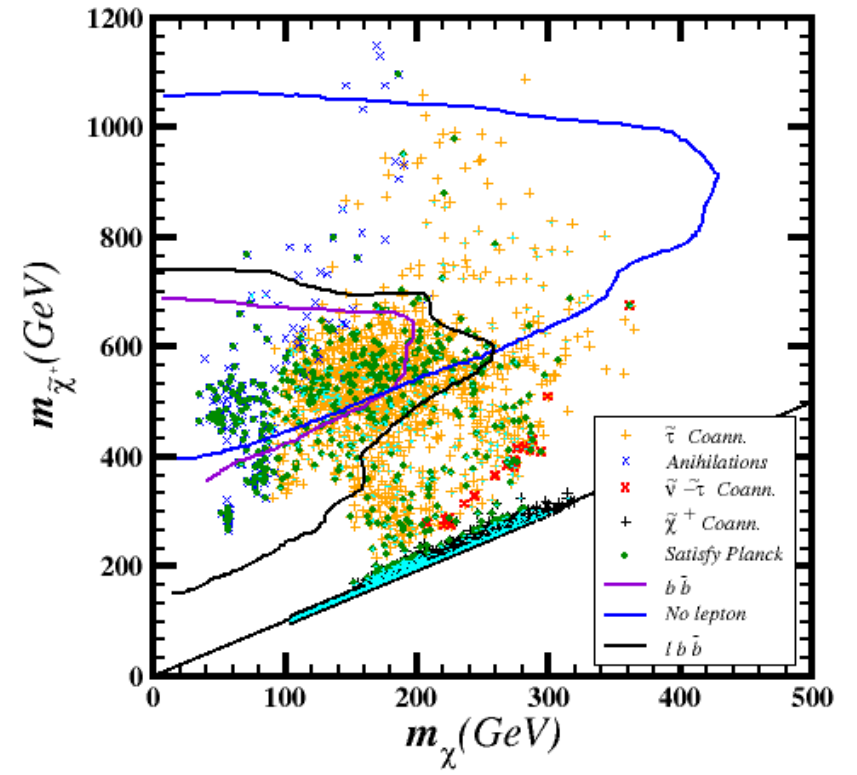
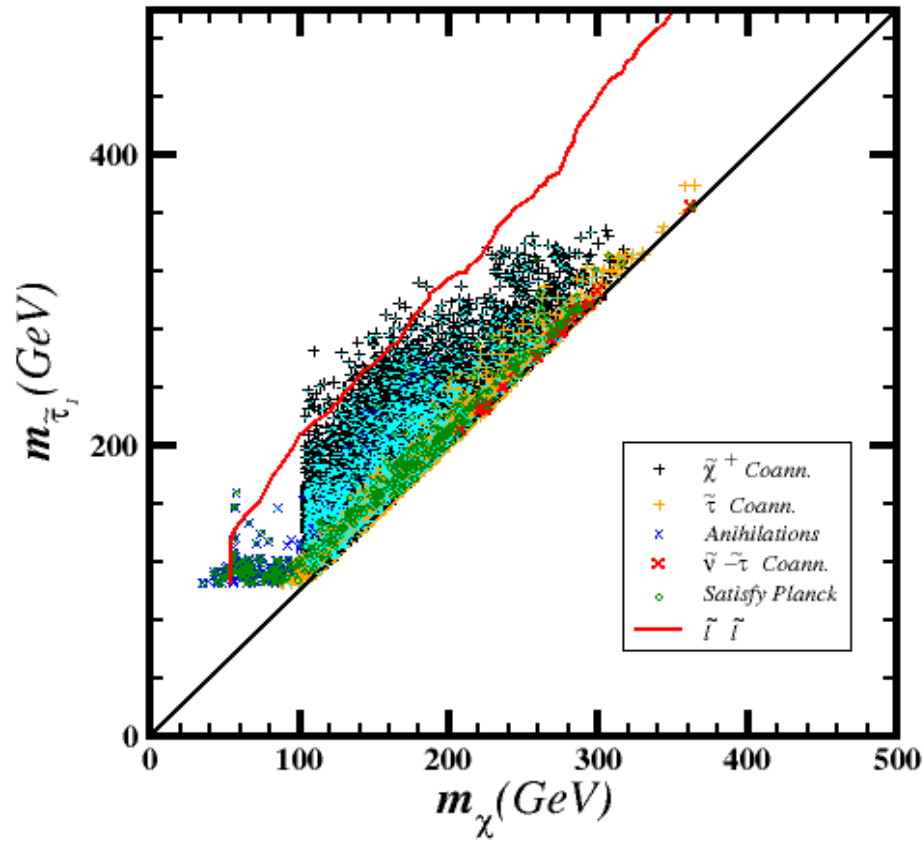
See-Saw



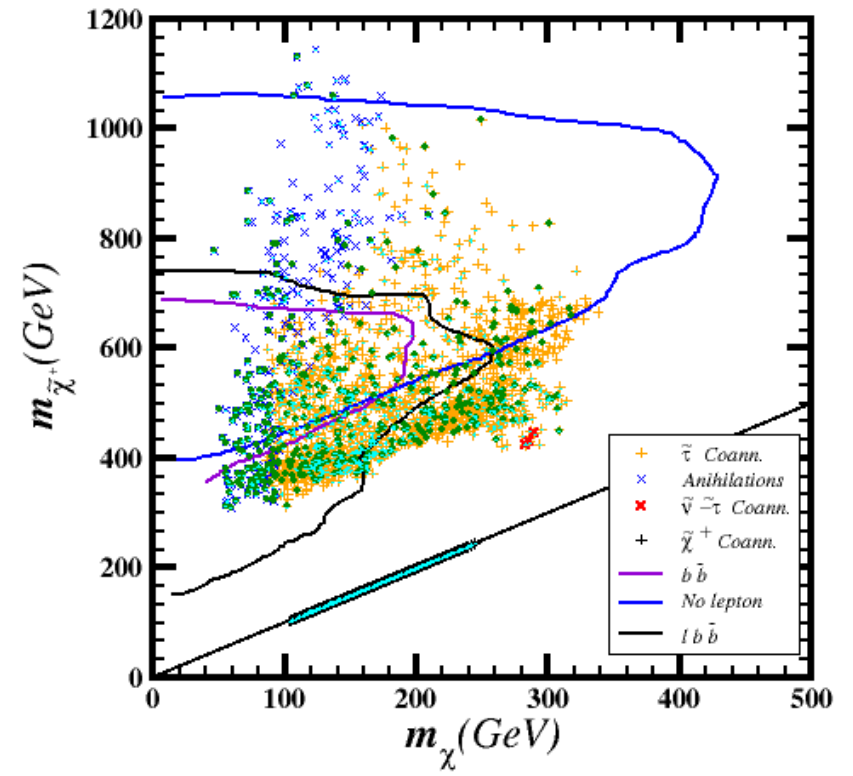
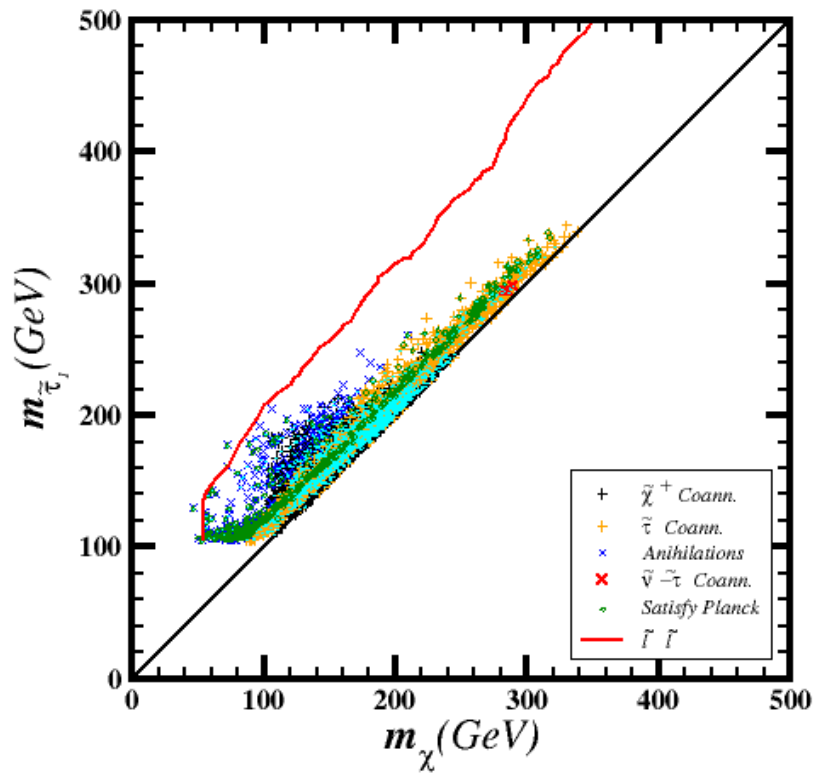
$$\varepsilon = 0.05$$



$$\varepsilon = 0.2$$

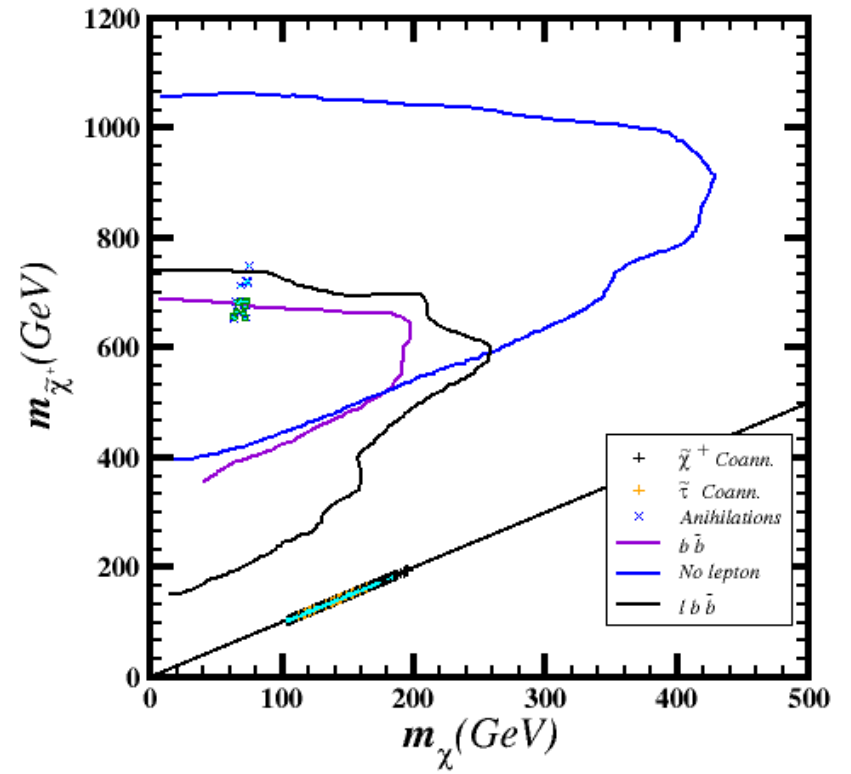
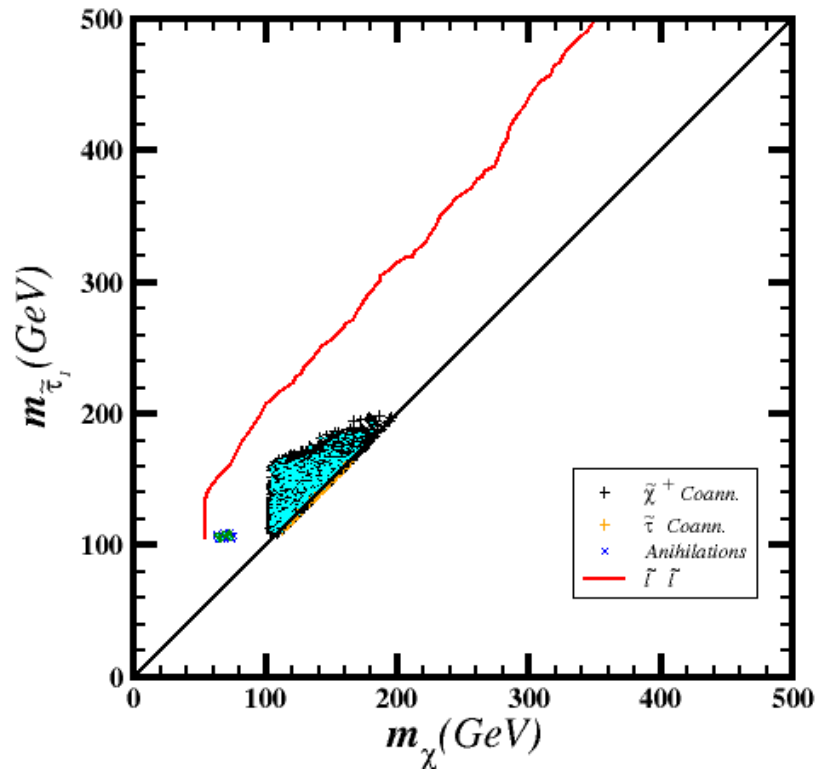


See-Saw



$$\epsilon = 0.05$$





$$\epsilon = 0.2$$

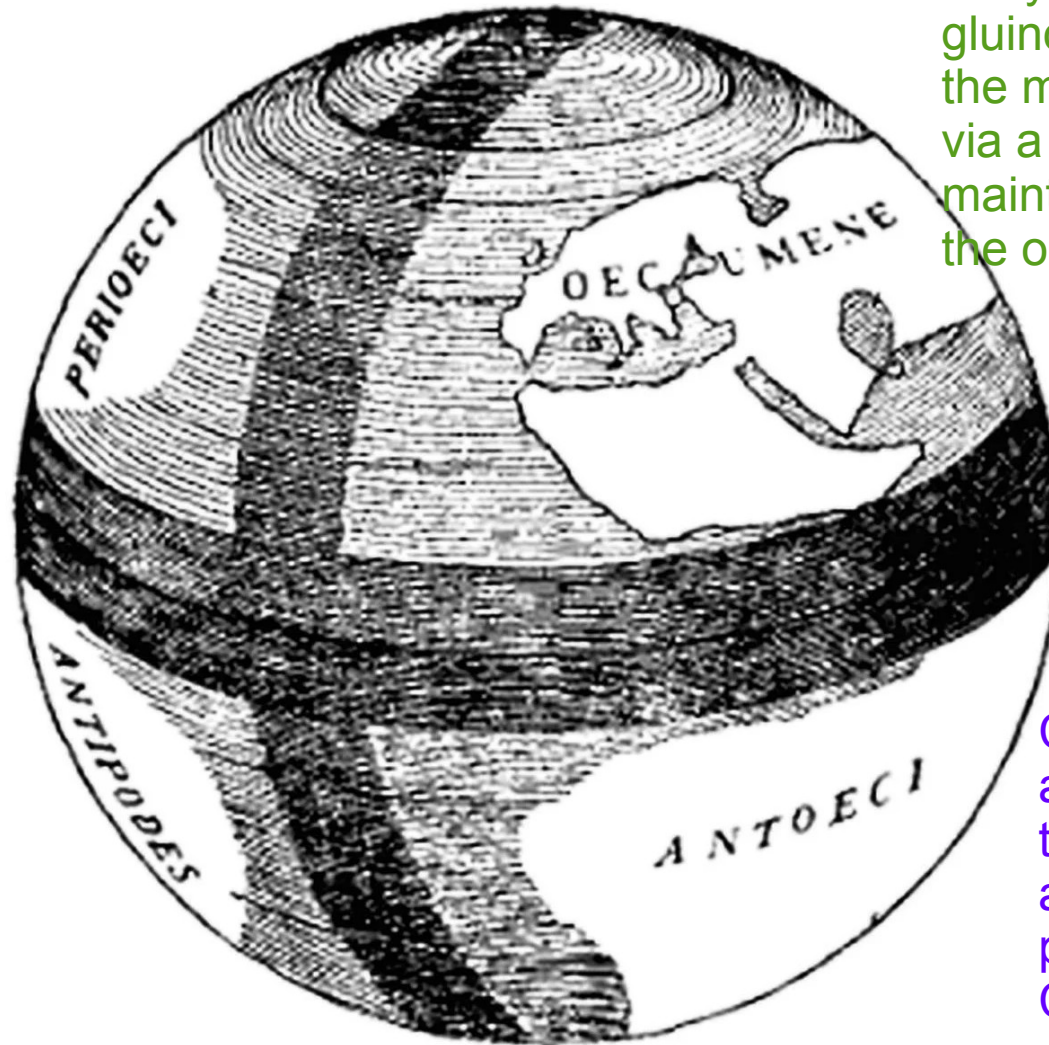
BSM

BR( $\mu \rightarrow e \gamma$ ) falls in the experimental range (i.e. MEG projected bound).

## CONCLUSIONS

SM

Susy Models with large gluino masses can explain the muon ( $g-2$ ) discrepancy via a SUSY contribution maintaining the prediction for the observed Higgs masses.



Accelerator

DM

Different kinds of LSP that satisfies the relic abundance condition.

Charginos and staus are not so heavy but they can not be identify at the LHC. Good prospects for Linear Collider..

Squarks are too heavy to be seen at the LHC

# murakoze!