GNSS Multipath: Characterization, Modeling & Mitigation

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Dr. Gary A. McGraw, PNT Consultant
Genova Technologies
Phone: +1-319-210-9707
Email: mcgraw.pnt@gmail.com

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Overview

Part 1: GNSS Multipath Characterization & Modeling

- GNSS multipath introduction & definitions
- Multipath effects on GNSS measurements
  - SNR
  - Pseudoranges (code)
  - Carrier Phase
- Multipath mitigation techniques
  - Code type
  - Antenna design & siting
  - Receiver signal processing
  - Measurement processing

Part 2: Multipath Mitigation via Measurement Processing

- GNSS measurement combinations
  - Wide and narrow-lane carrier phase
  - Ionospheric free
  - Ionospheric estimation
  - Divergence free
- Carrier smoothed code processing
  - Processing overview
  - Smoothing filter gain
  - Divergence-free smoothing

Part 1: GNSS Multipath Characterization & Modeling

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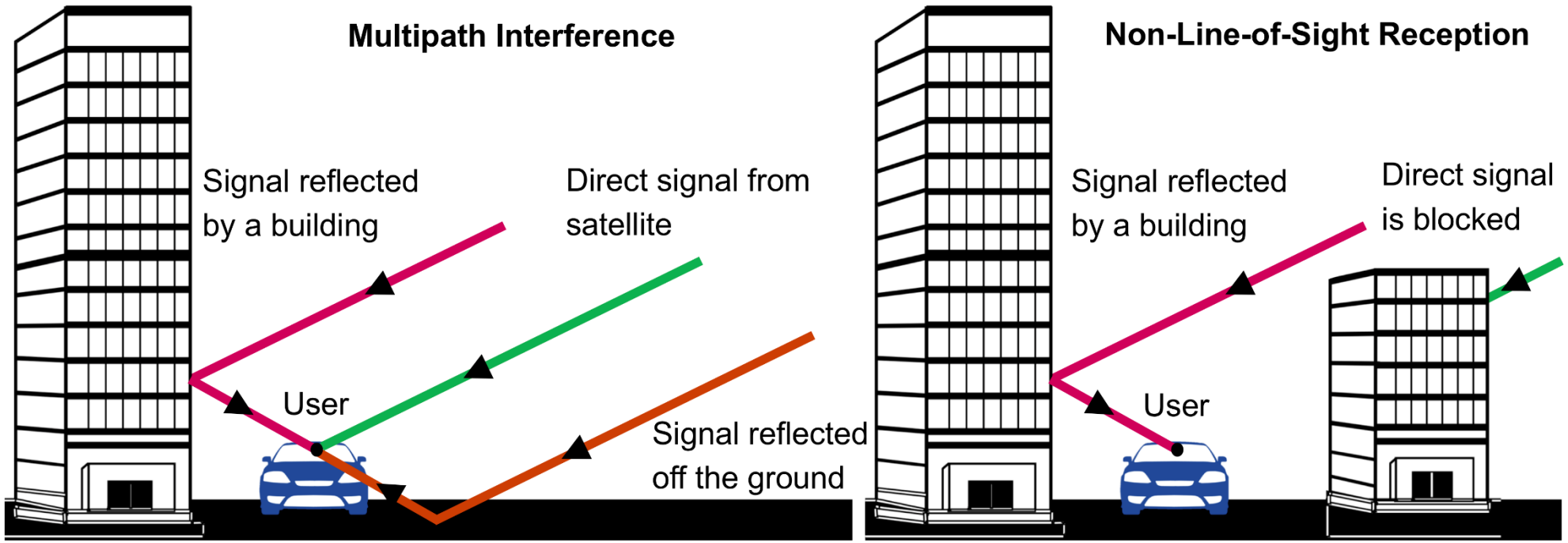
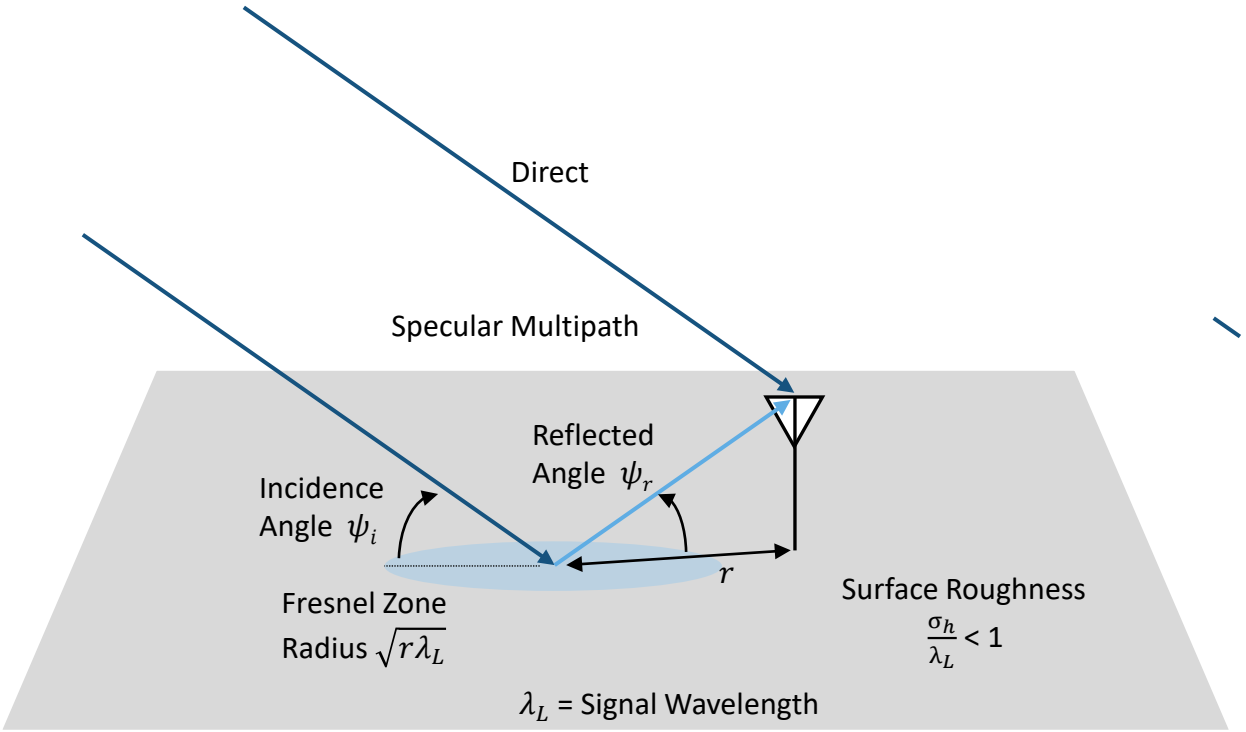
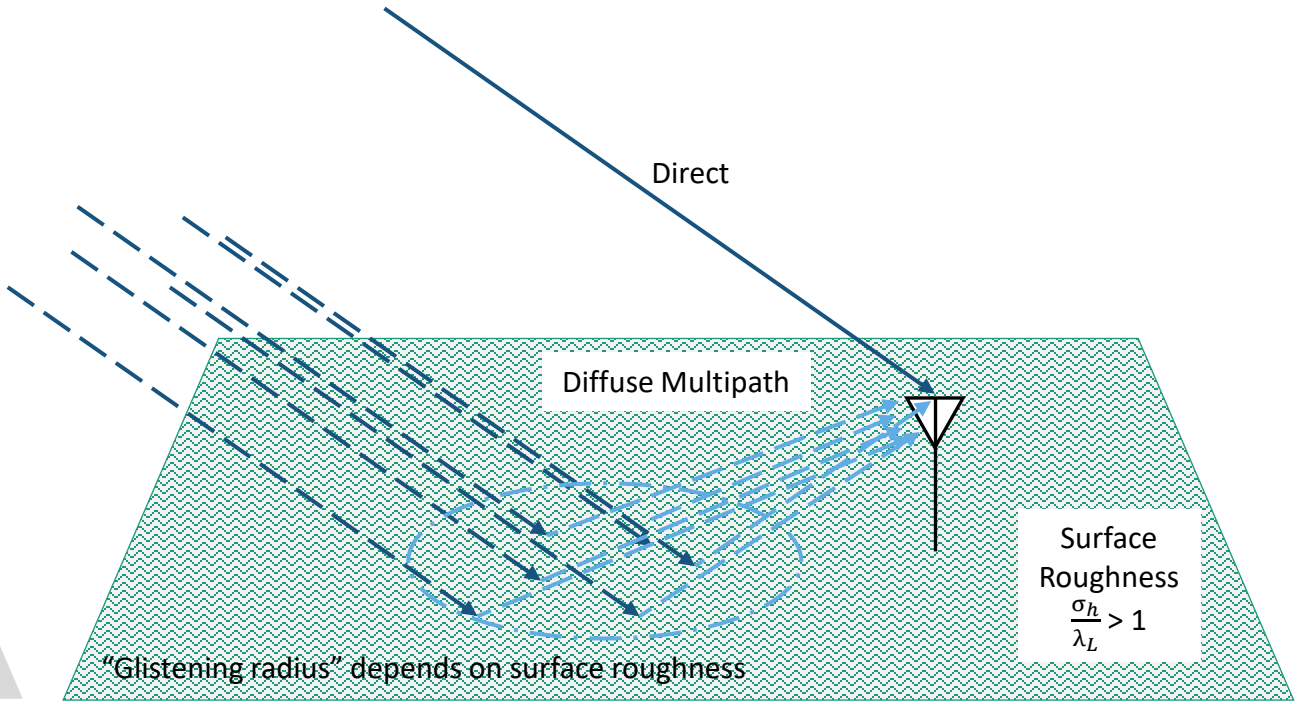
Multipath vs. Non-LOS Reception

- Multipath = Multiple signal propagation paths, including direct signal
- Non-LOS reception = Direct signal is blocked, but strong reflected signals are present



Specular vs. Diffuse Multipath

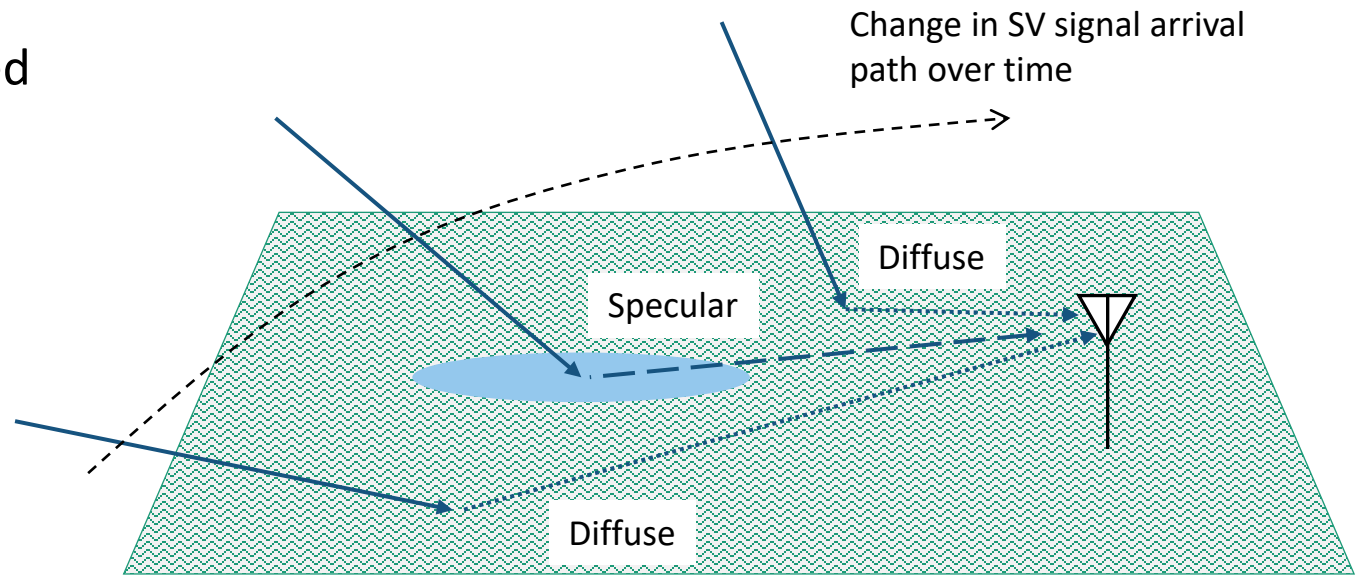


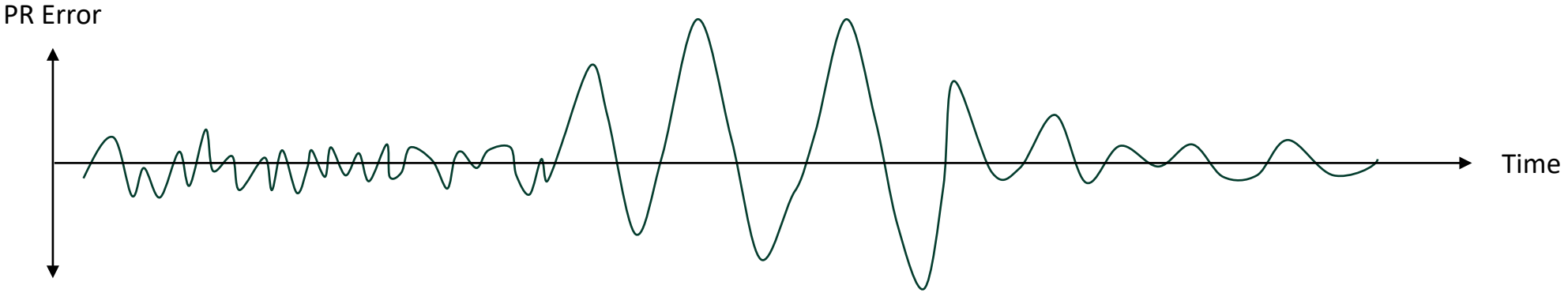


- For specular reflection $\psi_r = \psi_i$
- Amplitude of multipath dependent on surface composition
- GNSS signals are right-hand circularly-polarized (RHCP) signals; multipath usually dominated by left-hand circularly-polarized (LHCP) signals

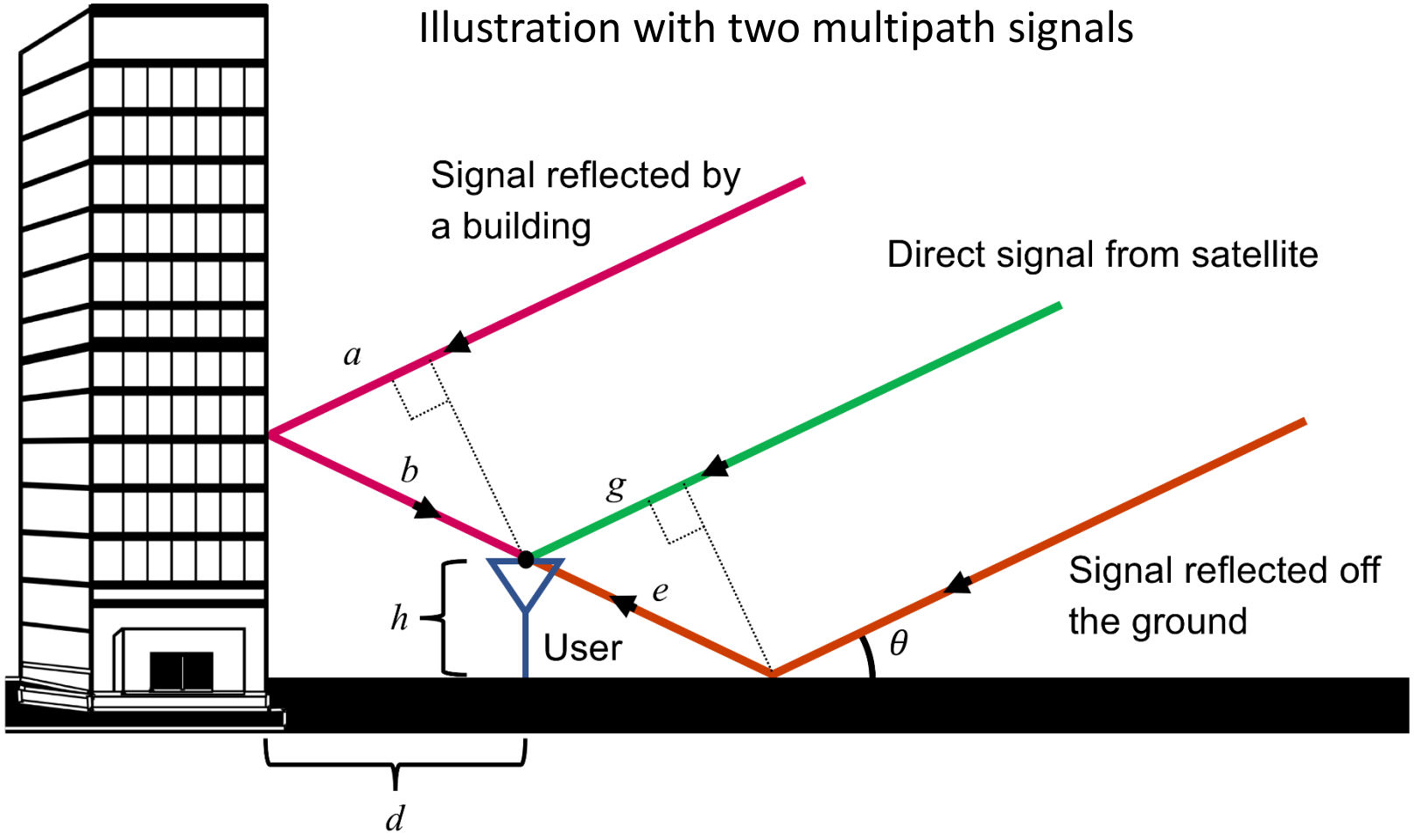
Multipath Error Characteristics

- Diffuse multipath appears like bandlimited noise
- Strong specular multipath has sinusoidal characteristics
- Often both types are present





Specular Multipath Characterization



Received Signal Model

$$r(t) = \sqrt{2P}\sum_{i=0}^{n}\Big\{\alpha_i C(t - t_0 - \Delta_i/c)D(t - t_0 - \Delta_i/c) \times \exp\Big(j\big[2\pi\big(f_L + f_D + \delta f_i\big)\big(t - t_0 - \Delta_i/c\big) + \phi_i\big]\Big)\Big\} + w(t)$$

P = direct signal power

n = number of reflected signals (i=0 is the direct signal)

α_i = relative amplitude of reflected signals ($\alpha_0 = 1$)

$C(\cdot)$ = pseudorandom noise (PRN) spreading code

$D(\cdot)$ = downlink data

t_0 = propagation delay for the direct signal (sec)

c = speed of light (m/s)

f_L = carrier frequency (Hz)

f_D = Doppler shift (Hz)

δf_i = relative multipath Doppler (Hz)

Δ_i = relative multipath delay (m)

ϕ_i = phase shift relative to direct (rad)

$w(\cdot)$ = bandlimited white Gaussian noise (WGN)

Relative Multipath Amplitude

$$\alpha_i = \sqrt{\frac{G_i R_i k_i}{G_0 R_0 k_0}}$$

G_0 = antenna gain for direct signal
G_i = antenna gain for the ith multipath component
R_i and R_0 = reflection coefficients (R_0 = 1 in our case)
k_i and k_0 = signal attenuation coefficients (due to foliage, etc.)

- Antenna gain for direct signal typically ranges from -6 dB to +3 dB
- Multipath antenna gain typically smaller than direct – but not true for mobile devices!
- Reflection coefficients depend on the properties of the reflecting surface
  - Calm water, metal, glass can have reflection coefficients as large as 0.5-0.7
  - Other surfaces will have lower reflection coefficients
- Attenuation coefficients ~1, unless foliage or scattering is present

Multipath Delay & Phase

Ground Reflected Signal Delay

$$\Delta_i = e - g = 2h\sin(\theta) \text{ (m)}$$

Building Reflected Signal Delay

$$\Delta_i = a - b = 2d\cos(\theta) \text{ (m)}$$

Phase Shift

$$\phi_i = \left(\frac{2\pi\Delta_i}{\lambda_L} + \phi_{Ri}\right) \text{MOD} 2\pi \text{ (rad)}$$

ϕ_{Ri} = Phase shift at reflection = π rad when incidence angle is less than the Brewster angle

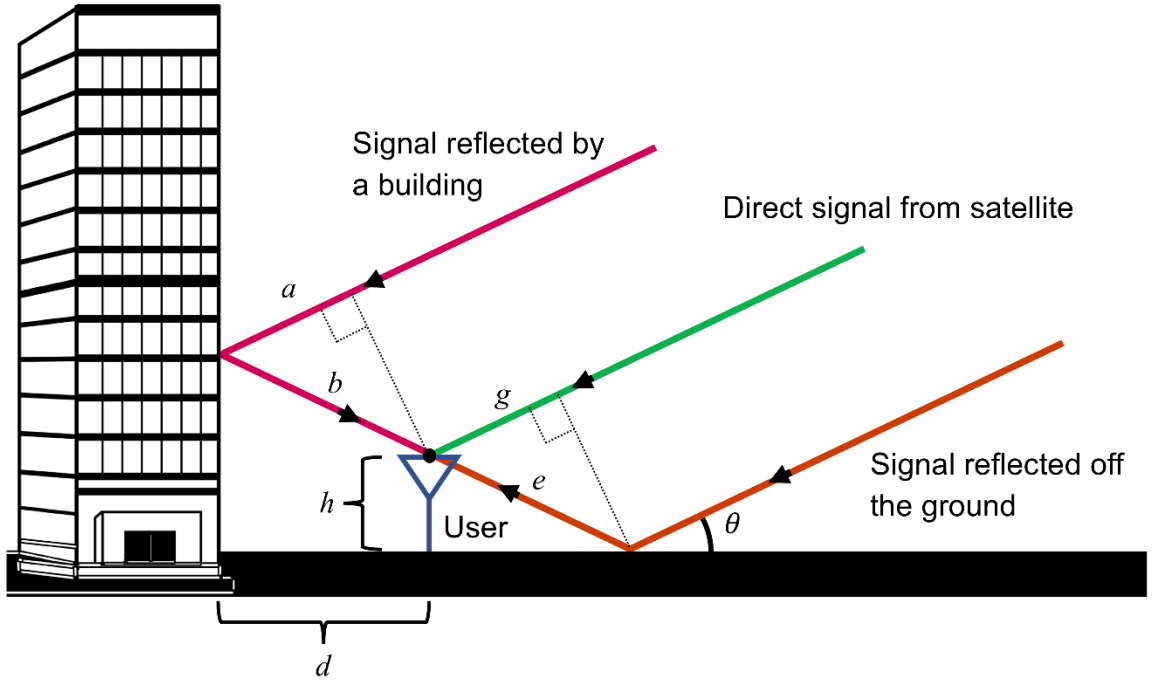
λ_L = wavelength (m)

Multipath delay in code chips

$$\delta_i = \Delta_i/\lambda_C = \Delta_i/cT_C$$

λ_C = PRN code chip length (m)

T_C = PRN code period (s)



- Delay increases with antenna height / distance
- Elevation angle greatly influences multipath characteristics

Multipath Fading Frequencies - Ground Reflection Relative Doppler

Ground Reflected Signal

$$\delta f_i = \left(\frac{2}{\lambda_L}\sin\theta\right)\frac{\partial h}{\partial t} - \left(\frac{2h}{\lambda_L}\cos\theta\right)\frac{\partial \theta}{\partial t}$$

Example:

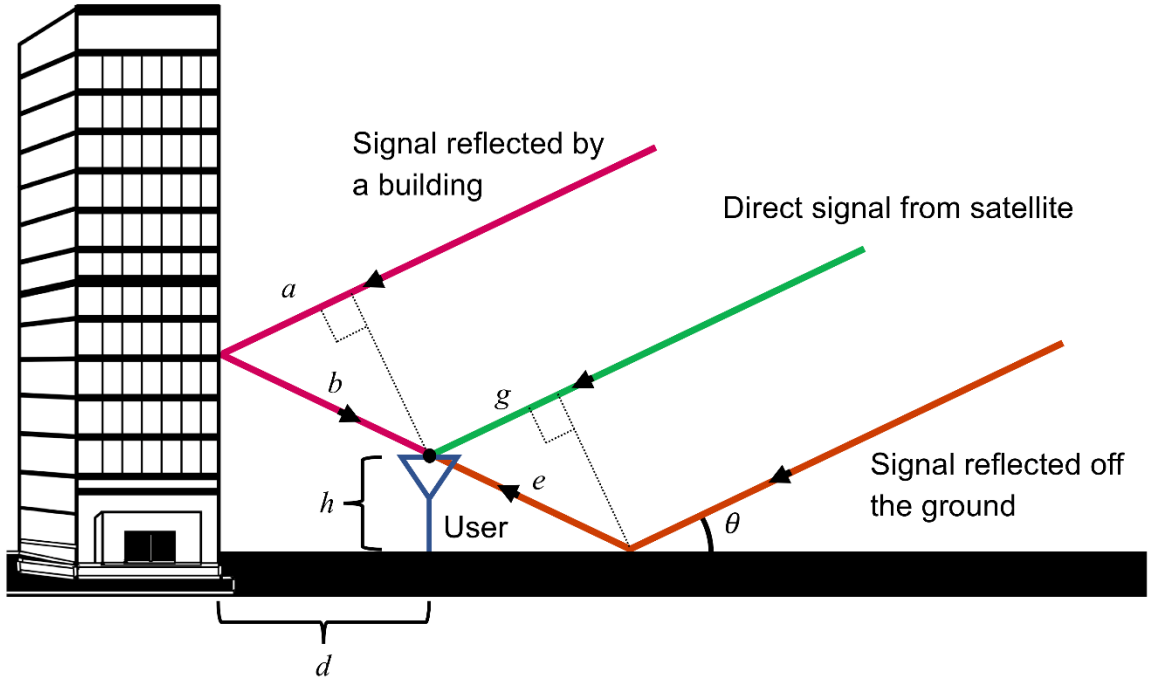
$h = 1$ m (fixed)

GNSS satellite angular rate:

$$\frac{\partial \theta}{\partial t} \approx 180\ \mathrm{deg}/\ 6\ \mathrm{hrs} \approx 0.15\ \mathrm{mrad/s}$$

L1 wavelength: λ_L=19 cm

$$\Rightarrow \begin{cases} \delta f_i \approx 1.6\ \text{mHz near the horizon} \\ \delta f_i \approx 0\ \text{near zenith} \end{cases}$$



- Frequencies dependent on relative satellite and antenna motion
- LEO satellite orbital angular rate ~10X faster than GNSS

Multipath Fading Frequencies – Building Reflected Relative Doppler

<u>Building Reflected Signal</u>

$$\delta f_i = \left(\frac{2}{\lambda_L}\cos\theta\right)\frac{\partial d}{\partial t} - \left(\frac{2d}{\lambda_L}\sin\theta\right)\frac{\partial \theta}{\partial t}$$

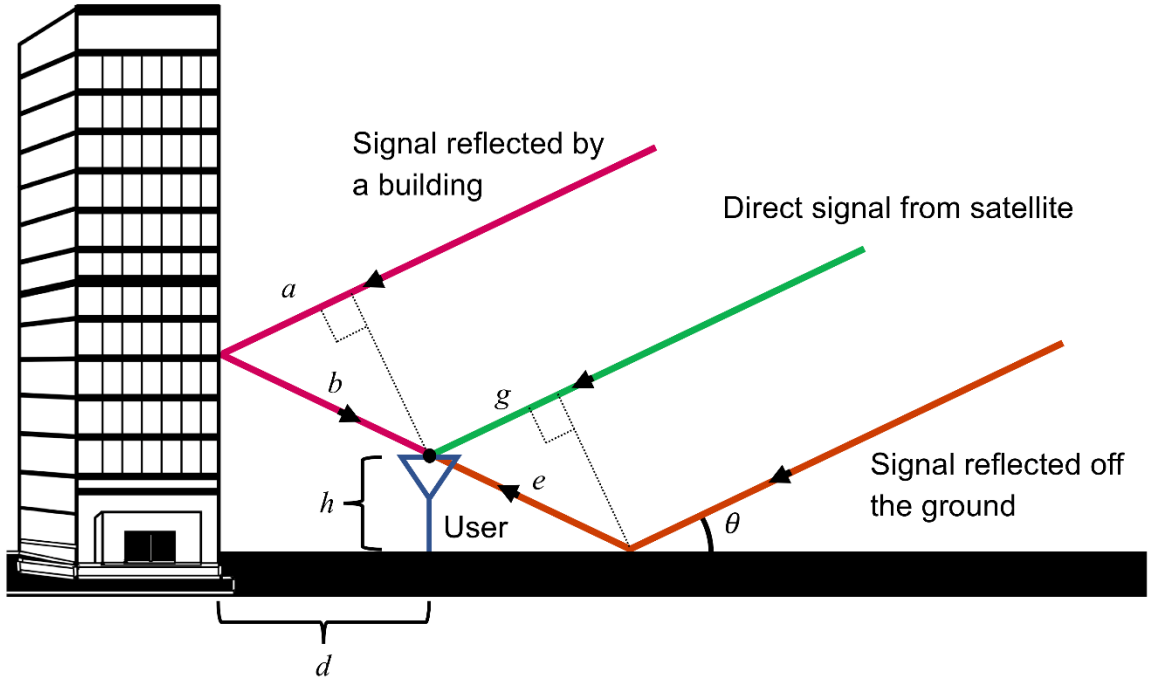
Example:

Antenna horizontal speed: $\frac{\partial d}{\partial t} = 1$ m/s

Satellite elevation angle: $\theta = 30°$

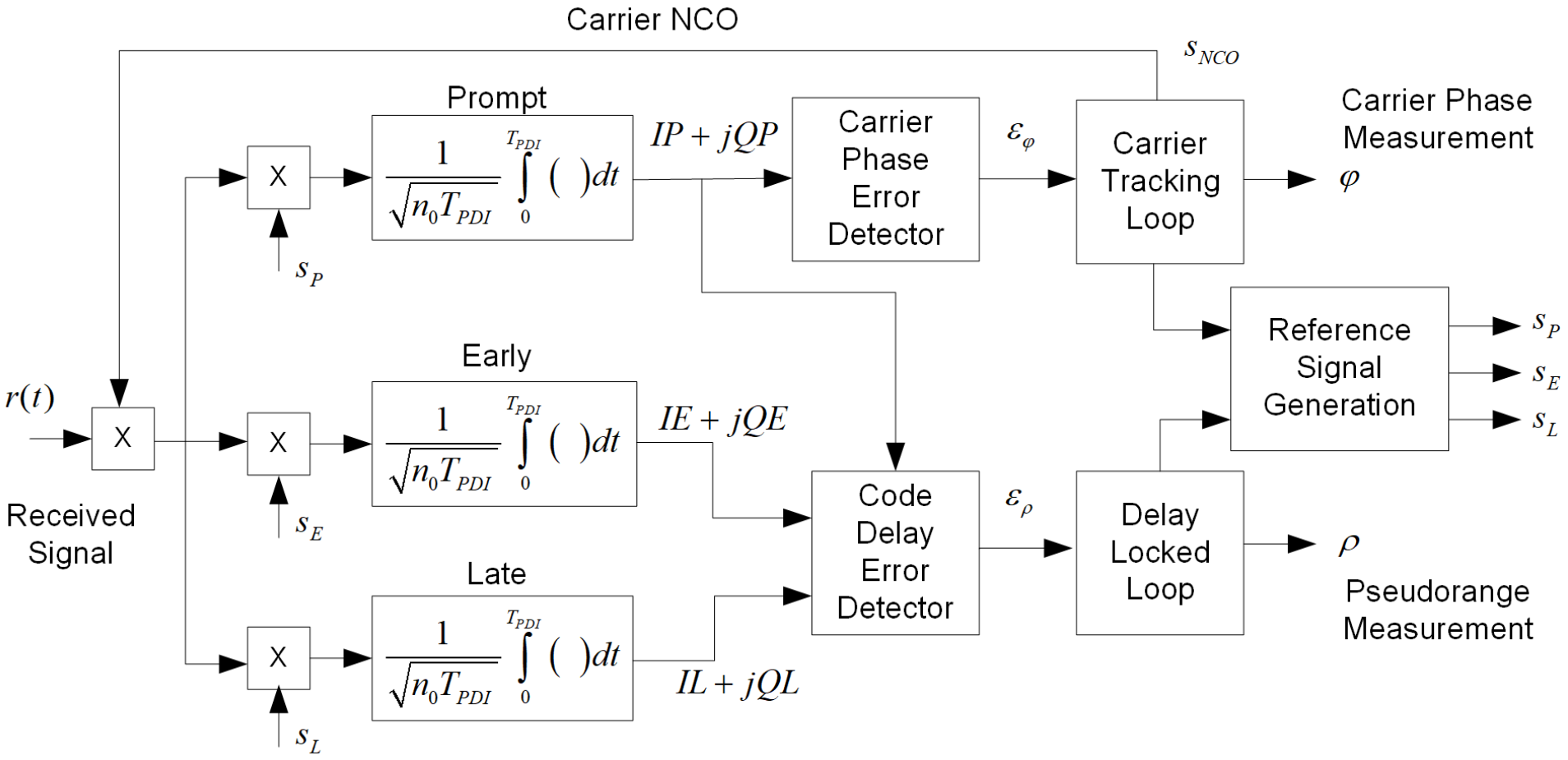
L1 wavelength: λ_L=19 cm

$\delta f_i \approx 5.3$ Hz



- At higher speeds, fading frequency would exceed carrier tracking loop bandwidth and would appear as noise
- Effects due to satellite motion similar to ground bounce case

Receiver Signal Processing Block Diagram



Correlator Output Signals

$$IE = \sqrt{2(c/n_0)T_{PDI}} \sum_{i=0}^{n} \left[\alpha_i R(\tau - \delta_i T_C + dT_C) \operatorname{sinc}\left(\pi(\delta f + \delta f_i) T_{PDI}\right) \cos(\delta\varphi + \phi_i) \right] + w_{IE}$$

$$IP = \sqrt{2(c/n_0)T_{PDI}} \sum_{i=0}^{n} \left[\alpha_i R(\tau - \delta_i T_C) \operatorname{sinc}\left(\pi(\delta f + \delta f_i) T_{PDI}\right) \cos(\delta\varphi + \phi_i) \right] + w_{IP}$$

$$IL = \sqrt{2(c/n_0)T_{PDI}} \sum_{i=0}^{n} \left[\alpha_i R(\tau - \delta_i T_C - dT_C) \operatorname{sinc}\left(\pi(\delta f + \delta f_i) T_{PDI}\right) \cos(\delta\varphi + \phi_i) \right] + w_{IL}$$

$$QE = \sqrt{2(c/n_0)T_{PDI}} \sum_{i=0}^{n} \left[\alpha_i R(\tau - \delta_i T_C + dT_C) \operatorname{sinc}\left(\pi(\delta f + \delta f_i) T_{PDI}\right) \sin(\delta\varphi + \phi_i) \right] + w_{QE}$$

$$QP = \sqrt{2(c/n_0)T_{PDI}} \sum_{i=0}^{n} \left[\alpha_i R(\tau - \delta_i T_C) \operatorname{sinc}\left(\pi(\delta f + \delta f_i) T_{PDI}\right) \sin(\delta\varphi + \phi_i) \right] + w_{QP}$$

$$QL = \sqrt{2(c/n_0)T_{PDI}} \sum_{i=0}^{n} \left[\alpha_i R(\tau - \delta_i T_C - dT_C) \operatorname{sinc}\left(\pi(\delta f + \delta f_i) T_{PDI}\right) \sin(\delta\varphi + \phi_i) \right] + w_{QL}$$

c/n_0 = carrier-power-to-noise-density ratio (ratio-Hz),

$R(\cdot)$ = PRN code autocorrelation function

$\tau = \hat{t}_0 - t_0$ = code tracking error (s)

δf = carrier frequency tracking error (Hz)

$\delta\varphi$ = carrier phase tracking error (Hz)

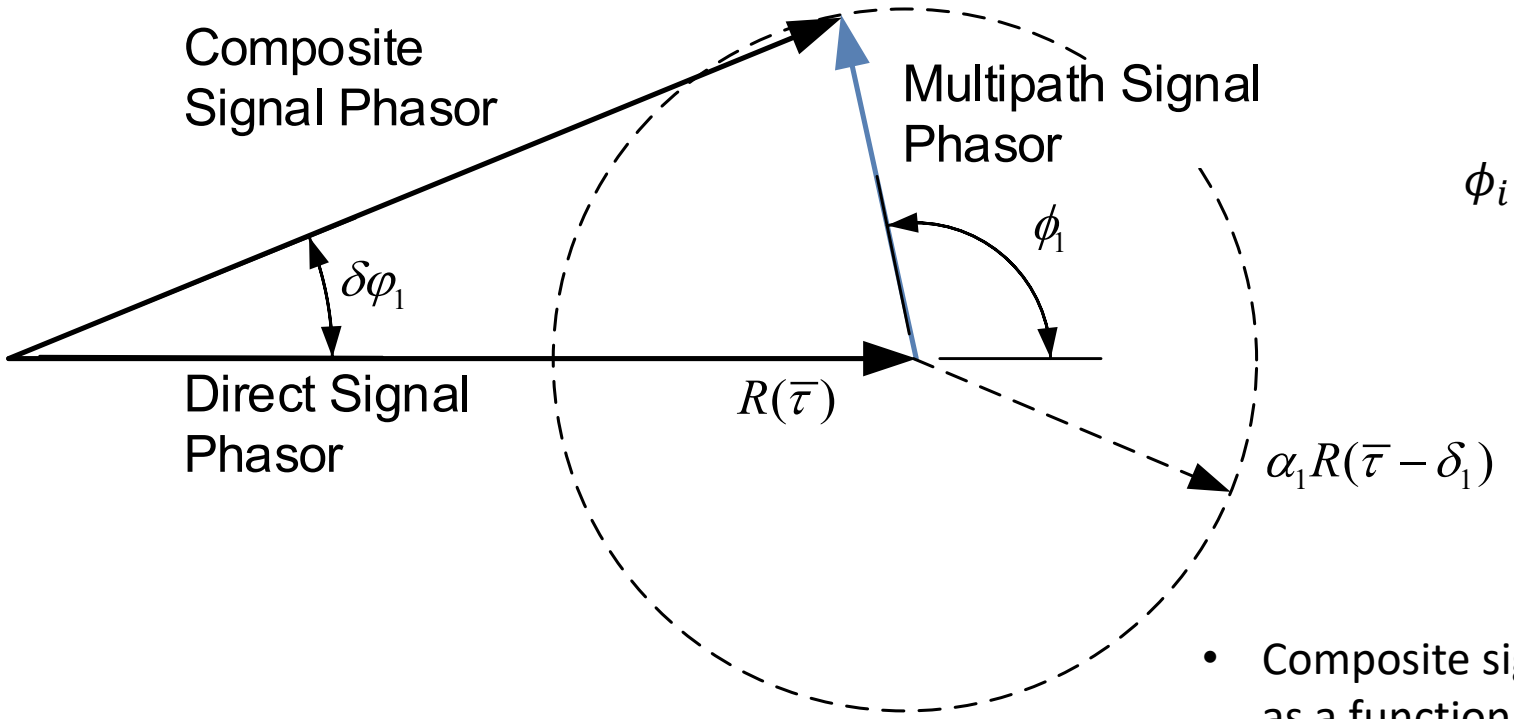
$$\operatorname{sinc}(\theta) = \begin{cases} \sin\theta/\theta, & \theta \neq 0 \\ 1, & \theta = 0 \end{cases}$$

I and *Q* denote in-phase and quadra-phase, *E*, *P* and *L* denote early, prompt and late

$w_{IE}, w_{IP}, w_{IL}, w_{QE}, w_{QP}, w_{QL}$ = I/Q WGN (zero mean and unit variance)

Ideal autocorrelation function for BPSK signals: $R(\tau) = E\{C(t)C(t-\tau)\} = \begin{cases} 1 - |\tau/T_C|, & |\tau| < T_C \\ 0, & |\tau| \geq T_C \end{cases}$

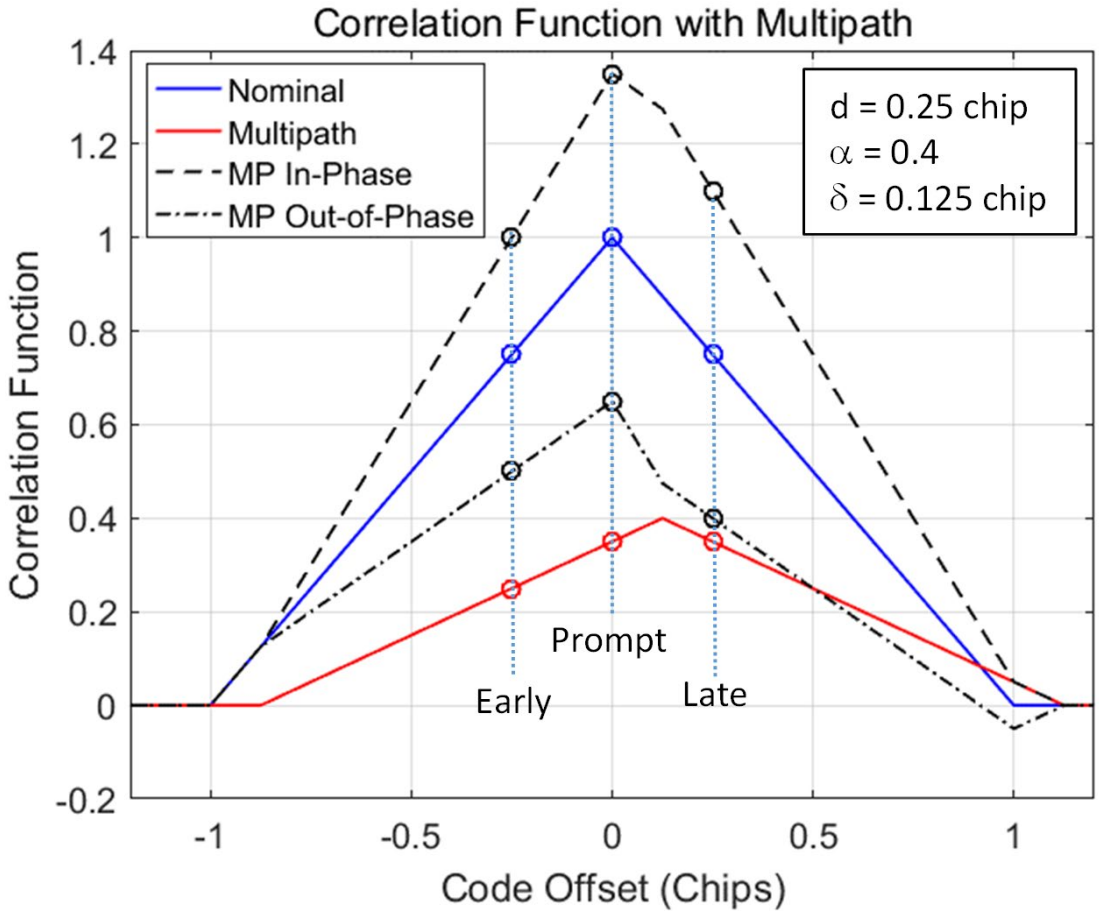
Composite Signal with Single Multipath



$$\phi_i = \left(\frac{2\pi\Delta_i}{\lambda_L} + \phi_{Ri}\right) \text{MOD} 2\pi \text{ (rad)}$$

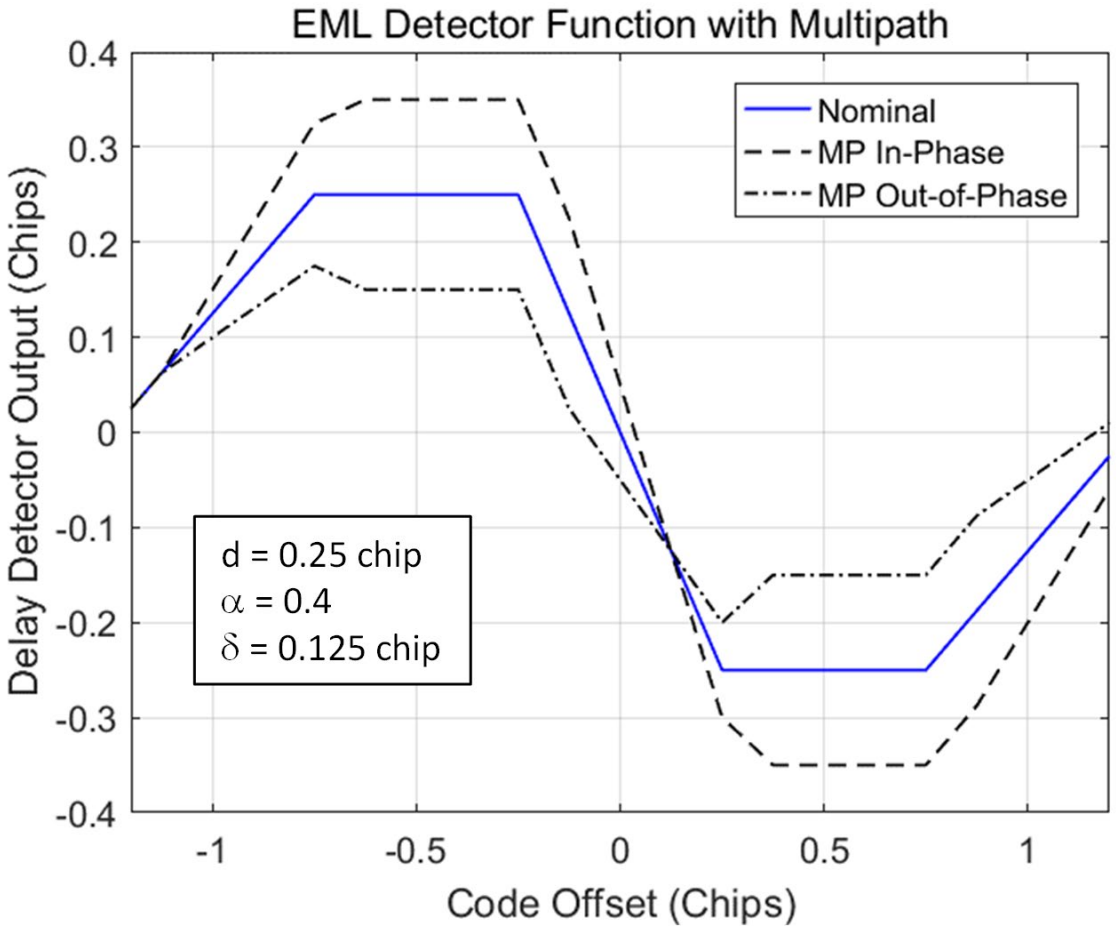
- Composite signal amplitude fluctuates as a function of multipath phase angle
- Deep fading can cause loss of lock

Ideal Code Correlation Functions for Single Multipath



- Binary Phase-Shift Key (BPSK) signal
- Infinite bandwidth
- Multipath distorts the shape of the correlation function

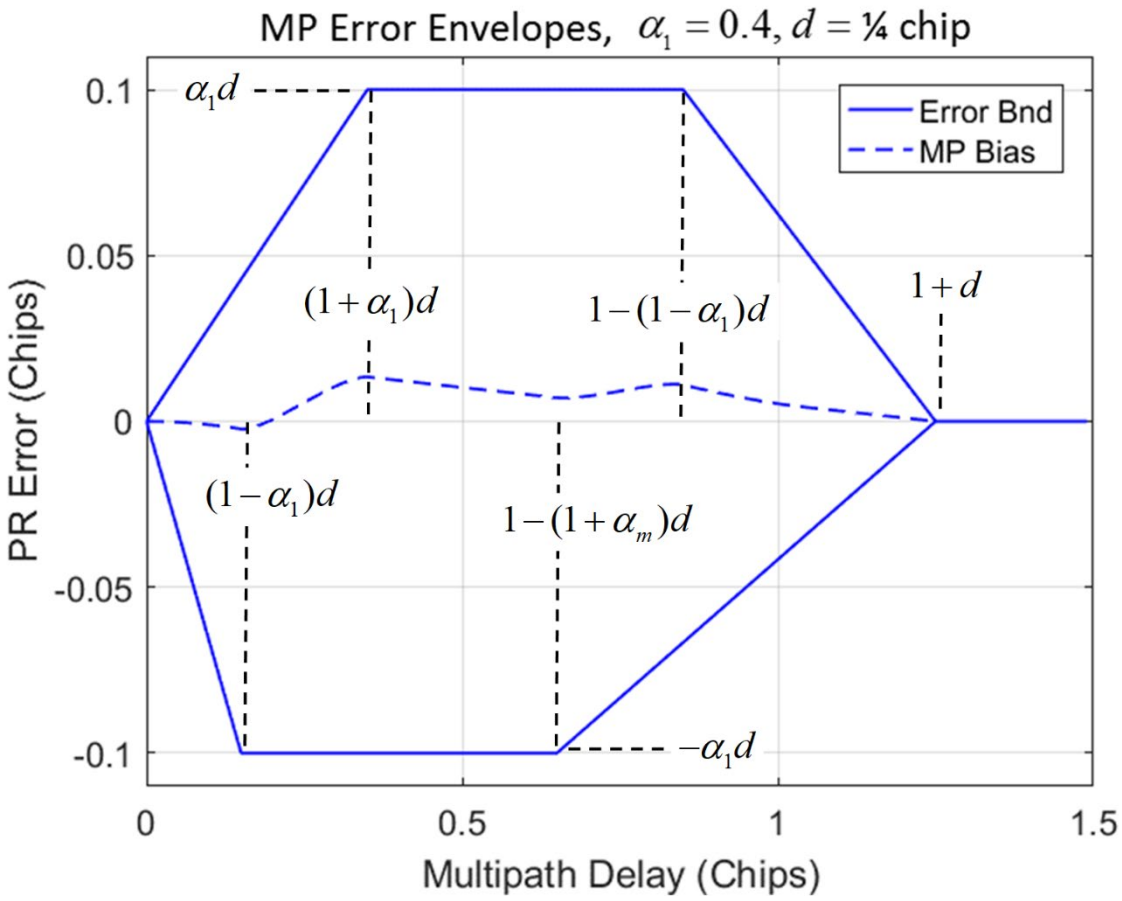
EML Code Tracking Error Discriminator



Early-Minus-Late (EML) Delay Lock Detector (DLD) function:

$$D_{EML}(\tau) = [R(\tau + dT_C) - R(\tau - dT_C)]/2$$

Ideal Code Tracking Error Envelopes



- Dot-product code tracking error detector:

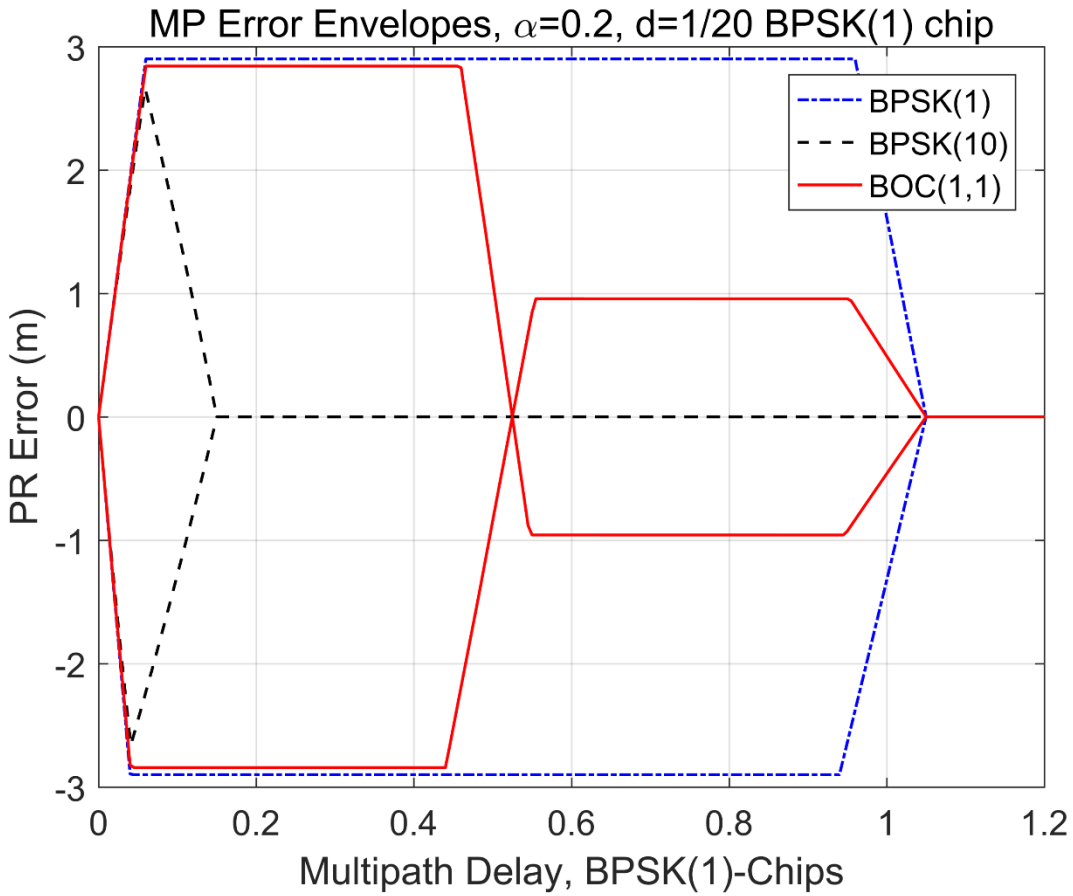
$$\varepsilon_D = \frac{ID_{EML} \cdot IP - QD_{EML} \cdot QP}{IP^2 + QP^2},$$

$$ID_{EML} = (IE - IL)/2, \quad QD_{EML} = (QE - QL)/2$$

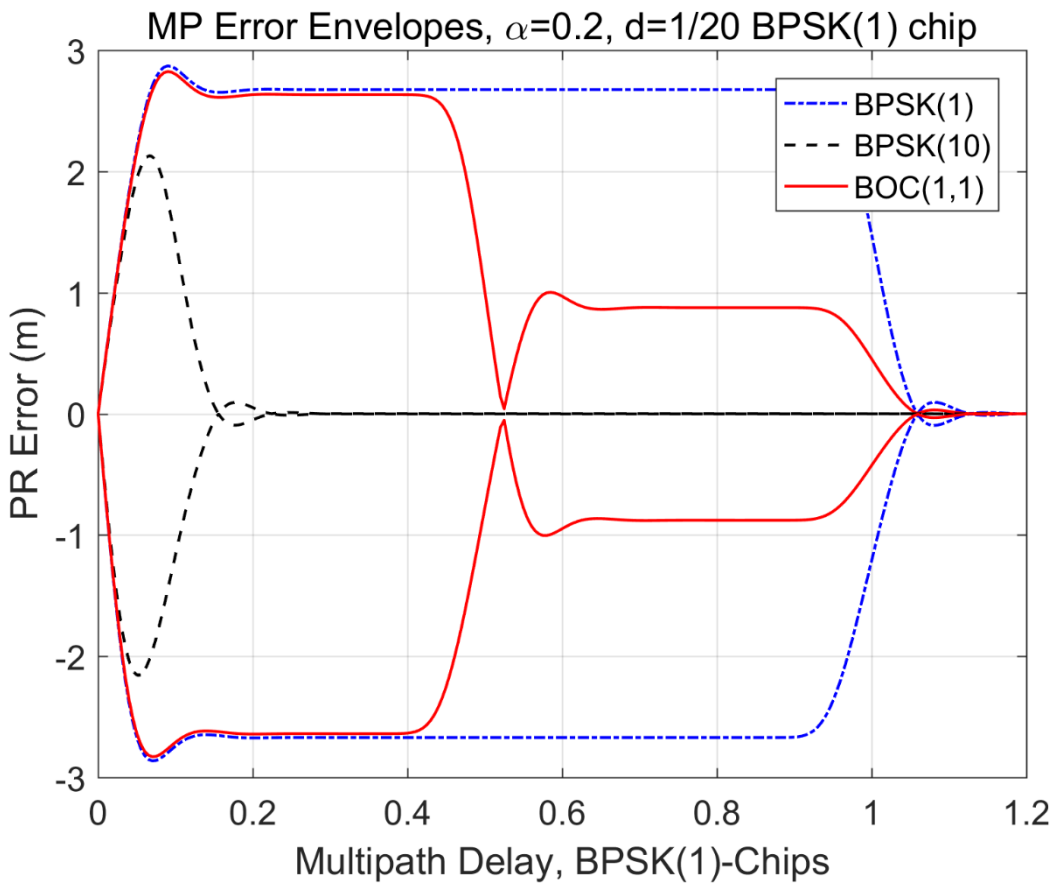
- Ideal PRN code and infinite receiver bandwidth assumed
- Bounds represent perfect in-phase or out-of-phase multipath cases ($\theta = 0, \pi$)
- Other multipath phases will lie in between these two bounds
- MP bias represents average over full phase cycle at a given MP delay – MP is not zero mean
- Multipath with delay bigger than 1+d chip has little or no effect on PR measurements

Multipath Code Tracking Error Envelopes for Different Code Types

Infinite bandwidth

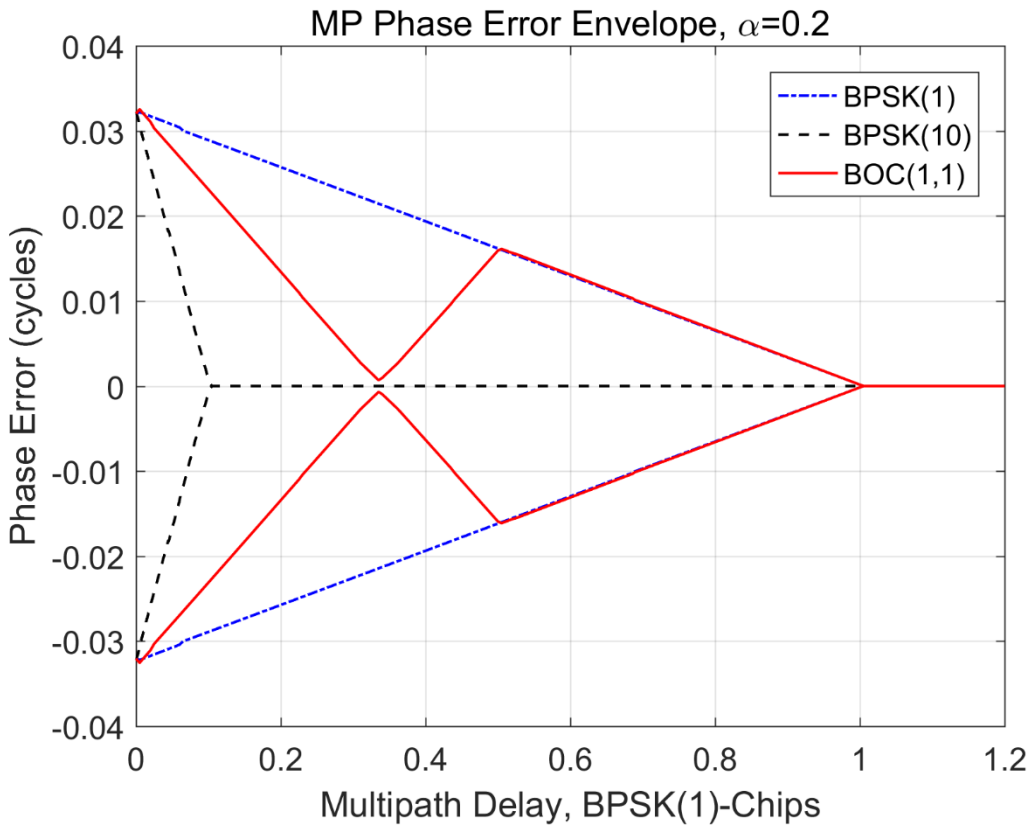


Band-limited with 10 MHz low-pass filter

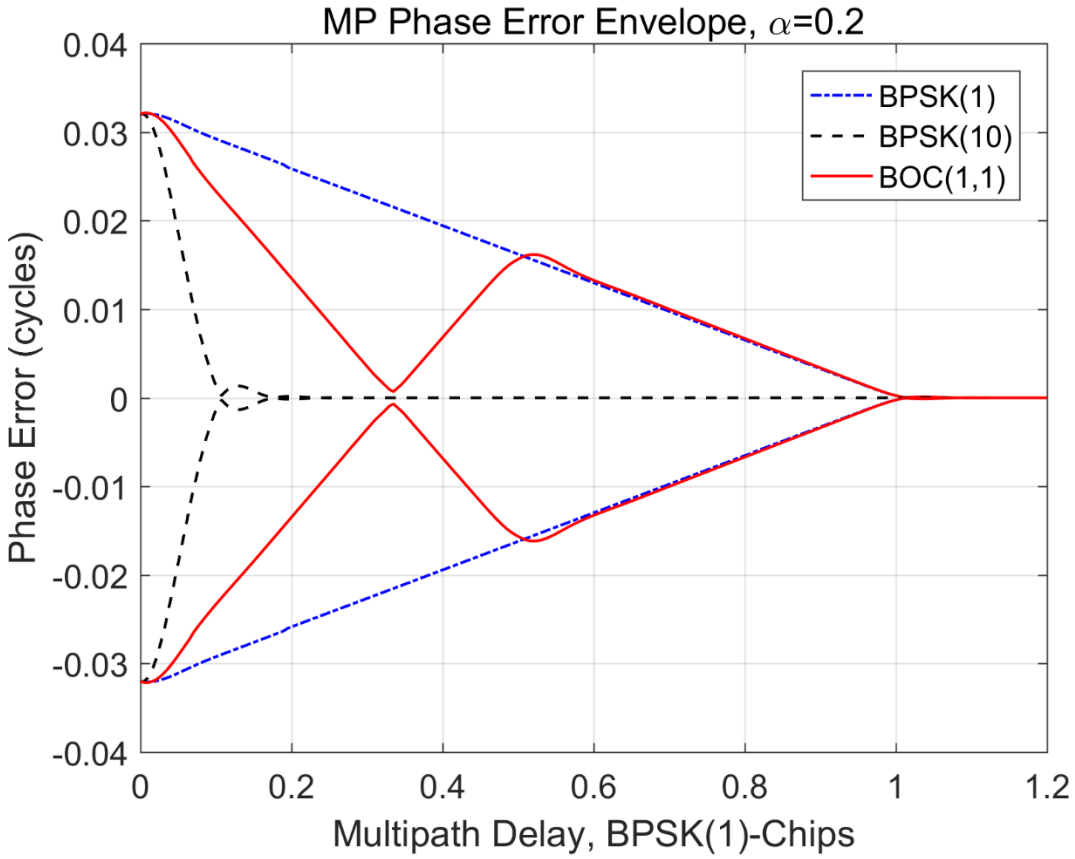


Multipath Phase Tracking Error Envelopes for Different Code Types

Infinite bandwidth



Band-limited with 10 MHz low-pass filter



GNSS Multipath Mitigation Techniques

- Code type
- Antenna design & siting
- Receiver design
  - Adaptive antenna array processing
  - Polarization processing
  - Correlator signal processing
  - Multipath estimation

  Not discussed here
- Measurement processing
  - Code & carrier combinations
  - Carrier smoothing

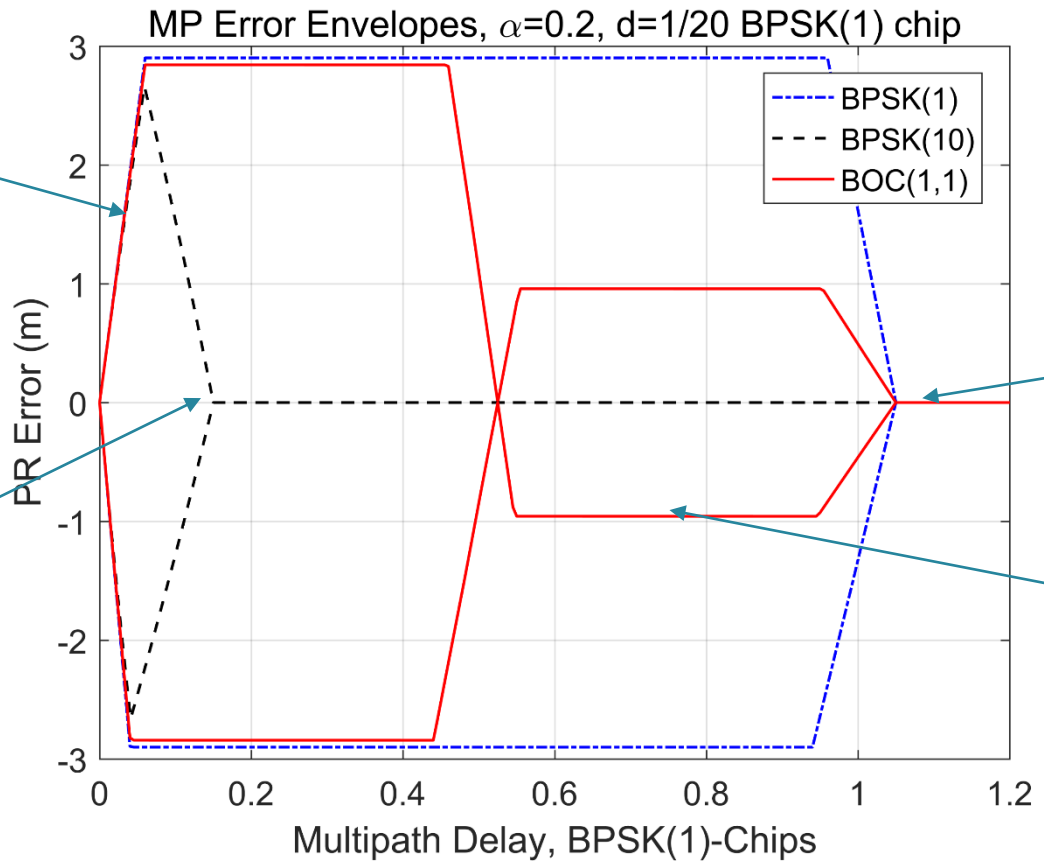
  Discussed in Part 2

Code Type

Higher chipping rates have improved multipath error characteristics

All code types have the same error for short multipath delays

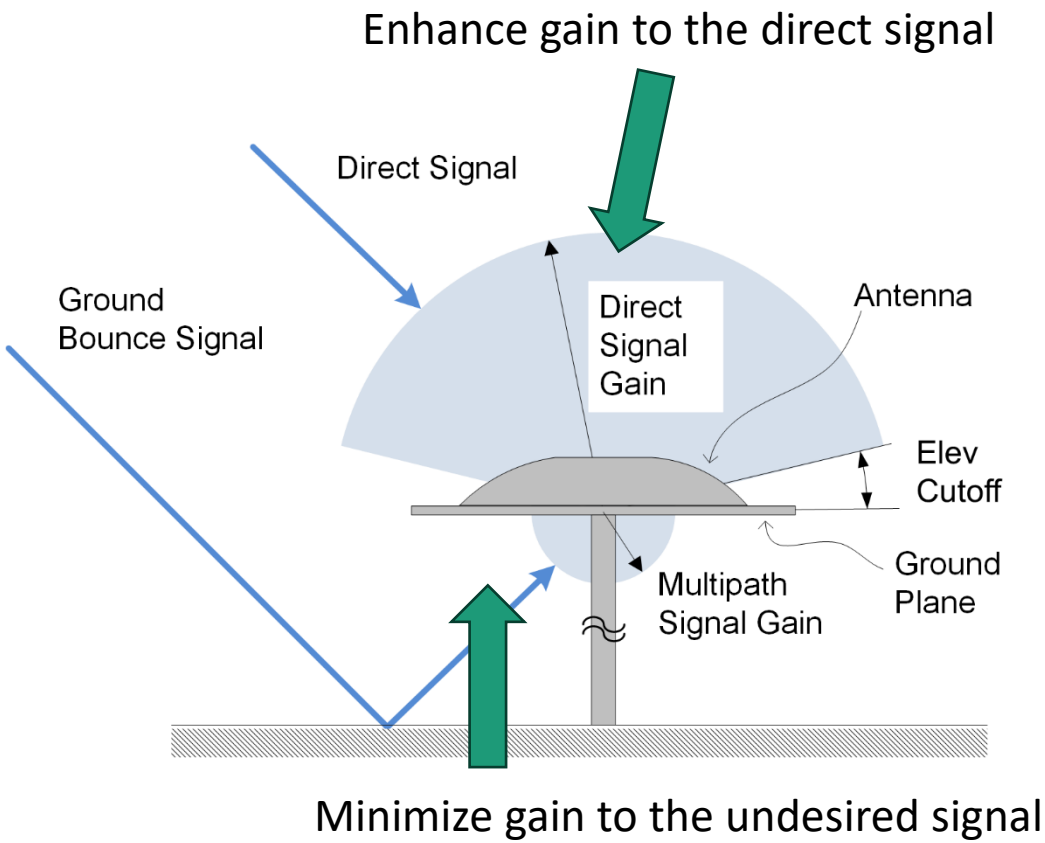
BPSK(10) signals immune to multipath with delays >40 m
- GPS P(Y) code L1 & L2
- GPS L5
- Galileo E5a, E5b
- Beidou B2a



BPSK(1) signals immune to multipath with delays >300 m
- GPS C/A code
- L1 SBAS

BOC(1,1) have reduced response to longer delay multipath
- GPS L1c
- Galileo E1 OS
- Beidou B1c

Antenna Design & Siting



Siting

- Move antenna away from strong reflectors
- Raise antenna above reflecting objects in the vicinity

Try to increase Direct/Undesired (D/U) signal ratio

Part 2: GNSS Measurement Processing & Carrier Smoothing

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Overview

- Measurement models
- Dual frequency code & carrier measurement combinations
  - Ionospheric-free
  - Wide-Lane (WL) / Narrow-Lane (NL)
  - Geometry-free
  - Divergence-free
- Code-Carrier smoothing
  - Single frequency
  - Dual frequency divergence free

Measurement Model @ f_L

$$\rho_L = r + \delta_T + \delta_R + I_L + T + \delta\rho_{ML} + \varepsilon_{\rho L}$$
$$\varphi_L = r + \delta_T + \delta_R - I_L + T + \delta\varphi_{ML} + \varepsilon_{\varphi L} + N_L\lambda_L$$

ρ_L= Code pseudorange measurement (in meters)
φ_L= Carrier phase measurement (in meters)
r = Geometric Line-of-Sight (LOS) range
δ_T= Satellite clock and ephemeris errors projected along LOS
δ_R= Receiver clock bias
I_L= K_I/f_L^2 = Ionospheric refraction at f_L
T = Tropospheric delay
$\delta\rho_{ML}, \delta\varphi_{ML}$ = Code and carrier multipath at f_L
$\varepsilon_{\rho L}, \varepsilon_{\varphi L}$ = Code and carrier receiver noise and other errors
$N_L\lambda_L$= Carrier phase ambiguity for the carrier with wavelength λ_L, where N_L is an integer.

Simplified Measurement Model @ f_L

$$\rho_L = r + I_L + \varepsilon_{\rho L}$$
$$\varphi_L = r - I_L + \varepsilon_{\varphi L} + N_L \lambda_L$$

ρ_L= Code pseudorange measurement (in meters)
φ_L= Carrier phase measurement (in meters)
r = Geometric Line-of-Sight (LOS) range (including SV & rcvr clocks and tropo)
I_L= K_I / f_L^2 = Ionospheric refraction at f_L
$\varepsilon_{\rho L}, \varepsilon_{\varphi L}$ = Code and carrier receiver noise, multipath and other errors
$N_L \lambda_L$= Carrier phase ambiguity for the carrier with wavelength λ_L, where N_L is an integer.

Code - Carrier Combinations

Iono−Free:

$$\rho_{IF} = \frac{f_1^2}{f_1^2 - f_2^2}\rho_1 - \frac{f_2^2}{f_1^2 - f_2^2}\rho_2,$$

$$\varphi_{IF} = \frac{f_1^2}{f_1^2 - f_2^2}\varphi_1 - \frac{f_2^2}{f_1^2 - f_2^2}\varphi_2$$

Wide−Lane Carrier Phase/Narrow−Lane Code:

$$\rho_{NL} = \frac{f_1}{f_1 + f_2}\rho_1 + \frac{f_2}{f_1 + f_2}\rho_2,$$

$$\varphi_{WL} = \frac{f_1}{f_1 - f_2}\varphi_1 - \frac{f_2}{f_1 - f_2}\varphi_2$$

Narrow−Lane Carrier Phase/Wide−Lane Code:

$$\rho_{WL} = \frac{f_1}{f_1 - f_2}\rho_1 - \frac{f_2}{f_1 - f_2}\rho_2,$$

$$\varphi_{NL} = \frac{f_1}{f_1 + f_2}\varphi_1 + \frac{f_2}{f_1 + f_2}\varphi_2$$

Divergence Free Carrier Combinations:

$$f_1\colon\ \rho = \rho_1, \quad \varphi = \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\varphi_1 - \frac{2f_2^2}{f_1^2 - f_2^2}\varphi_2$$

$$f_2\colon\ \rho = \rho_2, \quad \varphi = \frac{2f_1^2}{f_1^2 - f_2^2}\varphi_1 - \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\varphi_2$$

Geometry-Free (f_1 Iono-Estimation):

$$\rho = \frac{f_2^2}{f_1^2 - f_2^2}(\rho_2 - \rho_1),\ \varphi = \frac{f_2^2}{f_1^2 - f_2^2}(\varphi_2 - \varphi_1)$$

Iono-Free

PR noise amplification

$$\rho_{IF} = \frac{f_1^2}{f_1^2 - f_2^2}\rho_1 - \frac{f_2^2}{f_1^2 - f_2^2}\rho_2 = r + \frac{f_1^2}{f_1^2 - f_2^2}\epsilon_{\rho 1} - \frac{f_2^2}{f_1^2 - f_2^2}\epsilon_{\rho 2}$$

$$\varphi_{IF} = \frac{f_1^2}{f_1^2 - f_2^2}\varphi_1 - \frac{f_2^2}{f_1^2 - f_2^2}\varphi_2 = r + \frac{f_1^2}{f_1^2 - f_2^2}\epsilon_{\varphi 1} - \frac{f_2^2}{f_1^2 - f_2^2}\epsilon_{\varphi 2} + N_{IF}\lambda_{IF}$$

Frequencies	PR Noise Amplification	λ_{IF} (cm)
L1, L2	2.98	0.31
L1, L5	2.59	0.28

- Iono canceled
- PR and CP noise amplified
- Short effective CP wavelength

Wide–Lane Carrier Phase/Narrow–Lane Code

PR noise & multipath attenuation

$$\rho_{NL} = \frac{f_1}{f_1 + f_2}\rho_1 + \frac{f_2}{f_1 + f_2}\rho_2 = r + \frac{k_I}{f_1 f_2} + \frac{f_1}{f_1 + f_2}\epsilon_{\rho 1} + \frac{f_2}{f_1 + f_2}\epsilon_{\rho 2}$$

$$\varphi_{WL} = \frac{f_1}{f_1 - f_2}\varphi_1 - \frac{f_2}{f_1 - f_2}\varphi_2 = r + \frac{k_I}{f_1 f_2} + \frac{f_1}{f_1 - f_2}\epsilon_{\varphi 1} - \frac{f_2}{f_1 - f_2}\epsilon_{\varphi 2} + N_{WL}\lambda_{WL}$$

Frequencies	PR Noise Amplification	λ_{WL} (cm)
L1, L2	0.713	86.3
L1, L5	0.714	75.2

- PR noise & multipath attenuated
- CP noise amplified
- Long effective CP wavelength aids ambiguity resolution
- PR & CP iono have same sign

Narrow–Lane Carrier Phase/ Wide–Lane Code

PR noise amplification

$$\rho_{WL} = \frac{f_1}{f_1 - f_2}\rho_1 - \frac{f_2}{f_1 - f_2}\rho_2 = r + \frac{k_I}{f_1 f_2} + \frac{f_1}{f_1 - f_2}\epsilon_{\rho 1} - \frac{f_2}{f_1 - f_2}\epsilon_{\rho 2}$$

$$\varphi_{NL} = \frac{f_1}{f_1 + f_2}\varphi_1 + \frac{f_2}{f_1 + f_2}\varphi_2 = r + \frac{k_I}{f_1 f_2} + \frac{f_1}{f_1 + f_2}\epsilon_{\varphi 1} - \frac{f_2}{f_1 + f_2}\epsilon_{\varphi 2} + N_{WL}\lambda_{WL}$$

Frequencies	PR Noise Amplification	λ_{WL} (cm)
L1, L2	5.74	10.7
L1, L5	4.93	10.9

- PR & CP iono have same sign
- PR ~~and CP~~ noise amplified
- Reduced effective CP wavelength

, CP noise reduced

Geometry-Free (f_1 Iono Estimation)

PR noise amplification

$$\rho_{GF} = \frac{f_2^2}{f_1^2 - f_2^2}(\rho_2 - \rho_1) = I_1 + \frac{f_2^2}{f_1^2 - f_2^2}(\epsilon_{\rho 2} - \epsilon_{\rho 2})$$

$$\varphi_{GF} = \frac{f_2^2}{f_1^2 - f_2^2}(\varphi_1 - \varphi_2) = I_1 + \frac{f_2^2}{f_1^2 - f_2^2}(\epsilon_{\varphi 1} - \epsilon_{\varphi 2}) + N_{GF}\lambda_{GF}$$

Frequencies	PR Noise Amplification	λ_{GF} (cm)
L1, L2	2.19	0.25
L1, L5	1.78	0.21

- Iono delay measured
- PR and CP noise amplified
- Small effective CP wavelength

Divergence-Free Carrier Combinations for Single Frequency Code PR Measurements

$$f_1\colon\ \rho = \rho_1, \quad \varphi = \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\varphi_1 - \frac{2f_2^2}{f_1^2 - f_2^2}\varphi_2 = r + I_1 + \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\epsilon_{\varphi 1} - \frac{2f_2^2}{f_1^2 - f_2^2}\epsilon_{\varphi 2} + N_{D1}\lambda_{D1}$$

$$f_2\colon\ \rho = \rho_2, \quad \varphi = \frac{2f_1^2}{f_1^2 - f_2^2}\varphi_1 - \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\varphi_2 = r + I_2 - \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\epsilon_{\varphi 2} + \frac{2f_2^2}{f_1^2 - f_2^2}\epsilon_{\varphi 1} + N_{D2}\lambda_{D2}$$

Frequencies	PR Noise Amplification	λ_{DF}
L1, L2	1	<1 mm
L1, L5	1	<1 mm

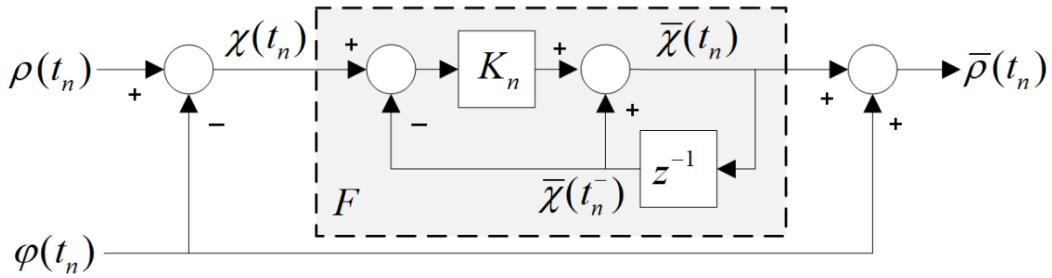
- PR & CP iono have same sign
- Tiny CP ambiguity wavelength

Code Carrier Smoothing

- Code PR have large noise+multipath errors (meter-level) but are unbiased
- Carrier phase measurements have small noise+multipath errors (cm-level) but have an integer cycle ambiguity
- Main idea: combine code and carrier measurements to yield a lower-noise, unbiased PR measurement
  - Low pass filter code and high pass filter carrier phase

Carrier Smoothing – Equivalent Formulations

Complementary Filter Formulation



Initialize: $\bar{\chi}(t_0^-) = 0$

For: $t_n, n = 1, 2, \ldots$

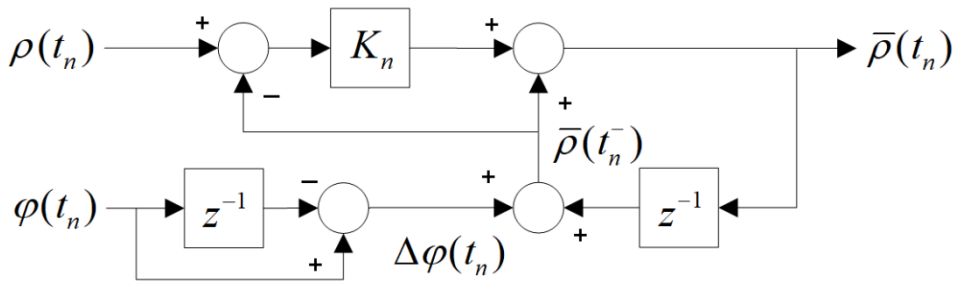
Input: $\chi(t_n) = \rho(t_n) - \varphi(t_n)$

Update: $\bar{\chi}(t_n) = \bar{\chi}(t_n^-) + K_n\left(\chi(t_n) - \bar{\chi}(t_n^-)\right)$

Output: $\bar{\rho}(t_n) = \bar{\chi}(t_n) + \varphi(t_n)$

Extrapolate: $\bar{\chi}(t_{n+1}^-) = \bar{\chi}(t_n)$

Hatch Filter Formulation



Initialize: $\bar{\rho}(t_0^-) = 0$

For: $t_n, n = 1, 2, \ldots$

Input: $\rho(t_n)$

Update/Output:

$$\bar{\rho}(t_n) = \bar{\rho}(t_n^-) + K_n\left(\rho(t_n) - \bar{\rho}(t_n^-)\right)$$

Extrapolate: $\Delta\varphi(t_n) = \varphi(t_n) - \varphi(t_{n-1})$

$$\bar{\rho}(t_n^-) = \bar{\rho}(t_{n-1}) + \Delta\varphi(t_n)$$

For time t_n, $n = 1, \ldots,$ the gain is: $K_n = \begin{cases} 1/n, & n = 1, \ldots, N_{max} \\ 1/N_{max}, & n \geq N_{max} \end{cases}$

Qualitative Error Analysis

Complementary filter operation: $\bar{\rho} = F(\rho - \varphi) + \varphi = F\rho + (1 - F)\varphi$

For the steady-state gain, $K = 1/N_{max}$, and the complementary filter iterative equations for F can be written as:

$$\bar{\chi}(t_n) = (1 - K)\bar{\chi}(t_{n-1}) + K\chi(t_n)$$

This discrete time equation can be written in terms of a Z-transform as:

$$F(z) = \frac{K}{1 - (1 - K)z^{-1}} = \frac{Kz}{z - (1 - K)}$$

This is a low-pass filter => *1-F* is high-pass

An error model for the smoothed single-frequency PR is:

$$\bar{\rho}_L = (r + \delta_T + \delta_R + T) + (2F - 1)I_L + F\left(\delta\rho_{ML} + \varepsilon_{\rho L}\right) + (1 - F)\left(\delta\varphi_{ML} + \varepsilon_{\varphi L} + N_L\lambda_L\right)$$

LOS range terms are unaffected

Iono delay is filtered

Code PR errors are low-pass filtered

Phase errors are high-pass filtered

Smoothing Filter Steady State Gain Calculation

The value for N_{max} can be determined by relating the CMC filter F to a first-order, continuous-time, low-pass filter:

$$F(s) = \frac{1}{T_0 s+1}, \; T_0 = \text{time constant (s)}$$

Discrete-time equivalent: $F(z) = \frac{(1-e^{-\Delta T/T_0})z}{z-e^{-\Delta T/T_0}} = \frac{Kz}{z-(1-K)}$

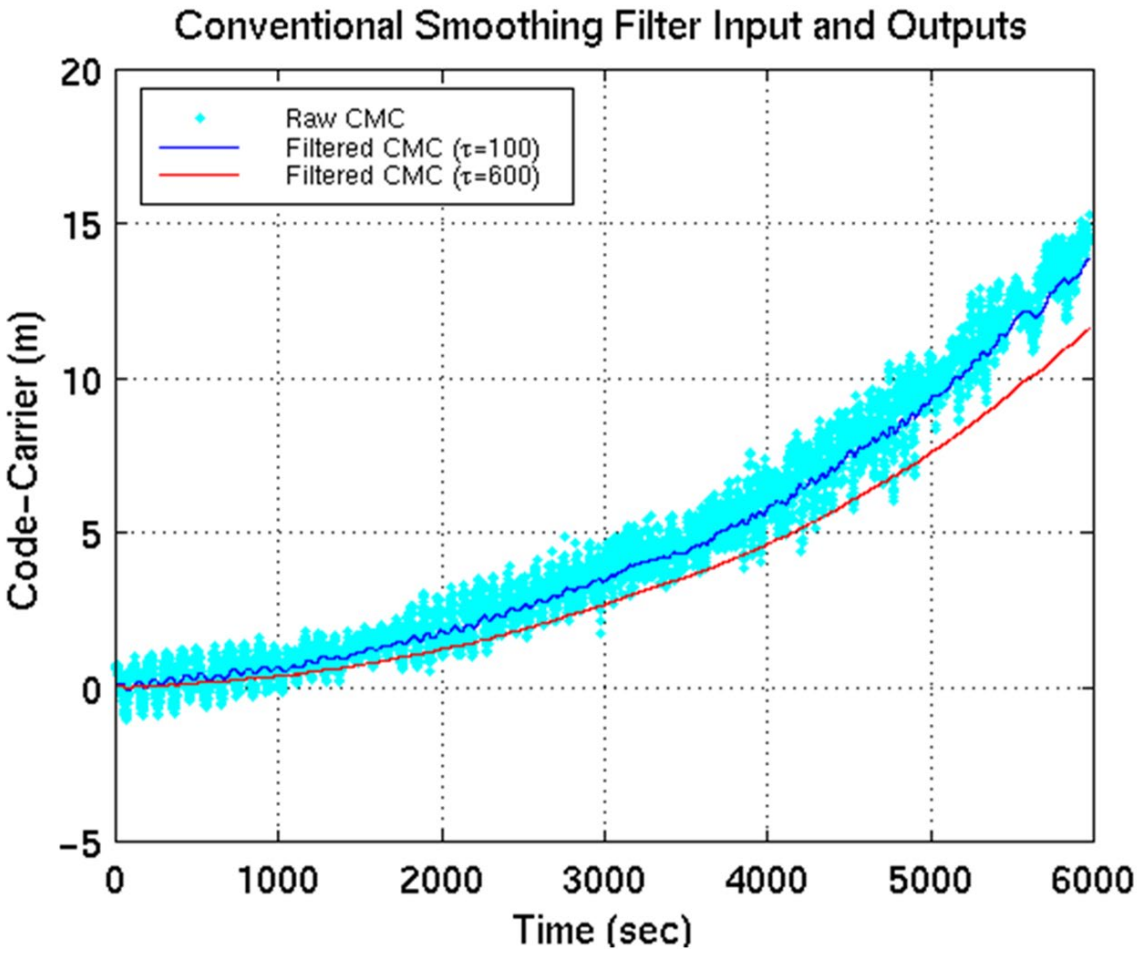
$$K = \frac{1}{N_{max}} = 1 - e^{-\Delta T/T_0} \approx \Delta T/T_0, \quad \Delta T << T_0$$

$$\Rightarrow N_{max} \approx T_0/\Delta T$$

For white noise, smoothed standard deviation given by:

$$\sigma_s = \sigma_\rho \sqrt{\frac{K}{2-K}} = \sigma_\rho \sqrt{\frac{1-e^{-\Delta T/T_0}}{1+e^{-\Delta T/T_0}}} \approx \sigma_\rho \sqrt{\frac{\Delta T/T_0}{2-\Delta T/T_0}} = \frac{\sigma_\rho}{\sqrt{2N_{max}-1}}$$

Single Frequency Smoothing Example Results

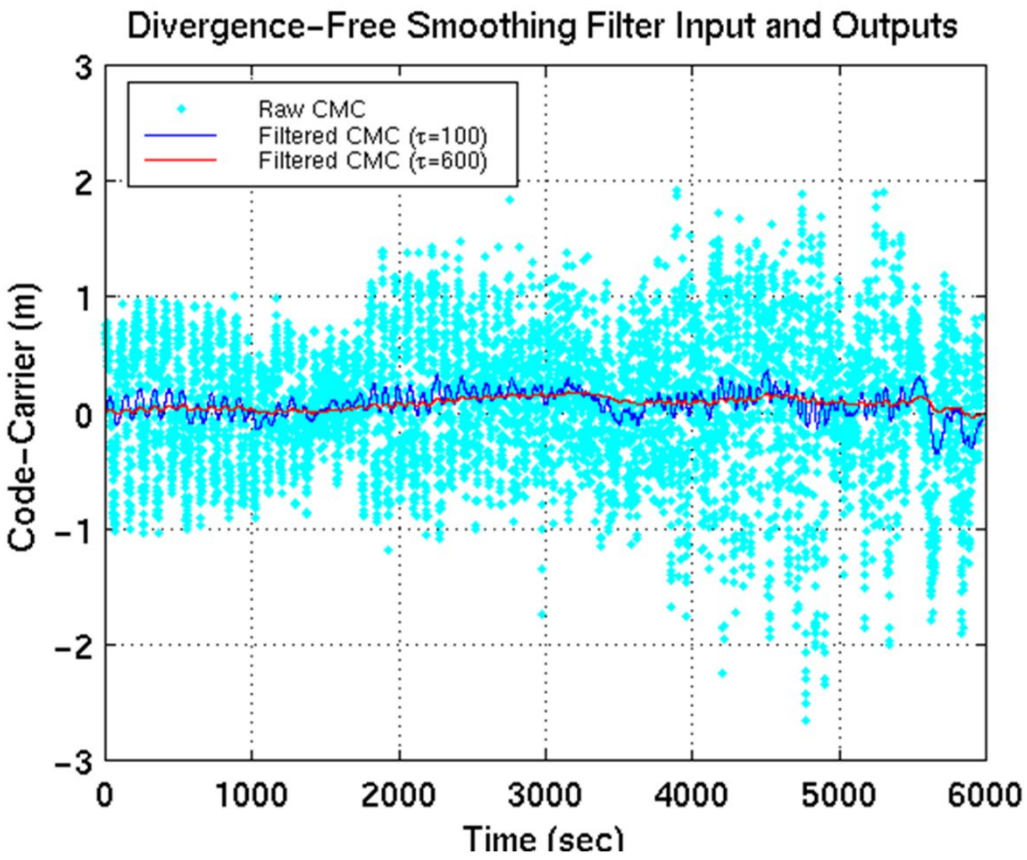


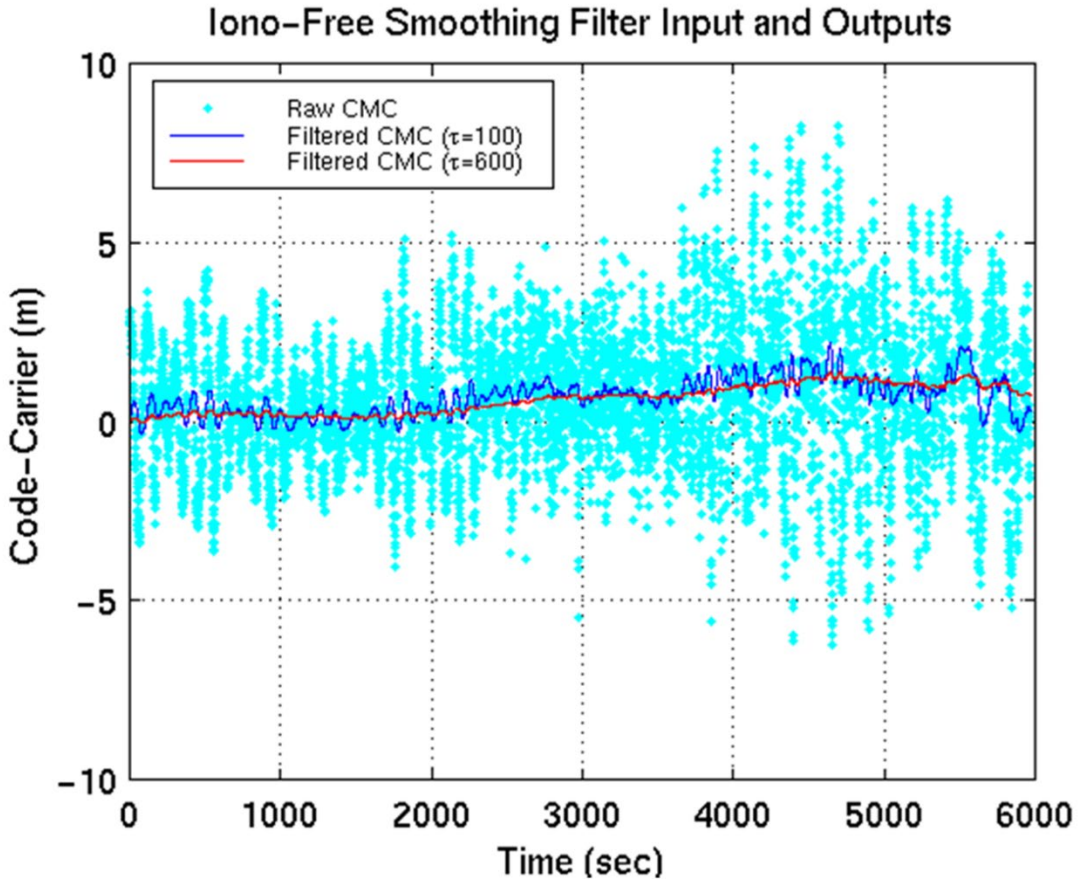
- Single frequency code and carrier phase
- Smoothing reduces meter-level noise to sub-decimeter level
- Longer smoothing time constant induces large bias due to iono divergence

Dual Frequency Smoothing

- Code-carrier iono divergence limits length of single frequency smoothing
  - Iono delays code and advances carrier phase
- Certain PR and CP combinations have equal iono delays (same sign)
  - Divergence free (single frequency code, dual frequency CP)
  - Iono Free
  - WL/NL
- Divergence free combinations enable extended carrier smoothing time constants

Dual Frequency Smoothing Example Results





- Examples use L1/L2 P(Y) code
- No iono divergence effects

- ~3X noise amplification due to iono-free combination is evident

Summary

- Multipath reception affects essentially all GNSS receiver applications
- For many applications it is the dominant error source
- Many techniques are available to mitigate multipath errors:
  - Antenna siting to avoid multipath
  - Antenna types that enhance direct signals and attenuate reflected signals, particularly for fixed sites
  - Adaptive antenna array processing
  - Correlation signal processing
  - Measurement processing techniques like carrier smoothing
  - Navigation processing to de-weight or exclude measurements impacted by multipath
  - Post-processing and modelling techniques that provide estimates to correct multipath errors
- Applicability of these techniques to different GNSS receiver types varies greatly, with mobile phones being especially constrained

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