Thermodynamics of a rotating and non-linear magnetic-charged black hole in the quintessence field

Presentation for the DSU 2023

By:

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Introduction

Introduction

Composition of the Universe

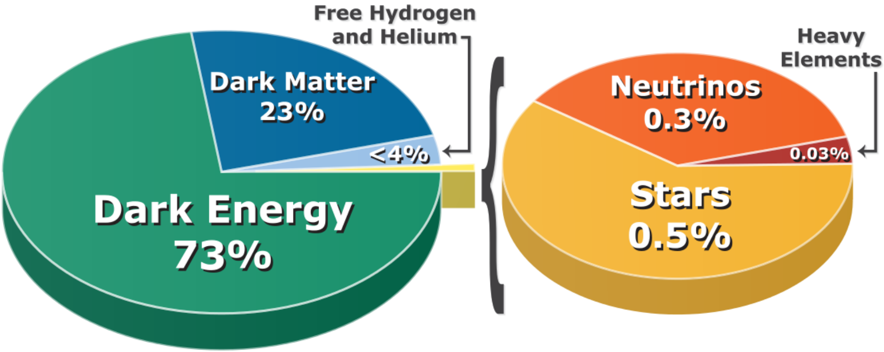

FIGURE 1 – The composition of the Universe

Introduction

Quintessence(1998) : a theoretical model of dark energy (**Limin Wang et Paul Steinhardt**)

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$$\Downarrow$$

Acccelerated expansion of the Universe

Quintessence dark energy

It is characterized by a parameter ϵ, which is the ratio of the pressure to the energy density of the dark energy, and the value of ϵ falls in the range $-1 \leq \epsilon \leq -\frac{1}{3}$. Then its equation of state is

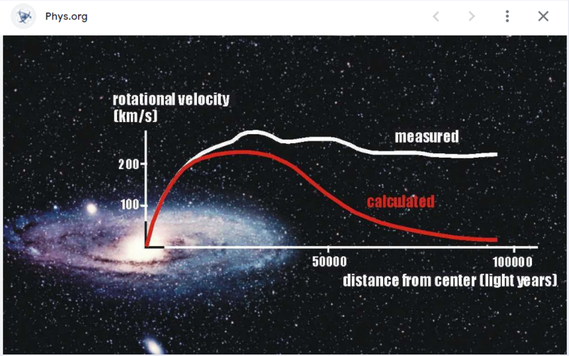
$$P_q = \epsilon \rho_q. \tag{1}$$

Introduction

Perfect fluid dark matter(PFDM) : PFDM preserves perfect fluid properties like isotropy of pressure and density.

$$\Downarrow$$

Possibility to explain the rotation speed curve of stars onto spiral galaxies, which is asymptotically flat (e.g. kiselev 2003, li et *al.* 2012)



Introduction

Another interesting subject in Astrophysics and Cosmology : **Black holes**

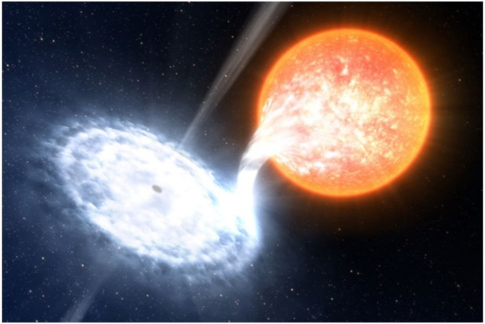


FIGURE 2 – Illustration of a star torn up by a black holes

Introduction

Beside the Gravitational waves detected by LIGO and VIRGO (2016)

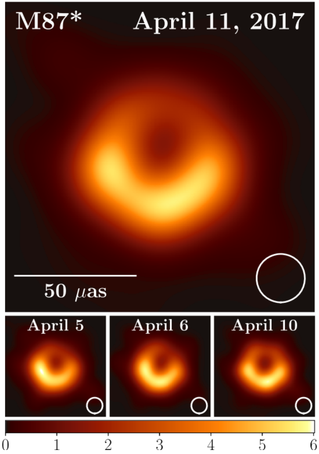


FIGURE 3 – First image of the supermassive black hole of M87 Galaxy(2019)

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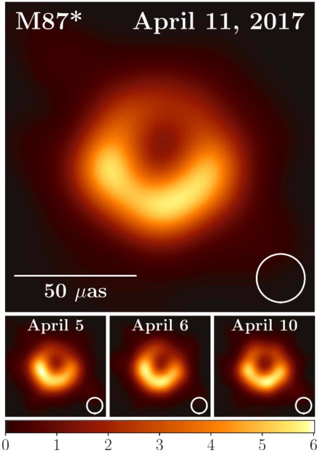


FIGURE 3 – First image of the supermassive black hole of M87 Galaxy(2019)

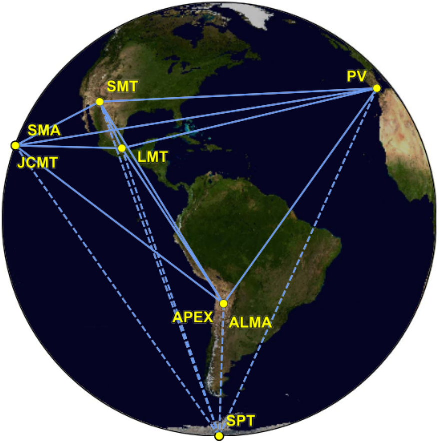


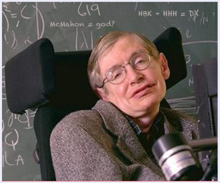
FIGURE 4 – The Event Horizon Telescope(EHT Collaboration)

Introduction

Thermodynamics of black holes

Stephen Hawking, James M. Bardeen, Brandon Carter : thermodynamic laws of black holes.

They allowed the scientific community to make a link between $A \approx S$ and $\kappa \approx T$.



Stephen Hawking (1970) : **Hawking Radiation**

Mathematical model

Mathematical model

Using the Newman-Janis algorithm(1965), Benavides et *al.*(2020) have derived the metrics of rotating and non-linear magnetic-charged black hole with quintessence, which is expressed as

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= -\left[1 - \frac{2\rho r}{\Sigma}\right] dt^2 + \frac{\Sigma}{\Delta} dr^2 - \frac{4a\rho r \sin^2\theta}{\Sigma} dt d\phi \\
&+ \Sigma d\theta^2 + \sin^2\theta \left[r^2 + a^2 + \frac{2a^2 \rho r \sin^2\theta}{\Sigma}\right] d\phi^2,
\end{aligned}
\tag{2}
$$

where

$$
\begin{aligned}
\Delta &= r^2 - 2\rho r + a^2, \\
\Sigma &= r^2 + a^2 \cos^2\theta, \\
2\rho &= \frac{2Mr^3}{Q^3 + r^3} + \frac{c}{r^{3\epsilon}}.
\end{aligned}
\tag{3}
$$

Here, the geometry of the black hole is expressed using spherical coordinates $(r,\ \theta,\ \phi)$, Q is the magnetic charge, a the rotating parameter, ϵ and c are quintessential parameters.

Mathematical model

Starting with the action of the system

$$S = \int d^4x\sqrt{-g}\left[\frac{c^4}{16\pi G}(R - 2\Lambda) - (\mathcal{L}_{\text{charge}} + 4\pi\mathcal{L}_{\text{PFDM}} - \mathcal{L}_{\text{quint}})\right] \tag{4}$$

where $\mathcal{L}_{\text{charge}} = \frac{3M}{|Q|^3}\frac{(2Q^2F)^{3/2}}{\left[1+(2Q^2F)^{3/4}\right]^2}$, $F \equiv F_{\mu\nu}F^{\mu\nu}/4$, $\mathcal{L}_{\text{quint}} = -\frac{1}{2}(\nabla\phi)^2 - V(\phi)$, Q -s the magnetic charge, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor et ϕ the scalar field related to quintessence.

Extermination of the action and Maxwell equations :

$\frac{1}{\sqrt{-g}}\frac{\delta S(\text{action})}{\delta g^{\mu\nu}} = 0$, $\nabla_\mu\left(\frac{\partial\mathcal{L}_{\text{charge}}}{\partial F}F^{\nu\mu}\right) = 0$ and $\nabla_\mu * F^{\nu\mu} = 0$.

Solution

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{5}$$

$$\text{with } f(r) = 1 - \frac{2Mr^2}{r^3 + Q^3} + \frac{\alpha}{r}\ln\frac{r}{|\alpha|} - \frac{c_q}{r^{3\epsilon+1}} - \frac{\Lambda}{3}r^2. \tag{6}$$

Results and Discussion : Thermodynamic study

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For a rotating black hole, the expression of entropy is found throughout the horizon area(Toshmatov et al. 2017) Therefore, we have

Entropy of the black hole

$$S = \frac{A}{4} = \frac{1}{4}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sqrt{g_{\theta\theta} g_{\phi\phi}}, \tag{7}$$

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which gives us

$$\begin{aligned} S &= \frac{\pi}{2}\int_0^{\pi}d\theta\sqrt{\Sigma\sin^2\theta\left[r_h^2+a^2+\frac{2a^2\rho r_h\sin\theta}{\Sigma}\right]} \\ &= \pi(r_h^2+a^2) \\ &= \pi(r_h^2+\frac{J^2}{M^2}),\ \ with\ \ a=\frac{J}{M}. \end{aligned}$$

This leads us to write the expression of the event horizon radius as

$$r_h=\sqrt{\frac{S}{\pi}-a^2}. \tag{8}$$

Entropy

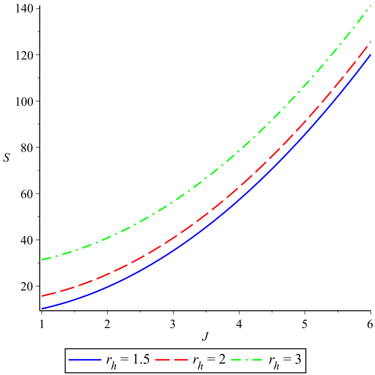

FIGURE 5 – Change of the entropy S in the presence of quintessence dark energy, with $M = 1$.

This Result tells us that a larger and more rotating black hole has a higher entropy.

the black hole mass M

To compute the mass M, we will use the event horizon property(Benavides et al. 2020) , then solving the following equation

$$g^{rr} = 0 \Rightarrow \frac{\Delta}{\Sigma} = 0. \tag{9}$$

This leads us to have

$$M = \frac{1}{2}(Q^3 + r_h^3)\left(\frac{1}{r_h^2} + \frac{a^2}{r_h^4} - \frac{c}{r_h^{3\epsilon+3}}\right). \tag{10}$$

Now, since we have $r_h = \sqrt{\frac{S}{\pi} - a^2}$, the mass M in Eq.(7) becomes

$$\begin{aligned} M &= \frac{1}{2}\left(Q^3 + \left(\frac{S}{\pi} - a^2\right)^{\frac{3}{2}}\right)\left(\frac{1}{\left(\frac{S}{\pi}-a^2\right)} + \frac{a^2}{\left(\frac{S}{\pi}-a^2\right)^2}\right. \\ &- \left.\frac{c}{\left(\frac{S}{\pi}-a^2\right)^{\frac{3\epsilon+3}{2}}}\right). \end{aligned} \tag{11}$$

Results and Discussion : Thermodynamic study

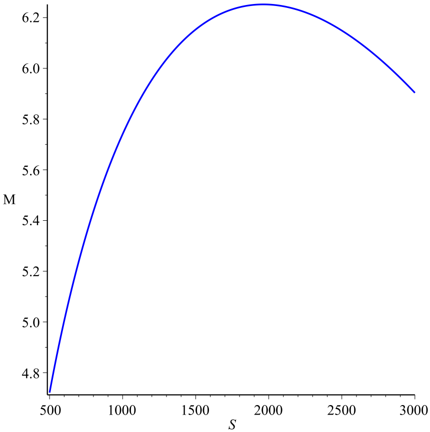

FIGURE 6 – for $c = 0.02$

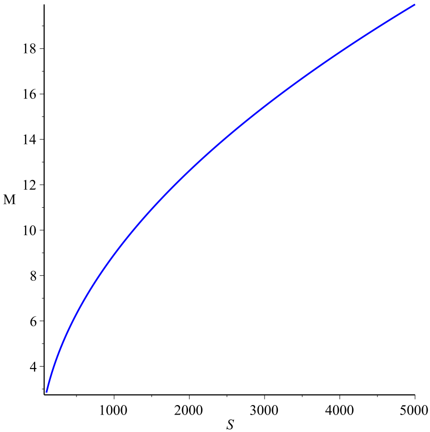

FIGURE 7 – for $c = 0$

Results and Discussion : Thermodynamic study

Thermodynamic quantities

Temperature

$$T = \left(\frac{\partial M}{\partial S}\right) = \frac{3}{4}\frac{\sqrt{\frac{S}{\pi}-a^2}\left(\frac{1}{\frac{S}{\pi}-a^2}+\frac{a^2}{\left(\frac{S}{\pi}-a^2\right)^2}-\frac{c}{\left(\frac{S}{\pi}-a^2\right)^{\frac{3\epsilon+3}{2}}}\right)}{\pi} + B\left(-\frac{1}{\pi\left(\frac{S}{\pi}-a^2\right)^2}-\frac{2a^2}{\pi\left(\frac{S}{\pi}-a^2\right)^3}+\frac{c\left(\frac{3\epsilon+3}{2}\right)}{\pi\left(\frac{S}{\pi}-a^2\right)^{\frac{3\epsilon+5}{2}}}\right) \quad (12)$$

with

$$B = \frac{1}{2}\left(Q^3+\left(\frac{S}{\pi}-a^2\right)^{\frac{3}{2}}\right).$$

Potential

$$\Phi = \left(\frac{\partial M}{\partial Q}\right) = \frac{3Q^2}{2}\left(\frac{1}{\frac{S}{\pi}-a^2}+\frac{a^2}{\left(\frac{S}{\pi}-a^2\right)^2}-\frac{c}{\left(\frac{S}{\pi}-a^2\right)^{\frac{3\epsilon+2}{2}}}\right). \quad (13)$$

First derivative of the free enthalpy : **first-order phase transition**(boiling water)

$$dG = -SdT + VdP \;\rightarrow\; \left(\frac{\partial G}{\partial T}\right)_P = -S \text{ and } \left(\frac{\partial G}{\partial P}\right)_T = V, \qquad (14)$$

First derivative of the free enthalpy : **first-order phase transition(boiling water)**

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second derivative of the free enthalpy : **second-order phase transition(ferromagnetic and paramagnetic transition)**

$$\begin{cases} \left(\frac{\partial^2 G}{\partial T^2}\right)_P &= -\frac{C_P}{T}, \\ \left(\frac{\partial^2 G}{\partial P^2}\right)_T &= -V\kappa_T, \\ \left(\frac{\partial^2 G}{\partial P \partial T}\right) &= V\beta_P, \end{cases} \tag{15}$$

with : C_p the heat capacity at constant pressure, κ_T the isothermal compressibility coefficient and β_p the isobaric expansion coefficient.

Results and Discussion : Thermodynamic study

$$C_a = -T\frac{\partial^2 G}{\partial T^2} = T\left(\frac{\partial S}{\partial T}\right). \tag{16}$$

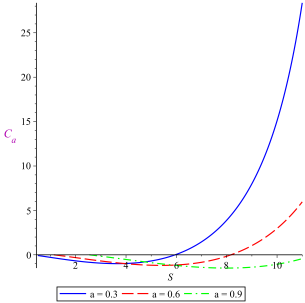


FIGURE 8 –

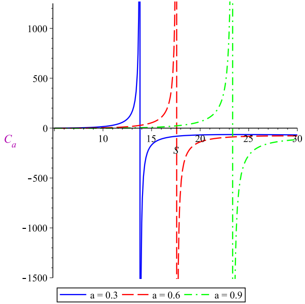


FIGURE 9 –

Change of the black hole heat capacity C_a in the presence of quintessence dark energy with characteristic $(c, \epsilon, Q) = (0.02, -\frac{2}{3}, 1)$.

Results and Discussion : Thermodynamic study(adding cosmological constant and dark matter)

Here, we consider that cosmological constant Λ will act as a dynamic pressure, hence :

$$P = -\frac{\Lambda}{8\pi}. \tag{17}$$

black hole mass

$$f(r_h) = 0 \Rightarrow M = \frac{(r_h^3 + Q^3)}{2r_h^2}\left(1 + \frac{\alpha}{r_h}\ln\frac{r_h}{|\alpha|} - \frac{c_q}{r_h^{3\epsilon+1}} + \frac{8\pi P}{3}r_h^2\right). \tag{18}$$

Hawking temperature

$$T = \frac{\kappa}{2\pi} = \frac{f'(r_h)}{4\pi}$$

$$\Rightarrow T = \frac{\left[\frac{r_h^3 - 2Q^3}{r_h} + \frac{3c_q\epsilon}{r_h^{3\epsilon+2}}\left(r_h^3 + Q^3\left(\frac{\epsilon+1}{\epsilon}\right)\right) + \frac{\alpha Q^3}{r_h^2}\left(1 - 3\ln\frac{r_h}{|\alpha|}\right) + \alpha r_h + 8\pi P r_h^4\right]}{4\pi(r_h^3 + Q^3)}. \tag{19}$$

Results and Discussion : Thermodynamic study(adding cosmological constant and dark matter)

Pressure P and the $P - r_h$ diagram

$$T = \frac{\left[\frac{r_h^3-2Q^3}{r_h} + \frac{3c_q\epsilon}{r_h^{3\epsilon+2}}\left(r_h^3 + Q^3\left(\frac{\epsilon+1}{\epsilon}\right)\right) + \frac{\alpha Q^3}{r_h^2}\left(1 - 3\ln\frac{r_h}{|\alpha|}\right) + \alpha r_h + 8\pi P r_h^4\right]}{4\pi(r_h^3 + Q^3)}.$$

$$\Downarrow$$

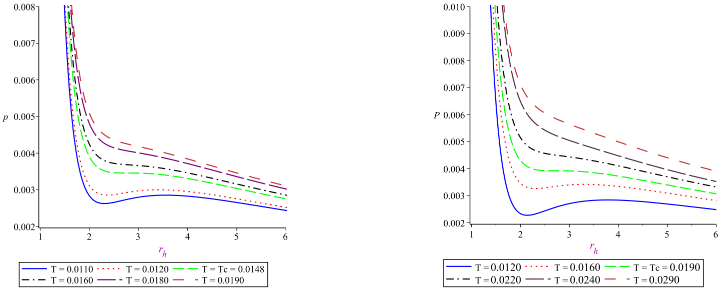
$$\begin{aligned} P &= \frac{1}{8\pi r_h^4}\left[4\pi(r_h^3 + Q^3)T - \frac{r_h^3-2Q^3}{r_h}\right. \\ &- \left.\frac{3c_q\epsilon}{r_h^{3\epsilon+2}}\left(r_h^3 + Q^3\left(\frac{\epsilon+1}{\epsilon}\right)\right) - \frac{\alpha Q^3}{r_h^2}\left(1 - 3\ln\frac{r_h}{|\alpha|}\right) - \alpha r_h\right]. \end{aligned} \tag{20}$$

Critical points (leading inflection) are found through

$$\left(\frac{\partial P}{\partial r_h}\right)_T = 0, \quad \left(\frac{\partial^2 P}{\partial r_h^2}\right)_T = 0. \tag{21}$$

Results and Discussion : Thermodynamic study(adding cosmological constant and dark matter)

$P - r_h$ diagram



(a) Pour $\alpha = 0.2$. (b) Pour $\alpha = 0.4$.

FIGURE 10 – Pressure for different values of temperature, with $(Q, c_q, \epsilon) = (1, 0.2, -2/3)$.

We observe a behaviour similar to the van der Waals fluid $\rightarrow$ Presence of **Small-large black hole phase transition**

Results and Discussion : Thermodynamic study(adding cosmological constant and dark matter)

Swallow tail on the Gibbs free energy evolution $$G = M - TS, \qquad (22)$$

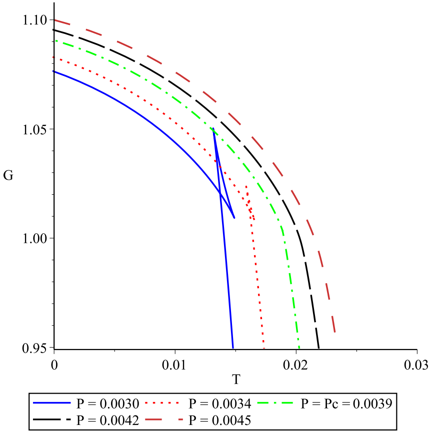


FIGURE 11 – Variation of the Gibbs free energy G for different values of P, with $(Q, c_q, \epsilon, \alpha) = (1, 0.2, -2/3, 0.4)$.

Conclusion and outlooks

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Make the thermodynamic study of rotating and non-linear magnetic-charged black hole surrounded by quintessence,

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  $\Rightarrow$ For the rotating case **the black hole mass could have a phase of decrease in the presence of quintessence energy**.
- Localization of the change of phase appears to the black hole through a second-order phase transition, remarked by the **discontinuity on the plot**.

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Make the thermodynamic study of rotating and non-linear magnetic-charged black hole surrounded by quintessence,

- We computed the entropy at the event horizon $\Rightarrow$ For the rotating case **the black hole mass could have a phase of decrease in the presence of quintessence energy**.
- Localization of the change of phase appears to the black hole through a second-order phase transition, remarked by the **discontinuity on the plot**.
- Localization of the swallow tail and the van der Waals fluid behaviour.

outlooks

1. **Include the thermal fluctuations and put out their effects on black holes**

Acknowledgement

- **ICTP / EAIFR**
- **ASFAP**(African Strategy for Fundamental and Applied Physics)
- My supervisors and Cameroon research group on Cosmology

Thank you for your kind attention ! ! !