

# Gravitational Positivity

## For

# Dark Gauge Bosons



Masahito Yamazaki

ICTP-EAIFR, Kigali, July 14, 2023



THE UNIVERSITY OF TOKYO

K. Aoki (Kyoto), T. Noumi (Kobe->Tokyo), R. Saito (Yamaguchi),  
S. Sato (Kobe/Tokyo), S. Shirai (IPMU), J. Tokuda (IBS) + MY,  
arXiv: 2305.10058 [hep-ph]

How dark is “dark”?



# Gravity to Rescue?

---

$$\| \left( \text{other forces} \right) \geq \left( \text{gravity} \right) \|$$

cf. • (gauge) weak gravity conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06

• scalar weak gravity conjecture

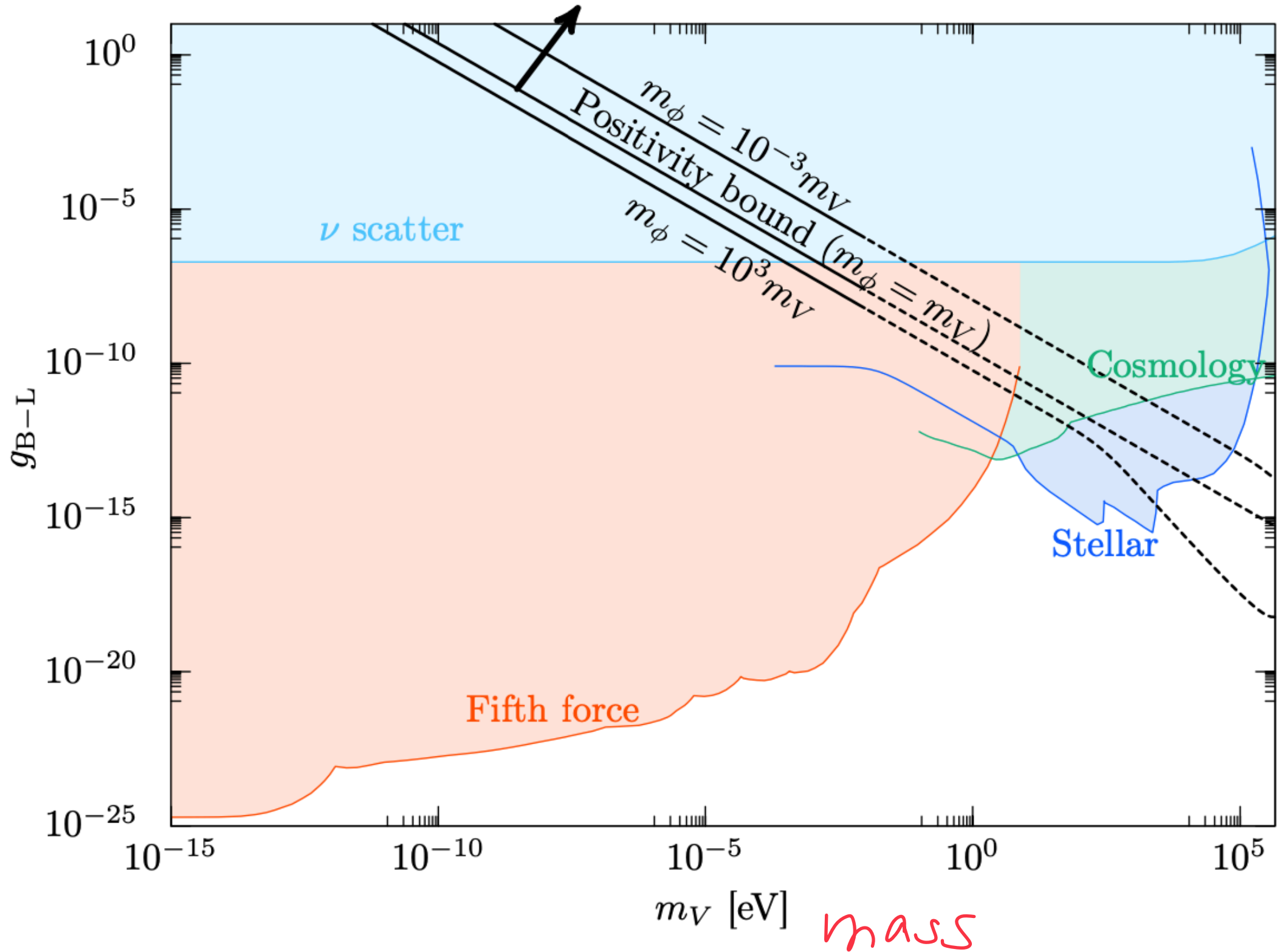
Palti '17, Shirai-MY '19, Kuzenko-Takhistov-Yamada-MY '19

• gravitational positivity

Tokuda, Aoki, Hirano '20, ....

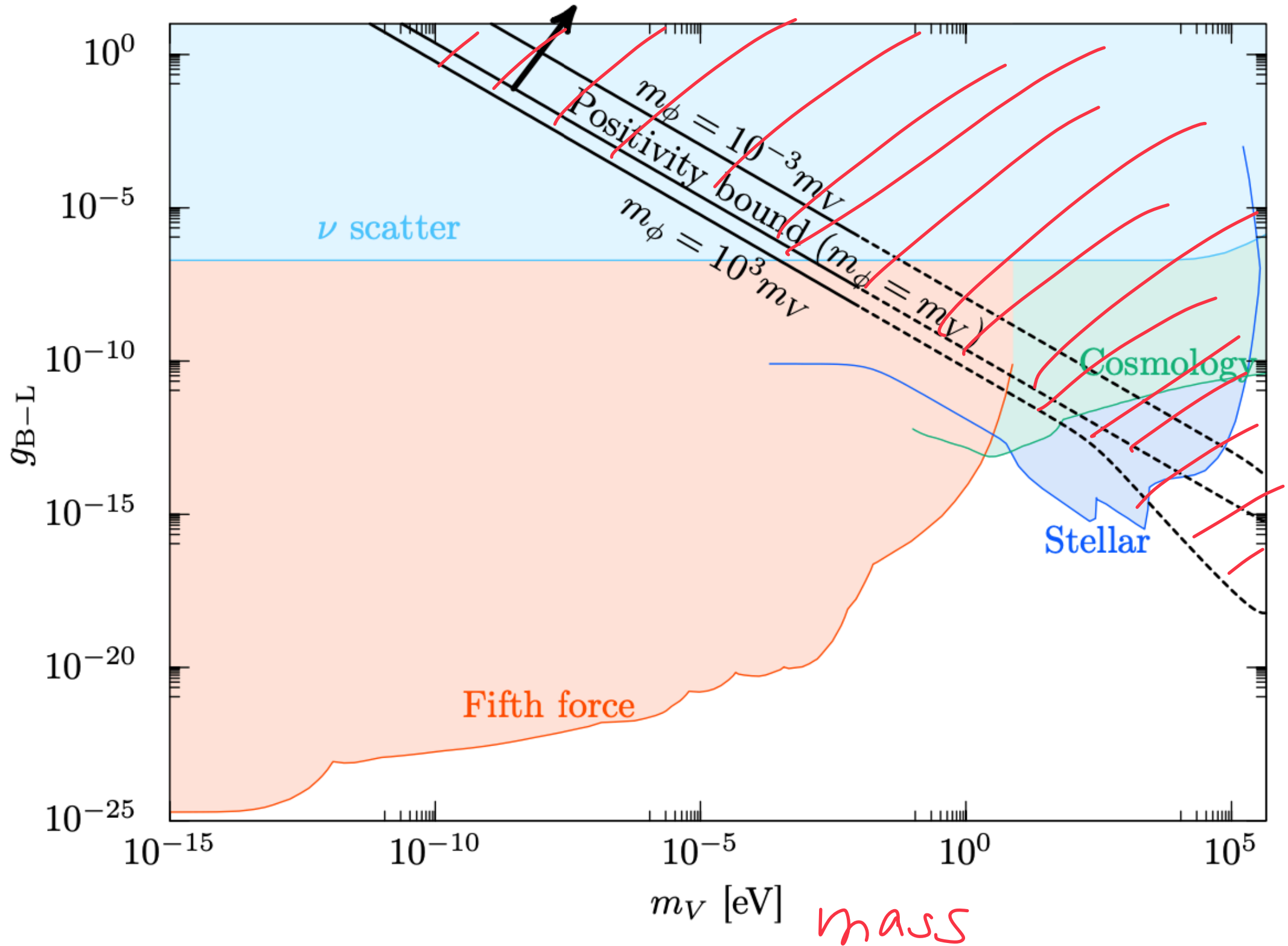
"Result" of gauged  $U(1)_{B-L}$

Coupling



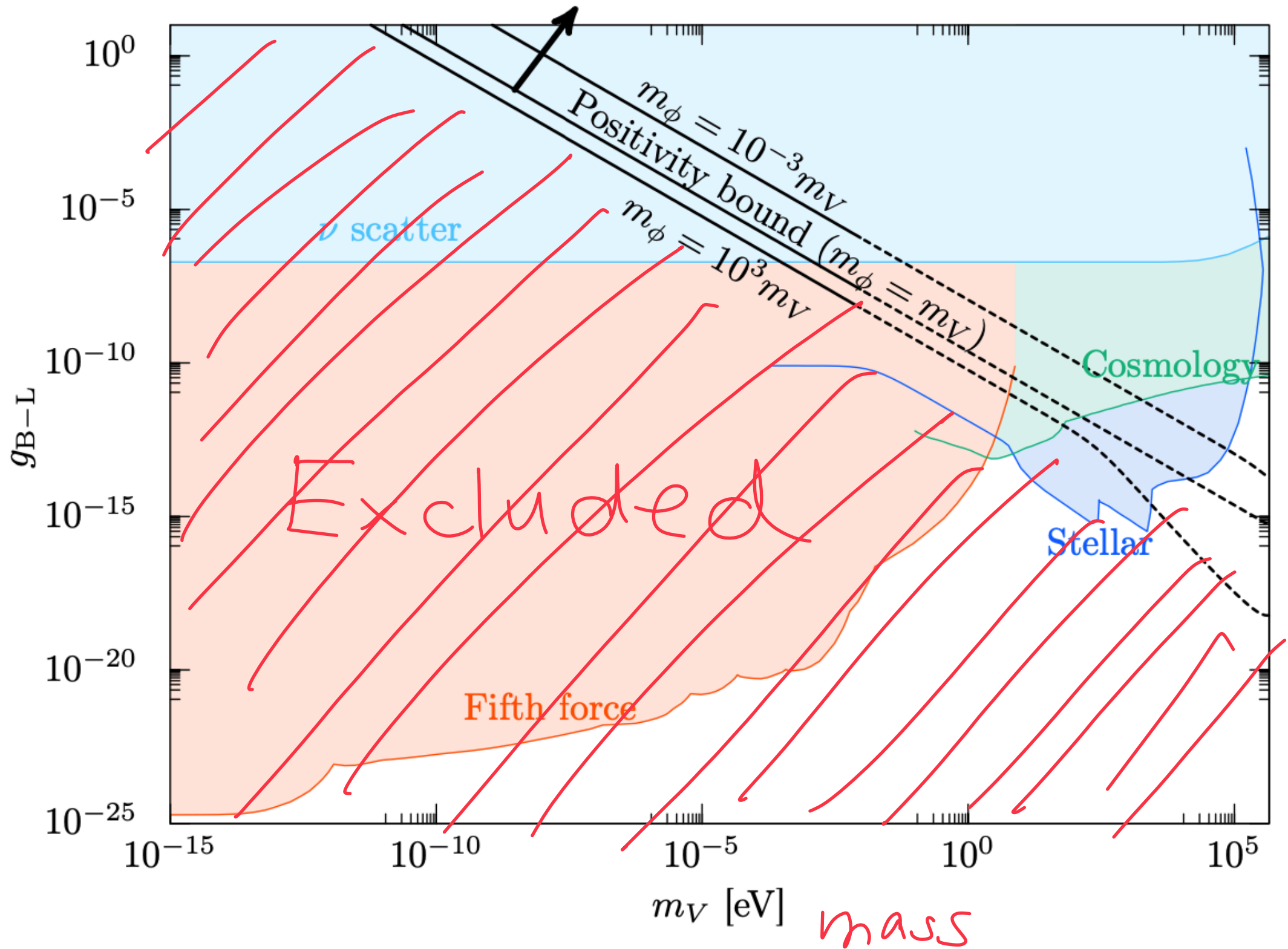
"Result" of gauged  $U(1)_{B-L}$

Coupling



Result of gauged  $U(1)_{B-L}$

Coupling



$$i\mathcal{M}_{\text{non-grav}}(s, t) = \begin{array}{c} \vee \\ A \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ B \\ \vee \end{array} + \dots$$

Assumptions

(i) analyticity

(ii) unitarity ( $\text{Im } \mu \geq 0$ )

(iii)  $s^2$ -boundedness  $\lim_{|s| \rightarrow \infty} \frac{|\mu(s, 0)|}{s^2} = 0$

["old" S-matrix theory since '60s]



$$i\mathcal{M}_{\text{non-grav}}(s, t) = \begin{array}{c} A \qquad A \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ B \qquad B \end{array} + \dots$$

$$a_{2i} = \left. \frac{\partial^2 \mathcal{M}(s, t=0)}{\partial s^2} \right|_{s=2m_V^2}$$

IR data

$$B_2 := a_2 - \frac{2 \cdot 2!}{\pi} \int_{m_V^2}^{\Lambda^2} ds \frac{\text{Im } \mathcal{M}(s, t=0)}{(s - 2m_V^2)^3}$$

$$= \frac{2 \cdot 2!}{\pi} \int_{\Lambda^2}^{\infty} ds \frac{\text{Im } \mathcal{M}(s, t=0)}{(s - 2m_V^2)^3} \geq 0$$

dispersion rel.

analyticity +  $s^2$ -boundedness

UV data

[ Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06  
 Bellazzini '16 de Rham, Melville, Tolley, Zhou '17, ... ]

$$i\mathcal{M}_{\text{grav},t\text{-channel}}(s, t) =$$

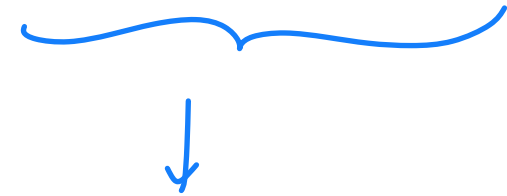
graviton



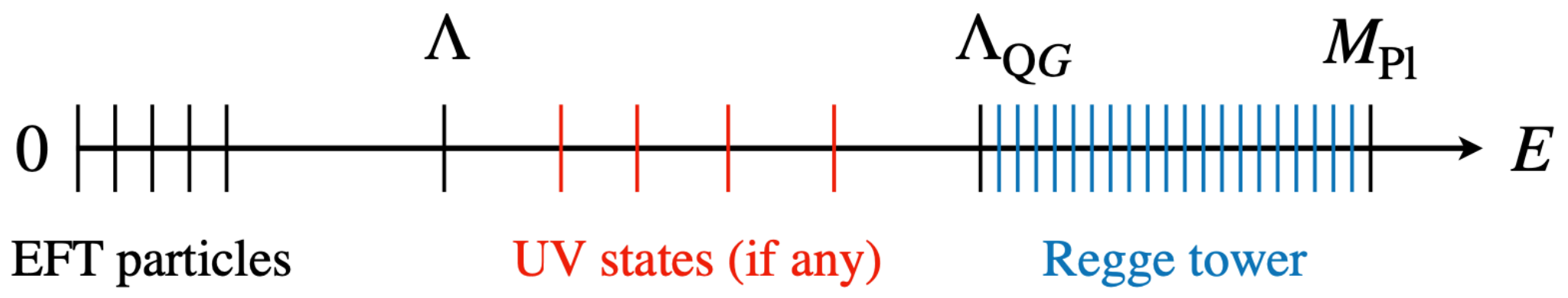
$\frac{s^2}{t}$  : violates  $s^2$ -boundedness?

$$i\mathcal{M}_{\text{grav},t\text{-channel}}(s, t) = \begin{array}{c} A \text{ --- } \text{---} \text{---} A \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ B \text{ --- } \bullet \text{ ---} B \end{array} + \dots$$

graviton



$\frac{s^2}{t}$  : violates  $s^2$ -boundedness?



recovers  $s^2$ -boundedness

$$B_{\text{non-grav}}(\Lambda) \simeq \frac{4}{\pi} \int_{\Lambda^2}^{\infty} ds \frac{\text{Im } \mathcal{M}_{\text{non-grav}}(s, t=0)}{(s - 2m_V^2)^3}$$

$$B_{\text{grav}}(\Lambda) \simeq \lim_{t \rightarrow -0} \left[ \frac{\partial^2 \mathcal{M}_{\text{grav}, t\text{-channel}}(s, t)}{\partial s^2} + \frac{2}{M_{\text{Pl}}^2 t} - (\text{kinematic singularity}) \right]_{s=2m_V^2}$$

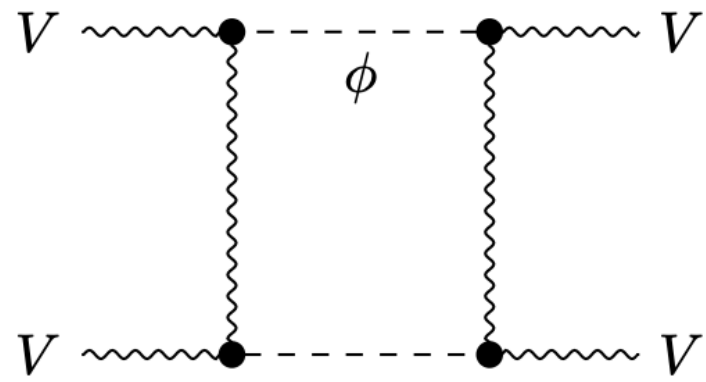
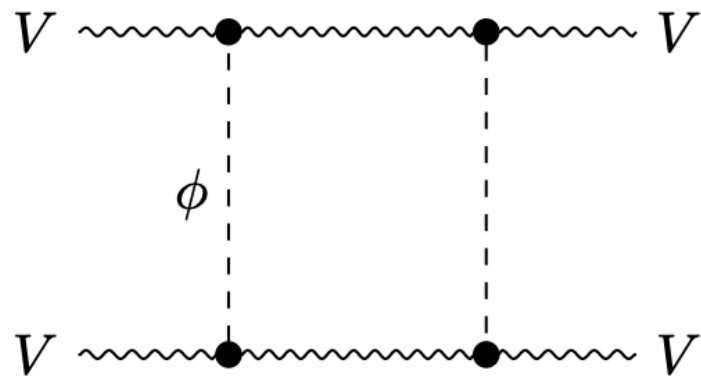
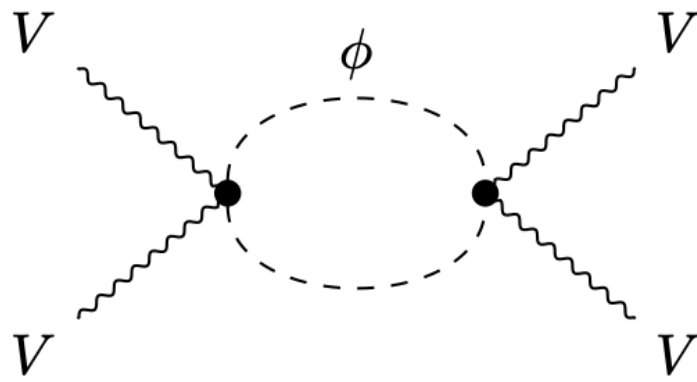
sum

$$B(\Lambda) \gtrsim \mathcal{O}\left(\frac{1}{M_{\text{pl}}^2 M^2}\right)$$

related to  $\Lambda_{\text{QG}}$

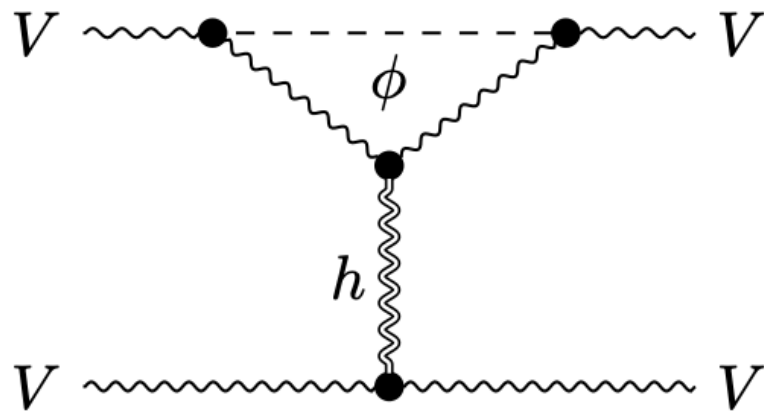
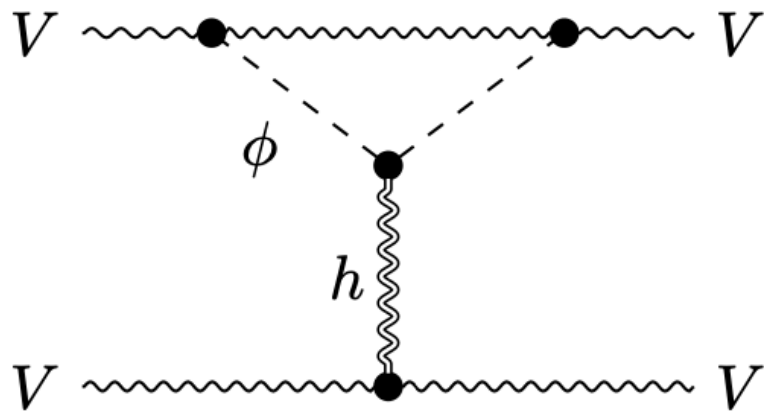
[ Tokuda, Aoki, Hirano '20  
cf. Alberte, de Rham, Jaitly, Tolley '20 ]

Example: U(1) gauge boson with Higgs mass



non-gravitational

$$B_{\text{non-grav}} \sim + \frac{g^4}{\Lambda^4} + \dots$$

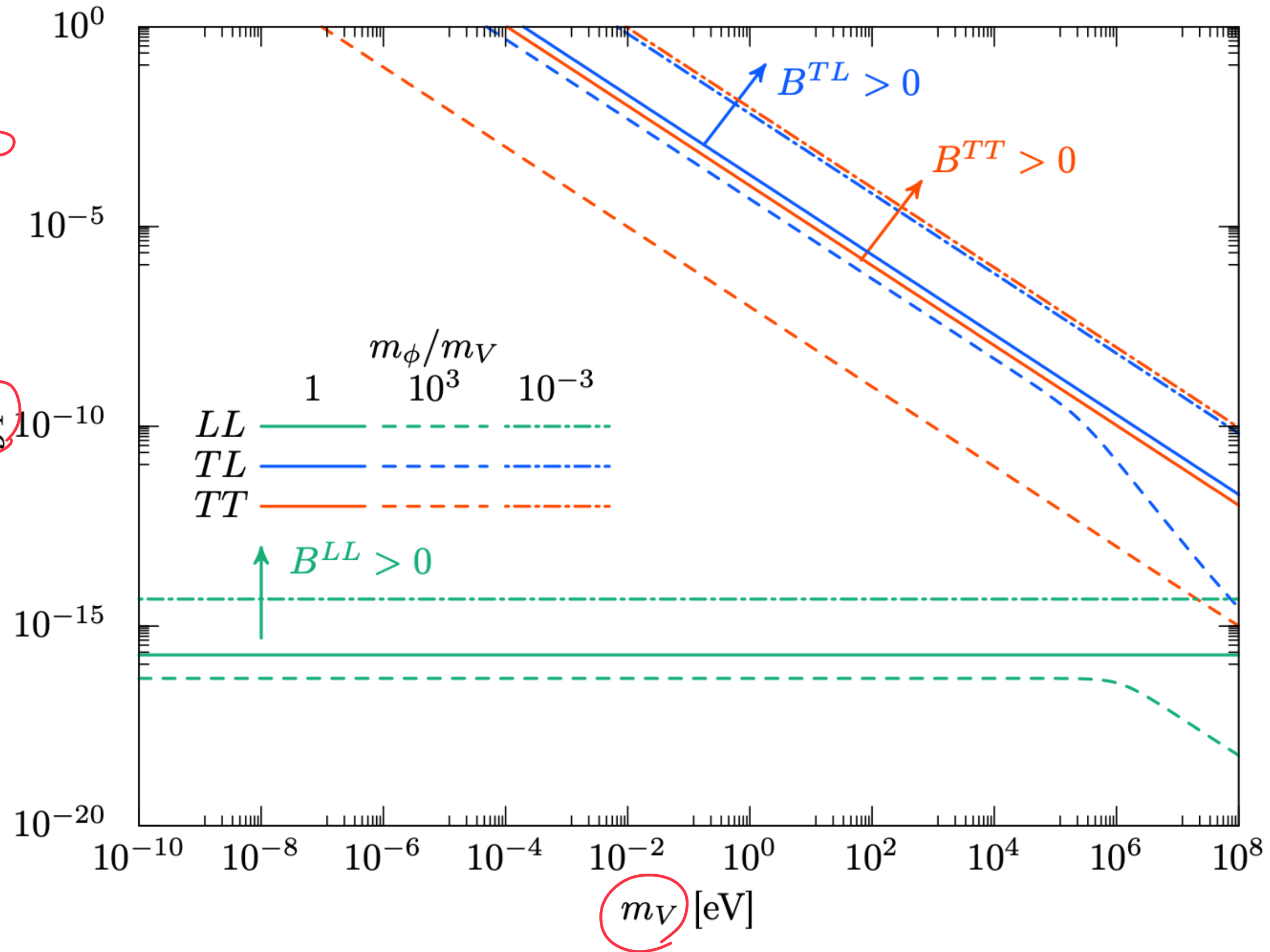


gravitational

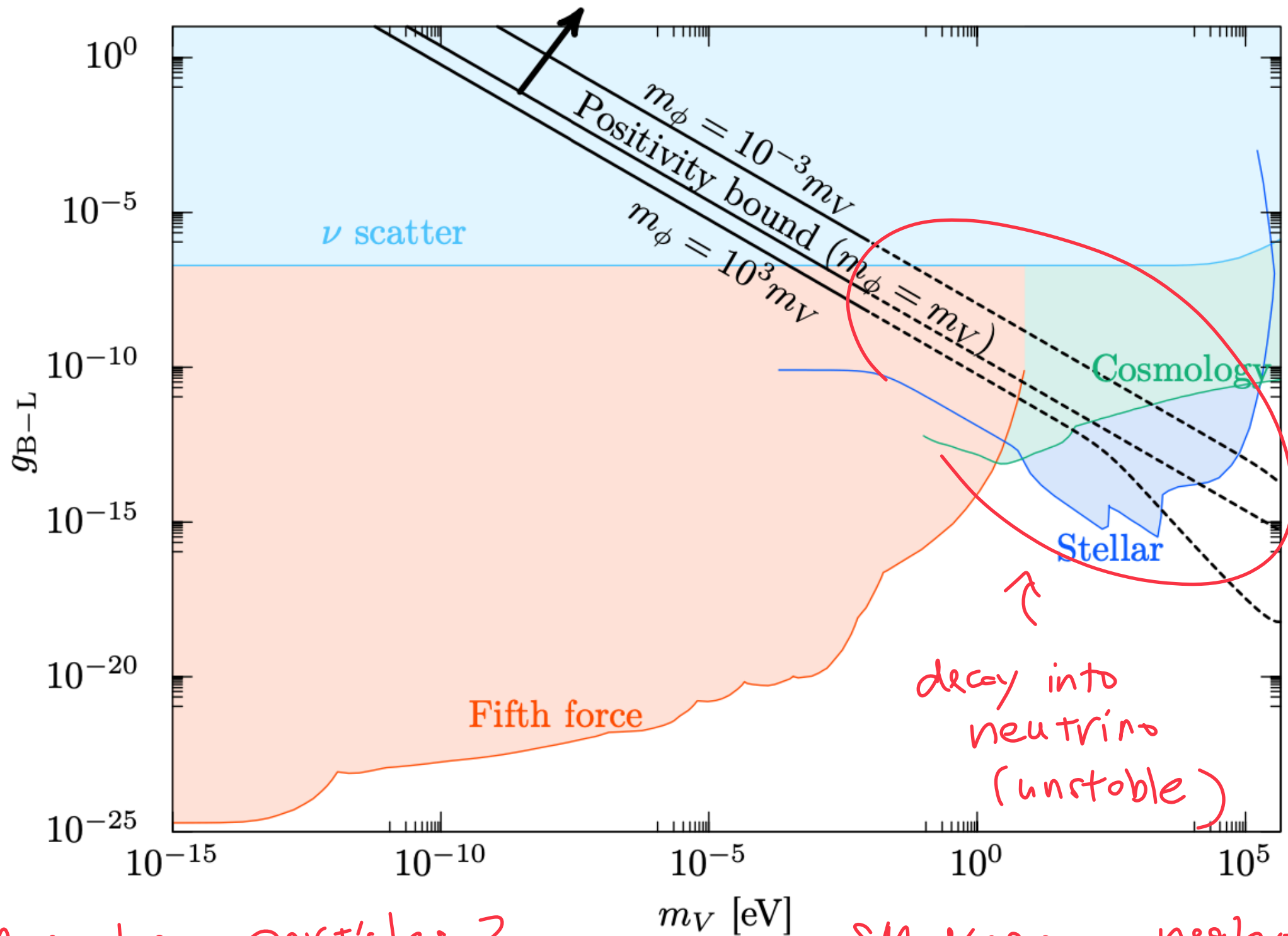
$$B_{\text{grav}} \sim - \frac{g^2}{m_\Phi^2 M_{\text{pl}}^2}$$

$U(1)$  charge of  $\Phi$

$g_\Phi$



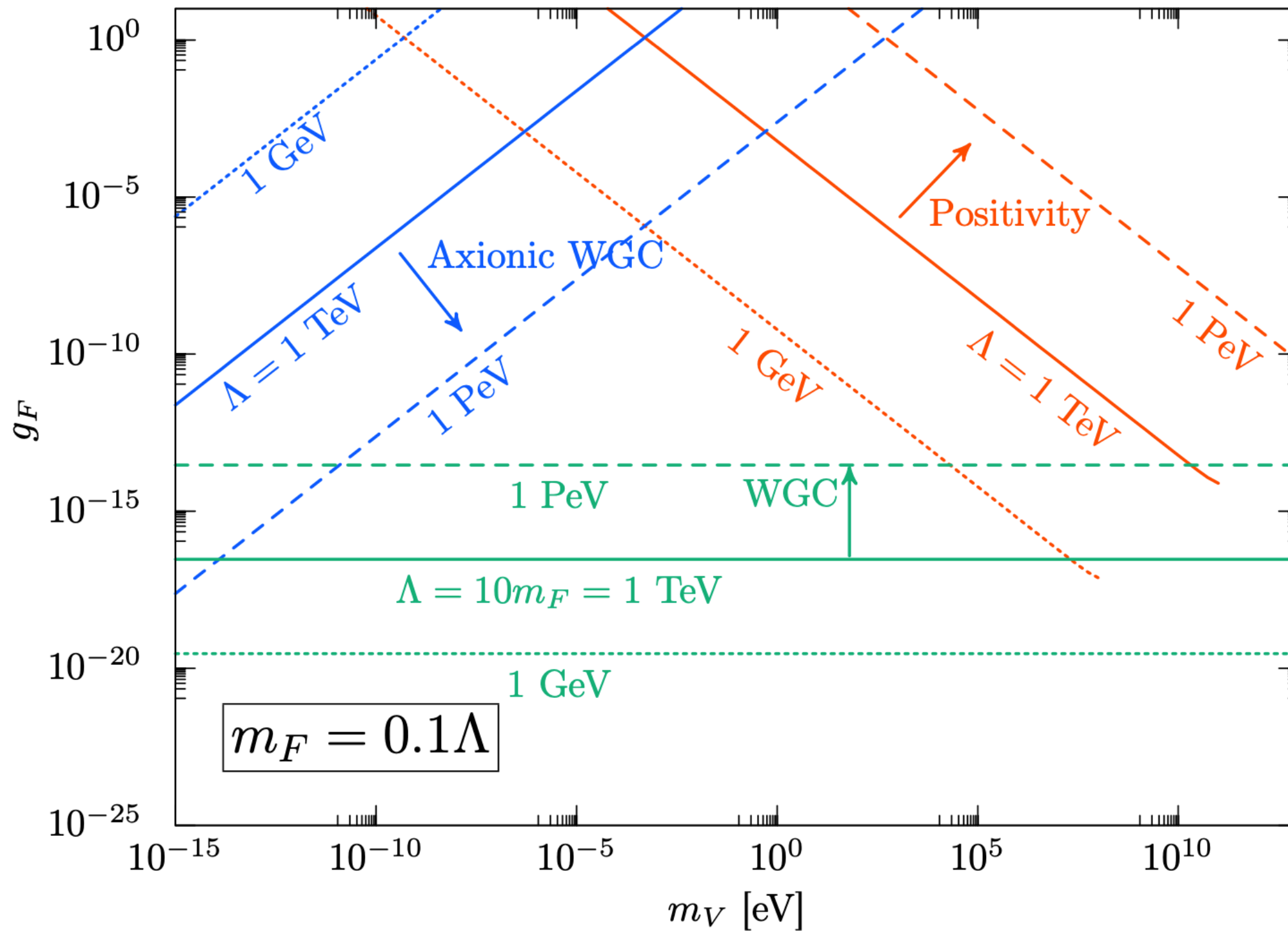
# Realistic Cases: Theoretically Subtle



massless particles?

SM diagrams neglected

Example: Stückelberg  $U(1)$





# Summary

( Gravity  
UV completion )



practical  
recipes

( lower bound  
on dark sector  
couplings )

Dark side of Universe

= Quantum - Gravity side of Universe