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T01

Affine Semigroups of Maximal Projective Dimension

Om Prakash Bhardwaj[1], **Kriti Goel**[2], **and Indranath Sengupta**[1]

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A submonoid of N^d is of maximal projective dimension (MPD) if the associated semigroup k-algebra has the maximum possible projective dimension. Such submonoids have a nontrivial set of pseudo-Frobenius elements. We generalize the notion of symmetric numerical semigroups, and pseudo-symmetric numerical semigroups to the case of MPD-semigroups in N^d, and give their characterizations. Under suitable conditions, we prove that these semigroups satisfy the Extended Wilf's conjecture. We also give a class of MPD semigroups in N^2, where the cardinality of the set of pseudo-Frobenius elements may not be a bounded function of its embedding dimension.

T02

Singularities of Nilpotent Type in Prime Characteristic and Their Blowups

A. Costantini[1], **K. Maddox**[2], **and L. E. Miller**[3]

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[3]*University of Arkansas*

When studying the singularities of a local ring R of prime characteristic, a well-established technique is to analyze the Frobenius actions on the local cohomology modules of R. In particular, nilpotent Frobenius actions on local cohomology modules identify various classes of F-singularities, which we refer to as F-singularities of nilpotent type. Among these, *F-nilpotent* and *weakly F-nilpotent singularities* were studied in [3, 4, 5, 6] and proved to enjoy similar properties as F-rational and Cohen-Macaulay singularities, respectively.

In this talk, I will discuss how weakly F-nilpotent singularities can be viewed as a subclass of a larger family of F-singularities, which share similar properties as generalized Cohen-Macaulay rings. Moreover, I will explain how these singularities behave under blow ups, generalizing a well-known result of Huneke [2] for Cohen-Macaulay singularities. This is part of joint work with Kyle Maddox and Lance Edward Miller [1].

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T03

Epsilon multiplicity in graded dimension two and density functions

Suprajo Das

Suppose that I is an ideal in Noetherian local ring $(R, \mathfrak{m})$ of Krull dimension d. Ulrich and Validashti have defined the *ε-multiplicity* of I to be

$$\varepsilon(I) := \limsup_{n\to\infty} \frac{l_R\left(H^0_{\mathfrak{m}}\left(R/I^n\right)\right)}{n^d/d!}.$$

The ε-multiplicity can be seen as a generalization of the classical Hilbert-Samuel multiplicity. Cutkosky showed that the '$\limsup$' in the definition of ε-multiplicity can be replaced by a limit if the local ring $(R, \mathfrak{m})$ is analytically unramified. An example due to Cutkosky et al. shows that this limit can be an irrational number even in a regular local ring. In the first half of this talk we shall describe a method to compute the epsilon multiplicity of a homogeneous ideal in a two-dimensional normal graded domain over a field. In some situations we have obtained explicit formulas. In the second half of this talk we shall restrict ourselves to homogeneous ideals in a standard graded domain over a field of positive characteristic. Inspired by Trivedi's approach to Hilbert-Kunz multiplicity via density functions, we shall introduce a density function that is related to the epsilon multiplicity by an integral formula. This talk is based on two ongoing joint projects with Dubey-Roy-Verma and Roy-Trivedi.

T04

Perturbations of ideals in local rings

Luís Duarte

University of Genova

Let I be an ideal of a Noetherian local ring R. We study how properties of the ideal change under small perturbations, that is, when I is replaced by an ideal J which is the same as I modulo a large power of the maximal ideal. In particular, assuming that R/J has the same Hilbert function as R/I, we show that the Betti numbers of R/J coincide with those of R/I. We also compare the local cohomology modules of R/J with those of R/I.

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T05

Abstract template for ...Activity Title...

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T06

THE FROBENIUS TEST EXPONENTS IN PRIME CHARACTERISTIC

Duong Thi Huong[1], **Pham Hung Quy**[2]

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Let $(R, \mathfrak{m})$ be a Noetherian local ring of prime characteristic p, I an ideal of R. The *Frobenius closure* of I is $I^F = \{x \mid x^{p^e} \in I^{[p^e]} \text{ for some } e \geq 0\}$, where $I^{[p^e]} = (r^{p^e} \mid r \in R)$ is the e-th Frobenius power of I. It is hard to compute I^F. By the Noetherianness of R there is an integer e, depending on I, such that $(I^F)^{[p^e]} = I^{[p^e]}$. We call the smallest number e satisfying the condition, the *Frobenius test exponent* of I, and denote it by $\mathrm{Fte}(I)$. The *Frobenius test exponent for parameter ideals*, denoted by $\mathrm{Fte}(R)$, is the smallest integer e such that $(\mathfrak{q}^F)^{[p^e]} = \mathfrak{q}^{[p^e]}$ for every parameter ideal $\mathfrak{q}$ of R, and $\mathrm{Fte}(R) = \infty$ if we have no such integer. In this talk, we discuss some results about the existence of an uniform bound of the Frobenius test exponents for some classes of ideals, including parameter ideals, ideals generated by filter regular sequences...We also give an upper bound of Hilbert-Samuel multiplicity of the ring in terms of the Frobenius test exponent for parameter ideals. This talk is based on joint works with Pham Hung Quy [4, 5, 6].

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T07

(Non)linearity of regularity of Tor over complete intersections in positive characteristic

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We investigate the asymptotic behavior of Castelnuovo-Mumford regularity of Ext and Tor, with respect to the homological degree, over complete intersection rings. We derive from a theorem of Gulliksen a linearity result for the regularity of Ext modules in high homological degrees. We show a similar result for Tor, under the additional hypothesis that high enough Tor modules are supported in dimension at most one. These results are characteristic independent. Finally, we analyze two non-trivial examples showing that the behavior could be pretty hectic when the latter condition is not satisfied and the base ring has positive characteristic.

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T08

Local cohomology of invariant rings

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[3]*University of Utah, USA*

Consider a finite group G acting on a polynomial ring R over a field. One of the fundamental problems of invariant theory is to establish a connection between the properties of the invariant ring R^G and the properties of the group action. However, the situation is generally better understood in the situation where the order $|G|$ of the group is invertible. In this talk, we will explore some questions on the local cohomology modules of the ring of invariants in the tricky situation where $|G|$ is not a unit.

T09

Image of Linear Derivations and Mathieu-Zhao subspaces

Sakshi Gupta, Divya Ahuja, and Surjeet Kour

Indian Institute of Technology Delhi, India

Let K be a field of characteristic zero. Zhao in [4] proposes the notion of Mathieu-Zhao subspaces, which establish a connection between the image of polynomial algebra derivations and the Jacobian Conjecture. The latter is a well-known, yet unsolved problem in polynomial algebra that states the invertibility of a polynomial map from K^n to K^n with a non-zero constant Jacobian determinant [1].

Recently in 2018, Zhao proposed the LNED conjucture in [5], which states that if d is a locally nilpotent K-derivation (or $K\mathcal{E}$-derivation) of $K[X]$, then d maps every ideal of $K[X]$ to a Mathieu-Zhao subspace of $K[X]$. In the same article, the author introduced the LFED conjecture, which asserts that for a locally finite K-derivation (or $K\mathcal{E}$-derivation) of $K[X]$, the image of the derivation is a Mathieu-Zhao subspace of $K[X]$.

In [8], it has been demonstrated that for a locally finite derivation of the polynomial algebra in two variables, its image forms a Mathieu-Zhao subspace. Thus, confirming that the LFED Conjecture holds for $n = 2$. For $n \geq 3$, the problem remains unresolved. van den Essen and Sun in [2] proved that the LFED and LNED conjectures hold for monomial preserving K-derivations (or $K\mathcal{E}$-derivations). Recently, Sun and Liu proved the LFED Conjecture for linear locally nilpotent derivations [3], rank two locally nilpotent derivations and rank three homogeneous locally nilpotent derivations [6] of $K[x_1, x_2, x_3]$. In [7] Tian, Du et al. proved that LFED conjecture holds for linear derivations and linear $\mathcal{E}$-derivations of $K[x_1, x_2, x_3]$.
In this talk, we aim to study the images of linear K-derivations (or $K\mathcal{E}$-derivations) of $K[x_1, x_2, x_3, x_4]$. We prove that if d is a linear K-derivation of $K[x_1, x_2, x_3, x_4]$ of rank less than or equal to two, then image of d is a Mathieu-Zhao subspace. We have further shown that, under certain conditions image of $K\mathcal{E}$-derivations of $K[x_1, x_2, x_3, x_4]$ forms a Mathieu-Zhao subspace.

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[2] A. van den Essen, X. Sun, Monomial preserving derivations and Mathieu–Zhao subspaces, Journal of Pure and Applied Algebra **222(10)**, 3219-3223 (2018).
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T10

The Weak Lefschetz Property of squarefree reductions of squarefree monomial ideals and Mixed Multiplicities of ideals

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The Weak and Strong Lefschetz properties have been studied from many different perspectives including Algebraic Geometry, Commutative Algebra and Combinatorics. In recent work of H. Dao and R. Nair, the authors classified the simplicial complexes that satisfy the WLP in degree 1 and characteristic zero after a squarefree artinian reduction. In our work, we relate the recent result of [1] to an older result [2] of A. Simis and R. Villarreal on the classification of Cremona monomial maps of degree 2. Using this new perspective, we give necessary and suficient conditions for the squarefree reduction of a simplicial complex to have the WLP in charactersitic $p \geq$ in terms of mixed multiplicities of ideals.

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T11

Betti Cones over Fibre Products

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In this talk, we shall discuss some of our recent results related to Betti cones of finitely generated modules over standard fibre products of standard graded algebras over a field. We provide a characterization of fibre product rings for which the extremal rays of the Betti cone are pure in terms of Cohen-Macaulayness and Hilbert series of the ring. We also give some results about the structure of syzygy modules over such rings.

This talk is based on a joint work with H. Ananthnarayan and Rajiv Kumar.

[1] H. Ananthnarayan, Omkar Javadekar, R. Kumar, *Betti Cones over Fibre Products*. (in progress)

T12

GRÖBNER DEFORMATIONS AND F-SINGULARITIES

MITRA KOLEY

Abstract: For a commutative ring R of prime characteristic p, the map $F : R \to R$ sending $r \mapsto r^p$ is a ring endomorphism, called the Frobenius morphism. Frobenius morphism plays an important role in studying singularities of such rings. The singularities that have been defined in terms of the Frobenius map are called F-singularities. In this talk we will look at the question how F-singularities behave along Gröbner deformation. This approach gives us a way to study F-singularities of various combinatorial algebras. This is a joint work with Matteo Varbaro.

T13

(A, d)-modules, Hochschild homology and Higher Derivations

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Let k be a commutative ring having characteristic zero and let A be a commutative k-algebra equipped with a k-linear derivation $d : A \longrightarrow A$. An (A, d)-module is a pair (M, d_M) which consists of an A-module M and a k-linear map d_M satisfying the condition $d_M(a \cdot m) = d(a) \cdot m + a \cdot d_M(m)$ for any $a \in A$ and any $m \in M$. Some basics of homological algebra over such objects were developed by Tanaka, who referred to them as "pre-(A, d)-modules". We build a theory of (A, d)-modules similar to that of modules over commutative rings. The motivation for this theory is heavily influenced by noncommutative geometry, especially by the observation that the pair $(HH_\bullet(A), L_d)$ consisting of the Hochschild homology $HH_\bullet(A)$ equipped with the Lie derivative L_d induced by the derivation d on A becomes an (A, d)-module. It was interesting to observe that the Galois descent of Hochschild homology of commutative rings extends to the (A, d)-module case. In continuation, a descent result for (A, d)-modules was obtained that extends Beauville-Laszlo descent from commutative algebra. Further, the theory was generalized to the modules over (A, Δ), where Δ is a higher derivation.

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T14

F-threshold of filtrations of ideals

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This abstract is for contributed talk for the *School on Commutative Algebra and Algebraic Geometry in Prime Characteristics* to be held from *May 2, 2023* to *May 5, 2023*.

The notion of F-threshold of an ideal was introduced by Mustata, Takagi and Watanabe for F-finite regular rings as a prime characteristic invariants of singularities [2]. Later, Huneke, Mustata, Takagi and Watanabe, defined this notion for arbitrary rings of prime characteristics [1]. In this talk, we introduce the notion of F-threshold for filtration of ideals with respect to an ideal and study their basic properties. Let R denote a Noetherian commutative ring of positive characteristic p, and I be a nonzero proper ideal of R. Let $\mathfrak{a}_\bullet = \{\mathfrak{a}_i\}_{i\geq 0}$ be a filtration of ideals in R. For every non-negative integer e, we define

$$\nu^I_{\mathfrak{a}_\bullet}(p^e) := \sup\{r \in \mathbb{Z}_{\geq 0} \ : \ \mathfrak{a}_r \not\subseteq I^{[p^e]}\}.$$

We define

$$\mathcal{C}^I_+(\mathfrak{a}_\bullet) := \limsup_{e\to\infty} \frac{\nu^I_{\mathfrak{a}_\bullet}(p^e)}{p^e} \text{ and } \mathcal{C}^I_-(\mathfrak{a}_\bullet) := \liminf_{e\to\infty} \frac{\nu^I_{\mathfrak{a}_\bullet}(p^e)}{p^e},$$

and if $\mathcal{C}^I_+(\mathfrak{a}_\bullet) = \mathcal{C}^I_-(\mathfrak{a}_\bullet)$, then we denote it by $\mathcal{C}^I(\mathfrak{a}_\bullet)$ and call it the *F-threshold of $\mathfrak{a}_\bullet$ with respect to I*.

We study finiteness of $\mathcal{C}^I_\pm(\mathfrak{a}_\bullet)$ and existense of $\mathcal{C}^I(\mathfrak{a}_\bullet)$. We mainly focus on the study of the F-threshold of symbolic power filtration of ideals. We obtain an upper bound for the F-threshold of symbolic power filtration of a non-zero ideal $\mathfrak{a}$ with respect to a non-zero proper ideal I. We prove that if $(R, \mathfrak{m})$ is a regular local ring and $\mathfrak{a}$ is a non-zero proper ideal of R, then $\mathcal{C}^{\mathfrak{m}}(\mathfrak{a}^\bullet) \leq \mathcal{C}^{\mathfrak{m}}(\mathfrak{a}^{(\bullet)}) \leq \mathrm{ht}(\mathfrak{a})$, where $\mathfrak{a}^\bullet$ and $\mathfrak{a}^{(\bullet)}$ denote the $\mathfrak{a}$-adic and symbolic filtration, respectively. When $\mathfrak{a}$ ia a square-free monomial ideal or ideals of minors of a generic matrix, or ideals of minors of symmetric matrix or ideals of a Henkel matrix or F-König ideal in corresponding standard graded poynomial ring, then we prove that $\mathcal{C}^{\mathfrak{m}}(\mathfrak{a}^{(\bullet)}) = \mathrm{ht}(\mathfrak{a})$. We also obtained a relation of F-threshold of a filtration with the Waldschmidt constant of the filtration.

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T15

On regularity of symbolic and ordinary powers of weighted oriented graphs

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Weighted oriented graphs are important because of their applications in many research areas like coding theory, algebraic geometry and others (see [1], [2], [3], [4],[5],[6]). For a weighted oriented graph D, one can associate a monomial ideal, known as the *edge ideal* of D denoted by $I(D)$. In this work, we are interested to study the algebraic invariant Castelnuovo-Mumford regularity of $I(D)^k$ and $I(D)^{(k)}$, where the latter one is known as the k^{th} symbolic power of $I(D)$.

In this talk, for any weighted oriented graph D, we compare the regularity of $I(D)^k$ and $I(D)^{(k)}$ and also discuss when they are equal.

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T16

INVARIANTS OF BINOMIAL EDGE IDEALS VIA LINEAR PROGRAMS

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In 2009 Herzog et al. [1] and independently Ohtani [2] introduced the notion of binomial edge ideal—a binomial ideal associated to a simple graph. Since their introduction there has been much research relating the combinatorial structure of the underlying graph to the algebraic structure of the associated ideal. I introduce a novel linear program for packings of vertex disjoint paths inside the graph. The optimal primal and dual values of this linear programming problem over the integers computes the binomial grade and height of the binomial edge ideal of a graph and has applications to the study of graphs of König type. Remarkably, the LP-relaxation of this linear program also relates to the log canonical threshold and F-threshold of the binomial edge ideal which are important invariants associated to the singularities of a variety in characteristic 0 and characteristic p. I show that the optimal value of the linear program computed over the rationals computes the F-threshold and the log canonical threshold of the binomial edge ideal if the graph is a block graph or of König type. I conjecture that the LP-relaxation of the linear program computes the log canonical threshold of the binomial edge ideal of any graph.

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T17

Upper bounds on two Hilbert coefficients

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New upper bounds on the first and the second Hilbert coefficients of a Cohen-Macaulay module over a local ring are given. Characterizations are provided for some upper bounds to be attained. The characterizations are given in terms of Hilbert series as well as in terms of the Castelnuovo-Mumford regularity of the associated graded module.

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T18

Symbolic powers of edge ideals of weighted oriented graphs

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Let D be a weighted oriented graph and $I(D)$ be its edge ideal. If D contains an induced odd cycle of length $2n+1$, under certain condition, we show that $I(D)^{(n+1)} \neq I(D)^{n+1}$. . We give necessary and sufficient condition for the equality of ordinary and symbolic powers of edge ideals of weighted oriented graphs having each edge in some induced odd cycle of it. We characterize the weighted naturally oriented unicyclic graphs with unique odd cycles and weighted naturally oriented even cycles for the equality of ordinary and symbolic powers of their edge ideals. Let D be the weighted oriented graph obtained from D after replacing the weights of vertices with non-trivial weights which are sinks, by trivial weights. We show that the symbolic powers of $I(D)$ and $I(D)$ behave in a similar way. We prove an analogue result of [2] which give necessary and sufficient condition for the equality of symbolic powers and ordinary powers for edge ideals weighted oriented bipartite graphs. All the results are from the paper [1]

[1] Mousumi Mandal and Dipak Kumar Pradhan, Journal of Algebraic Combinatorics, **56**, 453-473 (2022).
[2] A. Simis, W. Vasconcelos and R. H. Villarreal, On the ideal theory of graphs, J. Algebra 167 (1994), 389416..

T19

The Hilbert Functions of Artinian Local Complete Intersections

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In singularity theory or algebraic geometry, it is natural to investigate possible Hilbert functions for special algebras such as local complete intersections or more generally Gorenstein algebras. The sequences that occur as the Hilbert functions of standard graded complete intersections are well understood classically thanks to Macaulay and Stanley. Very little is known in the local case except in codimension two. In this talk we will discuss the possible Hilbert functions of Artinian local complete intersections of codimension three. This is a joint work with J. Jelisiejew and M. E. Rossi [1].

[1] J. Jelisiejew, S. K. Masuti and M. E. Rossi, Journal of Pure and Applied Algebra. **227**, 107326 (2023).

T20

Polynomial invariant rings in modular invariant theory

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Let V be a finite dimensional linear representation of a finite group $G \subseteq GL(V)$ over a field F and let S^G be the ring of invariants for the induced action of G on S, the symmetric algebra of V^*. It is known that if the invariant ring S^G is a polynomial ring then G is generated by elements which act as pseudoreflections on V. The converse is true if $|G| \neq 0$ in F. We shall discuss some necessary and sufficient conditions for S^G to be a polynomial ring when characteristic of $F = p > 0$ and G is a p-group. In this context the Shank-Wehalu-Broer conjecture states that if S^G is a direct summand of S, then S^G is a polynomial ring. We verify this conjecture for a class of p-groups and for representations of dimension 4 over prime field. This is joint work with Prof. Manoj Kummini.

[1] M. Kummini, M. Mondal, Proc. Amer. Math. Soc. **On Hilbert ideal for a class of p-groups in characteristic p**, 150 (2022).
[2] M. Kummini, M. Mondal, **On polynomial invariant rings in modular invariant theory**, https://arxiv.org/abs/2210.05945, submitted.

T21

On injective dimension of generalized Eulerian graded D-modules in char $p > 0$

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Let R be a polynomial ring in n indeterminates with coefficients in a field K of characteristic $p > 0$ such that $[K : K^p] < \infty$, and $\mathcal{D}$ be the ring of differential operators over R. In this paper, we prove that the minimal injective resolution of a generalized Eulerian graded $\mathcal{D}$-module M terminates in the direct sum of indecomposable with certain finiteness properties in case of characteristic $p > 0$. Consequently, we provide the best possible lower bound for the injective dimension of a graded $\mathcal{D}$-module $\mathcal{T}(R)$, where $\mathcal{T}$ is any graded Lyubeznik functor on the category of R-modules.

T22

Title. A density function for a Noetherian filtration of homogeneous ideals

Abstract. The objective of this talk is to discuss the behaviour of the Hilbert function of bigraded algebras over a field, generated in bidegrees $(1,0), (d_1, e_1), \ldots, (d_s, e_s)$, where d_i's (resp. e_i's) are non-negative (resp. positive) integers. It is well known that for a standard bigraded algebra R, the Hilbert function $H_R(u,v)$ is represented by a polynomial for all $u, v \gg 0$. However, N. D. Hoang and N. V. Trung showed that if all e_i's are one then there exist integers u_0, v_0 such that $H_R(u,v)$ is equal to a polynomial in u, v for $u \geq dv + u_0$ and $v \geq v_0$, where $d = \max\{d_1, \ldots, d_s\}$. In this talk, we will concentrate on how $H_R(u,v)$ behaves in the complementary region. Thereby we will define a density function for a Noetherian filtration of homogeneous ideals in a standard graded algebra with positive depth. Our main ingredient will be the structure theorem for vector partition functions due to B. Sturmfels. This talk is based on an ongoing joint work with Suprajo Das and Vijaylaxmi Trivedi.

T23

The v-number of Monomial Ideals

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We show that the v-number of an arbitrary monomial ideal is bounded below by the v-number of its polarization and also, find a criteria for the equality. By showing the additivity of associated primes of monomial ideals, we obtain the additivity of the v-numbers for arbitrary monomial ideals. We prove that the v-number $\mathrm{v}(I(G))$ of the edge ideal $I(G)$, the induced matching number $\mathrm{im}(G)$ and the regularity $\mathrm{reg}(R/I(G))$ of a graph G, satisfy $\mathrm{v}(I(G)) \leq \mathrm{im}(G) \leq \mathrm{reg}(R/I(G))$, where G is either a bipartite graph, or a (C_4, C_5)-free vertex decomposable graph, or a whisker graph. There is an open problem in [1], whether $\mathrm{v}(I) \leq \mathrm{reg}(R/I) + 1$ for any square-free monomial ideal I. We show that $\mathrm{v}(I(G)) > \mathrm{reg}(R/I(G)) + 1$, for a disconnected graph G. We derive some inequalities of v-numbers which may be helpful to answer the above problem for the case of connected graphs. We connect $\mathrm{v}(I(G))$ with an invariant of the line graph $L(G)$ of G. For a simple connected graph G, we show that $\mathrm{reg}(R/I(G))$ can be arbitrarily larger than $\mathrm{v}(I(G))$. Also, we try to see how the v-number is related to the Cohen-Macaulay property of square-free monomial ideals.

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T24

Bounds for the reduction number of primary ideals in dimension three

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A celebrated result of Rossi [2] gives an upper bound for the reduction number of an m-primary ideal in a Cohen-Macaulay local ring of dimension two. It is an open problem to extend Rossi's result in higher dimension. In this talk, we present bounds in dimension three and consequently gather some cases when Rossi's result holds true. We will discuss the major difficulties and the role of Ratliff-Rush filtration. In dimension three, the third Hilbert coefficient is present in our bound linearly. So, we also discuss bounds for the third Hilbert coefficient. The boundary conditions of these bounds are particularly interesting.

For the talk, some ideas will be borrowed from a joint work [3] with Anoot Kumar Yadav.

[1] Mousumi Mandal and Kumari Saloni, Bounds for the reduction number of primary ideal in dimension three, accepted, Proc. Amer. Math. Soc., 2023

[2] M. E. Rossi, A bound on the reduction number of a primary ideal, Proc. Amer. Math. Soc. 128(5) (1999), 1325-1332.

[3] Kumari Saloni and Anoot Kumar Yadav, Ratliff-Rush filtration, Hilbert coefficients and reduction number in dimension three. Manuscript in preparation.

T25

Betti sequence of projective closure of affine monomial curves

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We introduce the notion of star gluing of numerical semigroups and show that this preserves the arithmetically Cohen-Macaulay and Gorenstein properties of the projective closure. Next, we give a sufficient condition involving Gröbner basis for the matching of Betti sequences of the affine curve and its projective closure. We also study the effect of simple gluing on Betti sequences of the projective closure. Finally, we construct numerical semigroups by gluing, such that for every positive integer n, the last Betti number of the corresponding affine curve and its projective closure are both n.

[1] J. Saha, I. Sengupta, and P. Srivastava, *Betti Sequence of Projective Closure of Affine Monomial Curves*, arXiv:2111.13022 [math.AC], (2021)(Accepted in Journal of Symbolic Computation).