

The impact of dark matter self-scatterings on its relic abundance

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Based on [2204.07078](#)

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DARK SIDE
OF THE
UNIVERSE

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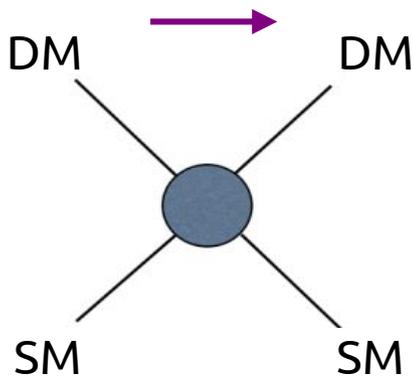
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Self-scatterings of DM in the early Universe

Self-scatterings are usually **neglected** during the calculation of the DM relic abundance

The **equilibrium shape** of DM momentum distribution is established via **elastic scatterings** on SM particles in the plasma

$$f_i = \frac{n_i}{n_i^{\text{eq}}} f_i^{\text{eq}}(E_i, T_{\text{SM}})$$



WIMPs typically go out of **kinetic equilibrium** much later than the freeze-out

$T \sim 1\text{-}10 \text{ MeV}$ ($x \gg 100$)

Bringmann, 0903.0189

Boltzmann equation

With this assumption, one solves the Boltzmann equation for the **DM number density (nBE)**

$$\frac{dY}{dx} = \frac{s(x)}{x H(\tilde{x})} \langle \sigma v \rangle(x) [Y_{\text{eq}}^2(x) - Y^2] \quad (\text{for } 2 \rightarrow 2 \text{ annihilations})$$

$$\langle \sigma v \rangle = \frac{g_\chi^2}{n_\chi^{\text{eq}} \cdot n_\chi^{\text{eq}}} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 \tilde{p}}{(2\pi)^3} \sigma_{\text{ann}} v \underline{f_\chi^{\text{eq}}(p) f_\chi^{\text{eq}}(\tilde{p})}$$

Can also include decays, coannihilations, etc.

Works both for **freeze-out** and **freeze-in**

Early kinetic decoupling

If the *annihilation rate* \gg the rate of *scatterings on SM* an **early kinetic decoupling** can occur \rightarrow the assumption about the shape of f doesn't hold!

Example: scalar singlet model (Z_2)

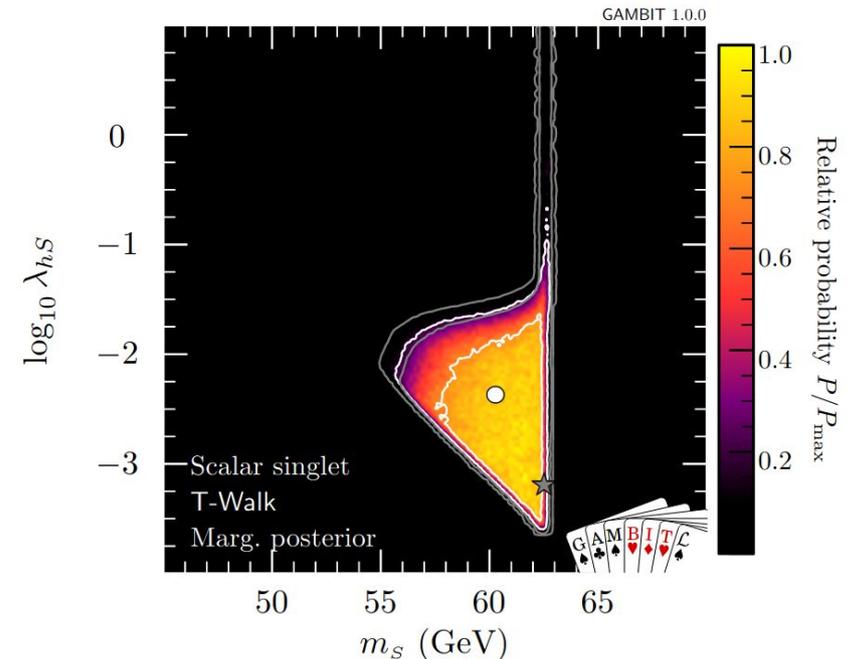
Binder et al., 1706.07433

- **Resonant** annihilation into SM fermions

$$\sigma v \propto \frac{\lambda_S^2}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2}$$

- Elastic scattering

$$|\mathcal{M}|^2 \propto \frac{\lambda_S^2}{(t - m_h^2)^2}$$



The best-fit region obtained by
GAMBIT Collaboration,
1705.07931

Deviation from equilibrium shape

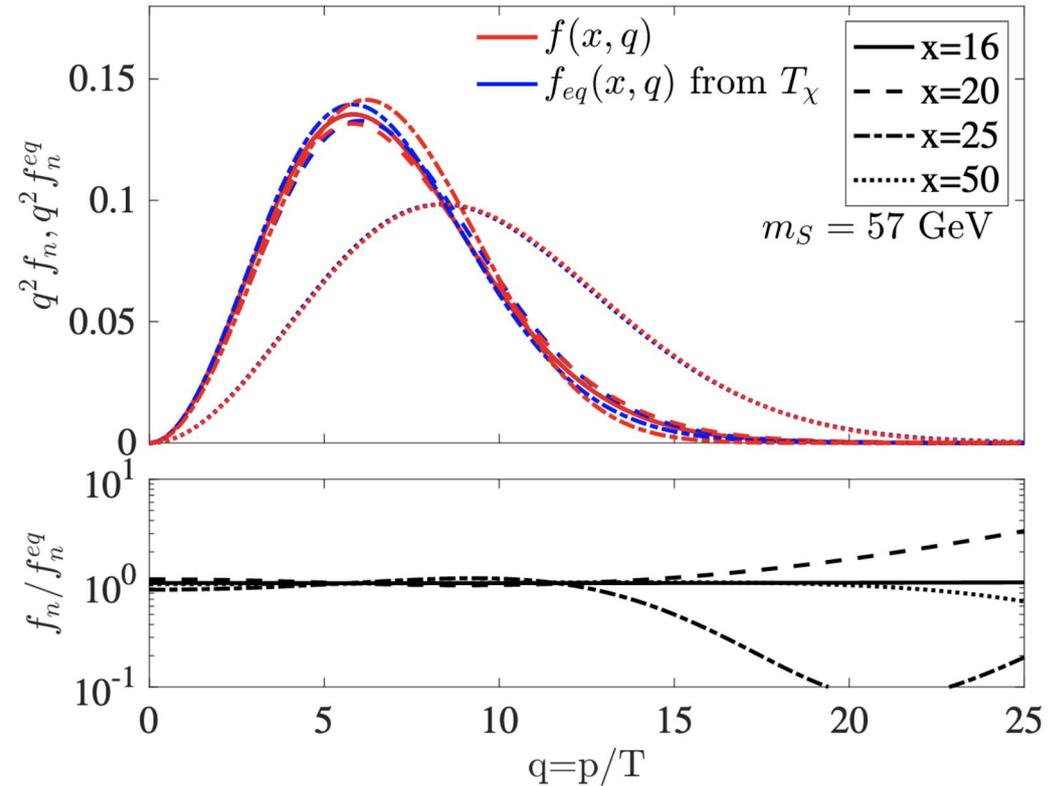
The **high-momentum tail** of the real distribution **enhances** the rate of annihilations

Works for models with **strong velocity-dependence**:

- Resonance
- Sommerfeld
- Threshold
- Etc.

$$\langle \sigma v \rangle = \frac{g_\chi^2}{n_\chi^{\text{eq}} \cdot n_\chi^{\text{eq}}} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 \tilde{p}}{(2\pi)^3} \sigma_{\text{ann}} v f_\chi^{\text{eq}}(p) f_\chi^{\text{eq}}(\tilde{p})$$

Binder et al., 1706.07433



Full Boltzmann equation

In fact, the **nBE** is derived from the general **Full Boltzmann equation (fBE)**

$$2E_i (\partial_t - H p \partial_p) f_i(p) = C[f_i]$$

DM **distribution function**   **Collision term**

The collision term takes into account all the processes

$$C[f_\chi] = \underbrace{C_{\text{ann}} + C_{\text{dec}}}_{\text{Number-changing processes (mainly affect the density)}} + \underbrace{C_{\text{el}} + C_{\text{self}} + \dots}_{\text{Equilibrating processes (affect the shape)}}$$

Number-changing processes (*mainly* affect the density)

Collision term

For DM + DM \rightarrow SM + SM :

$$\begin{aligned}
 C_{\text{ann}}[f_\chi] &= \frac{1}{2g_\chi} \int \frac{d^3 \tilde{p}}{(2\pi)^3 \tilde{E}} \int \frac{d^3 k}{(2\pi)^3 \omega} \int \frac{d^3 \tilde{k}}{(2\pi)^3 \tilde{\omega}} && \text{All momenta configurations} \\
 &\times (2\pi)^4 \delta^{(4)}(E + \tilde{E} - \omega - \tilde{\omega}) && \text{Energy and momenta conservation} \\
 &\times \left[|\mathcal{M}|_{\text{SM} \rightarrow \text{DM}}^2 f_{\text{SM}}(\omega) f_{\text{SM}}(\tilde{\omega}) [1 \pm f_\chi(E)] [1 \pm f_\chi(\tilde{E})] \right. \\
 &\quad \left. - |\mathcal{M}|_{\text{DM} \rightarrow \text{SM}}^2 \underbrace{f_\chi(E) f_\chi(\tilde{E})}_{\text{Unknown function}} [1 \pm f_{\text{SM}}(\omega)] [1 \pm f_{\text{SM}}(\tilde{\omega})] \right] \\
 &\text{Probability} && \text{x number} \\
 &\text{x number} && \text{of states}
 \end{aligned}$$

Leads to an **integro-differential** equation

Numerical difficulties

Collision terms have the following structure

$$C[f_\chi] \propto f_\chi(p) \int \dots \int \frac{d^3k}{(2\pi)^3 2\omega} \dots$$

Distribution function is not integrated over
Can be solved **explicitly**

$$C[f_\chi] \propto \int \dots \int \frac{d^3k}{(2\pi)^3 2\omega} f_\chi(k) \dots$$

Distribution function has to be integrated over
Integro-differential equation

fBE \rightarrow system of fBEs

$$f_\chi(k) \rightarrow \{f_1(k_1), \dots, f_n(k_n)\}$$

$$\int \frac{dk}{E_k} \rightarrow \sum_i \frac{(\Delta k)}{E_k^i}$$

Numerical integration

Self-scattering inevitably has **2 unknown functions** in the *lowest order* in f_χ

Why are self-scatterings important?

Self-scatterings can be more important than elastic scatterings in **shaping** the distribution:

- Momentum transfer $\Delta p/p \sim 1$ (for elastic $\Delta p/p \ll 1$) → **less collisions** required
- Couplings and vertices can be different (**enhanced**)
- **Less constrained** by observations

$$\sigma_{\text{el}}/m \lesssim 10^{-34} \text{ cm}^2/\text{GeV}$$

On electrons, from
structure formation
Nguyen+, 2107.12380

$$\sigma_{\text{self}}/m \lesssim 10^{-24} \text{ cm}^2/\text{GeV}$$

From cluster collisions
Kim+, 1608.08630

Setting

We study the role of self-scatterings on the example of a DM model with long-range interaction and a decaying long-lived heavier state

- Freeze-out
- Early kinetic decoupling
- (Late-time) DM injection from decays

The model

Decaying scalar singlet + DM fermion + dark U(1)*

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu S)^2 - V(S, H) + \underbrace{yS\bar{\chi}\chi + m_{\text{DM}}\bar{\chi}\chi}_{\text{orange}} + \bar{\chi}i\underbrace{\mathcal{D}_\mu\gamma^\mu\chi}_{\text{black}} - \underbrace{\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu}}_{\text{blue}} - \underbrace{\frac{\epsilon}{2}F'_{\mu\nu}F^{\mu\nu}}_{\text{purple}} + \frac{1}{2}m_A^2 A'_\mu A'^\mu$$

$\mathcal{D}_\mu = \partial_\mu - ie'A'_\mu$

$$V(H, S) = -\mu_H^2 |H|^2 - \frac{1}{2}m_S^2 S^2 + \lambda_H |H|^4 + \frac{\lambda_S}{4} S^4 + \frac{\lambda_{HS}}{2} |H|^2 S^2$$

Scalar **decays** → non-thermal component of DM

Dark U(1) → **self-** and **elastic-scatterings** + **annihilation into SM**

* this model without dark U(1) was studied in a similar context in [Ala-Mattinen+, 2201.06456](#)

The model

Decaying scalar singlet + DM fermion + dark U(1)

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu S)^2 - V(S, H) + \underline{yS\bar{\chi}\chi} + m_{\text{DM}}\bar{\chi}\chi + \bar{\chi}i\mathcal{D}_\mu\gamma^\mu\chi - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\epsilon}{2}F'_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2 A'_\mu A'^\mu$$

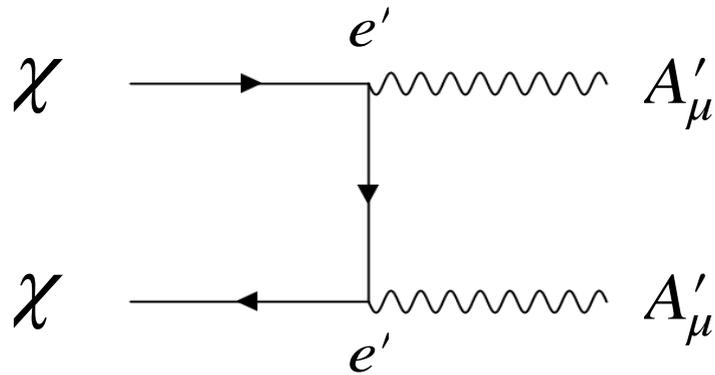
$$V(H, S) = -\mu_H^2 |H|^2 - \frac{1}{2}m_S^2 S^2 + \lambda_H |H|^4 + \frac{\lambda_S}{4} S^4 + \underline{\frac{\lambda_{HS}}{2} |H|^2 S^2} \quad \mathcal{D}_\mu = \partial_\mu - ie' A'_\mu$$

Z2 symmetry slightly **broken**:

- Small value of y → **long lifetime** (late injection)
- Loop-suppressed to SM → **no entropy dilution**
- Coupling to **Higgs** → always in thermal equilibrium

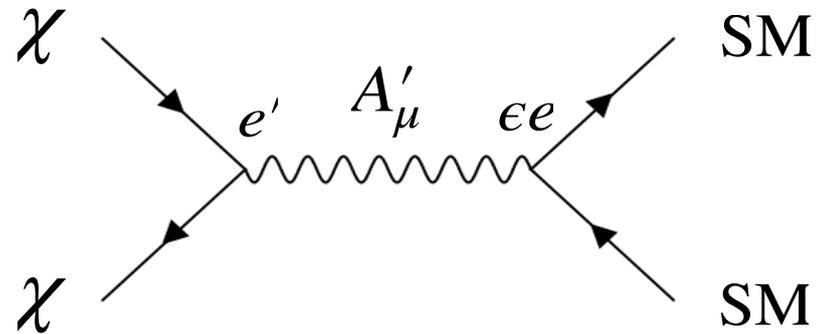
The model

Decaying scalar singlet + DM fermion + dark U(1)



Sub-threshold annihilation

$$m_\chi < m_{A'_\mu}$$



Resonant annihilation
(suppressed by the small ϵ)

Two DM annihilation processes are possible

In our case the strong velocity dependence comes from **sub-threshold**

How is fBE solved?

DRAKE code for the calculation
of DM abundance

Written in *Mathematica* language



<https://drake.hepforge.org> Binder+, 2103.01944

The current version solves **nBE** and **fBE** for the freeze out of 2-2 annihilation processes

For our study we **included**:

- Decays
- Self-scatterings – implemented a C++ patch for fast numerical intergration of the collision term integrals

$$C_{\text{self}}[f_{\chi}(p)] \propto \int d^3k \int d^3\tilde{p} \int d^3\tilde{k} f_{\chi}(\tilde{k}) f_{\chi}(\tilde{p})$$

Backward term for self-scattering

Density evolution

Kinetic equilibrium:

No decay – standard freeze-out

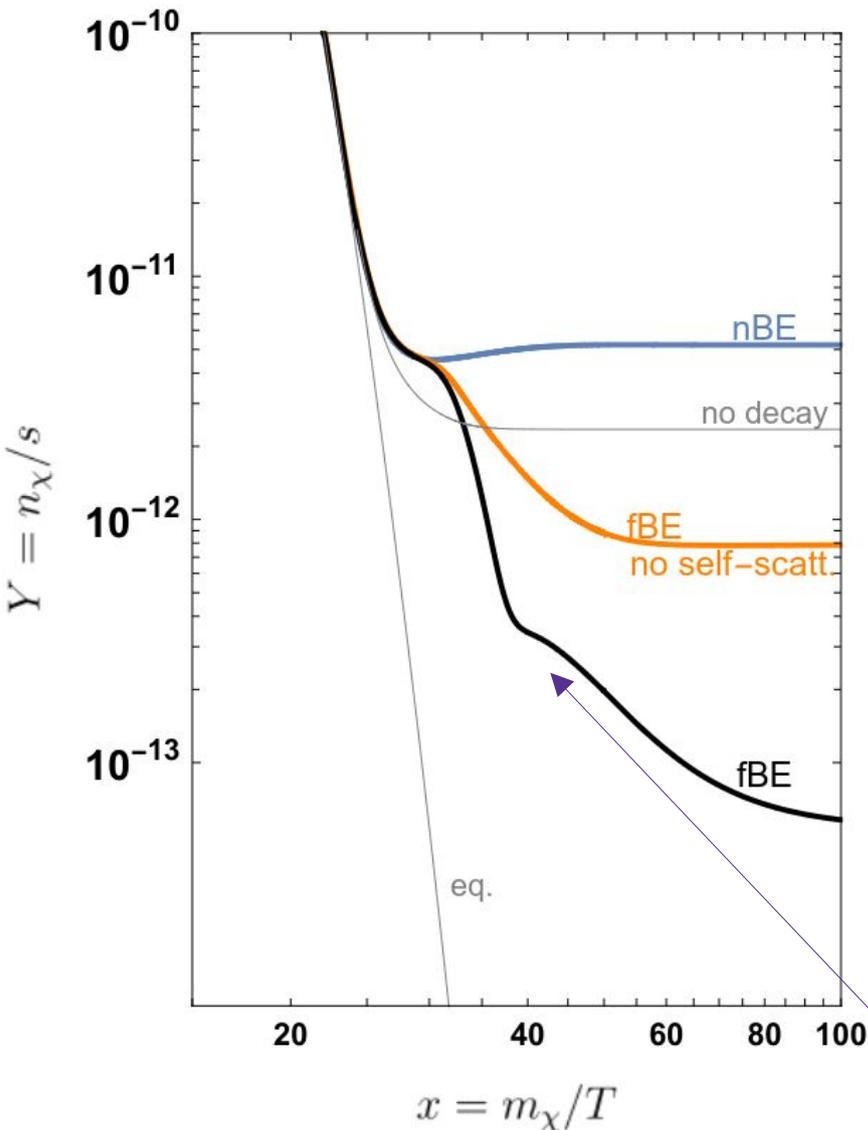
nBE – the density is increased by the late-time decay products

Early kinetic decoupling (fBE):

No self-scatterings – hot particles from decays extend the annihilation into SM and deplete the density

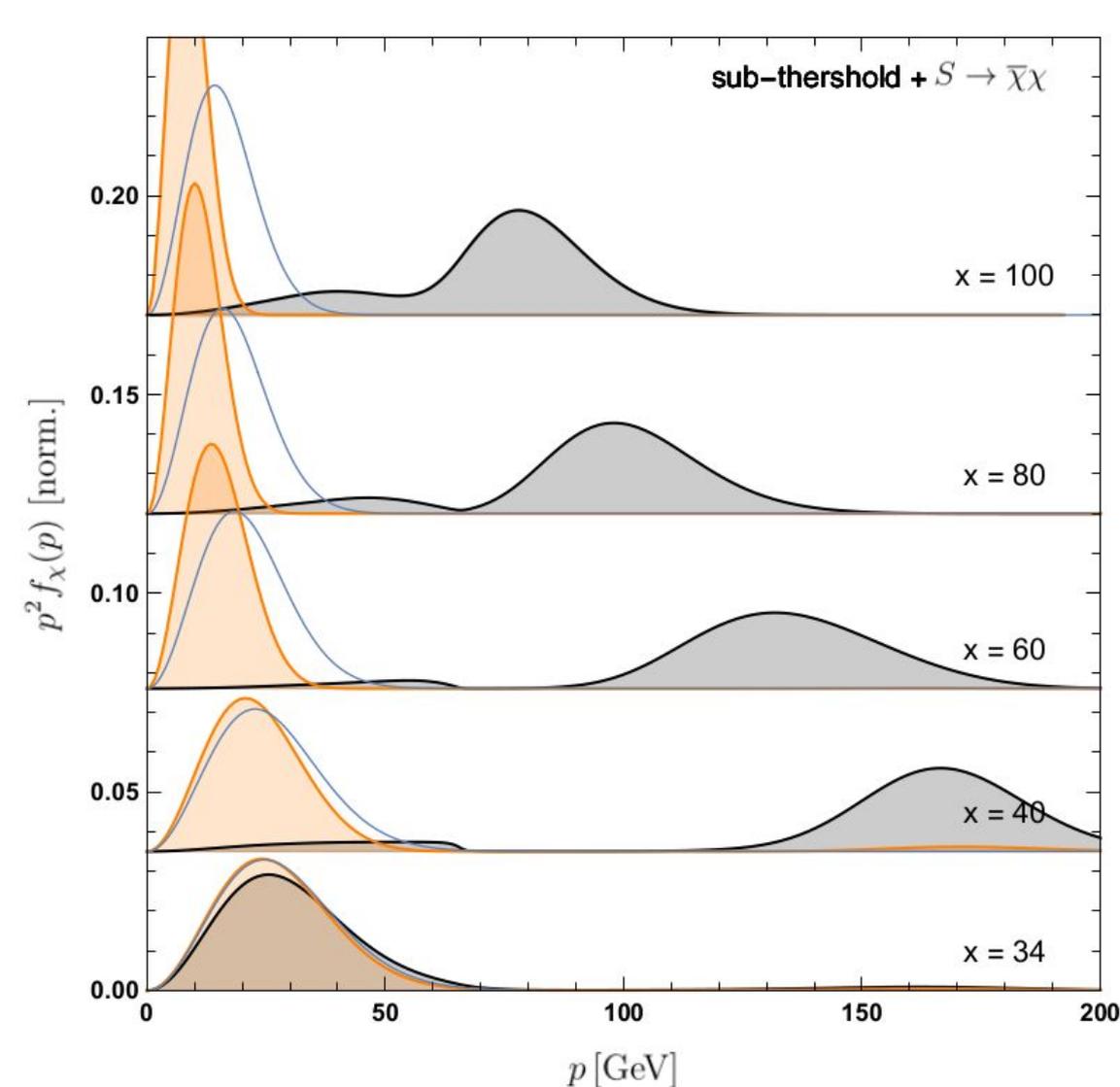
Self-scatterings redistribute the energy from decaying particles and extend the annihilation even longer

sub-threshold + $S \rightarrow \bar{\chi}\chi$
 $m_\chi = 100$ GeV
 $m_A = 108$ GeV
 $m_S = 400$ GeV
 $e' = 1.$
 $\epsilon = 0.001$



Decays essentially stop contributing

Distribution function



x

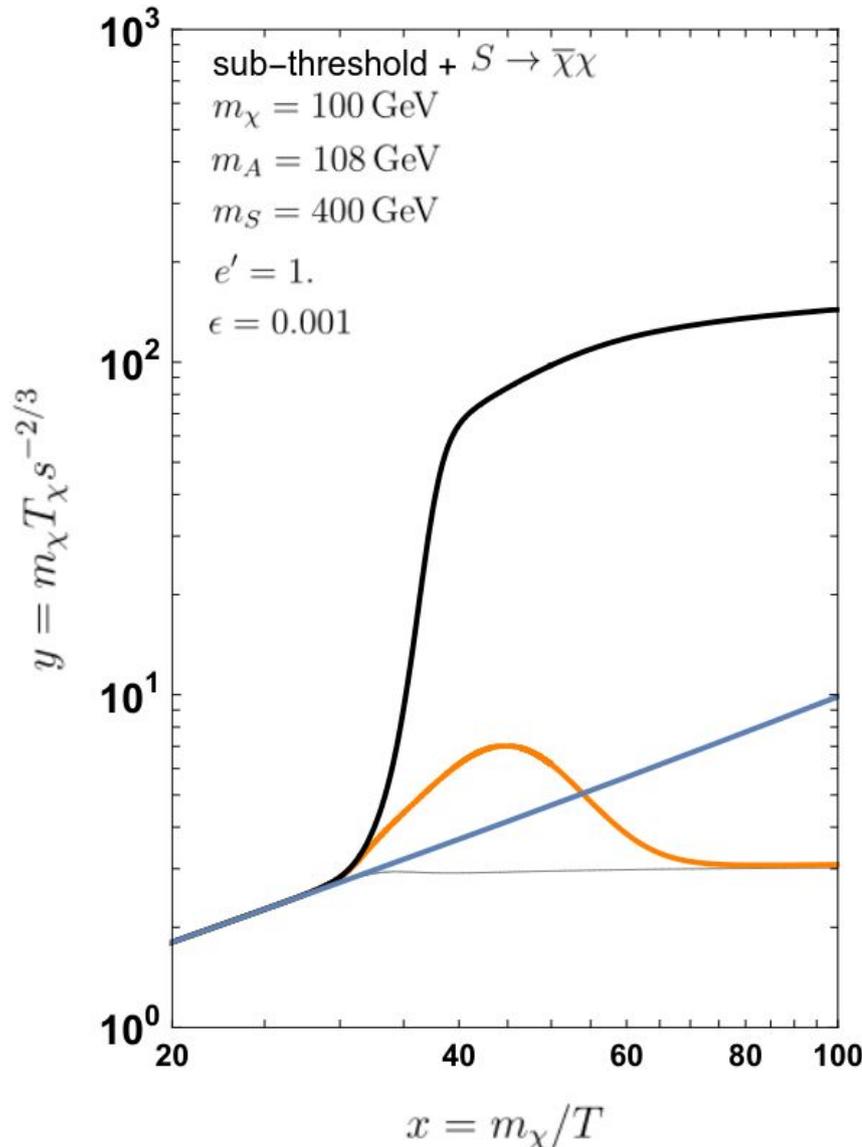
- nBE
- fBE (no scatterings)
- fBE (active scatterings)

Self-scatterings redistribute the heat and “move” the distribution towards larger momenta \rightarrow larger $\langle \sigma v \rangle$

Small component of high-energy particles (most of the energy is dumped into SM through annihilations)

Decays start to contribute to the density

Temperature evolution



$$y \propto T_\chi \cdot x^2$$

Significantly heated up

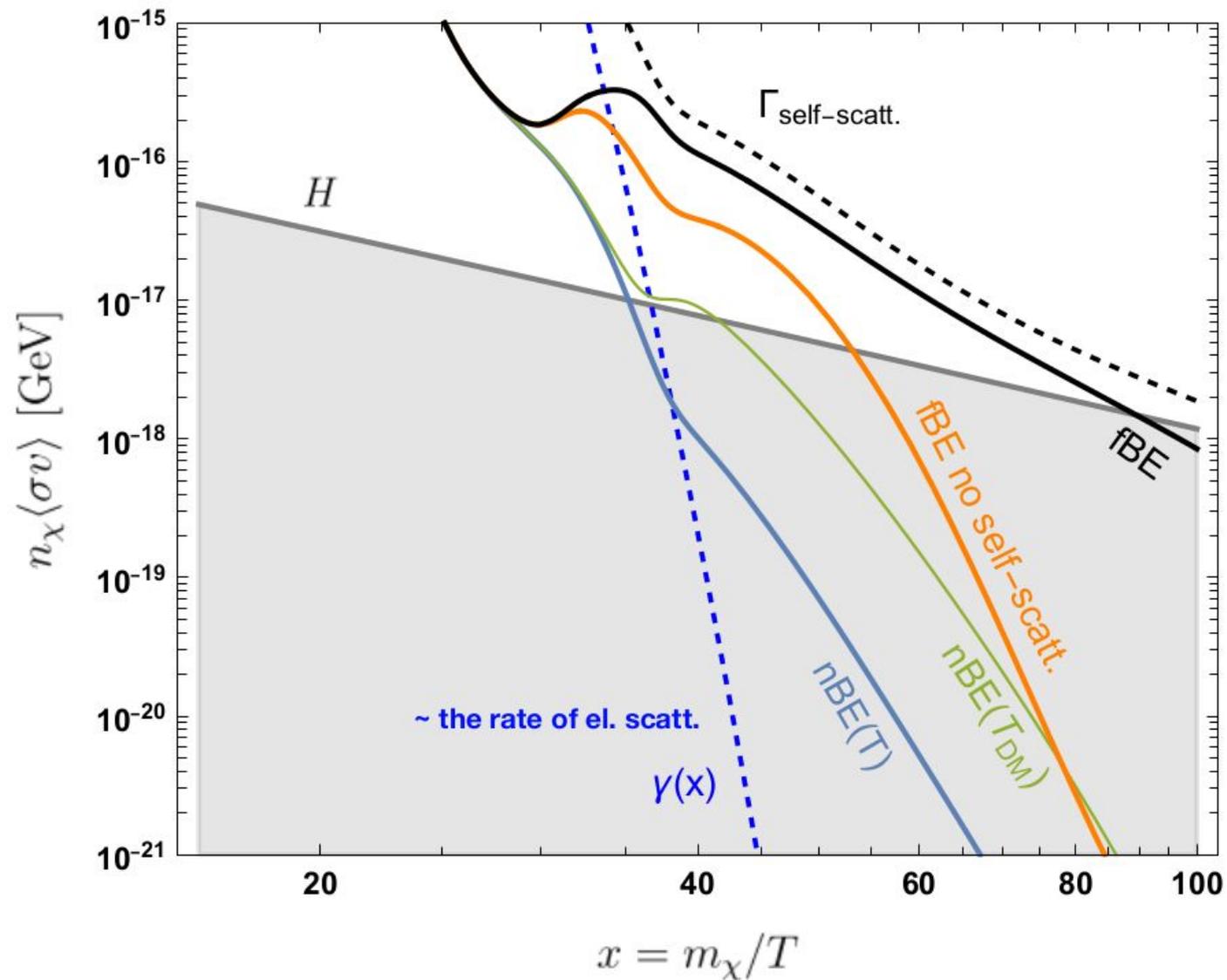
The heat from decays remains in the dark sector due to an efficient **redistribution by self-scatterings**

Temperature is equal to T_{SM} $T_{\text{SM}} \propto x^{-1}$

No self-scatterings – DM is slightly heated by decays and then the temperature decreases due to expansion

$$T_{\text{non-rel}} \propto x^{-2} \quad (\text{constant } y)$$

Rates of processes



Conclusion

The interplay of different interactions in some DM models can lead to a **deviation** of the DM energy distribution from the **equilibrium shape** that can affect the relic density of DM

We studied a model in which

- the self-scattering plays a crucial role **in shaping the energy distribution**
- the relic density can decrease with an injection of additional DM particles due to a **more efficient annihilation** of high-momentum particles (velocity-dependent cross section)