Soliton gas: from theory to water waves and optical fiber experiments

Pierre Suret, University of Lille (France)

Solitons are fundamental and ubiquitous objects in the field of nonlinear dispersive wave propagation [1]. In integrable systems (described by equations such as the one-dimensional nonlinear Schrödinger equation-1DNLSE), solitons exhibit particle-like properties such as elastic, pairwise interactions characterized by phase and position shifts [2]. Soliton gas (SG) can be defined as a large ensemble of interacting solitons characterised by random amplitude and phase distributions. The theoretical concept of SG has been introduced in 1971 by Zakharov who has considered a rarefied gas in which solitons are well separated and weakly interact [2]. The theory of SG has been extended to dense soliton gas in which the solitons significantly overlap and continuously and strongly interact [3].

The Inverse Scattering Transform (IST) is the main tool for the analysis of integrable nonlinear dispersive partial differential equations and of solitons. In this framework, solitons are associated with spectral parameters, the so-called IST discrete eigenvalues λ which are constants of motion. In the theory of SG, the central object is the density of state (DOS)-the distribution of spectral parameters λ- defined by using the IST [3]. It is only very recently that SGs experiments (performed in water tank) have been analysed in the framework of IST [4].

In this talk, we review the recent theoretical and experimental results in the field of SG. In particular, we will focus on the fundamental phenomenon of spontaneous modulation instability (MI) arising in the focusing regime of the 1DNLSE [5-8].

References

[1] M. Remoissenet, Waves Called Solitons. Concepts and Experiments (Springer, 2013), 4th ed.
[2] V. Zakharov, “Kinetic equation for solitons,” Sov. Phys. JETP 33, 538–540 (1971).
[3] G. El and A. Tovbis, “Spectral theory of soliton and breather gases for the focusing nonlinear Schrodinger equation,” Phys. Rev. E 101, 052207 (2020).
[4] P. Suret, A. Tikan, F. Bonnefoy, F. Copie, G. Ducrozet, A. Gelash, G. Prabhudesai, G. Michel, A. Cazaubiel, E. Falcon, G. El, and S. Randoux, “Nonlinear spectral synthesis of soliton gas in deep-water surface gravity waves,” Phys. Rev. Lett. 125, 264101 (2020).
[5] D. Agafontsev and V. E. Zakharov, “Integrable turbulence and formation of rogue waves,” Nonlinearity 28, 2791 (2015).
[6] A. E. Kraych, D. Agafontsev, S. Randoux, and P. Suret, “Statistical properties of the nonlinear stage of modulation instability in fiber optics,” Phys. Rev. Lett. 123, 093902 (2019).
[7] Gelash, A., Agafontsev, D., Zakharov, V., El, G., Randoux, S., & Suret, P.. Bound state soliton gas dynamics underlying the spontaneous modulational instability. Phys. Rev. Lett., *123*(23), 234102. (2019)
[8] A. Lebel, A. Tikan, S. Randoux, P. Suret, and F. Copie, “Single-shot observation of breathers from noise-induced modulation instability using heterodyne temporal imaging,” Opt. Lett. 46, 298–301 (2021).