Cosmological constraints of diffusive dark-fluid models

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14 July 2023

Introduction: tensions in standard cosmology

- Recent cosmological observations have shown that the universe is currently undergoing accelerated expansion
- Not conclusively known what caused this acceleration, the prevailing argument being that dark energy caused it
- Among the most widely considered candidates of dark energy is the vacuum energy of the cosmological constant Λ
- Some serious problems (tensions)
  - ✓ Cosmological Constant Problem [1](vacuum catastrophe): measured energy density of the vacuum over 120 orders of magnitude less than the theoretical prediction
    - Worst prediction in the history of physics (and of science in general)
    - Casts doubt on dark energy being a cosmological constant
  - ✓ Cosmic Coincidence Problem [2]: dark matter and dark energy densities have the same order of magnitude at the present moment of cosmic history, while differing with many orders of magnitude in the past and the predicted future

[1] Weinberg, S. The cosmological constant problem. Rev. Mod. Phys. 1989, 61 (1), 1

[2] Velten, H. E. et al. Aspects of the cosmological “coincidence problem". Eur. Phys. J. C 2014, 74 (11), 1

Tensions...

- Latest tensions vis-à-vis precise theoretical predictions and observational measurements:
  - ✓ H_0 CMB vs local measurements, more than 3σ discrepancy
    - Planck2018, ΛCDM model
$$H_0 = 67.27 \pm 0.60\ km/s/Mpc$$
    - Estimate using SNIa measurements (2016)
$$H_0 = 73.24 \pm 1.74\ km/s/Mpc$$
    - Parallax measurements of Milky Way Cepheids (2018)
$$H_0 = 73.48 \pm 1.66\ km/s/Mpc$$
  - ✓ S_8 vs cosmic shear data, more than 2.5σ discrepancy between Planck data and local measurements of
$$S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$$
    where σ_8 measures the amplitude of the linear power spectrum on the $8h^{-1}$Mpc scale
  - ✓ Ω_K, zero or not zero? ΛCDM assumes flat universe, but Planck temperature and polarisation power spectra give an above 3σ deviation:
$$\Omega_K \approx -0.044^{+0.018}_{-0.015}$$
- Several alternatives proposed, such as:
  - ✓ Interacting vacuum, $\Lambda = \Lambda(t)$
  - ✓ Interacting dark matter and dark energy → non-gravitational interactions
- Of particular interest for us here are those interacting models exchanging energy while the dark matter and dark energy components are not separately conserved

- The standard ΛCDM cosmology is a solution of the Einstein field equations (EFEs) derived from the action

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g}\,[R + 2\,(L_m - \Lambda)]$$

where R, L_m and Λ are the Ricci scalar, the matter Lagrangian density and the cosmological constant, respectively. The corresponding EFEs read:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

with the first (geometric) term represented by the Einstein tensor, and the RHS of the equation representing the total energy-momentum tensor (EMT) of matter fluid forms.

- Both $G_{\mu\nu}$ and $T_{\mu\nu}$ are covariantly conserved quantities. The EMT for perfect-fluid models is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

where ρ and p are the energy density and isotropic pressure of matter, respectively, often related by the barotropic equation of state (EoS) $p = w\rho$ for a constant EoS parameter w. The normalised vector u_α represents the four-velocity of fundamental observers comoving with the fluid

- The divergence-free EMT leads to the fluid conservation equation

$$T^{\mu\nu}{}_{;\mu} = 0 \implies \dot{\rho} + 3\frac{\dot{a}}{a}(1+w)\rho = 0$$

  where $a(t)$ is the cosmological scale factor whose evolution is given by the Friedmann equation

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}$$

  where k is the normalised spatial curvature parameter with values $-1\,,0\,,1$ depending on an open, flat or closed spatial geometry.

- In a multi-component fluid system, it is usually assumed that the energy density of each perfect-fluid component is assumed to evolve independently of the other fluids of the system:

$$\dot{\rho}_i + 3H(1+w)\rho_i = 0$$

  where here, we have introduced the Hubble parameter $H \equiv \frac{\dot{a}}{a}$ and in this case the total EMT is the algebraic sum of the EMTs of each fluid, so are the total energy density and total pressure terms are the algebraic sums of the individual components

- If we relax this assumption (of conservation) due to the presence of interactions such as diffusion, the individual components do not obey the matter conservation equation, but the total fluid still does.

- Diffusive fluids: non-conservation equation for the ith component fluid:

$$T_i^{\mu\nu}{}_{;\mu} = N_i^{\nu}$$

  where N_i^{ν} corresponds to the current of diffusion term for that fluid

Some "general" interaction models

- For more general interactions

$$\dot{\rho}_{\rm dm} + 3H\rho_{\rm dm} = Q \qquad ; \qquad \dot{\rho}_{\rm de} + 3H\rho_{\rm de}(1+\omega) = -Q$$

  where Q is the rate of energy exchange, which defines the direction of energy flow between the dark sectors such that:

$$Q = \begin{cases} > 0 & \text{dark energy} \rightarrow \text{dark matter} \\ < 0 & \text{dark datter} \rightarrow \text{dark mnergy} \\ = 0 & \text{No interaction } (\Lambda\text{CDM case}) \end{cases}$$

- Model 1: $Q_1 = \delta H \rho_{\rm dm}$

$$\rho_{\rm dm} = \rho_{({\rm dm},0)}\, a^{(\delta-3)}$$

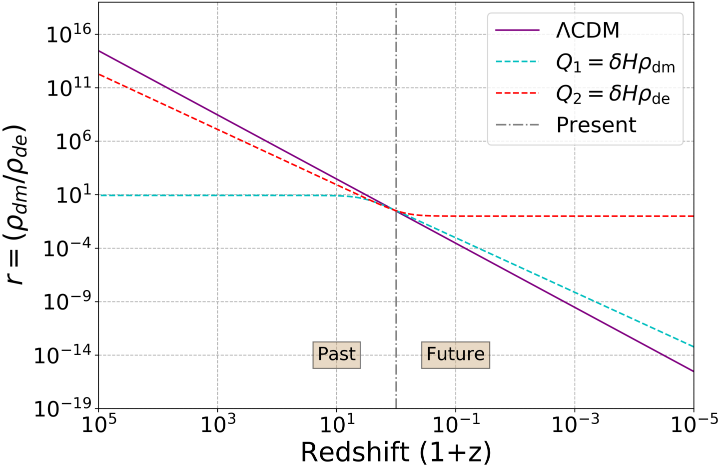
$$\rho_{\rm de} = \rho_{({\rm de},0)}\, a^{-3(1+\omega_{\rm de})} + \rho_{({\rm dm},0)} \frac{\delta}{\delta+3\omega} \left[a^{-3\omega} - a^{\delta}\right] a^{-3}$$

- Model 2: $Q_2 = \delta H \rho_{\rm de}$

$$\rho_{\rm dm} = \rho_{({\rm dm},0)}\, a^{-3} + \rho_{({\rm de},0)} \frac{\delta}{\delta+3\omega} \left[1 - a^{-(\delta+3\omega)}\right] a^{-3}$$

$$\rho_{\rm de} = \rho_{({\rm de},0)}\, a^{-(\delta+3\omega+3)}$$

The cosmic coincidence problem may now be addressed [3] by considering how the ratio of dark matter to dark energy $r = \rho_{\rm dm}/\rho_{\rm de}$ evolves with redshift z. Here it can clearly be seen that for the ΛCDM case, the current value of $r_0 \approx \frac{3}{7}$ seems fine-tuned and coincidental in comparison to Q_1 and Q_2, where r converges and becomes constant in the past and the future respectively. Thus, alleviating the cosmic coincidence problem



Cosmic Coincidence Problem

[3] M van der Westhuizen, AA (2023), Interacting dark energy: clarifying the cosmological implications and viability conditions, preprint arXiv:2302.11949

Event	$Q > 0$	$Q < 0$
Energy flow	DE $\rightarrow$ DM (iDEDM)	DM $\rightarrow$ DE (iDMDE)
Effective equations of state	$\omega_{\rm dm}^{\rm eff} < \omega_{\rm dm}$; $\omega_{\rm de}^{\rm eff} > \omega_{\rm de}$	$\omega_{\rm dm}^{\rm eff} > \omega_{\rm dm}$; $\omega_{\rm de}^{\rm eff} < \omega_{\rm de}$
Coincidence problem	Alleviates ($\zeta_{\rm IDE} < \zeta_{\Lambda{\rm CDM}}$)	Worsens ($\zeta_{\rm IDE} > \zeta_{\Lambda{\rm CDM}}$)
Hubble tension	Worsens	Alleviates
S_8 discrepancy	Alleviates	Worsens
Age of universe	Older	Younger
Radiation-matter equality	Later ($z_{\rm IDE} < z_{\Lambda{\rm CDM}}$)	Earlier ($z_{\rm IDE} > z_{\Lambda{\rm CDM}}$)
Cosmic jerk	Earlier ($z_{\rm IDE} > z_{\Lambda{\rm CDM}}$)	Later ($z_{\rm IDE} < z_{\Lambda{\rm CDM}}$)
Matter-dark energy equality	Earlier ($z_{\rm IDE} > z_{\Lambda{\rm CDM}}$)	Later ($z_{\rm IDE} < z_{\Lambda{\rm CDM}}$)

Consequences of interacting dark energy models (relative to uncoupled models)

$$\omega_{\rm dm}^{\rm eff} \equiv -\frac{Q}{3H\rho_{\rm dm}} \qquad \omega_{\rm de}^{\rm eff} \equiv \omega_{\rm de} + \frac{Q}{3H\rho_{\rm de}}$$

$$r \equiv \frac{\rho_{\rm dm}}{\rho_{\rm de}} = \frac{\rho_{({\rm dm},0)}\, a^{-3(1+\omega_{\rm dm}^{\rm eff})}}{\rho_{({\rm de},0)}\, a^{-3(1+\omega_{\rm de}^{\rm eff})}} = r_0\, a^{-\zeta_{\rm IDE}} \qquad \zeta_{\rm IDE} \equiv 3\left(\omega_{\rm dm}^{\rm eff} - \omega_{\rm de}^{\rm eff}\right)$$

- Ansatz: let's write the non-conservation equation for the fluid as [4]:

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(1+w)\rho_i = \frac{\gamma_i}{a^3}$$

where γ_i is a constant for that fluid such that $\sum_i \gamma_i = 0$

- Integrating the above equation gives

$$\rho_i = a^{-3(1+w_i)}\left[\rho_{i0} + \gamma_i \int_{t_0}^{t} a^{3w_i}\, dt'\right]$$

where ρ_{i0} is the present-day ($t = t_0$) value of the energy density of the ith fluid

- Using a late-time, i.e., $t - t_0 \ll t_0$, expansion and expressing $a(t) = a_0\left[1 - (t_0 - t)H_0) + \ldots\right]$, we can write [5] the last term of the above integrand as

$$\int_{t_0}^{t} a^{3w_i}\, dt = \int_{t_0}^{t} a^{3w_i}\left[1 - (t_0 - t)H_0) + \ldots\right]^{3w_i} dt'$$

[4] Maity, S., Bhandari, P., & Chakraborty, S. (2019). Universe consisting of diffusive dark fluids: thermodynamics and stability analysis. The European Physical Journal C, 79(1), 1-8.

[5] RR Mekuria, AA (2023), Observational constraints of diffusive dark-fluid cosmology, preprint arXiv:2301.02913

- Evaluating the previous integral and applying Taylor expansion around t_0 yields

$$\begin{aligned}
\int_{t_0}^{t} a^{3w_i}\,dt &= -\frac{1}{1+3w_i}\left[(1+(t_0-t)H_0)^{1+3w_i} - (1+(t_0-t)H_0)^{1+3w_i} + \dots\right] \\
&\approx \frac{1}{1+3w_i}\left[1-(1+(t_0-t)H_0)^{1+3w_i}\right] \\
&= \frac{1}{(1+3w_i)H_0}\left[1-(2-a)^{1+3w_i}\right]
\end{aligned}$$

  where in the last step, we have normalised the scale factor to unity today: $a_0 = 1$
- The energy density of each diffusive fluid component is then:

$$\rho_i = a^{-3(1+w_i)}\left\{\rho_{i0} + \frac{\gamma_i}{(1+3w_i)H_0}\left[1-(2-a)^{1+3w_i}\right]\right\}$$

- Assuming the well-known component of radiation, dust-like matter (baryons and dark matter) and vacuum energy, the above diffusive solution leads to:

$$\begin{aligned}
\rho_{\mathrm{r}} &= a^{-4}\left\{\rho_{\mathrm{r}0} + \frac{\gamma_{\mathrm{r}}}{2H_0}\left[1-(2-a)^2\right]\right\} \\
\rho_{\mathrm{m}} &= a^{-3}\left\{\rho_{\mathrm{m}0} + \frac{\gamma_{\mathrm{m}}}{H_0}\left[1-(2-a)\right]\right\} \\
\rho_{\Lambda} &= \rho_{\Lambda 0} - \frac{\gamma_{\Lambda}}{2H_0}\left[1-(2-a)^{-2}\right]
\end{aligned}$$

- Let's consider the Friedmann equation for the ΛCDM model for $k = 0$:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left\{\rho_{\mathrm{r}0}\, a^{-4} + \rho_{\mathrm{m}0}\, a^{-3} + \frac{\gamma_{\mathrm{m}}}{H_0}\left[1-(2-a)\right] a^{-3} + \rho_{\Lambda 0} - \frac{\gamma_\Lambda}{2H_0}\left[1-(2-a)^{-2}\right]\right\}$$

- We assume the diffusive interaction is limited between dark matter and dark energy, i.e., $\gamma_{\mathrm{r}} = 0$, and introduce the following dimensionless quantities:

$$\Omega_{\mathrm{i}} \equiv \frac{8\pi G}{3H_0^2}\rho_{\mathrm{i}}\ , \qquad \Delta_{\mathrm{i}} \equiv \frac{8\pi G}{3H_0^3}\gamma_{\mathrm{i}}\ , \qquad 1+z \equiv a^{-1}\ , \qquad h \equiv \frac{H}{H_0}$$

- We can then show that the Friedmann equation can be recast as

$$h^2 = \Omega_{\mathrm{r}0}(1+z)^4 + \Omega_{\mathrm{m}0}(1+z)^3 + \Omega_{\Lambda 0} - \Delta_{\mathrm{m}} z(1+z)^2 - \Delta_\Lambda\left[\frac{1}{2} - \frac{1}{2}\left(\frac{1+2z}{1+z}\right)^{-2}\right]$$

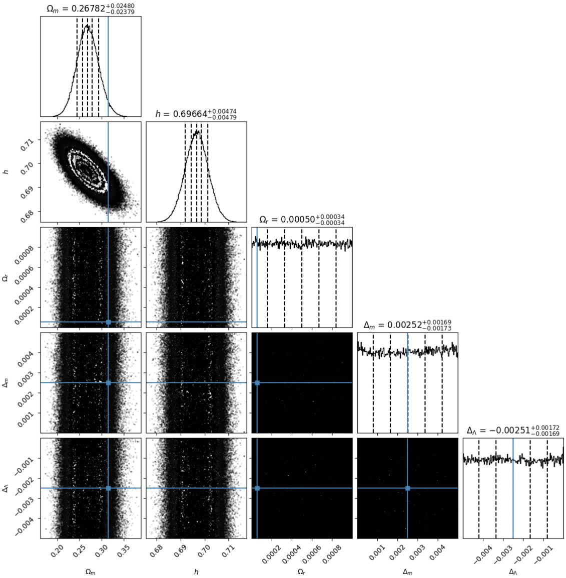
- Moreover, defining the deceleration parameter as

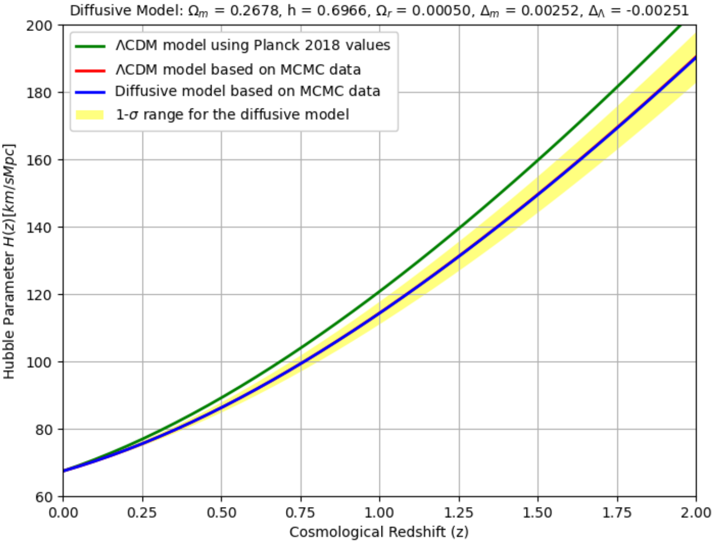
$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{4\pi G}{3H^2}\sum_i \rho_{\mathrm{i}}(1+3w_{\mathrm{i}})$$

we can show that for our current model, we have

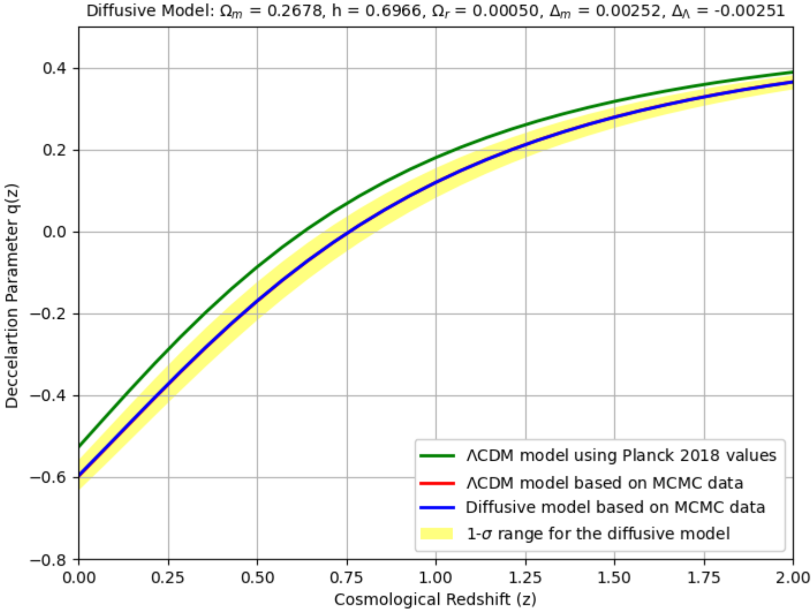
$$q = \frac{1}{2}\left\{\frac{2\Omega_{\mathrm{r}0}(1+z)^4 + \Omega_{\mathrm{m}0}(1+z)^3 - 2\Omega_{\Lambda 0} - \Delta_{\mathrm{m}} z(1+z)^2 + \Delta_\Lambda\left[1 - \left(\frac{1+2z}{1+z}\right)^{-2}\right]}{\Omega_{\mathrm{r}0}(1+z)^4 + \Omega_{\mathrm{m}0}(1+z)^3 + \Omega_{\Lambda 0} - \Delta_{\mathrm{m}} z(1+z)^2 - \Delta_\Lambda\left[\frac{1}{2} - \frac{1}{2}\left(\frac{1+2z}{1+z}\right)^{-2}\right]}\right\}$$

Case I: $\Delta_{\mathrm{m}} \sim 10^{-3}$



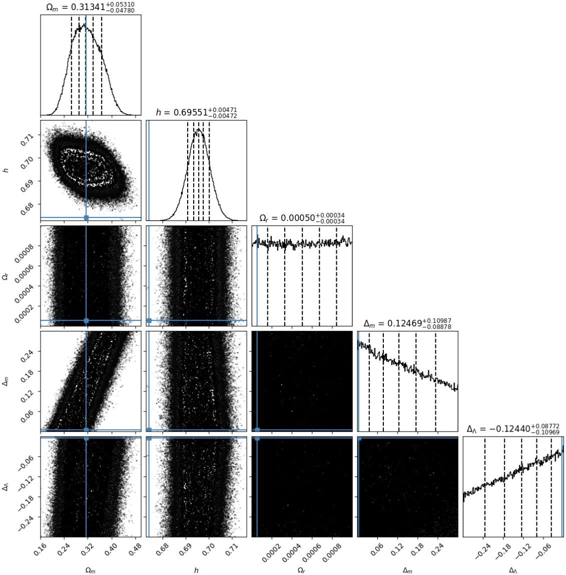


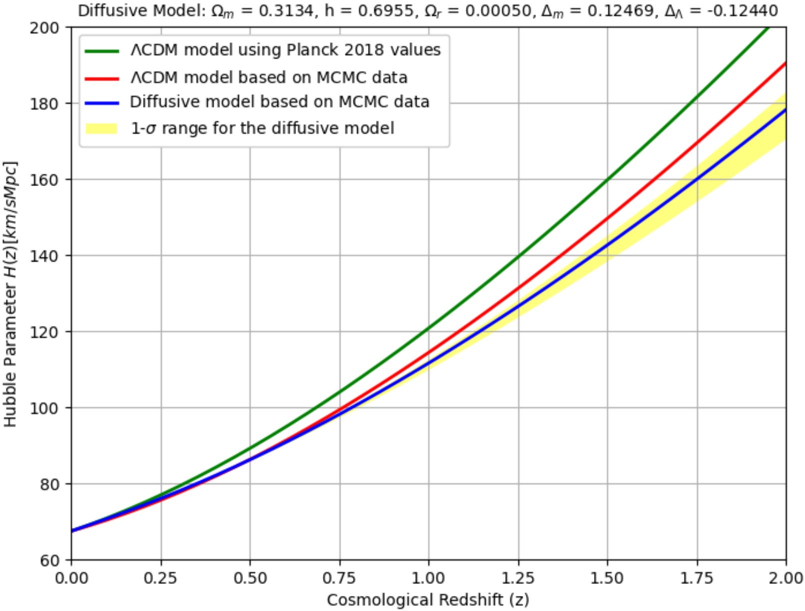
Diffusive Model: Ω_m = 0.2678, h = 0.6966, Ω_r = 0.00050, Δ_m = 0.00252, Δ_Λ = -0.00251
ΛCDM model using Planck 2018 values
ΛCDM model based on MCMC data
Diffusive model based on MCMC data
1-σ range for the diffusive model
Hubble Parameter $H(z)[km/sMpc]$
Cosmological Redshift (z)
200
180
160
140
120
100
80
60
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00



Diffusive Model: Ωm = 0.2678, h = 0.6966, Ωr = 0.00050, Δm = 0.00252, ΔΛ = -0.00251
Decelartion Parameter q(z)
Cosmological Redshift (z)
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
ΛCDM model using Planck 2018 values
ΛCDM model based on MCMC data
Diffusive model based on MCMC data
1-σ range for the diffusive model

Case II: $\Delta_{\mathrm{m}} \sim 10^{-1}$





Diffusive Model: Ωm = 0.3134, h = 0.6955, Ωr = 0.00050, Δm = 0.12469, ΔΛ = -0.12440
ΛCDM model using Planck 2018 values
ΛCDM model based on MCMC data
Diffusive model based on MCMC data
1-σ range for the diffusive model
Hubble Parameter H(z)[km/sMpc]
Cosmological Redshift (z)
200
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160
140
120
100
80
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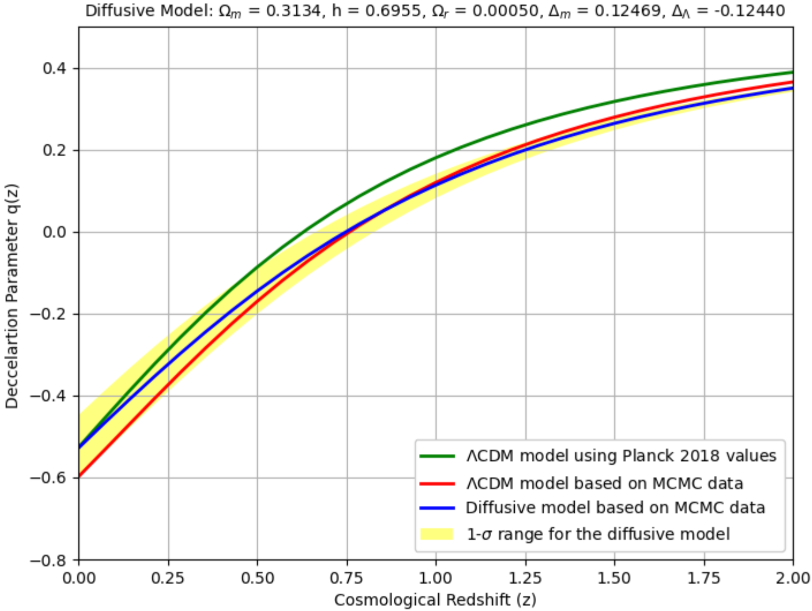
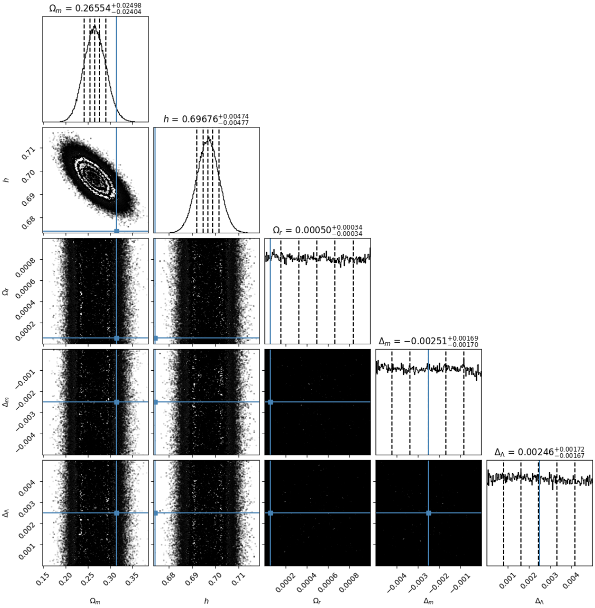
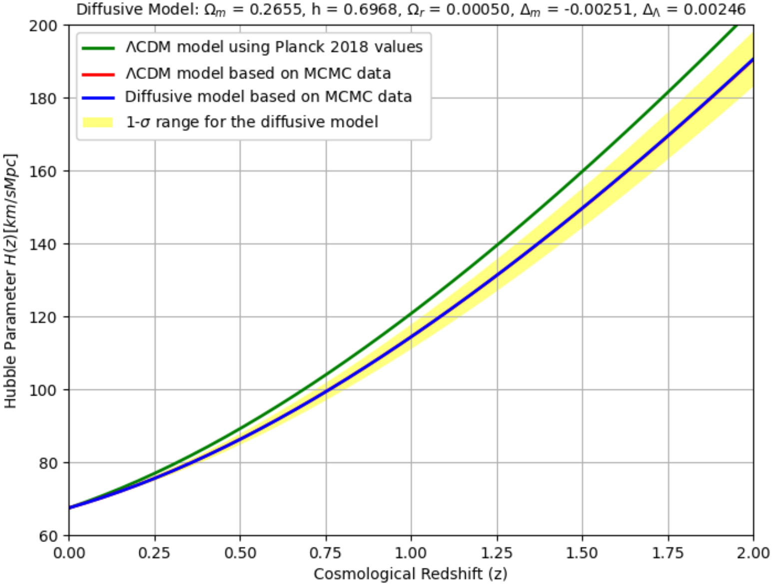
0.50
0.75
1.00
1.25
1.50
1.75
2.00



Diffusive Model: Ω_m = 0.3134, h = 0.6955, Ω_r = 0.00050, Δ_m = 0.12469, Δ_Λ = -0.12440
Deccelartion Parameter q(z)
Cosmological Redshift (z)
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
ΛCDM model using Planck 2018 values
ΛCDM model based on MCMC data
Diffusive model based on MCMC data
1-σ range for the diffusive model

Case III: $\Delta_{\mathrm{m}} \sim -10^{-3}$




Diffusive Model: Ω_m = 0.2655, h = 0.6968, Ω_r = 0.00050, Δ_m = -0.00251, Δ_Λ = 0.00246
ΛCDM model using Planck 2018 values
ΛCDM model based on MCMC data
Diffusive model based on MCMC data
1-σ range for the diffusive model
Hubble Parameter $H(z)[km/sMpc]$
Cosmological Redshift (z)
200
180
160
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120
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0.00
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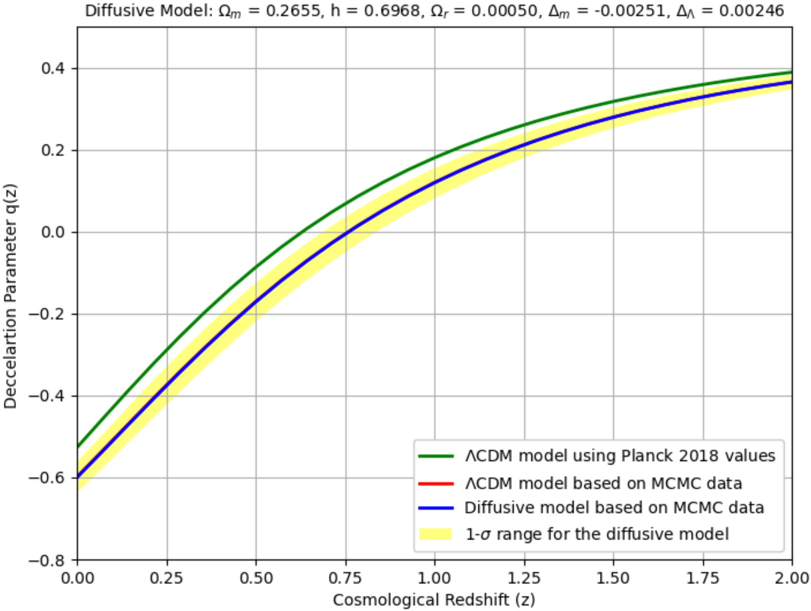
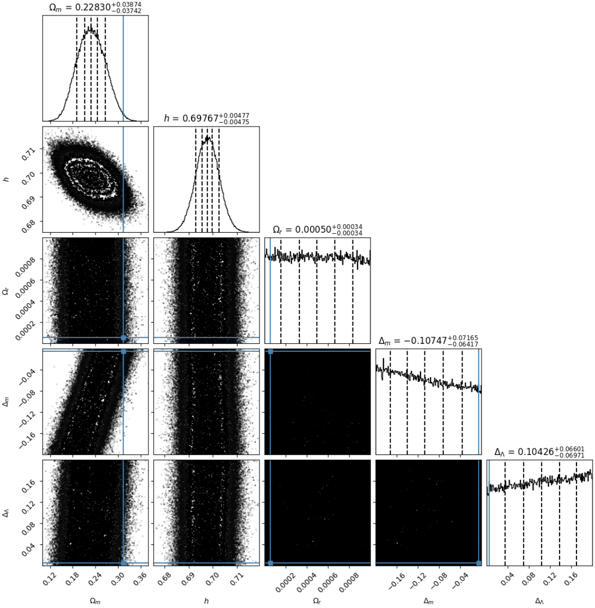
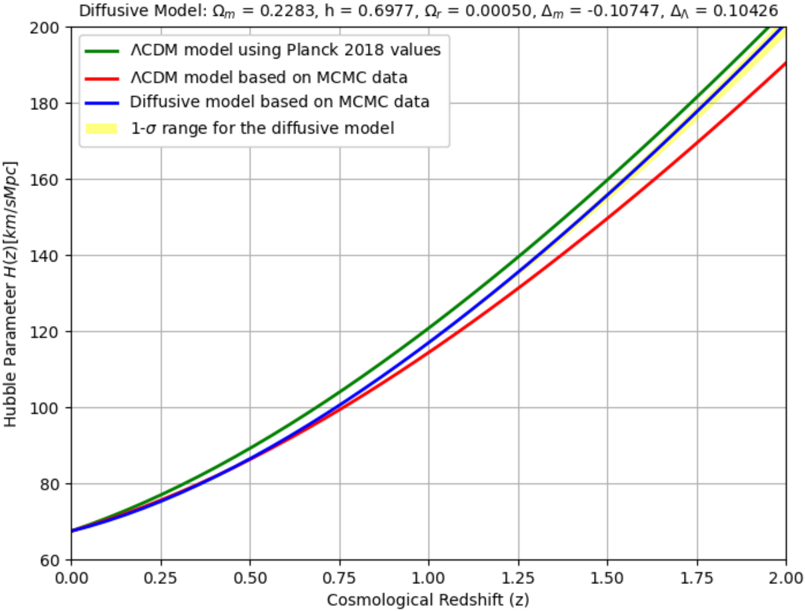
0.50
0.75
1.00
1.25
1.50
1.75
2.00

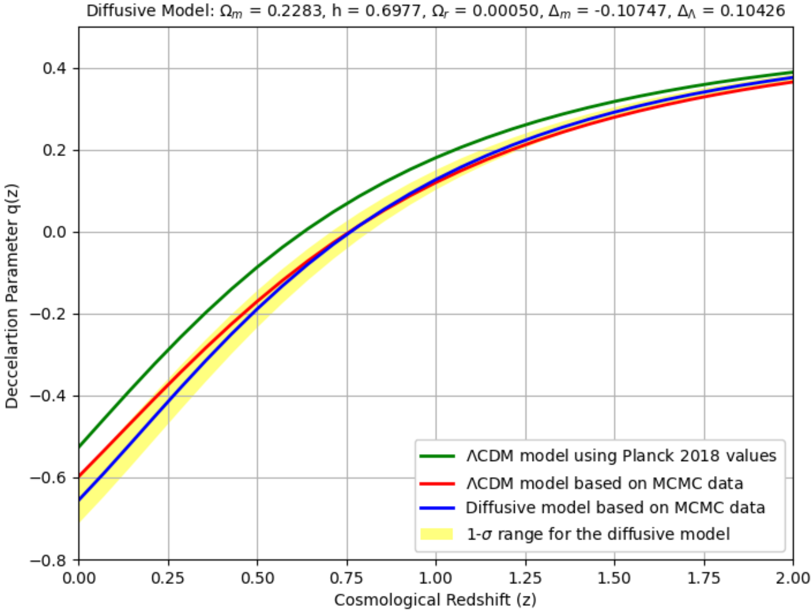


Diffusive Model: Ωm = 0.2655, h = 0.6968, Ωr = 0.00050, Δm = -0.00251, ΔΛ = 0.00246
Decceleartion Parameter q(z)
Cosmological Redshift (z)
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
ΛCDM model using Planck 2018 values
ΛCDM model based on MCMC data
Diffusive model based on MCMC data
1-σ range for the diffusive model

Case IV: $\Delta_{\mathrm{m}} \sim -10^{-1}$




Diffusive Model: Ω_m = 0.2283, h = 0.6977, Ω_r = 0.00050, Δ_m = -0.10747, Δ_Λ = 0.10426
ΛCDM model using Planck 2018 values
ΛCDM model based on MCMC data
Diffusive model based on MCMC data
1-σ range for the diffusive model
Hubble Parameter $H(z)[km/sMpc]$
Cosmological Redshift (z)
200
180
160
140
120
100
80
60
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00



Diffusive Model: Ωm = 0.2283, h = 0.6977, Ωr = 0.00050, Δm = -0.10747, ΔΛ = 0.10426
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
Deccelartion Parameter q(z)
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Cosmological Redshift (z)
ΛCDM model using Planck 2018 values
ΛCDM model based on MCMC data
Diffusive model based on MCMC data
1-σ range for the diffusive model

Some statistical parameters describing the diffusion models

Models	Δ_{m}	Δ_{Λ}	$\boldsymbol{L}(\hat{\theta}\vert\mathrm{data})$	χ^2	Red.χ^2	AIC	$\vert\Delta AIC\vert$	BIC	$\vert\Delta BIC\vert$
Diffusive Case II	+ve	-ve	-121.1677	242.3355	0.6845	252.3355	4.9405	271.7521	12.7072
Diffusive Case I	+ve	-ve	-120.7059	241.4118	0.6819	251.4118	4.0168	270.8285	11.7835
ΛCDM	0	0	-120.6975	241.3950	0.6780	247.3950	0	259.0449	0
Diffusive Case III	-ve	+ve	-120.6890	241.3781	0.6818	251.3781	3.9831	270.7947	11.7497
Diffusive Case IV	-ve	+ve	-120.3936	240.7872	0.6801	250.7872	3.3922	270.2039	11.1589

The reduced χ^2 -values are given as an indication of the goodness of fit for a particular model. The ΛCDM model is chosen as the "true model".

Summary

- Cosmology has a long history of tensions
- Potential solutions to solve - or at least alleviate - these tensions might lie somewhere beyond the standard cosmological model based on:
  - ✓ General Relativity
  - ✓ the Copernican (Cosmological) Principle
  - ✓ Noninteracting cosmological medium
  - ✓ Perfect fluids
- Relaxing these comes at a cost of more complexity, but it might be worth the extra effort
- Interacting fluid models can be viable cosmological alternatives to ΛCDM :
  - ✓ They may be potential models to alleviate the cosmic coincidence problem by stabilising the ratio of dark matter to dark energy in both the past and future
  - ✓ These models also predict a wide range of the values for H_0, thereby showing potential as a candidate for relieving the Hubble tension
- Cases having positive values of Δ_{m} were showing the largest values of likelihood function. Based on the analysis of likelihood, goodness of fit, AIC and BIC criteria, one can identify viable models most likely to be an alternative to the ΛCDM model.
- Current work is to provide a viability test of the different cases considered, but to reject or accept any of them more work is needed
- Future directions: putting more stringent constraints on the values of the defining parameters of the model:
  - ✓ With more data and statistical analysis – using existing, latest and upcoming cosmological data
  - ✓ Studying large-scale structure power spectrum, ISW effects, and other methods