The Abdus Salam
International Centre
for Theoretical Physics

School/Workshop on Wave Dynamics: Turbulent vs Integrable Effects | (SMR 3869)

28 Aug 2023 - 01 Sep 2023
ICTP, Trieste, Italy

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P01

Three-dimensional interfacial gravity-capillarity waves with short peaks in the presence of a parallel current

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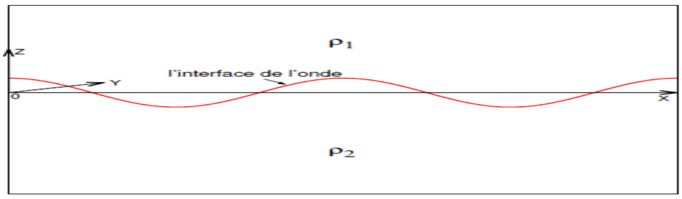
Abstract

Internal waves are waves that develop inside a stratified fluid medium whose thermodynamic properties vary with altitude. Two emblematic examples are to be cited The atmosphere, which is mainly stratified in temperature, and the ocean, whose temperature and salinity vary significantly with depth. Mention may also be made of astrophysical systems such as stars or galaxies, or even cocktails. When these waves appear at the interface of two layers of fluids of different densities, these are called interfacial waves. We are interested in the calculation of the three-dimensional gravity-capillarity waves which propagate at the interface of two layers of fluids of different densities and arbitrary thickness in the presence of a parallel current. The method used to solve the problem is that of disturbances. An analytical solution of order 3 was obtained using computer algebra software MAPLE. We study the properties of these waves in the vicinity of the harmonic resonance phenomenon

POSITION OF THE PROBLEM

Governing Assumptions and Equations

The flow is assumed to be irrotational and the fluids perfect, incompressible, homogeneous and immiscible in the presence of a parallel current. We consider the effects of voltage superficial



Schematic representation of an interfacial wave

Dimensionless equations and conditions

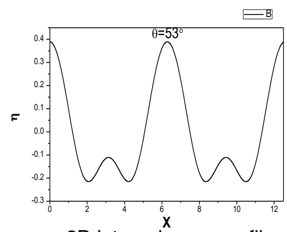
$$p^2\phi_{2XX} + q^2\phi_{2YY} + \phi_{2ZZ} = 0, \quad -d_2 < Z < \eta(X,Y)$$
$$p^2\phi_{1XX} + q^2\phi_{1YY} + \phi_{1ZZ} = 0, \quad \eta(X,Y) < Z < d_1$$

$$-\omega\eta_X + p^2\eta_X\phi_{1X} + q^2\eta_Y\phi_{1Y} - \phi_Z = 0, \quad Z = \eta(X,Y)$$
$$-\omega\eta_X + p^2\eta_X\phi_{2X} + q^2\eta_Y\phi_{2Y} - \phi_Z + pU\eta_X = 0, \quad Z = \eta(X,Y)$$

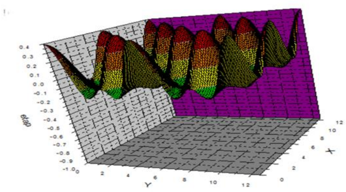
Perturbation method

$$\begin{cases} \phi_1(X,Y,z) = \sum_{r=1}^{\infty} h^r \phi_1^{(r)}(X, \\ \phi_2(X,Y,z) = \sum_{r=1}^{\infty} h^r \phi_2^{(r)}(X, \\ \eta(X,Y) = \sum_{r=1}^{\infty} h^r \eta_r(X,Y) \\ \omega = \sum_{r=0}^{\infty} h^r \omega_r \\ U = \sum_{r=0}^{\infty} h^r U_r \end{cases}$$

$$p^2\,\Phi_{1XX}{}^{(r)} + q^2\,\Phi_{1YY}{}^{(r)} + \Phi_{1ZZ}{}^{(r)} = 0 \quad Z > 0$$
$$p^2\,\Phi_{2XX}{}^{(r)} + q^2\,\Phi_{2YY}{}^{(r)} + \Phi_{2ZZ}{}^{(r)} = 0 \quad Z < 0$$
$$\omega_0\eta_X{}^{(r)} + \Phi_{1Z}{}^{(r)} = A_1^r \qquad Z = \eta(X,Y)$$
$$\omega_0\eta_X{}^{(r)} - pU\eta_X{}^{(r)} - \Phi_{2Z}{}^{(r)} = A_2^r \text{ en} \quad Z = \eta(X,Y)$$
$$\frac{-(\omega_0)}{\mu-1}\left(\mu\,\Phi_{1X}{}^{(r)} - \Phi_{2X}{}^{(r)}\right) + \frac{pU\,\Phi_{2X}{}^{(r)}}{\mu-1} + \eta^{(r)} + \kappa\left(p^2\eta_{XX}{}^{(r)} + a^2\eta_{YY}{}^{(r)}\right) = \frac{B^{(r)}}{\mu-1}$$



2D internal wave profile



Three-dimensional internal wave profile

The influence of the harmonic (2.0) results in the appearance of ripples in with 2 peaks along X

Conclusion

The calculation of the three-dimensional interfacial waves is carried out by the disturbance method, this method has made it possible to highlight the harmonic resonance. third-order analytical solutions were obtained using the computer algebra software Maple. In addition, we have shown the influence of current and the capillarity coefficient on the properties of the wave.and the behavior of these properties in the vicinity of harmonic resonance.

P02

About integration the loaded negative order KdV equation via inverse scattering transform method

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In 1967 American scientists Gargner, Green, Kruskal, and Miura [1] proposed the inverse scattering problem method for the Sturm-Liouville equation as a method for solving the Cauchy problem for the Korteveg-de Vries (KdV) equation. In the work [2] the loaded Korteweg—de Vries equation is solved, this equation plays an important role in hemodynamic processes.

Based on the regular KdV system, Qiao originally studied nKdV equations, particularly their Hamiltonian structures, Lax pairs, conservation laws, and explicit multisoliton and multikink wave solutions through bilinear Bäcklund transformations [3, 4].

This poster aims to study the integration of the loaded negative order Korteweg-de Vries equation in the "rapidly decreasing" class via the inverse scattering problem.

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P03

Isospectral Control of Self-Similar Rogue waves

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"Rogue waves", "freak waves" and "killer waves", refer to giant isolated waves that appear from nowhere, having amplitudes significantly larger than the background waves. Firstly observed in the oceans as extreme water waves, rogue waves (RWs) are ubiquitous in nature and appear in various contexts such as nonlinear optical systems, Bose-Einstein condensates, microwave cavities, etc. In particular, the study of rogue waves has gained fundamental significance in nonlinear optical systems, because of its potential applications in producing high-intensity optical pulses. Several models have been developed to study the dynamics of RWs, the nonlinear Schrödinger equation (NLSE) being the most studied one [1].

A significant aspect of the implementation of RWs in the communication industry is the manipulation of features like amplitude, speed etc. Recently, Dai and his collaborators have studied the dynamics of controllable rogue waves, modeled by variable coefficient NLSE, through dispersion and nonlinearity management [2]. Authors in [3] have shown that the equation governing rogue wave dynamics admits a wide class of self-similar solutions, whose amplitudes can be exactly controlled by tailoring gain and tapering profiles in optical fibers through a free parameter using the isospectral hamiltonian technique. In the present work, we extend this class of solutions, to control the amplitude by tailoring the gain and tapering profiles through two free parameters. In the paraxial regime, the beam propagation is governed by the inhomogeneous NLSE given by

$$i\frac{\partial U}{\partial Z} + \frac{1}{2}\frac{\partial^2 U}{\partial X^2} + F(Z)\frac{X^2}{2}U - \frac{i}{2}G(Z)U + |U|^2U = 0, \tag{1}$$

where $U(X,Z)$ represents the dimensionless complex field envelope, $F(Z)$ is the graded-index profile, and $G(Z)$ is the linear gain/loss function. Fig. 1 demonstrates the effect of

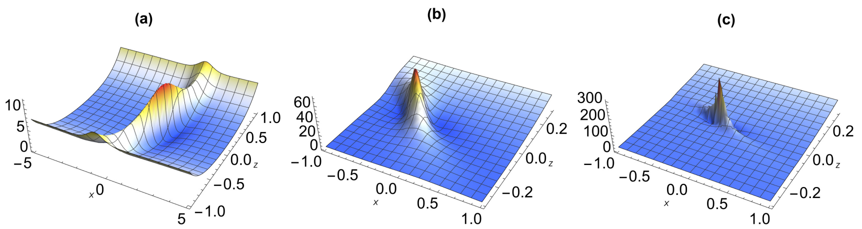

Figure 1: (a) Rogue wave intensity corresponding for undeformed case. Rogue wave intensity for (b) $\lambda_1, \lambda_2 = (0.1, -1.1)$ (c) $\lambda_1, \lambda_2 = (1, -1.1)$

Ricatti parameters (λ_1, λ_2) on intensity of the RW. It can be observed that the height of the RW can be effectively controlled through judicious combination of (λ_1, λ_2), increasing **up to 30 times** in Fig. 1(c).

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P04

Title： Well-posedness and wave-breaking for the stochastic rotation-two-component Camassa-Holm system

Abstract：We study the global well-posedness and wave-breaking phenomenon for the stochastic rotation-two-component Camassa-Holm (R2CH) system. First, we find a Hamiltonian structure of the R2CH system and use the stochastic Hamiltonian to derive the stochastic R2CH system. Then, we establish the local well-posedness of the stochastic R2CH system using a dispersion-dissipation approximation system and the regularization method. We also show a precise blow-up criterion for the stochastic R2CH system. Moreover, we prove that the global existence of the stochastic R2CH system occurs with high probability. At the end, we consider the transport noise case and establish the local well-posedness and another blow-up criterion.

P05

One-dimensional Optical Turbulence

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One dimensional Non-Linear Schrödinger Equation (1D NLSE), which governs different non-linear systems (optical fibers, liquid crystals, Bose-Einstein condensation), was proven to be integrable by Zakharov and Shabat in 1971 [1]. They used the Direct Scattering Transform (DST) method, a computation of the non-linear spectrum, equivalent of Fourier Transform. 1D NLSE is however just a model for most systems, the real equations contain high-order corrections. These new terms often break its integrability, allowing these systems to have a turbulent behaviour. The system studied here is the propagation of a laser beam into a liquid-crystals medium, governed by an non-integrable equation corresponding to a modified 1D NLSE. This system is thus turbulent and present interactions between linear waves and solitons. The main idea of our work is that we can apply DST to characterize solitons and thus study this turbulent system.

To understand these interactions, we run Direct Numerical Simulations and process the data obtained by using different methods to characterize the solitons, including the DST. We compare the results given by DST to the $x - t$ plot of the simulation and to $k - \omega$ plots. Doing this, we have three ways to identify solitons and to obtain their speeds and amplitudes. It appeared that the information given by DST is consistent with $x{-}t$ and $k-\omega$ plots which makes it a useful tool to study a non-integrable system. With these tools, we observed different sets of sech-profile solitons evolving with modified 1D NLSE and obtained interesting results, (i) these solitons move with an oscillating amplitude, (ii) some new solitons can be created and some annihilated, (iii) we saw evidences of soliton turbulence [2], a series of non-elastic collisions that leads to a big single soliton.

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[2] J. Laurie, U. Bortolozzo, S. Nazarenko, S. Residori, Physics Reports, **514**, 121-175, (2012).

P06

Interaction of soliton gas with dispersive hydrodynamic mean flows

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The soliton structure plays a fundamental role in many physical systems due to its fundamental feature: its shape remains unchanged after the collision with another soliton in the case of integrable dynamics. Such particle-like behavior has been at the origin of a new mathematical object: the soliton gas, consisting of an incoherent collection of solitons for which phases (positions) and spectral parameters (e.g. amplitudes) are randomly distributed. The study of soliton gas involves the description of the gas dynamics as well as the corresponding modulation of nonlinear wave field statistics, which makes the soliton gas a particularly interesting embodiment of the particle-wave duality of solitons.
Building on the recent developments of the theory of soliton-mean flow interaction, we investigate the interaction of a soliton gas with mean flows for the Korteweg-de Vries (KdV) equation. The existing results on the individual soliton's interaction with slowly varying mean fields (e.g. rarefaction wave) are generalized to the case of so-called cold soliton gases; we notably present a new hydrodynamic reduction of the kinetic equation based on the theory of KdV soliton condensates [1]. The obtained analytical results are compared with the numerical simulation of the KdV soliton gases propagating through simple hydrodynamic waves.

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P07

Dam-break problem in integrable and non-integrable discrete nonlinear Schrödinger equation, and applications to neuromorphic computing

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Integrability provides analytically treatable framework to study the formation and dynamics of solitonic and rogue wave excitations. A notable example is the the dam-break problem (box initial condition) for the integrable one-dimensional nonlinear Schrödinger (NLS) equation [El.*et.al.*, Nonlinearity **29**, 2798 (2016)]. We consider two discretized versions of the one-dimensional NLS – namely, the Ablowitz-Ladik and the discrete nonlinear Schrödinger lattices, where only the former one is integrable We discuss the dam-break problem in both nonlinear lattice cases, observing generation of Rogue waves in the integrable case, while condensation in the non-integrable one. We then introduce their applications of these two nonlinear lattices as reservoirs in neuromorphic computing.

P08

Integrability, solitons and invariant solutions of a generalized (2+1)-dimensional Hirota bilinear equation

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The present exposition investigates an extended version of the generalized (2+1)-dimensional Hirota bilinear equation, which is relevant to nonlinear wave phenomena in areas such as shallow water, oceanography, and nonlinear optics. The integrability properties [1] of the equation are studied using the Painleve analysis technique [2], which reveals that it is not completely integrable in the Painleve sense. The Bell polynomial form is introduced to obtain the Hirota bilinear form [3] and Backlund transformations [4], and Lax pairs are derived using the Cole-Hopf transformation and linearization of a coupled system of binary Bell polynomials. The paper also derives infinite conservation laws and illustrates one, two, and three soliton solutions from the Hirota bilinear form. Lie symmetry approach [5] is applied to analyze the Lie symmetries and vector fields of the problem, and similarity variables are used to obtain symmetry reductions and closed-form solutions such as parabolic wave and kink wave solutions.

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P09

Mean Field Evolution of Weakly Interacting Fermions

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A major problem in condensed matter is the derivation of effective theories to describe d-dimensional electron gases as a model for conducting materials. A conductor can be approximated as a three-dimensional electron gas but more sophisticated materials, like graphene and carbon nanotubes, are modelled in two and one dimensions. We are considering systems whith a number of particles in the order of 10^{23}, this sets a limit in the resolution of the many-body Schrödinger equation. To overcome this problem, effective evolution equations are introduced; in an idealized regime the many-body Schrödinger equation can be approximated by equations with fewer degrees of freedom. For a system of interacting fermions, the many-body dynamics can be approximated by the time-dependent Hartree-Fock equation. In 2014 Benedikter, Porta and Schlein studied the many-body dynamics for a three-dimensional system of weakly interacting fermions where a mean field regime is coupled to a semiclassical scaling. Considering the same regime and scaling limit, we generalize the model in d dimensions. The novelty of our approach lies in the introduction of an explicit formula for the unitary implementation of a particle-hole transformation. We prove that the error of this approximation tends to zero as the number of particles grows at infinity.

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Delta-shaped perturbations of the Laplace-Beltrami operator on a two-dimensional sphere

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Abstract

This work deals with delta-shaped perturbations of the Laplace-Beltrami operator on a Riemannian manifold. Some examples of the construction of the Laplace-Beltrami operator on a smooth Riemannian manifold with a smooth compact boundary are given. Green's functions in the two-dimensional sphere are constructed. Functionals in Sobolev space in the punctured sphere are found, such that regularization of an element allows to redefine its value at a singular point. Then the eigenvalues of the operator with the punctured point are estimated.

Introduction

Let $(\Omega, \partial\Omega, g)$ a smooth oriented Riemannian manifold with a metric g and a smooth compact boundary $\partial\Omega$. We denote by $C^{\infty}(\Omega)$ - the space of infinitely differentiable functions on a Riemannian manifold Ω and by $C_c^{\infty}(\Omega)$ - the space of infinitely differentiable functions with compact support strictly inside Ω.

Let Ω – n-dimensional smooth manifold, for any point $x \in \Omega$ we denote by $T_x\Omega$ - the tangent space to Ω at the point x. In the tangent space $T_x\Omega$ we define the scalar product of tangent vectors by the forms

$$\langle \xi, \eta \rangle_g = g_{ij}\xi^i \overline{\eta^j}, \quad \forall \xi, \eta \in T_x\Omega.$$

Let Λ_0 – an invertible restriction of the operator B. The operator Λ, is defined by the formula $\Lambda u = Bu$ on the domain of definition

$$D(\Lambda) = \left\{ u \in D(B) : u = \Lambda_0^{-1} f - \sum_{s=1}^{m} \varphi_s \cdot U_s\left(\Lambda_0^{-1} f\right), \forall f \in H \right\}$$

Main results

Theorem. The operator Λ – is an invertible operator, and

$$\Lambda^{-1} f = \Lambda_0^{-1} f - \sum_{s=1}^{m} \varphi_s \cdot U_s\left(\Lambda_0^{-1} f\right), \forall f \in H$$

$$(\Lambda - \lambda I)^{-1} f = (\Lambda_0 - \lambda I)^{-1} f - \sum_{s=1}^{m} \Lambda(\Lambda - \lambda I)^{-1} \varphi_s U_s((\Lambda_0 - \lambda I)^{-1} f)$$

Second generalized Hilbert identity for resolvents : $D(\Lambda) \neq D(\Lambda_0)$.

The Laplace-Beltrami operator is considered on a two-dimensional sphere S^2:

$$\Delta_{S^2}\Phi = -\left[\frac{1}{\cos^2\theta} \frac{\partial^2 \Phi}{\partial \varphi^2} - tg\theta \frac{\partial \Phi}{\partial \theta} + \frac{\partial^2 \Phi}{\partial \theta^2} \right].$$

The eigenvalues of the operator $-\Delta_{S^2}$, can only be numbers $\lambda_l = l(l+1)$, where $l \geq 0$ are integers. We introduce the function

$$\varepsilon(\varphi, \theta; \alpha, \beta) = \sum_{l=0}^{\infty} \frac{1}{l(l+1)} \sum_{m=-l}^{l} Y_l^m(\varphi, \theta) Y_l^m(\alpha, \beta)$$

$$P_l(\cos\theta') = \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}(\varphi, \theta) Y_{lm}^*(\alpha, \beta)$$

Lemma 1. For $\varepsilon(\varphi, \theta; \alpha, \beta)$ the representation of Green's function

$$\varepsilon(\varphi, \theta; \alpha, \beta) = \frac{1}{\sqrt{2\left(1 - \sin\alpha \sin\varphi - \cos\alpha \cos\varphi \cos(\theta - \beta)\right)}} = \frac{1}{|r_1 - r_2|}$$

Lemma 2.

$$\Phi(\varphi, \theta) = \int_{S^2} \varepsilon(\varphi, \theta; \alpha, \beta) f(\alpha, \beta) \cos\theta \, d\alpha \, d\beta$$

$$\Delta_{S^2}\Phi = f(\varphi, \theta).$$

It follows that the introduced function $\varepsilon(\varphi, \theta; \alpha, \beta)$ is the Green's function of the Laplace-Beltrami operator.

Let us introduce a set of functions in the Sobolev space on a punctured sphere $W^2_{2,(U_0,U_1,U_2)}(\Omega_0)$, for which the following requirements are fulfilled:

1) If $h \in W^2_{2,(U_0,U_1,U_2)}(\Omega_0)$, then h is defined everywhere on S^2, except a point (φ_0, θ_0).

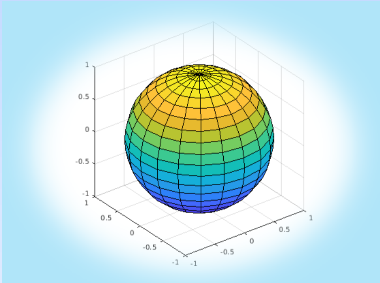
2) for $\forall \delta > 0$, function $h \in W_2^2\left(S^2 \setminus V_0(\delta)\right)$

3) functionals $U_0(h), U_1(h), U_2(h)$ take finite values.

4) $h - U_0(h)\varepsilon(\varphi, \theta; \varphi_0, \theta_0) - U_1(h)\frac{\partial}{\partial\alpha}\varepsilon(\varphi, \theta; \varphi_0, \theta_0) -$

$-U_2(h)\frac{\partial}{\partial\beta}\varepsilon(\varphi, \theta; \varphi_0, \theta_0) \in W_2^2(S^2)$

that is, regularization of an element h allows us to redefine its value at a point (φ_0, θ_0).



Conclusion

The true application of multivariate differential operator studies has been studied in medicine. Currently, there is a great need for important research in the field of biotechnology, i.e the development of gene interactions using polymers. This study aims to determine the type of mathematical model of polymers. Therefore, the ability to use Riemannian surfaces allows you to build a model.

Although Riemannian spaces were used in physics, mathematics, and chemistry, their most important role was identified in the theory of homology.

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P11

Resonant Dynamics of the Gross-Pitaevskii Equation

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This contribution will discuss how resonances influence the dynamics of the Gross-Pitaevskii equation with an external trapping potential

$$i\partial_t \psi = -\Delta\psi + V\psi + g|\psi|^2\psi. \tag{1}$$

The attention will be focused on highly resonant potentials, where the linear spectrum ω_k is non-dispersive, such as in the harmonic trap,

$$\omega_k = 2k + \frac{d}{2} \qquad k = 0, 1, 2, ..., \tag{2}$$

where d is the spatial dimension. In this case, the resonance condition $\omega_{k_1} + \omega_{k_2} = \omega_{k_3} + \omega_{k_4}$ is trivially satisfied by integers numbers, guaranteeing the presence of many resonances. It makes that GPEs with (2) display nontrivial dynamics even in the weakly nonlinear regime $|g| \ll 1$. The effects of these dynamics are "visible" in the time-scale $t \sim 1/g$ [1], which is much smaller than the kinetic scale $(1/g^2)$, showing the dominance of exact resonances. In this contribution we will show interesting phenomena associated with resonant dynamics that can be captured analytically as stationary and time-periodic energy-flows [2, 3, 4], or numerically as Fermi-Pasta-Ulam-Tsingou recurrences [5].

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P12

Unusual chaos and thermalization properties of confined hard rods

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In this work, we will present a comprehensive analysis of the dynamical properties and equilibration behaviour of hard rods confined to integrability-breaking confining traps. Specifically, we will explore the behaviour of hard rods in both harmonic and quartic traps and discuss how the harmonically trapped system shows non-chaotic to chaotic behaviour with increasing system size. We further investigate the underlying non-ergodic nature of the hard rod in the harmonic trap and contrast it with the ergodic and thermalizing behaviour observed in the quartic trap. Based on this we provide a heuristic understanding of the equilibration of harmonically trapped rods, highlighting the significant differences observed between the two types of confinement. Our findings shed new light on the complex dynamics of confined hard rod systems, with implications for Generalized hydrodynamics in the presence of confinement.

P13

Extreme wave generation due to exceptional bathymetry

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We present the numerical wave simulations over a particular seabed using Hawassi model. We investigate the amplification of a mild wave and a more energetic wave evolution over an abrupt bathymetry change. The existence of a shallow bathymetry in the open sea causes the extreme wave generation as the consequences of many physical aspects [1]. The amplification of the wave seems to behave only locally near the shallow bottom. Passing the shallow bottom, the wave transforms into a downward running flow toward the shore. The wave-bottom interaction incorporating wave shoaling, breaking waves, and nonlinearity also results in statistics wave transformation [2].

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P14

Novel nonlinear wave structure for some nonlinear equations with applications

P15

Semiclassical Trace Formula for Lieb-Liniger model

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In quantum chaos, an important tool to study the correspondence between a quantum system and its classical limit is the trace formula. This relates the energy density of a quantum model to the properties of the periodic solution of its classical limit. It is of relevance for our study particularly as it has been written decades ago for a fully integrable model by Berry and Tabor [1]. Here we apply the trace formula to the Lieb-Liniger model [2], [3] for a finite number of particles. This model has played a central role in mathematical physics and we believe it is particularly relevant for studying classical/quantum correspondence. Our motivation for using the trace formula here is that we believe it can be a valuable tool to give new insights as it pertains to the topic of thermalisation. The model deals with N 1-Dimensional Bose particles trapped in a box, interacting via a two-body potential chosen as the Dirac delta "function". More explicitly the stationary Schrödinger equation for the eigenstates reads:

$$\left(-\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2}+2c\sum_{1\le i\ne j\le N}\delta\left(x_i-x_j\right)\right)\psi=E\psi\ ,\quad R:\ 0\le x_i\le L,\quad 1\le i\le N \tag{1}$$

Where $c > 0$ is the interaction strength and R is the region of space. The original treatment by Lieb and Liniger uses the Bethe ansatz to find the energy spectrum and the corresponding eigenstates. Using the Bethe equations to count all the eigenvalues of the Hamiltonian (1) enables us to write a formula for the mean density of states as a volume integral over the phase space of the classical corresponding model. The oscillating part of the trace formula will relate the quantum energy spectrum to the classical periodic trajectories. We test our trace formula in two ways. First by solving numerically the Bethe equations for N and computing a long sequence of levels to get their density. Then comparing the obtained density by a sum over the classical periodic solutions with the shortest period obtained also numerically. In future we plan to expand this analysis to the continuum limit $N, L \to \infty$ when the phase space becomes infinite dimensional. The Lieb-Liniger model has been well studied in generalised hydrodynamics (GHD) [4], [5], [6] and we aim to make more explicit a connection between this and the trace formula.

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P16

Maxwell-Bloch equations without spectral broadening: the long-time asymptotics of an input pulse in a long two-level laser amplifier

P17

Soliton Shielding of the Focusing Nonlinear Schrödinger Equation

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We first consider a deterministic gas of N solitons for the focusing nonlinear Schrödinger (FNLS) equation in the limit $N \to \infty$ with a point spectrum chosen to interpolate a given spectral soliton density over a bounded domain of the complex spectral plane. We show that when the domain is a disk and the soliton density is an analytic function, then the corresponding deterministic soliton gas surprisingly yields the one-soliton solution with the point spectrum the center of the disk. We call this effect soliton shielding. We show that this behavior is robust and survives also for a stochastic soliton gas: indeed, when the N-soliton spectrum is chosen as random variables either uniformly distributed on the circle, or chosen according to the statistics of the eigenvalues of the Ginibre random matrix the phenomenon of soliton shielding persists in the limit $N \to \infty$. When the domain is an ellipse, the soliton shielding reduces the spectral data to the soliton density concentrating between the foci of the ellipse. The physical solution is asymptotically steplike oscillatory, namely, the initial profile is a periodic elliptic function in the negative x direction while it vanishes exponentially fast in the opposite direction.

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P18

Riemann problem for polychromatic soliton gases: a testbed for the spectral kinetic theory

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We use Riemann problem for soliton gas as a benchmark for a detailed numerical verification of the kinetic equation describing the evolution of the density states in the nonlinear Fourier spectral phase plane. We construct weak solutions of the kinetic equation describing collision of two dense, uniform soliton gases, each composed of a finite number of "quasi-monochromati" components. We extract the macroscopic physical observables of the associated nonlinear incoherent wave fields (integrable turbulence) for the focusing nonlinear Schrodinger equations from the the analytical spectral solution and compare them with the results of direct numerical simulations of respective polychromatic soliton gases. To numerically synthesize dense soliton gases we employ a novel method that combines advances in the spectral theory of the so-called soliton condensates and the recently developed effective algorithms for the numerical realization of N-soliton solutions with large N.

P19

Thermalization and hydrodynamics in an interacting integrable system: the case of hard rods

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(work with Anupam Kundu, Abhishek Dhar, Herbert Sphon)

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A classical Hamiltonian many-body system will generally thermalize to Gibbs Ensemble (GE) if left alone for a long time. However, there may exist systems that do not thermalize to GE, because of the existence of extra conservation laws which restrict their motion in the phase space. Thus, dynamical many-body systems can be thought of as constituting a spectrum, with systems having only Hamiltonian as the conserved quantity at one end of the spectrum, and systems having infinitely many conservation laws at the other end. The latter end consists of integrable many-body systems, which are believed to thermalize to the Generalized Gibbs Ensemble (GGE). They have a number of conservation laws equal to the number of degrees of freedom, and thus an infinity of them in the thermodynamic limit. Their non-equilibrium states close to local GGE is described by generalised hydrodynamics (GHD). In this poster, we will study thermalization to GGE of an interacting integrable system, which is that of hard rods, starting from an initial non-equilibrium state. We will also solve the GHD equations at the Euler level exactly by mapping it to a free particle Euler equation. We will compare our analytical results with those of molecular dynamics simulations.

P20

Formation of Rouge Waves and Modulational Instability Analysis in non-autonomous system

P21

Reducibility of a class of quasi-linear wave equation on the torus.

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We prove reducibility of a class of quasi-periodically forced linear wave equations of the form

$$\psi_{tt} - \psi_{xx} + \mathtt{m}\psi + \mathcal{Q}(\omega t)\psi = 0\,, \qquad x \in \mathbb{T} := \mathbb{R}/2\pi\mathbb{Z}$$

where $\psi = \psi(t, x)$, $\mathtt{m} \in [1, 2]$, $\mathcal{Q}$ is an unbounded pseudo-differential operator of order 2, provided that the forcing frequency $\omega \in \mathbb{R}^\nu$, $\nu \geq 1$ satisfies suitable non resonance conditions. In appropriate complex coordinates the equation reads as a first order system with a very weak, even, dispersion. At the highest order we split the dynamics into a backward/forward transport equation, with non-constant coefficients, on positive/negative modes. The key idea is to straighten such vector fields through a novel quantitative Egorov analysis.

P22

Creation and annihilation of solitons in the negative order Korteweg-de Vries equation

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Studying the negative order Korteweg de Vries equation (nKdV) plays an important role in the theory of peaked soliton (peakon) and cusp soliton (cuspon) [1, **?**].In the work [2] the inverse scattering method for the one-dimensional Schrodinger operator on a straight line is used to derive solutions for the Korteweg-de Vries equation with a self-consistent source which describes the creation and annihilation of solitons. In this work, the creation and annihilation of solitons in the system described by the negative order Korteweg-de Vries equation with a self-consistent source are studied.

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P23

Energy transfer in 3D DNS of turbulent systems. Case of study: space plasmas

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Recent in situ observations of the solar corona have corroborated, and revealed, a variety of details on its nature and its contribution to the solar-wind turbulence, e.g., [1, 2, 3, 4]. However, the existence of multiple components, scales, and regimes make this media extremely complex and holds open interesting questions [5]. In fact, one of this questions is related to the nature of the turbulent cascade of the solar-wind plasma, which is a quasi collisionless plasma because of its low density and high temperature [6]. In particular, the dissipation mechanisms of the electromagnetic fields and plasma flow fluctuations are not fully understood.
We study the energy transfer rate in three-dimensional direct numerical simulations of quasi collisionless space plasmas. The Yaglom-Politano-Pouquet law is analyzed in a turbulent scenario evolved using a Hall magnetohydrodynamic model. This law is computed using two different numerical schemes, where we evidence a scale-by-scale sign dependence to the integration method. The so-called Local Energy Transfer proxy is also studied in this simulation, showing no dependence to the integration method.

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P24

Kinetic equations for soliton gas: a Hamiltonian formulation

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The El's integro-differential kinetic equation for dense soliton gas was firsly derived in [1] as a thermodynamic limit of the KdV Whitham equations and reads as

$$\begin{gathered} f_t + (sf)_x = 0, \\ s(\eta) = S(\eta) + \textstyle\int_0^\infty G(\mu,\eta) f(\mu)[s(\mu) - s(\eta)]\, d\mu, \end{gathered} \tag{1}$$

where $f(\eta) = f(\eta, x, t)$ is the distribution function, $s(\eta) = s(\eta, x, t)$ is the associated transport velocity, $S(\eta)$ is the free soliton velocity and the kernel $G(\mu, \eta)$ is the phase shift due to pairwise soliton collisions.

In [2], the authors introduced a delta-functional ansatz

$$f(\eta, x, t) = \sum_{i=1}^{n} u^i(x,t)\ \delta(\eta - \eta^i(x,t)), \tag{2}$$

to reduce system (1) into a $2n \times 2n$ quasilinear system for $u^i(x,t)$ and $\eta^i(x,t)$.

In this poster, we present a Hamiltonian formalism for El's equations after delta-functional reduction. In particular, we find multiple first-order Hamiltonian structures for separable 2-soliton interaction kernels. We show explicitly the computed structures for the general separable case, for partially inhomogeneous hard rod gas and for hard rod gas, presenting the operators in arbitrary n number of components.

We finally discuss some future perspectives.

This is a joint work with E.V. Ferapontov and it is based on [3]

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P25

On existence, uniqueness and stability of multi-solitons for the generalized Benjamin Ono equations

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In this talk we will present our recent results concerning on the existence, uniqueness and stability of multi-solitons for the following generalized Benjamin Ono (gBO) equations

$$u_t + (-|D|u + |u|^{p-1} u)_x = 0, \qquad p \neq 3.$$

We constructed its strongly interacting N-solitons and showed the uniqueness of which for $N = 2$ and mass supercritical case $p > 3$. Compare to logarithmic relative distance of each solitons for the gKdV and gNLS equations, the relative distance of each soliton of gBO is $\sqrt{t}$. We will also consider the stability of N-solitons for the completely integrable BO equation. It is more likely a two dimensional integrable system, the recursion operator of which is implicit. By employing some IST, we finished the spectral anlysis of the recursion operators and second variation operator of the Lyapunov functional of N-solitons. Our approach in the spectral analysis consists in an invariant for the multi-solitons and new operator identities motivated by the bi-Hamiltonian structure of the BO equation. Our argument reveals that stability of one soliton in $H^{\frac{1}{2}}$ implies stability of N-solitons in $H^{\frac{N}{2}}$.This is a joint work with Yang Lan at Yau Center of Tsinghua University.