Emergent hydrodynamics of soliton gases in nonlinear dispersive waves

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Soliton gases represent large random ensembles of interacting solitons that display non-trivial emergent, macroscopic, behaviours ultimately determined by the properties of the elementary two-soliton collisions. Originally introduced by V. Zakharov in 1971 [1] the concept of soliton gas has recently attracted significant interest from both mathematics and physics communities. The hydrodynamics of non-equilibrium dense soliton gases in integrable dispersive systems such as the Korteweg-de Vries and nonlinear Schrödinger equations is described by a nonlinear integro-differential kinetic equation for the density of states in the spectral (Lax) phase space [2, 3, 4] The key ingredient of the spectral theory of soliton gases are the nonlinear dispersion relations arising as the thermodynamic limit of the classical wavenumber-frequency relations in the finite-gap theory of nonlinear multiphase waves. In my lecture, I will outline the main ideas of the spectral theory of soliton gases and its parallels with generalised hydrodynamics—the hydrodynamic theory for many-body quantum and classical integrable systems [5, 6]. The properties of the special class of soliton gases—the so-called soliton condensates, and their connections with the fundamental objects and phenomena of dispersive hydrodynamics such as dispersive shock and rogue waves will be discussed [7, 8].

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