A new observable for cosmic shear

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Work

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with Jérémie Francfort, RD & Giulia Cusin

Outline

Motivation

Measuring shear or convergence from weak lensing is very interesting for several reasons :

- It is sensitive to the *total matter* between source and observer, dark as well as luminous.
- Within ΛCDM there is a very close relation between shear and convergence, they have basically the same angular power spectrum. A deviation of this would be an interesting *signature beyond ΛCDM*.
- Shear is sensitive to the lensing potential, given by the sum of the Bardeen potentials. Combining it with velocity measurements e.g. RSD provides an important *test of gravity*.

So far, shear has been measured mainly by observing the shape of galaxies which is distorted by the shear induced by the gravitational field of the foreground matter. Correlating the ellipticity of far away galaxies one can measure the shear correlation function. Unfortunately this measurement is strongly affected by intrinsic alignment. In this talk I describe an alternative method to measure shear which does not suffer from intrinsic alignement (which comes in only once we compute the variance, i.e. in the error budget).

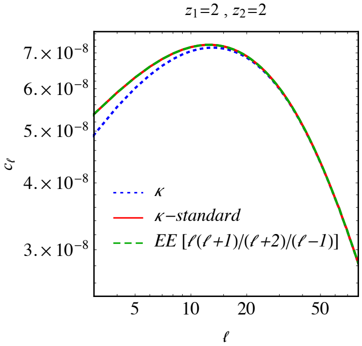
Introduction

A source is deformed by gravitational lensing. At first order in perturbation theory the linear Jacobi map J describing this deformation, $\mathbf{n} = \theta^1 \mathbf{e}_1 + \theta^2 \mathbf{e}_2$,

$$\begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix} = \boldsymbol{\theta} \mapsto J\boldsymbol{\theta}, \qquad J = \begin{pmatrix} 1-\kappa-\gamma_1 & \omega-\gamma_2 \\ -\omega-\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \simeq \mathbb{I} + (\partial_a \partial_b \phi(\boldsymbol{\theta}, z))$$

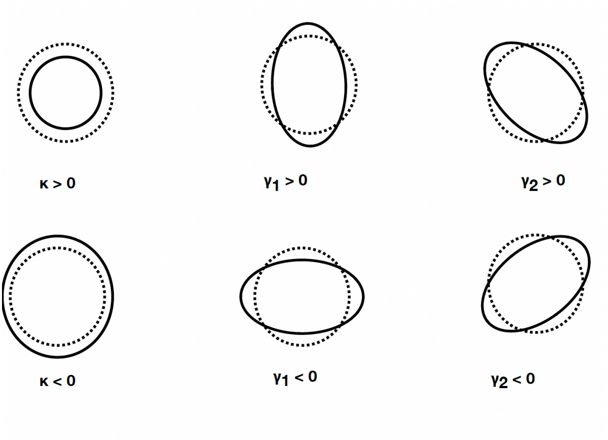
where ϕ is the lensing potential,

$$\phi(\boldsymbol{\theta}, z) = -\int_0^{r(z)} \mathrm{d}r \frac{r(z)-r}{r(z)r} \left[\Phi(r\boldsymbol{\theta}, t_0 - r) + \Psi(r\boldsymbol{\theta}, t_0 - r) \right] .$$



At large scales there are relativistic corrections to the Jacobi map, which e.g. induce differences between the convergence and shear power spectra and which request $\omega \equiv 0$.
[Fanizza, Di Dio, DR, Marozzi (2022)]

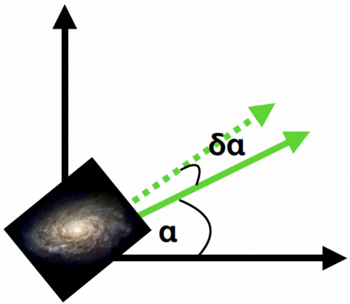
The deformation of a spherical shape by lensing :



However, galaxies are usually not spherical but rather elliptical.
If you shear an ellipse whose principle axes are not aligned with the principle axes of the shear map, this induces not only a change in the ellipticity but also a rotation. With respect to arbitrary parallel transported axes $(\mathbf{e}_1, \mathbf{e}_2)$, shear rotates the original orientation α of, say, the semi-major axis into $\alpha + \delta\alpha$ where (at first order in γ)

$$\delta\alpha = \frac{\varepsilon^2}{2-\varepsilon^2}\left(\gamma_2 \cos 2\alpha - \gamma_1 \sin 2\alpha\right) .$$

Here $0 \leq \varepsilon \leq 1$ is the ellipticity of the galaxy. (Note that $\delta\alpha = 0$ if the principle axes are aligned, i.e. $\alpha = 0$ (or π) and $\gamma_2 = 0$ or $\alpha = \pi/4$ (or $5\pi/4$) and $\gamma_1 = 0$.)



The observable

$$\delta\alpha = \frac{\varepsilon^2}{2-\varepsilon^2}\left(\gamma_2 \cos 2\alpha - \gamma_1 \sin 2\alpha\right) \,.$$

We can observe the ellipticity, ε and the orientation of the galaxy in the image plane, the screen, $\alpha + \delta\alpha$. But without further information, we cannot determine α, the orientation of the galaxy in the source plane, the screen at the position of the galaxy.

However, if the light coming from the galaxy is polarized and if the polarisation direction is related to the orientation of the galaxy, we can use the fact that polarisation is parallel transported, like our screen basis $(\mathbf{e}_1, \mathbf{e}_2)$, hence it keeps its original orientation.

This is exactly the situation we have for radio galaxies : The radio emission is mainly synchrotron emission from relativistic electrons in the magnetic field of the galaxy, which is (mainly) polarised in the orbital plane of the electrons hence normal to the magnetic field. On the other hand the magnetic field of the galaxy is expected to be aligned with its major axis. We can therefore use the direction of polarisation to measure α to obtain $\delta\alpha$.

The observable

Having observed ε, α and $\alpha + \delta\alpha$ we define the observable

$$\Theta = \frac{2 - \varepsilon^2}{\varepsilon^2}\delta\alpha = \gamma_2 \cos 2\alpha - \gamma_1 \sin 2\alpha$$

This is simply the shear in direction $\alpha - \pi/4$, i.e. in the direction most strongly mis-aligned with the orientation of the galaxy.

The correlation function of this observable is

$$\langle\Theta(\boldsymbol{n}_1, \alpha_1, z_1)\Theta(\boldsymbol{n}_2, \alpha_2, z_2)\rangle = \zeta_+(\mu, z_1, z_2)\cos(2(\alpha_1 - \alpha_2)) + \zeta_-(\mu, z_1, z_2)\cos(2(\alpha_1 + \alpha_2))$$

where $\mu = \mathbf{n}_1 \cdot \mathbf{n}_2$ and $\zeta_\pm$ are the positive and negative helicity correlation functions of the shear. These correlation functions are related to the power spectrum of the lensing potential $C_\ell(z_1, z_2)$ via ($\cos\theta = \mu$)

$$\int_{-1}^{+1} \zeta_\pm(\mu, z_1, z_2)\, {}_{\pm 2}Y_{\ell,-2}(\theta, \pi/2)\, \mathrm{d}\mu \propto C_\ell(z_1, z_2)\,.$$

An estimator for Θ

From the observable Θ we now construct an estimator for the correlation functions ζ_+ and ζ_- We consider two couples of galaxies both separated by the same angle θ and located at the same redshifts (within the resolution of our survey).
The galaxies of the first couple have the directions and redshifts $(\boldsymbol{n}_j, z_j)$ with orientations α_j ($j = 1, 2$), while the second couple of galaxies are located in different directions $\boldsymbol{n}_j'$ and with different orientations α_j' but inside the same redshift bins z_j. The two couples of galaxies, however should be separated by the same angle θ i.e. $\boldsymbol{n}_1 \cdot \boldsymbol{n}_2 = \boldsymbol{n}_1' \cdot \boldsymbol{n}_2' = \cos\theta = \mu$. We then compute

$$\Xi = \Theta(\boldsymbol{n}_1, \alpha_1, z_1)\Theta(\boldsymbol{n}_2, \alpha_2, z_2)\,,$$
$$\Xi' = \Theta(\boldsymbol{n}_1', \alpha_1', z_1)\Theta(\boldsymbol{n}_2', \alpha_2', z_2)\,.$$

This is an estimator for the Θ correlation function from which we can easily extract and estimator if $\zeta_\pm$,

$$\hat{\zeta}_\pm(\mu, z_1, z_2) = \Xi F_\pm(\alpha_1', \alpha_2', \alpha_1, \alpha_2) + \Xi' F_\pm(\alpha_1, \alpha_2, \alpha_1', \alpha_2')$$

For functions $F_\pm$ which are obtained by solving the expression $\langle\Theta(\boldsymbol{n}_1, \alpha_1, z_1)\Theta(\boldsymbol{n}_2, \alpha_2, z_2)\rangle$ for ζ_+ and ζ_-.

$$\langle\hat{\zeta}_\pm(\mu, z_1, z_2)\rangle = \zeta_\pm(\mu, z_1, z_2)\,.$$

The error bugdet

Note that in our estimator it does not matter whether the angles $\alpha_j\,,\ \alpha'_j$ are correlated due to intrinsic alignement.

An optimal estimator is obtained by summing over all estimators with fixed z_1, z_2 and μ, weighing them inversely proportional to the error.
For each pair of couples q, we compute an estimator $\hat{\zeta}_{\pm,q}(\mu)$ with its relative error $\tau_{\pm,q}$.
The total signal-to-noise ratio for the measurement of $\zeta_\pm(\mu, z_1, z_2)$ is given by

$$\mathrm{SNR}_\pm(\mu, z_1, z_2) = \sqrt{\sum_q \frac{1}{\tau^2_{\pm,q}}}\,.$$

Considering an average error

$$\tau_{\pm,q} \simeq \tau_0$$

we obtain

$$\mathrm{SNR}_\pm(\mu, z_1, z_2) \approx \frac{\sqrt{N_e(\mu, z_1, z_2)}}{\tau_0}\,,$$

where $N_e(\mu, z_1, z_2)$ is the number of estimators.

$$N_e(\varphi, z_1, z_2) \simeq \frac{1}{2}\left(N_g(z_1)N_g(z_2)\frac{8f_{\mathrm{sky}}\sin\theta}{\delta^3}\right)^2\,,$$

where δ is the angular resolution (pixel size) and $N_g(z)$ is the number of galaxies in a pixel at redshift z.

The error bugdet

Even if the indivitual errors are large, $\tau_0 > 1$, we can obtain a good SNR.

Assuming that galaxies in the same pixel are not independent we still have

$$N_e(\varphi, z_1, z_2) \simeq \frac{8 f_{\rm sky} \sin\theta}{\delta^3}$$

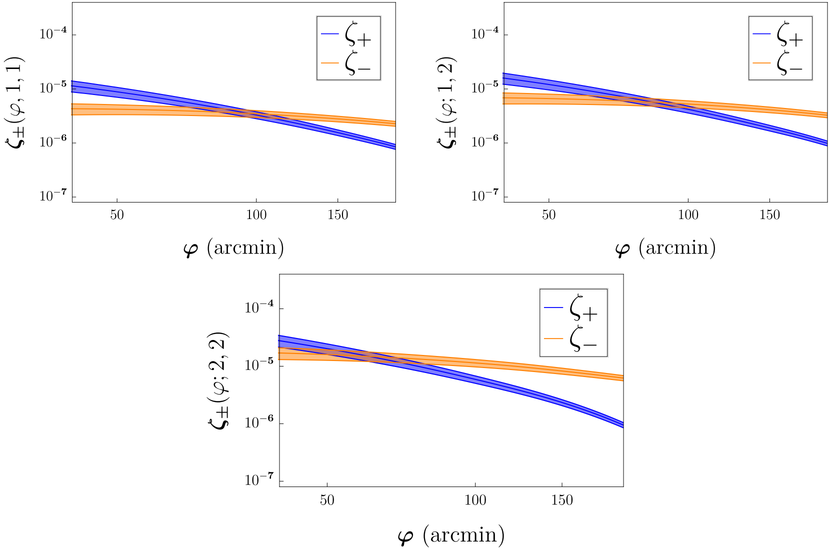
Using parameters are taken from SKA2, we choose a sky fraction and a pixel size of

$$f_{\rm sky} \approx 0.7\,,$$
$$\delta\theta = 5' \approx 1.4 \times 10^{-3}\,.$$

Assume a conservative relative error per galaxy pair of the order of $\tau_0 \approx 10^3$ this yields

$$\mathrm{SNR} \approx 45\sqrt{\sin\theta}\,.$$

Results



J. Francfort, RD, & G. Cusin (2022)

Conclusions

- If we can measure the *polarisation* of light emitted from a galaxy and it is correlated with the *semi-major axis* of the galaxy, we can use this to measure *shear*.
- *Radio galaxies* do provide this correlation via synchrotron radiation.
- Rough estimates of the expected S/N for SKA are very promising.
- This shear measurement is *local* and can in principle be used to generate *tomographic maps*.
- Is the correlation between polarisation and galaxy orientation strong enough and is its scatter sufficiently independent of the galaxy's location ?
- Can we measure this also for galaxies observed in the optical band ?

Conclusions

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