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Matter Power Spectrum in Scalar-Tensor Theory of Gravity

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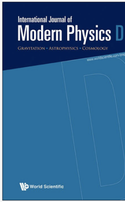

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Abstract

This work deals with the computation of the power spectrum of large-scale structure using the dynamical system approach for a multifluid universe in scalar–tensor theory of gravity. We use the $1+3$ covariant approach to obtain evolution equations and study the behavior of the matter power spectrum of perturbation equations. The study is based on the equivalence between $f(R)$ theory of gravity and scalar–tensor theory of gravity. We find that, for power-law (R^n) models, with $1 < n < 1.3$, we have the power spectrum evolving above general relativistic scale-invariant line. For $n \geq 1.3$, the power spectrum starts with constant amplitude then it experiences oscillations and eventually saturates at finite amplitude. Such behavior is consistent with other observations in the literature. The result supports the ongoing investigations of the equivalence between $f(R)$ and scalar–tensor theory at linear order.

Keywords: Dynamical systems ▪ modified gravity ▪ $f(R)$ gravity ▪ scalar–tensor models ▪ large-scale structure ▪ covariant perturbations ▪ power spectrum

History

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INTRODUCTION [GR Based Cosmology Model]

The standard big-bang cosmological model, is a model with

Success

- Formation and distribution of large scale structures
- BB nucleo synthesis
- Predicts Hubble law to hold for the entire Universe
- Predicts existence of CMB

Challenges

Early universe Problems

- The horizon problem
- The symmetry problem
- The flatness problem
- The inhomogeneity and anisotropy on small scales

Late universe problems

- The current cosmic acceleration
- The rotational curves of galaxies.

INTRODUCTION [Isotropic and homogeneous Universe]

- From Hilbert-Einstein action

$$I_{HE} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,(R + \mathcal{L}_m)\,, \tag{1}$$

where $\kappa = \frac{8\pi G}{c^4} = 1$, G being the gravitational constant and c is speed of light in vacuum.

- We make variation w.r.t metric $g_{\mu\nu}$ and get the field equations as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}\,, \tag{2}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy momentum tensor, $R_{\mu\nu}$ is the Ricci tensor and R is the Ricci scalar.

- The line element (general metric) is defined as

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}\,, \tag{3}$$

where $g_{\mu\nu}$ is spacetime metric.

- FLRW model is described by

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]\,, \tag{4}$$

where $a(t)$ is the scale factor, (t, r, θ, φ) are the spacetime coordinates of the Universe and K is the curvature constant which can be $-1, 0, 1$ for open, flat and closed spacetimes respectively.

INTRODUCTION
[Some quantities from the FLRW spacetime]

Applying symmetry property of FLRW spacetime, one has

The Ricci scalar R is given as

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right) . \tag{5}$$

Friedmann's equation

$$\frac{\dot{a}^2}{a^2} = \frac{\mu}{3} - \frac{K}{a^2} . \tag{6}$$

Raychadhuri's equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\mu + p) . \tag{7}$$

ALTERNATIVES TO GR
[Why modified gravity?]

From the astrophysical and cosmological point of view:

- To explain phenomena such as dark energy and dark matter from a **geometric** point of view, or **extra matter fields**.
- To explore the possibility that gravitational interaction depends on scales (cosmological scale or astrophysical scale).
- Using the extra degree of freedom manifested in those theories.

Changing geometry >>>>>> $f(R)$ theory.

Adding new form of matter >>>>>>>>> scalar-tensor theory.

Actions for $f(R)$ gravity and Scalar-Tensor Theory

- The action that represents $f(R)$ gravity is

$$I_{f(R)} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}[f(R) + \mathcal{L}_m]. \tag{8}$$

- The action in scalar-tensor (Brans-Dicke) theory

$$I_{BD} = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi}\nabla_\mu\phi\nabla^\mu\phi + \mathcal{L}_m\right], \tag{9}$$

the coupling parameter can be $\omega = 0$.

$$I_{f(\phi)} = \frac{1}{2\kappa}\int d^4\sqrt{-g}\left[f(\phi)+\mathcal{L}_m\right], \tag{10}$$

where $f(\phi)$ is the function of $\phi(R)$ and we consider the scalar field ϕ to be Frolov (2008)

$$\phi = f' - 1. \tag{11}$$

However ϕ is dimensionless.
$\phi = 0$ for GR.
ϕ is an extra degree of freedom.

Other versions of definition of ϕ by Thomas P Sotiriou at al. (2010):

$$\phi = f' \,. \tag{12}$$

There is yet another definition by Baojiu Li at al. (2007):

$$\phi = \ln f' \,, \tag{13}$$

for GR, we have $\phi = 0$.

- The equivalence of the two theories is covered in (Barrow et al. (1988), Baojiu Li et al. (2007), Frolov (2008), Thomas P Sotiriou et al. (2010)).
- For example in the work done by Thomas P Sotiriou et al. (2006), this equivalence was studied at action level.
- The work done by Castaneda et al. (2018) covered this equivalence at perturbation level using metric approach and $\phi = f'$ is used.
- **We need to study this equivalence of $f(R)$ with scalar-tensor theory using eq. (11), for which one has $\phi = f' - 1$.**

1+3 Covariant description

The 1+3 covariant approach is based on the decomposition of space-time in:

- Foliated hyper-surfaces of constant curvature (or constant scalar field).
- And the perpendicular worldline.
- This is the decomposition of the metric tensor into the projector tensor and a 4-velocity field u^a:

$$u^a = \frac{dx^a}{d\tau}, \tag{14}$$

where τ is the proper time, and

$$h_{ab} = g_{ab} + u_a u_b, \tag{15}$$

where h_{ab} is the projection tensor.

Gradient variables

Fluctuations in	Vector	Scalar	Harmonic
Energy density	$D^m_a = \frac{a\tilde{\nabla}_a \mu_m}{\mu_m}$	$\Delta_m = a\tilde{\nabla}^a D^m_a$	Δ^k_m
Expansion	$Z_a = a\tilde{\nabla}_a \Theta$	$Z = a\tilde{\nabla}^a Z_a$	Z^k
Scalar field	$\Phi_a = a\tilde{\nabla}_a \phi$	$\Phi = a\tilde{\nabla}^a \Phi_a$	Φ^k
Momentum of scalar field	$\Psi_a = a\tilde{\nabla}_a \dot{\phi}$	$\Psi = a\tilde{\nabla}^a \Psi_a$	Ψ^k

For the covariant Laplace-Beltrami operator, we have

$$\tilde{\nabla}^2 Q = -\frac{k^2}{a^2} Q \,, \tag{16}$$

and the order of harmonic (wavenumber) k is

$$k = \frac{2\pi a}{\lambda} \,. \tag{17}$$

DYNAMICAL SYSTEMS
[Dimensionless variables]

Friedmann equation (Sante Carloni et al., 2005):

$$1-\frac{R}{6H^2}+\frac{f}{6f'H^2}+\frac{\dot{R}f''}{f'H}-\frac{\mu_r}{3f'H^2}-\frac{\mu_d}{3f'H^2}=0\;. \tag{18}$$

Dimensionless parameters:

$$x=\frac{\dot{R}f''}{f'H}=\frac{\dot{\phi}(n-1)}{6n\phi' H^2}\;, \tag{19}$$

$$y=\frac{f}{6f'H^2}-\frac{R}{6H^2}=\frac{(1-n)\left(\frac{\phi+1}{n\beta}\right)^{1/(n-1)}}{6nH^2}\;, \tag{20}$$

$$\Omega_r=\frac{\mu_r}{3f'H^2}=\frac{\mu_r}{3H^2(\phi+1)}\;, \tag{21}$$

and

$$\Omega_d=\frac{\mu_d}{3f'H^2}=\frac{\mu_d}{3H^2(\phi+1)}\;. \tag{22}$$

So that we have $1+y+x-\Omega_d-\Omega_r=0$.

The evolution of the dimensionless variables are as follows (Amare et al. 2023)

$$-(1+z)\frac{dx}{dz} = -x - x^2 + \frac{(4-2n+nx)y}{n-1} + \Omega_d \ , \tag{23}$$

$$-(1+z)\frac{dy}{dz} = 4y + \frac{(x+2ny)y}{n-1} \ , \tag{24}$$

$$-(1+z)\frac{d\Omega_d}{dz} = \left(1 - x + \frac{2ny}{n-1}\right)\Omega_d \ , \tag{25}$$

$$-(1+z)\frac{d\Omega_r}{dz} = \left(- x + \frac{2ny}{n-1}\right)\Omega_r \ , \tag{26}$$

$$(1+z)\frac{dh}{dz} = \frac{h(2+ny)}{(n-1)} \ . \tag{27}$$

In the following equations, the prime indicates derivative with respect to z

$$\Delta_d^{''k} + \left[\frac{(2+ny)}{(1+z)(n-1)} + \frac{x-1}{(1+z)}\right]\Delta_d^{'k} - \frac{3\Omega_d}{(1+z)^2}\Delta_d^k - \frac{3(1-n)^{n-1}}{(1+z)\beta n(6nh^2y)^{n-1}}\Phi^{'k}$$
$$+\frac{1}{(1+z)^2}\left[\frac{(1-n)^{n-2}}{2h^2\beta n(n-1)(6nh^2y)^{n-2}} + \frac{3(1-n)^{n-1}}{\beta n(6nh^2y)^{n-1}}\left(\Omega_d - x - \frac{y}{n-1}\right)\right]\Phi^k = 0\ , \tag{28}$$

$$\Phi^{''k} + \left[\frac{(2+ny)}{(1+z)(n-1)} - \frac{2}{(1+z)} - \frac{x(n-2)}{(n-1)((1+z))}\right]\Phi^{'k} - \frac{(n-2)}{(1+z)^2}\left[-\frac{\Omega_d}{n-1} + \frac{4(1-q)}{3n(n-1)}\right.$$
$$\left. + \frac{(n^2-3n-4)x^2}{(n-1)^2} + \frac{3x}{(n-1)} - (n-2)x^2 - \frac{\hat{k}^2}{(n-2)}\right]\Phi^k - \frac{\Omega_d\beta n(6nh^2y)^{n-1}}{(1+z)^2(1-n)^{n-1}}\Delta_d^k \tag{29}$$
$$-\frac{\beta x n(6nh^2y)^{n-1}}{(1+z)(1-n)^{n-1}}\Delta_d^{'k} = 0\ .$$

In GR limit, we have (Amare et al., 2013)

$$\Delta_d^{''k} - \frac{1}{1+z}\Delta_d^{'k} - \frac{3\Omega_d}{(1+z)^2}\Delta_d^k = 0\ . \tag{30}$$

DYNAMICAL SYSTEMS
[Transfer function $T(k)$]

- Transfer function $T(k)$

$$T(k) = \left|\Delta_m^k\right|^2 . \tag{31}$$

- One can write power spectrum $P(k)$ given as

$$P(k) = \frac{P_k^{f(\phi)}}{P_k^{\Lambda CDM}\Big|_{eq}} = T(k) = \left|\Delta_d^k\right|^2 . \tag{32}$$

DYNAMICAL SYSTEMS
[Multifluid system, dust-dominated epoch]

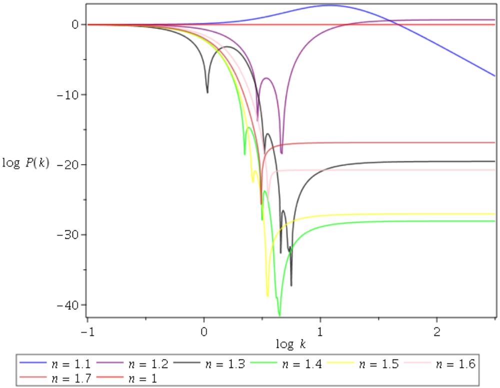

Figure 1: Power spectrum for $n = 1$, $n = 1.1$, $n = 1.2$, $n = 1.3$, $n = 1.4$, $n = 1.5$, $n = 1.6$, $n = 1.7$.

- For $n = 1$, we have the one for GR.
- For $n = 1.1$, the spectrum is above the invariant line before it turns down.
- For $n = 1.4$, the spectrum is the lowest. We need to investigate what is happening here.

[Multifluid system: dust-dominated epoch, around $n = 1.4$]

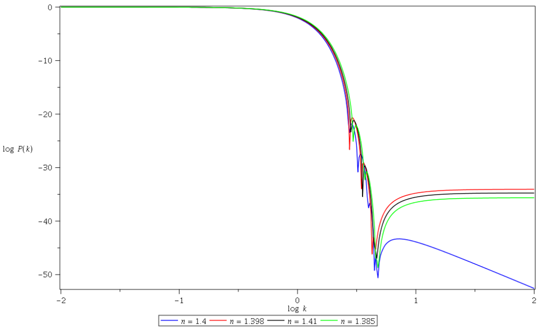

Figure 2: Power spectrum for $n = 1.41$, $n = 1.4$, $n = 1.385$, $n = 1.398$.

- Values of n near $n = 1.4$ are oscillating in the similar manner and saturate at close amplitude.
- But $n = 1.4$, after oscillation it keeps going down.
- This result is the same as the one obtained by Kishore et al. (2008) and Amare et al. (2013).

CONCLUSIONS

New gradient variables are used to develop perturbation equations

Dynamical systems approach is used to analyze power spectrum

- Dust dominated epoch is considered.
- We paid the attention to $n = 1.4$ since it is clear from the power spectrum that the values close to $n = 1.4$ are behaving similarly.
- It is shown that the power spectrum of $n = 1.4$ is lower than others for large values of k in all sets of initial conditions.

From the power spectrum behavior, the equivalence between $f(R)$ theory and scalar-tensor theory also applies to perturbation level , Since the other studies about $f(R)$ also produce similar power spectrum.

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THANK YOU
FOR
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