Confronting the Chaplygin gas with data (SNIa): background and perturbed cosmic dynamics

Renier Hough, Shambel Sahlu, Heba Sami, Maye Elmardi, Anna-Mia Swart, and Amare Abebe

Confronting the Chaplygin gas with data: background and perturbed cosmic dynamics

Renier Hough[1], Shambel Sahlu[2,3], Heba Sami[4], Maye Elmardi[1], Anna-Mia Swart[4], and Amare Abebe[4,5]
[1] *Centre for Space Research, North-West University, Potchefstroom, South Africa.*
[2] *Astronomy and Astrophysics Research and Development Department, Entoto Observatory and Research Centre.*
[3] *Department of Physics, College of Natural and Computational Science, Wolkite University, Wolkite, Ethiopia*
[4] *Centre for Space Research, North-West University, Mahikeng, South Africa. and*
[5] *National Institute for Theoretical and Computational Sciences (NITheCS), South Africa.*
(Dated: December 23, 2021)

In this paper, we undertake a unified study of background dynamics and cosmological perturbations in the presence of the Chaplygin gas. This is done by first constraining the background cosmological parameters of different Chaplygin gas models with SNIa data, and then feeding these observationally constrained parameters in the analysis of cosmological perturbations. Based on the statistical criteria we followed, none of the models has a substantial observational support but we show that the so-called 'original' and 'generalized' Chaplygin gas models have *some observational support* and *less observational support*, respectively, whereas the 'modified' and 'modified generalized' Chaplygin gas models miss out on the *less observational support* category but cannot be ruled out. The so-called 'generalized cosmic Chaplygin gas' model, on the other hand, falls under the *no observational support* category of the statistical criterion and can be ruled out. We follow the $1+3$ covariant formalism of perturbation theory and derive the evolution equations of the fluctuations in the matter density contrast of the matter-Chaplygin gas system for the models with some or less statistical support. The solutions to these coupled systems of equations are then computed in both short-wavelength and long-wavelength modes.

PACS numbers: 04.50.Kd, 98.80.Jk, 98.80.-k, 95.36.+x, 98.80.Cq

Talk's Plan

1. Introduction to Chaplygin Gas Cosmology
   - Supernova Cosmology in CG Models
2. Linear Cosmological Perturbations
   - The growth of fractional matter density in baryonic matter-Chaplygin gas
3. Conclusions

- The discovery of the accelerating expansion of the Universe and the existence of dark matter are the primary indicators for the limitation of our knowledge of physics laws and those two uncharted territories of the Universe have opened a new era for the development of modern cosmology.
- Different suggestions have been put forward to understand dark energy. The first suggestion is that the cosmological constant is the one responsible for the cosmic acceleration [1, 2, 3], and the second approach is the modification of General Theory of Relativity (GR) [4, 5].

Chaplygin gas (CG) Cosmology

- In this talk, we undertake a unified study of background dynamics and cosmological perturbations in the presence of the Chaplygin gas.
- This is done by first constraining the background cosmological parameters of different Chaplygin gas models with SNIa data,
- and then feeding these observationally constrained parameters in the analysis of cosmological perturbations.

- several CG models [6] but the most common of which are
  - the original Chaplygin gas (OCG),
  - generalized Chaplygin gas (GCG), modified
  - generalized Chaplygin gas (MGCG),
  - Extended Chaplygin Gas (ECG) and
  - generalized cosmic Chaplygin gas (GCCG)
- **Equation of state (ES) of each CG models are different**
- Based on their ESP each models have different,
  - fractional Hubble parameter $h(z) = \frac{H(z)}{H_0}$,
  - deceleration parameter $q(z)$,
  - fractional energy density Ω^i_{ch},
  - distance modulus and other cosmological parameters

Supernova Cosmolog in CG models

- The distance modulus that can be obtained by combining the different cosmological distance definitions, found in [7], is given by [1]

$$\mu = m - M = 25 - 5 \times \log_{10}\left[3000\bar{h}^{-1}(1+z)\int_0^z \frac{dz'}{h(z')}\right], \quad (1)$$

where m and M are the apparent and absolute magnitudes for a particular measured supernova.

- We also used the definition for the Hubble uncertainty parameter, which is given as $\bar{h} = \frac{H(z)}{100\frac{km}{s.Mpc}}$, with $H(z)$ being the Hubble parameter.

[1] This distance modulus is given in terms of Mpc.

- We obtained the normalized Friedmann equation $h(z)$, as a result from the definition for the line-of-sight co-moving distance [7] for each CG models.
- The distance modulus also incorporates these corrections.
- We use a Markov Chain Monte Carlo (MCMC) simulation that takes into account the error on our the distance modulus, when calculating the best-fitting parameters.
-

Best fit values

Table 1: Using MCMC simulation best fitting parameters values for each tested model

Model	$\bar{h}$	Ω_{m0}	$D_1/E_0/F_1/A$	$D_2/\mathcal{K}/\mathcal{K}_2$	α	B	ω
ΛCDM	$0.6967^{+0.0048}_{-0.0047}$	$0.2674^{+0.0249}_{-0.0239}$					
Original CG	$0.6500^{+0.0743}_{-0.0375}$	$0.0453^{+0.0356}_{-0.0308}$	$1.0506^{+0.2936}_{-0.3785}$	$0.1737^{+0.0675}_{-0.0693}$			
General CG	$0.6557^{+0.0741}_{-0.0408}$	$0.0475^{+0.0346}_{-0.0320}$	$0.9258^{+0.2476}_{-0.2669}$	$0.1976^{+0.0675}_{-0.0675}$	$0.6424^{+0.2516}_{-0.3594}$		
Modified CG	$0.6766^{+0.0738}_{-0.0564}$	$0.0483^{+0.0350}_{-0.0333}$	$1.0164^{+0.3670}_{-0.2904}$	$0.1292^{+0.0652}_{-0.0466}$	$0.5773^{+0.2915}_{-0.3445}$	$0.1499^{+0.1186}_{-0.1047}$	
Modified GCG	$0.6773^{+0.0751}_{-0.0557}$	$0.0461^{+0.0363}_{-0.0320}$	$0.8827^{+0.2847}_{-0.2464}$	$0.1506^{+0.0600}_{-0.0469}$	$0.5794^{+0.2880}_{-0.3460}$	$0.1448^{+0.1200}_{-0.1006}$	
Extended CG	$0.7172^{+0.0231}_{-0.0276}$	$0.0869^{+0.0095}_{-0.0170}$	$1.5750^{+0.1404}_{-0.1117}$	$-0.2105^{+0.0128}_{-0.0069}$			
GCCG	$0.7490^{+0.0689}_{-0.0729}$	$0.0641^{+0.0243}_{-0.0274}$	$1.0691^{+0.2961}_{-0.4024}$		$0.6934^{+0.2223}_{-0.3537}$	$0.2722^{+0.0417}_{-0.0535}$	$-0.1755^{+0.1282}_{-0.2224}$

Likelihood function

The models are listed in the order from the largest likelihood function value $\mathcal{L}(\hat{\theta}|data)$ to the smallest likelihood of being viable.

Model	$\mathcal{L}(\hat{\theta}\|data)$	χ^2	**Red.** χ^2	AIC	$\|\Delta AIC\|$	BIC	$\|\Delta BIC\|$
Original CG	-120.1494	240.2987	0.6769	248.2987	2.8978	263.8320	10.6645
Modified CG	-120.2718	240.5435	0.6814	252.5435	7.1426	275.8434	22.6759
Modified Generalised CG	-120.2832	240.5665	0.6815	252.5665	7.1656	275.8664	22.6989
Extended CG	-120.5708	241.1415	0.6793	249.1415	3.7406	264.6748	11.5073
Generalized Cosmic CG	-120.6540	241.3079	0.6836	253.3079	7.9071	276.6079	23.4404
General CG	-120.6823	241.3645	0.6818	251.3645	5.9636	270.7811	17.6136
ΛCDM	-120.7004	241.4009	0.6762	245.4009	0	253.1675	0

$q(z)$ for CG models

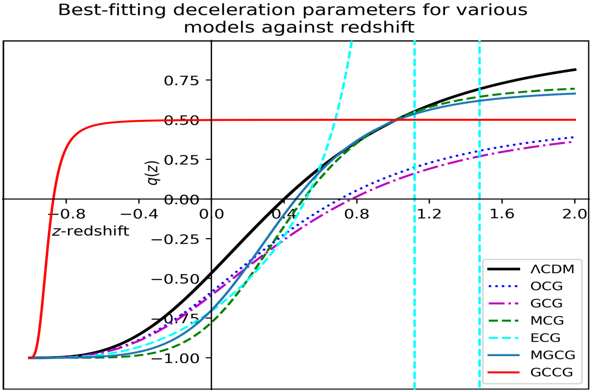

Figure 1: The deceleration parameters from each of the different tested models compared to the ΛCDM model basd on the best-fitting parameter values as predicted by the MCMC simulation on the

distance modulus in CG models

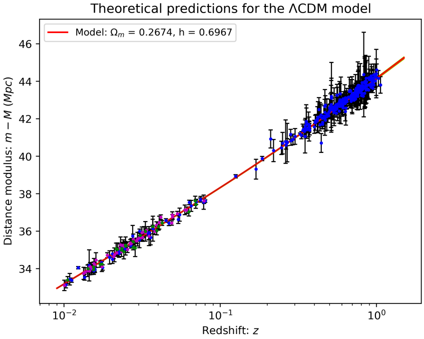

Figure 2: The ΛCDM model's best-fitting free parameters for the Supernovae Type 1A data with cosmological parameter values as $\bar{h} = 0.6967^{+0.0048}_{-0.0047}$ (constrained) and $\Omega_{m0} = 0.2674^{+0.0249}_{-0.0239}$ (constrained) as calculated by the MCMC simulation.

OCG model

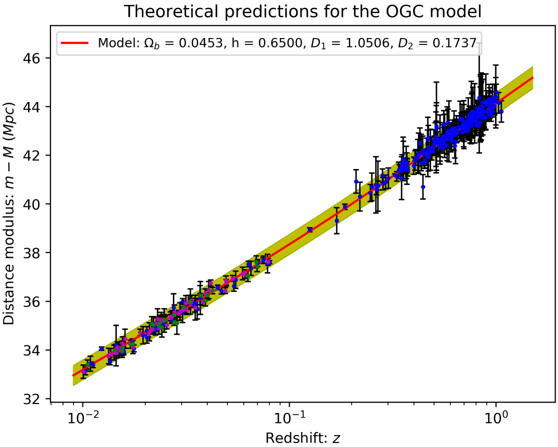

Figure 3: The original CG model the best-fitting free parameters for the Supernovae Type 1A data with cosmological parameter values.

GCG Model

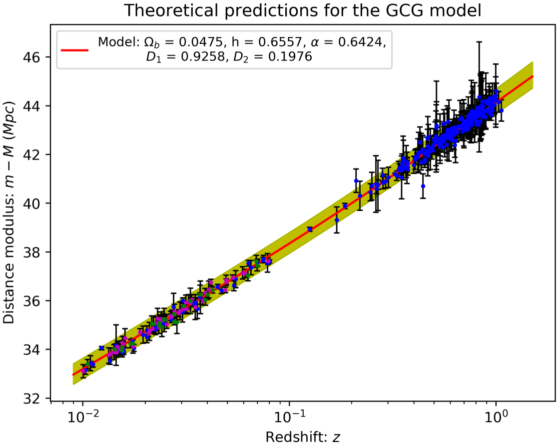

Figure 4: The general CG model of best-fitting free parameters for the Supernovae Type 1A data.

MCG Model

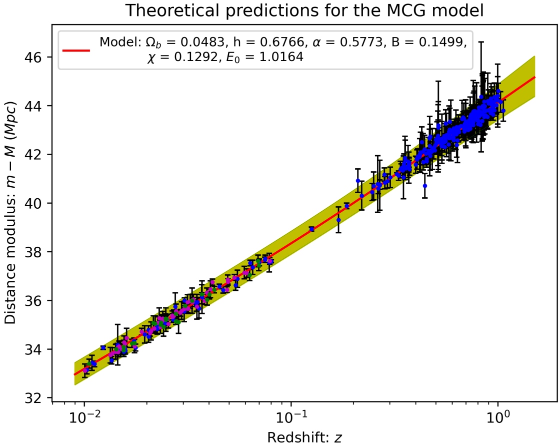

Figure 5: The modified CG model's of the best-fitting free parameters for the Supernovae Type 1A data

MGCG

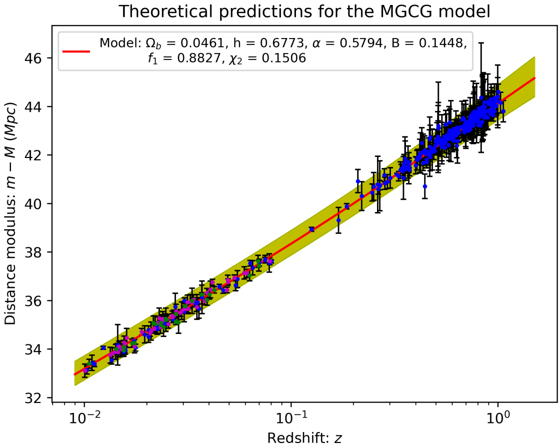

Figure 6: The modified generalized CG model best-fitting free parameters for the Supernovae Type 1A data

GCCG Model

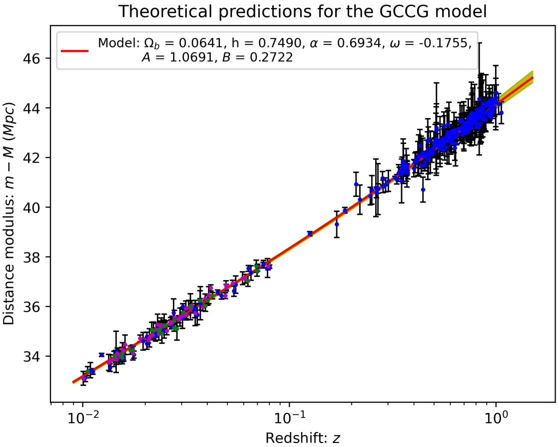

Figure 7: The generalized cosmic CG model best-fitting free parameters for the Supernovae Type 1A data

Linear Cosmological Perturbations

- The other aspect of this work focuses on linear cosmological perturbations in the $1+3$ covariant formalism in multi-fluid systems [8, 9, 10] adapted to include CG fluid models.
- After we explore the background cosmology of CG models, here we also extended our knowledge at a linear cosmological perturbation level to clearly see the contributions of CG for the growth matter density fluctuations.

- Based on our statistical results [11], we consider that OCG and GCG which are substantial observational support and less observational support of CG models.
- The density contrast is studied in GR, ΛCDM and baryoinc-Chaplygin gas approaches.
- Based on the statistical analysis as presented in **??**, GMCG and GCCG model are not favored with observation and not considered in this section to study the density contrasts.
- However MCG is considered to study at least this models is failed at the background and see how it behave at the perturbative level.

- To do this, we consider the non-interacting non-relativistic fluid ρ_b and relativistic fluid $\rho_{\rm r}$, $\rho_m \equiv \rho_r + \rho_b$ with exotic fluid ρ_{ch} in our Universe. In the background FLRW cosmological, we define spatial gradients of gauge-invariant variables such as

$$D^m_a \equiv \frac{a}{\rho_m}\tilde{\nabla}_a\rho_m\ , \quad Z_a \equiv a\tilde{\nabla}_a\theta\ , \qquad (2)$$
$$D^{ch}_a \equiv \frac{a}{\rho_{ch}}\tilde{\nabla}_a\rho_{ch}\ , \quad D^t_a \equiv \frac{a}{\rho_t}\tilde{\nabla}_a\rho_t\ ,$$

where the energy density D^m_a, volume expansion of the fluid Z_a as follows [12, 8], D^{ch}_a is the energy density for CG and D^t_a is the spatial energy density for total fluids [Matter fluid +CG].

- For a perfect multi-fluid system, the following conservation equations, considered in [13, 12] hold:

$$\dot{\rho}_i = -\theta(\rho_i + p_i) = -\theta(1 + w_i)\rho_i \,, \tag{3}$$

$$\dot{\rho}_t = -\theta(\rho_t + p_t) = -\theta(1 + w_t)\rho_t \,, \tag{4}$$

$$(\rho_t + p_t)\dot{u}_a + \tilde{\nabla}_a p_t = 0 \,. \tag{5}$$

- From these conservation equation, the 4-acceleration in the energy frame of the total fluid can be given as

$$\dot{u}_a = -\frac{\tilde{\nabla}_a p_t}{\rho_t + p_t} \tag{6}$$

$$= -\frac{1}{a\,(1 + w_t)\,\rho_t}\left[w_m \rho_m D^m_a + w_{ch}\rho_{ch} D^{ch} + a\rho_{ch}\tilde{\nabla}_a w_{ch}\right] \,,$$

with w_m and w_{ch} the equation of state parameters for matter fluid and Chaplygin gas, and w_t is the effective equation of state parameter for the total fluid.

- Another key equation for a general fluid is the so-called Raychaudhuri equation and it can be expressed as

$$\dot{\theta} = -\frac{1}{3}\theta^2 - \frac{1}{2}(\rho_t + 3p_t) + \tilde{\nabla}^a \dot{u}_a \ . \qquad (7)$$

The above-defined spatial gradient variables evolve according to:

$$\dot{Z}_a + \frac{2}{3}\theta Z_a + \left(\frac{1}{2}\left(1+3w_m\right)\rho_m - \frac{w_m\rho_m}{(1+w_t)\rho_t}\left(\frac{1}{3}\theta^2 + \frac{1}{2}(1+3w_t)\rho_t\right) + \frac{w_m\rho_m}{(1+w_t)\rho_t}\tilde{\nabla}^2\right)D^m_a + \left(\frac{1}{2}\left(1+3w_{ch}\right)\rho_{ch} - \frac{w_{ch}\rho_{ch}}{(1+w_t)\rho_t}\left(\frac{1}{3}\theta^2 + \frac{1}{2}(1+3w_t)\rho_t\right) + \frac{w_{ch}\rho_{ch}}{(1+w_t)\rho_t}\tilde{\nabla}^2\right)D^{ch}_a + \frac{\rho_{ch}}{(1+w_t)\rho_t}\left(\rho_t - \frac{1}{3}\theta^2 + \tilde{\nabla}^2\right)a\tilde{\nabla}_a w_{ch} = 0 \ . \qquad (8)$$

$$\dot{D}^m_a - \left(\frac{1+w_m}{1+w_t}\right)\frac{w_m\rho_m}{\rho_t}\theta D^m_a - \left(\frac{1+w_m}{1+w_t}\right)\frac{w_{ch}\rho_{ch}}{\rho_t}\theta D^{ch}_a - \left(\frac{1+w_m}{1+w_t}\right)\frac{\rho_{ch}}{\rho_t}a\theta\tilde{\nabla}_a w_{ch} + (1+w_m)Z_a = 0\,, \tag{9}$$

$$\dot{D}^{ch}_a - \left(\frac{1+w_{ch}}{1+w_t}\right)\frac{w_{ch}\rho_{ch}}{\rho_t}\theta D^{ch}_a - \left(\frac{1+w_{ch}}{1+w_t}\right)\frac{w_m\rho_m}{\rho_t}\theta D^m_a + \left(\frac{1+w_m}{1+w_t}\right)\frac{\rho_m}{\rho_t}a\theta\tilde{\nabla}_a w_{ch} + (1+w_{ch})Z_a = 0\,, \tag{10}$$

$$\dot{D}^t_a - w_t\theta D^t_a + (1+w_t)Z_a = 0\,, \tag{11}$$

- **Scalar Decomposition and**

$$\Delta_m = a\tilde{\nabla}^a D_a^m \;, \qquad Z = a\tilde{\nabla}^a Z_a \;, \qquad \Delta_{ch} = a\tilde{\nabla}^a D_a^{ch}$$

- **Harmonic decomposition techniques are applied**

$$X = \sum_k X^k(t)Q^k(\vec{x}) \;, \;\text{ and }\; Y = \sum_k Y^k(t)Q^k(\vec{x}) \;,$$

where k is the wave-number and $Q^k(x)$ is the eigenfunctions of the covariant derivative. Wave-number k represent the order of harmonic oscillator and relate with the scale factor as $k = \frac{2\pi a}{\lambda}$, where λ is the wavelength of the perturbations.

The growth of fractional matter density in baryonic matter-Chaplygin gas

- We explained the contribution of these coupling system (i.e., baryonic and Chaplygin gas)[2] for the formation of large scale structure.
- The long wavelength mode where $\kappa \ll 1$, and short-wavelength mode within the horizon, where $\kappa \gg 1$ are considered in the whole system to present the numerical results of density contrasts in OCG, GCG, MCG.
- In the following, Figs. 8 - 10 we present the behavior of the density contrast in short- and long-wavelength modes for baryonic fluid, total and Chaplygin gas or exotic fluids with cosmological red-shift.

[2] We assume the contribution of relativistic matter (radiation) is negelegible for the contribution of the formation of large-scale structures today.

OCG

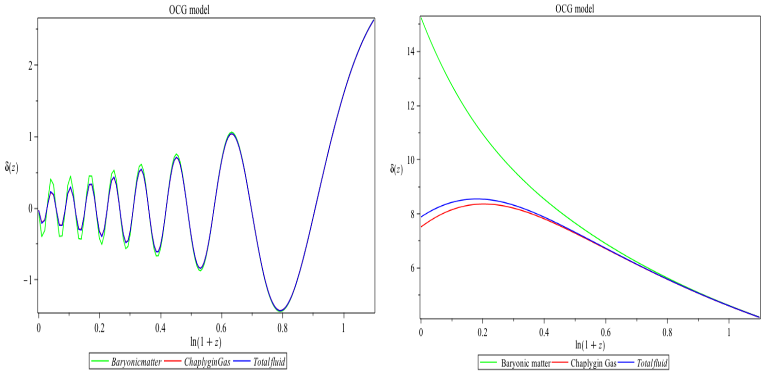

Figure 8: Left: The density contrasts of baryonic-Chaplygin gas with z for short-wavelength mode at $\kappa = 100$ for OCG model. Right: The density contrasts of baryonic matter-Chaplygin gas with z for long-wavelength mode at $\kappa = 0$ for OCG model.

GCG

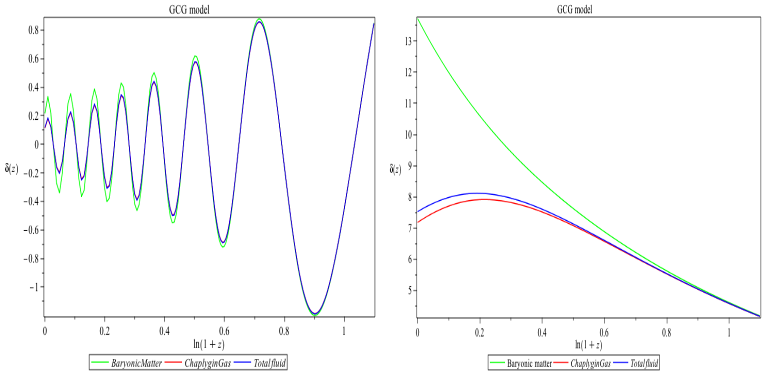

Figure 9: Left: The density contrasts of baryonic-Chaplygin gas with z for short-wavelength mode at $\kappa = 10$ for GCG model. Right: The density contrasts of baryonic matter - Chaplygin gas with z for long-wavelength mode at $\kappa = 0$ for GCG model. We use $\alpha = 0.6424$ from Table 1 for illustrative purpose.

MCG

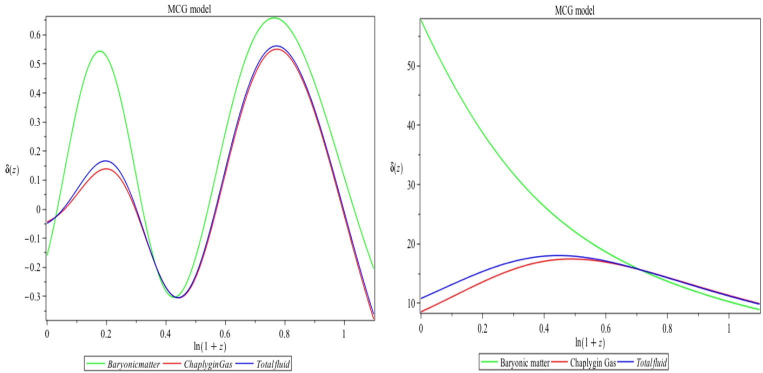

Figure 10: The growth of density contrasts in the baryonic matter-Chaplygin gas mixture for the short-wavelength mode at $\kappa = 100$ for MCG model (left) and long-wavelength mode at $\kappa = 0$ (right). We use $B = 0.1448$ and $\alpha = 0.5694$ from Table 1 for illustrative purpose.

Conclusions

- The background cosmological parameters have been studied and compared against supernova cosmological data for different Chaplygin gas models, namely: OCG, GCG, MCG, GMCG, ECG and GCCG.
- The comparison of the deceleration parameters of the various CG models with ΛCDM has been done to identify which model can best explain the late-time accelerating universe.
- The linear cosmological perturbations have also been investigated in a matter-Chaplygin gas multi-fluid setting around the FLRW background using $1+3$ covariant formalism.

End

Murakoze

[1] Allemandi G, Borowiec A, Francaviglia M and Odintsov S D 2005 *Physical Review D* **72** 063505

[2] Riess A G, Filippenko A V *et al.* 1998 *The Astronomical Journal* **116** 1009

[3] Frieman J A, Turner M S and Huterer D 2008 *Annu. Rev. Astron. Astrophys.* **46** 385–432

[4] Clifton T, Ferreira P G, Padilla A and Skordis C 2012 *Physics reports* **513** 1–189

[5] Faraoni V and Nadeau S 2005 *Physical Review D* **72** 124005

[6] Elmardi M Y 2018 *Topics in Relativistic Cosmology* Ph.D. thesis

[7] Deza M M and Deza E 2009 Encyclopedia of distances *Encyclopedia of distances* (Springer) pp 1–583

[8] Abebe A, Abdelwahab M, De la Cruz-Dombriz Á and Dunsby P K 2012 *Class. Quant. Grav.* **29** 135011

[9] Sahlu S, Ntahompagaze J, Abebe A, Cruz-Dombriz Á and Mota D 2020 *European Physical Journal C* **80** 1–19

[10] Ntahompagaze J, Sahlu S, Abebe A and Mbonye M R 2020 *International Journal of Modern Physics D* **29** 2050120

[11] Nesseris S and Garcia-Bellido J 2013 *Journal of Cosmology and Astroparticle Physics* **2013** 036

[12] Dunsby P K, Bruni M and Ellis G F 1992 *The Astrophysical Journal* **395** 54–74

[13] Dunsby P K 1991 *Class. Quant. Grav.* **8** 1785