Conference on Dynamics and Finance: from KAM Tori to ETFs | (SMR 3883)

09 Oct 2023 - 13 Oct 2023
ICTP, Trieste, Italy

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P01

Chaos control of a remanufacturing duopoly game with heterogeneous players and nonlinear inverse demand functions

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In this study, we explore the behavior of a discrete two-dimensional map that represents the interactions between two companies. The first company is an original equipment manufacturer (OEM) that exclusively produces and sells original products. The second company, referred to as the third-party remanufacturer, specializes in reconditioning returned goods to create distinct products.

The outcomes reveal that when we consider consumer willingness to pay and the OEM's relative speed of the output adjustment as bifurcation parameters, the system undergoes flip bifurcation and Neimark-Sacker bifurcation under specific circumstances. The Lyapunov exponents indicate that the system turns chaotic through each of these aforementioned bifurcations.

Additionally, we formulate a controller to mitigate the unpredictability into the market caused by chaotic behavior. This involves making minor adjustments to one of the system parameters within a short time-frame. Finally, we perform numerical simulations to visually illustrate and underscore the theoretical findings.

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P02

Abstract preparation for a Workshop

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Double rotations are the simplest subclass of interval translation mappings. A double rotation is of finite type if its attractor is an interval and of infinite type if it is a Cantor set. It is easy to see that the restriction of a double rotation of finite type to its attractor is simply a rotation. It is known due to Suzuki–Ito–Aihara and Bruin– lark that double rotations of infinite type are defined by a subset of zero measure in the parameter set.

In [1] we have introduced a new renormalization procedure on double rotations, which is reminiscent of the classical Rauzy induction. Using this renormalization, we prove that the set of parameters which induce infinite type double rotations has Hausdorff dimension strictly smaller than 3. Moreover, we construct a natural invariant measure supported on these parameters and show that, with respect to this measure, almost all double rotations are uniquely ergodic. In my poster I will outline the proof, that is based on the recent result by Fougeron for simplicial systems.

I will also discuss some work in progress about the weak mixing properties of double rotations of infinite type.

[1] M. Artigiani, C. Fougeron, P. Hubert, A. Skripchenko. A note on double rotations of infinite type. Trans. Moscow Math. Soc., **82** (2021), 157–172.

Complexity and Chaos Control of a Cournot Duopoly Game with Relative Profit Maximization and Heterogeneous Expectations

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The oligopoly market is dominated by a number of companies that offer homogeneous or differentiated products. The two classic oligopoly markets are named as Cournotmodel (production quantity competition) and Bertrand-model (price competition).

The first Cournot and Bertrand games studies marked milestones in the development of models, focusing on basic assumptions. Authors improved these models by differentiating approaches to company behavior, examining homogeneous agents and heterogeneous expectations in duopoly models [1].

In real markets, producers lack complete knowledge of the entire demand function, represented by cost functions. As a result, firms make local estimates of demand. Various economic models, such as oligopoly games, financial markets, and macroeconomic models, have been used to represent bounded rationality [2,3]. Firms update production strategies using local estimates of marginal profits.

The subject of this paper presents the dynamical analysis of a duopoly game of the Cournot type with differentiated products and heterogeneous expectations [4]. As demonstrated, changing the three key parameters k (players' speed of adjustment), α (second player's adaptation likelihood), and μ (percentage domestic energy adequacy) causes the model to produce complicated, chaotic, and unpredictable ways.

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P04

Geometric conditions to obtain Anosov geodesic flow for non-compact manifolds

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Geodesic flows appear naturally when we have a Riemannian metric on a complete manifold. These flows describe the evolution of vectors tangent along geodesics and their properties are closely related to the geometry of the manifold. For example, the curvature of the manifold can affect the behavior of the geodesics and certain geometric properties of the manifold can be deduced from the behavior of the geodesic flow. In this work, we consider a complete Riemannian manifold (M, g) without focal points and curvature bounded below. We prove that when the average of the sectional curvature in tangent planes along geodesics is negative and uniformly away from zero, then the geodesic flow is Anosov. This result is a version of Eberlein's result (see [2]) for non-compact manifolds and gives us a geometric characterization of the Anosov geodesic flows in non-compact manifolds without focal points. We use this result to construct a non-compact manifold of non-positive curvature with geodesic flow of Anosov type.

[1] A. Cantoral, S. Romaña, Geometric conditions to obtain Anosov geodesic flow for non-compact manifolds, preprint arXiv:2304.10606 (2023).

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P05

Probabilistic induction and the regularity of invariant densities

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It is well known that the study of the first return map, or *induced* map, can often yield interesting statistical information about the original system. In particular, a well known formula of Young's shows that an absolutely continuous invariant measure (acim) for the first map can be "spread out" to an acim for the original system. Despite this technique being very powerful, most information about the regularity of the acim is lost in the process of "spreading out" the measure from the induced system. This is due to the fact that induction is an inherently discontinuous process, with nearby orbits being split in small time. Together with A. Korepanov and D. Thomine we present a novel strategy for inducing in a *smooth* and *probabilistic* way. Moreover, we apply this probabilistic induction strategy to show that the density of the acim of certain intermittent maps is smooth.

P06

Fix an irrational number α and a smooth, positive function $\mathfrak{p}$ on the circle. If a particle is placed at a point $x \in \mathbb{R}/\mathbb{Z}$, then in the next step it jumps to $x + \alpha$ with probability $\mathfrak{p}(x)$ and to $x - \alpha$ with probability $1 - \mathfrak{p}(x)$. Sinai proved that if $\mathfrak{p}$ is asymmetric (in certain sense) or $\mathfrak{p}$ is symmetric and α is Diophantine then this random walk is mixing. Here we show it is mixing for every irrational frequency and generic symmetric absolutely continuous $\mathfrak{p}$. This can be rephrased as mixing of environment viewed from a particle. This partially answers a question posed by Dolgopyat, Fayad and Saprykina in 2019.

P07

Integrability and Chaos in Hamiltonian Systems

P08

Rigidity for multicritical circle maps

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The phenomenon of rigidity occurs when a weak equivalence between dynamical systems can be transformed into a stronger equivalence. In dimension one, a tool known as renormalization is often used to establish results about rigidity: topological conjugacies between certain dynamical systems in the interval or in the circle can be strengthened to smooth conjugacy.

We are interested in the rigidity phenomena for multicritical circle maps, i.e. homeomorphisms of the circle without periodic points and with more than one non-flat critical point. In this poster we will show that, under certains conditions on the renormalizations, two topologically conjugate multicritical circle maps are in fact C^1 conjugate.

P09

Sequence Spaces and Dynamical Systems

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Sequence space is a special case of function space when the domain is restricted to the set of natural numbers. On the other hand, the dynamical system deals with the study of how the variables change over time according to the mathematical rules. Sequence spaces can be viewed as a special case of discrete dynamical systems, where the evolution of a sequence of numbers over discrete time steps is studied. In this presentation, the relationship and different aspects of sequence spaces and dynamical systems will be exhibited along with their applications, especially in finance.

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P10

The Wade Formula and the oil price fluctuation: An Optimal Control Theory approach

Pierre Mendy, Babacar M. Ndiaye, Diaraf Seck, Idrissa Ly

Abstract. By considering the Wade Formula, we propose a model to study the evolution of the oil price per barrel. Our model shows that the policy of diversification of the energy is to be supported. This model is proposed to see how it is possible to control parameters so that the oil price should decrease.

Keywords. Wade Formula, optimal control, options, volatility, oil prices.

1. Introduction

The rapid increase in oil prices between 2002 and 2008 and their sharp decline in the second half of 2008 and 2014, (see [1], [2]), and the consequence of covid19 and Ukraine conflic on oil price, ([3], [4], [5]), has renewed the interest in the causes of oil price fluctuation [figure 1, curve(d)], and the effects of energy prices on the macroeconomy.
A large of studies prove that the oil fluctuations have a considerable consequence in economic activity ([6],[11],[7], [8],[9], [10], [13], [15], [14]).
While the oil price shock is asymmetric between oil exporters countries and oil importers countries ([12], [16], [17]), this asymmetric oil shock had inspired the Wade Formula.
Between 2005 and 2006, the full professor in Economics Abdoulaye Wade, former President of the Republic of Senegal, proposed a formula related to the evolution of the oil price per barrel. This formula translates the super-profits generated by the oil companies (selling in the world) and countries which produce oil.
At first, from the Wade Formula, we describ in section 2, the model. We resolve and show a necessary optimal condition which could give some hints for the orientations of the energy policies in section 3. We propose some simulations and interpretations of the obtained results, section 4. As conclusion in section 5, we present some variations of the obtained results and politics implications.

This work was completed with the support of the NLAGA project.

2. Model description

The formula should be stated as follows: Let be the time $t \in [T_0, T]$, where T_0 is the initial state. Since it is possible to find the structure of the oil price/barrel without financial speculation, President Wade studied and proposed a reasonable price (win-win price) $P_0 = 29\$$. For more details, see [18].
The Wade Formula for super profit is defined by the equation (2.1)

$$(p_t - 29)q_t = S_t \tag{2.1}$$

where p_t represents the price which is applied in the world for buying a barrel of oil, q_t is the quantity of oil bought by the countries, and S_t is the super profit for the oil companies or the super coast supported by the countries which don't produce oil.
It is important to remark that this formula is a conservation law.

3. Model resolution: Optimal control approach

In this section, we give a necessary optimal condition to minimize the super coasts, denoted[1], $S(t)$.
A reasonable model to describe the evolution of the reserves of oil in the world can be written as follows:

$$\frac{dR}{dt} = -v(t) + \alpha(t)R(t), t \geq 0 \tag{3.1}$$

where $\alpha(t) \in [0, 1[$ is the rate of the increase of oil in the world, $R(t)$ are the reserves and $v(t)$ is the demand in the world at time t.
In the following, we are going to take $\alpha(t) = \alpha$ a real constant. But the theorem below is satisfied even if α depends on the time t.

Theorem 3.1. *Let $a(t)$ be the demand of the country which doesn't produce oil, then a necessary optimal condition to minimize the super coasts for a country which doesn't product oil is:*

$$a^*(t) = \frac{c_0 \exp(-\alpha t)}{2(p(t) - 29)^2}.$$

Proof. Let T be a fixed time.

$$\min \int_0^T S(t)^m dt, \quad m \in \mathbb{N} \tag{3.2}$$

$$s.c \begin{cases} \frac{dR}{dt} & = & -v(t) + \alpha R(t) \quad on\ [0, T] \\ R(0) & = & Q \end{cases} \tag{3.3}$$

where, Q is the initial stock of the reserves,

$$S(t) = (p(t) - 29)a(t),$$

[1] For more details in optimal control theory see [19], [20], [21], [22]

$$v = a + w,$$

where a is the demand of country which doesn't produce oil.
Let's take the Hamiltonian defined by:

$$H(R(t), a(t), \lambda(t), t) = S(t)^m + \lambda(t)(-v(t) + \alpha R(t)) \tag{3.4}$$

and $\lambda(t)$ being the Pontryagin multiplier.
We have the necessary optimal conditions:

$$\begin{aligned} \frac{\partial H}{\partial R} &= \alpha\lambda(t) = -\frac{d\lambda}{dt} \\ \frac{\partial H}{\partial a} &= 0 \end{aligned}$$

The condition $\frac{\partial H}{\partial a} = 0$ is equivalent to $mS(t)^{m-1}(p(t) - 29) - \lambda = 0$.
This implies that:

$$S(t) = \left(\frac{\lambda}{m(p(t) - 29)}\right)^{\frac{1}{m-1}}$$

Since:

$$-\frac{d\lambda}{dt} = \alpha\lambda(t),$$

then $\lambda(t) = c_0 \exp(-\alpha t)$ where c_0 is a real constant.
Finally:

$$S(t) = \left(\frac{c_0 \exp(-\alpha t)}{m(p(t) - 29)}\right)^{\frac{1}{m-1}}$$

If $m = 2$, we obtain:

$$\begin{aligned} S(t) &= \frac{c_0 \exp(-\alpha t)}{2(p(t) - 29)} \text{ and by the following equality:} \\ S(t) &= (p(t) - 29)a(t) \text{ we have:} \\ a*(t) &= \frac{c_0 \exp(-\alpha t)}{2(p(t) - 29)^2} \quad \square \end{aligned}$$

□

3.1. Initial condition variation

We assume that $R(t)$ is Lipschitzian, so there exists a flow defined by:

$$\begin{cases} X \to X \text{ space of phases} \\ x_0 \to \phi_t(x_0) = x(t) \end{cases}$$

with $X = \mathbb{R}^n$, for n= 1,

$$\begin{cases} x(t) \to R(t) \\ x_0 \to Q \end{cases}$$

We consider a period of horizon h on the time interval $[t_0, t_0 + h]$ and $Q \in [Q_0, Q^*]$ with $Q_0 = Q(t0)$ and $Q^* = Q(t_{t0+h})$.
We define:

$$Q_k = Q_0 + \frac{k}{t_0 + h}(Q^* - Q_0)$$

The initial conditions vary with the variants of $k \in [t_0, t_0 + h]$.

3.2. Terminal condition variation

We still consider equations (3.3) and (3.4) and rewrite them by varying the terminal conditions. Let $s \in [0, T]$. Let $t = T - s$, $Y(s) = R(T - s)$. The optimization problem becomes:

$$min \int_0^T [S(T-s)]^m ds$$

$$sc \begin{cases} \frac{dY(s)}{ds} = -v(T-s) + \alpha Y(s) \\ Y(0) = R_T \end{cases}$$

where R_T is a real value sequence on $[Y_0, ..., Y^*]$ with $Y_0 = Y(t_0)$ and $Y^* = Y(t_0 + h)$.

$$R_T(k) = Y_0 + \frac{k}{t_0 + h}(Y^* - Y_0)$$

$$S(T-s) = (P(T-s) - 29)\, a(T-s)$$

In the next section we are going to use the curve below. This represents the plot of the result of the necessary optimal condition. The horizontal axis being the values of the prices while the vertical axis is the values of the demand a^*.

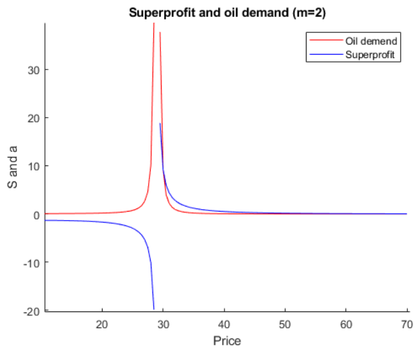
4. Numerical resolution and results interpretations

The simulation parameters are as follows. In the graphs of figure 1, we have taken as constant c the average of the percentage rate increase of the world fossil reserves. In figure 2, we consider the evolution of the rate of the resources over the study period in the numerical resolution of our optimal control problem The curves in Figures 1-2 are the results of the optimal control resolution. The result could be interpreted as follows:

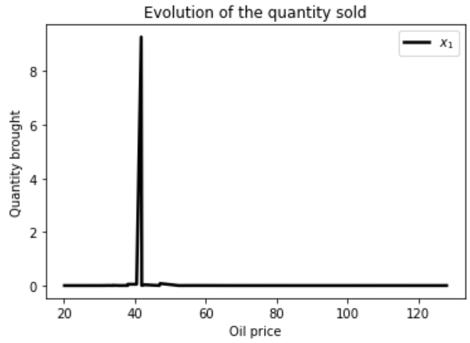
1. Between 1980 and 2021, [1d], there were two periods where the average price of oil was above 29 dollars, 1980 to 1983 and 2004 to 2021, and a period where it was below 29 dollars. In the first period (1980 to 1983), the average price of a barrel of oil fluctuated between 30 dollars and 37 dollars. It peaked at 39 dollars in 1981 due to the Reagan cut taxes. Two other causes of the high fluctuation of the average barrel price are the Iran embargo in 1980 and the end of the recession in 1982.

2. During the second period (2004-2021), the average price of a barrel of oil fluctuated between about 35 dollars in 2004 and 103 dollars in 2011. It reached a peak of 127 dollars in 2008 during the financial crisis. Other factors that explain the high fluctuation during this period are Hurricane Katrina in 2005, Bernanke becomes Fred Chair in 2006, the banking crisis in 2007, the great recession in 2009, Iran threatening the Straits of Hormuz, the increase of the 15 % dollars , the US shale oil increased 2015, the decrease of the dollars in 2016, the OPEC cut oil supply to keep prices stable and demand reduction of the pandemic since 2020.
3. From 1984 to 2003, the average price of a barrel oscillated between 14 dollars in 1986 and 27 dollars in 2003. During the Gulf War, the peak was reached with a barrel at almost 33 dollars . Other events during this period allowed this fluctuation are the prices doubled, the recession and the war in Afghanistan in 2002. The barrel reached the lowest price of 10 dollars in 1999.
4. The curves [1c] and [1d] have the shape of the sigmoid function. The world's demand for fossil energy cannot grow in an unlimited way. With climate change and the strong fluctuations of the oil price, many countries are diversifying their energy sources toward renewable energies. Fossil fuel reserves are also limited despite the discovery of new deposits.
5. The evolution of the Super profit (curve c fig1) is consecutive to the differential between the price 29 and the average price oil with a phase of strong fall between 1984 and 2003
6. The curves [2a] and [2b] have the shape of the sigmoid function. The world's demand for fossil energy cannot grow in an unlimited way. With climate change and the strong fluctuations of the oil price, many countries are diversifying their energy sources toward renewable energies. Fossil fuel reserves [2c] are also limited despite the discovery of new deposits.
7. The countries which don't product oil have to diversify their energy sources as soon as possible. It would be a good orientation even for the countries which product oil. Because the resources will disappear at a time T^*.
8. The strategies to develop the research in the other type of energy and their production have to be encouraged. This assertion is justified in the following sense: when we plot the necessary optimal condition (see Figure1 a)[2], it is easy to see that, if the price of the barrel goes far from 29 dollars, a^* decreases. And then the super coast $S(t)$ is minimized. But this quantity will not suffice for the consumption in the country because of growth of the needs in energy.

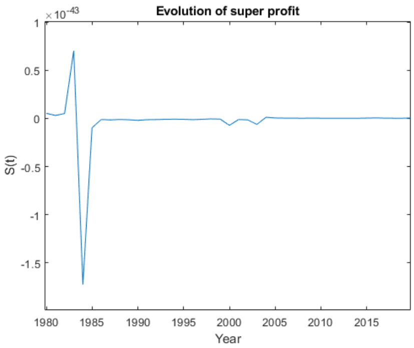
[2]The same interpretation can be done for the real data [Figure 1b)]



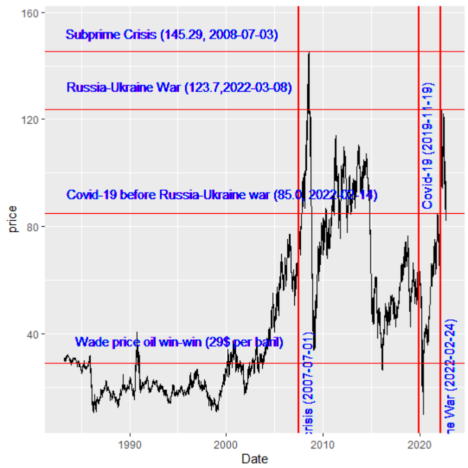
(A) Simulated data ($c0 = \%\bar{\alpha} = 0.2$, m=2)



(B) Evolution of world demand oil



(C) S(t) Real data from from International Energy Agency (IEA)

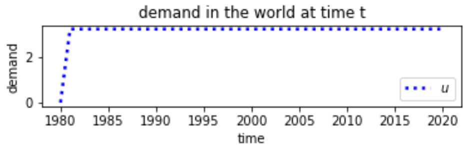


(D) Oil price data from EIA

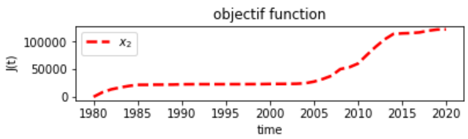
FIGURE 1. Wade Curve

5. Concluding and Remarks

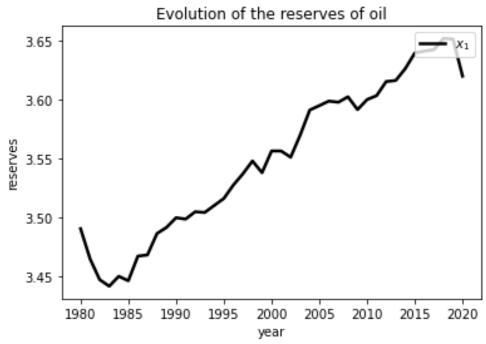
The Wade Formula is also satisfied if one day in the future the 29 dollars are not reasonable like a win win price. It suffices to replace 29 by $P_0(t)$ where $P_0(t)$ could satisfy the following system of equation and without financial speculation $\frac{dP_0(t)}{dt} = f(i(t)) - \mu P_0(t)$ and $P_0(t) = i(t) + pr$. The index i is the investment per barrel,



(A) oil wold demande optimal control result



(B) Objective optimal control function



(C) Reserves real data from IEA

Notes: The horizontal lines in Fig. 2 represent the maximum price of a barrel of oil during the different crises (Subprime around $146, covid19 before the Ukrainian war, $85 and Russo-Ukrainian war around $124).The vertical lines are the beginning of the different cries

$\mu \in [0, 1[$ is the depreciation rate, pr represents the reasonable profit and f is a function to be determined.

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P11

Piecewise Contractions

P12

Constructing New Diffeomorphisms with Non-Ergodic Generic Measures and Sets of Non-Trivial Hausdorff Dimensions

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Abstract

We discuss the combinatorial construction in Ergodic Theory. We briefly discuss a very special technique, "Approximation by conjugation," that allows the construction of exciting maps on the manifolds with prescribed interesting topological and measure-theoretic properties. We present an example of an Invariant measure for the smooth category, which is a generic but non-ergodic measure satisfying other topological, mixing and ergodic properties on the 2-Torus. Also, present an explicit collection of the set containing the generic points of the system with interesting values of its Hausdorff dimension. As such, this talk should interest a broad readership, including those interested in Smooth Ergodic Theory, the Existence of Invariant measure, the Anosov Katok Method, Generic measures of the smooth category and the Hausdorff Dimension of the set containing only generic points.

Keywords: Ergodic theory, Invariant measure, Generic and non-generic points, Hausdorff dimension, Anosov Katok method

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P13

Searching for computability in chaotic systems

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Chaotic Hamiltonian systems have garnered significant attention for their rich, unpredictable behavior. While chaos inherently lacks order and predictability, we delve into the intriguing possibility that certain subsets and orbits of these systems could be Turing-complete.

Our poster presents preliminary findings, showcasing specific examples that exhibit promising behavior and the methodologies we employ to identify Turing-completeness. While Turing-completeness can simulate chaotics systems, having chaotic systems (or subests-of) Turing-complete makes them open to results on inherent undecidability of certain questions, while also giving a geometry to questions of computability.

P14

Historical behavior and Takens' problem for Lotka-Volterra stochastic operators

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It is known that there is a problem related to the existence of persistent classes of smooth dynamical systems such that the set of initial states which give rise to orbits with historic behavior (i.e. the time averages of the topological dynamical system diverges) has a positive Lebesgue measure? The first example of historic behavior was given by Ruelle, where it is shown that the logistic family contains elements for which almost all orbits have historic behavior. In this talks we are going to discuss the mentioned problem within stochastic Lotka-Volterra operators defined on a finite-dimensional simplex. Note that stochastic Lotka-Volterra operators are generalizations of Volterra operators which appear to model the time evolution of conflicting species in biology.

P15

Local limit theorems, geodesic flows and regular covers by hyperbolic groups

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In the late 80ies, Ancona discovered that there is close relation between the geometry of a hyperbolic group and the probabilistic behaviour of a simple random walk on this group. That is, he showed that the visual boundary of the group coincides with the Martin boundary, and that this link is established through the set of minimal harmonic functions. By a refinement of Ancona's method, Gouezel and Lalley in 2012 were able to deduce a local limit theorem for symmetric and simple random walks.

These results were the motivation to study dynamical systems with strong mixing properties and cocycles with values in a countable, word hyperbolic group. In particular, if the dynamical system in the base is a subshift of finite type, the reference measure is an equilibrium state of a Hölder potential and the cocycle is generated by a locally constant map, then it is possible to extend the above results to this setting through a kind of geometric operator theorem, which also not was known before in the context of random walks.

From a probabilistic point of view, this means that the results for symmetric and simple random walks also hold for a class of random walks with non-independent increments. On the other hand, the generalisation also has the following application to hyperbolic geometry: Let $M = \mathbf{H}^2/G$ be a convex-cocompact, hyperbolic surface and G its associated Fuchsian group. Then the Poincaré series is defined by

$$P_G(s) = \sum_{g \in G} \exp(-sd(o, g(o))),$$

where o refers to some point in the hyperbolic plane and d to the hyperbolic metric. It is known that $P_G(s) \asymp 1/(s - \delta_G)$, where δ_G refers to the critical exponent of G. However, if N is a normal subgroup of G, that is, $\mathbf{H}^2/N$ is a regular (or Galois) cover of $\mathbf{H}^2/G$, then the situation is different.

If G/N is word hyperbolic, then G/N autoamtically is non-amenable, which implies that $P_N(\delta_N) < \infty$ (Zimmer 1978) and $\delta_G > \delta_N$ (Stadlbauer 2013). Moreover, the above results for cocycles with values in G/N translate to the following: the visual boundary of the group coincides with the ergodic conformal measures and $P'_N(s) \asymp (s - \delta_N)^{-1/2}$. Here, it is worth noting that these asymptotics are a consequence of a local limit theorem for the cocycle dynamics.

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P16

Localization of polynomial long-range hopping lattice operator with uniform electric fields

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Motivation:

There are numerous simulations show that dynamic localization (DL) holds true as soon as electric fields are included. Some rigorous proof of DL for Schrödinger operator can be seen in [1, 2]. We are interested in whether a similar result holds for nonlocal operators.

The model:

We consider the polynomial long-range hopping lattice operator with uniform electric fields in one dimension

$$\mathcal{H}_{\mathcal{V}}: \ell^2(\mathrm{Z}) \to \ell^2(\mathrm{Z}), \tag{1}$$

which is

$$(\mathcal{H}_{\mathcal{V}} u)(n) = (\mathrm{T}_a u)(n) + nu(n) + V_n u(n), \quad u := \{u(n)\}_{n\in\mathrm{Z}} \in \ell^2(\mathrm{Z}). \tag{2}$$

The long-range hopping T_a satisfies

$$(\mathrm{T}_a u)(n) = \sum_{m\in\mathrm{Z}} a_{n-m} u(m) \tag{3}$$

and

$$a_0 = 0, \quad a_m = \bar{a}_{-m}, \quad |a|_r = \sum_{n\in\mathrm{Z}} |a_n||n|^r < \infty. \tag{4}$$

The method:

Via the Fourier transformation and the KAM-like iteration, we can construct a unitary operator U, such that

$$\mathrm{U}\mathcal{H}_{\mathcal{V}}\mathrm{U}^* = \widehat{\mathrm{D}}, \tag{5}$$

where $\widehat{\mathrm{D}}$ is a diagonal operator. Besides, we know that $\mathrm{U}_n^m = \langle \delta_n, \mathrm{U}\delta_m \rangle$ satisfies

$$|\mathrm{U}_n^m| \le \mathcal{L}_s \langle n - m \rangle^{-s}. \tag{6}$$

The power-law localization:

Fix $r > 1$, Let $\frac{1}{2} < s < r - \frac{1}{2}$. There exists $\varepsilon_0 := \varepsilon_0(s, r, |\alpha|_r) > 0$ such that the following hold true. If $|V|_\infty < \varepsilon_0$, the linear operator $\mathcal{H}_{\mathcal{V}}$ has discrete pure point spectrum. Moreover, there exists a complete system of orthogonal eigenfunctions $\{\psi_n\}_{n\in\mathrm{Z}}$ obeying

$$|\psi_n(m)| \le \mathcal{L}_s \langle n - m \rangle^{-s}.$$

The dynamic localizaiton:

Fix $r > 1$. Let $0 \le q < s - \frac{1}{2} < r - 1$. There exists $\varepsilon_0 := \varepsilon_0(s, r, |\alpha|_r, q) > 0$ such that the following hold true. If $|V|_\infty \le \varepsilon_0$, for any $\varphi \in \ell_q^2(\mathrm{Z})$, we have

$$\sup_{t\in\mathrm{R}} \|e^{-\mathrm{i}\mathcal{H}_{\mathcal{V}}} \varphi\|_q < \infty.$$

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P17

Statistical instability and non-statistical dynamics

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Non-statistical dynamics are those for which a set of points with positive measure (w.r.t. a reference probability measure which is in most examples the Lebesgue on a manifold) do not have a convergent sequence of empirical measures. In this paper, we show that behind the existence of non-statistical dynamics, there is some other dynamical property: *statistical instability*. To this aim, we present a general formalization of the notions of statistical stability and instability and introduce sufficient conditions on a subset of dynamical systems to contain non-statistical maps in terms of statistical instability. We follow this idea and introduce a new class of non-statistical maps in the space of Anasov-Katok diffeomorphisms of the annulus. This presentation is mainly based on the paper [1].

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